The interaction of slow antinucleons with nucleons and nuclei

I. S. Shapiro
Institute of Theoretical and Experimental Physics, Moscow
Usp. Fiz. Nauk 109, 431-454 (March 1973)

The current data on scattering and annihilation of nonrelativistic antinucleons are reviewed. A description is given of the theoretical ideas on the relation between the nucleon-antinucleon and nucleon-nucleon interactions, which make it possible to organize the experimental data on the elastic scattering of nucleons into a picture of the forces which act between the nucleon and antinucleon. The main conclusion from the existing experimental facts and theoretical considerations is that we should expect a strong attraction due to the exchange of a neutral vector meson (the omega meson) in the nucleon-antinucleon system at distances of the order of 1–2 F. Calculations show that this attraction leads to the existence of bound and resonant nuclear-like states in the nucleon-antinucleon system, which should manifest themselves experimentally as heavy mesons of mass close to the mass of two nucleons (quasinuclear mesons). The lifetime of such mesons is determined by the probability of the annihilation process (which occurs at small distances of the order of the nucleon Compton wavelength and consequently has a small effect on the production of the quasinuclear state). A distinctive feature of quasinuclear mesons is their comparatively large probability of decay (virtual or real) into the nucleon-antinucleon channel. For this reason, the production of quasinuclear mesons should be seen in collisions of antinucleons with protons and nuclei. Another characteristic consequence of the existence of quasinuclear mesons is an anomalous energy dependence of the cross section for annihilation of slow antinucleons. The theoretical predictions are in agreement with the latest experimental results, which are discussed in detail in the paper.

CONTENTS

1. Introduction 173
2. The NN and N$\bar{\text{N}}$ Interaction Potentials 173
3. Annihilation 175
4. Quasinuclear Mesons 177
5. The Production of Quasinuclear Mesons by Slow Antinucleon Beams 179
6. Conclusions 182
Cited Literature 183

1. INTRODUCTION

In the present review we consider the collisions of nonrelativistic antinucleons ($\bar{\text{N}}$) with nucleons (N). The restriction to low energies enables us to apply the potential approach to the N$\bar{\text{N}}$ interaction, in spite of annihilation. This in turn makes it possible to ascertain theoretically certain properties of systems containing nucleons and antinucleons. The question as to whether there exist bound quasi-nuclear N$\bar{\text{N}}$ states is of particular interest in this respect. As we shall show, one should expect many such states to exist (approximately twenty) precisely because the forces of attraction are comparatively weak in the two-nucleon system, which has only a single bound state—the deuteron.

2. THE NN AND N$\bar{\text{N}}$ INTERACTION POTENTIALS

The relation between the electron-electron (e^-e^-) and electron-positron (e^-e^+) electromagnetic interactions is well known: they differ in sign. This is a consequence of the fact that the photon, which is exchanged by particles when they interact electromagnetically (Fig. 1), has charge parity $C = -1$. The repulsion of like charges is due to the vector character of the electromagnetic field, i.e., the fact that the spin and spatial parity of the photon are $J^P = 1^-$, as well as the fact that it is neutral[1)]. Thus, the complete set of quantum numbers of the photon determines the signs of the e^-e^- and e^-e^+ interactions.

The nuclear interaction of nucleons occurs as a result of the exchange of mesons (Fig. 2), and so the sign of the interaction in the NN and N$\bar{\text{N}}$ channels is determined by the quantum numbers of these mesons. However, the nuclear interaction differs from the electromagnetic interaction in three respects which are of interest to us here. First of all, we may regard it as established that mesons with various quantum numbers take part in producing the NN forces. Secondly, since the nuclear interaction is much stronger than the electromagnetic interaction, multi-meson exchanges can contribute appreciably to the nuclear potential (a two-meson exchange is shown in the diagram of Fig. 2b). Thirdly, the interacting nucleons exchange not only neutral but also charged mesons; the relation between the NN and N$\bar{\text{N}}$ potentials is therefore determined not by charge parity but by the so-called G-parity, which indicates how the sign of the meson field changes under the simultaneous action of two transformations—charge conjugation (conversion of a particle into an antiparticle, e.g., a π^+ meson into a π^- meson) and an isotopic rotation through 180° (e.g., $\pi^+ \rightleftarrows -\pi^-$, $\pi^0 \to -\pi^0$). For neutral particles with isospin I, the G- and C-particles are related by the equation

$$G = (-1)^I C.$$

The G-parity of a system of several mesons is equal to the product of the G-parities of the individual mesons; consequently, the G-parity of a potential corresponding to the exchange of a fixed number of mesons is as well determined as for single-meson exchange. If the interaction is due to the exchange of any set of mesons X

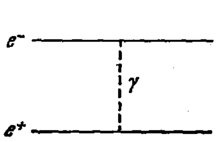

FIG. 1

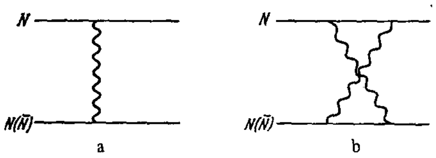

FIG. 2

FIG. 1. Photon exchange—the electromagnetic interaction between electrons.

FIG. 2. The NN and NN̄ nuclear interaction—single-meson (a) and two-meson (b) exchanges.

with G-parities G_X, then the relation between the parts of the potentials V_X and $\overline{V}_X$ corresponding to this exchange for the NN and NN̄ interactions is expressed by the equation[2)]

$$\overline{V}_X = G_X V_X. \tag{1}$$

The main experimental information which is used to determine the potential V is the NN elastic scattering data at nonrelativistic energies and the properties of the deuteron. For the scattering of protons by protons and neutrons up to kinetic energies $E_L = 300$ MeV in the laboratory system, there now exists the "complete experiment": measurements of all the required angular and spin correlations, making it possible to determine the scattering amplitude as a function of the kinematic variables—the energies of the colliding particles and the momentum transfer (see[1])[3)]. In principle it would be possible to uniquely reconstruct the potential (the inverse problem of scattering theory) from data of this kind, if they were very accurate. Unfortunately, the solution of the inverse problem is unstable—there exists a family of potentials which give a scattering amplitude in agreement with the experimental one within the errors of the experiments which have actually been performed (see the reviews[1,3] for a list of such potentials). The majority of these potentials have been constructed by means of a formal (fitted) parametrization of the invariant nonrelativistic interaction Hamiltonian of two identical particles with spins and isospins equal to ½. Of course, such potentials cannot be used for the transition to the NN̄ channel, since we do not know the G-parities of the terms which enter them.

Potentials which correspond to the exchange of the mesons which actually exist are free from this defect. In practice, one considers single-meson exchanges. As is well known, the range of the forces due to the exchange of a meson of mass μ_X is equal to $1/\mu_X$ (we take $\hbar = c = 1$). As to multi-meson exchange, one is guided here by the following considerations. The range corresponding to the exchange of n mesons is $1/n\mu_X$. On the other hand, since it is meaningless to consider very small distances (less than $1/m$, where m is the nucleon mass) in the nonrelativistic approximation, one usually takes into account only two- and three-meson exchanges of the lightest mesons, namely pions ($\mu_\pi \equiv \mu = 140$ MeV). For simplicity, the two- and three-pion forces are approximated by single-meson exchanges of fictitious particles σ_0 and σ_1 (the masses of these "mesons" and their coupling constants with nucleons are adjustable parameters to be chosen so as to obtain the best agreement between the theoretically calculated scattering amplitude and the experimental data). As to the G-parities of σ_0 and σ_1, they are obviously equal to the G-parities of the two- and three-pion systems. Since $G_\pi = -1$, we have $G_{\sigma_0} = 1$ and $G_{\sigma_1} = -1$. The spins and parities of the σ_0 and σ_1 mesons are taken to be equal to 0^+, and the isospins are 0 and 1, respectively. The remaining particles which contribute to the single-meson exchange potential exist in the free state, and their properties (quantum numbers and masses) are known from independent experiments not related to NN scattering. In addition to the above-mentioned fictitious particles σ_0 and σ_1, one usually also allows for the exchange of the mesons π, η, ρ and ω in the interaction potential of nonrelativistic nucleons. The part of the potential corresponding to the exchange of a scalar meson ($X = \sigma_0, \sigma_1$) is of the form

$$V_S = -g_X^2\left[\left(1-\frac{\mu_X^2}{4m^2}\right) - \mathbf{LS}\,\frac{1}{r}\frac{d}{dr}\right]\frac{e^{-\mu_X r}}{r}; \tag{2}$$

here g_X is the coupling constant of the meson X with a nucleon corresponding to the field Lagrangian

$$\mathcal{L} = g_X (4\pi)^{-1/2} (\bar{N}N)\, X,$$

r is the distance between the nucleons, **L** is the orbital angular momentum operator for the relative motion of the nucleons, and **S** is the spin operator

$$\mathbf{S} = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2,$$

where σ_1 and σ_2 are the Pauli matrices which operate on the spin variables of each of the nucleons. Similarly, for pseudoscalar mesons ($X = \pi, \eta$), we have

$$\begin{gathered} V_X = g_X^2\left(\frac{\mu_X}{2m}\right)^2\left\{\frac{1}{3}\sigma_1\sigma_2\left[\frac{1}{3}+\frac{1}{\mu_X r}+\left(\frac{1}{\mu_X r}\right)^2\right]S_{12}\right\}\frac{e^{-\mu_X r}}{r}, \\ \mathcal{L} = (g_X/\sqrt{4\pi})\,(\bar{N}\gamma_5 N)\,X. \end{gathered} \tag{3}$$

In this equation, S_{12} is the well-known tensor operator

$$S_{12} = 3\,(\sigma_1\hat{\mathbf{r}})\,(\sigma_2\hat{\mathbf{r}}) - \sigma_1\sigma_2,\quad \hat{\mathbf{r}} = \mathbf{r}/r.$$

For vector mesons ($X = \rho, \omega$), the Lagrangian and the potential have the following form:

$$\begin{aligned} \mathcal{L} &= \frac{g_X}{\sqrt{4\pi}}\,(\bar{N}\gamma_\mu N)\,X_\mu + \frac{f_X}{4m\sqrt{4\pi}}\,(\bar{N}\sigma_{\mu\nu}N)\,(\partial_\nu X_\mu - \partial_\mu X_\nu), \\ \sigma_{\mu\nu} &= i\,(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2, \\ V_X &= g_X^2\left\{1+\left(1+\frac{f_X}{g_X}\right)\frac{\mu_X^2}{2m^2}+\left(1+\frac{f_X}{g_X}\right)^2\frac{\mu_X^2}{6m^2}\,\sigma_1\sigma_2\right. \\ &\quad + \left(\bar{1}+\frac{4}{3}\frac{f_X}{g_X}\right)\frac{1}{6m^2}\,\mathbf{LS}\,\frac{1}{r}\frac{d}{dr} - \left(1+\frac{f_X}{g_X}\right)^2 \\ &\quad \left.\times\left[\frac{1}{3}+\frac{1}{\mu_X r}+\left(\frac{1}{\mu_X r}\right)^2\right]S_{12}\right\}\frac{e^{-\mu_X r}}{r}. \end{aligned} \tag{4}$$

For brevity, we have omitted the isospin operators of the nucleons and the isotopic indices of the meson field in Eqs. (2)—(4). If they are retained, the NN interaction potential which takes into account all the single-meson exchanges can be written in the form

$$V = U_0 + \boldsymbol{\tau}_1\boldsymbol{\tau}_2 U_1; \tag{5}$$

here $\tau_1/2$ and $\tau_2/2$ are the isospin operators of the nucleons, and U_0 and U_1 are the sums of the potentials (2)—(4) for isoscalar (η, σ_0, ω) and isovector (π, σ_1, ρ) mesons, respectively. Information on the coupling constants g_X and f_X for the mesons π, η, ρ and ω can be obtained not only from NN scattering data, but also from other experiments. Possible sets of values of these constants which give a satisfactory description of the scattering of nonrelativistic nucleons and which are consistent with other experimental facts are shown in Table I (we also give here the cut-off radius r_0 of the singular terms $\sim r^{-3}$[4)]). The transition from the potential (5) to the interaction potential $\overline{V}$ is carried out according to Eq. (1), allowing for the G-parities of the mesons which participate in the process ($G_\eta = G_{\sigma_0} = G_\rho = +1$, $G_\pi = G_{\sigma_1} = G_\omega = -1$).

From among the mesons which we have listed, the ω meson plays a special role in the interaction of nucleons,

TABLE I. OBEP*) parameters

Meson	Parameters	From [4]	From [23]
π	g^2	11.7	14.4
η	g^2	7.0	2.0—9.9
0+ (0+)	M, MeV	560	400; 700
σ_0	g^2	9.4	1.4; 5.7—7.0
σ_1	g^2	6.1	—
1⁻ (0+)	M, MeV	770	—
ρ	g^2	0.68	0.6—0.75
	f/g	4.4	4.4—5.0
ω	g^2	21.5	8.2—9.0
	f/g	0	0.1
	r_0, F	0.6	0.2

*OBEP is an abbreviation of the English "One Boson Exchange Potential".

owing to its large coupling constant $g_\omega^2 \approx 20$ (it is worth stressing that, in contrast with the pseusoscalar mesons, this constant enters the potential without the small factor $(\mu\chi/2m)^2$). This meson has the same quantum numbers as the photon and, consequently, gives a strong repulsion in the NN channel, which at distances less that 0.5 F is not compensated by the other terms in the potential. The total interaction becomes attractive at large distances, but this attraction still proves to be comparatively weak—it is hardly sufficient for a single discrete level. The short-range repulsion shows up experimentally most clearly in the energy dependence of the s-wave phase shifts in NN scattering; crudely speaking, they behave as if these waves experienced diffraction by an impermeable repulsive barrier (the phase shift is $\delta_S \approx -ka$, where k is the wave number and a is the radius of the barrier). We also note that the change of sign (with increasing distance) of the NN interaction potential is also required for the explanation of another fundamental fact—the saturation of the nuclear forces (i.e., the fact that nuclei have an approximately constant binding energy per nucleon, see[3]).

At the present time, we can scarcely doubt that there exists a repulsion between nucleons at distances of the order of 0.5 F. In the potential considered above, this repulsion is guaranteed primarily by the exchange of the ω meson. But there must then be a strong attraction in the $N\bar{N}$ channel (at least at the same distances). An attraction in the $N\bar{N}$ system due to the combined effect of all the meson exchanges actually occurs up to distances of the order of 1—2 F for most of the s, p and d states. For the above-mentioned reason, the effective potential well for the $N\bar{N}$ interaction (at the same range) turns out to be much deeper (by about a factor of 5) than for the NN system. We should therefore expect the spectrum of $N\bar{N}$ bound states to be much richer than for two nucleons.

The single-meson exchange (or, as it is called, the one-boson exchange) potential (OBEP) has been studied by several authors (see[5-8]) and, as can be seen from Table I in particular, it is not possible to suggest a unique set of constants if one takes the experimental errors which exist at the present time. In fact, bearing in mind the allowance for terms containing the square of the momentum, an even wider range of values of the OBEP parameters is allowed by the experimental data. In accordance with this, the energy spectra of states of the $N\bar{N}$ system calculated using different variants of the OBEP will differ from one another[5]. It is clear from this that at the present time we cannot expect an accurate prediction of the positions of the $N\bar{N}$ levels. However, the number of states, their quantum numbers and their multiplicities can be determined. But the main qualitative prediction of the OBEP is the relation between the strong attraction and repulsion in the $N\bar{N}$ and NN systems. This prediction is based on a simple physical hypothesis, namely the assumption that the exchange of an actually existing vector meson with the quantum numbers of the photon gives an appreciable contribution to the interaction. In other words, if this hypothesis is correct, then the $N\bar{N}$ and NN attraction and repulsion are related to each other essentially in the same way as the signs of the e^-e^+ and e^-e^- interactions. As we noted earlier, this means that, in a certain sense we should expect the presence of a spectrum of levels in the $N\bar{N}$ system precisely because two nucleons have only a single bound state, namely the deuteron.

As we have already noted, the OBEP gives an adequate description of the experimental data on the scattering of nonrelativistic nucleons by each other. Moreover, this potential differs from many others by the fact that it is based on a perfectly clear physical idea. It is therefore quite justified to use it to study the properties of the $N\bar{N}$ system, at least heuristically. However, in addition to the potential interaction, a nucleon and antinucleon have a large probability for annihilation, transforming into pions and other mesons. Annihilation should certainly show up in both elastic scattering and in the spectrum of discrete states, if the latter appear as a result of the potential interaction. This problem is discussed in the following section.

3. ANNIHILATION

The characteristic distance for annihilation is the Compton wavelength of the annihilating particles. We shall explain this on the basis of the uncertainty relation. The diagram of Fig. 3a, describing $N\bar{N}$ annihilation into several pions, necessarily contains a virtual single-pion annihilation $N\bar{N} \to \pi$ (the previous virtual emission of pions by the nucleon or antinucleon is not annihilation).

Let us consider this last process in the center-of-mass system (c.m.s.) of the annihilating N and $\bar{N}$, i.e., in the rest system of the pion which is produced. Its energy in this system is equal to its rest energy μc^2, while the total energy of the N and $\bar{N}$ is equal to $2(c^2p^2 + m^2c^4)^{1/2}$, where $p = p_N = p_{\bar{N}}$ is the momentum of each of the annihilating particles (for greater clarity, we indicate $\hbar$ and c explicitly here). Energy conservation is violated in this virtual process by an amount $\Delta E = 2(c^2p^2 + m^2c^4)^{1/2} - \mu c^2$ (we neglect quantities of the order $\mu/m \ll 1$). Consequently, according to the uncertainty relation between energy and time, the virtual annihilation in question must occur within a short time interval $\Delta t = \hbar/\Delta E$, not exceeding $\hbar/2mc^2$. The characteristic distance determined by this time interval will be less than or of the order $r_a = c\Delta t = \hbar/2mc$ (or 1/2m in the units adopted in this paper). We can arrive at the same conclusion in a more formal way by considering $N\bar{N}$ scattering as a result of the annihilation interaction (i.e., as a result of the virtual transition of $N\bar{N}$ into pions and their subsequent reannihilation into $N\bar{N}$; see the diagram of Fig. 3b). The "threshold" for the square of the momentum transfer for this diagram lies at $4m^2$, which corresponds to an interaction range[9] 1/2m. Thus, the annihilation interaction, being just as strong as the ordinary potential interaction (i.e., that corresponding to the diagrams of Fig. 2), is concentrated at much smaller distances. Therefore, if the potential interaction is strong enough

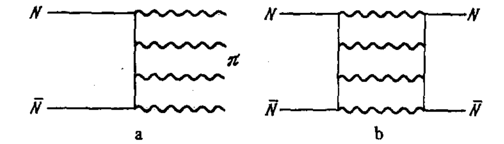

FIG. 3. The NN annihilation interaction–annihilation (a) and elastic scattering due to virtual annihilation (b).

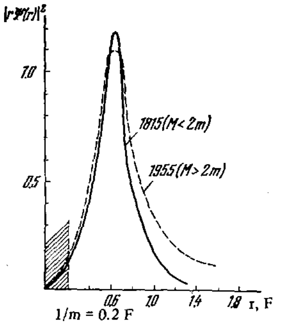

FIG. 4. The density of particles as a function of the distance between the N and $\overline{\text{N}}$ in a quasi-nuclear bound state (solid curve) and resonant state (dashed curve). The curves are obtained from a numerical solution of the Schrödinger equation [26] with a Hamiltonian of the OBEP type. The shaded area shows the annihilation region.

to produce a bound or resonant state, the "orbit" of the relative motion of the particles will lie outside the annihilation region (more precisely, the relative probability of finding the particles in the annihilation region must be small). The foregoing is illustrated by Fig. 4. We show here the annihilation region and the square of the modulus of the radial part of the wave function of the $N\overline{N}$ system as a function of the radius, for two states (a bound state and a resonant state) which appear in a number of other OBEPs in the scheme discussed above. As is clear from the figure, a small fraction of the normalized integral of the wave function falls in the annihilation region. On the basis of the foregoing, we should not expect annihilation to have a significant effect on the production of bound and resonant states, although it will completely determine their widths. This is confirmed by quantitative estimates (see Sec. 4).

The experimental data on the annihilation of slow antinucleons, $p\bar{p}$ elastic scattering and $\bar{p}p \to \bar{n}n$ charge exchange are in agreement with the theoretical ideas. Owing to the small value of the annihilation radius, the scattering cross section (σ_e) turns out to be less than the annihilation cross section (σ_a), while these cross sections are equal in the case of a "black" sphere (i.e., a uniform distribution of strong absorption throughout the interaction region). To explain the foregoing, it is useful to consider absorption and scattering by a purely imaginary square well. First of all, we note that from the well-known expressions

$$\sigma_e = \pi\lambda\!\!\!^{-2} \sum_l \sigma_e^{(l)}, \quad \sigma_a = \pi\lambda\!\!\!^{-2} \sum_l \sigma_a^{(l)}, \tag{6}$$

where

$$\sigma_e^{(l)} = (2l+1)\,|1 - S_e|^2, \quad \sigma_a^{(l)} = (2l+1)\,(1 - |S_e|^2), \tag{7}$$

$\lambda\!\!\!^{-}$ is the wavelength in the c.m.s. divided by 2π, and

$$S_e = \eta_e e^{2i\delta_e} \quad (\delta_e = \delta_e^*, \ 0 \leqslant \eta_e \leqslant 1), \tag{8}$$

it follows that

$$\sigma_a^{(l)}/\sigma_e^{(l)} = 1 - \eta_e^2/(1 + \eta_e^2 - 4\eta_e \sin^2\delta_e). \tag{9}$$

If $\sigma_a^{(1)}/\sigma_e^{(1)} > 1$, then according to (9) we have

$$\sin^2\delta_e < (1 - \eta_e)/2. \tag{10}$$

According to experimental data, for energies from 50 to 300 MeV in the laboratory system (l.s.) and for $l < 3$,

$$0{,}32 < \sin^2\delta_e < 0{,}40 \tag{11}$$

(in which case $\sigma_a/\sigma_e \approx 1.5$). To ensure that δ_e is small, as required by the inequalities (10) and (11), the radius of the absorption region must be small. For the s wave in the case of scattering by a purely imaginary square well, we have

$$\delta_0 = -kr_a + \delta_0', \tag{12}$$

where

$$\operatorname{tg} 2\delta_0' = k2 \operatorname{Re} L/(|L|^2 - k^2), \tag{13}$$

and the logarithmic derivative L of the wave function at the boundary of the well is determined by the equations

$$\operatorname{Re} L \approx \operatorname{Im} L \approx -K/2, \quad K = (mW/2)^{1/2}; \tag{14}$$

here r_a is the radius of the well, W is its depth, and k is the wave number of the particle in the c.m.s., where we have assumed that $Kr \gg 1$ and $k^2 \ll mW$. It is clear from (13) and (14) that $\delta_0' < 0$ everywhere; therefore, as implied by (12), to obtain a sufficiently small phase shift δ_0 we require a small radius r_a of the absorption (annihilation) region. In other words, a strong absorption shows up in the real part of the scattering phase shift as a repulsion, and to reduce the scattering we must reduce the radius of this effective reflecting sphere.

We have become practically convinced that the radius of the annihilation region is small by attempts to describe the experimental data on the interaction of nonrelativistic antiprotons with protons by means of an optical potential. As the real part of this potential, either a model with parameters chosen especially to describe the data on the $p\bar{p}$ interaction (as in[10], for example) or the OBEP[4] has been employed. The imaginary part of the potential, which in all such works is introduced to allow for annihilation, has been determined by fitting to some function which falls off exponentially with distance. In[10] the imaginary part of the potential was approximated by the expression

$$W = -W_0 e^{-(r/r_a)^4}.$$

In this case, to obtain agreement between the theoretical calculations and the experimental data it was necessary to take

$$r_a = 0.27 \text{ F}.$$

An analogous result was obtained in[4] for an imaginary part in the form of a Saxon-Woods potential:

$$W = -W_0/(1 + e^{r/r_a}). \tag{15}$$

The best value for the radius of the annihilation region in this variant is $r_a = 0.17$ F. All these values agree in order of magnitude with the theoretical prediction $r_a = 1/2m \approx 0.1$ F. Detailed reviews of the results of using the optical model for the $N\overline{N}$ interaction are contained in[11,12]. On the whole, they are not yet out of date; to avoid repetition, we shall only point out here that the OBEP, with the addition of the spin- and isospin-independent imaginary potential (15), gives a satisfactory description of the data on the total, elastic and differential elastic cross sections, as well as on $p\bar{p} \to n\bar{n}$ charge exchange. Unfortunately, there are only isolated polarization experiments with nonrelativistic antinucleons, and they are so far not sufficiently accurate for an effective comparison with the theoretical calculations (see[13]). Moreover, until recently, $\bar{p}$ annihilation at energies $E_L < 100$ MeV has also been studied only very roughly. Quite recently a number of new experimental results have been obtained in this region, which are not yet reflected in the review literature. These results, which are of great interest for the problems which we are considering and which we think give evidence for the existance of $N\overline{N}$ quasinuclear states, will be discussed in Sec. 5. We merely note here that the dependence of the annihilation cross section on the energy of the nonrelativistic nucleons is not simple. In particular, the annihilation cross section for slow particles does not conform to the $1/v$ law. Nevertheless, for rough estimates, we may suppose that in the range 50—300 MeV the total cross section for $N\overline{N}$ annihilation in each of the possible spin and isospin states is given by the equality

$$(k/m)\,\sigma_a \approx 50 \text{ mb}. \tag{16}$$

This figure is obtained from the experimental data on $p\bar{p}$ and $p\bar{n}$ annihilation under the assumption that σ_a is independent of the spin and isospin variables. On the average, the experimental errors in the measurement of the total cross sections for nonrelativistic $\bar{p}$ annihilation are 5—10 mb[12].

4. QUASINUCLEAR MESONS

The data on the interaction of nonrelativistic antinucleons with nucleons which we have quoted in the preceding sections enable us to make a theoretical study of the problem of $N\overline{N}$ bound and resonant states. We have in mind here nonrelativistic states, i.e., states which satisfy the conditions

$$R \gg 1/m, \tag{17}$$

$$|M - 2m| \ll m; \tag{18}$$

here R is the radius of the system (the mean distance between the particles) and M is its mass. The inequality (17) implies that the particles move with nonrelativistic velocities (since the average momentum is $\bar{k} \approx 1/R$). The inequality (18) is a necessary condition for the applicability of the quantum-mechanical potential approximation, without which the very concept of a composite system loses its meaning. The conditions (17) and (18) are satisfied for hadronic atoms (i.e., atoms with a negative hadron—a meson or baryon—instead of an electron), nuclei and hypernuclei. Since we shall be interested in $N\overline{N}$ states which arise as a result of the nuclear interaction, we shall call them quasinuclear. Since the baryon number (the number of baryons minus the number of antibaryons) is equal to zero for the $N\overline{N}$ system, $N\overline{N}$ quasi-nuclear states must also appear in different types of experiments as heavy mesons (with a mass close to the two-nucleon mass), which we shall also call quasinuclear[6)].

All quasinuclear states are unstable, owing to the possibility of annihilation of N and $\overline{N}$. The lifetime of a quasinuclear state, or its inverse quantity, the width, can be calculated in order of magnitude on the basis of the fact that, according to the condition (17), the mean distance between the particles R is much greater than the "annihilation radius" $r_a \lesssim 1/2m$ (see Sec. 3). Then the width Γ for the decay of a quasinuclear meson in the annihilation channels can be estimated to lowest order in the small parameter r_a/R according to the well-known formula (see, e.g.,[18,19])

$$\Gamma \approx (k/m)\,\tilde{\sigma}_a \rho, \quad \rho = |\Psi(0)|^2 \tag{19}$$

where k is the mean value of the momentum, and $\tilde{\sigma}_a$ is the annihilation cross section in the absence of the potential interaction between the particles. To obtain an estimate, we shall replace this unobservable quantity by the experimental value (16). The greatest uncertainty in a calculation according to Eq. (19) is due to the quantity ρ, the average value of the density of particles in the annihilation region (Ψ is the wave function of the quasinuclear state). The wave function of a quasinuclear system cannot be well known at distances of the order of $1/m$—it is strongly dependent on the details of the interaction, which at these distances cannot be given correctly by the OBEP or any other potential model, owing to the essential role of annihilation processes (we stress that the latter lead not only to absorption, but also to an additional attraction or repulsion due to the real part of the annihilation diagrams of the type of Fig. 3b). If at small distances ($r \lesssim r_a$) there is an attraction which grows not more rapidly than r^{-2} as $r \to 0$, then for the s states we have

$$\rho \approx 3/4\pi R^3. \tag{20}$$

Putting $R^3 \approx 1/\mu = 1.37$ F and substituting (20) and (16) in Eq. (19), we obtain

$$\Gamma_s \approx 100 \text{ MeV}. \tag{21}$$

This value of Γ (under the same assumptions about the interaction) is appreciably reduced for states with nonzero orbital angular momentum, owing to the centrifugal barrier, which prevents the penetration of the particles into the annihilation region. To obtain a rough estimate, we can take in this case

$$\rho_l \approx \{(Kr_a)^{2l}/[(2l+1)!!]^2\}\,\rho_s, \tag{22}$$

where $K = (mU_0)^{1/2}$ is the wave number in the effective potential well U_0 (we assume $U_0 \gg |2m - M|$). The relation (22) is valid when $Kr_a \gtrsim 1$ (ρ_1 does not differ from ρ_s in order of magnitude for large Kr_a). Taking this into account, we obtain the following estimate:

$$\Gamma_l \approx 100/[(2l+1)!!]^2 \text{ MeV}. \tag{23}$$

It follows from this that the annihilation widths Γ_1 for the p and d states can already be comparatively small, of the order of 10—1 MeV. The estimates which have been made are quite essential for the explanation of the experimental data on $p\bar{p}$ annihilation which are quoted in Sec. 5. These data can be explained by the presence of sufficiently sharp resonances in the $N\overline{N}$ system. The sharpness of a resonance is characterized by the ratio $\Gamma_{N\overline{N}}/\Gamma$, where $\Gamma_{N\overline{N}}$ is the width of the decay in the elastic channel, and Γ is the annihilation width. The value of $\Gamma_{N\overline{N}}$ for a quasi-nuclear s state can be estimated in order of magnitude by means of the well-known relation[7)]

$$\Gamma^{(s)}_{N\overline{N}} = 1/mR^2. \tag{24}$$

For $R \approx 1/\mu$, we obtain from (24)

$$\Gamma^{(s)}_{N\overline{N}} \approx \mu^2/m \approx 20 \text{ MeV}.$$

Comparing this figure with Γ_S, we find

$$\Gamma^{(s)}_{N\overline{N}}/\Gamma_s \approx 10^{-1}.$$

For $l \neq 0$ we will have the same and even a larger order of magnitude for the ratio $\Gamma_{N\overline{N}}/\Gamma$, since the penetration factor of the external centrigufal barrier in the elastic width $\Gamma_{N\overline{N}}$ is compensated by the abovementioned analogous factor in Γ, which reduces the probability for the particles to penetrate into the annihilation region. The penetration factor of the external barrier can be close to or equal to 1 for certain resonances (if $kR \lesssim 1$), in which case a suppression of the density of particles in the annihilation region will always occur for $l \neq 0$ and for potentials which are not too singular at the origin. Therefore, for quasinuclear mesons in the general case, we have

$$\Gamma_{N\bar{N}}/\Gamma \approx 10^{-1} - 1. \qquad (25)$$

The comparatively large probability of (real or virtual) decay of a quasinuclear meson in the $N\bar{N}$ channel is a distinctive feature of these particles (consisting of nucleons and antinucleons) which is not characteristic of heavy mesons of other types. For the latter, the decay into a nucleon and antinucleon is in no way distinguished from the other two-body decays. A crude estimate of $\Gamma_{N\bar{N}}/\Gamma$ for such mesons can be obtained by comparing the phase spaces of the two-body and multipion decay channels. For a meson of mass $M \approx 2m$, the ratio of phase spaces (measured by the same length) gives, for example (see[20]),

$$\Gamma_{2\pi}/\Gamma \approx 10^{-3}.$$

It is therefore clear that, for the decay in the $N\bar{N}$ channel of a heavy meson which is not of the quasi-nuclear type, we should expect

$$\Gamma_{\bar{N}\bar{N}}/\Gamma \leqslant 10^{-3},$$

i.e., values 2 to 3 orders of magnitude smaller than the expected value for quasinuclear mesons.

The estimate of Γ obtained above show that the widths of quasinuclear states are in any case not greater than (and most probably less than) the widths of the well-known boson resonances.

The mass spectrum and wave functions of the quasinuclear mesons can be obtained by solving the Schrödinger equation with the OBEP as the interaction Hamiltonian of the particles. Of course, the influence of annihilation effects on the positions of the levels will then not be taken into account. This influence can be estimated to lowest order in r_a/R according to a formula analogous to (19). For the annihilation level shift ΔE, we have the relation

$$\Delta E = (2\pi/m)\,\mathrm{Re}\,\tilde{f}_a\tilde{\rho}; \qquad (26)$$

here $\tilde{f}_a$ is the scattering amplitude resulting from the pure annihilation diagrams. Using the optical theorem, we can write

$$\operatorname*{Im}_{k\to 0}\tilde{f}_a = (k\tilde{\sigma}_a)_{k\to 0}/4\pi. \qquad (27)$$

Now expressing ρ in terms of Γ by means of (19) and using (27), we can rewrite (26) in the following form:

$$\Delta E = (\mathrm{Re}\,\tilde{f}_a/\mathrm{Im}\,\tilde{f}_a)\,\Gamma/2. \qquad (28)$$

It follows from (28) that ΔE should not exceed Γ in order of magnitude. This estimate is confirmed in those cases in which the annihilation shift can be evaluated more accurately (e.g., for the single-meson annihilation diagrams). All the results concerning the mass spectrum of quasinuclear mesons cited below can be correct with an accuracy up to the annihilation shift (28), i.e., up to a correction of the order of the level width.

In Table II we show the results of numerical calculations[21-25] of the mass spectrum of bound ($M < 2m$) quasinuclear $N\bar{N}$ states. These results were first obtained in[21,22] with the variant of the OBEP whose parameters are shown in Table I. Analogous results were subsequently obtained in[23]. This work differs from previous ones by the choice of the variant of the OBEP, the (smaller) cut-off radius of the singular terms, and the replacement of the Schrödinger equation by the Dirac equation. The authors used four different sets of

TABLE II. $N\bar{N}$ bound states ($M < 2m$)

Spectroscopic symbol	$I^G(J^P)$	Mass, MeV		Width, MeV [21,22]
		From [21,22]	From [23]*)	
$^1S_0$	$1^-(0^-)$	1722	240	93
	$0^+(0^-)$	1690	220	89
$^3S_1-{}^3d_1$	$1^+(1^-)$	1727; 1855	<0; 1850	94; 117
	$0^-(1^-)$	1414; 1382	<0; 940	71; 63
$^1P_1$	$1^+(1^+)$	1814	1250	104
	$0^-(1^+)$	1777	1150	100
$^3P_0$	$1^-(0^+)$	1724	1600	105
	$0^+(0^+)$	1289	1480	57
$^3P_1$	$1^-(1^+)$	1771	1500	107
	$0^+(1^+)$	1410	$>2m$	68
$^3P_2-{}^3f_2$	$1^-(2^+)$	1850; $>2m$	1380; $>2m$	88
	$0^+(2^+)$	1620; 1572	1475; 1740	107; 88
$^3d_2$	$1^+(2^-)$	1925; $>2m$	1820	
	$0^-(2^-)$	1608	1700	99
$^3g_3$	$1^+(3^-)$	$>2m$	$>2m$	
	$0^-(3^-)$	1839	$>2m$	148

*The variant which gives the best approximation to the pion mass for the $^1S_0$ state with quantum numbers $1^-(0^-)$.

OBEP parameters. The agreement between the results of the indicated independent calculations has been, on the whole, better than could be expected, bearing in mind the crude nature of the computational model and the rather strong difference between the variants of the OBEP which were used. Although the positions of the s levels obtained in these works differ quite strongly (in[23] they are found to be "overbound," sometimes even with $M < 0$), the masses of 11 other levels with $l \neq 0$ (of the total number of 13) differ by not more than 100–150 MeV[8)]. In addition to the masses, we show in Table II the orbital angular momenta and quantum numbers of the quasinuclear mesons. In this connection, we note that the P- and G-parities of states of the $N\bar{N}$ system are related to the other quantum numbers as follows:

$$P = (-1)^{l+1},\ G = (-1)^{I+S+l};$$

here S is the total spin of the N and $\bar{N}$, which is an integral of the motion, just as for the two-nucleon system. Owing to the presence of the tensor forces given by the OBEP (see Sec. 2), the orbital angular momentum is not conserved. However, estimates show that the contribution of the tensor forces to the energy of the triplet states is comparatively small (of the order of or less than 15–20%); therefore, owing to the crude nature of the model, they can, on the whole, be omitted (allowance for the tensor forces leads to a complication in the numerical computations which is unjustified in this case).

In Table II we also show the upper limits for the widths of the levels. For the states with $l \neq 0$, these values are obtained by truncating the centrifugal barrier at distances less than 0.6 F. When the centrifugal barrier is restored, the annihilation widths are reduced in accordance with Eq. (23). The positions of the levels themselves are changed only slightly in this case (10–50 MeV). This indicates that a basic role in the production of bound states of the $N\bar{N}$ system is played by the potential interaction at large distances, of the order of 1–1.5 F.

All the bound states listed in Table II correspond to wave functions without radial nodes. This means that the states with $l \neq 0$ appear as a result of a strong spin-orbit interaction. As we have already noted in Sec. 2, the presence of such an interaction is one of the characteristic features of the OBEP. Thus, the model for the $N\bar{N}$ interaction which we are considering leads to the following qualitative results for the spectrum of bound states:

a) there must exist in the $N\bar{N}$ system not one bound state, as in the pn system, but several;

b) there are no s states among the states close to the upper limit of the spectrum.

These qualitative features of the spectrum of bound states of the $N\bar{N}$ system remain unchanged when the OBEP parameters are varied, and are dependent on a perfectly clear physical principle: the substantial contribution to the NN and $N\bar{N}$ interactions from vector meson exchange processes. Since the attraction in the $N\bar{N}$ system is so strong that it leads to the appearance of a whole spectrum of bound states, it must also lead to the production of quasi-nuclear resonant levels ($M > 2m$). This is confirmed by the numerical solution of the wave equation[26]. The resonant levels in the $N\bar{N}$ system obtained with the same interaction Hamiltonian as the bound states are listed in Table III. In this table we also show the widths $\Gamma_{N\bar{N}}$ for the decay in the $N\bar{N}$ channel. As can be seen from Table III, the calculation confirms the estimates of $\Gamma_{N\bar{N}}$ which we made above.

The results on the resonant states of the $N\bar{N}$ system contained in Table III were obtained in[26] by calculating the Regge trajectories, i.e., the function $J(E)$ which expresses the dependence of the angular momentum of the system on its energy (see[27]). This function is real for $E < 0$, and the physical (experimentally observed) states correspond to integral values of J. The value of $J(E)$ is complex for $E > 0$, with $\mathrm{Im}\,J(E) > 0$. In this case, the physical states correspond to integral values of Re J. These states are quasistationary on account of the possibility of decay in the $N\bar{N}$ channel, and the width $\Gamma_{N\bar{N}}$ is determined by the value of Im J (see, e.g.,[27]):

$$\Gamma_{N\bar{N}} = 2\,\mathrm{Im}\,J\,[d\,\mathrm{Re}\,J/dE]^{-1}.$$

The widths $\Gamma_{N\bar{N}}$ shown in Table III are calculated according to Eq. (29). The Regge trajectories are calculated by solving the Schrödinger equation. The problem consists in finding a complex value of J which satisfies the equation for a given arbitrary E. If the trajectory $J(E)$ is known, the positions of the levels are determined by the integral values of $\mathrm{Re}\,J(E)$, and their widths $\Gamma_{N\bar{N}}$ by Eq. (29). As we have already mentioned, the total spin $S = 0, 1$ in the $N\bar{N}$ system is an integral of the motion. Therefore S is conserved along the Regge trajectory. In addition, the number $S' = J - 1 = 0 \pm 1$ for the triplet terms ($S = 1$) and the isospin of the system I are also conserved[9)]. The number of radial nodes of the wave function is also not changed along the trajectory. We note that the spatial parity and G-parity change along the trajectory in the nonrelativistic approximation. This does not occur in a relativistic theory, in which, owing to crossing symmetry (i.e., owing to the allowance for the annihilation interaction in our case), the analytic continuation of the orbital angular momentum $l(E)$ into the complex domain is carried out separately for even and odd values of l.

TABLE III. $N\bar{N}$ resonant states ($M > 2m$)

Spectroscopic symbol		M, MeV	$\Gamma_{N\bar{N}}$, MeV	Spectroscopic symbol		M, MeV	$\Gamma_{N\bar{N}}$, MeV
$^1d_2$	$1^-\ (2^-)$	1955	28	$^3d_3$	$1^+\ (3^-)$	2025	122
	$0^+\ (2^-)$	1930	15		$0^-\ (3^-)$	1880	0,0
$^3d_2$	$1^+\ (2^-)$	1925	10	$^3f_3$	$1^-\ (3^+)$	2165	76
	$0^-\ (2^-)$	$< 2m$	—		$0^+\ (3^+)$	1880	0,0

Thus, when relativistic corrections are taken into account, each of the Regge trajectories for the quasi-nuclear mesons splits into two (with opposite P- and G-parities).

One of the Regge trajectories for quasi-nuclear mesons is reproduced in Fig. 5 (in this figure we plot the square of the meson mass instead of the energy along the horizontal axis), As is clear from the figure, Re J grows with increasing mass, while Im J tends to zero after reaching a maximum. This behavior of the trajectory is due to the above-mentioned cut-off of the centrifugal potential. The behavior of $J(M)$ for large M is not of interest for our considerations, since, with the growth of M and Re J, the relative probability that the N and $\bar{N}$ remain at small distances rises, which shows up in the growth of ρ. This circumstance imposes an upper limit on the mass spectrum of quasi-nuclear mesons which can be studied theoretically within the framework of the quantum-mechanical potential approach.

As we see from Table III, the spectrum of resonant quasi-nuclear states of the $N\bar{N}$ system contains only states with $l \neq 0$. Thus, the theory predicts the presence of near-threshold resonances with non-zero orbital angular momenta. This result, as well as the occurrence of a large number of sub-threshold (bound) states with $l \neq 0$, is in agreement with the data of the latest experiments on $N\bar{N}$ annihilation which we discuss in Sec. 5 (which have appeared since the publication of the cited theoretical works).

The theoretically predicted spectrum of bound states and resonances can be compared with the experimental data on meson resonances[28]. Unfortunately, the experimental investigation of heavy meson resonances is still in its initial stages. At the present time, there are indications that there exist in this mass range (1400–2400 MeV) several resonances which could be identified with quasi-nuclear $N\bar{N}$ states. However, the quantum numbers of most of these resonances have not been established, and there are so far no quantitative data on the production of heavy mesons in collisions of antinucleons with nucleons and nuclei[10)]. Several recent experiments constitute an exception. We shall consider the comparison of their results with the theoretical data in the following section.

5. THE PRODUCTION OF QUASI-NUCLEAR MESONS BY SLOW ANTINUCLEON BEAMS

It has been established in the experiments of recent years that the partial waves with non-zero orbital angular momenta give a large contribution to the annihilation of slow antiprotons. It follows from the unitarity condition that the total interaction cross section

$$\sigma_t = \sigma_e + \sigma_a$$

and the elastic scattering cross section σ_e must satisfy

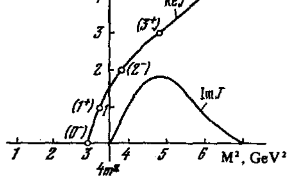

FIG. 5. A Regge trajectory for a family of quasi-nuclear mesons with isospin I = 1 and total spin S = 0 [26]. The points on the curve indicate the physical states (integral values of Re J).

the Rarita-Schwed inequality[29]

$$\sigma_t^2/\sigma_e \leqslant 4\pi\lambda^2 (L+1)^2, \qquad (29')$$

where L is the minimal orbital angular momentum which contributes to the interaction. In Fig. 6 we reproduce the results of one of the latest experimental works[30]. Fig. 6a shows the behavior of the annihilation cross section σ_a as a function of p_L. The maximally possible value of σ_a, as implied by the general equations (6) and (7), is equal to $\pi\lambda\!\!\!^{-2}(L+1)^2$. In Fig. 6b we plot on the vertical axis the quantity σ_t^2/σ_e for $\bar{p}p$ collisions, which appears on the left-hand side of the inequality (29′); on the horizontal axis we plot the antiproton momentum p_L in the l.s.; the solid curves correspond to the quantity $4\pi\lambda\!\!\!^{-2}(L+1)^2$ (the right-hand side of the inequality (29′)) for various L. As we can see from Fig. 6b, the partial waves with $l = 3$ give a substantial contribution to the $\bar{p}p$ interaction even at small momenta p_L = 100—200 MeV/c. It is also clear from Fig. 6a that orbital angular momenta up to 1 = 3 participate in the annihilation of slow $\bar{p}$. It is easy to see that such values of the angular momentum are anomalously large for short-range forces of radius $r \lesssim 1$ F. In fact, a momentum p_L = 200 MeV/c in the l.s. corresponds to a momentum $k = p_L/2$ = 100 MeV/c in the c.m.s., i.e., $L \approx kr \lesssim 0.5$.

The major role of non-zero orbital angular momenta in the annihilation of slow antiprotons is also shown directly by the results of another experiment, in which the relative probability of the annihilation $\bar{p}p \to 2\pi^0$ is measured. Annihilation into $2\pi^0$ from an s state is forbidden by parity conservation: P = −1 for the $N\bar{N}$ system in an s state, while only states with P = +1 are possible for two identical bosons with zero spin. Thus, the annihilation into $2\pi^0$ is an indication that partial waves with $l \neq 0$ are taking part in the $\bar{p}p$ interaction at low energies. On the other hand, the annihilation into a charged pair, $\bar{p}p \to \pi^+\pi^-$, can proceed even from an s state. The measured ratio of probabilities for the processes $\bar{p}p \to 2\pi^0$ and $\bar{p}p \to \pi^+\pi^-$ for slow antiprotons ($p_L \lesssim$ 150 MeV/c) shows that about 30% of all the events of two-pion annihilation into charged pions proceed from states of odd (and consequently non-zero) orbital angular momentum[31].

The large contribution to the annihilation of slow antiprotons from partial waves with non-zero orbital angular momenta can be explained by the presence of bound or resonant $N\bar{N}$ states with non-zero orbital angular momenta near threshold (i.e., with a mass close to the two-nucleon mass). The experimental data from[30,31] cited above indicate the existence of a whole serie of such states of the $\bar{p}p$ system. This circumstantial evidence is confirmed by direct experiments to detect bound and resonant states. In Fig. 7 we show one of the latest results[32] for the spectrum of proton recoil momenta in the annihilation of "stopped" antiprotons ($p_L \lesssim$ 150 MeV/c) in deuterium:

$$\bar{p}+d \to X^- + p, \quad X^- \to n\pi. \qquad (30)$$

According to the interpretation of the authors, the peak which occurs in the spectrum corresponds to a bound state of the $\bar{p}n$ system with isospin and G-parity $I^G = 1^+$ and with a mass of about 1800 MeV.

The main point to which we would like to call attention in connection with this experiment is that, owing to the comparatively large annihilation widths of the levels, the shape of this spectrum and in particular the locations and spreads of the maxima in it are determined not only by the conservation laws and the masses of the bound states, but also by their quantum numbers, especially their orbital angular momenta. We shall consider the reaction (30) on the basis of the pick-up mechanism (Fig. 8), which is well known in nuclear physics. We note that, for the main results outlined below, it is immaterial which concrete mechanism is chosen for the peripheral process, although some shift in the maximum of the spectrum in question may occur when, for example, we go over from the pick-up process to the reaction.

The differential cross section for the reaction (30) corresponding to the pick-up process can be written as follows:

$$\frac{k}{m}\frac{d\sigma}{dq\,d\Omega} = \frac{2J+1}{2\pi}\frac{q^2(q^2+\alpha^2)^2 F_d^2(q) F_{N\bar{N}}^2(s)\,\Gamma}{(q^2+\alpha^2-s^2-\varkappa^2)+(\Gamma^2 m^2/4)}; \qquad (31)$$

here k and q are the momenta of the antiproton and recoil proton, $d\Omega$ is the element of solid angle in the direction of q, $s = (k+q)/2$, J, κ^2/m and Γ are respectively the angular momentum, binding energy and annihilation width of the relevant state of the $N\bar{N}$ system, α^2/m is the deuteron binding energy, m is the nucleon mass, and F(a) are the Fourier components of the radial wave functions:

$$F(a) = \int_0^\infty \chi_l(r)\, j_l(ar)\, r\, dr$$

(l is the orbital angular momentum of the relative motion of the particles). In particular, for the Hulthén wave function of the deuteron and for stopped antiprotons (k = 0), we have

$$\frac{k}{m}\frac{d\sigma}{dq} = \frac{4(2J+1)\alpha\beta(\alpha+\beta)^3 q^2 F_{N\bar{N}}^2(q/2)\,\Gamma}{(q^2+\beta^2)^2\{[(3q^2/4)+\alpha^2-\varkappa^2]^2+(\Gamma^2 m^2/4)\}}, \qquad (32)$$

where β = 250 MeV/c is the Hulthén parameter. An obvious argument shows that in the region $q < 2l/R$ (R is the radius of the orbit) the function $q^2F_{N\bar{N}}^2(q/2)$ behaves like $(qR)^{2l+2}[(q^2/4)+\kappa^2]^{-2}$, i.e., it grows at sufficiently small q. Since the expression (32) also contains a monotonically decreasing Hulthén factor, $d\sigma/dq$ (as a function of q) may have a maximum in the interval $0 \leq q \lesssim 2l/R$ (henceforth we call it a kinematic maximum), even when the resonant denominator (the curly brackets in Eq. (32)) varies slowly in this inter-

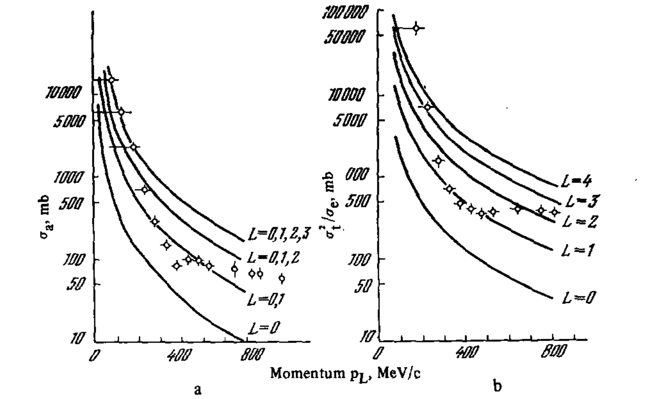

FIG. 6. The annihilation of slow antiprotons in hydrogen (according to the data of [30]). a) σ as a function of the momentum p_L in the l.s. (solid curves—the maximally possible cross sections $\sigma_a = \pi\lambda^2(2L+1)^2$); b) the Rarita-Schwed parameter σ_t/σ_e as a function of the momentum p_L (solid curves—the behavior of the quantity $4\pi\lambda^2(L+1)^2$).

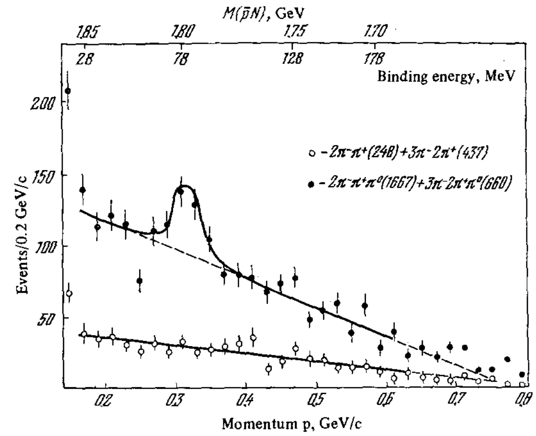

FIG. 7. The spectrum of recoil proton momenta in the reaction $\bar{p} + d \rightarrow X^- + p$, $X^- \rightarrow n\pi$ [32]. The upper and lower curves are for even and odd n, respectively.

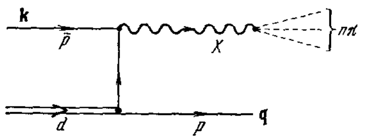

FIG. 8. Diagram of the pick-up mechanism for the reaction $\bar{N} + d \rightarrow X + N$.

val because of the comparatively large annihilation width. The position of a kinematic maximum is thus practically independent of the mass of the $N\bar{N}$ bound state and is determined principally by its orbital angular momentum and radius. This is confirmed by Table IV, in which we give the positions of the kinematic maxima for all the bound states which are predicted theoretically in[21,22]. Of course, a kinematic maximum can appear only when its width is substantially less than the annihilation width. Otherwise, the behavior of the curve for $d\sigma/dq$ will also be determined by the resonant factor in Eq. (32), and for a sufficiently small value of Γ the Briet-Wigner peak in the spectrum of recoil proton momenta must be dominant. In Fig. 9a we show the shape and position of the maximum corresponding to the $n\bar{p}$ $^3d_1$ state with mass 1855 MeV, for various values of Γ. It can be seen that when $\Gamma = 50$ MeV the position of the peak is very close to the Breit-Wigner maximum ($q_r = 175$ MeV/c; see Table IV), while for $\Gamma = 150$ MeV there appears a kinematic maximum ($q_m = 310$ MeV/c).

It follows from the foregoing that the presence of maxima in the spectrum of recoil proton momenta indicates at any rate that certain non-zero orbital angular momenta of the relative motion of the neutron and antiproton contribute significantly to the annihilation of stopped antiprotons by the deuteron. It is difficult to explain this fact otherwise than by the existence of near-threshold states with $l \neq 0$ in the $N\bar{N}$ system.

However, as the analysis carried out above shows, the positions and widths of the observed maxima are not always directly related to the masses and annihilation widths of these states.

In Fig. 9b we show the comparison of the spectra of recoil proton momenta in the reaction (30) for stopped antiprotons as calculated according to Eq. (32) (on the basis of[21,22]) and observed experimentally[32] (the theoretical curves are normalized on the vertical axis to the experimental numbers of events at the point $q = 0.29$ GeV/c). The curve for positive G-parity has

TABLE IV. Positions of the kinematic and Breit-Wigner maxima in the spectrum of recoil nucleon momenta in the pick-up reaction for stopped antiprotons

Isospin and G-parity I^G	Spin and parity J^P	Spectroscopic sybmol	Bound state mass, MeV	Breit-Wigner maximum q_r, MeV/c	Kinematic maximum q_m, MeV/c	Annihilation width Γ, MeV
1^+	1^-	$^3s_1$	1727	40	450	94
	1^+	$^3p_1$	1814	130	290	104
	1^-	$^3d_1$	1855	310	175	117
1^-	0^-	$^1s_0$	1722	40	450	93
	0^+	$^3p_0$	1724	120	450	105
	1^+	$^3p_1$	1771	130	360	107
	2^+	$^3p_2$	1850	130	190	88
0^+	0^-	$^1s_0$	1690	40	520	89
	0^+	$^3p_0$	1289	130	900	57
	1^+	$^3p_1$	1410	120	790	68
	2^+	$^3p_2$	1620	120	570	188
	2^+	$^3f_2$	1572	620	630	107
0^-	1^-	$^3s_1$	1414	40	780	63
	1^+	$^1p_1$	1717	130	350	100
	1^-	$^3d_1$	1382	760	815	71
	2^-	$^3d_2$	1608	560	580	99
	3^-	$^3g_3$	1839	750	230	148

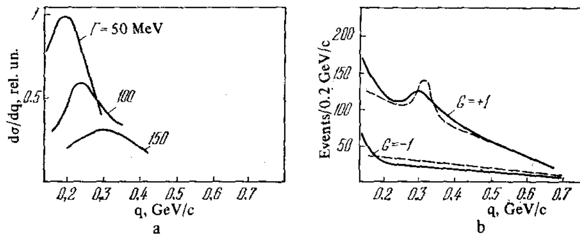

FIG. 9. a) The form and position of the maximum corresponding to the $^3d_1$ state of the $n\bar{p}$ system with a mass 1855 MeV, for various values of the annihilation width Γ; b) a comparison of the theoretical [32] and experimental [34] results for the annihilation of stopped antiprotons in deuterium. Solid curves—theory; dashed curves—experiment.

a kinematic maximum corresponding to the above-mentioned $^3d_1$ state (quantum numbers $J^{PG} = 1^{-+}$) with annihilation width $\Gamma = 150$ MeV. As can be seen from the figure, the position of this maximum coincides with the experimental peak. We note that, of the seven bound states with isospin 1 shown in Table IV, only one could appear under the conditions of the experiment[32]: the kinematic maxima due to the other states lie outside the range of the part of the spectrum which is studied in this experiment ($q \geq 150$ MeV/c).

We can conclude from the foregoing that the existing experimental data are in agreement with the theoretical predictions. At the same time, experiments to study the reaction (30) at various antiproton energies would be desirable. This would make it possible, first of all, to ascertain the nature of the observed maxima (it is easy to see from Eq. (31) that the Breit-Wigner and kinematic peaks will be shifted differently as s varies) and, secondly, to obtain more complete information about the spectrum of states of the $n\bar{p}$ system. The theory of the reaction (30) was given in[33-35].

As we have already noted in a previous section, a distinctive feature of quasinuclear resonances is the comparatively large widths $\Gamma_{N\bar{N}}$ for decay into N and $\bar{N}$. The direct measurement of $\Gamma_{N\bar{N}}$ for the observed heavy mesons (of mass $M > 2m$) is therefore of great interest. However, there are so far no direct and reliable (even in order of magnitude) measurements of $\Gamma_{N\bar{N}}$. Nevertheless, from the recently published data we may estimate the probable value of $\Gamma_{N\bar{N}}/\Gamma$ for the resonance of mass 1970 MeV mentioned in Sec. 4. The $N\bar{N}$ (1970) resonance was seen[36] in the annihilation $\bar{p}p \rightarrow K_L K_S$ and, even earlier (but less clearly), in elastic $\bar{p}p$ scattering[37]. In the annihilation experiment[36] the absolute value of the effective cross section for annihilation in the $K_L K_S$ channel was measured. It follows from the energy dependence of the cross section (Fig. 10) that it can be described by the sum of a resonant (Breit-Wigner) term and a nonresonant term. At the momentum $p_L = 600$ MeV/c, corresponding to a mass 1970 MeV, the experimental value of the resonant contribution $\sigma_{KK}^{(r)}$ is equal to $(75 \pm 20)\ \mu b$. On the other hand, $\sigma_{KK}^{(r)}$ is related to $\Gamma_{N\bar{N}}$ by the equation

$$\Gamma_{N\bar{N}}/\Gamma_t = (\Gamma_t/\Gamma_{KK})\, k^2 \sigma_{KK}^{(r)}/\pi\,(2J+1), \tag{33}$$

where $\Gamma_t = \Gamma + \Gamma_{N\bar{N}}$ is the full width of the resonance, and $k = 300$ MeV/c is the $\bar{p}$ momentum in the c.m.s. We can determine $\Gamma_{N\bar{N}}/\Gamma$ from this relation if Γ_{KK}/Γ_t and the spin of the resonance J are known. The value of $\Gamma_{KK}/\Gamma_t = \sigma_{KK}/\sigma_t$ (where σ_{KK} is the cross section for annihilation into two K mesons, and σ_t is the $p\bar{p}$ total interaction cross section) has not been directly measured in the region of the resonance. However, since the ratio of cross sections contains no resonance factors and must consequently be a comparatively smooth function of energy, we can estimate Γ_{KK}/Γ_t in order of magnitude from the value of σ_{KK}/σ_t at nonresonant energies. For the annihilation of antiprotons "at rest" ($p_L \le 150$ MeV/c), $\sigma_{KK}/\sigma_t = (0.61 \pm 0.09) \times 10^{-3}$ (see[20]). It is also known that $\sigma_{KK}/\sigma_{2\pi}$ is constant and equal to about 1/3 in a wide momentum range (0-400 MeV/c) and that $\sigma_{2\pi}/\sigma_t = 2 \times 10^{-3}$[11] at $p_L = 600$ MeV/c. Assuming that the ratio $\sigma_{KK}/\sigma_{2\pi}$ remains approximately the same at 600 MeV/c as at lower energies, we find

$$\sigma_{KK}/\sigma_t\ (600\ \text{MeV/c}) \approx 0.66 \cdot 10^{-3}.$$

This figure agrees with the result of direct measurement of σ_{KK}/σ_t for stopped antiprotons and supports the foregoing argument that σ_{KK}/σ_t has a weak energy dependence. We can therefore put

$$\Gamma_{KK}/\Gamma_t = \sigma_{KK}/\sigma_t = (0.6 \pm 0.1) \cdot 10^{-3}.$$

Then it follows from Eq. (33) and the value of $\sigma_{KK}^{(r)}$ that

$$\Gamma_{N\bar{N}}/\Gamma_t = (8 \pm 3)/(2J+1). \tag{34}$$

Apart from the charge parity $C = -1$, the quantum numbers of the resonance in question are unknown. The negative charge parity $C = (-1)^{l+S}$ and the conservation of spatial parity and angular momentum in the annihilation process exclude singlet states ($S = 0$), as well as all states for which $J \neq l \pm 1$. Thus, the possible values of J^P are

$$J^P = 1^-, 3^-, \ldots$$

The first of these values corresponds to the quasi-nuclear states $^3s_1$ and $^3d_1$, and the second to the state $^3d_3$. It is less probable that large orbital angular momenta participate in $\bar{p}p$ annihilation at $p_L = 600$ MeV/c (see Fig. 6). For J = 3 and 1, we obtain from (34) $\Gamma_{N\bar{N}}/\Gamma_t = 1.2 \pm 0.4$ and 3 ± 1, respectively. In both cases $\Gamma_{N\bar{N}}/\Gamma_t$ is equal to unity in order of magnitude, i.e. is in agreement with the value which is expected for quasi-nuclear mesons. Since $\Gamma_{N\bar{N}}/\Gamma_t$ by definition must be less than unity, we can infer from the figures given above that the value J = 3 is preferred. We note that the results of the angular distribution measurements carried out in[36] are consistent with this conclusion (see[38]), but that excessively large errors in the experiment prevent a definitive determination of J.

As we have already mentioned, the $N\bar{N}$ (1970) resonance also appears in backward elastic scattering. The theoretical analysis of the experimental data[37] carried out in[38] leads to the following values of $\Gamma_{N\bar{N}}/\Gamma_t$:

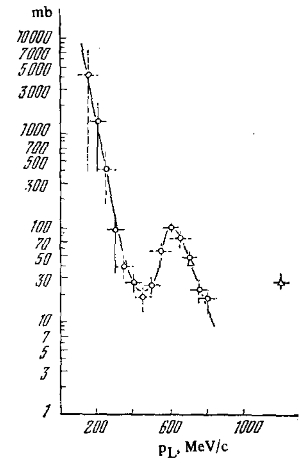

FIG. 10. Momentum dependence of the cross section for the process $\bar{p}p \to K_L K_S$. The resonant maximum corresponds to a state of the $\bar{p}p$ system of mass 1970 MeV and with a full width $\Gamma_t = 35$ MeV.

$$\Gamma_{N\bar{N}}/\Gamma_t = 0.8 \pm 0.1;\quad 0.4 \pm 0.1;\quad 0.22 \pm 0.05$$

for the states $^3s_1$, $^3d_1$ and $^3d_3$, respectively. Thus, this independent estimate of $\Gamma_{N\bar{N}}/\Gamma$ for the $N\bar{N}$ (1970) resonance leads to a result which is in agreement with the preceding one in order of magnitude. Everything that we have said supports the quasi-nuclear character of the $N\bar{N}$ (1970) resonance. In accordance with Table III, this resonance must have the quantum numbers $J^{PC} = 3^{--}$ and an isospin I equal to 0 (G = −1) or 1 (G = +1). Further experiments which can confirm or refute these predictions are quite essential for the class of problems considered here.

6. CONCLUSIONS

The levels predicted by the potential model can obviously undergo certain shifts for the allowed variations of the potential. The annihilation widths are extremely sensitive to these variations (by varying the potential at distances less than 0.5 F, we obtained values of the widths from 5 to 150 MeV without changing any of the characteristic features of the energy spectrum). However, it is important to stress that one essential fact remains unchanged in all cases: the presence of several near-threshold states with non-zero orbital angular momenta. This circumstance, which it would be especially desirable to subject to an experimental test, is a qualitative consequence of highly probable current hypotheses about the nature of the interaction of nucleons at nonrelativistic energies and is explained by the major role which the spin-orbit forces play in generating the spectrum of states of the $N\bar{N}$ system. As we have already emphasized above, another important qualitative prediction of the theory of quasi-nuclear $N\bar{N}$ systems is the comparatively large value of the quantity $\Gamma_{N\bar{N}}/\Gamma$. An immediate consequence of this fact is the possibility of observing quasi-nuclear $N\bar{N}$ resonances in the energy behavior of the cross sections for annihilation processes and elastic scattering. For bound states, the amplitude for virtual decay into $N\bar{N}$ is comparatively large. The corresponding quasi-nuclear mesons should therefore appear in various direct reactions between antinucleon beams and nuclear targets.

It should be stressed that the investigation of problems relating to the interaction of nonrelativistic antinucleons with nucleons is of direct interest not only for the determination of the physical character of heavy meson resonances—which is one of the problems of modern high-energy physics—but also for the theory of ordinary nuclei, i.e., for nuclear physics in the traditional sense of the word. In fact, we have seen that there exists a well-defined relation between the $N\bar{N}$ and NN interactions, so that the basic properties of nuclei with few nucleons and of quasinuclear mesons must, in the final analysis, be described by the same set of physical constants characterizing the "elementary" interaction of nonrelativistic particles. This means that, in order to establish the form of the potential interaction of nucleons, which many people now hope is the necessary starting point for the construction of a consistent theory, at least of nuclei with few nucleons, it is of paramount importance to study experimentally and theoretically the collision processes of slow antinucleons with nucleons, as well as the spectrum and properties of heavy meson resonances. The latter is due to the fact that the spectrum of discrete states of the $N\bar{N}$ system is much richer than the spectrum not only of the two-nucleon system, but also of the other lightest nuclei. Considering that, in addition to quasinuclear mesons consisting of the two particles $N\bar{N}$, three-particle baryon ($NN\bar{N}$) quasi-nuclear resonances are also possible, as shown by preliminary calculations (see[39,40]), it becomes clear what a great diversity of phenomena are to be understood on the basis of a unique starting point.

Thus, it is highly probable that, on the one hand, methods of nuclear physics will help us to gain an understanding of the properties and spectrum of the heavy "elementary" particle-resonances, while, on the other hand, antinucleon beams will help to answer certain "nasty" questions of nuclear physics.

The author is grateful to L. N. Bogdanova, O. D. Dal'karov, and B. O. Kerbikov for many discussions and assistance in the preparation of this review, and also to L. B. Okun' for constructive critical comments.

---

1)In the theoretical formalism, the quantum numbers of the photon (the vector character and neutrality of the electromagnetic field) show up in the sign of the interaction potential of identical nonrelativistic particles through the propagator (Green's function) of the free field (the dashed line in Fig. 1). For example, the parts of the propagators of the Coulomb field and any scalar particle (quantum numbers 0^+) or graviton (2^+) which are important in the approximation in question are opposite in sign. The signs of the interactions of identical particles due to the exchange of the photon (repulsion) and the exchange of a scalar particle or graviton (attraction) are therefore also opposite. The physical reason for the negative charge parity of the photon (and of any neutral vector field) is the fact that the source of the field is the current density, which is proportional to the difference of the numbers of particles and antiparticles at each point of space. If the particles are replaced by antiparticles, the source of the field changes sign, so that the field itself changes sign. But the sources of a scalar or gravitational field ($C = +1$) are proportional to the sum of the numbers of particles and antiparticles. These sums do not change under charge conjugation, so that the sign of the field also remains unchanged.

2)We stress that the G-parity (and not the charge C-parity) enters the relation (1) because this equation interrelates the NN and $N\bar{N}$ interactions in states with the same isospin. It is just this which forces us to make the isotopic rotation through 180° in addition to charge conjugation ($N \to \bar{N}$). For example, the deuteron ($I = 0$) is transformed into an $N\bar{N}$ state with isospin $I = 1$ when the neutron (n) is replaced by the antineutron ($\bar{n}$) (since the isospin projections of the antineutron and proton (p) are equal to 1/2 and, consequently, the total isospin projection is equal to 1). To obtain an $N\bar{N}$ system with $I = 0$ from the deuteron, the neutron must be replaced by the antiproton ($\bar{p}$), and this is achieved by just the Gp transformation (charge conjugation: $\bar{n} \to n$; isotopic rotation: $n \to -\bar{p}$).

3)There also exist less definite data on the NN scattering amplitude in the nonrelativistic domain at higher energies, above the threshold for inelastic processes (pion production) [2].

4)We note that in the potential approximation NN scattering is described least accurately in the s wave (orbital angular momentum $l = 0$). The single-meson exchange potential is no exception in this respect. The physical reason for this is that the details of the interaction at small distances show up more strongly in the s state (owing to the absence of the centrifugal barrier).

5)However, the (relative) shift in the levels is not greater than the change in the scattering phase shifts. In this sense, the problem is stable, and it makes sense to use potentials determined from the scattering data to find the spectrum.

6)The problem of whether there exist mesons consisting of a nucleon and antinucleon has quite a long history, going back to the well-known paper of Fermi and Yang [14]. In this work, which laid the foundations for a different kind of composite model of elementary particles, the pion was regarded as a bound state of N and $\bar{N}$. The works of Afrikyan [15] and Bethe and Hamilton [16] are closer to the ideas discussed in the present review (the use of the potential aporoximation only for nonrelativistic states with a small mass defect). In [15] there was posed the question of whether there exists a bound state analogous to the deuteron. Similar considerations were expressed in [16]. Finally, we should mention the work of Ball et al. [17], who obtained a spectrum of heavy mesons in the process of studying bootstrap-type equations (a set of self-consistent integral equations which are imposed with the idea of yielding as $N\bar{N}$ bound states the same mesons as those which produce the interactions in the NN channel). This idea was not realized in the cited work (light mesons were used as input, while heavy ones were obtained). Actually, the equations written in [17] correspond to the quantum-mechanical potential approach. Also, we note that the equation as to what effect strong annihilation has on the production of $N\bar{N}$ bound and resonant states was not considered in the papers which we have listed (there is no mention in them of the small value of the annihilation radius—the basic circumstance without which the entire scheme could hardly have physical meaning).

7)We recall that this formula is derived from a simple estimate of the probability per unit time that a particle is emitted from the region of radius R: the velocity of the particle is $v = \hbar/mR$, the probability of emission per second in the absence of a barrier is $\lambda = v/R = \hbar/mR^2$, and the width is $\Gamma_{N\bar{N}} = \hbar\lambda = \hbar^2/mR^2$.

8)This refers to the closest results of the cited works. As we mentioned above, several variants of the OBEP were considered in [23]. For the comparison, Table II shows the data corresponding to the variant which gives the best approximation to the pion mass for the $^1S_0$ state with the quantum numbers $1^-(0^-)$. In this case, 7 (or possibly 8) levels with $l \neq 0$ agree (with the indicated accuracy) with the data of [21,22]. The general regularity which is characteristic of the $N\bar{N}$ level spectrum—a large number of nonrelativistic bound states with $l \neq 0$—also appears in this variant.

9)The conservation of S′ is associated with the fact that the orbital angular momentum $l(E)$ and, through it, the total angular momentum $\mathbf{J} = \mathbf{l} + \mathbf{S}$ are actually analytical continued into the complex domain.

10)According to the existing data (see [28]), resonances of mass 1925 and 1970 MeV appear in $N\bar{N}$ backward (large-angle) elastic scattering. In addition, a resonant structure in the total cross section is seen in the region of masses 2190, 2345 and 2380 MeV.

11)$\sigma_{2\pi} = 300\,\mu b$ [20], $\sigma_t = 150$ mb [30].

1. A. E. Green and T. Savada, Rev. Mod. Phys. 39, 594 (1967).
2. R. Ya. Zul'karneev and A. M. Rozanova, Zh. Eksp. Teor. Fiz. 59, 1444 (1970) [Sov. Phys.-JETP 32, 788 (1971)].
3. F. Calogero, in: Problemy sovremennoĭ yadernoĭ fiziki (Problems of Contemporary Nuclear Physics), Moscow, Nauka, 1971.
4. R. A. Bryan and R. J. N. Phillips, Nucl. Phys. B5, 201 (1968).
5. N. Hochizake, I. Lin, and S. Machida, Progr. Theor. Phys. 26, 680 (1961).
6. D. Y. Wong, Nucl. Phys. 55, 212 (1964).

[7] A. E. S. Green and S. P. Sharma, Phys. Rev. Lett. 14, 380 (1965).
[8] R. A. Bryan and B. L. Scott, Phys. Rev. 164, 1215 (1967).
[9] A. Martin, ibid. 124, 614 (1961).
[10] P. É. Nemirovskiĭ and Yu. F. Strokov, Zh. Eksp. Teor. Fiz. 46, 1379 (1964) [Sov. Phys.-JETP 19, 932 (1964)].
[11] R. J. N. Phillips, Rev. Mod. Phys. 39, 681 (1967).
[12] D. Cline, Symposium on Nucleon-Antinucleon Interactions, Argonne, Ill., 1968, p. 79.
[13] M. Ceschia, Phys. Rev. D2, 2555 (1970).
[14] E. Fermi and C. N. Yang, ibid. 89, 1256 (1949).
[15] A. M. Afrikyan, Zh. Eksp. Teor. Fiz. 30, 734 (1956) [Sov. Phys.-JETP 3, 503 (1956)].
[16] H. Bethe and R. Hamilton, Nuovo Cimento 4, 1 (1956).
[17] J. S. Ball, A. Scotti, and D. Y. Wang, Phys. Rev. 142, 1000 (1966).
[18] M. L. Goldberger and K. M. Watson, Collision Theory, Wiley, New York, 1964.
[19] A. M. Badalyan and Yu. A. Simonov, Yad. Fiz. 11, 1112 (1970) [Sov. J. Nucl. Phys. 11, 618 (1970)].
[20] T. E. Kalogeropoulos and D. Cline, in[12], p. 17.
[21] O. D. Dalkarov, V. B. Mandelzweig, and I. S. Shapiro, Nucl. Phys. B21, 88 (1970).
[22] O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Zh. Eksp. Teor. Fiz. 59, 1363 (1970) [Sov. Phys.-JETP 32, 744 (1971)].
[23] G. Schierholz and S. Wagner, Nucl. Phys. B32, 306 (1971).
[24] O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, ZhETF Pis. Red. 10, 402 (1969) [JETP Lett. 10, 257 (1969)].
[25] O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Yad. Fiz. 11, 889 (1970) [Sov. J. Nucl. Phys. 11, 496 (1970)].
[26] L. N. Bogdanova, O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Zh. Eksp. Teor. Fiz. 61, 2242 (1971) [Sov. Phys.-JETP 34, 1200 (1972)].
[27] V. de Alfaro and T. Regge, Potential Scattering, North-Holland, 1965 (Russian Transl., Moscow, Mir, 1966).
[28] Particle Data, ed. by A. H. Rosenfeld, Rev. Mod. Phys. Suppl., 12 (1971).
[29] W. Rarita and P. Schwed, Phys. Rev. 112, 271 (1958).
[30] D. Cline, J. English, and D. Reeder, Phys. Rev. Lett. 27, 71 (1971).
[31] S. Devons, P. Nemethy, S. Shapiro, N. Horwitz, T. Kalogeropulos, J. Skelly, R. Smith, and H. Oto, ibid., p. 1614.
[32] L. Gray, P. Hagerty, and T. Kalogeropoulos, ibid. 26, 1491.
[33] L. N. Bogdanova, O. D. Dalkarov, and I. S. Shapiro, ibid. 28, 1418 (1972).
[34] O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Yad. Fiz. 13, 332 (1971) [Sov. J. Nucl. Phys. 13, 184 (1971)].
[35] L. N. Bogdanova, O. D. Dal'karov, and I. S. Shapiro, ZhETF Pis. Red. 15, 748 (1972) [JETP Lett. 15, 531 (1972)].
[36] A. Benvenutti, D. Cline, R. Rutz, D. D. Reeder, and V. R. Scherer, Phys. Rev. Lett. 27, 283 (1971).
[37] D. Cline, J. English, D. D. Reeder, S. Terrell, and P. Twitty, ibid. 21, 1268 (1968).
[38] O. D. Dal'karov, B. O. Kerbikov, and I. S. Shapiro, ZhETF Pis. Red. 15, 430 (1972) [JETP Lett. 15, 305 (1972)].
[39] O. D. Dal'karov, B. O. Kerbikov, V. B. Mandel'tsveĭg, and I. S. Shapiro, ibid. 14, 191 (1971) [14, 127 (1971)].
[40] I. S. Shapiro, Proceedings of the 4th International Conference on High-Energy Physics and Nuclear Structure (Dubna, 7-11 September, 1971), JINR, Dubna, 1972.

Translated by N. M. Queen