Hydrodynamics of plasma in a strong high-frequency field

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The article reviews the theory of a number of phenomena occurring in a plasma situated in a strong high-frequency field. The analysis is based on a relatively simply hydrodynamic collisionless plasma model. Principal attention is paid to the stability of the plasma in the strong high-frequency field. A single dispersion equation is used to consider different modifications of parametric instabilities (decay instabilities, parametric resonance, transparent-plasma instability of the stimulated scattering type). Also discussed are a number of problems in the equilibrium and dynamics of a plasma in a strong high-frequency field, particularly nonlinear penetration and reflection of electromagnetic waves from a plasma, stationary nonlinear waves, and spreading and contraction of a plasma layer under the influence of the pressure of a high-frequency field.

CONTENTS

I. INTRODUCTION

It is necessary in a number of experimental investigations of a fully ionized plasma to deal with electromagnetic fields whose pressure exceeds the thermal pressure of the particles. Fields that are so strong (in this sense) can result, on the one hand, from the development of instabilities, and can be produced on the other hand by external sources, as is the case, for example, in the radiative method of plasma acceleration[1]. It is obvious that in this case the character of the processes that occur in the plasma depends essentially on the electromagnetic fields, which in turn vary when the state of the plasma is altered. The traditional approach to the description of a plasma, in which the influence of the fields on its state is assumed to be weak and is taken into account by perturbation theory, turns out to be inapplicable in this case. It becomes necessary to consider simultaneously the change of the state of the plasma and of the electromagnetic fields in the plasma.

In many cases there is one important circumstance that permits appreciable simplification of the analysis of the processes occurring in a plasma situated in strong electromagnetic fields. Namely, at a sufficiently high frequency of variation of the fields one can assume that it is only electrons that execute rapidly-alternating motion, and that this occurs mainly under the influence of the electric field. The motion of the ions, on the other hand, and consequently also the change of the plasma density, is affected only by the time-averaged pressure of this high-frequency (hf) field. Therefore the amplitude of the field, which depends on the plasma density, varies much more slowly than its phase.

This subdivision of motions into rapidly-alternating and slow makes it possible to formulate a relatively simple system of self-consistent equations for the hydrodynamics of a collisionless plasma in an hf field. If the slow motion is quasineutral, this system reduces to three equations: the continuity equation, the equation of motion for a plasma with allowance for the high-frequency pressure, and the equation for a field (or for the hf velocity of the electrons) with slowly varying density. These equations are particular cases of the equations describing the motion of a transparent liquid when account is taken of the pressure of the hf field[2], which are usually formulated within the framework of macroscopic electrodynamics (see, for example,[3-5]) and are used in one form or another in many papers on nonlinear optics[6-8]. However, inasmuch as in a plasma, unlike a liquid or a solid, it is relatively easy to attain hf-field pressures that exceed the internal (thermal) pressure, and since the ensuing changes in the plasma density are far from small, the number of problems that can be considered with the aid of these equations becomes much larger. In particular, it is possible to solve hydrodynamic problems in which the hf pressure is of decisive significance for the processes occurring in the plasma, and the thermal motion of the particles can be neglected.

The purpose of the present review is to acquaint the reader with the equations of plasma hydrodynamics in a strong hf field and with a solution obtained with the aid of these equations for a number of different problems, most of which were considered in the Division of the Theory of Plasma Phenomena of the Physics Institute of the USSR Academy of Sciences.

Accordingly, the review is divided into four parts. In Chap. II we obtain the fundamental equations of the hydrodynamics of a plasma in an hf field and consider a number of cases when these equations can be simplfied. The properties of small perturbations in a plasma through which a strong electromagnetic wave of specified amplitude (pump wave) passes, and in particular the investigation of parametric resonance and decay instability, are the subjects of Chap. III of the review. Chapter IV deals with certain problems of the equilibrium of a plasma in an hf field. Finally, questions of plasma dynamics in hf fields are discussed in Chap. V.

II. FUNDAMENTAL EQUATIONS

In this part we formulate the fundamental equations of hydrodynamic theory of a plasma in a strong hf field

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(Sec. 1) and consider two cases when these equations can be greatly simplified (Secs. 2 and 3).

A procedure analogous to that used by us to derive the equations was considered in[9] in connection with an investigation of the drift instability of a plasma in a microwave discharge, and also in[10] in a determination of the stationary fields and currents in a plasma situated in an hf field. Another method of deriving the equations, based on considering the motion of individual particles, was discussed in[11].

It should be noted that in deriving the equations of the hydrodynamics of a plasma in an hf field we assume that all the quantities that characterize the state of the plasma and of the fields vary little in space over a distance on the order of the average displacement of the electrons in the hf field. This condition imposes constraints, on the one hand, on the characteristic scales of the variation of all the quantities and, on the other hand, on the amplitude and frequency of the hf fields, and is satisfied better the higher the frequency of the hf field.

1. Equations of hydrodynamics of a plasma in a high-frequency field. As a point of departure for the derivation of the equations of the hydrodynamics of a plasma in an hf electromagnetic field, we shall use a system of hydrodynamic equations for electrons and ions and the system of Maxwell's equations for the fields:

$$\partial \mathbf{v}_e/\partial t + (\mathbf{v}_e \nabla)\, \mathbf{v}_e = (e/m)\, \mathbf{E} + (e/mc)\, [\mathbf{v}_e \mathbf{B}] - (T_e/m)\, \nabla \ln n_e, \quad (1.1)^*$$

$$\partial n_e/\partial t + \operatorname{div}\, (n_e \mathbf{v}_e) = 0, \quad (1.2)$$

$$\partial \mathbf{v}_i/\partial t + (\mathbf{v}_i \nabla)\, \mathbf{v}_i = (e_i/m_i)\, \mathbf{E} + (e_i/m_i c)\, [\mathbf{v}_i \mathbf{B}] - (T_i/m_i)\, \nabla \ln n_i, \quad (1.3)$$

$$\partial n_i/\partial t + \operatorname{div}\, (n_i \mathbf{v}_i) = 0, \quad (1.4)$$

$$\operatorname{rot}\, \mathbf{E} = -c^{-1} \partial \mathbf{B}/\partial t, \quad (1.5)$$

$$\operatorname{rot}\, \mathbf{B} = c^{-1} \partial \mathbf{E}/\partial t + (4\pi/c)\, (e n_e \mathbf{v}_e + e_i n_i \mathbf{v}_i), \quad (1.6)$$

where $v_{e,i}$, $n_{e,i}$, and $T_{e,i}$ are respectively the velocities, concentrations, and temperatures of the electrons and ions, e_i and m_i are the charge and mass of the ions, e and m are the charge and mass of the electrons, and **E** and **B** are the vectors of the electric and magnetic field intensities.

We assume that the electric and magnetic fields, and also the concentration and velocity of the electrons, in addition to having a slow dependence on the time, contain also a rapid dependence, so that these quantities can be represented in the form

$$\mathbf{E} = \langle \mathbf{E} \rangle + \widetilde{\mathbf{E}},\ \ \mathbf{B} = \langle \mathbf{B} \rangle + \widetilde{\mathbf{B}},\ \ \mathbf{v}_e = \langle \mathbf{v}_e \rangle + \widetilde{\mathbf{v}}_e,\ \ n_e = \langle n_e \rangle + \widetilde{n}_e, \quad (1.7)$$

where the angle brackets denote averaging over the time:

$$a(t) = (1/t_0) \int_t^{t+t_0} a(t')\, dt'. \quad (1.8)$$

The time interval t_0 is large in comparison with the characteristic time τ of the fast variations and is small in comparison with the characteristic time t_{sl} of variation of the slow variables.

We neglect the rapid motion of the ions, and confine ourselves for simplicity to a plasma in which there are no slow (in particular, constant) magnetic fields ($\langle \mathbf{B} \rangle = 0$)[1)]. In addition, assuming that the collision frequency is small in comparison with t_{sl}^{-1}, we assume that the temperature of the plasma components is constant. Allowance for collisions calls in the general case for considering the variation of the temperatures of the plasma components. The nonlinear effects due to heating of the plasma in an hf field have been considered in a number of papers (e.g.,[13-20]).

We substitute relations (1.7) in (1.2), average the latter with respect to time, and subtract the results from the initial equation. If the slow and rapid quantities vary sufficiently smoothly in space (we denote the characteristic distances over which these quantities vary by L and λ, respectively), so that the following inequalities are satisfied

$$\lambda/\tau,\ L/\tau \gg |\langle \mathbf{v}_e \rangle|,\ |\widetilde{\mathbf{v}}_e|, \quad (1.9)$$

we obtain the continuity equation for the rapidly varying electron concentration:

$$\partial \widetilde{n}_e/\partial t + \operatorname{div}\, (\langle n_e \rangle\, \widetilde{\mathbf{v}}_e) = 0. \quad (1.10)$$

It follows from (1.10) that if inequality (1.9) is satisfied, the rapidly alternating changes of the electron concentration are always small in comparison with the slowly varying concentration ($|\widetilde{n}_e| \ll \langle n_e \rangle$). Using this fact, and also assuming that during the time of variation of the rapid quantities the thermal motion causes the electron to be displaced through a distance that is small in comparison with L or λ,

$$\lambda/\tau,\ L/\tau \gg (T_e/m)^{1/2} \equiv v_{Te}, \quad (1.9')$$

we obtain from (1.1), (1.5) and (1.6), with the aid of (1.7), a system of equations for the rapidly alternating quantities:

$$\partial \widetilde{\mathbf{v}}_e/\partial t = (e/m)\, \widetilde{\mathbf{E}}, \quad (1.11)$$

$$\operatorname{rot} \widetilde{\mathbf{E}} = -c^{-1}\, \partial \widetilde{\mathbf{B}}/\partial t, \quad (1.12)$$

$$\operatorname{rot} \widetilde{\mathbf{B}} = c^{-1}\, \partial \mathbf{E}/\partial t + (4\pi e/c)\, \langle n_e \rangle\, \widetilde{\mathbf{v}}_e. \quad (1.13)$$

From (1.11) and (1.12) it follows that

$$\widetilde{\mathbf{B}} = -(cm/e)\, \operatorname{rot} \widetilde{\mathbf{v}}_e. \quad (1.14)$$

Eliminating the fields in (1.13) with the aid of (1.11) and (1.14), we obtain for the electron oscillation velocity

$$\operatorname{rot}\operatorname{rot} \widetilde{\mathbf{v}}_e + c^{-2}\, \partial^2 \widetilde{\mathbf{v}}_e/\partial t^2 + (4\pi e^2/mc^2)\, \langle n_e \rangle\, \widetilde{\mathbf{v}}_e = 0. \quad (1.15)$$

Thus, by determining $\widetilde{\mathbf{v}}_e$ from (1.15) we can obtain the remaining rapidly oscillating quantities with the aid of (1.10), (1.11), and (1.14). The only slow quantity contained in these equations is the electron concentration.

The equations for the slow quantities are obtained by averaging the initial system of equations (1.1)–(1.6) with respect to time. When condition (1.9) is satisfied, these equations take the form

$$\partial \langle \mathbf{v}_e \rangle/\partial t + (\langle \mathbf{v}_e \rangle \nabla)\, \langle \mathbf{v}_e \rangle = -(e/m)\, \nabla\varphi - (1/2)\, \nabla \langle \widetilde{v}_e^2 \rangle - (T_e/m)\, \nabla \ln \langle n_e \rangle, \quad (1.16)$$

$$\partial \langle n_e \rangle/\partial t + \operatorname{div}\, (\langle n_e \rangle \langle \mathbf{v}_e \rangle + \langle \widetilde{n}_e \widetilde{\mathbf{v}}_e \rangle) = 0, \quad (1.17)$$

$$(\partial \mathbf{v}_i/\partial t) + (\mathbf{v}_i \nabla)\, \mathbf{v}_i = -(e_i/m_i)\, \nabla\varphi - (T_i/m_i)\, \nabla \ln n_i, \quad (1.18)$$

$$\partial n_i/\partial t + \operatorname{div}\, (n_i \mathbf{v}_i) = 0, \quad (1.19)$$

$$\Delta\varphi = -4\pi\, (e\, \langle n_e \rangle + e_i n_i), \quad (1.20)$$

where $\langle \mathbf{E} \rangle = -\nabla\varphi$, $\mathbf{v}_i \equiv \langle \mathbf{v}_i \rangle$, and $n_i \equiv \langle n_i \rangle$. Thus, the influence of the rapidly varying motion on the slow motion is taken into account in terms of the hf pressure in (1.16) and in terms of the drag flow in (1.17). The system (1.16)–(1.20) is given in[21], but with the drag flow neglected and under the assumption that the hf field is external and specified, and also with allowance for the hf motion of the ions. The question of the dragging of the electrons by the hf wave has been discussed in many papers (e.g.,[22-25]).

We are interested in plasma motions in which the quasineutrality condition

$$e\, \langle n_e \rangle + e_i n_i = 0$$

is satisfied and there is no slow current in the plasma:

$$e(\langle n_e\rangle\langle \mathbf{v}_e\rangle+\langle\widetilde{n}_e\widetilde{\mathbf{v}}_e\rangle)+e_i n_i \mathbf{v}_i=0.$$

Under these conditions, the system (1.16)–(1.20) reduces to equations for the average electron concentration $\langle n_e\rangle \equiv N$ and for the ion velocity $\mathbf{v}_i \equiv \mathbf{V}$, which form, jointly with Eq. (1.15) for the rapidly-varying electron velocity $\widetilde{\mathbf{v}}_e \equiv \mathbf{v}$, a closed system of hydrodynamic equations of the plasma in an hf field:

$$\partial N/\partial t+\operatorname{div}(N\mathbf{V})=0, \tag{1.21}$$

$$\partial \mathbf{V}/\partial t+(\mathbf{V}\nabla)\mathbf{V}=-(zm/2m_i)\nabla\langle v^2\rangle-(T/m_i)\nabla\ln N, \tag{1.22}$$

$$\operatorname{rot}\operatorname{rot}\mathbf{v}+c^{-2}\partial^2\mathbf{v}/\partial t^2+(4\pi e^2/mc^2)N\mathbf{v}=0, \tag{1.23}$$

where $T = zT_e + T_i$ and $z|e|$ is the charge of the ion.

It is frequently more expedient to use a system of equations containing not the rapidly varying electron velocity $\mathbf{v}$, but the intensity $\widetilde{\mathbf{E}} \equiv \mathbf{E}$ of the hf electric field. To this end it is necessary to establish, with the aid of (1.11), the connection between $\mathbf{v}$ and $\mathbf{E}$. In the general case this connection is integral with respect to time. However, if the hf field is almost monochromatic:

$$\mathbf{E}(\mathbf{r},\ t)=(1/2)\,[\mathbf{E}_0(\mathbf{r},\ t)\,e^{-i\omega_0 t}+\mathbf{E}_0^*(\mathbf{r},\ t)\,e^{i\omega_0 t}],$$

and if $\mathbf{E} = 0$ when $t = 0$, then we obtain from (1.22) and (1.23), using (1.11) and neglecting terms of order τ/t_{sl},

$$\partial \mathbf{V}/\partial t+(\mathbf{V}\nabla)\mathbf{V}=-(ze^2/4mm_i\omega_0^2)\nabla|\mathbf{E}_0|^2-(T/m_i)\nabla\ln N, \tag{1.24}$$

$$\operatorname{rot}\operatorname{rot}\mathbf{E}_0-(\omega_0^2/c^2)\mathbf{E}_0+(4\pi e^2/mc^2)N\mathbf{E}_0=0. \tag{1.25}$$

The average specific pressure of the hf field for particles

$$\mathbf{f}_E=-(ze^2/4mm_i\omega_0^2)\nabla|\mathbf{E}_0|^2 \tag{1.26}$$

which is contained in the right-hand side of (1.24), can be derived on the one hand from the more general relations of[2], and on the other hand by considering the motion of individual particles in an inhomogeneous hf field[26], as was done first in[27]. Therefore the force of the hf pressure in the plasma is frequently called the "Miller force." A detailed bibliography on this question, and also a generalization of (1.26) to include a magnetized plasma, can be found in the review[28] (see also[29]), and also in later papers[30,31]. The question of the hf pressure forces in a plasma with allowance for particle collisions is discussed in[11,32-34]. The pressure forces of low-frequency ion waves in a plasma layer have been considered in[35,36].

2. High-frequency pressure forces in a transparent plasma. In this section we consider forces acting on a plasma placed in an external hf field whose frequency is much higher than the Langmuir frequency of the electrons

$$\omega_0^2>\omega_{Le}^2=(4\pi e^2/m)\,N. \tag{2.1}$$

Under these conditions, the field in the plasma differs little from the field in vacuum, and can be determined by perturbation theory

$$\mathbf{E}_0=\mathbf{E}_0^{(0)}+\mathbf{E}_0^{(1)},$$

where the zeroth-order field $\mathbf{E}_0^{(0)}$ is simply the external field (i.e., the field in the absence of plasma), which, in accord with (1.25), satisfies the relation

$$[\Delta+(\omega_0^2/c^2)]\,\mathbf{E}_0^{(0)}=0,\quad \operatorname{div}\mathbf{E}_0^{(0)}=0. \tag{2.2}$$

The correction $\mathbf{E}_0^{(1)}$ to the external field is due to the presence of plasma and, as follows from (1.25), can be found from the equation

$$[\Delta+(\omega_0^2/c^2)]\,\mathbf{E}_0^{(1)}=(4\pi e^2/m)\,[c^{-2}N\mathbf{E}_0^{(0)}+(1/\omega_0^2)\nabla(\nabla N\cdot\mathbf{E}_0^{(0)})]. \tag{2.3}$$

In this case, according to (1.26), the hf-field pressure forces acting on the plasma are proportional, accurate to terms linear in $\mathbf{E}_0^{(1)}$, to the quantity

$$\nabla|E_0|^2\approx\nabla|E_0^{(0)}|^2+2\nabla|\mathbf{E}_0^{(0)}\mathbf{E}_0^{(1)}|. \tag{2.4}$$

If the first term in (2.4) is much larger than the second, then the hf-pressure forces acting on the plasma are determined by the external fields. The plasma can then be considered by using Eqs. (1.21) and (1.24), where the hf field is assumed given.

In many cases, however, the first term of (2.4) is much smaller than the second (for example, if $\mathbf{E}_0^{(0)}$ is the field of a plane wave, then the first term is equal to zero), and to find the forces acting on the plasma, it is necessary to know the correction to the external field. The general solution of Eq. (2.3) has been thoroughly investigated (see, for example, the book[37]), and can be used readily to express the hf-pressure forces acting on the plasma in terms of the plasma density and by the same token to reduce the system of hydrodynamic equations to equations for N and $\mathbf{V}$. Such forces are calculated for a number of cases in[38,39].

Let us consider the next example, which we shall need subsequently[40]. Assume that a plane linearly-polarized electromagnetic wave is incident at an angle φ on a plasma layer in which the density depends only on one coordinate x and on the time

$$\mathbf{E}_0^{(0)}=(E_0^{(0)}\cos\varphi,\ E_0^{(0)}\sin\varphi,\ 0)\,e^{ik_{0x}x+ik_{0y}y}. \tag{2.5}$$

From (2.2) we obtain the dispersion law for the electromagnetic waves in vacuum, $\omega_0^2 = k_0^2c^2$, while Eq. (2.3) determines $E_{0x}^{(1)}$ and $E_{0y}^{(1)}$. If the plasma density is different from zero at $L_2 \ge x \ge L_1$, then the solution of (2.3) should describe a plane wave propagating from left to right if $x > L_1$, and, to the contrary, from right to left if $x < L_2$. Using these conditions, we obtain the expressions for the components of the vector $\mathbf{E}_0^{(1)}$ with the aid of formula (2.4)—the per unit force of the hf pressure:

$$f_{E,\,x}=-\frac{2z\pi e^4}{m^2m_i\omega_0^2}|E_0^{(0)}|^2\times\Big[\frac{1}{\omega_0^2}\cos^2\varphi\,\frac{\partial N}{\partial x}+\frac{1}{c^2}(\cos 2\varphi)^2\int\limits_{L_1}^{x}N(x',t)\cos 2k_{0x}(x-x')\,dx'\Big]. \tag{2.6}$$

If the plasma has no sharp boundary and the density falls off continuously to zero as $x \to \infty$, then we can use $L_1 = \infty$ as the lower limit in (2.6).

3. Geometric-optics approximation. An approximate solution of (1.25) or (1.23) can also be obtained if the wavelength of the electromagnetic wave is small in comparison with the characteristic distance over which the plasma density varies ($\lambda \ll L$). In this case, the geometric-optics approximation is valid (see, for example,[41,42]).

We consider for simplicity the one-dimensional case[2)]. We assume that a linearly polarized wave propagates along the 0x axis and that the electric-field intensity vector is directed along the 0y axis; the plasma density depends on x and on the time. We seek the solution of (1.25) in the form

$$\omega_0\gg|\dot\varphi|,\ \ \omega_0E_0\gg\dot E_0,\ \ \omega_0N\gg\dot N, \tag{3.1}$$

where E_0 and φ are real functions (amplitude and phase). In accordance with the assumption made in Sec. 1 we have

$$E_{0y}(x,\ t)=E_0(x,\ t)\,e^{i\varphi(x,t)},$$

where the dot denotes differentiation with respect to time. Substituting (3.1) in (1.25) and recognizing that $|N/N'|$, $|E_0/E_0'| \ll c$ (the prime denotes differentiation with respect to x), we obtain the following equations for the determination of E_0 and φ:

$$E_0'' - \varphi'^2 E_0 + (\omega_0^2/c^2) E_0 = (4\pi e^2/mc^2) N E_0, \quad (3.2)$$

$$E_0^2 \varphi' = -M, \quad (3.3)$$

where M is generally speaking, a function of the time.

Thus, to find the field amplitudes we are left with a single equation that takes the form

$$E_0'' + (\omega_0^2/c^2) E_0 - (M^2/E_0^3) - (4\pi e^2/mc^2) N E_0 = 0. \quad (3.4)$$

So far we have not used the condition under which the geometric optics is valid, according to which the phase φ of the wave changes in space much more rapidly than the amplitude E_0. If this condition is taken into account, then the first term of (3.4) should be discarded because of its smallness[3]. As a result we get

$$E_0^2(x, t) = M/(\omega_0/c)[1 - (4\pi e^2 N(x, t)/m\omega_0^2)]^{1/2}. \quad (3.5)$$

If the plasma contains a region in which the density is constant or equal to zero, then the quantity M in (3.5) can be expressed in terms of the field amplitude in this region. Then the value of M is independent of the time and, in particular, if the hf wave is incident on the plasma from vacuum, we have

$$M = E_0^{(0)2}\omega_0/c, \quad (3.6)$$

where $E_0^{(0)}$ is the amplitude of the incident wave.

Using expressions (3.5) and (3.6) we obtain from (1.26)[44]

$$f_{E,x} = -(zE_0^{(0)2}/32\pi N_c^2 m_i)[(1 - (N/N_c)]^{-3/2}\partial N/\partial x, \quad (3.7)$$

where $N_c = m\omega_0^2/4\pi e^2$ is the critical density, i.e., the density at which the Langmuir frequency of the electrons is equal to the frequency of the hf field.

We note that in the geometric-optics approximation the connection between the force of the hf pressure and the density is local, in contrast to the transparent-plasma case considered in the preceding section. Furthermore, it follows from (3.7) that in the geometric-optics approximation the force of the hf pressure, like the force of thermal pressure, is always directed towards the region where the plasma density decreases.

The geometric-optics approximation is suitable far from turning points, i.e., at $N/N_c < 1$. If the plasma is transparent and $N/N_c \ll 1$, then we obtain from (3.7)

$$f_E = -(zE_0^{(0)2}/32\pi m_i N_c^2)\partial N/\partial x.$$

This expression also follows from formula (2.6) at $\cos\varphi = 0$ and in the limit as $k_{0x} \to \infty$.

III. SMALL PERTURBATIONS

The influence of strong electromagnetic fields becomes manifest, in particular, in an appreciable change of the dispersion properties of the plasma. The hf field may be the cause of plasma instability. To the contrary, there are conditions when the hf field stabilizes the instabilities existing in the plasma without the hf field. All these problems have recently been extensively investigated and are still far from their solution. However, some questions concerning mainly the linear theory of small perturbations of a plasma in an hf field have by now been made sufficiently clear[4].

The most complete theory was developed for small perturbations of a plasma that is homogeneous in the ground state and is situated in a homogeneous hf field (see the review[55]). It was shown even in the first paper of this group[56] that in a transparent plasma $\omega_0 \gg \omega_{Le}$) the hf field significantly alters the dispersion laws of the longitudinal perturbations[5], and particularly the dispersion law of the ion sound. (This result was later confirmed experimentally[58].) The same paper dealt with the stabilizing action of a hf field on the current instability of a plasma.

Next, using a two-fluid hydrodynamic collisionless plasma model as an example, an important result was obtained in[45], namely that an instability sets in when the hf field frequency approaches the plasma frequency (the so-called "parametric resonance"). In this case, both in[56] and in[45], unlike many succeeding investigations and the present review in particular (see the inequality (1.9)), it was not assumed that the amplitude of the electron oscillations in the hf field is low in comparison with the perturbation wavelength.

The results of[45] were confirmed in laboratories[59] (see also[60-73]) as well as in numerical experiments[74-77], and have been the subject of extensive discussions. In[78,79], using the perturbation theory for the Green's function, which is valid in the approximation of weak hf fields (the thermal energy of the electrons is large in comparison with the energy of the hf oscillations), an approximation that is the opposite of that considered in[45], the parametric-instability threshold connected with the dissipative effects was obtained. These results were subsequently discussed in a number of other papers in which various methods were used[80-88]. In particular, it was shown in[81] that the results of[45] correspond to plasma instability in fields much stronger than the threshold value.

It should be noted that in weak hf fields, which have practically no influence on the wave-dispersion laws in the plasma, and at sufficiently small damping of these waves, the parametric instability is closely related with decay processes in nonlinear wave interaction. In these processes, one wave (in this case it is the external wave) breaks up into two other waves whose amplitudes increase, and the plasma state is then unstable. This instability was first considered in[89] and is called decay instability (see also[90-92]).

In strong hf fields, to the contrary, a situation obtaining in[45] and in a number of other investigations, the dispersion laws for small perturbations are themselves determined by the hf field, and the usual concept of decay processes does not hold. In this case it is customary to speak of a strong parametric coupling of the perturbations in the plasma[93].

In the cited papers[45,56,57,78-87] they considered electrostatic (potential) perturbations of the fields. Under definite conditions, however, it becomes important to take the perturbations of the magnetic field into account, and it is precisely this nonpotential character of the perturbations which determines the plasma instability. This question was considered in[94-98], and in many cases it was important to take into account the finite length of the pump wave[94,99-104].

The theory of small perturbations is generalized to include the case of a magnetized plasma situated in an hf field in[105-115] for potential perturbations, and in[116-124] for nonpotential perturbations. In[125,126] they consider parametric instability of surface waves. The influence of relativistic effects when electrons move in an hf field are discussed in[127,128].

A special group of papers is devoted to small oscillations in an inhomogeneous plasma situated in an hf field. On the one hand, the inhomogeneity of the plasma, and also that of the hf field, cause special parametric instabilities[129-141], and on the other hand it affects the development of the instabilities known to exist in a homogeneous plasma[142-144].

Besides hydrodynamic instabilities, a plasma situated in an hf field can contain kinetic instabilities due to radiation of waves by a definite group of particles. This question is discussed in[145-148] and is outside the scope of the hydrodynamic theory considered by us.

The possibility of stabilizing a plasma with the aid of hf fields, first indicated in[56], was subsequently discussed in[149-160] (see also the review[161]).

It is not our purpose to discuss in any detail all the questions of the theory of small perturbations within the framework of the model in question, and we shall confine ourselves only to certain linear-theory problems that can demonstrate the principal features of small perturbations in a plasma situated in an hf field.

4. The dispersion equation. In this section we derive the dispersion equation that describes the spectrum of small perturbations in a homogeneous and unbounded plasma through which a plane monochromatic wave with specified amplitude (pump wave) passes. The unperturbed state of the plasma is characterized by a constant concentration N_0 and by an hf velocity of the electron oscillations

$$\mathbf{v}_0 = \mathbf{v}_E \cos(\omega_0 t - \mathbf{k}_0\mathbf{r}), \tag{4.1}$$

where the frequency ω_0 and the wave vector $\mathbf{k}_0$, as follows from (1.23), are connected by the dispersion relations ($\omega_{Le}^2 = 4\pi N_0 e^2/m$)

$$\omega_0^2 = \omega_{Le}^2 + k_0^2 c^2, \quad \mathbf{k}_0 \perp \mathbf{v}_E, \tag{4.2}$$

$$\omega_0^2 = \omega_{Le}^2, \quad \mathbf{k}_0 \parallel \mathbf{v}_E. \tag{4.3}$$

We note that the amplitude of the oscillations of the electron velocity $\mathbf{v}_E$ can be expressed in terms of the amplitude of the electric field in the pump wave with the aid of the obvious relation $\mathbf{v}_E = e\mathbf{E}_0/m\omega_0$.

In the linear approximation, for small perturbations of the plasma concentration δN and perturbations of the hf electron velocity $\delta\mathbf{v}$ we obtain from (1.21)–(1.23) the system of equations

$$\partial^2\delta N/\partial t^2 - s^2\Delta\delta N = zN_0 (m/m_i)\ \Delta\ \langle \mathbf{v}_0\delta\mathbf{v}\rangle, \tag{4.4}$$

$$\operatorname{rot}\operatorname{rot}\delta\mathbf{v} + c^2\partial^2\delta\mathbf{v}/\partial t^2 + (\omega_{Le}^2/c^2)\,\delta\mathbf{v} = -(4\pi e^2/mc^2)\,\mathbf{v}_0\delta N, \tag{4.5}$$

where $s^2 = T/m_i$.

Equations (4.4) and (4.5) describe field perturbations that are coupled parametrically via the pump wave (more accurately, perturbations of the velocity of the electron oscillations) and perturbations of the plasma density. The general theory of parametrically coupled perturbations of two scalar quantities with damping in a homogeneous hf field has been constructed in[93]. The question of the connection between the low-frequency and high-frequency waves in a plasma was also discussed in[162-164].

To obtain the dispersion equation, we assume that the concentration perturbations depend on the coordinates and time like $\delta N = \delta N_0 \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$. Then, using expression (4.1), we obtain from (4.5) for the perturbation of the electron oscillation velocity

$$\delta\mathbf{v} = (\delta\mathbf{v}_- e^{i\omega_0 t - i\mathbf{k}_0\mathbf{r}} + \delta\mathbf{v}_+ e^{-i\omega_0 t + i\mathbf{k}_0\mathbf{r}})\, e^{-i\omega t + i\mathbf{k}\mathbf{r}}, \tag{4.6}$$

where

$$\delta v_{\pm,j} = \frac{1}{2}\omega_{Le}^2 v_{E,i}\frac{\delta N_0}{N_0}\left[\frac{k_{\pm,i}k_{\pm,j}}{k_\pm^2}\frac{1}{\omega_\pm^2\varepsilon_\pm} + \frac{(k_{\pm,i}k_{\pm,j}/k_\pm^2) - \delta_{ij}}{k_\pm^2 c^2 - \omega_\pm^2\varepsilon_\pm}\right],$$
$$\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0,\ \omega_\pm = \omega \pm \omega_0,\ \varepsilon_\pm = \varepsilon(\omega_\pm),\ \varepsilon(\omega) = 1 - (\omega_{Le}^2/\omega^2). \tag{4.7}$$

It is seen from (4.6) that the perturbations of the rapidly varying electron velocity depend significantly on the accuracy which the resonance conditions $k_\pm c^2 = \omega_\pm^2\epsilon$ or $\epsilon_\pm = 0$ are satisfied.

Substituting (4.6) in (4.4) and averaging, we obtain a dispersion equation that connects ω with the wave vector $\mathbf{k}$:

$$\omega^2 = k^2\left\{s^2 + \frac{1}{4}\omega_{Li}^2\left[\frac{[\mathbf{k}_-\mathbf{v}_E]^2}{k_-^2}\frac{1}{\omega_-^2\varepsilon_- - k_-^2c^2} + \frac{(\mathbf{k}_-\mathbf{v}_E)^2}{k_-^2}\frac{1}{\omega_-^2\varepsilon_-} + \frac{[\mathbf{k}_+\mathbf{v}_E]^2}{k_+^2}\frac{1}{\omega_+^2\varepsilon_+ - k_+^2c^2} + \frac{(\mathbf{k}_+\mathbf{v}_E)^2}{k_+^2}\frac{1}{\omega_+^2\varepsilon_+}\right]\right\}, \tag{4.8}$$

where $\omega_{Li}^2 = 4\pi e^2 zN_0/m_i$ is the Langmuir frequency of the ions.

Dispersion equation (4.8), obtained in[99] by a kinetic approach, determines the spectrum of coupled perturbations of the density and the hf field in the plasma, which are called electroacoustic waves in a number of papers[94,104].

For perturbations with a wavelength that is small in comparison with the pump wavelength ($|\mathbf{k}_0| \ll |\mathbf{k}|$), and under the condition $\omega_0 \gg \omega_{Le}$, we obtain from (4.8) the dispersion relation

$$\omega^2 = k^2s^2 + (1/2)\,\omega_{Li}^2\{[(\mathbf{k}\mathbf{v}_E)^2/\omega_0^2] - ([\mathbf{k}\mathbf{v}_E]^2/k^2c^2)\}. \tag{4.9}$$

In the limit $c \to \infty$ (potential perturbations of the hf field), formula (4.9) corresponds to the result of[56] relative to the spectrum of the low-frequency potential oscillations. When the last term is taken into account, formula (4.9) can describe unstable perturbations (see (sec. 6 below); it is analyzed in detail in[95].

Formula (4.9) can easily be obtained from simple physical reasoning. Let us analyze the time variation of a small periodic quasineutral density perturbation $\delta\rho = \delta\rho_0\cos(kx - \omega t)$ in a plasma situated in a homogeneous hf field $\mathbf{E}_0 = \mathbf{E}_0\sin\omega_0 t$ ($\omega_0 \gg |\omega|$) (Fig. 1).

From the continuity equation and from the equation of motion it follows that

$$(1/\rho_0)\,\partial^2\delta\rho/\partial t^2 = -\partial f/\partial x, \tag{4.10}$$

where f is the force acting on a plasma ion as a result of the forces of the thermal pressure and the hf pressure.

The expression for the thermal-pressure force is obvious:

$$f_T = -(1/\rho_0)\,\partial\delta p_T/\partial x = (T/m_i)\,k\delta\rho_0\sin(kx - \omega t)/\rho_0. \tag{4.11}$$

Let us examine the forces connected with perturbations of the electric and magnetic fields. The component $E_{0\parallel}$ of the hf field, which is directed along the 0x axis, causes the separation of the charge and leads to the appearance of an inhomogeneous rapidly alternating electric field, which, as follows from the condition for the continuity of the normal component of the induction vector, is equal at $\omega_0 \gg \omega_{Le}$ to

$$\delta E = -(\delta\varepsilon/\varepsilon)\,E_{0\parallel} = (4\pi e^2 z/mm_i\omega_0^2)\,E_{0\parallel}^{(0)}\delta\rho\sin\omega_0 t. \tag{4.12}$$

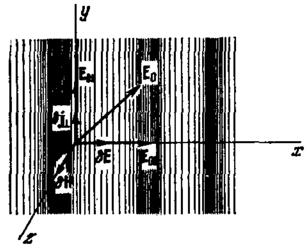

FIG. 1. Perturbations produced in high-frequency electric (δE) and magnetic (δH) fields by periodic perturbation of the plasma density.

The perturbed field, which is stronger wherever the density perturbations are larger, leads to the appearance of an additional pressure force, which equals, in accordance with formula (1.26),

$$f_E = (ze^2/2mm_i\omega_0^2)\,(E_{0\|}^{(0)})^2\,(4\pi e^2/m\omega_0^2)\,k\delta\rho_0 \sin(kx-\omega t), \tag{4.13}$$

and the direction of the action of the force (4.13) coincides with the direction of the force (4.11).

The component $E_{0\perp}$ of the hf field, directed along the 0y axis, does not cause separation of the charge, but leads to the appearance of perturbations of the rapidly alternating current of the electrons, and consequently to the appearance of a perturbed hf magnetic field directed along the 0z axis:

$$-\partial\delta H/\partial x = (4\pi/c)\,\delta j_\perp = -(4\pi e/c)\,\delta\rho\,(eE_{0\perp}^{(0)}/mm_i\omega_0)\sin\omega_0 t.$$

This magnetic field, acting on the electrons that execute hf oscillations, produces an average force f_H that tends to increase the initial density perturbations (in analogy to the attraction between parallel-flowing currents):

$$f_H = \frac{z}{c\rho_0}\langle j_{0\perp}\delta H\rangle = -\frac{z}{2c^2}\,\frac{4\pi e^2}{m\omega_0^2}\,\frac{e^2N_0}{m}\,\frac{E_{0\perp}^{(0)2}}{km_i}\,\frac{\delta\rho_0}{\rho_0}\sin(kx-\omega t). \tag{4.14}$$

With the aid of (4.11) and (4.13), (4.14), and (4.10) we obtain formula (4.9).

It is obvious that the terms in formula (4.8) containing the longitudinal dielectric constants $\epsilon_\pm$ in their denominators correspond to allowance for the pressure forces of the hf electrostatic field produced as a result of the separation of the charges in the pump-wave field. On the other hand, the terms whose denominators contain the expressions $\omega_\pm^2\epsilon_\pm - k_\pm^2c^2$ take into account the forces due to the perturbation of the hf eddy currents in the plasma. It is important to emphasize here that the expressions in the denominators of (4.8) contain ω, and that when the dispersion equation is solved this dependence is particularly important in resonances, when the denominators are small.

5. Weak coupling approximation. Decay instabilities. In this section we establish a connection between the developed theory and the usual theory of nonlinear wave interaction. We note that in the derivation of the usual theory of nonlinear wave interaction one uses a procedure wherein the currents in the plasma are expanded in powers of the field[165]. It follows then from the linear approximation that the dispersion laws of all the waves are determined by the properties of the plasma in the absence of these waves, and the nonlinear currents describe the interaction between the waves. In our case, such an approximation corresponds to a solution of Eqs. (4.4) and (4.5), which describe the coupled perturbations of the density and of the field, using perturbation theory with respect to terms proportional to v_E (the approximation of weak parametric coupling). It turns out here that in the zeroth approximation the density and field perturbations are given by two types of independent proper waves in the plasma, and that their dispersion laws are determined by the usual linear theory. In the first approximation, on the other hand, allowance for the terms proportional to v_E leads to a slow change in the amplitudes of the coupled waves. Therefore, when weak parametric coupling of waves is considered one can use instead of (4.4) and (4.5) a system of abbreviated equations for the slowly varying amplitudes (see, e.g.,[6,7,91]). However, many of the principal results are easier to obtain from the dispersion relation (4.8) by solving it by perturbation theory with respect to terms containing v_E.

We seek the solution of (4.8) in the form

$$\omega = \omega_s + \omega^{(1)}, \tag{5.1}$$

where $\omega_s^2 = k^2s^2$, and in a nonisothermal plasma ($T_e \gg T_i$) this is simply the dispersion law of the ion-acoustic waves with a wavelength that is large in comparison with the Debye radius of the electrons[166].

For the determination of $\omega^{(1)}$ we get from (4.8) the equation

$$2\omega_s\omega^{(1)} = \frac{k^2}{4}\omega_{Li}^2\Big[\frac{[\mathbf{k}_-\mathbf{v}_E]^2}{k_-^2}\,\frac{1}{(1-\omega^{(1)}\partial/\partial\omega_-^{(0)})\,\omega_-^{(0)2}\varepsilon_-^{(0)}-k_-^2c^2} + \frac{(\mathbf{k}_-\mathbf{v}_E)^2}{k_-^2}\,\frac{1}{(1-\omega^{(1)}\partial/\partial\omega_-^{(0)})\,\omega_-^{(0)2}\varepsilon_-^{(0)}} + \frac{[\mathbf{k}_+\mathbf{v}_E]^2}{k_+^2}\,\frac{1}{(1+\omega^{(1)}\partial/\partial\omega_+^{(0)})\,\omega_+^{(0)2}\varepsilon_+^{(0)}-k_+^2c^2} + \frac{(\mathbf{k}_+\mathbf{v}_E)^2}{k_+^2}\,\frac{1}{(1+\omega^{(1)}\partial/\partial\omega_+^{(0)})\,\omega_+^{(0)2}\varepsilon_+^{(0)}}\Big], \tag{5.2}$$

$$\omega_\pm^{(0)} = \omega_0 \pm \omega_s, \quad \varepsilon_\pm^{(0)} = \varepsilon(\omega_\pm^{(0)}).$$

The form of the correction to the frequency depends on whether or not the relations customarily called the decay conditions are satisfied:

$$\omega_\pm^{(0)2}\varepsilon_\pm^{(0)} - k_\pm^2c^2 = 0, \tag{5.3}$$

$$\varepsilon_\pm^{(0)} = 0. \tag{5.4}$$

If either (5.3) or (5.4) is satisfied, and the subscript in these formulas is minus, then $\omega^{(1)}$ is imaginary. In this case the perturbations increase in time and instability sets in; following[89], where it was first considered, this is called decay instability. Let us consider this question in greater detail.

Assume that condition (5.3) with a minus sign is satisfied. We write it down in explicit form:

$$(\omega_0-\omega_s)^2 - \omega_{Le}^2 - c^2(\mathbf{k}_0-\mathbf{k})^2 = 0. \tag{5.5}$$

The largest term in the right-hand side of (5.2) is the first, and its solution is

$$\gamma = i\omega^{(1)} = \pm[k\omega_{Li}/4(\omega_0\omega_s)^{1/2}]\,|[(\mathbf{k}-\mathbf{k}_0)\mathbf{v}_E]|/|\mathbf{k}-\mathbf{k}_0|. \tag{5.6}$$

The minus sign in (5.6) corresponds to perturbations that increase with time, i.e., the amplitudes of the ion-acoustic and transverse waves with frequency $\omega_0 - \omega_s$ and wave vector $\mathbf{k}_0 - \mathbf{k}$ grow simultaneously. At a specified dispersion law for the pump wave, relation (5.5) determines those values of the wave vectors for which instability sets in. If the pump wave has a transverse polarization and the frequency ω_0 is connected with the wave vector $\mathbf{k}_0$ by relation (4.2), then the considered instability describes the thoroughly investigated transformation of a transverse wave into a transverse wave and an ion-acoustic wave (see, for example,[167]). From (5.5) it follows that the unstable perturbations are those having the wave vectors

$$k = 2[k_0\cos\varphi - (s\omega_0/c^2)] > 0, \tag{5.7}$$

where $\cos\varphi = \mathbf{k}\cdot\mathbf{k}_0/kk_0$. In particular, if $\omega_0 \approx k_0c$ and, consequently, $\omega_0 \gg \omega_{Le}$, then we find from (5.7) that the unstable perturbations are those with the wave number $k = 2k_0\cos\varphi$ ($\cos\varphi > s/c$).

When condition (5.4) with minus subscript is satisfied,

$$(\omega_0-\omega_s)^2=\omega_{Le}^2, \quad (5.8)$$

the largest term is the third term in the right-hand side of (5.2) and the solution takes the form

$$\gamma=i\omega^{(1)}=\pm[k\omega_{Li}/4(\omega_0\omega_s)^{1/2}]\,|(\mathbf{k}-\mathbf{k}_0)\,\mathbf{v}_E\,|/|\,\mathbf{k}-\mathbf{k}_0\,|. \quad (5.9)$$

In this case the longitudinal Langmuir wave with frequency $\omega_0-\omega_s$ increases together with the sound in the plasma, and since $\omega_s \ll \omega_0$, ω_{Le}, if (5.8) is to be satisfied it is necessary that the frequency of the pump wave be close to the plasma frequency. If the pump wave is longitudinal and $\omega_0 \approx \omega_{Le}$, then there follows from (5.9) the increment obtained in[89] (see also[165,168]). However, to determine the wave vectors of the unstable perturbations it is necessary to take the thermal motion of the particles in the dispersion laws of Langmuir waves into account in relation (5.8) (see[89]), and this is beyond the scope of our approximations (see inequality (1.9′)).

On the other hand, if the pump wave has transverse polarization and the pump wavelength is much larger than the perturbation wavelength $(k \gg k_0)$, then it follows from (5.8) that the unstable perturbations are those with wave numbers

$$k=k_0^2c^2/2s\omega_{Le} \quad (5.10)$$

and the increment is equal to

$$\gamma=\omega_{Li}\,|\,\mathbf{k}\mathbf{v}_E\,|/4\,(\omega_s\omega_{Le})^{1/2}. \quad (5.11)$$

It is easy to show that neglect of thermal motion of particles in the dispersion law of the Langmuir waves is permissible in this case if the inequality $c/v_T \gg k/k_0$ is satisfied $(v_T^2=T/m)$.

In this section, when solving the dispersion equation (4.8), we used perturbation theory and have assumed by the same token that $|\omega^{(1)}| \ll \omega_s$. It is precisely this condition that allowed us to state that the instability corresponds to a slow growth of the wave amplitudes, whereas their dispersion law is specified. Now, having at our disposal expressions (5.6) and (5.9), it is easier to obtain an explicit criterion for the suitability of this approximation:

$$v_E/v_T \leqslant (\omega_0\omega_s)^{1/2}/\omega_{Le}. \quad (5.12)$$

In particular, at $\omega_0 \approx \omega_{Le}$ it follows from (5.12) that one can speak of ion-acoustic perturbations in a plasma with an ordinary dispersion law only for fields whose pressure is much lower than the thermal pressure of the plasma particles, namely $(v_E/v_T)^2 \ll \omega_s/\omega_{Le}$.

6. Short-wave perturbations at $\omega_0 \approx \omega_{Le}$. Parametric resonance. In this section we consider the solution of the dispersion equation (4.8) without using perturbation theory with respect to the hf field. We are interested in the case of short-wave perturbations $(k \gg k_0)$ in the field of a transverse pump wave, the frequency of which is close to the Langmuir electron frequency $(\omega_0 \approx \omega_{Le} \gg k_0c)$. In addition, we shall confine ourselves to consideration of perturbations whose phase velocity is small in comparison with the velocity of light $(kc \gg |\omega|)$, while the wave numbers are sufficiently large $(kc \gg \omega_{Le})$. When all these conditions are satisfied, Eq. (4.8) is greatly simplified and takes the form

$$(\omega^2-k^2s_1^2)\,[(k_0c)^4-4\omega^2\omega_0^2] = \omega_{Li}^2\,(\mathbf{k}\mathbf{v}_E)^2\,[(1/2)\,(\omega^2+k_0^2c^2)-2\omega\omega_{Le}\,(\mathbf{k}\mathbf{k}_0/k^2)], \quad (6.1)$$

where

$$s_1^2=s^2-(\omega_{Li}^2\,[\mathbf{k}\mathbf{v}_E]^2/2k^4c^2). \quad (6.2)$$

For perturbations with wave vectors that are almost perpendicular to the direction of the hf field, the expression on the right in formula (6.1) is close to zero. The influence of the pump-wave field on the perturbation dispersion law reduces in this case to a replacement of the square of the sound velocity s^2 by the quantity (6.2) (see also (4.9)), which can be negative if

$$v_T^2/v_E^2 \leqslant \omega_{Le}^2/k^2c^2 \ll 1 \quad (6.3)$$

This means that the frequency is imaginary and the plasma is unstable. The physical cause of this instability was discussed in Sec. 4.

However, the condition (6.3) is satisfied only for very strong hf fields $(v_T \ll v_E)$. Furthermore, as shown in[95], the unstable perturbations are those with wave vectors lying in the solid angle kc/ω_{Le}, which is very small under the assumptions made.

Of greater interest is the solution of the dispersion equation (6.1) under those conditions when an inequality inverse to (6.3) is satisfied, and the instability indicated above is impossible. In this case, the only terms of (4.8) contributing to (6.1) are those containing ϵ_+ and ϵ_- in the denominators. In other words, all the effects discussed later on in this section are due to perturbations of the electrostatic field.

To simplify the investigation of (6.1), we assume that

$$k_0^2c^2 \gg (1/4)\,\omega_{Le}\,|\,\omega\,|\,|\,\mathbf{k}\mathbf{k}_0\,|/k^2. \quad (6.4)$$

It is clear that the inequality (6.4) is satisfied if $\mathbf{k} \perp \mathbf{k}_0$. However, as will be shown below, it is also satisfied for arbitrary angles between $\mathbf{k}$ and $\mathbf{k}_0$ under the condition that $kc \gg \omega_{Le}$, which we assume to be satisfied. The condition (6.4) therefore does not in fact change the general character of the results.

Using the notation

$$\Delta=\frac{k_0^2c^2}{2\omega_{Le}^2} \ll 1, \quad \cos\theta=\frac{\mathbf{k}\mathbf{v}_E}{kv_E}, \quad q_E^2=\frac{1}{8}\left(\frac{v_E\omega_{Li}\cos\theta}{\omega_{Le}^2}\right)^2, \quad q_T^2=\frac{s^2}{\omega_{Le}^2}, \quad (6.5)$$

we express the solution of (6.2) in the form

$$\omega/\omega_{Le}=\pm(1/2)\,\{[\Delta^2+k^2(q_T^2-q_E^2)+2k\,(\Delta^2q_T^2+2\Delta q_E^2)^{1/2}]^{1/2} \pm [\Delta^2+k^2(q_T^2-q_E^2)-2k\,(\Delta^2q_T^2+2\Delta q_E^2)^{1/2}]^{1/2}\}. \quad (6.6)$$

Let us analyze this expression for different pump-wave field intensities.

We start with relatively weak fields, when the inequality

$$q_E \ll 4q_T \quad ((v_E/v_T)\sqrt{z}\cos\theta \ll 8\sqrt{2}). \quad (6.7)$$

is satisfied. In this case, solutions that increase in time are obtained if the expression under the square root in the second term of (6.6) is negative:

$$\Delta^2+k^2q_T^2-2k\,(\Delta^2q_T^2+2q_E^2\Delta)^{1/2}<0. \quad (6.8)$$

The first term in (6.6) then determines the real part of the frequency, and the second the imaginary part.

Given the amplitude and frequency (and consequently also the wave vector) of the pump wave, the inequality (6.8) determines the wave-number interval for which instability takes place:

$$(1/q_T^2)\,[(2\Delta q_E^2+\Delta^2q_T^2)^{1/2}-(2q_E^2\Delta)^{1/2}]<k <(1/q_T^2)\,[(2\Delta q_E^2+\Delta^2q_T^2)^{1/2}+(2q_E^2\Delta)^{1/2}]. \quad (6.9)$$

This interval is symmetrical with respect to the wave number

$$k = k_{max} = (1/q_T^2)(2\Delta q_E^2 + \Delta^2 q_T^2)^{1/2}, \tag{6.10}$$

at which, as seen from (6.9), the increment is maximal and equal to

$$\gamma_{max} = \gamma(k_{max}) = (1/\sqrt{2})\,\omega_{Le}\sqrt{\Delta}\,q_E/q_T = (1/4\sqrt{2})\,\omega_{Li}\,(k_0 v_E/\omega_{Le})\,(c/s)\cos\theta. \tag{6.11}$$

At $k = k_{max}$, the real part of the frequency, according to (6.6), is[8)]

$$\omega'_{max} = \mathrm{Re}\,\omega(k_{max}) = \omega_{Le}\,[\Delta^2 + (3\Delta q_E^2/2q_T^2)]^{1/2}. \tag{6.12}$$

To establish the connection between the results obtained above and those obtained in Sec. 5 by the weak-coupling approximation, let us discuss first the limit in which the fields are so weak that the inequality

$$\Delta q_T^2 \gg q_E^2 \quad ((v_E/v_T)\,z^{1/2}\cos\theta \ll 2k_0c/\omega_{Le} < 1) \tag{6.13}$$

is satisfied. It follows then from (6.9) that the region of unstable wave numbers becomes very much narrower, so one can say that the only unstable perturbations are those with one definite value of the wave number

$$k = \Delta/q_T = k_0^2c^2/2s\omega_{Le}. \tag{6.14}$$

As expected, expression (6.14) coincides with formula (5.10) for the value of the unstable wave vectors in the decay of the pump wave into a Langmuir wave and an acoustic wave. Formula (6.13) then goes over into (5.12), and from (6.11) we can easily obtain the increment (5.11). Thus, the inequality (6.13) is the criterion under which the weak-parametric-coupling approximation is valid.

When the inequality opposite to (6.13) is satisfied, the approximation of weak parametric coupling becomes unsuitable. But if the thermal pressure of the particle is nevertheless larger than the pressure of the hf field (i.e., the condition (6.7) is satisfied), then, in accordance with (6.9), the wave-number interval in which the perturbations are unstable is given by

$$(1/2\sqrt{2})\,\Delta^{3/2}/q_E < k < 2\,(2\Delta)^{1/2}\,q_E/q_T^2.$$

The maximum increment of (6.14) then occurs at the wave number

$$k = k_{max} = (q_E/q_T^2)\,(2\Delta)^{1/2} = (1/4\sqrt{2})\,(v_E/s)\,(\omega_{Li}/s)\,(k_0c/\omega_{Le})\cos\theta,$$

and the real part of the frequency, according to (6.12), is by no means close to the sound frequency, but equals

$$\omega'_{max} = \omega_{Le}\,(q_E/q_T)\,(3\Delta/2)^{1/2} = (1/4)\,(3/2)^{1/2}\,(v_E/v_T)\,k_0cz^{1/2}\cos\theta.$$

Figure 2 shows the dependence of the increment on the wave number for different values of the field in the pump wave. It is seen from it that in weak fields, only perturbations with a single wave number are unstable. When the field increases to values in which the approximation of weak parametric coupling is no longer suitable, the region of unstable wave numbers broadens together with a change of the dispersion law for the sound. The maximum increment then shifts towards shorter wavelengths, and its value increases linearly with increasing field. When account is taken of thermal pressure in the dispersion of the Langmuir waves, as is done in[85], it is seen that further increase of the field makes the increase of the maximum increment more complicated.

We proceed now to the case considered in[45], where the hf fields are so strong that the thermal motion of the particles can be neglected in comparison with the hf field pressure (parametric resonance):

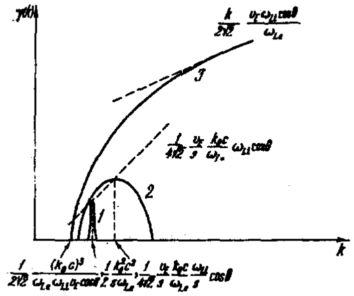

FIG. 2. Dependence of the increment on the wave number at different hf field intensities. 1 – $(v_E/v_T)z^{1/2}\cos\theta \ll 2k_0c/\omega_{Le}$ – decay instability; 2 – $2k_0c/\omega_{Le} \ll (v_E/v_T)z^{1/2}\cos\theta \ll 8\sqrt{2}$; 3 – $(v_E/v_T)z^{1/2}\cos\theta \gg 8\sqrt{2}$ – parametric resonance.

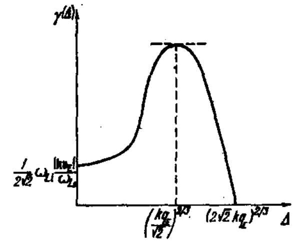

FIG. 3. Dependence of the increment on the hf field frequency (on $\Delta = (\omega_0 - \omega_{Le})/\omega_{Le}$) in the case of parametric instability.

$$q_E \gg q_T.$$

According to (6.6), we have

$$\omega/\omega_{Le} = \pm(1/2)\,\{[\Delta^2 - k^2q_E^2 + 2k\,(2\Delta q_E^2)^{1/2}]^{1/2} \pm [\Delta^2 - k^2q_E^2 - 2k\,(2\Delta q_E^2)^{1/2}]^{1/2}\}. \tag{6.15}$$

Following[45], we consider the solutions (6.15) for different values of the parameter Δ, which characterizes the frequency (or wavelength) of the pump wave. If the condition $\Delta < kq_E$ is satisfied, then the nonzero solution of (6.15) takes the form

$$\omega = \pm ikq_E\omega_{Le} = \pm(i/2\sqrt{2})\,(|\mathbf{k}\mathbf{v}_E|/\omega_{Le})\,\omega_{Li}$$

and describes, when taken with the plus sign, aperiodically growing perturbations. If, to the contrary, $\Delta > kq_E$, then we get from (6.15)

$$\omega/\omega_{Le} = \pm(1/2)\,\{[\Delta^2 + 2kq_E\,(2\Delta)^{1/2}]^{1/2} \pm [\Delta^2 - 2kq_E\,(2\Delta)^{1/2}]^{1/2}\}$$

and instability sets in only if the following inequality is satisfied:

$$\Delta^{3/2} < 2\sqrt{2}\,kq_E.$$

The increment is then maximal for $\Delta = (kq_E/\sqrt{2})^{2/3}$ and is equal to

$$\gamma_{max} = (\sqrt{3}/2^{4/3})\,\omega_{Le}\,(kq_E)^{2/3}.$$

Figure 3 shows a plot of the increment against the frequency of the hf field (more accurately, against Δ); this plot agrees with the results of[45] at $\Delta > 0$. It is important that the increment increases with increasing wave number (see Fig. 2). (It must be borne in mind here, of course, that our analysis is valid if $|\mathbf{k}\cdot\mathbf{v}_E|/\omega_{Le} < 1$.)

Let us discuss briefly the physical cause of parametric resonance. Assume that a static density perturbation $\delta\rho \approx m_i\delta N_i$ has occurred in a plasma placed in a homogeneous external hf field. Then the hf field gives rise to a rapidly-alternating separation of the charges and to an associated electric field. From the equation div D = 0 it follows that this field is equal to (see (4.12))

$$\delta \mathbf{E} = -(\delta\varepsilon/\varepsilon)\, \mathbf{E}_{0\parallel} = (4\pi z e^2 \delta N_i / m\omega_0^2 \varepsilon(\omega_0))\, \mathbf{E}_{0\parallel}.$$

If $\omega_0 > \omega_{Le}$ and $\epsilon(\omega_0) > 0$, then the charge-separation field has the same direction as the external hf field. Consequently, the amplitude of the total field is larger where the plasma density is larger. The hf pressure forces, which are proportional to $\partial(\delta \mathbf{E} \cdot \mathbf{E}_{0\parallel})/\partial x$, tend to decrease the existing density perturbations (see (4.13)). It is this which makes it possible to use hf fields to stabilize instabilities[149, 161].

On the other hand, if $\omega_0 < \omega_{Le}$, then $\epsilon(\omega_0) < 0$ and the field $\delta\mathbf{E}$ is directed opposite to the direction of the external field. Therefore the amplitude of the total field in the plasma is smaller where the density is larger. The hf pressure forces tend to increase the initial density perturbation, and instability sets in.

Actually, the density perturbations are not static and the frequency of the charge-separation field does not coincide with the frequency of the external hf field. This does not change the physical gist of the phenomenon, but causes the picture of the onset of parametric instability, which is considered in this section, to become more complicated.

7. Perturbations in a tenuous plasma ($\omega_0 \gg \omega_{Le}$). In this section we consider the consequences ensuing from the dispersion equation (4.8) in the case when the frequency of the transverse pump wave ω_0 is much larger than the Langmuir frequency of the electrons and it can be assumed that $\omega_0 \approx k_0 c$. Then, unlike the preceding section, it is important to take into account in (4.8) the terms connected with the perturbations of the eddy currents (see Sec. 4), and it takes the form

$$\omega^2 = k^2 \left\{ s^2 - \frac{1}{2} \frac{\omega_{Li}^2}{c^2} k^2 \frac{v_E^2 - [(\mathbf{k v}_E)^2/k_0^2]}{k^4 - 4(\mathbf{k k}_0)^2 + 8\mathbf{k k}_0 (\omega k_0/c)} \right\}. \tag{7.1}$$

To trace the influence of the increase of the pump-wave amplitude on the character of the decay instability, we begin the analysis of (7.1) with the case of relatively weak hf fields, for which the condition (5.12) is satisfied, and which have little effect on the soundwave dispersion law. We assume that

$$\omega = ks(1 + \delta).$$

From (7.1) we obtain for determination of δ the equation

$$\delta(2+\delta)\left[1 + \delta + \frac{k^4 - 4(\mathbf{k k}_0)^2}{8\mathbf{k k}_0 (k k_0 s/c)}\right] + \frac{\omega_{Li}^2}{2c^2} k^4 \frac{v_E^2 - [(\mathbf{k v}_E)^2/k_0^2]}{8\mathbf{k k}_0 (k_0/c)(ks)^3} = 0. \tag{7.2}$$

In the theory of decay instabilities, the lengths of the growing waves are determined from the condition of simultaneous satisfaction of the dispersion relations for all the waves participating in the decay process (see Sec. 4). In this case, only one pair of growing waves can propagate in a definite selected direction. These results can also be obtained from (7.2) if it is assumed that $|\delta| \gg 1$ and the condition (5.5) is satisfied for the decay of a transverse pump wave into an acoustic wave and a transverse wave, a condition equivalent to the vanishing of the expression $1 + [k^4 - 4(\mathbf{k} \cdot \mathbf{k}_0)^2] / 8\, \mathbf{k} \cdot \mathbf{k}_0\, (s k k_0/c)$.

However, when solving (7.2) it is also possible not to determine the lengths of the unstable waves beforehand, but to find them from the condition $\operatorname{Im} \delta > 0$. If this is done, recognizing that $|\delta| \ll 1$, then we obtain the following limitation on the lengths of the unstable waves:

$$2\left[\cos\varphi - \frac{s}{c} - \frac{1}{2\sqrt{2}} \frac{v_E}{s} \frac{\omega_{Li}}{\omega_0} \left(\frac{s}{c}\right)^{1/2} \left(\frac{1 - 4\cos^2\varphi \cos^2\theta}{\cos^2\varphi}\right)\right]^{1/2} < \frac{k}{k_0} < 2\left[\cos\varphi - \frac{s}{c} + \frac{1}{2\sqrt{2}} \frac{v_E}{s} \frac{\omega_{Li}}{\omega_0} \left(\frac{s}{c}\right)^{1/2} \left(\frac{1 - 4\cos^2\varphi \cos^2\theta}{\cos^2\varphi}\right)\right]^{1/2}, \tag{7.3}$$

where $\cos\varphi = \mathbf{k} \cdot \mathbf{k}_0 / k k_0$, and $\cos\theta = \mathbf{k} \cdot \mathbf{v}_E / k v_E$. If $\cos\varphi > s/c$, then the maximum growth rate is possessed by waves having the wave number

$$k_{\max} = 2k_0 \left(\cos\varphi - \frac{s}{c} + \frac{1}{64} \frac{\omega_{Li}^2 v_E^2}{\omega_0^2 s^2} \frac{1 - 4\cos^2\varphi \cos^2\theta}{\cos\varphi}\right). \tag{7.4}$$

Thus, the form of the decay instability changes with increasing pump-wave amplitude. The region of unstable wavelengths broadens, and the wave length for which the increment is maximal decreases (cf. Sec. 6).

We now stop to discuss the solutions of (7.1) that differ essentially from the acoustic dispersion law. We consider in succession two regions of wave numbers, confining ourselves for simplicity to the case $\cos\varphi = 0$, i.e., $\mathbf{k} \parallel \mathbf{k}_0$ (a more general case is considered in[169]). We start with nonresonant waves, for which the condition

$$|k^2 - 4k_0^2| > 8|\omega| k_0^2/ck. \tag{7.5}$$

is satisfied. The solution of (7.1) is obvious:

$$\omega^2 = k^2 \{s^2 - [(\omega_{Li}^2/2c^2) v_E^2/(k^2 - 4k_0^2)]\}. \tag{7.6}$$

This formula was first obtained in[94] and was discussed in[99, 104]. It follows from it immediately that at

$$(k^2 - 4k_0^2) \leqslant (\omega_{Li}^2/2c^2) v_E^2/s^2 \tag{7.7}$$

aperiodic instability sets in. Therefore, from the condition of simultaneous satisfaction of the inequalities (7.5) and (7.7), when (7.6) is taken into account, it follows that this instability sets in when the inequality opposite to (5.12) is satisfied. The increment increases as k^2 approaches $4k_0^2$. However, the condition (7.5) is violated starting with a certain value of the wave number. We therefore proceed to solve Eq. (7.1) in the other limiting case, when the inequality inverse to (7.5) is satisfied (the resonant case). The dispersion equation is

$$\omega^3 - \omega k^2 s^2 + (1/4)\, \omega_{Li}^2 k v_E^2 / c = 0. \tag{7.8}$$

In order for this cubic equation to have complex roots, it is necessary to satisfy the relation

$$\omega_{Li}^2 v_E^2 / c > (32/3)\, k_0^2 s^3. \tag{7.9}$$

One of the roots will then describe unstable perturbations. Therefore the inequality (7.9) at a specified pump-wave frequency (and consequently also k_0) determines the frequency starting with which instability sets in. This criterion differs by a numerical factor from the approximate expression obtained in[99]. The increment at resonance ($k = 2k_0$), according to (7.8), is equal to

$$\gamma = (\sqrt{3}/2)\, [(1/2)\, \omega_{Li}^2 (v_E^2/c)\, k_0]^{1/3}. \tag{7.10}$$

This equation was obtained in[99], and then by other methods in[102, 103].

In conclusion, let us dwell on the physical cause of the considered instability. An electromagnetic wave scattered by plasma-density perturbations that have a spatial periodicity λ such that $2\lambda = \lambda_0$ (λ_0 is the pump wavelength), forms, together with the pump wave, a field structure of the standing-wave type. The antinodes of the field occur at the minima of the density, and the force of the hf pressure tends to increase the initial perturbations. If this force is small in comparison with the force of the thermal pressure of the plasma, then it hardly changes the oscillation frequency, but may turn out to be sufficient to increase the amplitude of the initial density perturbations.

This case corresponds approximately to weak coupling, or to the so-called decay instability. On the other hand, if the pump-wave field exceeds a certain value, say the one determined by formula (7.9), then the hf pressure forces exceed the thermal-pressure forces. In this case the concept of ion-sound waves with the usual dispersion law becomes meaningless, and an aperiodic instability sets in, with the increment (7.10).

An order-of-magnitude expression for the increment (7.10) can easily be obtained from simple physical reasoning, if it is recognized that at $k = 2k_0$ the waves scattered in a spatial region of length on the order of c/ω add up coherently, and the force f_H in (4.14) should be increased by a factor kc/ω. The expression (7.10) then follows from formula (4.10), apart from a constant on the order of unity.

IV. EQUILIBRIUM OF A PLASMA IN A HIGH-FREQUENCY FIELD

Interest in the study of the equilibrium of a plasma in an hf field arose initially in connection with the problem of plasma confinement in installations for thermonuclear research. The first papers on this subject were published about 10 years ago[170-179] (see the review[28]). Special mention should be made of[171], which gives the most complete and consistent solution of the one-dimensional problem of plasma equilibrium in the field of a plane standing wave (see Sec. 8). Among the later papers in this field, we mention the papers[180-181], dealing with plasma confinement in a combination of a constant magnetic field and hf fields.

Recently, the question of the equilibrium of a plasma in an hf field has attracted attention also for another reason, namely that stationary self-maintaining radiation channels can be produced in a plasma, as in all other nonlinear media[182]. Many of the previously obtained results, and primarily the results of[171], turned out to be applicable also for description of this phenomenon. Extensive material concerning the self-trapping of radiation not only in a plasma, but also in other media, is contained in a number of reviews[183]. At the presently attainable hf field intensities, a plasma, unlike other media, can turn out to be a strongly nonlinear medium, and therefore the effects in it should become more distinctly manifest[184,185] and are of particular interest[186].

Another problem pertaining to the investigation of the equilibrium state of a plasma in an hf field is that of nonlinear penetration of an electromagnetic wave into a plasma. For the case of normal incidence of a wave on the boundary of an arbitrary conductor, this problem was considered in[187] (see also[188]). It is shown that, in particular, the redistribution of the plasma density, which occurs under the influence of hf pressure[7)], not only alters the character of the penetration and reflection of the wave, but also leads to a decrease of the critical frequency, above which the plasma is transparent (see Sec. 9). A generalization of the results of[187] to the case of oblique incidence is contained in[190,191]. A number of papers have discussed the question of the propagation of intense electromagnetic waves along[192,193] and across[194,195] a plasma layer.

Mention should also be made of a number of interesting papers dealing with the nonlinear theory of plasma resonance[44,196]. A strong increase of the field in the resonance region leads to a redistribution of the plasma near this region, and this in turn leads to a change of the field. This problem was considered in[197] (see also[198-201]).

There is also one other group of problems that have been considered recently, the solution of which reduces to an equation for the equilibrium of a plasma in an hf field. We have in mind stationary (steady-state) nonlinear waves, and in particular solitary waves (solitons). As applied to the case of waves with sufficiently small wavelength propagating along the hf field, the problem of determining the shape of the steady-state waves was solved in[202]. To the contrary, the case of stationary waves propagating across an hf field is analyzed in detail in[104,203-205], and will be discussed in Sec. 10 below.

To be able to speak of practical realizability of equilibrium of a plasma and an hf field, it is necessary to clarify the question of stability of this equilibrium. Many conclusions obtained for a homogeneous plasma seem to remain in force in this case (see Chap. II). There are also, however, a number of differences connected with the inhomogeneity of the plasma and of the hf field. The stability of certain equilibrium configurations is discussed in[207-210]; steady-state multidimensional solutions of the equilibrium equations that result from unstable one-dimensional solutions are discussed in[211].

8. Equilibrium of a plasma in the field of a standing wave. In the equilibrium state, the hydrodynamic equations (1.21, 1.24, and 1.25) of a plasma in an hf field become much simpler. The plasma as a unit has zero velocity, and the particle concentration and the hf-field amplitude do not depend on the time. Therefore Eq. (1.21) is satisfied, and it follows from (1.24) that the plasma density and the hf-field amplitude are connected by the simple relation

$$N = C_0 \exp\left(-ze^2E_0^2/4Tm\omega_0^2\right), \tag{8.1}$$

where the constant C_0 is expressed in terms of the plasma concentration at the point where the hf field is known.

Using (8.1), we rewrite (1.25) in the form

$$(c^2/\omega_0^2)\,\mathrm{rot\,rot}\,\mathbf{E}_0 - \mathbf{E}_0 + (4\pi e^2/m\omega_0^2)\,C_0\mathbf{E}_0 \exp\left(-ze^2E_0^2/4Tm\omega_0^2\right) = 0. \tag{8.2}$$

This is the fundamental equation for the investigation of equilibrium states of a plasma in an hf field.

In this section, following mainly[171], we consider the possible equilibrium configurations of a plasma in the field of a transverse plane standing wave

$$\mathbf{E}_0 = (0,\ E_0(x),\ 0).$$

In this case, using the following dimensionless variables:

$$E = E_0 ez^{1/2}/\omega_0\,(2Tm)^{1/2}, \quad \eta = x\omega_0/c, \quad C = C_0/N_c, \quad N_c = m\omega_0^2/4\pi e^2,$$

we obtain from (8.2) the first integral, which corresponds to constancy of the total pressure at each point[8)]:

$$(dE/d\eta)^2 + E^2 + 2Ce^{-E^2/2} = 2C_1, \tag{8.3}$$

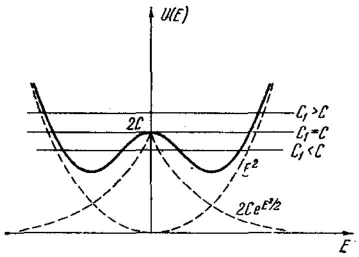

FIG. 4. Potential (8.5) for a dense plasma.

where C_1 is the constant that characterizes this pressure. Solving Eq. (8.3) relative to $dE/d\eta$, we obtain in implicit form the dependence of the hf field amplitude on the coordinate:

$$\eta-\eta_0=\pm(1/2)\int^{E^2/2}\tau^{-1/2}(C_1-\tau-Ce^{-\tau})^{-1/2}d\tau. \qquad (8.4)$$

The character of the possible solutions of (8.3) can be simply understood by using the formal mechanical analogy proposed in[178] (see also[28]). We regard the intensity of the field E as the coordinate of the particle, and the coordinate η as the time. Then relation (8.3) can be identified with the energy conservation law for a particle moving in a potential well:

$$U(E)=E^2+2Ce^{-E^2/2}. \qquad (8.5)$$

Depending on the constant C, the potential can vary in character. Indeed, taking the derivative dU/dE and equating it to zero, we find that at $C>1$ (this means that there is a point at which the plasma concentration exceeds the critical value N_c) the potential has a maximum at $E=0$ and has a minimum at $E=\pm(2\ln C)^{1/2}$ (Fig. 4). On the other hand, if $C\le 1$ and the plasma concentration is everywhere smaller than the critical value, then the potential curve has a single minimum at $E=0$. Let us consider these cases in succession.

The three horizontal lines in Fig. 4, which shows the potential (8.5) at $C>1$, correspond to three different values of the constant C_1, which characterizes the total energy of the particle (the total pressure of the hf field and of the plasma). The intersections of these lines with the potential curve determine those particle coordinates at which the particle velocity is zero. Depending on the relation between C and C_1, three different variants of particle motion are possible; in other words, there are three different hf field configurations.

1) If the plasma density characterized by the constant C is low enough so that $C<C_1$, then, as seen from Fig. 4, there are only two points at which the particle velocity is equal to zero and, in addition, the particle passes through the origin. This means that the particle oscillates in the potential well and that the field is a periodic function of the coordinate. Analytic expressions for the field can easily be obtained in two limits[9)]. If $E<1$, then it follows from (8.4) that

$$E=\pm[2(C_1-C)]^{1/2}(\eta-\eta_0).$$

On the other hand, if the maximum (or minimum) field E_m determined by the relation

$$2C_1=U(E_m)=E_m^2+2Ce^{-E_m^2/2},$$

corresponds to the coordinate η_m then, by expanding the quantities that enter in (8.4) in the vicinity of this point, we also obtain the law governing the field variation:

$$E=E_m\{1-(1/2)(\eta-\eta_m)^2(1-Ce^{-E_m^2/2})\}.$$

On the basis of the available data, it is possible to construct a qualitative picture of the variations of the field and the concentration (Fig. 5).

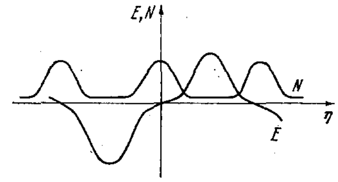

FIG. 5. Variation of the field and of the concentration with the coordinate at $C_1>C$.

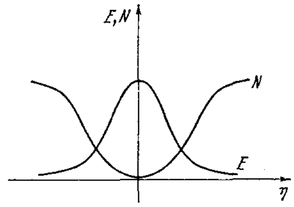

FIG. 6.

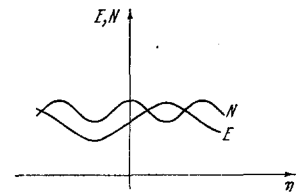

FIG. 7.

FIG. 6. Variation of field and of concentration with the coordinate at $C_1=C$.

FIG. 7. Variation of the field and of the concentration with the coordinate at $C_1<C$.

2) Let $C=C_1$. In Fig. 4, this case corresponds to the straight line passing through the maximum of the potential. A particle moving with such energy will take an infinite time to reach the point $E=0$. This means that the field vanishes at infinity. To find the law governing the field variation as $\eta\to\pm\infty$, we consider formula (8.4). Putting $\tau\ll 1$ in it, we obtain

$$E\sim\begin{cases}\exp[-(C-1)^{1/2}\eta] & \text{as}\quad \eta\to+\infty,\\ \exp[(C-1)^{1/2}\eta] & \text{as}\quad \eta\to-\infty.\end{cases} \qquad (8.6)$$

Let the field assume its maximum value E_m at $\eta=0$. A simple relation can then be established between the value of E_m and the plasma density N_∞ at infinity. Inasmuch as $E\to 0$ as $\eta\to\pm\infty$, in accordance with (8.6), it follows from (8.1) and (8.3) that the constant C is connected with the density at infinity by the simple relation $C=C_1=N_\infty/N_c$, and in accordance with (8.3) we can write

$$E_m^2=2(N_\infty/N_c)(1-e^{-E_m^2/2}).$$

We denote the plasma density at $\eta=0$ by N_m and recognize that $N_m=N_\infty\exp(-E_m^2/2)$. From this we obtain the connection between the plasma densities at infinity and at the maximum of the field:

$$N_m/N_\infty=\exp[(N_m-N_\infty)/N_c].$$

If $N_\infty/N_c\gg 1$ and the plasma density at infinity is much larger than the critical value (the plasma is opaque to the hf field), then $(N_m/N_\infty)\ll 1$ and

$$E_m^2/2\approx N_\infty/N_c \quad (zE_{0m}^2/16\pi TN_\infty\approx 1).$$

It follows therefore that, with exponential accuracy, the field pressure at the maximum coincides with the pressure of the particles at infinity.

Plots that describe qualitatively the dependence of the field and of the plasma density on the coordinates are given in Fig. 6.

The case when the minimum of the plasma density is balanced not by the field of one monochromatic wave but by a set of short waves with different frequencies was considered recently in[206].

3) We consider now the case $C>C_1$. The particle

energy is in this case insufficient to pass through the point $E = 0$. This means that the field amplitude can never vanish, and ranges from E_{min} to E_{max} (Fig. 7). It follows from the requirement $(dE/d\eta)^2 > 0$, according to (8.3), that the possible values of C_1 are subject to the limitation

$$2C_1 \geqslant U_{min} = 2(\ln C + 1).$$

At the minimum value of C_1, which equals $\ln C + 1$, the particle in our mechanical analogy is at rest at the minimum of the potential energy. This means that the field amplitude and the plasma concentration do not depend on the coordinates. If the concentration is equal to N, then the field is obviously equal to $E = [2 \ln (4\pi e^2 N/m\omega_0^2)]^{1/2}$.

Let us dwell briefly on the case $C < 1$ (tenuous plasma). As already noted, the potential $U(E)$ has in this case a single minimum at $E = 0$ and tends to infinity as $\eta \to \pm\infty$ like E^2. It is clear that only periodic changes of the amplitude of the hf field are possible for all the permissible values, and that there are always points at which the field is equal to zero. In the case of a transparent plasma, the field has in fact a structure similar to that of the field in vacuum.

9. Penetration and reflection of a strong electromagnetic wave. The linear theory of reflection and refraction of electromagnetic waves by a plasma boundary is valid in those cases when the electromagnetic-field pressure is small in comparison with the thermal pressure. If this condition does not hold and the field pressure becomes comparable with the thermal pressure, then we can no longer neglect the changes that the incident wave produces in the characteristics of the medium. In other words, the problem of reflection and refraction of waves becomes nonlinear in this case.

As applied to conductors (a plasma or metals), the stationary nonlinear theory of reflection and penetration of waves was constructed in[187], where the case of sufficiently rare collisions is considered alongside the case when the particle collisions are completely neglected. In the present section, following mainly the cited reference[187], we consider the one-dimensional problem of reflection and penetration of an electromagnetic wave in a plasma, but remain within the framework of the model investigated by us and neglect the particle collisions altogether.

Let the plasma occupy the half-space $x > 0$ and let a linearly polarized electromagnetic wave be incident normal to its surface on the boundary $x = 0$. We represent the wave field in the plasma in the form

$$\mathbf{E}_{in} = (0,\ E_0(x) \cos[\omega_0 t + \varphi(x)],\ 0). \qquad (9.1)$$

For the field outside the plasma, which is the sum of the incident and reflected waves, we use the expression

$$\mathbf{E}_{out} = (0,\ E_{inc}[\sin(k_0 x - \omega_0 t) + R \sin(k_0 x + \omega_0 t + \psi)],\ 0), \qquad (9.2)$$

where E_{inc} is the amplitude of the incident wave, R is the reflection coefficient, and ψ is the initial phase of the reflected wave.

From the condition for the continuity of the electric and magnetic fields on the plasma boundary[41] we obtain

$$R = 1 - (M^2/k_0^2 E_{inc}^2), \qquad (9.3)$$

$$E_0(0) = E_{inc}(1 + R^2 + 2R \cos\psi)^{1/2}, \qquad (9.4)$$

$$\operatorname{ctg} \varphi(0) = -R \sin\psi/(1 + R\cos\psi), \qquad (9.5)$$

$$\sin\psi = -E_0(0) E_0'(0)/2k_0 E_{inc}^2 R, \qquad (9.6)$$

where $M = -E_0^2(0)\varphi'(0)$ and the prime denotes differentiation with respect to the coordinate.

We substitute (9.1) in Eq. (8.2), which determines the field in the plasma, and equate the sine and cosine terms. As a result we obtain two equations, which are expressed in the notation of Sec. 8 in the form

$$-E^2(\eta)\varphi'(\eta) = M, \qquad (9.7)$$

$$d^2E/d\eta^2 - E(d\varphi/d\eta)^2 + E - CE\exp(-E^2/2) = 0. \qquad (9.8)$$

We are interested in the case when the field does not penetrate the plasma and at $\eta \to +\infty$ we have $E \to 0$ and $E' \to 0$. Then, according to (9.7) and (9.3), we have $M = 0$ and $R = 1$, i.e., the wave is completely reflected from the plasma. Using this condition, we write down the first integral of (9.8) in the form

$$(dE/d\eta)^2 + E^2 + 2C(e^{-E^2/2} - 1) = 0, \qquad (9.9)$$

where the constant C is connected with plasma density at $\eta = \infty$ by the relation $C = 4\pi e^2 N_\infty/m\omega_0^2$. Expression (9.9) corresponds to the second case with $C > 1$ considered by us in Sec. 8, when $C = C_1$. Unlike the solution investigated in Sec. 8, however, where the field decreased both as $\eta \to +\infty$ and as $\eta \to -\infty$, in the present case the solution at $\eta = 0$ must satisfy definite boundary conditions, which is not possible at arbitrary values of the parameters of the wave incident on the plasma.

At $R = 1$ it follows from the boundary conditions (9.4) and (9.6) that

$$E_0(0) = 2E_{inc}\cos(\psi/2), \quad E_0'(0) = -2k_0 E_{inc}\sin(\psi/2), \qquad (9.10)$$

and from formula (9.9) we obtain for the phase of the scattered wave

$$\cos^2(\psi/2) = -(N_c/N_\infty)\,\beta \ln[1 - (1/\beta)], \qquad (9.11)$$

where N_c is the critical density and $\beta = 4\pi T N_\infty/zE_{inc}^2$ is the ratio of the thermal pressure of the particles at $\eta = \infty$ to the pressure of the field in the incident wave. From (9.11) we see that $\cos^2(\psi/2) > 0$ only if $\beta \geq 1$, and if the field is to have a stationary distribution in the plasma it is necessary that the pressure of the incident wave be smaller than the thermal pressure of the plasma. In addition, it follows from (9.11) that the critical frequency ω_c, below which the field can decrease in the plasma as $\eta \to +\infty$, is equal to

$$\omega_c^2 = (4\pi e^2 N_\infty/m\beta)\{-\ln[1 - (1/\beta)]\}^{-1}.$$

Figure 8 shows a plot of the dependence of the critical frequency on β. It follows from it that with increasing amplitude of the incident wave (i.e., with decreasing β), the region of plasma opacity shifts towards lower frequencies.

To obtain the dependence of the field in the plasma on the coordinate, we use formula (8.4), in which we put $C = C_1$ and make the substitution $\tau = u^2/2$:

$$\eta = \int_E^{E(0)} [2C(1 - e^{u^2/2}) - u^2]^{-1/2}\, du. \qquad (9.12)$$

The upper limit $E(0)$ in (9.12) corresponds to the field at the plasma boundary, and it can easily be expressed in terms of the amplitude of the incident wave with the aid of formulas (9.10) and (9.11). Figure 9 shows plots constructed with the aid of (9.12), which illustrate the variation of the field in the plasma and the concentration of the particles with the coordinate for different values of the constant β characterizing the amplitude of the incident wave, and at a constant concentration of the unperturbed plasma at infinity.

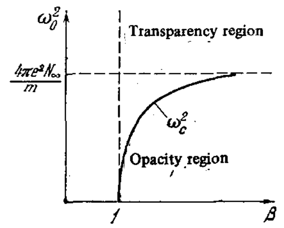

FIG. 8.

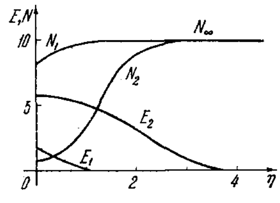

FIG. 9.

FIG. 8. Dependence of the critical frequency on the ratio β of the thermal pressure of the particles to the pressure of the incident-wave field.

FIG. 9. Variation of the plasma concentration N and of the amplitude E of the hf field with variation of the coordinate η for different amplitudes of the incident wave ($E_{\text{inc. }2} > E_{\text{inc. }1}$).

10. Nonlinear stationary waves. In Chap. III we have considered problems of the linear theory of coupled perturbations of the density and of the hf field (electroacoustic waves). In this section we dwell briefly on the results of nonlinear theory of electroacoustic stationary waves, i.e., waves propagating in a plasma at constant velocity without changing their shape.

We consider the one-dimensional problem for simplicity and assume that all the quantities in (1.21)–(1.23) depend on the coordinate and on the time in the following fashion:

$$N(x,\ t) = N(x-ut),\quad V(x,\ t) = V(x-ut),\quad v(x,\ t) = v_0(x-ut)\cos[\omega_0 t - \varphi(x-ut)]. \qquad (10.1)$$

Substituting (10.1) in (1.21)–(1.23), we obtain

$$NW = C_1, \qquad (10.2)$$

$$(W^2/2) + (zm/4m_i)v_0^2 + s^2 \ln N = C_2, \qquad (10.3)$$

$$d\varphi/d\xi = (C_3 + u\omega_0 v_0^2)/v_0^2(c^2+u^2), \qquad (10.4)$$

$$\frac{d^2 v_0}{d\xi^2} + \frac{\omega_0^2 c^2}{(c^2+u^2)^2} v_0 + \frac{1}{v_0^3}\frac{C_3^2}{(c^2+u^2)^2} - \frac{4\pi e^2}{m(c^2+u^2)} N v_0 = 0, \qquad (10.5)$$

where $\xi = x - ut$, $W = V - u$; C_1, C_2, and C_3 are constants, and $s^2 = T/m_i$.

Thus, the problem of investigating the structure of stationary electroacoustic waves reduces to a solution of Eq. (10.5), in which the concentration is expressed in terms of the amplitude of the hf oscillations of electrons with the aid of a relation that follows from (10.2) and (10.3), namely

$$(C_1^2/2N^2) + (zm/4m_i)v_0^2 + s^2 \ln N = C_2. \qquad (10.6)$$

A qualitative analysis of the possible types of solutions of (10.5) and (10.6) is presented in[205]. We confine ourselves to one example, in which it is possible to obtain analytic expressions for N and v_0. Following[104], we consider small perturbations of the plasma density $N = N_0 + \delta N$. From (10.6) we have

$$N = N_0\left[1 + \frac{C_2 - (zm/4m_i)v_0^2}{s^2 - (C_1^2/N_0^2)}\right], \qquad (10.7)$$

and the first integral of (10.5) takes the form ($\omega_{Le}^2 = 4\pi e^2 N_0/m$)

$$\left(\frac{dv_0}{d\xi}\right)^2 + v_0^2\frac{\omega_0^2}{c^2+u^2}\left[\frac{c^2}{c^2+u^2} - \frac{\omega_{Le}^2}{\omega_0^2}\left(1 - \frac{C_2}{s^2-(C_1^2/N_0^2)}\right)\right] + \frac{1}{2}\frac{\omega_{Le}^2}{c^2+u^2}\frac{zm}{4m_i}\frac{v_0^4}{s^2-(C_1^2/N_0^2)} - \frac{C_3^2}{v_0^2(c^2+u^2)^2} = C_4. \qquad (10.8)$$

Among the possible types of solutions of (10.8), special interest attaches to the solutions considered in[104,203-205,209], which have the form of solitary waves. For these solutions, the hf electric field and the magnetic field vanish at infinity, as do also δN and $V(\delta N,\ v_0,\ V,\ dv_0/d\xi \to 0$ as $\xi \to \pm\infty)$, and the phase of the wave is constant. These conditions allow us to conclude that the constants C_2, C_3, and C_4 in (10.8) are equal to zero, and $C_1 = -N_0 u$. As a result we have

$$\left(\frac{dv_0}{d\xi}\right)^2 + v_0^2\frac{\omega_0^2}{c^2}\left(1 - \frac{\omega_{Le}^2}{\omega_0^2}\right) + \frac{1}{2}\frac{\omega_{Le}^2}{c^2}\frac{zm}{4m_i}\frac{s^2}{s^2-u^2}v_0^4 = 0. \qquad (10.9)$$

We have taken into account here the fact that $u \ll c$, and assumed the condition $|1 - (\omega_{Le}^2/\omega_0^2)| > u^2/c^2$.

Equation (10.9) can have a solution that is bounded in space only in the case of a opaque plasma in which $\epsilon(\omega_0) = 1 - (\omega_{Le}^2/\omega_0^2) < 0$. This solution is

$$v_0 = A\,\mathrm{sech}\,\{[(\omega_{Le}^2 - \omega_0^2)^{1/2}/c](x-ut)\}, \qquad (10.10)$$

where the maximum amplitude A of the electron oscillations is connected with the soliton propagation velocity u by the relation

$$u = s\,[1 - (\omega_{Le}^2/2\omega_0^2|\varepsilon(\omega_0)|)(zm/4T)A^2]^{1/2}. \qquad (10.11)$$

According to (10.7), this is a rarefaction wave and the density varies like

$$N = N_0\left\{1 - 2\frac{\omega_{Le}^2 - \omega_0^2}{\omega_0^2}\,\mathrm{sech}^2\left[\frac{(\omega_{Le}^2-\omega_0^2)^{1/2}}{c}(x-ut)\right]\right\}. \qquad (10.12)$$

If it is recognized that the amplitude of the electron oscillations is connected with the amplitude of the hf wave by the relation $A = eE_0/m\omega_0$, then expressions (10.10)–(10.12) go over into the results of[212] (see also[5]). The evolution of the solitons with time and the boundary conditions that determine their formation when an electromagnetic wave is incident on the plasma have been considered in[212,204]. In[203] they took into account the influence of weak dissipative effects on the development of the solitons. The question of soliton stability is discussed in[209].

V. PLASMA MOTION IN A HIGH-FREQUENCY FIELD

The study of plasma motion in strong hf fields was initiated more than 10 years ago and was stimulated by work on the radiation method of plasma acceleration[213]. In the first studies[214-217] (for details see[218,219]) the plasma was regarded as a formation with a given and invariant configuration (sphere, ellipsoid, torus, etc.), and the forces acting on this formation in different hf fields were investigated (see also[220]). Of course, the conclusions drawn concerning the motion of the plasma on the basis of such a simplified model were only qualitative in character.

Possible deformations of plasma formations have been discussed in[38,39], in which the distribution of the hf pressure forces was obtained for a given distribution of the density of a transparent plasma. The possibility of changing the plasma configuration was deduced on the basis of these data. However, the change in the hf pressure forces was not taken into account.

A more realistic model of the acceleration of a layer of opaque plasma was used by the authors of[221] (see also[222]). In their model, the hf field played the role of a piston and the internal motion of the plasma particles reflected from this piston was taken into account. However, the self-consistent problem of the plasma motion was not solved in these studies.

A number of very simple cases in which the nonlinear equations describing the joint variation of the plasma density and of the hf fields in the plasma could be solved were considered in[40,44] and will be discussed here in greater detail.

In Secs. 11 and 12 we consider, in a quasistatic approximation, two variants of motion of a layer of transparent plasma in the field of a traveling electromagnetic wave of sufficiently large amplitude. We shall show that under certain conditions, the plasma spreads out, while under other conditions it is accelerated and contracts. In Sec. 13 we solve the self-similar problem of the spreading of a plasma boundary on which an electromagnetic wave of wavelength much shorter than the characteristic thickness of the boundary is incident.

11. Spreading of a plasma layer. In this section we consider the spreading of a thin layer of a transparent plasma under the influence of hf pressure forces[40]. We assume that the plasma density depends only on the coordinate x and on the time, and that the wave propagates along the 0y axis, the electric field intensity vector being directed along the 0x axis. At such a geometry, it is necessary to put in (2.5) $k_{0x} = 0$, $\sin\varphi = 0$, and $\cos\varphi = 1$. Then, if the wavelength c/ω_0 is large in comparison with the transverse layer dimension, we obtain the corresponding expression for f_E from (2.6), and, neglecting the thermal pressure, we write down Eqs. (1.21) and (1.24) in the form

$$\partial V/\partial t + V\,\partial V/\partial x = -\gamma\,\partial N/\partial x, \qquad (11.1)$$

$$\partial N/\partial t + \partial NV/\partial x = 0, \qquad (11.2)$$

where $\gamma = (4\pi e^2 z/2m_i)v_E^2/\omega_0^2$ and $v_E = eE_0/m\omega_0$.

The solution of Eqs. (11.1) and (11.2) under the initial condition

$$V(0,\ x) = 0,\quad N(0,\ x) = N^{(0)}\,\mathrm{ch}^{-2}(x/a) \qquad (11.3)$$

takes the form[223] (other types of solutions of this system are considered in[224])

$$V = (2\gamma t/a)\,N\,\mathrm{th}\,[(x - Vt)/a], \qquad (11.4)$$

$$N = [N^{(0)} - (\gamma t^2/a^2)\,N^2]\,\mathrm{ch}^{-2}\,[(x - Vt)/a], \qquad (11.5)$$

where $N^{(0)}$ is the initial plasma density at $x = 0$, and a is a quantity characterizing the initial width of the plasma layer.

The plots in Fig. 10 were obtained with aid of formulas (11.4) and (11.5), and show the variation of the dimensionless density $\nu = N/N^{(0)}$ against the coordinate $\xi = x/a$ for different values of the time $\tau = {}^1/_2\sqrt{z}\,(V_E/a\omega_0)\,t\omega_{Li}$, where $\omega_{Li}^2 = 4\pi ze^2N^{(0)}/m_i$. It is seen from the figure that the plasma layer spreads out in the course of time. The maximum density is retained at the center of the layer and is equal to

$$\nu_{\max} = \nu(0,\ \tau) = (1/2\tau)\,[(1 + 4\tau^2)^{1/2} - 1]. \qquad (11.6)$$

It is important that at a certain instant of time τ_0, which can be obtained from formulas (11.4) and (11.5), namely

$$\tau_0 \approx 1{,}6\quad (t \approx 3{,}2\sqrt{z}\,(a\omega_0/v_E)/\omega_{Li}), \qquad (11.7)$$

a density discontinuity is produced. The flow velocity of the plasma at the discontinuity point is $0.95\,(\omega_{Li}/\omega_0)v_E/\sqrt{2z}$. It must be borne in mind that Eq. (11.1) becomes meaningless at the discontinuity point, as does (1.24), since the theory considered by us is based on the assumption that the characteristic distance over which the density varies is much larger than the amplitude of the electron oscillations in the hf field.

As applied to the problem considered here, neglect

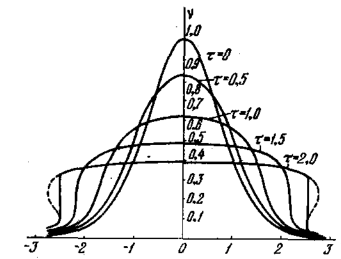

FIG. 10. Spreading of the plasma layer an hf field.

of the thermal-pressure forces, in accordance with (1.24), is justified provided the following conditions are satisfied:

$$1 \gg (T/z\mathscr{E}_E)\,\omega_0^2/\omega_{Le}^2,$$

where $\mathscr{E}_E = mv_E^2/4$ is the average energy of the electron oscillations in the hf field and $\omega_{Le}^2 = 4\pi e^2N^{(0)}/m$.

12. Acceleration and contraction of a plasma layer. In this section we consider the motion of a transparent thin plasma layer on the surface of which an electromagnetic wave is normally incident. As in the preceding section, we assume that the density depends only on a single space variable x and on the time, but the electromagnetic wave propagates not along the layer, but perpendicular to the layer, and the electric field intensity vector is directed along the 0y axis. The hf pressure force corresponding to this case can easily be obtained from (2.6). Neglecting thermal pressure, the equation of motion (1.24) then takes the form ($L_1 = \infty$)

$$\partial V/\partial t + V\,\partial V/\partial x = -\beta_0\int_{\infty}^{x} N(x',\ t)\cos 2k_0(x - x')\,dx', \qquad (12.1)$$

where $\beta_0 = (zv_E^2/2c^2)\cdot 4\pi e^2/m_i$.

By multiplying (12.1) by $N(x,\ t)$ and integrating with respect to the coordinates, we easily obtain the equation of motion of the center of gravity of the layer

$$\frac{du}{dt} \equiv \frac{d}{dt}\left[\frac{\int_{\infty}^{-\infty} N(x,\ t)\,V(x,\ t)\,dx}{\int_{\infty}^{-\infty} N(x,\ t)\,dx}\right] = \frac{z\pi e^4E_0^2}{m^2m_i\omega_0^2c^2N_s}$$

$$\times\left[\left(\int_{-\infty}^{\infty} N(x,\ t)\cos 2k_0x\,dx\right)^2 + \left(\int_{-\infty}^{\infty} N(x,\ t)\sin 2k_0x\,dx\right)^2\right], \qquad (12.2)$$

where $N_s = \int_{\infty}^{-\infty} N(x,\ t)dx$ is the time-independent number of particles integrated over the layer thickness. It is seen from (12.2) that the acceleration of the center of gravity of the layer depends on the density distribution over the cross section. In the particular case of wave lengths $k_0^{-1} = c/\omega_0$ that are larger than the layer thickness, it follows from (12.2) that the acceleration du/dt is constant and is independent of the density distribution:

$$du/dt = N_sz\pi\,(e^2/mc^2)^2\,(c^2E_0^2/m_i\omega_0^2).$$

The ensuing change in the layer configuration can be traced with the aid of Eqs. (1.21) and (12.1), which take the form

$$\partial V/\partial t + V\,\partial V/\partial x = -\beta_0\int_{\infty}^{x} N(x',\ t)\,dx',$$

$$\partial N/\partial t + \partial NV/\partial x = 0.$$

The general solutions of this system of equations are given in[40], and under the initial conditions (11.3) they are

$$\xi = w\tau + \mathrm{arcth}\,[1 - (w/\tau)], \qquad (12.3)$$

$$\nu = [\mathrm{ch}^2(\xi - w\tau) - \tau^2]^{-1}, \qquad (12.4)$$

where the following dimensionless variables are used:

$\xi = x/a,\ \nu = N/N^{(0)},\ \tau = (1/2\sqrt{z})\,(v_E/c)\,t\omega_{Li},\ w = (c\sqrt{z}/v_E)\,V/a\omega_{Li}.$

In accordance with the result (12.2), it follows from (12.3) and (12.4) that the coordinate at which the density is maximal varies in proportion to the square of the time $\xi = \tau^2$, and that the density at the maximum is equal to

$$\nu_{max} = 1/(1 - \tau^2).$$

Figure 11 shows a plot of the function $\nu(\xi, \tau)$ based on formulas (12.3) and (12.4). We see from it that the plasma layer not only accelerates as a unit, but also contracts. This is due to the fact that the force acting on the plasma is larger on the side of the incident wave.

The foregoing results are valid only so long as our assumption that the plasma is transparent, $\nu_{max} \ll m\omega_0^2/4\pi e^2 N^{(0)}$, remains in force. In addition, the conditions for neglecting the thermal pressure in this case leads to the inequality

$$1 \leqslant (za^2\omega_0^2/c^2)\,(2\mathscr{E}_E/T)\,\omega_{Le}^2/\omega_0^2.$$

13. Rarefaction wave. Let us examine how a plasma on the boundary of which a plane electromagnetic wave is normally incident spreads in a vacuum[44]. We assume that the plasma density depends only on the coordinate x and varies little over the wavelength. Then Eq. (1.24), in which the hf pressure force is given by (3.7) and in which the thermal pressure force of the plasma is neglected, and also the continuity equation (1.21), take the forms

$$\partial N/\partial t + \partial NV/\partial x = 0, \qquad (13.1)$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = -\frac{zE_0^{(0)2}}{32\pi N_c^2 m_i}\,\frac{1}{[1-(N/N_c)]^{3/2}}\,\frac{\partial N}{\partial x}. \qquad (13.2)$$

Let the quantities N and V depend only on one self-similar variable $\xi = x/t$. From (13.1) and (13.2) it follows that

$$V = \xi - s(N), \qquad (13.3)$$

where

$$s^2(N) = (zE_0^{(0)2}/32\pi N_c^2 m_i)\,N/[1-(N/N_c)]^{3/2}$$

From (13.3) we obtain with the aid of (13.1) the connection between the plasma velocity and its density:

$$V = \int [s(N)/N]\,dN. \qquad (13.4)$$

Further, using an expression derived from (13.4), we can obtain the functions $N(\xi)$ and $V(\xi)$ from (13.3). The simplest expressions are obtained in the limit of a tenuous plasma, when $(N/N_c) < 1$:

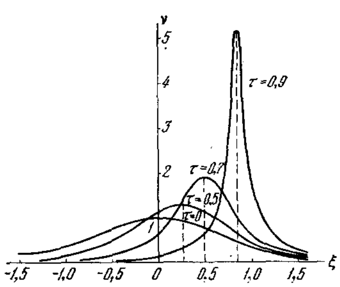

FIG. 11. Acceleration and contraction of a plasma layer.

$$N/N_0 = (1/9)\,[(\xi/s_0) + 2]^2, \qquad (13.5)$$

$$V = (2/3)\,(\xi - s_0), \qquad (13.6)$$

where $s_0^2 = E_0^{(0)2} zN_0/32\pi N_c^2 m_i$ is the propagation velocity of small perturbations (see formula (7.6) with $s^2 = 0$ and $k_0^2 \gg k^2$).

The plasma flow velocity is maximal at $N \to 0$ and is given, in accord with (13.5) and (13.6), by

$$V_{max} = -2s_0 = -\sqrt{2}\,(\omega_{Li}/\omega_0)\,(eE_0^{(0)}/m\omega_0).$$

A more general analysis of the rarefaction wave, particularly with allowance for the constant magnetic field, is contained in[44].

VI. CONCLUSION

In connection with the progress made in microwave and laser technology, the investigation of plasma properties in strong hf fields has recently become particularly timely not only from the scientific point of view, but also in practice. Naturally, one can expect to obtain the most complete and consistent description of the plasma under these conditions only with the aid of kinetic theory[10)], and the hydrodynamic theory considered in the present review is quite crude. It does not enable us, for example, to consider dissipative effects that arise in the kinetic description; the thermal motion of particles is taken into account only via the temperature, which is assumed to be constant. For many phenomena, however, the relatively simple hydrodynamic theory can yield a perfectly satisfactory description. This pertains to the questions of hydrodynamic stability of the plasma and of the parametric wave coupling, to questions of plasma equilibrium in hf fields, reflection and penetration of waves, and to questions of plasma dynamics in hf fields, which were considered in the present review. A number of the problems considered are relevant only for a plasma in which it is relatively easy to produce hf fields whose pressure exceeds the internal pressure (in this case, the thermal pressure). On the other hand, problems in which the density changes produced by the hf field are small can easily be dealt with also in connection with other transparent media. Conversely, all problems usually considered phenomenologically for arbitrary transparent media (envelope shock waves and modulation waves[226], transients occurring during instability development[227], etc.) situated in an hf field fall within the scope of the described hydrodynamic theory and pertain also to plasma. There is also a large group of problems of interest and practical importance, the solution of which is possible with the aid of the hydrodynamic theory of a tenuous plasma in a strong hf field.

*$[V_e B] = V_e \times B$.

1)The equations of the hydrodynamics of a plasma in a constant magnetic field and in hf fields were discussed in[11, 12].

2)The general case of field equations in the geometric-optics approximation, for an inhomogeneous and nonstationary medium, is considered in[43] with allowance of both the frequency and the spatial dispersions.

3)Relation (3.3) in fact determines the energy flux of the hf wave in the plasma[42]. If the hf wave is a standing wave, then M = 0 and Eq. (3.5) is not suitable. To determine the field amplitude it is necessary to use Eq. (3.4), from which the next-to-last term must be discarded (for details, see Chap. III).

4)Quasilinear effects for a plasma in an hf field were discussed in[45–50]; nonlinear interactions of waves that are unstable in an hf field were taken into account in[51, 52]; nonlinear effects connected with the action of unstable waves on the field of a pumping wave are the subject of[53, 54].

[5)]Since the frequencies of the waves depend in this case on the amplitude of the pump wave, this suggests the interesting possibility of exciting these waves parametrically by modulating the amplitude of the pump wave[57].

[6)]It should be noted that all the expressions considered by us are valid only if it is legitimate to neglect the thermal motion in the dispersion law of the Langmuir waves. As already noted in Sec. 5, this requires satisfaction of the inequality $k_0 c \gg k v_T$, which can be expressed with the aid of (6.12) in the form $\Delta \gg (m_i/m)\,[\Delta^2 + 2\Delta (q_E/q_T)^2]$.

[7)]The question of the change of plasma density under the influence of pressure of the natural surface wave is considered in[189].

[8)]In the case of wave traveling along the 0z axis, the equation for the field differs from (8.3) in that the second term on the left is preceded by a constant factor $1 - (c^2 k_{0z}^2/\omega_0^2)$, where k_{0z} is the projection of the wave vector on the 0z axis.

[9)]If the pressure of the hf field is much smaller than the thermal pressure of the plasma, then the exponential in (8.4) can be expanded and the connection between the coordinate and the field can be obtained in explicit form (see[187, 201], and also Sec. 10).

[10)]A kinetic equation for a weakly inhomogeneous plasma in an hf field, in which an averaged force acting on the particle is introduced, has been considered in[225].

[1]V. I. Veksler, Atomnaya Énergiya 2, 427 (1957).
[2]L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. 39, 1450 (1960) [Sov. Phys.-JETP 12, 1008 (1961)].
[3]S. A. Akhmanov and R. V. Khokhlov, Usp. Fiz. Nauk 88, 439 (1966) [Sov. Phys.-Usp. 9, 210 (1960)].
[4]V. S. Starunov and I. L. Fabelinskiĭ, Usp. Fiz. Nauk 98, 441 (1969) [Sov. Phys.-Usp. 12, 463 (1970)].
[5]B. B. Kadomtsev and V. I. Karpman, Usp. Fiz. Nauk 103, 193 (1971) [Sov. Phys.-Usp. 14, 40 (1971)].
[6]N. Bloembergen, Nonlinear Optics, Benjamin, 1965.
[7]S. A. Akhmanov and R. V. Khokhlov, Problemy nelineĭnoĭ optiki (Problems of Nonlinear Optics), Moscow, VINITI, 1964, Chaps. 1 and 4.
[8]I. L. Fabelinskiĭ, Molekulyarnoe rasseyanie sveta (Molecular Scattering of Light), Moscow, Nauka, 1965, Sec. 34.
[9]E. L. Lindman, Phys. Fluids 10, 396 (1967).
[10]N. Ya. Kontsarenko and A. M. Fedorchenko, Zh. Tekh. Fiz. 36, 460 (1966) [Sov. Phys.-Tech. Phys. 11, 340 (1966)].
[11]R. Klima, Zh. Eksp. Teor. Fiz. 53, 882 (1967) [Sov. Phys. JETP 26, 535 (1968)]; Czech. J. Phys. B18, 1280 (1968).
[12]R. Klima and V. A. Petrzilka, ibid., p. 1292.
[13]V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk 70, 201, 393 (1960) [Sov. Phys.-USP. 3, 115, 175 (1960)].
[14]A. V. Gurevich, Geomagn. i Aeron. 5, 70 (1965).
[15]D. T. Farlay, J. Geophys. Res. 68, 401 (1963).
[16]A. G. Litvak, 8th Intern. Conference on Phenomena in Ionized Gases. Contributed Papers, Vienna, IAEA, 1967.
[17]F. G. Bass, Yu. G. Gurevich, and M. V. Kvimsadze, Zh. Eksp. Teor. Fiz. 60, 632 (1971) [Sov. Phys.-JETP 33, 343 (1971)].
[18]A. V. Gurevich, ibid. 48, 701 (1965) [21, 462 (1965)].
[19]M. A. Liberman and A. T. Rakhimov, ibid. 61, 1047 (1971) [34, 559 (1972)].
[20]A. G. Litvak, ibid. 57, 629 (1969) [30, 344 (1970)].
[21]A. G. Litvak, Canadidate's Dissertation, Scientific Research Institute of Radiophysics (NIRFI), Gor'kiĭ, 1967.
[22]V. A. Golovko, Zh. Eksp. Teor. Fiz. 47, 1765 (1964) [Sov. Phys.-JETP 20, 1189 (1965)].
[23]V. P. Sidorov and T. R. Soldatenkov, Zh. Tekh. Fiz. 35, 1749 (1965) [Sov. Phys.-Tech. Phys. 10, 1354 (1966)].
[24]N. Ya. Kotsarenko, S. M. Levitskiĭ, A. M. Feodorchenko, and V. F. Virko, ibid. 37, 1148 (1967) [12, 827 (1967)].
[25]B. Chakraborti, Zh. Eksp. Teor. Fiz. 59, 457 (1970) [Sov. Phys.-JETP 32, 250 (1971)].
[26]A. M. Morozov and L. S. Solov'ev, in: Voprosy teorii plasmy (Problems of Plasma Theory), Vol. 2, Moscow, Gosatomizdat, 1963.
[27]A. V. Gaponov and M. A. Miller, Zh. Eksp. Teor. Fiz. 34, 242 (1958) [Sov. Phys.-JETP 7, 168 (1958)].
[28]H. Motz and C. I. H. Watson, Adv. Electr. and Electron Phys. 23, 154 (1967).
[29]Ya. L. Al'pert, A. V. Gurevich, and L. P. Pitaevskiĭ, Iskusstvennye sputniki v razrezhennoĭ plazme (Artificial Satellites in Tenuous Plasma), Moscow, Nauka, 1964, Chap. 4, Sec. 18.
[30]R. Klima and V. A. Petrzilka, Czech. J. Phys. B19, 35 (1969); Phys. Lett. A27, 20 (1968).
[31]J. Teichmann, II-eme Colloque Intern. sur Interactions entre les Champs Oscillants et les Plasmas (Saclay, 15—19 janvier 1968), v. II, P., L'Institut National des Sciences et Techniques Nucleaires-La Direction de la Physique, 1968, p. 511.
[32]T. W. Johnson, RCA Rev. 21, 4 (1960).
[33]K. S. Golovanivskiĭ and A. A. Kuzovnikov, Zh. Tekh. Fiz. 34, 1714 (1964) [Sov. Phys.-Tech. Phys. 9, 1325 (1965)].
[34]V. I. Perel' and Yu. M. Pinskiĭ, Zh. Eksp. Teor. Fiz. 54, 1889 (1968) [Sov. Phys.-JETP 27, 1014 (1968)].
[35]A. F. Aleksandrov, A. A. Kuzovnikov, N. Nikolov, and A. A. Rukhadze, Nucl. Fusion 9, 137 (1969).
[36]V. V. Sever'yanov, Zh. Tekh. Fiz. 41, 504 (1971) [Sov. Phys.-Tech. Phys. 16, 392 (1971)].
[37]L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), Moscow, Nauka, 1965, Sec. 62.
[38]L. M. Kovrizhnykh, Zh. Eksp. Teor. Fiz. 33, 72 (1957) [Sov. Phys.-JETP 6, 54 (1958)].
[39]V. V. Yankov, Zh. Tekh. Fiz. 36, 438 (1966) [Sov. Phys.-Tech. Phys. 11, 324 (1966)].
[40]L. M. Gorbunov, Zh. Eksp. Teor. Fiz. 56, 1693 (1969) [Sov. Phys.-JETP 29, 907 (1969)].
[41]L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred, (Electrodynamics of Continuous Media), Moscow, Gostekhizdat, 1957, Sec. 65.
[42]V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in Plasma), Moscow, Fizmatgiz, 1960, Sec. 16.
[43]Yu. A. Kravtsov, Zh. Eksp. Teor. Fiz. 55, 1470 (1968) [Sov. Phys.-JETP 28, 769 (1969)].
[44]L. M. Gorbunov, ibid. 54, 205 (1968) [27, 110 (1968)].
[45]V. P. Silin, ibid. 48, 1679 (1965) [21, 1127 (1965)].
[46]V. P. Silin, ibid. 56, 574 (1969) [29, 317 (1969)].
[47]A. Yu. Kiriĭ, ibid. 60, 955 (1971) [33, 517 (1971)].
[48]E. J. Valeo and C. Oberman, Bull. Am. Phys. Soc. 15, 1477 (1970).
[49]O. M. Gradov and B. M. Markeev, FIAN SSSR Preprints Nos. 35 and 36, Moscow, 1971.
[50]V. V. Pustovalov and V. P. Silin, ZhETF Pis. Red. 14, 439 (1971) [JETP Lett. 14, 299 (1971)].
[51]V. V. Pustovalov and V. P. Silin, Zh. Eksp. Teor. Fiz. 59, 2215 (1970) [Sov. Phys.-JETP 32, 1198 (1971)].
[52]E. Valeo, C. Oberman, and F. W. Perkins, Phys. Rev. Lett. 28, 310 (1972).
[53]V. E. Zakharov, V. S. L'vov, and S. S. Starobinets, Zh. Eksp. Teor. Fiz. 59, 1200 (1970) [Sov. Phys.-JETP 32, 656 (1971)].
[54]V. E. Zakharov and V. S. L'vov, Zh. Eksp. Teor. Fiz. 60, 2066 (1971) [Sov. Phys.-JETP 33, 1113 (1971)].

[55] V. P. Silin, in: A Survey of Phenomena in Ionized Gases, Vienna, IAEA, 1968; Usp. Fiz. Nauk 108, 625 (1972) [Sov. Phys.-Usp. 15, 742 (1973)].
[56] Yu. M. Aliev and V. P. Silin, Zh. Eksp. Teor. Fiz. 48, 901 (1965) [Sov. Phys.-JETP 21, 601 (1965)].
[57] Yu. M. Aliev and D. Zyunder, Zh. Eksp. Teor. Fiz. 61, 1057 (1971) [Sov. Phys.-JETP 34, 567 (1972)].
[58] S. Takamura, S. Aihara, and K. Takayama, J. Phys. Soc. Japan 31, 925 (1971).
[59] R. A. Stern and N. Tzoar, Phys. Rev. Lett. 17, 903 (1966).
[60] A. V. Wong, M. V. Goldman, F. Mai, and R. Rowberg, ibid. 21, 518 (1968).
[61] H. Kubo and S. Nakawa, J. Phys. Soc. Japan 22, 1304 (1967).
[62] S. Hiroi and H. Ikegami, Phys. Rev. Lett. 19, 1414 (1967).
[63] I. R. Gekker and O. V. Sizukhin, ZhETF Pis. Red. 9, 408 (1969) [JETP Lett. 9, 243 (1969)].
[64] G. M. Batanov, K. A. Sarksyan, and V. A. Silin, FIAN SSSR Preprint No. 7, Moscow, 1968.
[65] K. F. Sergeĭchev, FIAN SSSR Preprint No. 61, Moscow, 1969.
[66] H. Dreicer, D. B. Henderson, and I. C. Ingrahm, Phys. Rev. Lett. 26, 1616 (1971).
[67] H. Eubank, Phys. Fluids 14, 2551 (1971).
[68] A. M. Messian and P. E. Vandeuplas, Phys. Lett. A25, 389 (1967).
[69] M. Fitaire and A. M. Pointa, 9th Intern. Conference on Phenomena in Ionized Gases, Contributed Papers, Bucharest, Academy of Science, 1969.
[70] M. Porkolab and R. P. H. Chang, Phys. Rev. Lett. 22, 826 (1969); Phys. Fluids 43, 2766 (1970).
[71] R. A. Demirkhanov, G. L. Kharasanov, and I. K. Sidorova, Zh. Eksp. Teor. Fiz. 59, 1873 (1970) [Sov. Phys.-JETP 32, 1013 (1971)].
[72] R. Sugaya, K. Mizuno, and S. Tanaka, J. Phys. Soc. Japan 30, 253 (1971).
[73] D. Phelps, S. Ryme, and G. von Hoven, Phys. Rev. Lett. 26, 1688 (1971).
[74] W. L. Kruer, P. K. Kaw, J. M. Dawson, and C. O. Oberman, ibid. 24, 987 (1970).
[75] W. L. Kruer, P. K. Kaw, and J. M. Dawson, Bull. Am. Phys. Soc. 15, 1407 (1970).
[76] I. Katz and J. S. de Groot, Bull. Am. Phys. Soc. 15, 1472 (1970).
[77] W. L. Kruer and J. M. Dawson, Phys. Rev. Lett. 25, 1174 (1970).
[78] M. Goldman, Ann. Phys. (N.Y.) 88, 95, 117 (1966).
[79] D. F. DuBois and M. Goldman, Phys. Rev. Lett. 14, 544 (1965).
[80] Y. C. Lee and C. H. Su, Phys. Rev. 152, 129 (1966).
[81] E. A. Jackson, ibid. 153, 235 (1967).
[82] D. F. DuBois and M. Goldman, ibid. 164, 207.
[83] K. Nishikawa, Progr. Theor. Phys. 37, 769 (1967).
[84] K. Nishikawa, J. Phys. Soc. Japan 24, 1152 (1968).
[85] N. E. Andreev, A. Yu. Kiriĭ, and V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 1024 (1969) [Sov. Phys.-JETP 30, 559 (1970)].
[86] V. P. Silin, ZhETF Pis. Red. 7, 242 (1968) [JETP Lett. 7, 187 (1968)].
[87] D. F. DuBois, in: Statistical Physics of Charged Particles Systems, ed. by R. Kubo and T. Kihara, N. Y., W. A. Benjamin, 1969.
[88] I. R. Sunmartin, Phys. Fluids 13, 1533 (1970).
[89] V. N. Oraevskiĭ and R. Z. Sagdeev, Zh. Tekh. Fiz. 32, 1291 (1962) [Sov. Phys.-Tech. Phys. 7, 955 (1963)].
[90] V. N. Oraevskiĭ, Doctorate Dissertation, Nucl. Phys. Inst. Ukrainian Acad. Sci., 1969.
[91] N. Bloembergen, Am. J. Phys. 35, 989 (1967).
[92] A. Sjölund and L. Stenflo, J. Appl. Phys. 38, 2676 (1967).
[93] K. Nishikawa, J. Phys. Soc. Japan 24, 916 (1968).
[94] T. F. Volkov, in: Fizika plazmy i problemy upravlyaemykh termoyadernykh reaktsiĭ (Plasma Physics and Problems of Controlled Thermonuclear Reactions), Vol. 4, Moscow, Izd-vo AN SSSR, 1958.
[95] L. M. Gorbunov and V. P. Silin, Zh. Eksp. Teor. Fiz. 49, 1973 (1965) [Sov. Phys.-JETP 22, 1347 (1966)].
[96] D. Montgomery and I. Alexeff, Phys. Fluids 9, 2075 (1966).
[97] R. Prasad, ibid. 11, 1768 (1968).
[98] K. P. Das, J. Phys. Soc. Japan 28, 1541 (1970).
[99] L. M. Gorbunov, Zh. Eksp. Teor. Fiz. 55, 2298 (1968) [Sov. Phys.-JETP 28, 1220 (1969)].
[100] A. Yu. Kiriĭ, ibid. 58, 1002 (1970) [31, 538 (1970)].
[101] N. E. Andreev, ibid., p. 2099 [1132].
[102] A. G. Litvak and V. Yu. Trakhtengerts, ibid. 60, 1702 (1971) [ibid. 33, 921 (1971)].
[103] V. E. Zakharov, V. V. Sobolev, and V. S. Synakh, ibid., p. 136 [77].
[104] V. Ts. Gurovich and V. I. Karpman, ibid. 56, 1952 (1969) [29, 1048 (1969)].
[105] Yu. M. Aliev, V. P. Silin, and H. Watson, ibid. 50, 943 (1966) [23, 626 (1966)].
[106] I. Amano and H. Okamoto, J. Phys. Soc. Japan 23, 1432 (1967).
[107] V. I. Domrin, Zh. Tekh. Fiz. 38, 1259 (1968) [Sov. Phys.-Tech. Phys. 13, 1036 (1969)].
[108] T. Ohinuma and Y. Hatta, Phys. Lett. D26, 620 (1968).
[109] Yu. M. Aliev and D. Zyunder, Zh. Eksp. Teor. Fiz. 57, 1324 (1969) [Sov. Phys.-JETP 30, 718 (1970)].
[110] N. Tzoar, Phys. Rev. 178, 356 (1969).
[111] O. M. Gradov and D. Zyunder, Zh. Eksp. Teor. Fiz. 58, 624 (1970) [Sov. Phys.-JETP 31, 335 (1970)].
[112] N. E. Andreev, Izv. Vuzov (Radiofizika) 14, 1160 (1971).
[113] N. E. Andreev and A. Yu. Kiriĭ, Zh. Tekh. Fiz. 41, 1080 (1971) [Sov. Phys.-Tech. Phys. 16, 854 (1971)].
[114] B. M. Markeev, PMTF, No. 5, 12 (1971).
[115] D. Zyunder and A. A. Rukhadze, Izv. Vuzov (Radiofizika) 14, 36 (1971).
[116] Y. Pomean, Phys. Fluids 10, 2695 (1967).
[117] L. M. Gorbunov and V. P. Silin, Zh. Tekh. Fiz. 39, 3 (1969) [Sov. Phys-Tech. Phys. 14, 1 (1969)].
[118] N. E. Andreev, ibid., p. 1650 [1237 (1970)].
[119] A. G. Litvak, Magnitn. gidrodin. 1, 30 (1965).
[120] C. S. Chen, and G. J. Lewak, J. Plasma Phys. 4, 357 (1970).
[121] M. V. Goldman, Phys. Fluids 13, 1281 (1970).
[122] R. Prasad, ibid., p. 1310.
[123] K. P. Das, J. Phys. Soc. Japan 28, 1310 (1970).
[124] R. Prasad, J. Plasma Phys. 5, 291 (1971).
[125] Yu. M. Aliev and É. Ferlengi, Zh. Eksp. Teor. Fiz. 57, 1623 (1969) [Sov. Phys.-JETP 30, 877 (1970)].
[126] Yu. M. Aliev, O. M. Gradov, and A. Yu. Kiriĭ, ibid. 63, 112 (1972) [36, 59 (1973)].
[127] A. L. Berkhoer and V. E. Zakharov, ibid. 58, 903 (1970) [31, 486 (1970)].
[128] N. L. Tsintsadze, ibid. 59, 1251 (1970) [32, 684 (1971)].
[129] I. H. Krenz and G. S. Kino, J. Appl. Phys. 36, 2387 (1965).
[130] A. Takeo, J. Phys. Soc. Japan 22, 1282 (1967).
[131] I. Okutani, ibid. 23, 110.

[132] R. R. Ramazashvili, Zh. Eksp. Teor. Fiz. 53, 2168 (1967) [Sov. Phys.-JETP 26, 1225 (1968)].
[133] T. Amano and M. Okamoto, J. Phys. Soc. Japan 26, 529 (1969).
[134] M. Yomaduchi, S. Hagimawa, and S. Kojima, ibid. 25, 1743 (1968).
[135] T. Hiroshi and W. Shinoune, J. Phys. Soc. Japan 24, 969 (1968).
[136] K. Kato and M. Yoseli, ibid. 23, 671 (1967).
[137] M. A. Hassen and P. W. McDonald, Phys. Fluids 11, 2775 (1968).
[138] A. Tanaca and S. Watanabe, J. Phys. Soc. Japan 24, 969 (1968).
[139] K. Kato, M. Yoseli, S. Kijama, and S. Watanabe, ibid. 20, 2097 (1965).
[140] C. S. Chen, J. Plasma Phys. 5, 107 (1971).
[141] Y. Amagishi, K. Yamagiwa, H. Kozima, and K. Kato, Phys. Lett. A36, 241 (1971).
[142] F. W. Perkins and I. Flivk, Phys. Fluids 14, 2012 (1971).
[143] A. D. Piliya, in: Konferentsiya po teorii plazmy. Annotatsii dokladov (Conference on Plasma Theory. Abstracts of Papers), Kiev, 1971.
[144] M. Porkolab and R. P. H. Chang, Phys. Fluids 13, 2054 (1970).
[145] V. P. Silin, Zh. Eksp. Teor. Fiz. 51, 1842 (1966) [Sov. Phys.-JETP 24, 1242 (1967)].
[146] E. Rebhan, Phys. Fluids 12, 192 (1969).
[147] V. P. Silin, Zh. Tekh. Fiz. 37, 989 (1967) [Sov. Phys.-Tech. Phys. 12, 712 (1967)].
[148] P. L. Nanez, Phys. Fluids 14, 633 (1971).
[149] Ya. B. Faĭnberg and V. D. Shapiro, Zh. Eksp. Teor. Fiz. 52, 293 (1967) [Sov. Phys.-JETP 25, 189 (1967)].
[150] V. D. Shapiro, Doctorate dissertation (FTI AN USSR, 1967).
[151] J. Teichmann, Phys. Lett. A26, 328 (1968).
[152] R. A. Demirkhanov, T. I. Gutkin, and S. N. Lozovskiĭ, Zh. Eksp. Teor. Fiz. 55, 2195 (1968) [Sov. Phys.-JETP 28, 1164 (1969)].
[153] R. A. Demirkhanov, G. L. Kharasanov, and I. K. Sidorova, ZhETF Pis. Red. 6, 861 (1967) [JETP Lett. 6, 300 (1967)].
[154] D. Lepechinsky, P. Rolland, and J. Teichmann, Nucl. Fusion 11, 297 (1971).
[155] T. I. Gutkin and G. I. Boleslavskaya, Ukr. Fiz. Zh. 16, 1415 (1971); Nucl. Fusion 11, 167 (1971).
[156] S. Aihara and S. Takamura, Appl. Phys. Lett. 18, 375 (1971).
[157] M. Okamoto, T. Amano, K. Kitao, J. Phys. Soc. Japan 29, 1041 (1970).
[158] L. M. Gorbunov and V. P. Silin, Yadernyĭ sintez 6, 118 (1966).
[159] Yu. S. Kostyukova and F. M. Nekrasov, Nucl. Fusion 10, 277 (1970).
[160] F. M. Nekrasov, in: Fizika plazmy i upravlyaemye termoyadernye reaktsii (Plasma Physics and Controlled Thermonuclear Reactions), FTI GKAE Preprint No. 3, Sukhumi, 1970, pp. 162 and 165.
[161] Ya. B. Faĭnberg, Usp. Fiz. Nauk 93, 593 (1967) [Sov. Phys.-Usp. 10, 737 (1968)].
[162] A. A. Vedenov, Fizika plazmy (Plasma Physics), Moscow, VINITI, 1965, Chap. 6.
[163] A. S. Bakaĭ, Zh. Eksp. Teor. Fiz. 55, 266 (1968). [Sov. Phys.-JETP 28, 140 (1969)].
[164] V. E. Zakharov and A. M. Rubenchik, PMTF No. 5, 84 (1972).
[165] B. B. Kadomtsev, see ref. 26, Vol. 4, 1964.
[166] V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasma-Like Media), Moscow, Atomizdat, 1961, Sec. 14.
[167] L. M. Kovrizhnykh, Tr. FIAN 32, 173 (1965).
[168] R. Z. Sagdeev, see ref. 26, Vol. 4, 1964.
[169] L. M. Gorbunov, FIAN SSSR Preprint No. 174, Moscow, 1969.
[170] E. B. Knox, Austr. J. Phys. 10, 565 (1957).
[171] T. F. Volkov, see ref. 94, Vol. 3, p. 336.
[172] R. Z. Sagdeev, ibid., p. 346.
[173] M. A. H. Boot, S. A. Self, R. B. R.-Shersby-Harvie, J. Electron. and Control 4, 434 (1958).
[174] E. S. Weibel, ibid. 5, 435.
[175] R. T. P. Whipple, Rept. A.E.R.E. R 2917, London, 1959.
[176] S. A. Self, Phys. Fluids 3, 488 (1960).
[177] R. B. R.-Shersby-Harvie, J. Electron. and Control 8, 421 (1960).
[178] H. Motz, Phys. Fluids 6, 308 (1963).
[179] T. W. Johnston, RCA Rev. 21, 570 (1960).
[180] Yu. A. Berezin, T. I. Gutkin, S. N. Lozovskiĭ, and T. R. Soldatenkov, Zh. Tekh. Fiz. 34, 448 (1964) [Sov. Phys.-Tech. Phys. 9, 350 (1964)].
[181] R. Klima, Czech. J. Phys. B16, 16 (1966).
[182] G. A. Askar'yan, Zh. Eksp. Teor. Fiz. 42, 1567 (1962) [Sov. Phys.-JETP 15, 1088 (1962)].
[183] B. I. Bespalov, A. G. Litvak, and V. I. Talanov, in: Nelineĭnaya optika (Trudy 2-go Vsesoyuznogo seminara po nelineĭnoi optike) (Trans. Second All-Union Seminar on Nonlinear Optics) Novosibirsk, Nauka, 1968; S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, ibid. p. 348.
[184] Yu. A. Brodskiĭ, B. G. Eremin, A. G. Litvak, and Yu. A. Sakhanchik, ZhETF Pis. Red. 13, 136 (1971) [JETP Lett. 13, 95 (1971)].
[185] G. M. Batanov and V. A. Silin, ibid. 14, 445 (1971) [14, 303 (1971)].
[186] V. I. Talanov, Izv. Vuzov (Radiofizika) 7, 564 (1964); Yu. R. Alanakyan, Zh. Eksp. Teor. Fiz. 55, 1338 (1968) [Sov. Phys.-JETP 28, 700 (1969)]; A. G. Litvak, Izv. Vuzov (Radiofizika) 8, 1148 (1965); 9, 629, 900 (1966).
[187] V. P. Silin, Zh. Eksp. Teor. Fiz. 53, 1662 (1967) [Sov. Phys.-JETP 26, 955 (1968)].
[188] R. B. Hall and A. Gerwin, Phys. Rev. A3, 1151 (1971).
[189] Yu. R. Alanakyan, Zh. Tekh. Fiz. 37, 817 (1967) [Sov. Phys.-Tech. Phys. 12, 587 (1967)].
[190] V. M. Eleonskiĭ and V. P. Silin, ZhETF Pis. Red. 13, 167 (1971) [JETP Lett. 13, 117 (1971)]. Zh. Eksp. Teor. Fiz. 60, 1927 (1971) [Sov. Phys.-JETP 33, 1039 (1971)].
[191] V. V. Demchenko and A. Ya. Omel'chenko, Ukr. Fiz. Zh. 15, 2065 (1970).
[192] V. V. Demchenko and V. V. Dolgopolov, Izv. Vuzov (Radiofizika) 12, 504 (1969); Zh. Tekh. Fiz. 38, 1856 (1968); 41, 839 (1971) [Sov. Phys.-Tech. Phys. 13, 1495 (1969); 16, 661 (1971)].
[193] V. V. Demchenko, V. V. Dolgopolov, and A. Ya. Omel'chenko, FTI AN USSR Preprint 70-57, Khar'kov, 1970.
[194] V. A. Mironov, Izv. Vuzov (Radiofizika) 12, 1765 (1969); 14, 1450 (1971).
[195] V. V. Demtchenko and A. M. Hussein, J. Plasma Phys. 13, 996 (1971).
[196] A. V. Gurevich and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. 45, 1243 (1963) [Sov. Phys.-JETP 18, 855 (1964)].
[197] V. B. Gil'denburg, ibid. 46,2156 (1964) [19,1456 (1964)].

[198] V. V. Golgopolov, Zh. Tekh. Fiz. 36, 273 (1966) [Sov. Phys.-Tech. Phys. 11, 198 (1966)].
[199] V. B. Gildenburg, I. Y. Kondrat'ev, and M. A. Miller, 8th Intern. Conference on Phenomena in Ionized Gases. Contributed Papers, Vienna, IAEA, 1967.
[200] V. B. Gil'denburg, I. G. Kondrat'ev, and G. A. Markov, Izv. Vuzov (Radiofizika) 12, 655 (1969).
[201] V. V. Demchenko and V. V. Dolgopolov, Zh. Tekh. Fiz. 38, 1843 (1968) [Sov. Phys.-Tech. Phys. 13, 1486 (1969)].
[202] R. R. Ramazashvili, Kr. Soobshch. fiz. (FIAN SSSR), No. 1, 15 (1971).
[203] V. Ts. Gurovich and V. I. Karpman, IYaF SO AN SSSR Preprint No. 310, Novosibirsk, 1969.
[204] V. I. Karpman, J. Plasma Phys. 13, 477 (1971).
[205] V. V. Vas'kov and V. I. Karpman, Zh. Tekh. Fiz. 41, 676 (1971) [Sov. Phys.-Tech. Phys. 16, 529 (1971)].
[206] B. A. Trubnikov, ZhETF Pis. Red. 14, 472 (1971) [JETP Lett. 14, 322 (1971)].
[207] V. E. Zakharov, Zh. Eksp. Teor. Fiz. 53, 1735 (1967) [Sov. Phys.-JETP 26, 994 (1968)].
[208] V. I. Domrin and R. R. Ramazashvili, Kr. soobshch. fiz. (FIAN SSSR), No. 7, 62 (1970).
[209] V. V. Vaskov and V. I. Karpman, Nucl. Fusion 11, 215 (1971).
[210] V. Ts. Gurovich, V. I. Karpman, and R. N. Kaufman, IYaF SO AN SSSR Preprint No. 262, Novosibirsk, 1968.
[211] V. M. Eleonskiĭ and V. P. Silin, Zh. Eksp. Teor. Fiz. 56, 574; 57, 478 (1969) [Sov. Phys.-JETP 29, 317 (1969); 30, 262 (1970)].
[212] V. I. Karpman, ZhETF Pis. Red. 9, 480 (1969) [JETP Lett. 9, 291 (1969)].
[213] V. I. Veksler, Proc. of the Symposium on High-Energy Accelerators and Pion Physics, v. 1, Vienna, IAEA, 1956.
[214] M. L. Levin, Trudy RTI AN SSSR 1, No. 1, 5 (1969).
[215] G. A. Askar'yan, M. L. Iovnovich, M. L. Levin, and M. S. Rabinovich, Nucl. Fusion Suppl. 2, 797 (1962).
[216] M. L. Levin, Tr. TRI AN SSSR 1, No. 2, 36 (1959).
[217] M. A. Miller, Zh. Eksp. Teor. Fiz. 36, 1909 (1959) [Sov. Phys.-JETP 9, 1358 (1959)].
[218] V. I. Veksler, I. R. Gekker, É. Ya. Gol'ts, B. P. Kononov, G. S. Luk'yanchikov, M. S. Rabinovich, K. A. Sarksyan, K. F. Sergeĭchev, V. A. Silin, and L. É. Tsopp, Tr. FIAN 32, 60 (1966).
[219] R. Z. Muratov, Candidate's Dissertation (FIAN, 1969).
[220] M. L. Levin and R. Z. Muratov, Zh. Tekh. Fiz. 39, 1712 (1969) [Sov. Phys.-Tech. Phys. 14, 1285 (1970)].
[221] A. V. Gurevich and V. P. Silin, Yad. Fiz. 2, 250 (1965) [Sov. J. Nucl. Phys. 2, 179 (1966)].
[222] V. I. Domrin and R. R. Ramazashvili, ibid. 5, 312 (1967) [5, 219 (1967)].
[223] S. A. Akhmanov, A. P. Sukhorukov, and R. V. Khokhlov, Usp. Fiz. Nauk 93, 19 (1967) [Sov. Phys.-Usp. 10, 609 (1968)].
[224] A. V. Gurevich and A. B. Shvartsburg, Zh. Eksp. Teor. Fiz. 58, 2012 (1970) [Sov. Phys.-JETP 31, 1084 (1970)].
[225] A. G. Litvak, M. I. Petelin, and E. I. Yakubovich, Zh. Tekh. Fiz. 35, 108 (1965) [Sov. Phys.-Tech. Phys. 10, 81 (1965)].
[226] L. A. Ostrovskiĭ, see ref. 183, p. 301; A. V. Gaponov, L. A. Ostrovskiĭ, and M. I. Rabinovich, Izv. Vuzov (Radiofizika) 13, 163 (1970).
[227] N. M. Kroll, J. Appl. Phys. 36, 34 (1965); Chen-Show Wang, Phys. Rev. 182, 482 (1969); L. M. Gorbunov, in: ref. 143; Zh. Eksp. Teor. Fiz. 62, 2141 (1972) [Sov. Phys.-JETP 35, 1119 (1972)].

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