SOVIET PHYSICS

USPEKHI

A Translation of Uspekhi Fizicheskikh Nauk

Vol. 15, No. 4, pp. 375-531 (Russian Original Vol. 107, Nos. 3 and 4) January-February 1973

533.9.01

KINETICS OF IMPACT-RADIATION IONIZATION AND RECOMBINATION

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Usp. Fiz. Nauk 107, 353–387 (July, 1972)

The theory of impact-radiation recombination (ionization) in a low-temperature plasma is based on the representation of this process as a random walk of a recombining (released) electron in the discrete space of the atom's energy levels. Different methods of studying recombination (ionization) are considered. The described modified diffusion approximation, combining the possibilities of the previously developed approaches, takes into account the real energy structure of the atom, the influence of the radiative transitions, and the relationship between the non-equilibrium distributions of the atoms over the levels and of the electrons over the energies. A solution is presented for the Fokker-Planck equation expressed in finite-difference form, and analytic expressions are obtained for the ionization and recombination coefficients. The results are compared with the published experimental data.

CONTENT

1. INTRODUCTION

RECOMBINATION of charged particles in a plasma may be the consequence of different elementary processes. Several recombination coefficients (radiative, impact, etc.) were introduced earlier in this connection. Each of these coefficients is in essence a characteristic of a definite elementary process and does not depend on the concentration of the recombining particles. The ionization coefficients were introduced analogously. It became clear later on, however, that this approach calls for a qualitative review. Appreciable progress in this direction was made by determining the decisive role played by the excited atoms in the kinetics of the appearance and vanishing of the charged particles. Thus, under a wide range of conditions, the highly-excited atoms are ionized directly (the ionization cross sections and the statistical weights of these states are the largest). Consequently, each ionization act is the result of a number of successive elementary acts that cause the atoms to become excited, change their excitation, and only then become ionized. The same pertains to recombination. An electron captured on one of the excited levels covers a long and ragged path before it reaches the ground state. The individual stages of this path are governed, generally speaking, by different elementary processes. In this connection, the distinction between impact, radiative, etc. ionization or recombination becomes meaningless. The problems of the kinetics of ionization and recombination, excitation and de-excitation of a set of energy states must be solved simultaneously with allowance for the different elementary processes. The solution should yield ionization and recombination coefficients that take into account the entire aggregate of the processes, and depend generally speaking not only on the temperature but also on the concentration of the participating particles.

Different variants of electron motion in the ionization and recombination processes are traced schematically in Fig. 1. The most general problem of determining

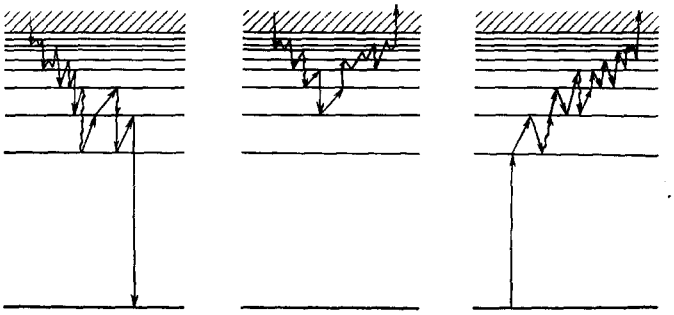

FIG. 1. Different variants of electron motion in energy space of the atom during the process of ionization (recombination).

the recombination and ionization coefficients on the basis of the kinetic balance equations was formulated by Bates and co-workers[1,2]. With a number of simplifying assumptions, the complicated system of equations was solved numerically for a certain range of conditions. At the same time, the motion of a bound electron over the energy levels of the atom (Fig. 1) recalls the meandering of a diffusing particle. In fact, the energy of a bound electron can, with almost equal probability, either increase or decrease under the influence of collisions with other plasma particles. The analogy becomes more complete if it is recognized that the probability $w_{m,n}$ of collisions between the excited atoms and the free electrons is maximal for transitions to neighboring levels, $n = m \pm 1$. With increasing difference between the energies of the final and the initial states, $w_{m,n}$ decreases noticeably, $w_{m,n} \sim (E_m - E_n)^{-4}$. This means that the electron energy changes little when averaged over several collisions. In such a situation, it is natural to use the diffusion approximation. If the electron spends the greater part of the recombination time passing through strongly excited states (where $|E_n - E_{n-1}| < T$), then the energy spectrum can be regarded as quasicontinuous. The recombination coefficient is proportional to the particle diffusion flux calculated by solving the Fokker-Planck equation. This approach to the problem has been intensively developed starting with the work of Belyaev and Budker[3] and Pitaevskiĭ and Gurevich[4-6].

In many cases, however, replacement of the discrete spectrum by a continuous one leads to large errors, particularly when the principal events take place between the ground state and low-lying excited states of the atom. To simplify the problem, the real level scheme of the atom was simulated in some cases by two or three effective levels.

The authors of the present review have recently developed a so-called modified diffusion approximation (MDA)[7,8a,b]. This approximation is based on the diffusion-approximation principle, but unlike the latter it takes into account the discrete character of the change of energy in the transitions. From the point of view of the MDA, recombination is the meandering of an electron over discrete energy levels of the atom. Corresponding to this process is a diffusion equation expressed in finite differences, which goes over into the Fokker-Planck equation in the limiting case of infinitesimally small energy change.

Within the framework of the MDA, the plasma is regarded as a unified system of its interrelated components. Account is taken of the influence of the inelastic collisions on the electron energy distribution and of the opposing influence of the non-equilibrium character of this distribution on the populations of the excited atoms and on the ionization and recombination coefficients. It is known that conditions under which a Maxwellian distribution cannot be maintained (especially at energies much lower than thermal) correspond to low degrees of ionization and are frequently realized. It is also important that the modified diffusion approximation takes into account not only the collision processes but also radiation processes. Both optically transparent and dense plasma are described. If the plasma volume in question is homogeneous enough, the local character of the problem is preserved.

The present review is devoted almost entirely to impact-radiative recombination and ionization. The range of parameters covered corresponds to electron concentrations $\sim 10^{10}-10^{19}$ cm^{-3}, electron temperatures $\sim 10^3-10^5$°K, linear plasma dimensions $\sim 10^{-2}-10^2$ cm, and relaxation times $\sim 1-10^4$ μsec. These ranges are presently the most important from the point of view of a number of applications.

2. FUNDAMENTAL EQUATIONS

We consider a volume element of a non-equilibrium plasma containing n heavy particles (atoms and ions), whose translational temperature is T_a. The electron concentration (n_e) and the concentrations of the atoms in different states (n_1, n_2, ..., n_k, ...) at the initial instant are specified. The problem is to determine the rate of change of n_e with time. It turns out to be closely related to the problem of the distribution of the atoms over the states in a non-equilibrium plasma. The average electron energy $\langle \epsilon \rangle = 3T_e/2$ is assumed known, although the electron energy distribution function, generally speaking, must still be determined.

From balance considerations, we write down the rate of change of the electron concentration

$$\frac{\partial n_e}{\partial t} = \sum_{k,\,q} (n_k w_{ke}^q - n_e^2 w_{ek}^q) - \nabla \mathbf{J}_e. \qquad (2.1)$$

The quantities w_{ke}^q characterize the ionization probabilities of an atom on the level k as a result of the q-th elementary process, and w_{ek}^q characterize the recombination probabilities. These quantities can be functions of T_e, n_e, and of other parameters, depending on the process q described by them. The quantities $\nabla \cdot \mathbf{J}_e$ take into account transport in space. The bulk of the present review deals with volume ionization and recombination. Effects due to transport in space will be discussed only in certain cases. As a rule, therefore, we omit $\nabla \cdot \mathbf{J}_e$ in the equation for $\partial n_e/\partial t$.

Equation (2.1), which contains the unknown quantities n_k, must be supplemented by a system of balance equations for the atoms in different energy states:

$$\frac{\partial n_k}{\partial t} = \sum_{n,\,q} (n_n w_{nk}^q - n_k w_{kn}^q); \qquad (2.2)$$

w_{nk}^q characterize the probabilities of transitions between different states; $\sum_n$ implies summation over all the bound states and integration over all the states of the free electrons.

Since an important role among the processes that determine the kinetics of the ionization and excitation is played by collisions with electrons (q = e), and their energy distribution function $f(\epsilon)$ can be non-Maxwellian, it is necessary to supplement the system (2.1) and (2.2) with the Boltzmann equation

$$\frac{\partial f(\varepsilon)}{\partial t} = \sum_q S_q(\varepsilon). \tag{2.3}$$

The right-hand side is the sum of the sources of electrons with energy ϵ. The distribution $f(\epsilon)$ is normalized by the condition $\int_0^\infty f(\epsilon)d\epsilon = 1$.

The solution of Eqs. (2.1)—(2.3) is a rather complicated task. In addition, it is very closely connected with the concrete initial conditions. However, if we are not interested in not too short elapsed times from the start of the development of the process, we can use the so-called "quasistationary" approximation. Then, solving the system (2.2)—(2.3) and substituting the solutions in (2.1), we can reduce the problem to a solution of only one equation

$$\frac{\partial n_e}{\partial t} = n_1 n_e \beta - n_e^3 \alpha, \tag{2.4}$$

where α and β are the generalized ionization and recombination coefficients, to the calculation of which the main part of the review is devoted.

a) "Quasistationary" approximation. Quasistationarity of the distribution of the atoms over the excited states means that during the evolution of the ionization (recombination) the quantities n_k have time to "attune themselves" to the other relatively-slowly-varying parameters of the plasma (m, n_e, T_e, etc.). There is therefore no explicit time dependence of n_k, and the system (2.2) can be written in the form

$$\sum_{n,q} (n_n w_{nk}^q - n_k w_{kn}^q) = 0. \tag{2.5}$$

The possibility of using (2.5) is connected with the fact that the inequality

$$\sum_{k>1} n_k \ll n_1, n_e \tag{2.6}$$

is satisfied under a wide range of conditions. This is connected to a considerable degree with the small value of the configuration weight of the excited state in comparison with the weight of the ground state and of the continuous spectrum. If (2.6) is valid, then the states n_1 and n_e are, so to speak, reservoirs of particles; particles flow from one reservoir to another through a channel produced by the excited states.

The times required to establish quasistationarity of n_k are usually 10^{-7}—10^{-9} sec and become larger only at small values of n_e or very small T_e. It is obvious that $\tau_k \approx (\sum_{q,n} w_{kn}^q)^{-1}$. We note that for certain problems the quasistationarity approximation may not be valid[1)]. We have in mind processes whose evolution time is comparable with the relaxation time of an individual excited state (10^{-7}—10^{-9} sec). Such conditions arise, e.g., in the investigation of the possible inversion of the populations of some pair of levels during the plasma decay process. In this case it is necessary to solve the nonstationary system of balance equations. Examples of numerical calculations of this kind can be found in the papers of Gordiets et al.[9] and Son[10].

It will be convenient to introduce the concept of the electron flux in energy space j_k, which is proportional to the number of particles passing per unit time through a section drawn between the pair of levels with k — 1 and k. Obviously,

$$j_k = \sum_{n<k} \sum_{m \geqslant k} (n_n w_{nm}^q - n_m w_{mn}^q), \tag{2.7}$$

and (2.2) takes the form

$$\frac{\partial n_k}{\partial t} = j_k - j_{k+1} = \Delta j_k. \tag{2.8}$$

The quasistationarity approximation is equivalent to the assumption that j_k does not depend on the number of the level k, and is therefore sometimes called the "constant outflow" approximation. Obviously, the flux in the energy space of the atom determines the ionization (recombination) rate: $\partial n_e/\partial t = j$. If excitation and ionization predominates we have $j > 0$, and if radiation, quenching, and recombination predominate we have $j < 0$. Finally, a stationary regime with flux j = 0 is possible. In this case the total number of ionization acts in a given volume element is equal to the number of recombination acts. But such a regime is not necessarily in equilibrium, for the balance may be upset by radiation to the outside.

In the presence of transport of electrons in coordinate space we have $\partial n_e/\partial t = j + \nabla \cdot J_e$. In the stationary regime we have $j = -\nabla \cdot J_e$, i.e., stationarity is insured by the outflow (or influx) of electrons and ions from the volume element in question and the influx (outflow) of atoms in the ground state. In this case the circuit for the flux in energy space is closed in coordinate space.

In analogy with the concept of quasistationarity of n_k one introduces the approximation of quasistationarity of the electron energy distribution $f(\epsilon)$. It makes it possible to replace Eq. (2.3) by the simpler relation $\sum_q F_q(\epsilon) = 0$. The times $\tau(\epsilon)$ required to establish quasistationarity of $f(\epsilon)$ are determined, naturally, by the frequencies of the most intensive collisions, usually between the electrons. $\tau(\epsilon)$ depends on the electron energy ϵ and is usually quite small for thermal electrons (owing to the large value of the Coulomb cross section). We should be concerned primarily, however, with the "tail" of $f(\epsilon)$ at the threshold excitation energy $\epsilon \approx E_1 - E_2$. The corresponding time

$$\tau^{-1}(\varepsilon)_{\varepsilon=E_1-E_2} = 2\pi e^4 \lambda n_e/(2m)^{1/2}(E_1-E_2)^{3/2}$$

may not be small enough ($\tau \sim 10^{-7}$ sec at $n_e = 10^{13}$ cm^{-3}). In any case, this circumstance must be taken into account in those essentially non-quasistationary problems mentioned above. Apparently, however, no such studies have been reported as yet.

b) Structure of transition probabilities. For excitation by electron impact we have

[1)] If the principal role in the kinetics is played by collisions with electrons and by radiation, then a simple estimate can be obtained by using [8c] $\tau_k^{-1} = A_k^* + [4(2\pi)^{1/2}\Lambda_{k-1}E_k e^4 n_e][mT_e)^{1/2}(E_{k-1}-E_k)]^{-1}$, where E_k is the energy of the k-th level reckoned from the continuum. A_k^* is the effective probability of the radiative transitions, and the constants are $\Lambda_1 = 0.01$—0.05 and Λ_k 0.2 at $k \geqslant 2$.

$$w_{nk}^e = \int_{E_n-E_k}^{\infty} f(\varepsilon)\, w_{nk}(\varepsilon)\, d\varepsilon = n_e \int_{E_n-E_k}^{\infty} f(\varepsilon)\, \sigma_{nk}(\varepsilon)\, (2\varepsilon/m)^{1/2}\, d\varepsilon,$$

where $\sigma_{nk}(\epsilon)$ is the cross section of the n → k transition and depends on the energy ϵ of the incoming electron (see[11a] among the latest reviews on the cross sections of inelastic transitions); $(E_n - E_k)$ is the threshold energy; E_n and E_k are the level energies reckoned from the continuum.

If $f(\epsilon)$ is identical with the Maxwellian $f^{(0)}(\epsilon)$ (the superscript 0 will always denote equilibrium quantities), then

$$n_k^0 (w_{kn}^e)^0 = n_n^0 (w_{nk}^e)^0.$$

The probabilities w_{nk}^a of the transitions resulting from collisions between heavy particles are expressed in similar fashion. Although the σ_{nk} for these collisions are small, they can be significant under certain conditions.

Let us consider the probabilities of the radiative transitions. If n > k, then $w_{nk}^R = A_{nk}$, where A_{nk} is the Einstein spontaneous radiation probability. Analogously, $w_{ek}^R = \alpha_{ek}$ is the probability of radiative recombination at the level k. On the other hand, if n < k, then w_{nk}^R characterizes a transition to a higher level as a result of absorption of the radiation. Allowance for radiation absorption calls for the use of the radiation transport equation. This, generally speaking, does not make it possible to determine α and β in terms of local quantities.

Radiation transport theory[12], however, admits of an approximate analysis, according to which A_{nk} is replaced by an effective quantity A_{nk}^* that takes into account the difference between the radiative transitions n → k and k → n. In this approximation we have

$$w_{nk}^R = A_{nk}^* = A_{nk}\Theta_{nk} \quad (n>k), \quad w_{nk}^R = 0, \quad (n<k), \qquad (2.9)$$

where Θ_{nk} is the probability of non-absorptive photon emission from a given point outside the volume occupied by the plasma. The dependence of Θ_{nk} on the coordinate should be weak. In the absolute case, such an approach is not valid (boundary-layer plasma, in discontinuities accompanying the passage of shock waves, etc.).

We present two simple expressions for Θ_{nk}. If the characteristic dimension of the problem is R, then we have for a dispersion line shape

$$\Theta_{nk} \approx 1/3\, [\pi (k_0)_{nk} R]^{1/2},$$

and for a Doppler shape

$$\Theta_{nk} \approx 1/4 \sqrt{\pi}\, (k_0)_{nk} R \{\ln [(k_0)_{nk} R]\}^{1/2},$$

where $(k_0)_{nk}$ is the absorption coefficient at the center of the given line. Reabsorption is significant for transitions to the ground state and to much lesser degree for the remaining transitions.

c) Structure of the collision integral. The sources $S_q(\epsilon)$ are customarily divided into $S_{el}^q(\epsilon)$ and $S_{in}^q(\epsilon)$, determined by elastic and inelastic collisions, respectively. In more specific conditions, $f(\epsilon)$ can also be affected by radiative transitions and by external fields (e.g.,[13,14]). Since only a small fraction of energy is exchanged in each elastic collision, the $S_{el}^q(\epsilon)$ have a simple Fokker-Planck form (see, e.g., the splendid review[13]). For collisions between electrons

$$S_{el}^e(\varepsilon) = \frac{\partial}{\partial\varepsilon}\left\{\frac{2\pi e^4\lambda}{(2m)^{1/2}}\left[\frac{f}{\varepsilon^{1/2}}\int_0^{\varepsilon} f\, d\varepsilon + \frac{2}{3}\frac{\partial}{\partial\varepsilon}\left(\frac{f}{\varepsilon^{1/2}}\right)\left(\int_0^{\varepsilon} f\varepsilon\, d\varepsilon + \varepsilon^{3/2}\int_{\varepsilon}^{\infty}\frac{f}{\varepsilon^{1/2}}\, d\varepsilon\right)\right]\right\},$$

where $\lambda = \log(9T_e^3/8\pi n_e e^6) + 1$ is the usual Coulomb logarithm. We are interested primarily in conditions under which the "core" of $f(\epsilon)$ is Maxwellian, and $f(\epsilon)$ is significantly non-equilibrium only in the far "tail" ($\epsilon \gg T_e$). In this case a fast electron collides principally with Maxwellized electrons; $S_{el}^e(\epsilon)$ takes on the usual linearized form

$$S_{el}^e(\varepsilon) = \frac{\partial}{\partial\varepsilon}\left\{\frac{\pi e^4\lambda}{(2m)^{1/2}}\left[\frac{f}{\varepsilon^{1/2}} + T_e\frac{\partial}{\partial\varepsilon}\left(\frac{f}{\varepsilon^{1/2}}\right)\right]\right\}. \qquad (2.10)$$

For elastic collisions with ions and atoms we have

$$S_{el}^a = \frac{\partial}{\partial\varepsilon}\left\{\frac{2m}{M}(\nu_a + \nu_i)\,\varepsilon^{3/2}\left[\frac{f}{\varepsilon^{1/2}} + T_a\frac{\partial}{\partial\varepsilon}\left(\frac{f}{\varepsilon^{1/2}}\right)\right]\right\},$$

where ν_a and ν_i are the frequencies of the elastic collisions determined by the values of the cross sections ($\nu_a = n(2\epsilon/m)^{1/2}\sigma_{ea}(\epsilon)$, $\sigma_{ea}(\epsilon)$ is the cross section and depends on the energy).

The source $S_{in}(\epsilon)$, due to the inelastic collisions, can be easily expressed in a sufficiently general form. Difficulties arise when one considers the influence exerted on $f(\epsilon)$ by transitions between numerous excited states. This question is discussed in[8c], where criteria have been found that make it possible to evaluate the role of these effects. In the present review we confine ourselves to the case when the electrons cause inelastic transitions from the ground state of the atom to the first excited state 1 ⇄ 2. As is well known, it is precisely these processes that can exert a strong influence on $f(\epsilon)$ at $\epsilon \gg T_e$. We write $S_{in}(\epsilon)$ for this case in the form

$$S_{in}(\varepsilon) = n_1 f(\varepsilon)\, w_{12}(\varepsilon) - n_2 f(\varepsilon')\, w_{21}(\varepsilon'), \qquad (2.11)$$

where $\epsilon' = \epsilon - E_1 + E_2$.

3. METHODS OF CALCULATING THE IONIZATION AND RECOMBINATION COEFFICIENTS

There exist numerical and model methods for calculating the ionization coefficients (IC) and the recombination coefficients (RC). The former have gained widespread use in the last decade in connection with the development of computers. Model methods can be schematically divided into two groups corresponding to two alternate notions concerning the kinetics of ionization and recombination. In one case, by simplifying the real discrete structure of the level scheme, the authors have tended to obtain a model with two or three "effective" levels. The second trend is based on the notion of electron diffusion during the recombination (ionization) in energy space, which is assumed continuous. Let us stop to discuss these methods in detail.

a) "Prompt ionization," "block of excited states," and "bottleneck" approximations. It can be assumed that during the ionization process the electron spends a long time in passing through the energy gap between the ground and the first excited states of the atom, and that the passage through the entire aggregate of the excited states is rapid. We then arrive at the so called "prompt

ionization'' approximation. According to Petschek and Byron[15], the ionization coefficient takes in this case the form

$$\frac{\partial n_e}{\partial t} = n_1 n_e \beta, \quad \beta = \sum_{k \geqslant 2} w_{1k} + w_{1e}. \tag{3.1}$$

The inverse processes are not taken into account in (3.1). This can be easily done by using the method of the ''block of excited states''[16], according to which the excited states are in relative equilibrium with one another and with the continuum at an electron temperature T_e. Such a situation is possible as a result of the high frequency of the impact transitions between the excited states. Within the framework of this approach it is easy to take into account the radiative corrections to the ground state. The expressions for α and β are

$$\alpha = \sum_{k \geqslant 2} \{(n_k^0 w_{k1}/n_1^0 K_1) + w_{e1} + n_e^{-1}[(n_k^0 A_{k1}/n_1^0 K_1) + \alpha_{e1}]\}, \tag{3.2}$$

$$\beta = \sum_{k \geqslant 2} w_{1k} + w_{1e}, \tag{3.3}$$

where K_1 is the ionization-equilibrium constant (see (4.13) below), and n_k^0 and n_1^0 are determined by the temperature T_e. This method was used to consider a number of problems, e.g., ionization relaxation behind a strong shock wave[17], non-isothermal plasma decay[18a], deviation from thermodynamic equilibrium as a result of outgoing radiation[7b], and the state of plasma in a thermoelectric converter[67]. However, as will be shown subsequently, the region of applicability of (3.2) and (3.3) is bounded on the side of low temperatures and small n_e.

In the ''bottleneck'' method proposed by Byron et al.[19], the entire aggregate of the energy levels is divided into two parts. It is assumed that several of the low-lying excited levels are in relative equilibrium with the ground state, while the group of the highly-excited states is in equilibrium with the continuum. The rate of transition of the recombining electron between these two groups determines the recombination rate. The problem reduces to a determination of the position of the level k* that separates these groups of states, or, as is customarily stated, to a determination of the ''bottleneck'' in the energy spectrum. To determine k*, it is proposed to find the minimum deactivation rate D_k calculated as a function of k assuming relative equilibrium with the electrons:

$$D_k = (n_k^0/n_1^0 K_1)\, n_e^2 \sum_{n<k} (n_e w_{kn} + A_{kn}), \quad (D_k)_{\min} = D_{k*}.$$

α is then calculated in the following manner:

$$\alpha = D_{k*}/n_e^3. \tag{3.4}$$

It is clear that, depending on the plasma parameters, the position of the ''bottleneck'' can change. This method was used in[20,21] and elsewhere to describe the development and fall-off of ionization in ''inert gas + alkali additive'' mixtures.

The distribution of the atoms over the excited states, proposed in this method, is actually far from always realized. There is an entire group of levels that are in no partial equilibrium whatever. The authors of[19-21] have therefore introduced in formula (3.4) an indeterminate factor $\gamma = 0.25$—1, which must be determined by comparison with experiment.

The ''bottleneck'' concept becomes particularly pronounced in the so-called ''single-quantum'' approximation[7a]. Even though the energy levels in the atom are not equidistant, an analysis of the data on the transition cross sections shows that the most frequent transitions are those between neighboring energy levels. Retaining all of these transitions and neglecting all others, we obtain the single-quantum approximation. The flux (2.7) in energy space between the levels k and k + 1 is written in this case in the simple form $j = w_{k,k+1} n_k^0 (y_k - y_{k+1})$, where $y_k = n_k/n_k^0$. Expressing y_k in terms of y_{k-1}, ..., y_1, and y_{k+1} in terms of y_{k+2}, ..., $y_e^2 = n_e^2/n_e^{02}$, we obtain

$$j = (y_1 - y_e^2)/\sum_{k \geqslant 1} (n_k^0 w_{k,k+1})^{-1}. \tag{3.5}$$

A certain analogy can be drawn between (3.5) and Ohm's law for a section of a circuit consisting of resistors $(w_{k,k+1} n_k^0)^{-1}$ connected in series. The conductivity of the circuit elements has a minimum, since $w_{k,k+1}$ increases with increasing k, and n_k^0 decreases. The value of k corresponding to the minimum of $(w_{k,k+1} n_k^0)$ determines the position of the ''bottleneck''. The bottleneck is more or less sharply pronounced, depending on the conditions.

Let us discuss the influence of violation of equilibrium of the free electrons on the IC and the RC. This effect was investigated many times in the ''prompt ionization approximation[18b,22-25]. The rate of excitation and ionization from the ground state is averaged over the obtained non-Maxwellian distribution $f(\epsilon)$. The effect of the velocity dependence of the cross section $\sigma_{12}(v)$ on the resultant IC was explained in[24]. Kagan and Lyagushchenko[23], in a large cycle of papers, investigated in detail the influence of the non-Maxwellian distribution on the kinetics of the processes in the plasma of a positive-column discharge of medium pressure. It was assumed in[18b,22-25] that the opposing processes, impacts of the second kind, can be neglected in the expression for $S_{in}(\epsilon)$. This greatly simplifies the problem, since the ionization rate turns out to be independent of the distribution over the excited states. At the same time, this assumption greatly limits the region of applicability of the results.

b) Electron diffusion in energy space. Let us examine the motion of a bound electron in energy space. A characteristic feature of this motion is that the electron energy changes little in each collision. We introduce the distribution function of the bound electrons over the energies f(E) and write a kinetic equation for f(E). Taking the foregoing into account, we can use a differential representation for the collision integral (see[3]). We obtain

$$g(E)\frac{\partial f}{\partial t} = \frac{\partial}{\partial E}\left[B(E)\, g(E)\left(\frac{\partial f}{\partial E} + \frac{f}{T}\right)\right], \tag{3.6}$$

where g(E) is the density of states for the bound electrons. In the classical limit, g(E) is equal to the volume of the configuration space for a bound electron with energy E:

$$g(E) = 2\int (2\pi\hbar)^{-3}\, \delta\left(p^2(2m)^{-1} - (e^2/r) + E\right) d\mathbf{p}\, d\mathbf{r} = \mathrm{Ry}^{3/2}/E^{5/2}, \tag{3.7}$$

where $\mathrm{Ry} = me^4/2\hbar^2$. An equation of the (3.6) type was investigated by Pitaevskiĭ and Gurevich[4-6].

$B = \partial\langle E\rangle^2/\partial t$ is the mean squared change of the energy of the bound electron per unit time due to collisions with other electrons. This quantity was calculated in[6]:

$$B = 4\,(2\pi)^{1/2}e^4 n_e \Lambda E/3\,(mT_e)^{1/2}, \qquad (3.8)$$

where Λ is the so-called Coulomb logarithm for bound states.

Solving (3.6) for stationary conditions, assuming that $f(E) \sim e^{E/T}$ as $E \to 0$ and $f(E) \to 0$ as $E \to \infty$, we find

$$\alpha = (\pi^{3/2}e^6/4T_e^{3/2})\left|\int_0^\infty \exp(-E/T_e)\,dE/g(E)\,B(E)\right|^{-1},$$

or, using (3.8), we obtain[5]

$$\alpha = (4\,(2\pi)^{1/2}e^{10}\Lambda/9m^{1/2})\;T_e^{-9/2}. \qquad (3.9)$$

We note that a similar dependence of α on the plasma parameters is obtained on the basis of Thomson's crude model[26]. Following Thomson, we consider the recombination of an electron and an ion with kinetic energy on the order of T. For recombination of these particles it is necessary that they approach each other to a distance r_c, at which the interaction potential is $e^2/r_c \gtrsim T$. In order for the recombination act to take place it is necessary that at this instant a third particle (electron) be situated within the radius r_c. The probability of this event is $\sim n_e r_c^3$. We write for these conditions the rate of vanishing of the electrons $\partial n_e/\partial t \sim n_e n_i v \sigma n_e r_c^3$, or, recognizing that for particles interacting in accordance with Coulomb's law the energy-transfer cross section is $\sigma \sim e^4/T^4$, and also that $v \sim (T/m)^{1/2}$ apart from a numerical coefficient that is not determined within the framework of this analysis, we obtain (3.9).

An analysis of the recombination kinematics, based on a solution of a differential equation of the Fokker-Planck equation, is contained in[11b,27-30]. The main assumption, connected in one way or another with the form of the differential equation describing the transitions between discrete levels, consists of smearing out the latter into a certain continuous distribution of the density of the bound states. This is justified under conditions when the recombining electron spends most of the time in passing over strongly excited states, which are indeed close to one another. Otherwise the diffusion approach can lead to an error whose magnitude is difficult to estimate.

c) Numerical methods. The numerical methods of determining the IC and RC are based on a system of kinetic balance equations, written out with respect to a large number of excited states with allowance of both the collision and radiative processes. The problem of calculating the RC of this case was formulated by Bates, Kingston, and others (see[2]). A Maxwellian energy distribution of the free electrons was assumed.[2)] The radiation is taken into account in the approximation of an optically thin layer. In certain cases, data were presented for the case when a part of the spectral lines was completely trapped in the volume. A calculation performed in the quasistationary approximation yielded tables for the IC and RC as functions of n_e and T_e. The tabulated quantities were defined in the following manner:

$$\frac{\partial n_e}{\partial t} = -\gamma n_e^2 = S n_e n_a - \alpha^* n_e^2.$$

S is equal to the coefficient β introduced by us in (2.4), and $\alpha^* = \alpha n_e$.

Figure 2 shows the dependence of the IC for an optically thin hydrogen plasma in accordance with the data of[1]. It is clearly seen that at large n_e the quantity $\alpha = \alpha^*/n_e$ ceases to depend on n_e, since the recombination becomes purely collisional. At small n_e, radiative corrections are quite important.

Since they cover a wide range of conditions, the tables of Bates, Kingston, and McWhirter can serve as a reliable test of different approximate model methods. Similar calculation methods were performed for an H plasma[31], for a He plasma[32], for a Cs plasma[33], and for an N plasma[34]. The shortcomings of the numerical method of determining the IC and RC become significant when complex problems are solved, in which the determination of the IC and RC is only the initial stage of the solution, or the IC and RC are determined during the course of solving the problem.

4. MODIFIED DIFFUSION APPROXIMATION

The main shortcoming of the diffusion approximation discussed in Sec. b of Chap. 3 is the complete neglect of the discrete character of the energy structure of the atoms. Such an assumption is justified if the recombining electron spends most of the time in highly-excited states. In a wide range of conditions, this assumption is not valid. At the same time, the character of the motion of the electron through levels separated by large energy gaps remains in a certain sense the same, in that transitions between neighboring levels are the most probable. This indicates that one can attempt to generalize the diffusion approximation in such a way as to take into account the discrete structure of the scheme of the atomic terms. Such a generalization was carried out by us in[7a,8]. It consists of regarding the ionization and recombination processes as diffusion in a discrete energy space. This called for a derivation and solution of a finite-difference Fokker-Planck equation.

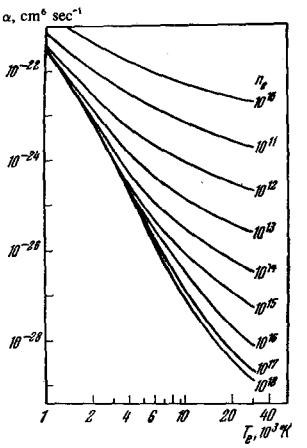

FIG. 2. Dependence of the IC α and T_e in an optically thin hydrogen plasma at different electron concentrations.

[2)] For certain particular conditions, the system of the balance equations for the population of the atomic levels was solved in [22] simultaneously with the kinetic equation. The influence of the non-equilibrium distribution of the electron energy on the distribution of the atoms over the levels and on the ionization equilibrium was investigated.

a) Kinetics of collision ionization and recombination. Among the processes leading to ionization and recombination, we take into account first only the collisions of free electrons with atoms and triple electron-electron-ion collisions. We assume an equilibrium energy distribution function. The problem consists of obtaining an equation describing the diffusion of the bound electrons in the discrete spectrum and its subsequent solution.

Equation (2.2) for $\partial n_k/\partial t$ was already rewritten in the form (2.8), i.e., in the form of a "divergence" of the particle flux in the discrete energy spectrum Δj_k. We transform Δj_k to the diffusion form. We begin with the formal device used in the derivation of the Fokker-Planck equation[35]. We set up the quantity

$$\sum R_i \Delta j_i = \sum_i R_i \left[\sum_k (n_k w_{ki} - n_i w_{ik})\right] = \sum_i n_i \left[\sum_k (R_k w_{ik} - R_i w_{ik})\right], \quad (4.1)$$

where R_i is a certain auxiliary function that vanishes together with its differences at finite summation points. We expand the function R_k in (4.1) in a series of finite differences at the point $i = k$[36]:

$$R_k \approx R_i + (\Delta R_{i-1}/\Delta E_{i-1})(E_k - E_i) + [\Delta (\Delta R_{i-1}/\Delta E_{i-1})/(E_{i-1} - E_{i+1})](E_k - E_i)(E_k - E_{i-1}) + \ldots \quad (4.2)$$

Substituting (4.2) in (4.1), we obtain

$$\sum R_i \Delta j_i = \sum_i n_i \{(\Delta R_{i-1}/\Delta E_{i-1}) a_i + [\Delta (\Delta R_{i-1}/\Delta E_{i-1})/(E_{i-1} - E_{i+1})] b_i\}, \quad (4.3)$$

where a_i and b_i are the first and second moments of the transition probability

$$a_i = \sum_k w_{ik} (E_k - E_i), \quad (4.4)$$

$$b_i = \sum_k w_{ik} (E_k - E_{i-1})(E_k - E_i). \quad (4.5)$$

If we were to retain the third difference in the expansion (4.2), we would get the third moment

$$c_i = \sum_k w_{ik} (E_k - E_{i-1})(E_k - E_i)(E_k - E_{i+1}). \quad (4.6)$$

We see from (4.4)–(4.6) that single-quantum transitions contribute only to the first two moments and make no contribution to the succeeding ones. This enables us to discard in the expansion (4.2) the terms with third and higher differences. Applying the Abel transformation for summation by parts[36] ($\sum_i x_i \Delta y_i = x_i y_i|_i - \sum_i y_{i+1} \Delta x_i$) once to the first term in the right-hand side of (4.3) and twice to the second term, we obtain ultimately

$$\Delta j_k = -\Delta (n_k a_k/\Delta E_{k-1}) + \Delta\{(\Delta E_{k-1})^{-1} \Delta [n_{k-1} b_{k-1}/(E_{k-2} - E_k)]\} = \Delta \{[-a_k + \Delta (b_{k-1}/(E_{k-2} - E_k))] (n_k/\Delta E_{k-1}) + [b_{k-1}/(E_{k-2} - E_k)] \Delta n_{k-1}/\Delta E_{k-1}\}, \quad k > 1. \quad (4.7)$$

In (4.7), Δ is the operator forming the finite difference. For example, $\Delta E_k = E_k - E_{k+1}$. Equation (4.7) is the diffusion equation in discrete space, expressed in terms of finite differences.

It is easy to verify that on going to the continuous energy spectrum, Eq. (4.7) is transformed into the divergence of the flux, written in the usual Fokker-Planck form (cf. (3.6)). On the other hand, the expression in terms of finite differences makes it possible to take exact account of transitions between close-lying states, the so-called single-quantum transitions as $k \to k \pm 1$. The latter becomes clear if we rewrite (4.7) in a somewhat different form:

$$\Delta j_k = \Delta (z_{k-1,\,k} n_{k-1} - z_{k,\,k-1} n_k), \quad (4.8)$$

where

$$z_{k-1,\,k} = b_{k-1}/\Delta E_{k-1} (E_{k-2} - E_k),$$
$$z_{k,\,k-1} = [b_k/\Delta E_{k-1} (E_{k-1} - E_{k+1})] + a_k (\Delta E_{k-1})^{-1}.$$

In the limiting case when transitions are possible only between neighboring levels (the single-quantum approximation), we obtain a flux j_k in the form of a difference between the transition numbers $k-1 \to k$ and $k \to k-1$. It is therefore convenient to call $z_{k-1,\,k}$ and $z_{k,\,k-1}$ the effective probabilities of the single-quantum transitions.

Multiquantum transitions are taken into account in (4.8) approximately. The resultant error is offset to a certain degree by means of a procedure customarily used in the diffusion approximation. Under equilibrium conditions, the expression for the flux should vanish. This requirement leads to the relation $z_{k-1,\,k} n^0_{k-1} = z_{k,\,k-1} n^0_k$ between the moments. Using this relation, we obtain

$$\Delta j_k = \Delta \{z_{k-1,\,k} [n_{k-1} - (n_k n^0_{k-1}/n^0_k)]\} = \Delta (z_{k-1,\,k} n^0_{k-1} \Delta y_{k-1}). \quad (4.9)$$

In the last equation of (4.9) we have introduced the "relative" populations $y_k = n_k/n^0_k$, which characterize the degree of deviation from the equilibrium values calculated for T_e. Similarly, we introduce below $y_e = n_e/n^0_e$ as the degree of deviation from the value determined by the Saha equation.

In the quasistationary approximation we have $\Delta j_k = 0$, and we get for the flux

$$j = z_{k-1,\,k} n^0_{k-1} \Delta y_{k-1}, \quad \Delta y_{k-1} = y_{k-1} - y_k. \quad (4.10)$$

Thus, the system of balance equation has been reduced to a system of pairwise coupled equations, which can be solved directly. This yields not only the RC and IC, but also the non-equilibrium distribution of the atoms over the excited states:

$$\left.\begin{aligned} j &= (y_1 - y_e^2)/\sum_{k=1}^{e} S_k, \\ y_k &= \left(y_1 \sum_{i=k}^{e} S_i + y_e^2 \sum_{i=1}^{k-1} S_i\right)/\sum_{i=1}^{e} S_i, \\ S_k &= (n^0_k z_{k,\,k+1})^{-1}. \end{aligned}\right\} \quad (4.11)$$

The explicit forms of the IC and RC are

$$\beta = (n_e n_1^0 \sum_{k=1}^{e} S_k)^{-1}, \quad \alpha = [n_e (n_e^0)^2 \sum_{k=1}^{e} S_k]^{-1} \quad (4.12)$$

The quantities n^0_1 and n^0_e, as already mentioned, are calculated from the Saha formula with the electron temperature T_e:

$$(n_e^0)^2 = n_1^0 K_1, \quad n_1^0 = n - n_e^0,$$

where K_1 is the ionization equilibrium constant

$$K_1 = 2\Sigma_i (g_1 h^3)^{-1} (2\pi m T_e)^{3/2} \exp(-E_1/T_e); \quad (4.13)$$

Σ_i is the partition function of the residual ion, and g_1 is the statistical weight of the ground state.

b) Calculation of effective transition probabilities. The calculation of $z_{k,\,k+1}$ reduces to a determination of the moments a_k and b_k, which are connected with the

average energy transferred to the atom upon collision with the electron, and its square. It is known that frequently the latter quantities are easier to calculate than the individual cross sections that define them.

Let us calculate the first moment. Prior to averaging over the electron distribution $f(\epsilon)$, we have for the first moment, by definition,

$$a_k(\varepsilon) = n_e \sum_n w_{kn}(\varepsilon)(E_n - E_k) = n_e \sum_n \sigma_{kn}(\varepsilon)(2\varepsilon/m)^{1/2}(E_n - E_k).$$

Substituting here the cross section $\sigma_{kn}(\epsilon)$ in the Born approximation and taking into account the sum rule[37]

$$\sum_n (E_k - E_n)\,|\,(\sum_a e^{-i\mathbf{q}\mathbf{r}_a})_{kn}\,|^2 = -\hbar^2 q^2/2m$$

($\mathbf{q}$ is the transferred momentum and $\mathbf{r}_a$ is the radius vector of the electron), we obtain

$$a_k(\varepsilon) = -4\pi e^4 n_e \Lambda_k/(2m\varepsilon)^{1/2},$$

where Λ_k is the Coulomb logarithm for the k-th bound state.

The moment $b_k(\epsilon)$ is conveniently represented in the form

$$b_k(\varepsilon) = \sum_n w_{kn}(\varepsilon)(E_n - E_k)^2 - (E_{k-1} - E_k)\sum_n w_{kn}(\varepsilon)(E_k - E_n).$$

The quantity $\sum_n w_{kn}(\epsilon)(E_n - E_k)^2$ is calculated in the Bethe-Born approximation with allowance for the sum rule[37]

$$\sum_n (E_n - E_k)^2\,|\,(x)_{kn}\,|^2 = 2mE_k/3,$$

where $(x)_{kn}$ is the matrix element of the dipole-moment projection. As a result we obtain

$$b_k(\varepsilon) = [4\pi e^4 \Lambda_k n_e/(2m\varepsilon)^{1/2}]\,(^4/_3 E_k - E_k + E_{k-1}) \approx 4\pi e^4 \Lambda_k n_e E_{k-1}/(2m\varepsilon)^{1/2}.$$

The moments calculated in this manner enable us to find the effective transition probabilities $z_{k,k-1}(\epsilon)$. These expressions are valid, strictly speaking, at sufficiently high energies, when all the transitions from the level under consideration are possible and the sum rules are satisfied. We can verify, however, that extrapolation of $z_{k,k+1}(\epsilon)$ to lower energies does not lead to appreciable errors.

We can now calculate the probabilities $z_{k,k+1}$[8C] averaged over the Maxwellian distribution $f^{(0)}(\epsilon)$

$$z_{k,k+1} = [4(2\pi)^{1/2}\Lambda_k e^4 n_e E_{k-1}/(mT_e)^{1/2}(E_{k-1} - E_{k+1})(E_k - E_{k+1})] \times \exp[-(E_k - E_{k+1})/T_e], \quad k > 1.$$

For k = 1 we have (4.14)

$$z_{12} = [4(2\pi)^{1/2} e^4 \Lambda_1 n_e/(mT_e)^{1/2}(E_1 - E_2)] \exp[-(E_1 - E_2)/T_e]. \quad (4.15)$$

We note that the moment b_n practically coincides at low energies E_n with the corresponding quantity obtained in[6] from quasiclassical considerations.

The values of Λ_n and Λ_1 in (4.14) and (4.15) are consequently quantities averaged over the energy distribution of the electrons. These quantities can be calculated on the basis of the experimental data available in the literature concerning the excitation cross sections. It turns out that they depend primarily on the relative threshold energy $\Delta E/T_e$, and this dependence is universal to a sufficient degree. It is shown in Fig. 3.

We note that the disappearance of an explicit dependence on the cross sections of the individual processes and the expression of all the quantities in terms of certain moments that are averaged over a large number of transitions are the consequence of the main premises of the diffusion approximation. After using the sum rules, all the calculations can be carried out without using data on the individual cross sections. Of course, it is necessary to stipulate here the method of calculating the Coulomb logarithms. Only in some particular cases (e.g., when the Bethe-Born approximation is not valid, when there are forbidden transitions that play a noticeable role in the kinetics, etc.) must the moments be calculated from the initial formulas, using measured or calculated cross sections.

c) Ionization and recombination kinetics for a non-equilibrium electron energy distribution. A non-equilibrium distribution of the atoms over the states should violate the Maxwellian distribution of the electrons. This violation can be small if the role of the Maxwellizing collisions is sufficiently large. In the opposite case, the absence of equilibrium in the electron distribution is essential and influences in turn the population of the atomic levels. In the general case it is quite difficult to obtain self-consistent atom and electron energy distributions. We consider below a particular but very important case, when the predominant role in the kinetic equation is played by interelectron elastic collisions (as the Maxwellizing factor) and inelastic collisions between the electrons and the atoms. Since the frequency of the interelectron collisions decreases sharply with increasing energy, the inelastic collisions can exert the strongest influence in the "tail" of the Maxwellian distribution ($\epsilon \sim E_1 - E_2$). We confine ourselves therefore to allowance for transitions between the ground and excited states.

Using (2.10) and (2.11), we write down the kinetic equation in the quasistationary approximation. Instead of $f(\epsilon)$ we introduce tentatively $y(\epsilon) = f(\epsilon)/f^0(\epsilon)$, which is the deviation of $f(\epsilon)$ from the Maxwellian value

$$f^0(\varepsilon) = 2\pi(\pi T_e)^{-3/2}\sqrt{\varepsilon}\exp(-\varepsilon/T_e).$$

Then

$$\frac{\partial}{\partial\varepsilon}\left[\frac{2(2\pi)^{1/2}\lambda n_e^2}{(2mT_e)^{1/2}} e^{-\varepsilon/T_e}\frac{\partial y(\varepsilon)}{\partial\varepsilon}\right] - n_1^0 f^0(\varepsilon) z_{12}(\varepsilon)[y_1 y(\varepsilon) - y_2 y(\varepsilon')]\,\theta(\varepsilon') = 0. \quad (4.16)$$

In (4.16) we have $\theta(\epsilon') = 1$ at $\epsilon' \geq 0$ and $\theta(\epsilon') = 0$ at $\epsilon' < 0$. The frequency of the inelastic transitions $1 \rightleftarrows 2$, which is contained in (2.11), is replaced, in accordance with the general MDA by the effective value $z_{12}(\epsilon)$, which was calculated in the preceding section:

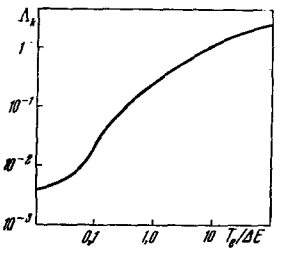

FIG. 3. Coulomb logarithm Λ_k for bound states as a function of the relative transition energy $T_e/\Delta E$.

$$z_{12}(\varepsilon) = 4\pi e^4 \Lambda_1 n_e/(2m\varepsilon)^{1/2}\,(E_1 - E_2).$$

$z_{12}(\epsilon)$ takes into account the transitions from the ground state into all the excited states. As a result, (4.16) is a second-order equation with constant coefficients. The solution of the linear equation (4.16) must be sought in the form of a superposition of exponentials that contain in their arguments the roots of the characteristic equations in the two energy regions $\epsilon \leq E_1 - E_2$ and $\epsilon \geq E_1 - E_2$. The values of the coefficients of the exponentials are determined from the conditions of the continuity of $y(\epsilon)$ and $y'(\epsilon)$ at $\epsilon = E_1 - E_2$, and the normalization condition (which reduces to $y(0) \approx 1$). The value of $y(\infty)$ follows from the balance of the inelastic collisions at high energies.

Without presenting $y(\epsilon)$, which has a very simple form[8C], let us consider the result of its substitution into Eq. (4.10), written for the interval $1 \rightleftarrows 2$ with allowance for the possible absence of a Maxwellian distribution

$$j = n_1^0 \int_0^\infty [y_1 y(\varepsilon) - y_2 y(\varepsilon - E_1 + E_2)]\, f(\varepsilon)\, z_{12}(\varepsilon)\, d\varepsilon = n_1^0 z_{12} F (y_1 - y_2) \quad (4.17)$$

Carrying out the integration in (4.17), we obtain an expression for F:

$$F = c^{-1}\,[(1 + 4c)^{1/2} - 1]/[(1 + 4c)^{1/2} + 1], \quad (4.18)$$

where

$$c = n_1 z_{12}/n_e z_{ee} = (2n_1/n_e)\,[T_e/(E_1 - E_2)]\,\Lambda_1/\lambda.$$

The structure of the system (4.10) remains by the same token unchanged. Allowance for the non-Maxwellian character has reduced formally to the appearance of the function F in that equation which characterizes the energy interval $1 \rightleftarrows 2$ (4.17). This allows us to use the solution (4.11), (4.12). By merely redefining the quantity $S_1 = (n_1^0 F z_{12})^{-1}$ we obtain the IC and RC, and also an atomic-level distribution that takes into account the non-equilibrium character of $f(\epsilon)$.

If y_1 exceeds y_2 appreciably, and consequently the particle flux in the interval $1 \rightleftarrows 2$ is due mainly to the excitations $1 \rightarrow 2$, then F can be easily interpreted. It takes into account the decrease of the number of $1 \rightarrow 2$ transitions as a result of the shortage of fast electrons in the "tail" of the function $f(\epsilon)$. The function F interpreted in this manner was calculated by many authors. It is very important, as shown by Wojaczek[24], that F depends little on the behavior of the $1 \rightarrow 2$ excitation cross section with changing ϵ; all that matters is the correct value[3] of the cross section at the energy $\epsilon \approx E_1 - E_2 + T_e$.

The character of the influence of the non-Maxwellian distribution on the IC and RC will be discussed in detail in Chap. 5. The interrelated distributions of the atoms over the levels and of the electrons over the energies may be of independent interest. By way of illustration, these distributions are shown in Fig. 4 for a hydrogen plasma. y_i and $y(\epsilon)$ are plotted in the regions of negative and positive energies E, respectively ($y(\epsilon)$ characterizes the deviation from the Maxwellian distribution). The curves were plotted for $T_e = 8000°$ K and $n = 10^{16}$ cm^{-3} at different degrees of ionization: $n_e/n = 10^{-3}$ (1), 10^{-4} (2), and 10^{-5} (3). The same figure shows, for comparison, the behavior of y_i (dashed lines) calculated under the assumption that the distribution is Maxwellian. As seen from Fig. 4 (particularly in case 3), the deviation from Maxwellian distribution greatly decreases the population of the excited levels. The non-equilibrium behavior is determined primarily by the coefficient c, which is the ratio of the number of inelastic $1 \rightarrow 2$ collisions to the number of interelectron collisions z_{ee}. Under typical conditions we have $\lambda \sim 10$ and $\Lambda_1 \sim 0.01$–0.05. This means that at $T_e \sim 1$ eV the absence of a Maxwellian distribution is important if the degree of ionization is small, $n_e/n_1 \lesssim 10^{-4}$–10^{-6}.

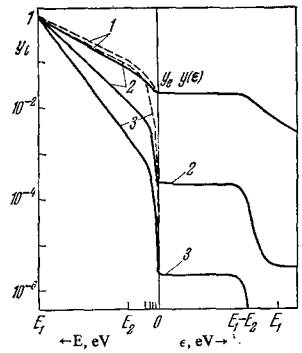

FIG. 4. Electron distribution functions in the discrete and continuous spectra.

We call attention to the fact that the obtained formulas take into account only one type of elastic collisions—interelectron collisions, and are not valid at very small degrees of ionization. For example, when the Maxwellizing collisions are elastic electron-atom collisions we have

$$n_e/n_1 \ll (m/M)\,\sigma_{ea}/\sigma_{ee}\ (\varepsilon \approx E_1 - E_2). \quad (4.19)$$

Here σ_{ea}/σ_{ee} is the ratio of the electron-atom and electron-electron elastic collision cross sections, taken at an electron energy $\epsilon = E_1 - E_2$. Usually $n_e/n_1 \lesssim 10^{-7}$–10^{-8}. Such conditions can be realized, e.g., in a positive-column plasma. In spite of the fact that the number of electrons is small, $T_e \gg T_a$ and the ionization is determined just by the electrons. Such a plasma was investigated by a number of workers. The most detailed investigations were performed by Kagan and Lyagushchenko[23]. When the inequality (4.19) is satisfied, the function F must be modified and should depend on the concrete form of $\sigma_{ae}(\epsilon)$. However, reasonable estimates can be made by using the following simple expressions:

$$F = (1 + c)^{-1}, \quad c = z_{12}/z_{ea} \approx 2\pi e^4 \Lambda_1/(E_1 - E_2)\, T_e \sigma_{ae}\ (\varepsilon = E_1 - E_2),$$

the meaning of which is obvious.

d) Kinetics with allowance for radiative processes. Turning to (2.7), we write down the contribution made to the flux j by the radiative transitions:

$$j^R = -\sum_{l<n} \Big(\sum_{k \geq n} n_k A_{kl}^* + n_l^2 \alpha_{el}\Big). \quad (4.20)$$

The motion of the bound electron over the energy spectrum, as a result of radiation acts, has a directional character, and at first glance the diffusion approximation cannot be used. It is possible, however, to simplify

[3] Therefore Λ_1 in (4.18) can differ somewhat from the value average over $f(\epsilon)$ in (4.15). This circumstance can be easily taken into account if more accurate calculations are desired. See the solution obtained for the similarly formulated problem in [68].

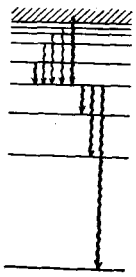

FIG. 5. Scheme for taking the radiative transitions into account (for any atomic level n).

expression (4.20) and include it in the MDA scheme.

We use the fact that at not too large l and k we have $A_{kl} \sim k^3 l^{-1}/(k^2 - l^2)^{-1}$. If k is fixed, A_{kl} changes relatively little with increasing l. To the contrary, if l is fixed, A_{kl} decreases rapidly with increasing k. In other words,

$$A_{l+1,l} \gg A_{l+k,l}, \quad A_{l,l-k} \approx A_{l,l-1} \quad (k \geqslant 2). \tag{4.21}$$

Then (4.20) takes the form

$$j^R \approx -n_n a_n^R - n_e^2 a_n^e,$$

where

$$a_n^R = \sum_{k \geqslant n} (n_k^0/n_n^0) \sum_{l<k} A_{kl}, \quad a_n^e = \sum_{l<n} \alpha_{el}.$$

If we substitute j_R in the expression for $\partial n_n/\partial t$, then we can verify that the radiative transition n + 1 → n, as well as the transitions from the level n to all the lower states with $l < n$, is taken into account exactly. The transitions with m + n → n ($m \geq 2$) are taken into account approximately, but in view of the inequality (4.21) their role in the kinetics is small. Figure 5 shows the transitions that are taken into account exactly by thick wavy lines, and all others by thin lines.

The inequalities in (4.21) are valid, as noted above, for not too large l. However, when l is increased the cross sections of the inelastic processes increase sharply, and the radiative-transition probabilities decrease. Therefore the direct influence of the radiative transitions on the populations of the higher levels is small. By the same token, the method used to take into account the radiative transitions at large l ceases to be important. Naturally, the method used by us to take into account the radiative transitions calls for refinement when an extremely rarefied plasma is considered. On the other hand, in an extremely dense plasma, the inequalities (4.21) may cease to be valid when A_{kl} is replaced by A^*_{kl} in accordance with (2.9). In[8a] there are given expressions that are valid for a_n^R and a_n^e under these conditions.

We now write down an expression for the flux j in discrete energy space, taking into account both the impact and the radiative processes. We obtain the following system of coupled equations:

$$j = n_k z_{k,k+1} - n_{k+1}(z_{k+1,k} + a_{k+1}^R) - n_e^2 a_k^e. \tag{4.22}$$

Solving this system in analogy with the solution obtained in the section on collision ionization and recombination, we obtain the impact-radiation IC and RC

$$\beta = (n_e n_i^0 \Pi_i \sum_{k \geqslant 1} S_k)^{-1}, \tag{4.23}$$
$$\alpha = [1 + \sum_{k \geqslant 1} a_{k+1}^e (n_e^0)^2 S_k]/[n_e (n_e^0)^2 \sum_{k \geqslant 1} S_k],$$

$$\Pi_k = \prod_{n \geqslant k} [1 + (a_{n+1}^R n_n^0/z_{n,n+1} n_{n+1}^0)],$$
$$S_1^{-1} = z_{12} F n_1^0 \Pi_1, \quad S_k^{-1} = z_{k,k+1} n_k^0 \Pi_k \quad (k \geqslant 2), \tag{4.24}$$

Π_k is a factor that takes into account the influence of the emergence of the radiation in the spectral lines on the population of the (k + 1)-st level. On the other hand, the first factor in α takes into account the role of the recombination continuum. From (4.23) there follows a relation between α and β:

$$\beta = \alpha (n_e^2/n_i)^0 [1 + \sum_{k \geqslant 1} (n_e^0)^2 a_{k+1}^e S_k]^{-1} \Pi_i^{-1}.$$

A detailed discussion of the expressions obtained and of their use will be given in the next chapter of the review. In concluding this section we note that within the framework of the MDA it is easy to take into account the inelastic collisions with heavy particles, say with atoms. This can be done in universal fashion by assuming that the probabilities of the collisions with the atoms satisfy the requirement that the MDA be applicable. Things are not completely clear here, although for transitions between strongly excited states the situation seems to be as described here[4]. Thus, it is necessary to add in (4.22) a term that takes into account the atom-atom collisions: $n_k z^a_{k,k+1} - n_{k+1} z^a_{k+1,k}$, where $z^a_{k,k\pm1}$ are the corresponding effective probabilities. The values of α and β with allowance for these processes are given in[8a].

Expression (4.27) pertain to the stepwise ionization and recombination coefficients. The total IC and RC are the sums of the stepwise and direct transitions

$$\beta = \beta_{st} + \beta_d, \quad \alpha = \alpha_{st} + \alpha_d;$$

β_d and α_d are determined by the probabilities of the elementary acts of ionization from the ground state and recombination to the ground state, and are well known (see, e.g.,[30]).

e) Comparison of the results of the modified diffusion approximation with experiment. The MDA described above was compared in[7a,8] with various experimental data. It is important to note that the comparison concerned not the RC and IC (such a comparison is given in Chap. 5), but to the distribution of the atoms over the excited states, which are more sensitive to various kinds of inaccuracies in the description of the phenomenon. The comparison was both for stationary[7a] and nonstationary conditions[38-40]. The comparison revealed a number of characteristic regularities in the distribution of the atoms over the excited levels in the process of recombination (ionization), and this factor is apparently of interest. Table I contains a brief summary of the considered conditions. In all these cases, the agreement between theory and experiment was perfectly satisfactory. By way of example, Fig. 6 shows a comparison of the calculated data[8a] with the experimental ones[45].

The figure shows the relative concentrations y/y_e^2 of helium atoms in ^3D states as functions of the binding energy E (reckoned from the continuum) and of the principal quantum number n. In[45] they considered three regimes: 1 – T_e = 3140° K, $n_e = 5.6 \cdot 10^{13}$ cm^{-3}; 2 – 2200° K, $1.8 \cdot 10^{13}$ and 3 – 1510° K, $6.2 \cdot 10^{12}$ cm^{-3}. For the regime with $n_e = 6.2 \times 10^{12}$ cm^{-3} and T_e

Table I

Reference	Plasma composition	n_e, cm^{-3}	T_e, 10^3 °K	Character of kinetics	Plasma source
			Stationary Plasma		
41	H	10^{15}	9	Electron-atom collisions	Arc discharge
42	Ar	10^{13}	13	Strongly non-Maxwellian distribution	Low-pressure discharge
43	Ar + Cs	10^{12}—10^{13}	3	Radiation	Medium-pressure discharge
44	H	10^{13}	100	Radiation	Phillips manometer
			Nonstationary Plasma		
45	He	10^{13}—10^{14}	1,5—3	Radiation, plasma decay	Stellarator
38	N	10^{14}—10^{16}	10—16	Electron-atom collision, development of ionization	Shock tube
39	Cs	10^{13}—10^{14}	2—3	Electron-atom collision, decay	Medium-pressure discharge
21	Ar + K	10^{12}—10^{14}	3	Electron-atom collision, development of ionization	Voltage pulse

= 1510° K, three theoretical curves were plotted, namely: 4—without allowance for radiation, 5—complete reabsorption of the lines of the resonance series, 6—optically transparent plasma. For the remaining two cases, only curves 5 were plotted. As seen from Fig. 6, the levels with $n \gtrsim 5$ are in relative equilibrium with the continuum, $y/y_e^2 = 1$. The populations of the lower-lying levels are not in equilibrium, and agreement with experiment was reached when account is taken of the radiative transitions between the excited states.

5. CALCULATION OF THE IMPACT-RADIATION IONIZATION AND RECOMBINATION COEFFICIENTS

The general expressions obtained in the preceding sections for α and β are quite complicated. They can be greatly simplified, however. This chapter is devoted to a derivation of simple expressions for α and β, in which the main features impact-radiative recombination (ionization) are reflected. To this end we stop first to discuss the qualitative features both in the case of pure collision and under conditions of noticeable influence of radiative transitions.

a) Qualitative features of the kinetics of impact-radiation recombination and ionization. It was noted above that usually a relatively small group of atomic levels forms a "bottleneck" for the flow of recombining electrons. Let us determine the position of the "bottleneck" in the case of purely collisional kinetics. We simplify first the expression for the distribution of the atoms over the levels, changing over to continuous variation of the energy. The sums in (4.23) must then be replaced by integrals in accordance with the following scheme:

$$\sum_{k \geq m} (z_{k,\,k+1} n_k^0)^{-1} \sim \int_0^{E_m} [e^{-E/T_e}/Eg(E)]\, dE,$$

where $g(E) = Ry^{3/2}/E^{5/2}$ is the density of the hydrogen-like states (3.7). From this we readily obtain the distribution of the atoms over the excited states:

$$y(E) = y_1 \chi(E_1/T_e) + y_e^2 [1 - \chi(E/T_e)], \qquad (5.1)$$

where

$$\chi(x) = (4/3\sqrt{\pi}) \int_0^x e^{-t} t^{3/2} dt.$$

A plot of $\chi(x)$ is shown in Fig. 7. It was assumed in (5.1) that $E_1 \gg T_e$, i.e., that $\chi(E_1/T_e) \approx 1$.

Figure 8 shows a characteristic plot of y(E). The group of strongly excited states is in relative equilibrium with the electrons. The lower excited levels are closely coupled to the ground state. The group of intermediate levels is the "bottleneck." In fact, the flux j has in the same approximation the form $j \sim \mu(E)\, dy/dE$, $\mu(E) \sim Eg(E) e^{-E/T_e}$. The quantity $\mu(A)$ characterizes the mobility of the electrons in energy space. Obviously, it is minimal if dy/dE is large. Therefore, putting $d^2y/dE^2 = 0$, we obtain the position of the bottleneck $E^* = 3T_e/2$. The lower limit of the bottleneck is determined from the requirement that dy/dE decrease by a factor e in comparison with the value at the maximum. This yields a value $\sim 7T_e/2$. Thus, at low temperatures the "bottleneck" lies in the region of highly excited states and gradually drops with increasing temperature. In the former case one can replace the discrete spectrum with a quasicontinuum and in the second we can confine ourselves to transitions between several low-lying energy states. We arrive accordingly at simplified pictures of the kinetics—the diffusion approximation and the prompt ionization approximation, considered in Chap. 2.

In the presence of radiative transitions, the picture becomes distorted. Let us consider the case of low

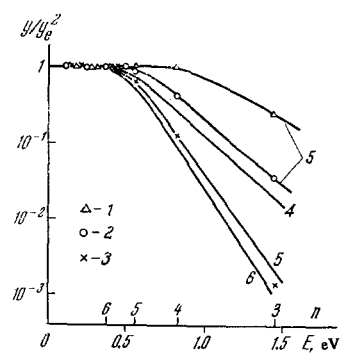

FIG. 6. Comparison of experimental and calculated populations of different excited levels of the helium atom.

temperatures, when the bottleneck lies in the region of highly excited levels. The intensity of the radiative transitions decreases sharply with decreasing binding energy E_k (the oscillator strength is $f_k \sim k^{-3}$), whereas the intensity of the impact processes increases sharply (cross section $\sigma_k \sim k^4$). This enables us to separate distinctly the energy interval into two regions: 1) $E < E_R$, where impact transitions predominates, and 2) $E > E_R$, where radiative transitions predominate. The position of the level E_R is determined from the condition that the frequencies of the impact and radiative transitions be approximately equal at $\Pi_{E_R} \approx 2$. Assuming the strongly excited levels to be hydrogen-like and neglecting the reabsorption of the radiative transitions between them, we can obtain[8] the following expression for the estimate of E_R:

$$E_R \approx \mathrm{Ry}\, n_e^{1/4} (\pi^{1/2} e^4 \bar{\Lambda}/\mathrm{Ry}\,(mT_e)^{1/2} c)^{1/4} \quad (c = (3 \div 4)\cdot 10^{10}\ \mathrm{sec}^{-1}).$$

It is convenient to rewrite this formula in the form

$$E_R \approx T_e (n_e/n_e^*)^{1/4}, \quad n_e^* = 4.5\cdot 10^{13}\, T_e^{9/2}\ \mathrm{cm}^{-3} \quad (T_e \text{ in eV}).$$

Various cases are possible, depending on the ratio of the lower limit $7T_e/2$ of the "bottleneck" to E_R.

At high temperatures, the bottleneck corresponds to the interval between the ground and first excited level. The emission connected with transitions to the ground state can be quite appreciable. Allowance for this emission reduces primarily to an increase in the number of impacts of the second kind by a factor Π_1. The two-level model would yield $\Pi_1 = 1 + (A_{21}^*/w_{21})$. The MDA modifies this quantity (sometimes quite significantly): $\Pi_1 = [1 + (a_2^R/z_{21})]\,[1 + (a_3^R/z_{32})]\ldots$

The influence of the non-Maxwellian distribution on the kinetics reduces formally to replacement of z_{12} by $z_{12}F$. If $F \ll 1$, then z_{12} is replaced by z_{ee} and the number of $1 \rightleftarrows 2$ transitions is determined by the rate of appearance of fast electrons with energy $\epsilon \sim E_1 - E_2$. In the presence of a non-Maxwellian distribution the position of the "bottleneck" can change. Indeed, the resistance to the electron motion in energy space, exerted by the individual sections of the spectrum is $\sim [n_k^0 z_{k,k+1}]^{-1}$, and therefore when $F \ll 1$ the most difficult interval for the passage of electrons may be the section $1 \to 2$. In the presence of a noticeable deviation from Maxwellian distribution, the dependence of the IC and RC on the plasma parameters can be strongly modified.

b) Electronic ionization and recombination. In this case $\Pi_k = 1$, $a_k^e = 0$, and from (4.23) and (4.24) we get

$$\alpha = [n_e' (n_e^0)^2 \sum_{k\geq 1} S_k]^{-1}, \quad \beta = \alpha (n_e^0)^2/n_1^0. \tag{5.2}$$

We see that α and β are connected via the ionization-equilibrium constant $K_i = (n_e^0)^2/n_1^0$, and this holds true also in the absence of a Maxwellian distribution[4]. If $F = 1$, then α and β are only functions of T_e. On the other hand if $F < 1$, then α and β depend also on n_e. To obtain approximate expressions for α and β, we proceed as follows. We separate the term with $k = 1$ in the sums

[4]This result is due to the fact that we are considering one specific cause of violation of the equilibrium distribution of the electron energies at large energies, namely elastic collisions with atoms.

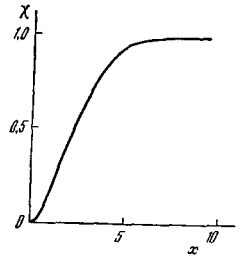

FIG. 7

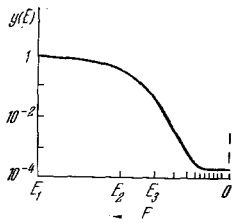

FIG. 8

FIG. 7. Plot of the function $\chi(x)$.

FIG. 8. Characteristic form of the distribution of the levels in the regime where ionization predominates ($Y_e^2 < 1$).

(5.2) and change over in the remaining group of terms to a quasicontinuous variation of the energy, as was done above. As a result we get

$$\beta^{-1} = \beta_1^{-1} + \beta_2^{-1}\chi\,(E_2/T_e), \tag{5.3}$$

$$\alpha^{-1} = \alpha_1^{-1} + \alpha_2^{-1}\chi\,(E_2/T_e), \tag{5.4}$$

where the subscript 1 marks the IC and IR contributions of the transitions between the ground and first excited level, and the subscript 2 marks the contributions of the transitions between the excited states.

The case $\beta_1^{-1} \gg \beta_2^{-1}\chi(E_2/T_e)$ and $\alpha_1^{-1} \gg \alpha_2^{-1}\chi(E_2/T_e)$ corresponds to the immediate ionization approximation, and the inverse sign of the inequalities means that the diffusion approximation is applicable.

In formula (5.3) we put

$$\begin{aligned} \beta_1 &= \Gamma\Lambda_1\,[\mathrm{Ry}^{3/2}F/T_e^{1/2}\,(E_1 - E_2)]\, e^{-(E_1-E_2)/T_e}, \\ \Gamma &= 4\,(2\pi)^{1/2} e^4/m^{1/2}\,\mathrm{Ry}^{1/2} = 1{,}7\cdot 10^{-7}\ \mathrm{cm}^3/\mathrm{sec} \end{aligned} \tag{5.5}$$

In the limit of strong deviation from Maxwellian distribution we have $F \ll 1$, so that the rate of the transitions $1 \rightleftarrows 2$ is determined by the rate of the appearance of electrons with $\epsilon \sim E_1 - E_2$, and

$$\beta_1 = \Gamma\,(\mathrm{Ry}/T_e)^{3/2}\,(\lambda n_e/2n_1)\, e^{-(E_1-E_2)/T_e}. \tag{5.6}$$

The contribution made to α by the $1 \rightleftarrows 2$ transitions is

$$\alpha_1 = \Gamma_1(\Lambda_1 g_1/\Sigma_i)[\mathrm{Ry}^3 F/\,(E_1 - E_2)\, T_e^3)]\, e^{E_2/T_e},\ \Gamma_1 = h^3 e^4/\pi m^3\,\mathrm{Ry}^3. \tag{5.7}$$

Actually α_1 is proportional to the rate of deactivation of the first level. It is curious to note that deviation from Maxwellian distribution can decrease the value of α_1.

The contribution of the transitions between the excited states is determined from the expressions

$$\alpha_2 = \Gamma_1(2\bar{\Lambda}/3\sqrt{\pi})\,(\mathrm{Ry}/T_e)^{9/2},\ \beta_2 = \Gamma(2\bar{\Lambda}\Sigma_1/3\pi^{1/2}g_1)(\mathrm{Ry}/T_e)^3\, e^{-E_1/T_e}, \tag{5.8}$$

where $\bar{\Lambda} \approx 0.2$ is the mean value of the Coulomb logarithm for the excited states, and Σ_i is the partition function of the ion. Naturally, we obtain $\alpha \sim T_e^{-9/2}$ in this case. A similar dependence is obtained from the diffusion approximation, where the discreteness is disregarded from the very beginning. Expression (5.8) for α does not depend on the type of atoms. This circumstance is natural, for in this case the rate of recombination (ionization) is determined by the passage of the electron through strongly excited hydrogen-like states.

Figure 9 compares the values of α calculated for a hydrogen plasma in accordance with (5.4), with the re-

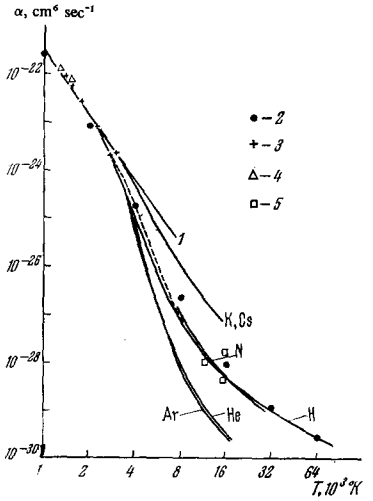

FIG. 9. Coefficient of triple recombination (in accordance with the electron-electron-ion shceme) as a function of T_e for different elements.

Table II

T_e, 10³°K	2	4	6	8	10	12	14
α : K	1.2-23	4.9-25	4.3-26	9.8-27	3.4-27	1.7-27	—
N	1.2-23	1.9-25	6.8-27	1.1-27	3.3-28	1.6-28	8.0-29
Ar	1.2-23	8.0-26	9.8-28	7.2-29	2.0-29	8.3-30	4.5-30
H	1.2-23	1.9-25	6.0-27	8.0-28	2.6-28	1.2-28	6.8-29

T_e, 10³°K	16	18	20	24	28	32
α : K	—	—	—	—	—	—
N	5.9-29	3.9-29	3.7-29	1.5-29	1.0-29	8.0-30
Ar	3.2-30	2.3-30	1.7-30	1.4-30	1.1-30	1.0-30
H	9.1-29	5.0-29	2.7-29	1.7-29	1.3-29	1.1-29

1.3-23 denotes 1.3×10^{-23} cm⁶ sec⁻¹, etc.

sults of Bates[1], which are marked by the black circles. We point out that in[1] the cross sections of the inelastic collision transitions are taken after Gryzinski. Our calculations were performed in an entirely different method—they are based on cross sections taken in the Bethe-Born approximation. Nonetheless, the agreement is on the whole quite good, thus indicating once more that α is not very sensitive to the cross sections of the individual transitions. An exception is the temperature $T_e = (8-16) \times 10^{3\circ}$K, precisely where the dominant contribution to α is made by only one transition $1 \rightleftarrows 2$ (the "bottleneck" is at the ground state). Some discrepancy in the RC is due to the difference in the cross section of the $1 \rightleftarrows 2$ transition, particularly in the behavior near the threshold. At higher temperatures, $T_e > 16 \times 10^{3\circ}$K, the influence of the near-threshold behavior of the $1 \rightleftarrows 2$ cross section levels out, and this decreases the discrepancy.

The foregoing pertained to hydrogen. Let us see what differences occur when the theory is applied to complicated atoms. According to the MDA formulas, each level is characterized by an energy E_k and by a statistical weight g_k. In a real level scheme there are many levels and sublevels having close values of the energy. Allowance for all the possible transitions between them in the MDA approach would be an exaggeration of the accuracy, so that it is advisable to make the energy spectrum "somewhat rougher" combining a group of nearby states into a single level, to which we ascribe a summary statistical rate and an average energy. For many elements, it is reasonable to combine into a single level most states with given principal quantum number n (as a rule, this is a state with large l). After obtaining such a "crude term scheme," we have a set of data $\{E_1, g_1\}$, $\{E_2, g_2\}$, $\{E_3, g_3\}$ etc. (the arrangement is in order of decreasing energy).

Usually $\{E_1, g_1\}$ is the ground state of the atom[5], $\{E_2, g_2\}$ is a group of low-lying excited states or an individual separated level, $\{E_3, g_3\}$ is the next group of excited states, etc. On going over to more excited states, such a unification becomes more and more natural, since the levels become hydrogen-like. The ionization kinetics is now determined by the effective single-quantum transitions between these groups of levels with probabilities $z_{k,k+1}$ calculated in accordance with formula (4.14). With this taken into account, Fig. 9 shows the values of α for a number of elements, and Table II gives the numerical values.

Let us examine the dependence of α on the specifics of the arrangement of the atomic levels. At $T_e < 3000^\circ$K as seen from Fig. 9, $\alpha \sim T_e^{-9/2}$ for all elements. The straight line 1 in Fig. 9 corresponds to this dependence. With increasing T_e, deviations from the $T_e^{-9/2}$ begin. For atoms with a relatively uniform level density in the spectrum (K, Cs), the $T_e^{-9/2}$ law is naturally more applicable. For He, which has the maximum gap $E_1 - E_2$, the deviations from $T_e^{-9/2}$ begin much earlier (the case of Ar will be discussed later on) and the "prompt ionization" approximation is quite suitable. Hydrogen and nitrogen occupy intermediate positions. The specifics of the atom become manifest not only in the value of $E_1 - E_2$. At large T_e, the value of α depends noticeably on the ratio Σ_i/g_1. Formula (5.7) explains why the values of α practically coincide for the pairs H, N and He, Ar. These atoms have close values of $\Sigma_i(E_1 - E_2)/g_1$ and E_2.

Without performing the calculations, we can not easily trace the course of $\alpha(T_e)$ also for a few other elements. For Xe, for example, α should be very close to that of Ar. For Mercury, which has a relatively uniform level density, $\alpha(T_e)$ should recall the variation of α for alkali metals, although the deviations from the $T_e^{-9/2}$ law should appear at somewhat lower temperatures.

Figure 10 shows the ionization coefficients β as functions of T_e/E_1 for Argon (curves 1—4) and potassium (5—9). Curve 9 for potassium corresponds to the diffusion approximation (5.8), curve 8 corresponds to the prompt-ionization approximation (5.5), and curve 5 is plotted in accordance with formula (5.3). We see that the diffusion approximation for potassium is valid for almost the entire range of T_e/E_1 under consideration. The situation with argon is different. Curve 1, constructed in accordance with (5.3), coincides with the curve constructed in accordance with (5.5), i.e., the immediate ionization approximation operates in the entire T_e/E_e interval.

[5]If a group of closely-lying states is located near the ground state, it is advisable to carry out the unification indicated above. For example, for the nitrogen atom near the ^4S ground state there are the levels ^2D and ^2P.

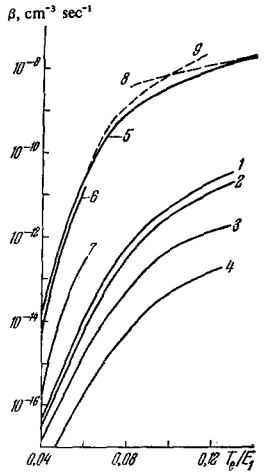

FIG. 10. Dependence of the ionization coefficient β on the relative temperature.

The influence of the deviation from Maxwellian distribution on the rate of ionization of an argon plasma is shown in Fig. 10 (E_1 is the ionization energy). We see that at low degrees of ionization β is proportional to the concentration of the electrons and decreases sharply in magnitude, corresponding to the limiting formula (5.6). Curves 1—4 in Fig. 10 correspond to $n_e/n_1 = 10^{-2}$, 10^{-3}, 10^{-4}, and 10^{-5}, respectively.

Let us examine the influence of the discreteness of the atomic levels on the values of α and β. If the term with k = 1 is not separated in 5.2, and integration is carried out with respect to the energy up to the ground level, we obtain

$$\beta^{-1} = \beta_2^{-1}\chi(E_1/T_e), \quad \alpha^{-1} = \alpha_2^{-1}\chi(E_1/T_e).$$

If T_e is not too large, then $E_1/T_e \gg 1$ and $\chi(E_1/T_e) \approx 1$ (see Fig. 7), and actually $\beta = \beta_2$ and $\alpha = \alpha_2$. Consequently, α is described by the "9/2 law." Its applicability can be readily traced in Fig. 9. The largest deviation (up to several orders of magnitude) occurs for elements having a large energy gap between the ground and the excited levels (He and Ar, followed by H and N). For K and Cs, the approximation that neglects the discreteness gives better results, since the level energies of these atoms are more uniformly distributed.

c) Impact-radiation recombination and ionization. When the influence of radiation becomes appreciable, α and β are no longer connected by the detailed-balancing relation, and it is necessary to calculate both α and β, since these quantities depend differently on the plasma parameters. In the presence of reabsorption, the additional difference is connected with the influence of the linear dimensions of the plasma and of other parameters that determine the effective lifetimes. Allowance for these effects is contained in formula (4.23). Let us discuss the extreme cases and the possible simplifications. If we decrease n_e and by the same token increase the role of the radiation $\Pi_k \gg 1$, then in the limit there follows from (4.2) the so-called "radiative recombination" coefficient

$$\alpha = \alpha_R = (n_e n)^{-1} \sum_k \alpha_{ek}.$$

In this extreme case, α is determined by the summary number of primary acts of recombination with radiation, $A^+ + E \to A + h\nu$. For all k we should have in this case $\Pi_k \gg 1$. For large k, reabsorption can be neglected and this inequality can be rewritten in the form

$$n_e \ll 10^{17} (\ln k/k^7) (T_e/\mathrm{Ry})^{1/2}. \tag{5.9}$$

A criterion similar to this was obtained by Griem[46] from qualitative considerations.

If the inequality (5.9) is reversed, then the radiation emitted becomes negligible and we return to the formulas of the preceding section. In the intermediate conditions, impact-radiation recombination (ionization) takes place. Let us write down approximate expressions for α and β in the case of impact-radiation recombination, starting from the same considerations as in the derivation of (5.3) and (5.4). We obtain

$$\beta^{-1} = \beta_1^{-1} + \beta_2^{-1}\chi(\tilde{E}/T_e)\,\Pi_1, \tag{5.10}$$

$$\alpha^{-1} = (\alpha_1\Pi_1)^{-1} + \alpha_2^{-1}\chi(\tilde{E}/T_e), \tag{5.11}$$

where $\tilde{E} = \min(E_2, E_R)$. These formulas differ from (5.3) and (5.4) in that they contain Π_1 and that χ has a different argument. The factor Π_1 follows naturally from the general expressions (4.23), and reflects the influence of the radiative transitions on the rate of passage of the electrons through the interval $1 \rightleftarrows 2$. It slows down the ionization and accelerates the recombination. When $E_R < E_2$, the factor $\chi(E_R/T_e)$, reflects the influence of the radiative process on the kinetics of the transitions between the excited states. Its appearance can be explained by starting from the following considerations. The radiation begins to play a role in the kinetics of transitions between excited states, if the level $E_R < 7T_e/2$ is the lower limit of the bottleneck. In this case the recombining electron spends most of the time at the levels with $E < E_R$, and then falls rapidly, via a cascade of radiative transitions, to a lower excited state (the contribution of the transitions $2 \rightleftarrows 1$ is taken into account separately). In such a situation, the length of the bottleneck reduces, as it were, a fact that can be accounted for by replacing $\chi(E_2/T_e)$ in (5.4) by $\chi(E_R/T_e)$ at $E_R < E_2$. We note that a procedure based on a similar idea was used earlier by Hinnov and Hirschberg[45].

Various factors connected with radiation were taken into account, with different degrees of accuracy, in a number of papers. For example, Kuznetsov and Raĭzer[28], in their analysis of recombination, took into account factors that are analogous in the appearance of Π_1, while Abramov and Smirnov[29] actually introduced the analog of $\chi(E_R/T_e)$. The regions of applicability of the results of these papers correspond to high[28] and low[29] temperatures.

The radiation affects α and β quite differently. At low temperatures, while having little effect on the RC, the radiation can radically change the IC: $\beta = \beta_2/\chi(E_R/T_e)\,\Pi_1$. This result is qualitatively understandable, since the electron, before it is released, must negotiate the energy interval $E_1 - E_R$. On this path there is an appreciable probability of returning to the ground state. Only a fraction of electrons $\sim [\chi(E_R/T_e)\,\Pi_1]^{-1}$ reaches the energy level E_R.

By way of example, Fig. 11 shows, for an argon-potassium plasma, a plot of Π_1 against the parameter $R\xi$, where R is the linear dimension of the plasma and

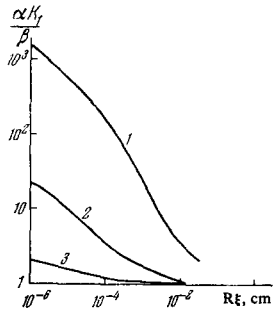

FIG. 11. Dependence of Π_1 on the parameter $R\xi$ for the mixture Ar + K.

ξ is the ratio of the number of potassium atoms to the number of argon atoms. Curves 1–3 were constructed for different $n_e = 10^{12}$, 10^{13} and 10^{14} cm^{-3}, respectively. It follows from Fig. 11 that Π_1 can reach large values, especially at small n_e.

Figure 10 shows plots of the IC β for a potassium plasma. Curves 5–7 were constructed in accordance with formula (10) for $n_e = 10^{14}$, 10^{13}, and 10^{12} cm^{-3} respectively ($R\xi = 10^{-4}$). At large n_e ($n_e \gtrsim 10^{14}$ cm^{-3}), the radiation is negligible and we return to formula (5.3). At smaller n_e, the role of the radiative transitions is large. It is curious to note that if in addition there is also a strong deviation from the Maxwellian distribution, then $\beta \sim n_e^2$ or even $\sim n_e^3$, which can greatly stretch out the initial stage of ionization development[39].

Figure 12 shows a plot of the RC α against T_e for an argon plasma at different degrees of influence of the radiative transitions (formula (5.11)). For the solid curves the value of Π_1 is indicated in the figure, and $\chi(E_R/T_e) = 1$. For the dashed lines, the figure shows both the values of Π_1 and the values of E_R (in E_V). The thick line corresponds to $\alpha \sim T_e^{-9/2}$. We note that Π_1 depends mainly on n_1 and on the optical depth k_0R, whereas $\chi(E_R/T_e)$ depends to a larger degree on T_e and comes into play at much smaller n_e.

Thus, if we decrease n_e or R, then Π_1 increases, and a curious situation arises. The first them in (5.11) vanishes, and we return at high temperatures to the $\alpha \sim T_e^{-9/2}$ law. The specific features of the atom drop out. This is now the consequence of the strong radiation of the lower excited states (the "bottleneck" shifts towards the region of the hydrogen-like levels). At sufficiently low n_e, the factor $\chi(E_R/T_e)$ comes into play at low temperatures. The RC begin to exceed the values given by the formula $\alpha \sim T_e^{-9/2}$. Then the result is again independent of the specifics of the atom. In the case of very strong de-excitation, when $\Pi_1 \gg 1$ and $\chi(E_R/T_e) \ll 1$ simultaneously, the specifics of the atom come into play again at high temperatures. Then the RC also exceed the values given by the formula $\alpha \sim T_e^{-9/2}$. Such a dependence of the RC on the factors describing the radiation is typical of a plasma made up of many elements.

A similar analysis can be carried out for β. In the case of development of ionization as a result of strong radiation, the approximation of "prompt ionization" for β_1 may no longer be valid at very high temperatures. Formulas (5.10) and (5.11) determine the limits of applicability of the previously obtained expressions and makes possible estimates and calculations in the intermediate cases.

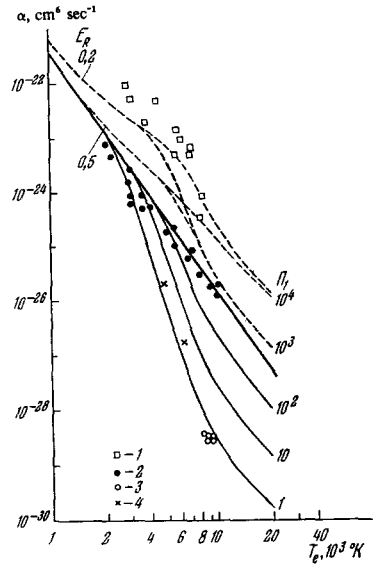

FIG. 12. Dependence of the coefficient of impact-radiative recombination for Ar on T_e (formula (5.11)).

With the aid of (5.10) and (5.11) it is also easy to find the conditions under which the processes of direct ionization or recombination come to the forefront. The direct ionization can become noticeable at very high temperatures, when there is no longer a difference between the exponential $\exp[-(E_1 - E_2)T_e]$, which is contained in β_1, and $\exp[-E_1/T_e]$ to which the direct-ionization coefficient β_d is proportional. However, if the radiation in the lines ($\Pi_1 \gg 1$) is very intense and the continuum is still reabsorbed, then β_d can become appreciable also at noticeably lower temperatures. This obviously follows from the expression for the total IC: $\beta = [\beta_1^{-1} + \Pi_1 \chi(E_R/T_e)\beta_2^{-1}]^{-1} + \beta_d \to \beta_d$, $\Pi_1 \gg 1$.

In concluding this section, we call attention to the fact that, strictly speaking, the foregoing analysis of the influence of radiation pertains to the kinetics in the central regions of the plasma. In boundary layers or in other regions where the gradients of the parameters are large, the expressions for θ, given in Sec. b of Chap. 2, no longer hold. In this case it is necessary to use for θ more complicated formulas[47] or to use approximate methods for solving the transport equation for the radiation in the plasma. The character of the influence of the radiation on the kinetics can become entirely different. "Illumination" with radiation from "hotter" regions of the plasma can accelerate the ionization or slow down the recombination, e.g., during the first stage of ionization relaxation behind a shock wave[48], at the entrance into the channel of an MHD generator[49], or for passage of a shock wave in T-tubes[50].

d) Influence of collisions with atoms on the ionization and recombination coefficients. The three-particle recombination coefficient in ion-atom-electron collisions was obtained in the diffusion approximation by Pitaevskiĭ[4]:

$$\alpha = [32\,(2\pi)^{1/2}/3]\, m^{1/2} M^{-1} e^6 \sigma_{ae} T^{-5/2} n_1/n_e, \qquad (5.12)$$

σ_{ae} is the cross section for elastic scattering of an electron by an atom, M is the mass of the atom. This

result was later obtained by a different method by Bates and Khare[51]. In[52], formula (5.12) was extended to include the case of a non-isothermal plasma, when $T_a \neq T_e$:

$$\alpha = [32\,(2\pi)^{1/2}/3]m^{1/2}e^6\sigma_{ae}(T_a/MT_e^{7/2})n_1/n_e. \qquad (5.13)$$

As already noted, within the framework of the MDA, collisions with heavy particles can be taken into account and included in the general scheme. In the limiting case of an optically dense and weakly-ionized gas, expressions analogous to (5.1) are[8a,c]

$$\alpha = [\sum_k (K_k^a n n_e/(z_{k,\,k+1}^a))]^{-1}, \qquad \beta = K_1^a \alpha n. \qquad (5.14)$$

In the low-temperature limit, when the atomic term scheme can be regarded as quasicontinuous, we obtain (5.12) from (5.14) after specifying the concrete $z^a_{k,\,k+1}$. Atom-atom collisions can be important also during the first stage of ionization development at high atomic temperatures. In fact, if $T_e = T_a$ then, inasmuch as the cross sections for the excitation and ionization of atom-atom collisions are of the order of 10^{-20}–10^{-19} cm^2, these processes are important at degrees of ionization $\lesssim 10^{-5}$–10^{-7}. If $T_a > T_e$, then the atom-atom collisions are important also at higher degrees of ionization.

We present below an expression for β, in which allowance is made not only for collisions with electrons but also for atom-atom and radiative transitions $1 \rightleftarrows 2$:

$$\left.\begin{aligned} \beta &= (\beta_1\xi_1)^{-1} + (\beta_2\xi_2)^{-1}, \\ \xi_1 &= (n_e z_{12} + n_a\, z_{12}^a)/n_e z_{12}; \\ \xi_2 &= (n_e n_1 z_{12} + n_a^2 z_{12}^a)\,(n_e z_{21} + A_{21}^* + n_a\, z_{21}^a)^{-1}[n_e^0\,(T_e)]. \end{aligned}\right\} \qquad (5.15)$$

As before, the first term in (5.15) corresponds to the limit of "prompt ionization," and ξ_1 reflects the growth in the number of excitation acts as a result of atom-atom collisions. The second term in (5.15) characterizes the rate of passage of the electron over the excited states. ξ_2 is approximately the ratio of the population of the second level n_2 when account is taken of atom-atom collisions and radiation, to the Boltzmann population $n_2^0(T_e)$ at the temperature T_e. By the same token, ξ_2 takes into account the influence of these processes on the rate of stepwise ionization.

6. COMPARISON OF EXPERIMENTAL AND CALCULATED RECOMBINATION AND IONIZATION COEFFICIENTS

We will recall the equations for the ionization kinetics:

$$dn_e/dt = n_1 n_e\beta - n_e^3\alpha + \nabla \mathbf{J}_e. \qquad (6.1)$$

(6.1) describes the rate of change of n_e as a result of generation in the volume and transport in coordinate space. Stationary and nonstationary conditions are possible.

Non-equilibrium stationary conditions. In a number of experiments, non-equilibrium stationary conditions $dn_e/dt = 0$ are realized. The generation of the electrons inside the volume is offset by their influx (outflow) resulting from transport in space. To determine $\nabla \cdot \mathbf{J}_e$ it is necessary, generally speaking, to solve the problem of the spatial distribution of the plasma parameters with concrete conditions on the boundaries. In a number of cases, however, it is possible to use also simpler estimates. For example, if the transport in determined by ambipolar diffusion, then for the central regions of the plasma we have $\nabla \cdot \mathbf{J}_e \approx -n_e/\tau_D$, where $\tau_D = R^2/pD_a$ is the characteristic diffusion time, D_a is the coefficient of ambipolar diffusion, R is the linear dimension of the system, and p is a numerical coefficient that depends on the geometry of the volume.

A detailed solution of Eq. (6.1) for non-equilibrium stationary conditions is given in the last paper of[8a], where a number of relations that determine n_e in a non-equilibrium plasma was obtained (they are sometimes called the generalized Saha equations). For example, if the only cause of the non-equilibrium distribution is diffusion of the charged particles from the central parts of the volume, then this equation takes the form

$$n_e^2 + K_1 n_e - K_1 n\,[1 - (\beta\tau_D n)^{-1}] = 0. \qquad (6.2)$$

Equation (6.2) is valid of $\beta\tau_D n \gtrsim 1$ (on the other hand, if $\beta\tau_D n \gg 1$, then Eq. (6.2) yields a small correction to the Saha formula). The experimental conditions under which (6.2) is valid were realized in[41]. When other factors of the non-equilibrium conditions are taken into account, the equation for n_a becomes more complicated. Thus, if the radiative transitions between the ground and first-excited levels plays an appreciable role, with account taken of the non-equilibrium distribution of the free electrons, n_e is given by a fourth-degree equation.

Table III, which is taken from[8a], gives the experimental and calculated values of $\varphi_e = n_e/n$ under stationary conditions.

In[41], the main cause of the deviation of the distribution from equilibrium was the diffusion towards the boundaries of the arc, whereas in[43] the causes were diffusion and radiation, while in[42,53] there was an appreciable deviation from a Maxwellian distribution of the electron energy. As seen from Table III, in all cases the agreement between the experimental and calculated values is quite satisfactory. We note that if n_e is calculated under the conditions of[42,53] neglecting the violation of the Maxwellian distribution, then the calculated values exceed the experimental ones by several orders of magnitude.

b) Nonstationary conditions. If the term $\nabla \cdot \mathbf{J}_e$ in (6.1) is small, and either ionization ($y_1 \gg y_e^2$) or recombination ($y_e^2 \gg y_1$) predominates, then α and β can be determined by measuring the time dependence of $n_e(t)$. On the other hand, if both the ionization and the recombination term are significant in (6.1), then α and β can not be determined directly from the dn_e/dt dependence. However, when one of the coefficients is known, the other can be determined.

In[45,54] they investigated the decay of H and He plasma obtained with installations of the stellarator type. The quantity $n_1 n_e\beta$ was negligibly small under the conditions of these experiments, so that the RC was determined directly from the decrease of dn_e/dt. T_e was determined by measuring the relative population of the highly-excited levels, and n_e was determined by the method of the phase shift of a microwave beam. The results are in good agreement with theory. Some of the many experimental points are shown in Fig. 9, where 3 corresponds to measurement[45,54] for hydrogen and 4 for helium. They fall in the low-temperature region

Table III

Reference	Composition	n, cm^{-3}	n_e, cm^{-3}	T_e, 10^3 °K	R, cm	φ_e, experiment	φ_e, calculation	φ_e, Saha equation
41	H+Ar	$2{,}2^{18}$	2^{15}	9,3	0,15	$0{,}9^{-3}$	$1{,}3^{-3}$	$8{,}0^{-3}$
43	Cs+Ar	$1{,}6^{13}$	5^{12}	3,1	0,8	0,31	0,45	0,9
43	Cs+Ar	$4{,}3^{12}$	2^{12}	3,9	1,0	0,46	0,53	1,0
42	Ar	$6{,}0^{16}$	$2{,}7^{11}$	25	0,6	$4{,}5^{-6}$	$4{,}5^{-6}$	1,0
42	Ar	$6{,}0^{16}$	$4{,}6^{12}$	13	0,6	$7{,}8^{-5}$	$8{,}7^{-5}$	$4{,}7^{-1}$
53	He	$6{,}7^{17}$	4^{11}	66	0,6	$6{,}0^{-6}$	$1{,}7^{-5}$	1,0

A × 10^r is designated A^r.

$T_e \lesssim 3000°K$. The experiment was performed at low pressures (p ~ 1 mm Hg) and the lines corresponding to transitions to the ground state are weakly reabsorbed, $\Pi_1 > 1$. This leads to a strongly unbalanced population of the lower excited states of the atom (see Fig. 6, which was discussed above), but does not influence the recombination rate, which is determined by the time required to pass through the highly-excited states. Under these conditions we have $\chi(E_R/T_e) \approx 1$, and therefore $\alpha \sim T_e^{-9/2}$ (see (5.8)).

Approximately at the same n_e and somewhat larger T_e, Chen[55] performed measurements under conditions in which the plasma was produced by discharging a capacitor bank through a low-pressure chamber. The "bottleneck" determined by the collision with the electrons shifted towards lower levels, where radiative transitions are important. As a result of the latter, the size of the "bottleneck" was reduced ($\chi(E_R/T_e) < 1$) and the rate of recombination increased and exceeded the values given by the relation $\alpha \sim T_e^{-9/2}$. Figure 12 (points 1) shows a group of points from[55], pertaining to different n_e, and the theoretical values of α with $E_R = 0.2$ and 0.5 eV and $\Pi_1 \approx 10^3$—10^4, which corresponds approximately to the experimental conditions. We see that there is reasonable agreement.

Plasma with higher T_e (up to 10^4° K) and with larger n_e ($n_e > 10^{15}$ cm^{-3}) was obtained by Funahashi and Takeda[56] in a shock T-tube at an initial gas pressure p ~ 10 μ Hg. The plasma decay began after the initiating discharge was turned off and the shock wave was reflected. The results obtained in hydrogen[56b] agree fairly well with the calculated values from[1]. The measurements performed in argon correspond to $n_e = 10^{13}$—10^{15} cm^{-3}. The authors of[56a] have noted that the measured values of the RC can be fitted well to a plot of $T_e^{-9/2}$ (see the group of points 2 in Fig. 12). This result is directly connected with the discussion given in Sec. c of Chap. 4. Owing to the intense transitions between the ground and excited states (as in[45], we have here $\Pi_1 \gg 1$, but $\chi(E_r/T_e) \approx 1$), the "bottleneck" shifts towards the higher levels. As a result, the individuality of the plasma is lost. The RC is practically independent of the composition of the plasma and is described by the "9/2 law." Under the conditions of of[56a] $\Pi_1 = 10$—10^2 and $\chi(E_R/T_e) = 1$.

Let us discuss the experiments performed at the highest temperatures and densities of the electrons. In these cases, the radiation does not influence the IC and the RC. Generalov et al.[57] measured the growth rate of cascade ionization behind a strong shock wave in xenon. The measurements of the absorption and transmission of microwave radiation gave the values of n_e and T_e ($n_e \sim 10^{16}$—10^{17} cm^{-3}); n_1 was known from the conditions behind the shock. The obtained value of β was used to calculate α. The results of[57] are in good agreement with the MTA (see the group of points 3 on Fig. 12)[6].

The values of α in hydrogen at high T_e and n_e were obtained by Craggs et al.[58]. The plasma was produced by a strong-current spark discharge. The pure decay regime was investigated. The results are shown in Fig. 9 (group of points 5). It is seen that they are in full agreement with our calculation.

Several investigations were devoted to the measurement of the RC in alkali-metal vapor[21,59-61]. Since the "low-temperature" region for alkali metal is the broadest, all the measurements lie in this region. At small n_e, however, radiative transitions come into play ($\chi(E_R/T_e) < 1$). On the whole, the theory describes satisfactorily the experiment. The solid lines 1—3 in Fig. 13 are plots of formula (5.11) for different values of the electron concentration, 10^{14}, 10^{13}, and 10^{12} cm^{-3}, respectively. The figure shows also the experimental data of[59] (4),[61] (5), and[21] (6). The figures on Fig. 3 next to certain experimental points correspond to the values of n_e measured under these conditions (in units of 10^{13} cm^{-3}). We note that the theoretical data given by the MDA are in fair agreement with the numerical results of Norcross and Stone[33], which are shown dashed, and also with the results of[68].

In[62] they investigated a potassium plasma with increasing ionization under the influence of a field pulse. The ionization coefficient β was measured. It turned out that these quantities contradict strongly the values of α obtained in a decaying plasma[21,59-61,63]. The cause of the discrepancy remained unclear.

Desai and Corcoran[64] and Aleksandrov et al.[65] measured the RC in an argon plasma at atmospheric

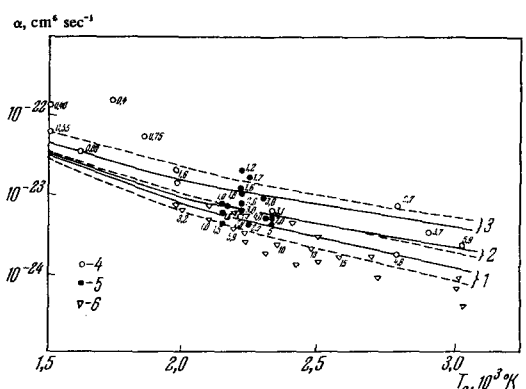

FIG. 13. Recombination functions in K and Cs plasmas.

[6] The values of α under these conditions were quite close for Ar and Xe. This is due to the closeness of the values of E_2 and the excitation cross sections for the transition 1 → 2.

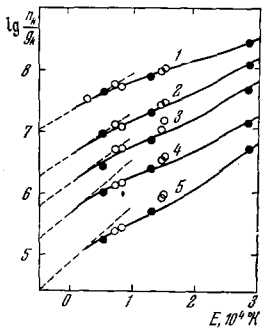

FIG. 14. Populations of the excited levels of argon vs the binding energy for different coordinates along the tube axis: 1–4.3, 2–6.2, 3–8.1, 4–10, and 5–12 cm.

pressure. However, as shown in[66], the values of α obtained by them do not agree with the directly measured quantities. This was the consequence of an incorrect determination of T_e from the spectral-line intensities[7]. The authors of[64,65] determined T_e from the slope of the straight-line plot of the function $\ln(n_n/g_n) \approx f(T_e)$. Such a procedure is legitimate only for levels that are in relative equilibrium with the continuum. In the general case this method yields a certain distribution temperature $T_{dist} \neq T_e$. In the case of ionization $T_{dist} < T_e$, and in the case of recombination $T_{dist} > T_e$. The appendix gives the relations obtained on the basis of the MDA, which make it possible to determine n_e and T_e under non-equilibrium conditions from the populations of three excited states.

Figure 14 shows the distribution of the atoms over the excited states, obtained in[64]. The dark circles correspond to the core configurations $3p^5 (^2P^0_{1/2})$, and the light ones to $3p^5 (^2P^0_{3/2})$, while the solid line is the calculated curve[66]. The slope of the dashed straight line corresponds to T_e. We see that the levels whose populations were measured are not in relative equilibrium with the continuum, and the line formally drawn through the experimental points does not correspond to T_e. After correcting T_e, the values of the RC agree satisfactorily with MDA calculations ($\Pi_1 > 1$ under the conditions of[64]). The RC values obtained in[66] by reducing the measurements of[65] are shown in Fig. 12 (group of points 4).

7. CONCLUSION

The theory of impact-radiation ionization and recombination, which is based on the idea that this process is a random wandering of the electron over discrete energy levels, has made it possible to obtain simple analytic expressions for the corresponding coefficients. We took into account here the real energy structure of the atoms and the relation between the non-equilibrium distribution of the atoms over the levels and of the electrons over the energies. The obtained expressions are in satisfactory agreement for the experimental data pertaining to a great variety of conditions.

Among the ionization (recombination) mechanisms and the low-temperature plasma, the impact-radiation ionization (recombination) has apparently been investigated to the greatest extent. However, in a molecular plasma or in an atomic plasma containing molecular ions, an important role is played also by the mechanism of dissociative recombination and associative ionization. It has been established by now that the products of the dissociative recombination are predominantly excited atoms, while associative ionization can proceed from excited states: $A_2^+ + e \rightleftarrows A^* + A$.

The presence of excited atoms in the mechanism of the process is evidence that associative ionization and dissociative recombination must be considered in connection with the kinetics of the atomic excited states. If, e.g., the dissociative recombination is accompanied by the appearance of atoms on excited levels, lying above the "bottleneck" in the atomic spectrum, then the resultant recombination rate is determined by the rate of impact-radiation recombination. On the other hand, if these excited levels are below the "bottleneck," then it is precisely dissociative recombination which determines the rate of vanishing of the charge particles. The construction of some general scheme is made difficult at present by the lack of data on the probabilities of the elementary processes.

APPENDIX

The determination of n_e and T_e under non-equilibrium conditions from the absolute intensity of the spectral lines can be based on the following relations[66]. We write down the equation connecting the concentration at three arbitrary levels m, k, and l (in increasing order of binding energy), which follows from (4.22)

$$(y_k/\Pi_k)(x_m - x_l) = (y_m/\Pi_m)(x_k - x_l) + (y_l/\Pi_l)(x_m - x_k) + n_e^2[(X_m - X_k)(x_k - x_l) + (X_k - X_l)(x_m - x_l)], \quad (A.1)$$

where

$$x_m = \sum_{n \geq m} (z_{n,n+1} n_n^0 \Pi_n)^{-1}, \quad \text{and} \quad X_m = \sum_{n \geq m} \alpha_n^e (z_{n+n+1} n_n^0 \Pi_n)^{-1}. \quad (A.2)$$

If the inequality ($E_R > E_l$) is satisfied and the kinetics of the transitions between the level m, k, and l is determined only by the collisions, then (A.1) can be simplified by assuming all $\Pi_k = 1$ and $\alpha_k^e = 0$. If in addition $E_m, E_k, E_l \leq T_e$, then one can replace all the sums in (A.2), without a substantial error, by integrals, after which we obtain

$$(n_k/g_k) e^{u_k} [\chi(u_m) - \chi(u_l)] = (n_m e^{u_m}/g_m)[\chi(u_n) - \chi(u_l)] + (n_l e^{u_l}/g_l)[\chi(u_m) - \chi(u_k)], \quad (A.3)$$

where $u_m = E_m/T_e$, and $\chi(u)$ is determined by the relation (5.1).

Making in (A.3) the substitution of $n_l e^{u_l} g_l \rightarrow n_e^2 h^3/2\Sigma_i (2\pi m T_e)^{3/2}$, we connect n_k and n_m with n_e^2:

$$(n_k/g_k) e^{u_k} \chi(u_m) = (n_m e^{u_m}/g_m)\chi(u_k) + [n_e^2 h^3/2\Sigma_i (2\pi m T_e)^{3/2}][\chi(u_m) - \chi(u_k)]. \quad (A.4)$$

Relations (A.3) and (A.4) makes it possible to determine n_e and T_e in a nonequilibrium plasma from the populations of three excited states. We note that in the case of equilibrium they are satisfied identically.

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Translated by J. G. Adashko