Turbulence and microstructure in the ocean

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The possibility of distinguishing between turbulence and wave fluctuations of the hydrodynamic fields is discussed. Possible mechanisms of generation of turbulence in the ocean are considered, and theoretical information as well as experimental data are given concerning the spectra of small-scale turbulence in the upper layer of the ocean. Intermittent turbulence in the main body of the ocean is described. The laws governing the turbulent diffusion in the ocean are briefly considered. A hypothesis is advanced concerning the spectral transport of the mean-squared vorticity in the field of quasi-two-dimensional turbulence. The small-scale convection produced by the difference between the diffusion coefficients of heat and salt in sea water is considered. Experimental data are presented on the vertical thin-layer microstructure of the ocean, and possible mechanisms whereby it is generated are considered, such as shear instability, internal waves, lateral convection, and differences between the diffusion coefficients for momentum, heat, and salt. The distortions produced in the internal-wave spectra by the presence of vertical thin-layer microstructure are analyzed.

CONTENTS

1. INTRODUCTION

Recent extensive measurements of the characteristics of ocean turbulence and of the fine vertical structure of hydrophysical fields in the ocean (temperature, electrical conductivity, speed of sound, refractive index, current velocity) have yielded many unexpected results.

It has turned out that the ocean is practically always and everywhere stratified into quasihomogeneous layers with thicknesses ranging from dozens of meters down to decimeters and centimeters, separated by very thin layers with abrupt vertical changes (jumps) of the hydrophysical parameters and appreciable lifetimes, at least in the dozens of minutes and hours. The turbulence is usually weak, incapable of destroying the indicated stratification, develops only within quasihomogeneous layers, has a local character (does not depend directly on the depth), and is characterized by small Reynolds numbers.

These results, besides being of direct importance (knowledge of the nature and properties of short-period fluctuations of the hydrophysical fields in the ocean), are apparently also of broad general-oceanological significance, since they alter appreciably the hitherto prevailing notions concerning the vertical structure of the ocean and the natural processes of its vertical mixing, and hence of the propagation of various admixtures in it (dissolved salts in gases, mineral suspensions, plankton, radioactive substances).

Moreover, these results call for a new understanding of the physical nature of the small-scale internal motions in a very stably stratified liquid, in which the buoyant forces suppress turbulence and the latter can develop only locally, in regions with local enhancement of the velocity gradients (which are apparently produced primarily in internal gravity waves, which develop intensively in stably-stratified liquids). It is probable that under these conditions the relative role of molecular transport of momentum, heat, and admixtures increases, and that the differences between the diffusion coefficients of these properties can become appreciable.

There is urgent need for coordinating the theoretical-physics treatment of the results of measurements of small-scale fluctuations of the hydrophysical fields in the ocean with further improvement of the apparatus and the measurement procedures. The purpose of the present review is to contribute to the solution of this problem.

It is very difficult to measure the turbulent fluctuations of current-velocity components, of temperature, electrical conductivity, the speed of sound, refractive index, or other hydrodynamic parameters in the ocean, since this calls for highly sensitive low-inertia marine turbulence meters. When these are towed by a moving vessel, the records of the natural fluctuations are distorted by instrument oscillations due to the rolling and pitching of the vessel, yawing of the pods carrying the turbulence meters, and vibration of the towing cables; at high frequencies, electrical noise is added. Owing to these difficulties, serious study of ocean turbulence has been undertaken only during the last decade, and the accumulated data are still scanty. The available information on ocean turbulence is reviewed in[1-5].

2. DIFFERENCE BETWEEN TURBULENCE AND WAVES

Another difficulty is that the frequency intervals of the turbulent fluctuations and of the surface and internal waves overlap to a considerable degree, so that in the frequency interval that is common to the turbulence and the waves the characteristics of the turbulence as such (defined as that fraction of the natural fluctuations which is not coherent with the waves) can be estimated only by filtering out from the readings of the marine turbulence meter not only the aforementioned noise of mechanical and electrical origin, but also the fluctuations produced by the waves.

Fluctuations produced by surface waves can be filtered out by recording, in synchronism with the total natural fluctuations $\xi(t)$ (where t is the time) also the oscillations $\zeta(t)$ of the sea level (or the oscillations of the pressure at a certain depth), by using some wave-plotting device. This procedure was first used by Bowden and White[6], and was developed in greater detail by Benilov and Felyushkin[7], who used Yaglom's[8] method of linear filtering of stationary random processes. According to this theory, the root-mean-squared fluctuations $\eta(t)$ produced by the surface waves can be approximated by sequences of finite sums in the form $\sum_k \beta_k \zeta(t - t_k)$, where the coefficients β_k are determined from the equations

$$B_{\xi\zeta}(t_l) = \sum_k \beta_k B_{\zeta\zeta}(t_l - t_k),$$

in which $B_{\zeta\zeta}(\tau)$ is the correlation function of the waves $\zeta(t)$, while $B_{\xi\zeta}(\tau)$ is the cross correlation function of the natural fluctuations and the waves. The rms filtering error is determined in this case by the formula

$$\sigma^2 = \int_{-\infty}^{\infty} \left| \frac{f_{\xi\zeta}(\omega)}{f_{\zeta\zeta}(\omega)} - \sum_k \beta_k e^{-it_k\omega} \right|^2 f_{\zeta\zeta}(\omega)\, d\omega,$$

where $f_{\zeta\zeta}(\omega)$ and $f_{\xi\zeta}(\omega)$ are the Fourier transforms of the aforementioned correlation functions, i.e., the corresponding spectral densities. Figure 1a shows a typical filtering after[7], in which they used synchronous records of natural fluctuations $\xi(t)$ of the temperature at a depth of 0.5 m and of the sea level $\zeta(t)$, as obtained with a buoy in the Mediterranean. In this sample, the "wave noise" $\eta(t)$ was maximal (on the order of 45%) at frequencies $\omega \sim 1.1-1.4$ rad/sec, which correspond to the maximum of the wave spectrum, and decreased rapidly with increasing frequency. In[7] they also compared other statistical characteristics of the natural fluctuations of the temperature $\xi(t)$ and of the "filtered" fluctuations $\xi(t) - \eta(t)$. Figure 1b shows the probability distributions of these fluctuations (with 65 terms in the finite sum approximating $\eta(t)$, corresponding approximately to a 1% filtering error σ^2/σ_η^2); we note that the probability distributions for the heights of the waves $\zeta(t)$ and for the "wave noise" $\eta(t)$ were approximately Gaussian in this case.

It is impossible to filter out the fluctuations produced by internal waves in the same manner, since the latter cannot be registered separately. To assess the feasibility in principle of resolving a horizontally homogeneous random vector field $\mathbf{u}(x, y, z, t)$ (for example, a velocity field) into turbulent and wave components, A. N. Kolmogorov proposes to consider the spectral representation of this field in a horizontal plane; this representation is defined by a vector random spectral measure $\mathbf{Z}(M)$ (where M are sets in the plane of the horizontal wave vectors $\mathbf{k}$; the measure $\mathbf{Z}$ also depends on the vertical coordinate z and on the time t), and to separate in this measure, for each fixed $\mathbf{k}$, the component*

$$\mathbf{Z}^{(0)} = k^{-2} ([\mathbf{nk}]\, \mathbf{Z})\, [\mathbf{nk}]^*$$

in a horizontal plane perpendicular to $\mathbf{k}$ ($\mathbf{n}$ is a unit vector in the vertical direction), and the component**

$$\mathbf{Z}^{(1)} = k^{-2} (\mathbf{kZ})\, \mathbf{k} + (\mathbf{uZ})\, \mathbf{n}$$

in a vertical plane containing $\mathbf{k}$. Then the field

$$\mathbf{u}^{(0)} = \int e^{i(k_1x + k_2y)} \mathbf{Z}^{(0)}(dk)$$

will describe the horizontal turbulence (we shall return to its properties in Chap. 6 below), while the field

$$\mathbf{u}^{(1)} = \int e^{i(k_1x + k_2y)} \mathbf{Z}^{(1)}(dk)$$

will contain both the turbulence and the waves, which can be distinctly separated only in nonintersecting sections of their frequency spectra. We then have $\overline{|\mathbf{u}|^2} = \overline{|\mathbf{u}^{(0)}|^2} + \overline{|\mathbf{u}^{(1)}|^2}$ (the superior bar denotes the mathematical expectation value), but the components $\mathbf{u}^{(0)}$ and $\mathbf{u}^{(1)}$ can, generally speaking, be correlated (although in the linear-dynamics approximation this correlation should apparently attenuate with time).

The principal hopes of separating turbulence from internal waves must apparently be placed in the use of phase relations (the phase-shift spectra) between fluctuations of different spatial components of the velocity and scalar fields, which are fixed in the internal waves and are arbitrary in the turbulence. The development of a suitable program for filtering internal waves from plots of the total natural fluctuations still remains a task for the theoreticians.

3. GENERATION OF OCEANIC TURBULENCE

One can point to a number of mechanisms capable of generating small-scale turbulence in the ocean[5]. These include the hydrodynamic instability of the horizontal velocity gradients in small-scale quasihorizontal motions (determined by their Reynolds number); the instability of the vertical velocity gradients in large (usually geostrophic) ocean currents, in drift currents in the upper layer of the ocean, in the boundary layer at the bottom (for example, tidal currents) and in the field of internal gravity waves (the latter mechanism is apparently the principal one in the main body of the ocean); the breaking of surface and internal waves; convection in layers with unstable stratification of the density (produced, for example, by cooling of the surface of the ocean during the cold seasons, and sometimes perhaps by accumulation of salts in the surface waters during periods of intensive evaporation).

In the cases of stable density stratification usually encountered in the ocean, $\partial\rho/\partial z > 0$ (where z is the depth; it would be more accurate to use the difference between the actual density gradient and its value at constant entropy), the vertical velocity gradients $\partial u/\partial z$

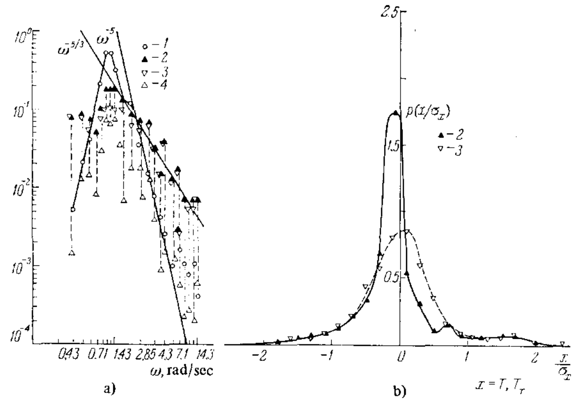

FIG. 1. a) Typical plots of spectral densities of the wave heights $f_{\xi\xi}(\omega)/\sigma_\xi^2$ (1), of the natural temperature fluctuations $f_{\zeta\zeta}(\omega)/\sigma_\xi^2$ at a depth of 0.5 m (2), of the filtered fluctuations $[f_{\zeta\zeta}(\omega) - f_{\eta\eta}(\omega)]/\sigma_\xi^2$ (3), and of the "wave noise" $f_{\eta\eta}(\omega)/\sigma_\xi^2$ (4) (from[7]); b) probability densities p of the natural (2) and filtered (3) temperature fluctuations (normalized to σ_ξ) in the same example as in Fig. a).

turn out to be unstable and generate turbulence, if the Richardson criterion,

$$\mathrm{Ri} = (g/\rho)\ (\partial\rho/\partial z)\ (\partial u/\partial z)^{-2} = [N/(\partial u/\partial z)]^2 < \mathrm{Ri}_{\mathrm{cr}},$$

is satisfied, where $N = [(g/\rho)(\partial\rho/\partial z)]^{1/2}$ is the so-called Vaisala frequency and g is the acceleration due to gravity. In geostrophic currents, corresponding to equilibrium between the horizontal pressure gradient and the Coriolis force,

$$\partial u/\partial z \sim (g/l\rho)\ \partial\rho/\partial x \sim KN^2/l$$

(where x is the coordinate along the current, l is the Coriolis parameter, and K is the slope of the isopycnic surface), and the Richardson criterion takes the form

$$KN > (\mathrm{Ri}_{\mathrm{cr}})^{-1/2} l.$$

In the internal waves in the pycnocline (the layer of the sharpest growth of the density with depth, usually at depths of 100—500 meters), the minimum local Richardson number is equal to

$$\mathrm{Ri} = a^{-2}k^{-2}\,[(N_m/n) - (n/N_m)]^{-2},$$

where a, k, and n are the amplitude, wave number, and frequency of the wave, while N_m is the maximum of N(z); according to Miles and Howarth, the sufficient criterion for stability is the condition $\mathrm{Ri} > 1/4$, so that at

$$ak > 2\ |\ (N_m/n) - (n/N_m)\ |^{-1}$$

the waves can be unstable near the crests and troughs. It is there where the spots of turbulence in the main body of the ocean are apparently generated.

With respect to the turbulence properties, the ocean can be broken up into three layers: 1) the upper mixing layer (above the interface layer), with a thickness on the order of 100 meters, which is continuously filled with the turbulence constantly generated by the atmospheric action with the aid of the breaking of the surface waves, drift currents, and convection; 2) the internal layer (almost the entire thickness of the ocean), in which only intermitted turbulence exists in the form of individual spots or "pancakes" probably formed in the regions of hydrodynamic instability of the internal waves; 3) a bottom layer of approximate thickness 10 m, analogous to the boundary layer of the atmosphere and continuously filled with turbulence. The upper and bottom layers are apparently separated from the internal layer by distinct boundaries of irregular shape, produced by the large-scale turbulent vortices (with scales on the order of the thickness of the layer) and by internal waves.

4. TURBULENCE SPECTRA IN THE UPPER LAYER OF THE OCEAN

In the upper mixing layer of the ocean, the mean-squared fluctuations of the velocity are usually of the order of 1 cm/sec and decrease on the average with depth. The intensity of the velocity fluctuations can also be assessed from the rate of dissipation of the turbulent energy ϵ, which at large Reynolds numbers is the only parameter of the kinetic-energy spectrum of the turbulence in the inertial subrange of wave numbers k (sometimes observed in the turbulence spectra in the upper mixing layer of the ocean), where the three-dimensional spectrum E(k) is described by the Kolmogorov-Obukhov "five-thirds law"

$$E(k) = C_1\varepsilon^{2/3}k^{-5/3};$$

here C_1 is a numerical constant, close to 1.4 according to the experimental data (see Secs. 21 and 23 of[9]); the frequency spectrum of the longitudinal velocity component, as recorded by a turbulence meter towed with velocity U, is then given by

$$E_1(\omega) = C_2\,(\varepsilon U)^{2/3}\omega^{-5/3},$$

where $C_2 \approx 0.48$. On the surface of the ocean, ϵ is usually of the order of $1—10^{-1}$ cm^2/sec^3 and decreases on the average with depth to values of the order of $10^{-3}—10^{-4}$ cm^2/sec^3 at the thermocline.

The mean-squared fluctuations of the temperature behave quite differently; they can decrease with depth initially, but apparently have a maximum (usually on the order of 10^{-1} °C) in the thermocline, where there are very large vertical temperature gradients. One can frequently expect the temperature-fluctuation spectra to contain an inertial-convective wave-number range in which these spectra are described by the Obukhov-Corrsin "five-thirds law"

$$E_{\mathrm{T}}(k) = B_1\varepsilon_{\mathrm{T}}\varepsilon^{-1/3}k^{-5/3}, \qquad (2)$$

where ϵ_T is the rate of equalization of the temperature-field inhomogeneities, and B_1 is a numerical constant, close to 1.1 in accord with the experimental data (see again Secs. 21 and 23 of[9]); the frequency spectrum of the temperature fluctuations, obtained from the records of a towed turbulence meter, is then given by

$$E_{\mathrm{T}}(\omega) = B_2\varepsilon_{\mathrm{T}}\varepsilon^{-1/3}U^{2/3}\omega^{-5/3},$$

where $B_2 \approx 0.7$. The values of ϵ_T apparently range from 10^{-3} to 10^{-8} (°C)^2/sec. In the region of very large wave numbers (frequencies), the turbulence spectra fall off rapidly because of the action of molecular forces. According to the Kolmogorov similarity hypotheses for locally-isotropic turbulence, the decrease in the velocity fluctuation spectra is described by a universal dimensionless function

$$\varphi(k\eta) = (\varepsilon\nu^5)^{-1/4}E(k),$$

where ν is the kinematic coefficient of molecular viscosity and $\eta = (\nu^3/\epsilon)^{1/4}$ is the Kolmogorov turbulence microscale; at small $k\eta$, the function φ is proportional to $(k\eta)^{-5/3}$, i.e., the "five-thirds law" (1) is valid. These predictions were confirmed by measurements of the spectra of ocean turbulence in a tidal current with very large Reynolds numbers, 3×10^8, by Stewart, Grant, and Moilliet[10-12]; *the universal function obtained by them* (φ) is shown in Fig. 2 (the abscissas show, in logarithmic scale, an argument proportional to $k\eta$). The logarithmic scale of φ may mask the scatter, but the latter is in fact small, as can be seen from Fig. 3a, which shows, now in natural scale, the same function φ multiplied by $(k\eta)^2$, which gives the dimensionless energy-dissipation spectrum. The maximum of the energy-dissipation spectrum occurs at a wave number $k \approx 1/8\eta$; at the same point, the one-dimensional longitudinal spectra begin to deviate noticeably from the "five-thirds law." Nasmyth[13] has noted that the spectra of oceanic turbulence in the dissipation range are sometimes higher than the "universal curve" of Fig. 2. He proposes to attribute this fact to the action of buoyant forces on liquid parcels that differ in density from the surrounding medium. In large scales, the buoyant forces can be comparable with the inertial forces and give rise to deviations from the "five-thirds law" (see formulas (4) below); with decreasing inhomogeneity scales, these forces decrease, but in the region of minimal scales the inertial forces become completely insignificant, while the buoyant forces can be statistically balanced by the molecular forces, thus producing the effect noted by Nasmyth.

A somewhat different behavior is exhibited in the region of large wave numbers by the temperature and salinity fluctuation spectra. The universal function

$$\varphi_T(k\eta,\ \mathrm{Pr}) = (\varepsilon/\varepsilon_T)(\varepsilon\chi^5)^{-1/4}E_T(k),$$

which is proportional to $(k\eta)^{-5/3}$ (Eq. (2)) at small $k\eta$, depends here on the Prandtl number $\mathrm{Pr} = \nu/\chi$ (χ is the kinematic coefficient of molecular diffusion of heat or salt). According to Batchelor's theory[14], at large values of Pr (in sea water we have $\mathrm{Pr} = 7$ for heat and 700 for salt), the spectrum in the region $k\eta \gg 1$ can depend on ϵ not directly (since the main energy dissipation is concentrated at the smaller wave numbers), but only via the quantity $\tau_\eta^{-1} = (\epsilon/\nu)^{1/2}$, i.e., via the typical value of the deformation rates that give rise to convective mixing *via rotation and via the coming together of* the isothermal (or isohaline) surfaces. Hence

$$\varphi_T \sim (\mathrm{Pr})^{3/4}\,\Phi\,[k\eta\,(\mathrm{Pr})^{-1/2}].$$

So long as the argument of the function Φ is small here (i.e., in the visco-convective range of the spectrum $1 \ll k \ll (\mathrm{Pr})^{1/2}$), the diffusion coefficient χ should not affect $E_T(k)$, for which purpose it is necessary to have $\Phi(x) \sim x^{-1}$.

We thus have $E_T(k) \sim k^{-5/3}$ in the inertial-convective spectrum range, $E_T(k) \sim k^{-1}$ in the visco-convective range, and only in the viscous-diffuse range $k\eta \gg (\mathrm{Pr})^{1/2}$ *does the* $E_T(k)$ *spectrum fall off sharply*, owing to the smoothing action of molecular diffusion. These predictions were confirmed by measurements of the temperature fluctuation spectra made in the ocean by Grant, Hughes, and Vogel[15]. One example of their $E_T(k)$ spectra (compared with $E(k)$) is shown in Fig. 3b (curve 1).

As already noted, in the region of small wave numbers the turbulence spectra in the stratified ocean can deviate from the "five-thirds law" owing to the action of the buoyant forces on the turbulence. According to the Obukhov-Bolgiano similarity theory (see Sec. 21.7 of the book[9]), *this action becomes noticeable in large* scales

$$L \gg L_* = \varepsilon^{5/4}\varepsilon_T^{-3/4}(\alpha g)^{-3/2}, \tag{3}$$

where $\alpha \sim 2 \times 10^{-4}\ \mathrm{deg}^{-1}$ is the coefficient of thermal expansion of water. At $k \lesssim 1/L_*$, the factors C_1 and B_1 *in (1) and (2) become functions of* kL_*; *these functions* were calculated by semiempirical methods in[16], in the case of stable stratification of the density, the large energy lost to the work against the buoyant forces should make the rate of viscous dissipation of the energy ϵ much smaller than the rate of energy transfer over the spectrum in the region of small k, and should therefore cease to influence the shapes of the spectra in this region. To this end we must have $C_1 \sim (kL_*)^{-8/15}$ and $B_1 \sim (kL_*)^{4/15}$, so that

$$E(k) \sim \varepsilon_T^{2/5}(\alpha g)^{4/5}k^{-11/5},\qquad E_T(k) \sim \varepsilon_T^{4/5}(\alpha g)^{-2/5}k^{-7/5}. \tag{4}$$

Of course, such a range can appear in the spectrum only if L_* is smaller than the integral turbulence scale. On the other hand, if L_* turns out to be a small quantity of the order of η, then the interval in which the laws (4) hold will occupy the entire space of the inertial subrange. Figure 4 shows a typical spectrum of the velocity fluctuations in the upper layer of the ocean, which agrees with (4) and was obtained on the second voyage of the ship "Akademik Kurchatov" in the Atlantic (it is impossible here, of course, to estimate the contribution of the internal waves, which frequently have a spectrum $E(k) \sim k^{-3}$; this also applies to other spectra frequently observed *in the ocean*, $E(k) \sim k^{-m}$ *with* $m > 5/3$).

We note, however, that in a region with not too small scales, the universal-type spectra (1), (2), and (4) are predicted by similarity theory only for turbulence with very large Reynolds numbers. Real ocean turbulence, however, apparently usually appears to develop only in

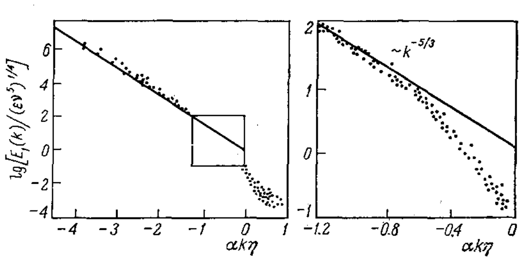

FIG. 2. a) Dimensionless longitudinal spectrum of velocity fluctuations in a tidal current in the sea (after[10]); b) enlargement of the square inside Fig. a.

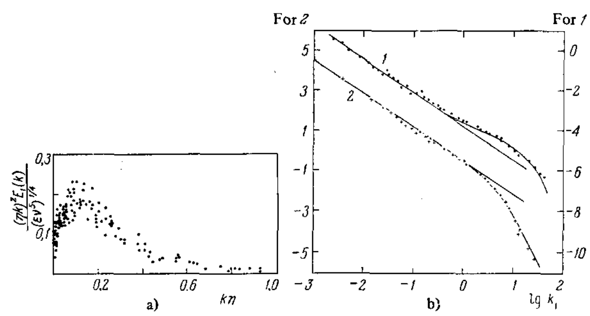

FIG. 3. a) Dimensionless energy-dissipation spectrum (after[10]); b) typical spectrum of the temperature fluctuations in the ocean (curve 1, right-hand scale) compared with the velocity-fluctuation spectrum (curve 2, left-hand scale) (after [15]).

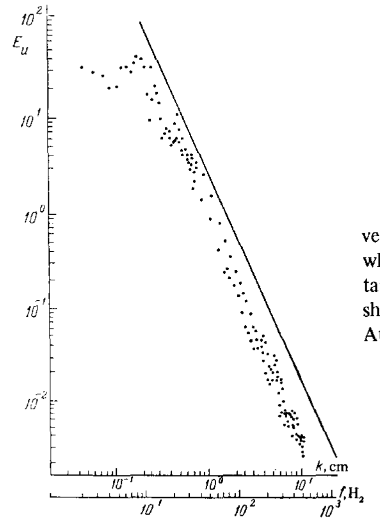

FIG. 4. Typical spectrum of the velocity fluctuations in the ocean, which obeys the law (r) and was obtained on the second voyage of the ship "Akademik Kurchatov" in the Atlantic.

quasi-uniform layers with thicknesses on the order of, say, several meters, in which the velocity drops are of the order of several cm/sec and the Reynolds numbers are of the order of 10^4—10^5, i.e., only one or two orders of magnitude higher than the critical number $\mathrm{Re}_{cr} \sim 3000$. At such small values of Re, there will no longer be any wave-number intervals with universal spectral laws (1), (2), or (4), and the turbulence spectra in scales comparable with the thickness of the quasi-homogeneous layers will be determined by a larger number of large-scale parameters. Their calculation calls for the development of a concrete hydrodynamic theory for quasiuniform layers in the ocean (or generally in a stably stratified liquid), which is still a matter for the future.

5. INTERMITTENT TURBULENCE IN THE MAIN BODY OF THE OCEAN

In the internal layer of the ocean, a distinguishing feature of the turbulence is its intermittent character. It can be characterized by an intermittence coefficient p(z), defined as the average fraction of the area occupied by the turbulence at the depth z. In submarine measurements by Grant, Moilliet, and Vogel[17], the coefficient p(z) was equal to unity in the upper mixing layer of the ocean of thickness 50 meters, decreased to 0.05 at 100 m, and then changed little down to a depth of 300 m (ϵ was equal to 2.5×10^{-2} cm^2/sec^3 at a depth of 15 meters and 1.5×10^{-4} at z = 90 m, while ϵ_T was 5.6 $\times 10^{-7}$ (°C)^2/sec at a depth of 15 m, reached a maximum of 6.7×10^{-6} in the density interface layer at z = 43 m, and then decreased to 7.2×10^{-8} at z = 90 m). Typical values of p(z) in the inner layer of the ocean are apparently of the order of 10^{-2}.

Kolomgorov has proposed to use, for more detailed description of the spatial intermittence of the small-scale fluctuations, a more frequent (say, every three seconds) calculation of the structure functions or spectra of the fluctuations, using records taken with towed turbulence meters over many minutes. An experiment with such a calculation, using the recorded fluctuations of the electrical conductivity of water on the seventh voyage of the USSR Academy vessel "Dmitriĭ Mendeleev," has shown that the three-second structure functions vary with periods on the order of several minutes, i.e., on the order of the Vaisala period, which apparently points to a connection between the intermittence and the internal waves, which possess this period. It would be even more interesting to perform such calculations on the basis of records of the current-velocity fluctuations.

There are at present practically no quantitative measurements of the turbulence characteristics at large depths in the ocean. One can attempt to extract some preliminary estimates of certain characteristics from information on the stratification of the density (which is formed under the influence of vertical turbulent exchange). It has been established in[18] that in an appreciable lower part of the internal layer of the ocean, the vertical profile of the Vaisala frequency N can frequently be approximated by the function Ah, where h is the height above the bottom (see one of the examples in Fig. 5), and the coefficient A varies at different hydrological stations in the range from 10^{-7} to 10^{-6} m^{-1}sec^{-1}. If an attempt is made to apply the theory of similarity for turbulence in a stably stratified medium to the internal layer of the ocean (see Sec. 7 of the book[9]), then the formula N = Ah corresponds to exchange-coefficient ratios for heat and momentum that decrease with height like h^{-2}, and the coefficient A is then given by the formula

$$A = (\varkappa/\mathrm{Ri}_m)\,(gM/\rho)^2\,u_*^{-5},$$

where $\kappa \approx 0.4$ is the Karman constant, $\mathrm{Ri}_m \sim 10^{-1}$ is the maximum value of the dynamic Richardson number, u_* is the friction velocity, and $M = \overline{\rho' w'} = p\langle\rho' w'\rangle$ is the vertical turbulent mass flux (w is the vertical velocity, the primes denote the turbulent fluctuations, the superior bar denotes averaging over the area, and the angle brackets denote averaging over only the turbulent spots). From this we obtain at $u_* \sim 1$ cm/sec the value $M \sim 10^{-8}$ g/cm^{2}sec. On the other hand, according to[18], in the upper part of the inner layer of the ocean the dependence of N on the depth z can frequently be approximated by the formula $N = w_*/z$, where $w_* \approx 2.2$ m/sec varies little from station to station. Equating the two expressions for N in the middle of the ocean with z = H/z, it is possible to obtain the following estimate for the relative fluctuations of the density in the turbulent spots[19]

$$\rho'/\rho \sim Bu_*^2/gH;$$
$$B = (\varkappa r_{\rho w})^{-1}\,[(4\varkappa\,\mathrm{Ri}_m/p)\,r_{uw}\,w_*/u_*]^{1/2},$$

where $r_{\rho w}$ and r_{uw} are the correlation coefficients between the corresponding fluctuations, which are apparently of the order of 10^{-1}. Therefore $\rho'/\rho \sim 10^{-6}$ and $T' \sim \rho'/\alpha\rho \sim 10^{-2}$ deg (these estimates depend, of course, on the assumed value $u_* \sim 1$ cm/sec). Finally, from the relation $u_*^2 \sim p r_{uw} w'^2$ at $p \sim 10^{-2}$ we see that the fluctuations w′ in the turbulent spots can exceed the values of u_* by at least one order of magnitude.

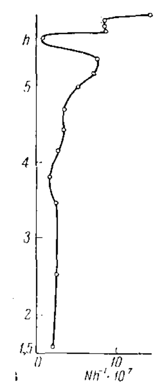

FIG. 5. Plot [3] of the Vaisala frequency N vs. height h above the bottom at the "Vityaz'" hydrological station No. 4371 in the Pacific Ocean (27°07′ northern latitude, 153°45′ eastern longitude ocean depth 6 km).

There have been practically no measurements of turbulence in the bottom layer of the ocean. We note, however, measurements of the low-frequency fluctuations (with frequencies on the order of 10^{-1}—10^{2} cycle/h) in the bottom layer of a deep ocean, performed by Munk and Winbusch[20], who have interpreted their results with some success within the framework of similarity theory for turbulence in the case of unstable stratification (see Sec. 7 of[9]) for the purpose of estimating the ability of the bottom layer to transmit geothermal heat upward.

6. TURBULENT DIFFUSION IN THE OCEAN

The turbulent character of the ocean greatly accelerates diffusion of various admixtures in it. A study of this phenomenon becomes particularly timely in connection with the growing danger of contamination of the ocean with oil, radioactivity, DDT, lead, and other harmful impurities. It should be borne in mind that the horizontal diffusion produced by the medium-scale horizontal turbulence is much more rapid, especially in the presence of the velocity gradients of the large-scale currents, than the vertical diffusion, which is produced by small-scale turbulence. The latter is characterized by depth-dependent (and strongly stratification-dependent) eddy diffusivity D_z, which are of the order of 10 cm^2/sec in the upper layer of the ocean; the rms displacements of the impurity particles then increase with time like $(2D_zt)^{1/2}$. The horizontal turbulence in the ocean is usually represented as a result of the breaking up of large eddies into small ones. This process is characterized primarily by the rate ϵ of energy redistribution over the scale spectrum, and satisfies the "five-thirds law" (1) and (2) (and the analogous "four-thirds law" of Richardson for the dependence of the coefficient of horizontal diffusion on the dimension L of the diffusing impurity spot, $D_h \sim \epsilon^{1/3}L^{4/3}$); the rms displacements of the impurity particles are then proportional to $(\epsilon t^3)^{1/2}$.

This representation is described in the article[21] and in Ozmidov's book[22]. It is confirmed, for example, by measurements of the inhomogeneities of the temperature field of the surface of the ocean[23] (the structure functions of which, in scales 10^0—10^1 km, turned out in many cases to satisfy a "two-thirds law" equivalent to (28), $(\delta_rT)^2 \sim \epsilon_T\epsilon^{-1/3}r^{2/3}$, where r is the distance), and by a direct empirical verification of the "four-thirds law" and its consequences from data of diffusion experiments in the ocean[24-25] (see Fig. 6, where the estimates obtained for ϵ are 10^{-4} cm^2/sec^3 at scales 10^1—10^3 m and 10^{-5} cm^2/sec^3 at scales 10^1—10^3 km). Figure 7 shows the frequency spectrum obtained in[25] for the impurity-concentration fluctuations at a fixed point of the sea in the presence of a continuous point source; at frequencies higher than 0.03 Hz, it obeys the "five-thirds law" (and at lower frequencies the ω^{-2} law, which the authors of[25] propose to represent by the formula $E_T(k) \sim \epsilon_T u_*^{-1}k^{-2}$, where u_* is the friction velocity). The joint action of the vertical diffusion (with different models for the diffusivity D_z) and of the horizontal diffusion that satisfies the "four-thirds law" has been analyzed theoretically in[26].

We note that the diffusion in accordance with the $(\epsilon t^3)^{1/2}$ law should not be confused with ordinary $(2D_zt)^{1/2}$ diffusion, which is accelerated in the presence of a gradient Γ of the large-scale current velocity in the direction of this flow, and follows the law $(2/3\Gamma^2D_zt^3)^{1/2}$ (see Sec. 10.4 of the book[9]). This effect may be responsible for the frequently observed stretching of the spots of the diffusing impurity in the flow direction.

7. SPECTRAL TRANSPORT OF ENSTROPHY

We recall finally, a representation of the horizontal turbulence different from that given above, based on the fact that two-dimensional flow is subject not only to the kinetic-energy conservation law, but also to the vorticity conservation law, so that the structure of the two-dimensional turbulence in the inertial subrange of the spectrum can be determined not only by the rate of energy degeneracy ϵ, but also by the rate of degeneracy of the enstrophy (i.e., the mean-squared vorticity) ϵ_1 (see the papers of Batchelor[27] and Kraichnan[28]). This gives rise to a length scale $L_1 = (\epsilon/\epsilon_1)^{1/2}$, and the constants C_1 and B_1 in the "five-thirds laws" (1) and (2) become functions of kL_1. If it is assumed that at one of the ends of the inertial subrange of the spectrum only the parameter ϵ is of importance, and at the other end only the parameter ϵ_1, then on this second end we obtain in place of the "five-thirds laws" the laws

$$E(k) \sim \varepsilon_1^{2/3}k^{-3}, \quad E_T(k) \sim \varepsilon_T\varepsilon_1^{-1/3}k^{-1}, \tag{5}$$

and instead of Richardson's "four-thirds law" we obtain $D_h \sim \epsilon_1^{1/3}L^2$. The law (5) for E(k) has found some confirmation in numerical experiments on two-dimensional turbulence and in the statistics of large-scale atmospheric motions, but no corresponding data have been published as yet on horizontal turbulence in the ocean.

8. DOUBLE DIFFUSION

Back in 1956, Stommel and co-workers[29] called attention to the possible development of convection in a layer of salt water with stable stratification of the density, but with oppositely directed gradients of the temperature and salinity, as a result of "double diffusion," i.e., a difference between the diffusion coefficients for heat and salt in the water (the diffusion coefficient for heat in sea water is 100 times larger than that for salt). This idea, subsequently developed by Stommel[30], stimulated a number of theoretical[31-38] and laboratory[39-47] investigations of thermohaline convection.

In the laboratory experiments of Turner and Stommel[39-40] it was established that when cold and

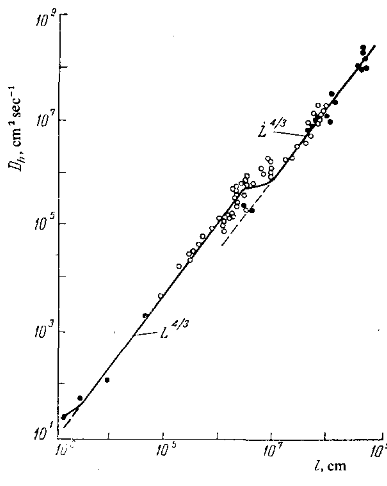

FIG. 6. Dependence of the coefficient of horizontal turbulent diffusion on the scale of the phenomenon from data of two experiments with diffusion of tracer spots in the ocean (from[24]).

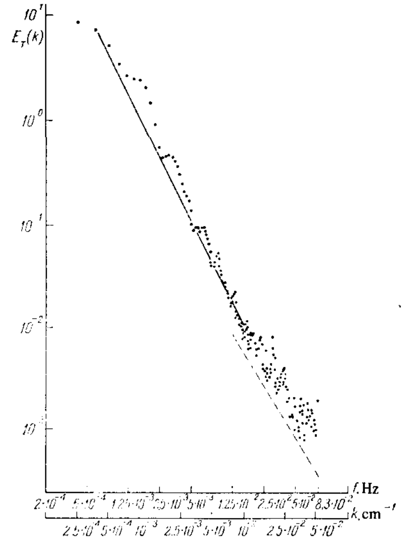

FIG. 7. Frequency spectrum of the fluctuations of the rhodamine concentration in sea water at a distance 150 m from a continuous point source (delivering 0.12 g/sec) near the Baltic shore (from[25]).

relatively fresh water is placed over warm, salt (denser) water, stratification takes place, i.e., successive convective and laminar layers are produced, since the relatively rapid diffusion of the heat upward induces convection at certain levels, but the upward penetration of the latter is limited by the stable salinity gradient, which is preserved in the laminar layers because of the slowness of salt diffusion. Besides the convective vertical heat flux H, there is also produced a convective salt flux F. When recast in dimensionless form by dividing respectively by $D_T(\delta T/h)$ and $D_S(\delta S/h)$ (where D_T and D_S are the diffusion coefficients for heat and salt, while δT and δS are the vertical differences of the temperature and salinity and h is the layer thickness), these fluxes are proportional to $(\mathrm{Ra})^{1/3}$, where $\mathrm{Ra} = \alpha g h^3 \delta T/\nu\chi_T$ is the Rayleigh number (α is the coefficient of thermal expansion of the water, g is the acceleration of gravity, and ν and χ_T are the kinematic viscosity and thermal-conductivity coefficients), with proportionality coefficients that depend on the ratio $\beta\delta S/\alpha\delta T$ of the contributions of salinity and temperature to the vertical density difference ($\beta = \rho^{-1}\partial\rho/\partial s$). Experiments have shown that the ratio $\beta F/\alpha H$ of the changes of the potential energy as a result of the transport of salt and heat first decreases rapidly with increasing $\beta\delta S/\alpha\delta T$, and at $\beta\delta S/\alpha\delta T > 2$ it assumes an approximately constant value 0.15, i.e., 15% of the potential energy released in the convective heat transfer goes to lift the salt.

Turner and Stern[41,44] have established in their experiments that when warm salt water is placed over cold and less salt (denser) water, convection develops in the form of long and narrow vertical cells called "salt fingers," since the relatively rapid horizontal equalization of the temperature anomalies gives rise to density anomalies, while the salinity anomalies are preserved because of the slow diffusion of the salt. The experiments have shown that in this case the ratio $\alpha H/\beta F$ depends little on the stability parameter $\alpha\delta T/\beta\delta s$, and is approximately equal to 0.56, i.e., more than half of the potential energy released by the salinity convection goes to heat transport, namely, the "salt fingers" are an effective mechanism for vertical transport not only of salt but also of heat. The convective layers should look like vertical steps on the temperature and salinity profiles. According to Stern's approximate theory[36], the maximum thickness of the steps is estimated by the formula

$$h_m \approx \nu^2 (\chi_T\chi_s)^{-3/4} (g\beta\partial S/\partial z)^{-1/4},$$

from which we get $h_m \sim 20$ meters at $\nu = 1.5 \times 10^{-2}$ cm^2/sec, $\chi_T = 1.3 \times 10^{-3}$ cm^2/sec, $\chi_S = 1.3 \times 10^{-5}$ cm^2/sec, and $\beta\partial S/\partial z \sim 10^{-8}$ cm^{-1}. The theory of layers with "salt fingers" still requires further development.

9. OBSERVATIONS OF THE MICROSTRUCTURE IN THE OCEAN

If we start from the idea of double diffusion, we can expect to find a microstructure in ocean layers with oppositely directed vertical temperature and salinity gradients, i.e., alternating laminar and convective layers, which are seen as steps on the temperature and salinity profiles. To reveal these steps, the oceanologists had, of course, to abandon the tradition of plotting smooth temperature and salinity profiles based on measurements of water samples from different depths with the aid of so-called bathometers in favor of low-inertia continuously recording temperature-salinity-depth (TSD) probes. Their use has made it possible in many cases to observe a steplike microstructure in the ocean[48-83], which subsequently turned out to be an almost ubiquitous phenomenon.

Stratification under conditions of temperature inversions (where the temperature increases with increasing depth) was observed by Ragotzkie and Likens[48] and by Hoare[50] under the ice in the Antarctic salt lake Wanda (several steps of thickness 1—1.5 m). It was also observed over hot salt water wells at the bottom of the Red Sea by Swallow and Crease[49] (homogeneous layers of thickness 70—150 m at depths 1900—2100 meters) and by Krause and Zeigenbein[51] (homogeneous layers of thickness 20—40 m at the same depths), and also over the warm and salt Red-Sea waters in the Gulf of Aden by Siedler[58] (steps of approximate thickness 10 m at depths 250—320 meters) and along the Somali shore by Krause[59] (homogeneous layers with thicknesses on the order of 100 meters at depths 300—600 m). Siedler[60] has also observed them over the Mediterranean waters in the eastern Atlantic (homogeneous layers 100—200 m thick at depths 500—1000 m). These data were summarized by Fedorov[64,72]; Fig. 8 shows examples taken from[72].

The stepwise microstructure observed in stratifications corresponding to the conditions of formation of "salt fingers" (decrease of the temperature and salinity with depth) was observed in the principal thermocline of the Sargasso Sea off the Bermuda Islands by Cooper and Stommel[55] (numerous homogeneous layers of thickness 3—5 m, separated by laminar layers 10—15 m thick with temperature drops 0.3—0.5°C and salinity drops 0.04—0.1%, traced to distances 400—1000 m in the horizontal direction). Under the Mediterranean waters in the eastern Atlantic they were observed by Tait and Howe[56,73], by Pingree[66], and by Zenk[71]. Pingree[56] found up to nine steps of thickness 15—30 m with temperature drops between them on the order of 0.25% C and salinity drops 0.04% at depths 1280—1500 meters; see, for example, Fig. 9. Zenk[71] found on the average 7.6 steps 21.6 m thick with temperature and salinity drops 0.37°C and 0.069% at depths 1300—1545 m (see the example in Fig. 10). In the Northern Trade current in

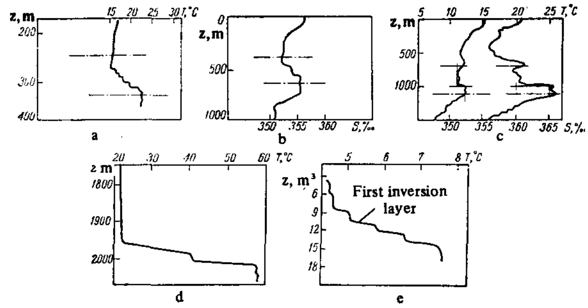

FIG. 8. Examples of stepped thermohaline structure of temperature inversions: a) "Meteor" station No. 49 in the Gulf of Aden[58]; b) "Meteor" station No. 130 at the Somali shore[59]; c) "Meteor" station No. 52 in the eastern Atlantic[60]; d) "Meteor" station No. 385 in the Red Sea[51]; e) Wanda Lake in the Antarctic[50].

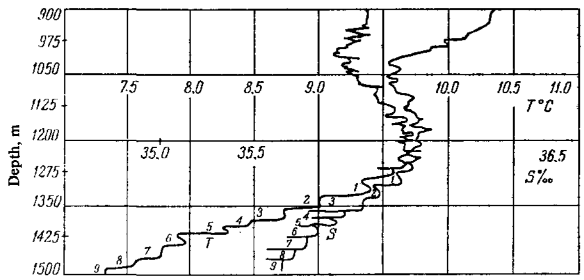

FIG. 9. Stepwise microstructure under the Mediterranean waters in the Atlantic, observed at the "Discovery" station No. 15 (after[56].

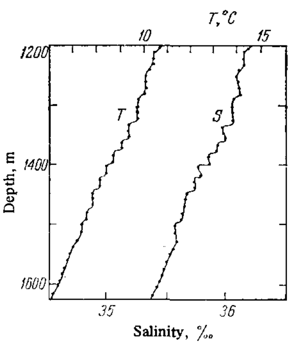

FIG. 10. Stepwise microstructure under the Mediterranean waters in the Atlantic as observed at the "Meteor" station No. 36 (from[71]).

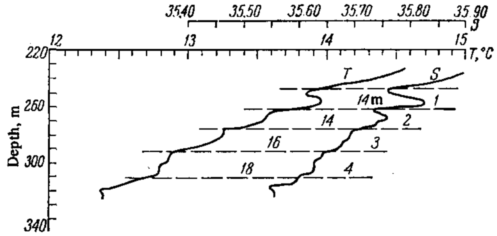

FIG. 11. Steplike microstructure in the Northern Trade current in the Atlantic at the "Akademik Kurchatov" station No. 603 (from[81]).

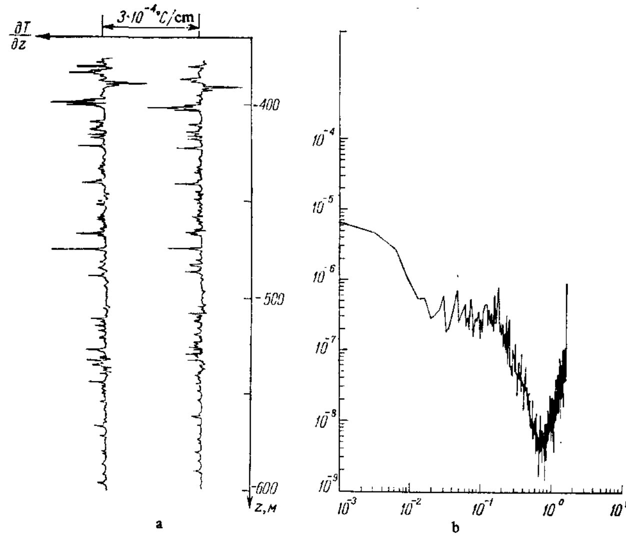

FIG. 12. a) Profiles of $\partial T/\partial z$ in the San Diego trough, recorded with two TD probes[65] 50 meters apart; b) spatial specturm of vertical temperature gradient $\partial T/\partial z$ (from[65]; at $k > 10^{-1}$ cycles/cm the noise is electronic).

the Atlantic, Fedorov[79-82] found a few steps 15–30 m thick with temperature drops 0.36–0.65°C and salinity drops 0.07–0.14% at depths 250–350 m; see the example in Fig. 11. These data are summarized in[82]. We note that the stability parameter $\alpha\delta T/\beta\delta S$ in the layers with the steps ranged in this case only from 1.30 to 1.55.

Woods[52,61-63] observed a rather thin-layered microstructure in the seasonal thermocline of the Mediterranean Sea by taking underwater motion pictures of dyed layers. Nasmyth[70] registered the microstructure in the upper layer of the ocean by using a pickup placed on a cyclically diving pod towed by a ship. Since the microstructure records are subject to some distortion when a TSD probe is dropped by cable from shipboard owing to the rolling of the ship, Woods[69,74-75] and Cox, Nagata, and Osborn[65] observed the microstructure by using freely falling low-inertia TD probes. Figure 12a shows typical profiles of the vertical temperature gradient $\partial T/\partial z$ at depths 400–600 meters, obtained 40 km west of San Diego with probes[65] falling at a rate of 50 cm/sec at a time constant 0.25 sec. Figure 12b shows the spatial spectrum of $\partial T/\partial z$ obtained from records of a probe[65] falling at a velocity in the interval 3–15 cm/sec and with a time constant 0.02 sec; one can clearly see the rapid decrease of the spectrum at $k > 10^{-1}$ cycle/cm, which shows that this probe registered practically the entire fine structure of the temperature profile.

The accumulated data show that double diffusion is far from always responsible for the formation of the stepwise microstructure (and apparently is involved only in a few special cases). Thus, Simpson and Woods[74] have discovered a steplike microstructure on the temperature profiles in the fresh-water lake Loch Ness (see the examples in Fig. 13) where, owing to the absence of salinity, there can be no double-diffusion mechanism. On the fifth voyage of the USSR Academy of Sciences ship "Dmitriĭ Mendeleev" in the Pacific, a steplike microstructure was observed in a number of cases with stable stratification both with respect to temperature and with respect to salinity (see the example in Fig. 14), cases in which this mechanism is likewise inoperative. It can be assumed that in layers with vertical gradients of the temperature and of the current velocity, microstructure is produced as a result of differences between the coefficients of molecular diffusion of heat and of momentum (the diffusion coefficient for momentum in water is seven times larger than the diffusion coefficient for heat), but this hypothesis has not yet been dealt with theoretically.

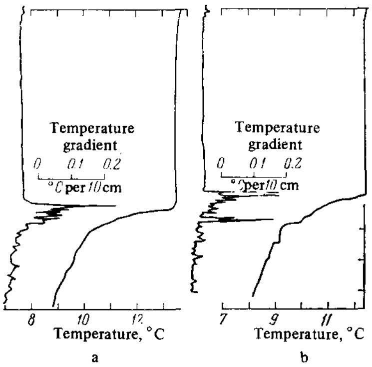

FIG. 13.

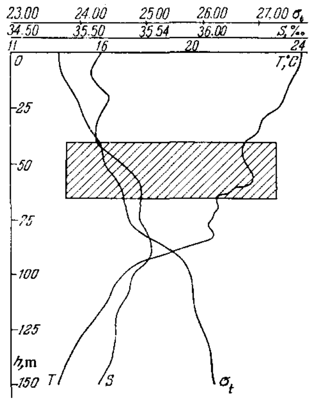

FIG. 14.

FIG. 13. Profiles of the temperature T(z) and of the temperature gradient $\partial T/\partial z$ in the fresh-water lake Loch Ness in the warm southern zone (a) and in the colder northern zone of the lake (b) (from[74]).

FIG. 14. Steplike microstructure in stable stratification, both with respect to temperature and with respect to salinity, observed at "Dmitriĭ Mendeleev" station No. 378.

10. LATERAL CONVECTION

The third and perhaps simplest hypothesis concerning the origin of the steplike microstructure is "lateral convection," i.e., the equalization of the horizontal differences between neighboring differently stratified water columns by quasihorizontal displacements of individual layers or lenses of water (owing to the independent meanderings of the currents in different layers, for example as a result of baroclinic instability of the currents or disturbance of their geostrophic equilibrium in the upper layer of the ocean by the intermittent wind stresses on the water surface, or else because the layers that are heavier than the neighboring horizontal ones glide along inclined isopycnic, or, more readily, isoentropic surfaces, giving rise to a more stable stratification than in the initial water columns).

The concept of lateral convection was advanced by Stommel and Fedorov[53], who proposed to explain a temperature inversion of thickness 10 m that was observed under the 120-meter upper mixing layer in the Timor Sea as being due to the sliding down of thermal waters, made more salty by evaporation, from the Australian shelf to horizontal distances of hundreds of miles. A similar explanation for the layers of Arctic waters in the North Atlantic was proposed by Cooper[54]. The idea of lateral convection was confirmed by the laboratory experiments of Thorpe, Hutt, and Soulsby[84] and of Gubin and Khaziev[85]. In[77], Fedorov used it to explain the formation of inversions at a depth 45—50 m in the Atlantic, at the stations "Crawford" No. 308 and "Atlantic" No. 5806, and in[76] it was used to trace a thin layer of reduced salinity at a depth of 75 m in the Northern Trade current in the Atlantic at two "Akademik Kurchatov" stations, No. 561 and No. 567 (Fig. 15). Woods and Wiley[75] proposed to attribute to lateral convection of gravitational or dynamic character the formation of quasihomogeneous layers of several meters thickness (separated by "sheets" of 10—20 cm thickness with abrupt drops of the temperature, salinity, and velocity).

On the seventh voyage of the "Dmitriĭ Mendeleev" (January-March 1972 in the Indian Ocean), the "lateral convection" hypothesis obtained its first direct confirmation through measurements of the mesostructure of

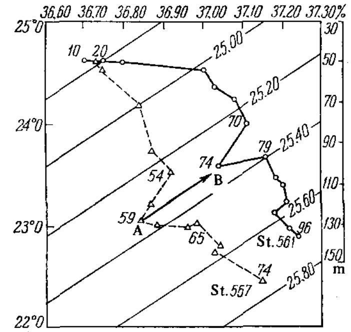

FIG. 15. Temperature-salinity curves obtained at the "Akademik Kurchatov" stations No. 567 (dashed curve) and No. 561 (solid curve) with layers of decreased salinity A and B on one and the same isopycnic level (from[78]).

the current-velocity field. These measurements have revealed the presence in this field of vertical inhomogeneities with scales down to 5—10 m and less, which correlated fairly well with the inhomogeneities of the temperature and of the salinity of the water, so that different quasihomogeneous layers apparently do indeed move with horizontal velocities that differ both in magnitude and direction. The theory of such an independent motion of different layers still has to be developed.

11. FORMATION OF MICROSTRUCTURE FOLLOWING LOSS OF STABILITY OF INTERNAL WAVES

The fourth hypothesis concerning the origin of quasihomogeneous layers (vertical steps of the temperature and salinity profiles) is advanced in the theory of Bretherton[86] and Orlansky and Bryan[87] concerning the breaking of internal waves of finite amplitude, which leads to mechanical mixing of a certain layer of water and to subsequent density convection in this layer. According to[87] (in which, to be sure, they consider only thermal stratification), the internal wave breaks when the orbital velocity on its crest exceeds its phase velocity; this criterion is reudced to the form $\mathrm{Ri} \ll 1 + (k^2/m^2)$, where k and m are the horizontal and vertical wave numbers.

A similar idea, but applied not to quasihomogeneous layers (which are regarded here as specified and laminar), but to the fine structure of the "sheets" that separate them, is developed in a number of papers by Woods and co-authors[52,61-63,67-69], and especially in[75] (see also its discussion in the article by Fedorov[83]). According to this idea, when passing across a "sheet" of an internal wave in the vicinities of its crests and troughs, the velocity gradients can become unstable (at $\mathrm{Ri} < 1/4$), and the "sheet" becomes turbulent; as a result of the entrainment of water into the turbulized layer, the latter becomes thicker (by 4—5 times), and when Ri grows to an approximate value of unity, the turbulence in the layer degenerates, and two new "sheets" are produced on its boundaries. Repetitions of this process produce entire ensembles of "sheets." The action of this mechanism is illustrated by Woods[61] with examples of underwater motion-picture photography of the loss of stability of packets of steep internal waves with lengths on the order of 5 m, periods of several minutes, and phase velocities of several centimeters per second, with artificially dyed "sheets" of several centimeters thick in the thermocline of the Mediterranean. The doubling of the "sheets" was confirmed by Woods and Wiley[75] with the aid of a freefalling (with velocity 5 cm/sec) TG probe with a time constant 0.06 sec. Figure 16 shows one example of a doubled "sheet," while Fig. 17 shows an example of an ensemble of "sheets."

Meteorologists have long suspected that turbulence in a stably stratified medium can develop only in the form of individual layers or "pancakes," which produce steps on the temperature profiles (see[88]). Ideas similar to those of Woods, as applied to the "clear air turbulence," were developed by Ludlam[89] and confirmed by a number of measurements (primarily by radar), as summarized in[75] and in an article by Phillips[90].

12. FLUCTUATION SPECTRA IN THE PRESENCE OF MICROSTRUCTURE AND INTERNAL WAVES

In the absence of internal waves the vertical profile of the temperature (or some other hydrodynamic characteristic) at a given station in the ocean is described in a certain layer about a fixed depth z_1 by the function $T(z) = (z - z_1)\Gamma + \vartheta(z)$, where Γ is the average vertical gradient of the temperature and $\vartheta(z)$ describes the microstructure (in the presence of turbulence, ϑ and T can also depend on the time t), but once internal waves appear, waves characterized in the considered layer by vertical displacements $\zeta(t)$, a pickup placed at a depth z_1 will register fluctuations

$$\Theta(t) = T[z_1 - \zeta(t)] = \Gamma\zeta(t) + \vartheta[z_1 - \zeta(t)],$$

that depend both on the internal waves and on the microstructure. Approximating the fluctuations $\Theta(t)$ by a sequence of random uncorrelated jumps $\delta\Theta$, Phillips[90] obtained for them the spectrum

$$E(\omega) = \left(\frac{\omega_0\overline{(\delta\Theta)^2}}{2\pi}\right)\omega^{-2} \qquad (6)$$

(where ω_0 is the average frequency of the jumps), in the frequency interval $\tau_l^{-1} \ll \omega \ll \tau_S^{-1}$, where τ_l is the average interval between the jumps, and τ_S is the average width of the jumps. The spectrum of the small-scale turbulence, roughly speaking, is simply added to (6). Reid[91] has confirmed (6) in the particular case of a two-layer model with Gaussian displacements $\zeta(t)$. A more detailed general calculation was made by Garrett and Munk[92], who regarded $\zeta(t)$ and $\vartheta(z)$ as stationary random processes ($\zeta(t)$ is Gaussian) with correlation functions $B_{\zeta\zeta}(\tau)$ and $B_{\vartheta\vartheta}(\zeta)$, and assumed $\zeta(t)$ and $\vartheta[z - \zeta(t)]$ to be uncorrelated, so that the correlation function of the fluctuations $\Theta(t)$ took the form

$$\Gamma^2 B_{\zeta\zeta}(\tau) + \langle B_{\vartheta\vartheta}(\zeta_1 - \zeta_2)\rangle,$$

where the angle brackets denote averaging over the probability distribution for $\zeta_1 = \zeta(t)$ and $\zeta_2 = \zeta(t + \tau)$. The calculation was made under the assumption that the typical scale of the microstructure is small in comparison with the typical height Z of the internal waves. The formula obtained for the contribution of the microstructure to the spectrum of the fluctuations $\Theta(t)$ was

$$E(\omega) = (2/\pi)^{1/2}(SZ/\omega^2)\int_0^\infty e^{-x}F(\omega/SZ\sqrt{2x})\,dx,$$

where F(x) is the spectrum of the microstructure gradient $\partial\vartheta/\partial z$, and S is a certain typical frequency of the internal waves (S^{-1} is their Taylor time scale). From this, for example, at a spectrum F(k) which is constant in the interval $k_l \le k \le k_S$ and equal to zero outside this interval, one obtains the Phillips formula (6), and at high frequencies the contribution of the microstructure to the spectrum of $\Theta(t)$ turns out to be larger than the contribution of the internal waves. The coherence between the fluctuations $\Theta(t)$ at two levels, even in the case of total coherence of the internal waves, turns out to attenuate with distance and with frequency (constant at a distance inversely proportional to the frequency).

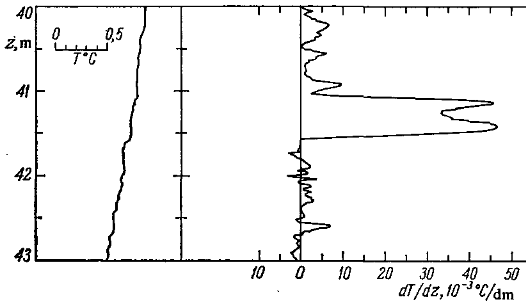

FIG. 16. Example of doubled "sheet" (from[75]).

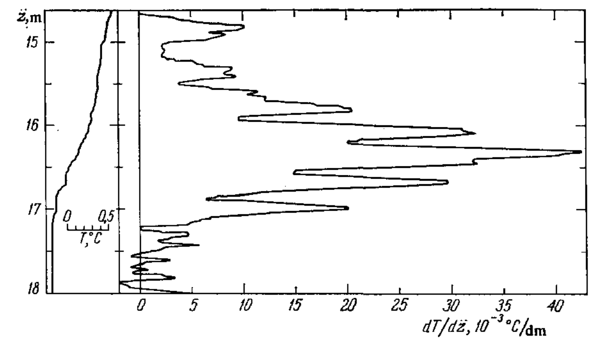

FIG. 17. Example of ensemble of "sheets" (from[75]).

*[nk] = n × k.
**(nk) = n · k.

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