# Application of scale invariance to elementary particle physics

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Roman Jackiw (Physics Today 25(1), 23, 1972) discussed in a simple and attractive form one of the most interesting phenomena in elementary particle physics, the property of scale invariance, which is discussed in connection with the results of a study of deep inelastic lepton-hadron interaction. During the year and a half elapsed since the writing of this article, new important results were obtained in the study and application of scale invariance, and it is appropriate to discuss them here briefly. I also want to acquaint the reader with some of the principal studies made by Soviet physicists, who have made an appreciable contribution to this field.

The idea of scale invariance is not a new one in physics. Contributions to the development of this idea, which is connected with the idea of the independence of the laws of nature on the choice of the measurement units, were made by such classical scientists as Newton, Fourier, Rayleigh, and others. A rigorous mathematical formulation of this invariance was made in the 1950s, as applied to problems in gasdynamics and hydrodynamics, for example the problem of a strong "point" explosion, by Sedov, Landau, and Stanyukovich[1] and was named the self-similarity principle. In problems of this type, in regions where the values of the kinematic variables (e.g., the coordinates) differ strongly in their scale from the dimensional parameters, the solution ceases to depend on these parameters. Then, under transformation of the dimensions, i.e., when quantities with the dimension of length are enlarged, it suffices to transform only the coordinates. Such transformations have a geometrical meaning and are called scale transformations. This procedure, which is quite general and serves of the basis of the so-called mathematical simulation, makes possible not only hydro- and gasdynamic phenomena that range in scale from atomic-bomb explosions to the breakdown of the medium at the focus of a laser beam, but also phenomena in a great variety of fields of physics. Mention can be made, for example, of the evolution of a star with a nuclear energy source, in which hydrodynamic equations and the radiation laws are used[2], or Kolmogorov's model of turbulent motion[3].

The latest applications of the principle of scale invariance arose in high-energy physics in connection with the study of processes of deep-inelastic interactions of leptons with hadrons, and the multiple production of particles in hadron-hadron collisions at high energies.

Back in 1963, M. A. Markov[4a] pointed out the possibility of a "pointlike" behavior of the cross sections of inelastic neutrino-nucleon interaction, a behavior connected with the vanishing of the parameters that characterize the dimensions of the nucleon when summation is carried out over all the open channels of the reaction. The possibility of investigating deep inelastic processes was noted, following Markov, by a number of Soviet and foreign scientists. Indeed, the results obtained at SLAC for the reaction[5] $e^- + p \to e^- +$ hadrons at an approximate incident-electron energy 20 GeV, and then at CERN for the reaction[6] $\nu_\mu + N \to \mu^- +$ hadrons, have confirmed the "pointlike" picture of the interaction. By the same token, they have revealed a new phenomenon, the scale-invariant behavior of the form factors of the indicated processes.

Starting with 1967, A. A. Logunov and co-workers[7a] have investigated, within the framework of quantum field theory, the processes of deep-inelastic hadron interaction in those cases when one registers at the end of the reaction one, two, etc. particles. These processes were subsequently named inclusive.

The experimental study of inclusive processes[7b] with one singled-out type of particles, $p + p \to \pi$, K + hadrons, carried out with the Serpukhov accelerator at incident proton energies up to 70 GeV, has also revealed scaling properties typical of the dynamics of strong interactions.

The importance of the role of scale transformations in deep inelastic processes was pointed out by N. N. Bogolyubov, who indicated a possible analogy between the process of deep inelastic electroproduction and the dynamics of a "point" explosion. Starting with this analogy, Matveev, Muradyan, and Tavkhelidze[8a] formulated the principle of approximate self-similarity, whereby it is assumed that scale invariance is possessed by all deep inelastic lepton-hadron interaction processes. In this approach, using dimensional-analysis and similarity methods, investigations of the asymptotic behavior of various deep inelastic interaction processes were made in a unified manner, and a number of sum rules were established. The results obtained in this scheme[8b] describe qualitatively correctly the experimental data of Lederman's group[8c] on muon-pair production in hadron-hadron collisions at high momentum transfers, $p + U \to \mu^+\mu^- +$ hadrons ($E_p = 30$ GeV).

The self-similarity principle was subsequently generalized for the case of pure hadron-hadron collisions, by starting from the analogy with a "planar" explosion in hydrodynamics[9]. The colliding hadron is regarded, in the limit of extremely high energies, as an extended object that is oblated by the Lorentz contraction into an infinitesimally thin disk with finite transverse dimensions. The physical quantities observable in hadron-hadron collisions at high energies are characterized by definite scaling properties when the scale of the longitudinal momenta $p_z$ is dilated along the collision axis z, namely, $p_z \to \lambda p_z$, $p_\perp \to p_\perp$, $p_0 \to \lambda p_0$, $(m^2 + p_\perp^2)/2p_z^2 \ll 1$, where $p_0$ is the energy and $p_\perp$ is the transverse momentum of the secondary particle, bounded by the experimentally known value ~0.4 GeV/c.

A similar lucid physical picture of the high-energy hadron interaction was developed by many authors, starting with Heisenberg. It was used by Yang and co-workers[10] and by Feynman[11] to construct models of multiple particle production ("limiting fragmentation" and the parton model, respectively). The self-similarity hypothesis and dimensional analysis with two length scales $p_\perp$ and $p_z$ make it possible to obtain[9] by a model-independent method the main predictions in the approaches of Yang and Feynman (e.g., the constancy of the total cross sections, the regularities of the single-particle and two-particle distributions of the secondary particles, etc.).

Ideas concerning the applicability of scale invariance to collisions of composite systems were advanced by Baldin[12a]. For a concrete process, namely the collision of relativistic nuclei, he predicted the effect of the transfer of energy from a group of nucleons to one secondary particle, called the cumulative effect. This effect, greatly exceeding the effect of a pulsed distribution of the nucleons in the nucleus, was observed experimentally[12] with the Dubna proton synchrotron in the reaction $d + Cu \rightarrow \pi^- + \ldots$ ($E_{\pi^-}^{kin} > E_d^{kin}/2 \approx 4$ GeV).

It should be noted that the regularities exhibited by inclusive reactions appear also in the study of multiple production processes in cosmic rays. Thus, scale invariance of the high-energy part of the spectrum of the secondary ("leading") particles has been known since the early 1950s, since it had been established that the inelasticity coefficient ($K = \langle E\rangle/\langle E_0\rangle$, where E is the energy carried away by the secondary particles) and the excess of positive muons in the energy interval $E_0 \sim 10-10^4$ GeV are constant. Those wishing to become aquainted with this field are advised to read Feinberg's review[13a] and also his review[13b], in which more detailed models of multiple particle production are given. Among the works devoted to the theoretical interpretation of scale-invariant and self-similar behavior of deep inelastic processes, mention should be made of papers[14] in which, speaking picturesquely, "physics on the light cone" is developed.

As noted by Ioffe et al.[14], assuming an asymptotic regime, these processes enable us to measure directly the commutator of two electromagnetic currents that are specified in the vicinity of the light cone in configuration space. This is of great interest from the theoretical point of view, since in the Lagrangian models of field theory the local operators are most singular just on the light cone, and the character of their behavior on the cone explains one of the most important principal questions, namely whether the commutation relations are altered in the theory of interacting fields, or can they be written down in analogy with the theory of free fields.

In recent years, a new trend has developed in the study of the properties of scale invariance and self-similar behavior of deep inelastic processes of leptons with hadrons, based on the use of only the most general principles of local quantum field theory[15]. A rigorous formulation of this trend on the basis of the axiomatic approach was given in the papers of Bogolyubov, Vladimirov, and Tavkhelidze[16]. They have developed methods that make it possible to use effectively such a fundamental requirement as the causality principle and to prove rigorously on its basis the existence of self-similar asymptotic forms. The results of these papers yielded a consistent verification of the connection between the self-similar behavior of the form factors of deep inelastic processes and the character of the singularities of the commutator of the local currents on the light cone. The results of these researches were subsequently developed in a number of later papers[17].

Logunov and co-workers[17] have recently also continued the study of the laws governing multiple production of particles in strong interactions. Thus, in[18a], starting from the general requirements of unitarity and analyticity, they investigated the role of "soft" particles produced in collisions between high-energy hadrons, and established definite scaling relations of the self-similar type. A "statistical" method of investigating inclusive spectra was developed in[18b]. The progress made in this direction is reviewed in[18c].

We proceed now to the connection between scale invariance of quantum field-theoretical models and their renormalizability properties. In scale-invariant models one postulates the existence of a set of unitary transformations of the type $U(\rho)\,\Phi(x)U^{-1}(\rho) = \rho^d\,\Phi(\rho x)$. The constant d is called in this case the scale dimension of the field $\Phi(x)$ and is obtained by commuting the infinitesimal dilatation operator D (introduced in Jackiw's article) with the operator $\Phi(x)$. Wilson has noted[19] that the scale dimension of the renormalized field $\Phi_R(x)$, calculated approximately by summing the perturbation-theory graphs or obtained in an exactly solvable model of quantum field theory, can become anomalous, $d = d_0 + f(g)$, where $d_0$ is the usual or canonical dimension and f is a function of the coupling constant g. A distinction should be made between the dimension concept introduced in this manner and the concept of the usual dimension. Usual scale dimension shows how the field varies if all the quantities with dimension of length are dilated, i.e., the coordinates, the Compton lengths of the particles, the momenta etc., and thus coincides with the physical scale dimension of the field. Anomalous dimension characterizes by variation of the field when only the coordinates (or momenta) are dilated. In many cases, *the field acquires upon renormalization a dimensional* factor $m^{f(g)}$ that remains unchanged under the coordinate dilatation considered above, and therefore the *anomalous dimension differs from the usual one* by a certain increment f(g).

We recall that phenomena of the type of anomalous dimensions have been in fact known for a long time. It was observed in the investigation of the infrared behavior of the matrix elements in quantum electrodynamics[20], in the determination of the exact solutions of the Thirring model (see[21]), in an analysis of the renormalization invariance of the theory of interacting fields[22a]. The latter question was analyzed in detail by Bogolyubov, Shirkov, and Ginzburg[22b] with the aid of the general method of the renormalization group.

In a recent paper by Shirkov[23], the idea of the possible relation between the scale-invariant character of the ultraviolet asymptotic equations in quantum field theory and the hypothesis of finite charge renormalization was based on an analysis of the equations of the renormalization group. Anomalous dimensionalities were investigated also by Gribov and Migdal[24] in the reggeon approach in connection with the problem of the self-consistency of the Pomeranchuk pole, and by Polyakov[25a] and Migdal[25b] as applied to phase transitions in various statistical systems.

Several papers dealt with the character of the asymptotic behavior of the form factors in the deep inelastic region by starting from an analysis of perturbation-theory diagrams[26]. Gribov and Lipatov[26b] have shown that in the logarithmic approximation, i.e., when $g^2 \ln(q^2/M^2) \sim 1$ (where g is the coupling constant and $g^2 \ll 1$), the summation of the perturbation-theory leads to a deviation from the Bjorken behavior of the structure functions. The noted behavior corresponds to the "zero charge" situation. The opposite case was considered by Efremov and Ginzburg[26c], who concluded, within the framework of the assumption of finite charge renormalization, that a self-similar asymptotic behavior of the amplitudes determined by the entire aggregate of perturbation-theory diagrams is possible. They have subsequently[26d] developed a general method based on the analysis and summation of perturbation-theory diagrams in the high-energy region, and obtained a number of qualitative predictions of the hypothesis of scale invariance for the hadron-scattering processes, multiple production, and deep inelastic scattering.

We note further that scale invariance is not the only symmetry that can appear at high energies. In addition to the chiral theories mentioned by Jackiw, we note theories that are invariant to the group of conformal transformations[27] (15-parameter group of space-time transformation, isomorphic to the group O(4, 2)).

Conformal symmetry may turn out to be an approximate theory in nature, valid in the limit of large momenta, when the rest masses of the particles can become negligible. This circumstance was pointed out by Wigner[28] and related by him to the exact conformal invariance of Maxwell's equations in vacuum (describing particles with zero rest mass), which was observed already at the beginning of the century[29].

Let us make a few remarks concerning studies of conformally-invariant theories. Apart from the results that follow already from the scale symmetry, considerations of conformal symmetry have not yet yielded significant predictions that could be verified in experiments. Yet from the purely theoretical point of view there is definite progress. We recall that within the framework of the Lagrangian formulations of quantum theory the onset of anomalous dimensions, mentioned by Jackiw, is connected with the infinite renormalization of the field-theory operators. The requirement of invariance to the conformal group of transformations in conjunction with the idea of the anomalous dimensions has made it possible to determine practically uniquely (accurate to one constant) the vertex functions in quantum field theory[30]. (It is curious to note that this result was first obtained by Polyakov[31] not in the theory of elementary particles, but in the field-theoretical exposition of the hypothesis of conformal invariance of fluctuations at a phase-transition point.) To find the higher Green's function, Mak and Todorov[32] formulated a "skeleton" diagram technique, which contains no ultraviolet divergences.

It would be difficult to describe in a short article the status of this vigorous development of elementary-particle physics. More extensive and specialized information concerning the development of conformally- and scale-invariant theories can be found, for example, in the Proceedings of the International Conference on Mathematical Problems of Quantum Field Theory and Quantum Statistics (Moscow, Inst. of Appl. Math. USSR Acad. Sci., December 1972)[33] and in reviews[34].

We note in conclusion that the hypothesis of scale invariance and conformal symmetry is a fruitful physical idea. As an approximate principle in the physics of elementary particles, scale invariance comes into play in processes when the masses of all particles become insignificant, and the search for conformal symmetry can play a heuristic role in the construction of more highly perfected theories.

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