FROM THE HISTORY OF PHYSICS

Kepler's works in optics[1)] (On the 400th anniversary of his birth)

V. P. Linnik

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Johannes Kepler was not only a great astronomer who unraveled the true laws of the motion of the planets, but also an outstanding optical physicist. A significant portion of his works (no less than a quarter) deals with optics. There are so many propositions and suppositions, not always correct ones, advanced in these works, that a brief exposition of their contents is very difficult to accomplish and it cannot be exhaustive. Being an astronomer, Kepler always strove to point out the applications which optics could have in astronomy. Kepler's first work on optics is Paralipomena in Vitellionem (supplements to the Optics of the Polish scientist Witelo, which was written in the 13th century and which was the basic work on optics in Kepler's day). Appended to it was the treatise on optics written by the Arab scientist Al-Hazen. Basically, these were compilations which used the results accomplished by the Ancient Greek scientists Euclid, Ptolemy, and others.

Kepler's second work is Astronomiae Pars Optica (The Optical Part of Astronomy).

The third opus, perhaps the most interesting one, is Dioptrics, which, contains among others, the description of Kepler's famous astronomical telescope. Finally, many problems of optics are discussed in Kepler's correspondence with various scientists of his day.

The Paralipomena opens with general considerations of the nature of light (De natura lucis). These considerations are partially philosophical in nature and sometimes are simply theological. Light is one of the factors that causes material bodies to interact, because bodies by themselves are incapable of motion. The speed of light is infinite, because it has no mass. Light is indestructible in space and is distributed over the surface of a sphere, from which follows inverse-square. The change in the direction of the distribution of light incident on a material body depends on the properties of the surface of the body.

Color is light dormant in a translucent body and interacting with the incident light, in general agreement with Aristotle's views.

When the light is reflected, the incidence angle remains equal to the reflection angle, and both rays lie in the same plane as the normal to the surface of the body. An analogous situation arises in refraction.

Light heats up bodies, sometimes destroys them, and also destroys the color of bodies.

The section entitled "De figuratione lucis" (Concerning Images Produced by Light) deals with the relationship between image sharpness in a pinpoint camera (camera obscura) on the one hand, and the size and the form of the opening, as well as the form and distance of the image, on the other. This problem is very important when solar eclipses are observed in a dark room into which sunbeams are allowed to enter through an opening. Kepler found the reason for the curving of the horns in the image of the moon-eclipsed sun disc and furnished a precise calculation of the magnification of the sun disc visible on the screen.

"De fundamentis catoptricis et loco imaginis" (Concerning the Foundations of Catoptrics and the Location of the Image) is the section that deals with reflections from flat and spherical mirrors and with the location of the reflected image. Here, by the way, Kepler disproves Euclid's assertion that the image allegedly disappears altogether if the mirror is in the region near the normal drawn from the object to the mirror, and consequently passing through the image as well. To prove this, Kepler investigated inclined rays. In the discussions of the image location consideration is given to the possibility of naked-eye estimates of the distance to the images by viewing it with both eyes or one eye.

Finally, Kepler considers the location of the image in convex and concave mirrors. It is interesting that Kepler tackles so complicated a case as the viewing of an image by an eye located comparatively close to a tangent to the mirror surface. This is apparently the reason why there is not even a hint here of the relations, well familiar to us, between the distances to the object, image, and the surface of the mirror on the line connecting the object, the image, and the center of the mirror.

"De refractionum mensura" (Concerning the Measurement of Refraction) is the section where Kepler attempts to find a refraction law based on the experimental data available to him. It consists of Ptolemy's table as quoted by Witelo, and Tycho Brahe's refraction tables.

In the past, the opinions regarding the refraction of rays in the atmosphere were most contradictory. The approach selected by Kepler is a very complicated one and is of interest because of its originality; but it is precisely this originality that had hindered the discovery of the true law, which was later determined by Snell and Descartes. Kepler made an attempt to find the inner connection between the laws of refraction and the laws of reflection. This was stimulated by the familiar phenomenon that angular dimensions of an object located at the bottom of a vessel filled with water seem smaller than the dimensions of the object seen from the same distance in air. This is analogous to the

β	α	α_K	$\alpha-\beta$	$(\alpha-\beta)_K$	Δ
10°	13°23′	13°18′	3°23′	3°18′	5′
20°	27°7′	27°1′	7°7′	7°1′	6′
30°	41°48′	41°48′	11°48′	11°48′	0
40°	58°57′	58°45′	18°57′	18°45′	12′
45°	70°29′	68°43′	25°29′	23°45′	1°46′

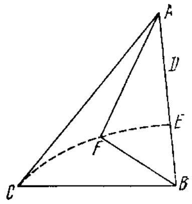

FIG. 1

appearance of an object when it is observed in a convex mirror.

Since Kepler attempts subsequently to simulate refraction with the aid of conic sections, he prefaces his discussion by a section on conic sections, which is the best exposition of the problem for its day. The term "focus" is used in this section for the first time.

Kepler then attempted to find with the aid of a geometrical model the relation between the incidence refraction angles. From two points A and B (Fig. 1) on a certain axis, he drew two angles α and β, which are respectively the angle of incidence and the angle of refraction of a ray passing from air into water. Let the rays intersect at some point C. Kepler constructed a hyperbola passing through point C and with foci at the points A and B. His hope was that the other points on the hyperbola would also produce values corresponding to the experimental data. The hope proved to be a vain one. He tried to use an ellipse, and then stopped altogether trying to relate points A and B with the foci of a conic section. The result was the same. Then he wrote: "Inquisitionis methodus nulla geometrica est, pericilitanda fortuna (The geometric method is unsuitable, we must keep on trying). Then Kepler abandoned attempts at constructing a model and assumed that the $\alpha - \beta$ ray deflection by refraction depends on two factors, on the density of the substance and on the incidence and refraction angles. Kepler's expression for the dependence of the ray deflection angle on the incidence and refraction angles would be $\alpha - \beta = \lambda\alpha \sec \beta$, where λ is a constant coefficient. This dependence agrees rather well with the experimental data (for water) which Kepler took from Witelo.

We present by way of illustration a table of values computed for water according by Kepler and according to Descartes, assuming that the results coincide at $\beta = 30°$. The refractive index of water is assumed to equal 1.33.

Here α_K and $(\alpha - \beta)_K$ are values obtained using Kepler's relation, and Δ is the difference between the deflection angles according to Kepler and Descartes. This difference becomes appreciable only at $\beta = 45°$, i.e., at an incidence angle $\alpha = 70°29'$. It is interesting that Kepler gives for water the extreme values $\alpha = 90°$ and $\beta = 53°30'$, i.e., the threshold at which total internal reflection sets in. Then Kepler moves on to atmospheric refraction.

Tycho Brahe had compiled an atmospheric-refraction table on the basis of his observations, but his data were not sufficiently reliable. When Kepler made an attempt to obtain constants for his own formula with the aid of Tycho's data on the refraction at the zenith angles 89° and 90°, he could not obtain a sufficiently precise value for the zenith angle 76°; later, however, he found other figures in Tycho's papers and his confidence in his own theory grew.

From his ideas regarding the dependence of refraction on the density of the substance he obtained an atmospheric density 1 :1177.7. Thus, he is the first to speak of the weight of the air, and his estimate of its density is not all that bad for a first try.

Kepler's refraction table is sufficiently precise for his times.

The last chapter of Paralipomena examines the eye as an optical instrument. In this chapter Kepler examines the ray diagram in the different media of the eye and arrives at the conclusion that the image of an object is formed on the retina not as it would be in a camera obscura with a simple aperture (stenope') but is equivalent to a camera with a lens. He then examines the action of spectacles that correct vision, and also of accomodation, pointing out the existence of near and far points of a sharp image. However, he sees the mechanisms of accomodation as involving either changes in the distance between the lens and the retina, or changes in the density of the vitreous humor. It had never crossed his mind that the shape of the lens can change.

All of Kepler's work on optics have in final analysis applications to astronomy as their goal. Thus, in the case of the eye he considers the influence of its defects on the quality of astronomical observations. He considers the effect of the position of the eye on the precision with which angles between the stars can be measured, and speaks about apparent increases in the angular dimensions of bright objects (irradiation) and about the disappearance of a faint body when it is next to a bright one.

The second significant work of Kepler's is Astronomiae Pars Optica (The Optical Part of Astronomy). Here there are three main sections: 1) The light from and illumination of celestial bodies and the shadow cast by non-self-luminous luminaries. 2) Changes in the apparent location of a luminary (parallax in general). 3) Application of optics to the observation of eclipses. Kepler considered the sun, the planets, and the stars to be self-luminous, and the moon and earth to be dark bodies. In light of the prevailing opinions of his time, Kepler's views of the physical structure of the sun are of some interest. The sun is supposed to be composed of a substance of immense density. Its mass is equal to the sum of the masses of all other luminaries, because it dominates over all of space. The structure of the solar matter is very simple and is extraordinarily homogeneous. It has the nature of a completely transparent fluid. The sun is the heart of the universe. It emits light and warmth. However, like the heat emitted by the heart of an animal, the sun's heat radiation is not the result of combustion. Therefore sunlight can be caused only by the existence of a soul (anima) or of a vital faculty (facultas vitalis). The soul dwells in the entire body of the sun and for this reason forces all of its parts to shine. The sun's index of refraction is extraordinarily great; therefore a ray of light emerging from it emanates from the center, reaches the surface radially and is distributed by the surface in all directions. Thus, we have three factors: the center, the radius, and the surface—the symbol of the Holy Trinity (according to Nicholas of Cusa).

He considers next the illumination of the moon by the sun and the position of the terminator in the limiting cases. Some ideas are expressed regarding the lunar surface, which is considered to be analogous to the surface of the earth: the bright spots are seas, the dark spots are continents. Two chapters are devoted to a detailed analysis of the distribution of illumination in the earth's shadow during lunar eclipses, with allowance for the refraction of the sun's rays in the earth's atmosphere. This refraction shortens the cone of the shadow and contributes to the red coloring of the moon during the full phase of the eclipse. Causes of total and annular solar eclipses are also examined.

Kepler includes in the section on optical phenomena also the parallax, which was given to men by the Creator as a means of determining celestial distances. To make it easier to determine the altitudinal parallaxes, Kepler compiled tables for zenith angles ranging from 1° to 90° and for parallaxes from $1'$ to $66'$ and he also provided instructions on how to find longitudinal and latitudinal parallaxes with the help of these tables.

Kepler devotes the next section to apparent motions of planets as seen from a given location. Visible conditions of the displacements of objects observed from a stationary base are examined together with instances when the observer also moves. The relative nature of the concept of a stationary observer is pointed out. Kepler says that if we were on the Moon we would consider it to be a stationary body.

In the end of Astronomiae Pars Optics Kepler dedescribes the ecliptic instrument, instrumentum eclipticum, which he had invented, and demonstrates its applications with a number of examples. The instrument consists of a ruler that can be moved in azimuth and up and down. These changes in the position of the ruler can be marked. Fastened to the one end of the ruler and perpendicular to it is a plate with an aperture. At a certain rather large distance from the plate is located a screen which is perpendicular to the ruler and which can be moved, if necessary, along the ruler. If we aim the ruler at the sun, then we obtain on the screen an image of the sun formed by the rays passing through the aperture as in a camera obscura. We can measure the diameter of the image of the sun on the screen and, knowing the distance between the aperture and the screen, we can find the angular dimension of the sun.

Measurements of the diameter of the sun carried out with the aid of the ecliptic instrument were more accurate than the earlier measurements of Tycho and Maestlin. This instrument also turned out to be useful for observing solar eclipses and for measuring the various phases of the eclipse. Although the moonlight is much weaker than sunlight, the instrument could nonetheless be used for observing the moon and its eclipses as well.

Dioptrics. The third, most significant, and perhaps most interesting work of Kepler's is Dioptrics. One can say that the stimulus for writing this book came from the remarkable discoveries made by Galileo with the aid of the telescope and published by him in the book Nuncius Siderous (Stellar Messenger).

Upon reading this book, Kepler printed a tract addressed to Galileo entitled Dissertatio cum Nuncio Sidereo (Conversation with the Stellar Messenger), where he says that he is doing new work on optics in which he also examines the telescope as well.

Kepler's Dioptrics differs from the two preceding works on optics in its strict and objective style reminiscent somewhat of Euclid's Geometry. It consists of 144 points which bear such titles as "Definitio" "Axioma", "Problema" and others. They present a precise presentation of various properties, for example a lens with the explanation of its operation.

At the outset the book examines the refraction of rays in transparent bodies bounded by planes. After three sections describing refraction, a problem is raised: how can one measure the refraction of a ray in a solid bounded by a flat surface? Then follow two variants of the solution to this problem. Further follows a series of premises deduced from experience. For example, refraction in glass and in quartz is approximately the same. Up to an incidence angle of 30° the deflection of a ray is proportional to the angle, and amounts to about one-third the incidence angle. The precisely measured deflection of a ray is not strictly proportional to the angle of incidence.

The largest deflection of a ray in a crystal is about 48°. A ray of light passing inside a crystal and reaching its surface may not leave the crystal (total internal reflection).

He deals next with passage of beams through a prism with a cross section in the form of an isosceles triangle, and points out that the emerging beam is colored. The treatment of lenses begins with definitions of converging and diverging beams from lenses—doubly convex, doubly concave, plano-convex, plano-concave, and, as he calls them, "mixed" lenses.

The focal length of a doubly convex lens with surfaces of equal curvature is equal to the radius of curvature of the surfaces; for a plano-convex lens it is equal to twice the radius of curvature. In the intermediate cases, the focal point has other locations that are indicated only qualitatively. There are no quantitative expressions in algebraic form at all.

"Concerning the Action of a Lens." Here he deals with the properties of the image produced by a positive lens. The image produced by a lens is inverted relative to the object. The size of the object and of the image are inversely proportional to their distances from the lens. Experimental problems are then posed: find the radius of curvature of a lens having equal sides. For this purpose one must measure the distance from the lens to the image of a remote object, this distance being the sought for radius. Some of these problems are quite curious: For example, how to light a fire with a doubly-convex lens or light a fire with a plano-convex lens; how to concentrate the light of a bright star with a convex lens enough to be able to read a text at night; how to use a convex lens, to project light at night as far as possible. To this end, the flame of a candle is placed in the focal point of a convex lens. This is a prototype of the modern-day searchlight. What is remarkable is that in addition to the convex lens it is proposed to place a concave mirror in such a way that the candle be located in the center of its curvature. This is a complete scheme of the lighting system used in modern-day projectors (Fig. 2). Further, a range-finding system is proposed in which the distance between the object and the lens is determined by measuring the distance from the lens to the image. Finally, proof is presented of the impossibility of igniting a distant object by projecting

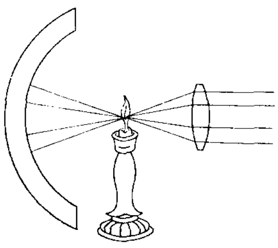

FIG. 2

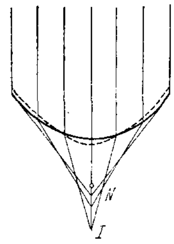

FIG. 3

on it the image of an incandescent body. This is a refutation of a proposal made by Porta, a contemporary of Kepler's.

The next section is devoted to the eye and to combination of the eye with a lens, to properties of spectacles and of the magnifying glass, and to visual examination of the image produced by a lens. What turns out to be particularly interesting in this section is the discussion of the convergence of rays in the focal point of a plano-convex lens or of the focusing of a parallel beam of rays passing from a refracting medium through a spherical boundary of this medium. It is shown that the rays are not gathered in one point but that different zones have different foci, the central zone having the focus farthest from the surface while the edge zone has the focus nearest to the surface. In other words, what is described is the phenomenon of spherical aberration. Kepler sought to find the shape of a curve capable of gathering all of the rays in one point. He considers this to be a hyperbola (Fig. 3).

Having finished with the properties of a single lens, Kepler moved on to a system consisting of two lenses and in the very first problem provides a description of his telescope. The problem reads: with the aid of two convex lenses obtain a magnification of an object with complete sharpness but in an inverted position. A drawing is used to show how the lens must be mounted in order to get a sharp image. However, no numerical data are provided. Kepler himself did not construct this system. It was made and adapted for astronomical observations by Scheiner and described by him in his treatise Rosa Ursina.

Among the problems that follow is: obtain on paper, with the aid of two convex lenses, an upright image of a distant object. This is the method now used for observing the sun on a screen, but Kepler said nothing about this possibility.

The next problem reads as follows: use two convex lenses to obtain a magnified sharp and upright image of an object. This is what is now known as a terrestial telescope, but it was first constructed by Scheiner.

Further follows a qualitative description of the properties of concave lenses, mainly used on conjunction with the eye.

In the next section, combinations of convex and concave lenses are discussed. Here he starts from the very outset with a detailed discussion of Galileo's telescope.

The 51st definition in Dioptrics says: The telescope (tubus) is a dark hollow cylinder both ends of which are covered with transparent glasses (lenses), i.e., it is an instrument with the aid of which distant objects appear to be situated close by.

The 52nd definition: One of the openings with its glass is turned toward the eye, the other—toward the object.

The 53rd postulate: the lines that pass through the centers of convexities and concavities of the two lenses must coincide, so that the lenses can be mounted in the tube perpendicular to the axis. It is pointed out further that the front (convex) lens produces an image of the object, but a concave lens is placed in the path of this image, and through the latter lens, under certain conditions of convergence of the emerging beam, the eye can see a magnified upright image. This is how Galileo's telescope functions.

The next problem is: Obtain on a paper, with the aid of a system consisting of a positive and a negative lens, a sharp inverted image. From the interpretation of this problem one can come to the conclusion that Kepler had arrived speculatively at the design of a modern-day telephoto lens.

From among remaining items in this section, the following are of interest:

1. Refutation of Porta's opinion that it is possible to obtain with the aid of an optical system a very thin beam of light capable of propagating over a great distance.

2. The image in the central part of the field of view of a telescope is sharper than around the edges.

3. Images produced by a small portion of an objective are sharper than the one obtained with full aperture (the action of a diaphragm). However, nowhere is there a mention of the fact that the field of views in a Galilean telescope depends on the aperture of the objective.

4. A method for determining the magnification of a telescope by simultaneously observing the same object directly with one eye and through the telescope with the other. This method was proposed also by Galileo, probably earlier.

The last section of Dioptrics deals with various combinations of convex and concave lenses, as well as of memisei.

The last, the 141st item in Dioptrics contains the problem of building a telescope in which eyepiece is convex and the object lens is concave. This system is now used to decrease the focal length of a motion picture camera lens.

Kepler lived in financially straitened and trying conditions and could not, therefore, test in practice a number of his interesting ideas. In particular, the best known of his optical ideas, now called the Keplerian telescope, was not constructed and sufficiently studied by him, and for this reason its remarkable sighting properties were not noted by Kepler. It was only almost forty years after Kepler's death that Auzout and Picard, by introducing cross-hairs in the focus of the objective of Kepler's telescope, began a new era in measurement astronomy.

[1)] Information from this article was used in author's paper delivered at the Symposium Dedicated to the 400th Anniversary of Kepler's Birth (Leningrad, August 1971) which was a part of the 13-th International Congress on the History of Science.