

# Focused-image holography

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The physical principles underlying the focused-image holography are considered. An analysis is given of the processes involved in holographic recording of focused images and of their reconstruction by illumination of a hologram with light of different spectral compositions, including white light. The problem of the depth of a scene, which is reconstructed satisfactorily by white light illumination, is discussed. The special features of the recording of focused-image holograms are considered in the case when the reference wave is complex. The quality of the reconstructed image is independent of the dimensions and shape of the reference source and the reasons for this are discussed. Conditions are given for the observation of reconstructed images using reference and reconstructing waves of different forms and spectral compositions. The principles and new applications of holographic focused-image interferometry are considered.

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## 1. INTRODUCTION

The holographic methods for recording and reconstruction of wave fronts, based on the pioneering work of Gabor,<sup>[1-3]</sup> Denisjuk,<sup>[4-6]</sup> and Leith and Upatnieks,<sup>[7-10]</sup> have been developed considerably in recent years. These methods have been considered on many occasions in the present journal. Possible applications of holography in the various branches of science, technology, and national economy have been discussed widely in the scientific literature. Many of the possible applications have already been realized or are in the course of being put into practice.

The well-known work of Leith and Upatnieks,<sup>[7-10]</sup> who were the first to use a laser as the radiation source, converted holography into a lensless method for obtaining high-quality three-dimensional images of objects. The term "holography" has been restricted to the lensless photography in coherent light, especially as the first papers in the Russian language<sup>[11,12]</sup> have established this terminology. The recording of the interference field of a light wave in a Fresnel or a Fraunhofer diffraction zone imparts unique imaging properties to the holographic method. Therefore, it is not surprising that much detailed work has been done on these properties.

In the conventional holography the light field of an object, disturbed considerably by diffraction (in the case of transmission) or diffuse scattering (in the case of reflection), is recorded in the presence of a coherent reference beam. However, the principal feature of holography, which is associated with the retention of information on the phases of the waves scattered by the object, is also a feature of the arrangement in which a reference beam is applied in the plane of a sharp (focused) image.<sup>[13,14]</sup>

Focused-image holograms not only retain the usual properties of holographic recording but also have a number of special features which are very interesting from the theoretical point of view and have important applications.

A considerable literature now exists on focused-image holograms and their features but all of them, with almost the only exception of the excellent paper by Brandt,<sup>[15]</sup> are in the form of laconic original announcements. The results given in these announcements are fragmentary and not always comparable with those reported by others workers.

The aim of the present review is to consider the main results obtained in investigations of focused-image holograms and to consider the special features as well as some practical applications of the focused-image holography.

## 2. HOLOGRAPHIC RECORDING AND RECONSTRUCTION OF FOCUSED IMAGES

In the focused-image holography the steady-state interference field resulting from the superposition of the coherent object and reference waves is recorded in or close to the plane of the sharp (focused) image of the object (Fig. 1). In contrast to the conventional holographic methods, the reference beam need not be a simple (plane or spherical) wave but it can be produced by a source of arbitrary structure. The reasons for this freedom in the selection of the reference source will be considered later.

The general arrangement used in the focused-image holography requires the addition of a coherent reference wave to the image of an object focused in the normal manner. The hologram can be recorded in the

focused-image plane or outside it. Illumination of a focused-image hologram with a wide laser beam in the direction of the diffraction maxima produces a three-dimensional image of the object which can be "tuned" in depth so that different sections of the object are focused in turn.

The main features of focused-image holograms result from the fact that the information is recorded locally, i.e., that each point of the object gives rise to a point or a small region in the hologram so that the image carried by the reconstructing wave is produced in the hologram plane or close to it.

If the image is reconstructed with monochromatic laser radiation,<sup>[16,17]</sup> the reconstructed images are three-dimensional and their depth is limited only by the coherence length of the source used in the recording stage.

The three-dimensional properties are exhibited either by the virtual image, in the case when the image plane is in front of the hologram, or by the real image in front of the photographic plate, when the plane of the image is located behind the hologram. If the plane of the hologram intersects the image of a three-dimensional scene, one part of the reconstructed scene is observed as a virtual image and the other as a real one; in the first-order conjugate beam these parts of the scene are interchanged.<sup>[16]</sup>

The conditions for the observation of reconstructed images depend on the direction of incidence of the reconstructing beam.<sup>[15]</sup> If the beam used in reconstruction is opposite in direction to the reference beam, the positive lens (or some other focusing system) used in the recording stage is reconstructed by pseudoscopic inversion in the form of an image of the same positive lens located on the observer side (Fig. 2); this increases the field of view. Illumination of the hologram with an exact copy of the reference beam reproduces an image of the lens on the other side of the hologram, which reduces the field of view quite considerably.<sup>[15,16,18]</sup>

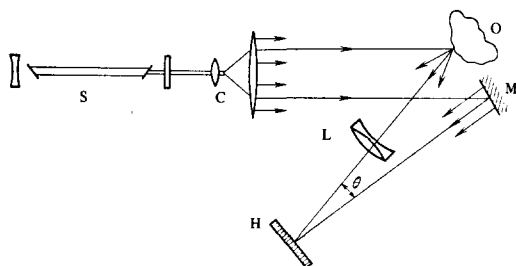


FIG. 1. Arrangement used in the recording of focused-image holograms. S is a laser source, C is a collimator, O is an object, M is a mirror, L is a lens, and H is a hologram.

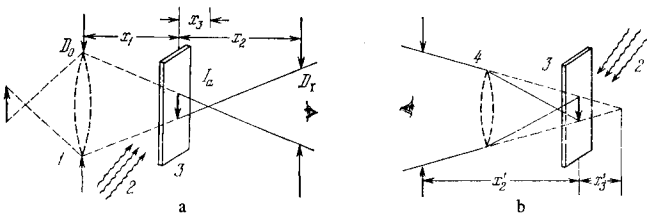


FIG. 2. Influence of the reconstruction geometry on the field of view in focused-image holography: reconstruction by a copy of the reference wave (a) and by a wave conjugate to the reference wave (b): 1—virtual image of the lens; 2—reconstructing beam; 3—focused-image hologram; 4—real image of the lens.

The local nature of the recording of information in focused-image holograms, which results in the reconstruction of images in the plane of a photographic plate, reduces considerably the stringency of the requirements in respect of the monochromaticity of the radiation employed so that reconstruction can be performed even with polychromatic radiation.<sup>[17-21]</sup> In this case the spectral components of the polychromatic beam reconstruct (by diffraction), from a "thin" hologram, a series of multicolored images which are observed—because of spatial dispersion—at different angles with respect to the hologram axis. If the reconstructing beam is white, the reconstructed pattern forms a spectrally colored image of the object.

It is well known that the Fresnel holograms cannot be reconstructed with white light because spatial dispersion blurs strongly the images which are localized at considerable distances from the photographic plate. In the case of the focused-image holograms the localization of the images near the plane of the photographic plate ensures that the blurring of the images due to dispersion is not very significant. Strictly speaking, a perfectly sharp image is obtained only in the hologram plane. However, some residual chromatism is permissible because of the finite aperture of the human eye and its limited resolving power. Therefore, it is in practice possible to obtain high-quality reconstructions of objects or scenes in white light and these reconstructions can be up to several centimeters deep.<sup>[15,18]</sup>

Focused-image holograms can be obtained using an in-line reference beam, as in the Gabor arrangement,<sup>[1-3]</sup> and this can be done for diffusely reflecting objects (Fig. 3). The images reconstructed from such holograms are observed in the axial direction but the observations should be made at some angle to the axis of the illuminating beam because otherwise the strong background of the reconstructing source is superimposed on the image.<sup>[22]</sup> The reconstructed images obtained from such holograms in white light are characterized by a very small spectral dispersion and the intensity distributions in these images are the same as in black-and-white photographs. This is due to the fact that the curvatures of the object and the reference waves are practically coincident in the recording stage so that the phase information is stored by local spatial modulation of the intensity and the usual carrier does not appear. This practically complete absence of the dispersion makes it possible to obtain high-quality reconstructed images of objects of depth in excess of 10 cm<sup>[23]</sup> because the dispersion is the cause of the blurring of the images reconstructed from holograms recorded using a simple reference wave.

The possibility of white-light reconstruction of focused-image holograms, which is the consequence of

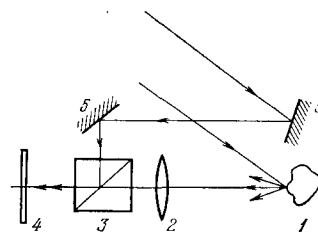


FIG. 3. Arrangement used in the recording of focused-image holograms using an in-line reference beam: 1—object; 2—lens; 3—beam splitter; 4—hologram; 5—mirrors.

the local recording of the phase information, makes it permissible to carry out the whole holographic process in white light.<sup>[24]</sup> In this case one must use an achromatic holographic arrangement in the hologram recording stage.<sup>[25,26]</sup> A system for recording focused-image holograms in white light is shown in Fig. 4. The spatial carrier is the achromatic image of a diffraction grating. The focused image of an object (transparency) is formed in the first-order diffracted beam produced by this grating and the zeroth-order beam is used as the reference wave. In this case a periodic pattern of fringes, modulated in amplitude in accordance with the transmission function of the transparency, is formed in the image plane of the focusing system. Satisfactory images can be reconstructed in white light from a hologram recorded in this way.

### 3. MECHANISM OF RECONSTRUCTION OF OFF-AXIS IMAGES IN WHITE LIGHT

The processes used in the reconstruction of focused-image holograms in white light have been considered in detail in<sup>[15,18,27,28]</sup>. Experiments have shown that polychromatic reconstruction of images from such holograms is possible because of the formation of a sharp image in the recording plane. If an object has a considerable spatial depth, a high-quality image is obtained only of that part of the object which is focused in the hologram plane and in its immediate vicinity. The processes used in the recording of focused-image holograms and in obtaining off-axis reconstructed images can be described analytically<sup>[27]</sup> by assuming that the object is planar and focused exactly in the hologram plane. We shall assume that the optical system forming the focused image produces no distortions and transfers the image of the object to the hologram plane in the form of a plane wave. If the object is characterized by the amplitude transmission or reflection  $T(x, y)$  and a plane reference wave is incident on the  $(x, y)$  plane at an angle  $\theta_0$ , the recorded intensity distribution is

$$I(x, y) = |T(x, y) + e^{-ik_0 x \sin \theta_0}|^2 = (|T(x, y)|^2 + 1) + T(x, y) e^{ik_0 x \sin \theta_0} + T^*(x, y) e^{-ik_0 x \sin \theta_0}, \quad (1)$$

where  $k_0 = 2\pi/\lambda_0$  and  $\lambda_0$  is the wavelength of the radiation used in the process.

If the interference pattern is recorded on a photographic plate and then analyzed subject to the condition that the transparency of the plate is proportional to  $I(x, y)$ , Eq. (1) describes—to within a constant factor—the amplitude transmission function of a focused-image hologram. Equation (1) can be rewritten conveniently in the form

$$\tau(x, y) = (|T(x, y)|^2 + 1) + T(x, y) e^{-i(2\pi/d_0)x} + T^*(x, y) e^{i(2\pi/d_0)x}, \quad (2)$$

where  $d_0 = 2\pi/k_0 \sin \theta_0$  is the period of the recorded interference pattern (i.e., the period of the spatial carrier of the focused-image hologram).

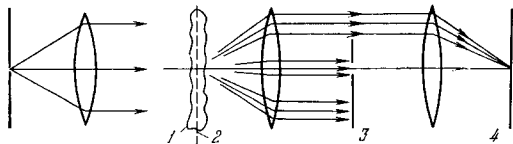


FIG. 4. Arrangement used in the recording of achromatic focused-image holograms: 1—object; 2—diffraction grating; 3—screen; 4—hologram.

Let us assume that a focused-image hologram, recorded using radiation of wavelength  $\lambda_0 = 2\pi/k_0$ , is illuminated with a plane-wave beam of monochromatic radiation of arbitrary wavelength  $\lambda \neq \lambda_0$ . The distribution of the amplitudes in the reconstructed wave, i.e., in the  $(x, y)$  plane, is now given by the expression

$$U_r(x, y) = \tau(x, y) e^{-ikz} = (|T(x, y)|^2 + 1) e^{-ikz} + T(x, y) e^{-ik[z - (2\pi/kd_0)x]} + T^*(x, y) e^{-ik[z + (2\pi/kd_0)x]}.$$

Omitting the propagation factor (the constant phase factor  $e^{-ikz}$ ) and bearing in mind that in the first-order diffraction  $\sin \theta = 2\pi/kd_0$ , we obtain the following expression for the reconstructed wave

$$U_r(x, y) = (|T(x, y)|^2 + 1) + T(x, y) e^{ikx \sin \theta_k} + T^*(x, y) e^{-ikx \sin \theta_k}, \quad (3)$$

here, the index  $k$  indicates that an image reconstructed with light of wavelength  $\lambda = 2\pi/k$  is observed at an angle  $\theta_k$ . Thus, in addition to the zeroth-order beam (the first term), we have a pair of symmetric images of the object localized in the  $(x, y)$  plane. The angle between the axis of the illuminating beam and the direction along which an image is observed varies with the wavelength of the monochromatic radiation used for reconstruction. In contrast to the conventional Fresnel holograms, which record the wavefront at some distance from the object, the focused-image holograms produce images whose scale is independent of the wavelength of the reconstructing radiation. Therefore, illumination of such holograms with white light produces high-quality colored images. Let us now assume that a focused-image hologram is illuminated with a plane-wave polychromatic beam which can be described by

$$U_0 = \int_{k_1}^{k_2} e^{-ikz} dk,$$

where  $k_1$  and  $k_2$  are the wave numbers corresponding to the extreme wavelengths of the monochromatic components of the beam. Then, the wave reconstructed from the hologram is given by

$$U_r^{(2)}(x, y) = \int_{k_1}^{k_2} (|T(x, y)|^2 + 1) e^{-ikz} dk + \int_{k_1}^{k_2} T(x, y) e^{-ik(z+x \sin \theta_k)} dk + \int_{k_1}^{k_2} T^*(x, y) e^{-ik(z-x \sin \theta_k)} dk. \quad (4)$$

Thus, each sinusoidal component of the radiation incident on a focused-image hologram creates a monochromatic wave of the type

$$T(x, y) e^{-ikx \sin \theta_k}.$$

The combined wave emerging in the direction of the first maximum is described by the second or third term in Eq. (4), i.e., the observed pattern is the result of superposition of the images reconstructed by the monochromatic components of the beam. This result is obtained on the assumption that an optical field corresponding to the focused image of a two-dimensional (plane) object is produced in the hologram plane and the reconstructed wave reproduces the amplitude distribution in that plane. In planes which do not coincide with the  $(x, y)$  plane the reconstructed image is blurred by dispersion. Therefore, strictly speaking, an object which has even a slight relief (depth) cannot be reconstructed perfectly in white light: those parts of the object which are outside the image plane are unavoidably blurred in the reconstruction by a polychromatic beam. This blurring is naturally stronger for those parts of the object which are further from the hologram plane. However, the physiological attributes of the human eye

ensure that the images of relatively "shallow" objects are quite satisfactory when viewed directly.

#### 4. DEPTH OF RECONSTRUCTION IN WHITE LIGHT

Blurring of the image of an object resulting from the spectral dispersion of polychromatic radiation is slight for subjects of certain depth and it does not exceed the dimensions of a scattering circle which appears to the eye as a point. The blurring  $\Delta x$  of an arbitrary point in the reconstructed image located at a distance  $h$  from the plane of a focused-image hologram illuminated by a polychromatic point source is given by the self-evident relationship<sup>[15,18]</sup>

$$\Delta x = h (\Delta\lambda/\lambda_0) \sin \theta_0, \quad (5)$$

where  $\Delta\lambda$  is the range of the wavelengths emitted by the polychromatic source. Equation (5) can be used to determine the resolution which can be attained at a specified depth in an image reconstructed using non-monochromatic radiation. For example, a hologram recorded using radiation of wavelength  $\lambda_0 = 6000 \text{ \AA}$  and a reference beam incident at an angle  $\theta_0 = 30^\circ$  on the hologram plane produces an image which is blurred by  $\Delta x = 50 \mu$  at a depth  $h = 1 \text{ cm}$  if the reconstructing source has the spectral width  $\Delta\lambda = 60 \text{ \AA}$ . In practice it is found that the blurring is still tolerable at much greater distances from the hologram plane.<sup>[15,25]</sup>

Usually the images reconstructed from focused-image holograms are observed visually and the eye is an optical system with a finite aperture. For certain relationships between the hologram dispersion and the diameter of the eye's pupil the eye receives only a very restricted part of the spectrum, i.e., the blurring of each point in the image is considerably less than the value  $\Delta x$  given by Eq. (5). Therefore, it is possible to observe high-quality images with a considerable depth.

An optical system of diameter  $D_0$ , separated by a distance  $R$  from a hologram, receives light from each point in a reconstructed image but this light is restricted to the range of angles given by  $\theta_{\text{lim}} = D_0/R$ . This means that the system receives from each point only the radiation in the wavelength range  $\Delta\lambda_{\text{eff}}$  even if the hologram is illuminated with polychromatic (white) light. The spectral width of the wavelengths received in this way is given by

$$\Delta\lambda_{\text{eff}} = \theta_{\text{lim}} \partial\lambda/\partial\theta = (D_0/R) \partial\lambda/\partial\theta, \quad (6)$$

where  $\partial\lambda/\partial\theta$  is the angle of dispersion of the hologram. The blurring described by Eq. (5) can be written in the form

$$\Delta x = h\Delta\lambda \partial\theta/\partial\lambda, \quad (7)$$

so that, substituting  $\Delta\lambda_{\text{eff}}$  from Eq. (6) into Eq. (7), we find that the blurring limit is

$$\Delta x_{\text{lim}} = hD_0/R.$$

The angular blurring of a hologram can be written in the form

$$\Delta\theta_b = hD_0/R^2.$$

Hence,  $h = R^2\Delta\theta_b/D_0$ . Thus, the blurring due to the nonmonochromaticity of the reconstructing beam is independent of the hologram dispersion if the image is viewed by a system with a limited aperture. A sharp image can then be observed within a certain distance from the hologram plane. The depth of the field in which

the image retains its high quality can be quite large. For example, if the minimum resolution for viewing from a distance  $R = 1 \text{ m}$  through a pupil of  $D_0 = 4 \text{ mm}$  diameter is  $\Delta\theta_b = 4 \times 10^{-4} \text{ rad}$ , the depth of the field for high-quality imaging is  $\pm 10 \text{ cm}$ .

#### 5. USE OF EXTENDED REFERENCE SOURCES IN FOCUSED-IMAGE HOLOGRAPHY

The possibility of using an extended source in holographic recording of focused images was first considered by Rosen.<sup>[29]</sup> It was found that in recording holograms of focused images and in reconstructing these images one can use arbitrarily shaped and quite extended coherent radiation sources and there is no need to compensate for the extended nature of the reference source.<sup>[30]</sup> Later it has been found<sup>[31]</sup> that this applies also to holograms recorded in reflected light in the direct vicinity of an object. These experimental results are explained in<sup>[29]</sup> by the following relationships. If the plane reference wave is

$$U_{\text{ref}}(x) = A_{\text{ref}}e^{-ikx \sin \theta},$$

and the object wave (proceeding from a point object) is

$$U_{\text{ob}}(x) = A_{\text{ob}}e^{i(k/2R)(x-x_0)^2}, \quad (8)$$

where  $R$  is the distance between the object and the hologram plane  $x$ , the transparency function of a Fresnel hologram is proportional to the expression

$$I(x) = \{ |A_{\text{ref}}|^2 + |A_{\text{ob}}|^2 \} + A_{\text{ref}}A_{\text{ob}}^* e^{i[(k/2R)(x-x_0)^2 + kx \sin \theta]} + A_{\text{ref}}^*A_{\text{ob}} e^{-i[(k/2R)(x-x_0)^2 + kx \sin \theta]}.$$

Illumination of this hologram with a plane wave

$$U_{\text{r}}(x) = A_{\text{r}}e^{-ikx \sin \varphi}$$

produces a reconstructed wave and a component of this wave described by the expression

$$U_{\text{r}}(x) = C e^{i(k/2R)[(x-x_0)^2 + 2Rx(\sin \theta - \sin \varphi)],}$$

represents a virtual image of a point object [see Eq. (8)] deviated through an angle  $\gamma$ , where

$$\sin \gamma = \sin \theta - \sin \varphi.$$

If the reconstructing beam is produced by an extended source, the angle  $\varphi$  ranges from  $\varphi_{\text{min}}$  to  $\varphi_{\text{max}}$ . Consequently, the reconstructed image of a point source is observed in an angular range  $\Delta\gamma$  from  $\gamma_{\text{min}}$  to  $\gamma_{\text{max}}$ .

The blurring of the image of a point is

$$\Delta x = R \Delta\gamma$$

and  $\Delta x \rightarrow 0$  when  $R \rightarrow 0$ , which corresponds to focused-image holograms.

The processes of the recording of focused-image holograms using an extended reference source of coherent radiation and of the reconstruction of high-quality images with radiation sources of arbitrary shape and spectral composition are considered in detail in<sup>[32]</sup>. The same paper discusses reconstruction with an extended polychromatic (white) radiation source, i.e., the case when the blurring in the reconstructed image is not only due to illumination from many directions but also due to dispersion.

In the analysis given in<sup>[32]</sup> it is assumed that the hologram is recorded in the plane of a focused image of a two-dimensional object. The reference wave is assumed to consist of plane waves reaching the hologram plane (Fig. 5) within a certain range of angles, i.e.,

$$U_{\text{ref}}(x) = \sum_{n=1}^N e^{-ik_n x \sin \theta_n},$$

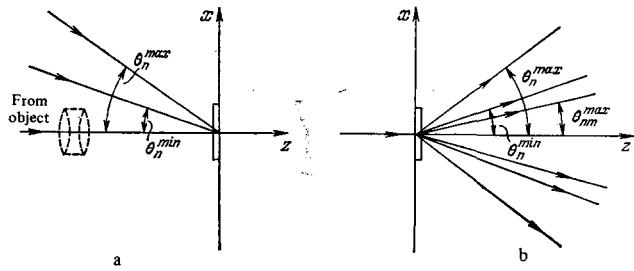


FIG. 5. Ray paths in the recording of focused-image holograms using an extended reference source (a) and in the reconstruction with a plane monochromatic wave (b).

here,  $k_0 = 2\pi/\lambda_0$  and  $\lambda_0$  is the wavelength of the laser radiation employed in the recording stage.

The intensity distribution in a focused-image hologram recorded using an extended reference source can be described, to within a constant coefficient, by the following expression:

$$I_{\text{rec}}(x) = |T(x) + \sum_{n=1}^N e^{-ik_0 x \sin \theta_n}|^2 = (|T(x)|^2 + N) + \sum_{n=1}^N T(x) e^{ik_0 x \sin \theta_n} + \sum_{n=1}^N T^*(x) e^{-ik_0 x \sin \theta_n} + \sum_{\substack{n=1 \\ n \neq m}}^N \sum_{m=1}^N e^{-ik_0 x (\sin \theta_n - \sin \theta_m)} + \sum_{\substack{n=1 \\ n \neq m}}^N \sum_{m=1}^N e^{ik_0 x (\sin \theta_n - \sin \theta_m)}$$

By analogy with Eq. (2), the hologram equation can be rewritten in the form

$$I_{\text{rec}}(x) = (|T(x)|^2 + N) + \sum_n T(x) e^{i(2\pi/d_n)x} + \sum_n T^*(x) e^{-i(2\pi/d_n)x} + \sum_{\substack{n \\ m \neq n}} e^{-i(2\pi/d_{nm})x} + \sum_{\substack{n \\ m \neq n}} e^{i(2\pi/d_{nm})x}$$

If the reconstruction is performed using a plane monochromatic wave of wavelength  $\lambda \neq \lambda_0$  traveling along the  $z$  axis,

$$U_0(x) = e^{-ikx},$$

the optical field formed after passing through the hologram is

$$U_T(x) = (|T(x)|^2 + N) e^{ikx} + \sum_n T(x) e^{-ik[z - (2\pi/d_n)k]x} + \sum_n T^*(x) e^{-ik[z + (2\pi/d_n)k]x} + \sum_{\substack{n \\ m \neq n}} e^{-ik[z + (2\pi/d_{nm})k]x} + \sum_{\substack{n \\ m \neq n}} e^{-ik[z - (2\pi/d_{nm})k]x}. \quad (9)$$

If allowance is made for the diffraction by periodic structures with periods  $d_n$  and  $d_m$ , the reconstructed optical field becomes

$$U_T(x) = (|T(x)|^2 + N) + \sum_n T(x) e^{ikx \sin \theta'_n} + \sum_n T^*(x) e^{-ikx \sin \theta'_n} + \sum_{\substack{n \\ m \neq n}} e^{-ikx \sin \theta'_{nm}} + \sum_{\substack{n \\ m \neq n}} e^{ikx \sin \theta'_{nm}}$$

where the angles  $\theta'_n$  and  $\theta'_{nm}$  are given by the diffraction conditions [see, for example, Eq. (3)]. Thus, in the image plane we have an axial wave (the first term), a pair of symmetric images observed in the angular range from  $\pm\theta'_{n \min}$  to  $\pm\theta'_{n \max}$  (the second and third terms), and a scattered background resulting from all the plane waves (the fourth and fifth terms) traveling on both sides of the  $z$  axis right up to angles  $\pm\theta'_{nm \min}$ ; this background partly overlaps the axial beam.

Let us assume that the reconstructing beam is in the form of a plane polychromatic wave

$$U_0(x) = \sum_{l=1}^L e^{-ik_l x}.$$

Then, the optical field in the hologram plane becomes

$$U'_T(x) = L(|T(x)|^2 + N) + \sum_{l=1}^L \sum_{n=1}^N T(x) e^{-ik_l x \sin \theta_{nl}} + \sum_{l=1}^L \sum_{n=1}^N T^*(x) e^{ik_l x \sin \theta_{nl}} + \sum_{l=1}^L \sum_{\substack{n=1 \\ n \neq m}}^N \sum_{m=1}^N e^{ik_l x \sin \theta_{nml}} + \sum_{l=1}^L \sum_{\substack{n=1 \\ n \neq m}}^N \sum_{m=1}^N e^{-ik_l x \sin \theta_{nml}}.$$

$$+ \sum_{l=1}^L \sum_{\substack{n=1 \\ n \neq m}}^N \sum_{m=1}^N e^{-ik_l x \sin \theta_{nml}}.$$

The range of angles in which reconstructed images are observed broadens, because of the dispersion, at right-angles to the direction of the spatial carrier of a focused-image hologram; moreover, the individual monochromatic components of the reconstructed wave, which are diffracted by gratings with different periods, become mixed, i.e., they coincide in direction. Thus, images reconstructed by components of wavelengths  $\lambda'_l$  and  $\lambda''_l$  diffracted by elementary holograms of periods  $d'_n$  and  $d''_n$  are observed at the same angle if  $\lambda'_l d''_n = \lambda''_l d'_n$ . The degree of diffuseness of the scattered background increases because a larger number of diffracted plane waves travels in the same angular range. Finally, in the case when the image is reconstructed by a wave generated in an extended polychromatic source,

$$U_0(x) = \sum_{p=1}^P \sum_{l=1}^L e^{-ik_l x \sin \theta_p},$$

the off-axis images in the expression for the reconstructed wave are represented by terms of the type

$$U''_r(x) = \sum_{p=1}^P \sum_{l=1}^L \sum_{n=1}^N T(x) e^{-ik_l x (\sin \theta_p - \sin \theta_n)},$$

$$U''_r(x) = \sum_{p=1}^P \sum_{l=1}^L \sum_{n=1}^N T^*(x) e^{ik_l x (\sin \theta_p - \sin \theta_n)}.$$

Hence, it follows that the range of angles in which reconstructed images are observed broadens additionally because of the diffraction on the gratings associated with different spatial components of the incident wave  $U_0''(x)$ .

Thus, the special nature of the recording of focused-image holograms, which results in the localization of the reconstructed images in the hologram plane, ensures high-quality imaging even when the resultant optical field is an assembly of a large number of identical images carried by diffracted beams of different wavelengths traveling along different directions. The use of an extended reference source in the recording of focused-image holograms simply increases the number of such beams and makes some of them coincide in direction, which is a consequence of the recording of a set of spatial carriers.

Similar conclusions follow from a discussion of holographic processes<sup>[15]</sup> based on the well-known Meier relationships<sup>[33]</sup> which give the positions of the image as a function of the geometric parameters of the recording and reconstruction processes. Thus, the position of an off-axis reconstructed image point is given by the expression

$$a_R = \frac{m^2 x_C z_0 z_r - \mu m x_0 z_C z_r - \mu m x_r z_C z_0}{m^2 z_0 z_r - \mu z_C z_r + \mu z_C z_0}, \quad (10)$$

and the distance of this point from the originating hologram is

$$L_R = \frac{m^2 z_C z_0 z_r}{m^2 z_0 z_r - \mu z_C z_r + \mu z_C z_0}, \quad (11)$$

where  $x_C$ ,  $x_0$ , and  $x_r$  are the off-axis coordinates of the reconstructed image, the object, and the reference source;  $z_C$ ,  $z_0$ , and  $z_r$  are the corresponding distances from the hologram plane;  $m$  is the magnification between the recording and reconstruction stages;  $\mu$  is the ratio of the wavelengths of the radiations employed in the reconstruction and recording operations.

The blurring of an image point resulting from the finite size of a reconstructing source of dimensions  $\Delta x_C$  is

$$\Delta a_R = \Delta x_c da_R/dx_c.$$

Using Eq. (10), we find that

$$\Delta a_R = \frac{\Delta x_c}{1 - \mu(z_c/m^2 z_0) + \mu(z_c/m^2 z_r)}.$$

In the limit, for  $z_0 \ll z_c$  and  $\mu z_c/m^2 z_r \sim 1$ , we obtain

$$\Delta a_R = \Delta x_c m^2 z_0 / \mu z_c,$$

so that the blurring approaches zero as the distance between the object and the hologram also approaches zero ( $z_0 \rightarrow 0$ ). A similar result is easily obtained also for a focused-image hologram recorded using an extended reference source and reconstructed employing a point source.

Experiments involving recording of focused-image holograms using an extended reference source and reconstruction of these images using extended white-light sources<sup>[15,32,34]</sup> have confirmed the properties of the images deduced theoretically. This applies to focused-image holograms obtained using part of the radiation scattered by the object as the reference beam<sup>[15,34]</sup> as well as to holograms obtained employing an independent diffuse reference source.<sup>[32]</sup> The experimental arrangements used in these two cases are shown schematically in Fig. 6. The reconstructed images (Fig. 7) are observed in a wide range of angles; the colors in the images are strongly mixed because of coincidence of the directions of light beams of different color generated by the diffraction—in the hologram—of polychromatic beams of different directions (produced by an extended source). A diffusely scattered white-light field is observed in a fairly wide range of angles in the direction of the zeroth diffraction maximum. This field results from the diffraction of the illuminating beam by the spatial carriers of periods  $d_{mn}$  [see Eq. (9)].

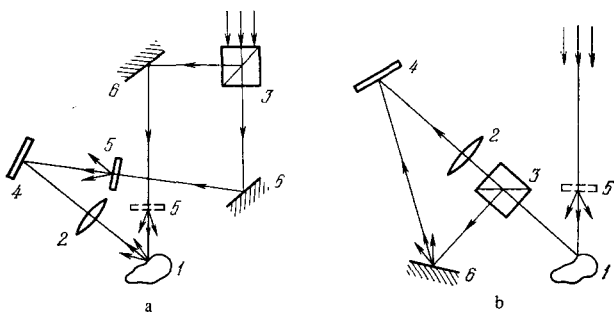


FIG. 6. Arrangement used in the holographic recording of focused images using a diffusely scattered reference wave (a) and a reference wave scattered by the object itself (b): 1—object; 2—lens; 3—beam splitter; 4—hologram; 5—diffuse scatterer; 6—mirrors.

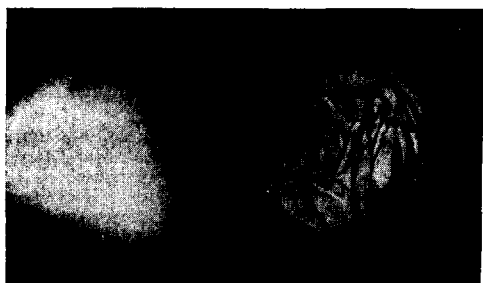


FIG. 7. Photograph of the reconstruction of a diffusely scattering bas-relief formed by a hologram obtained using a reference source.

## 6. CONDITIONS FOR OBSERVATION OF IMAGES RECONSTRUCTED WITH WHITE LIGHT FROM FOCUSED-IMAGE HOLOGRAMS

The conditions for the observation of off-axis reconstructed images in the hologram plane<sup>[27]</sup> depend strongly on the spatial structure of the object, reference, and reconstructing beams and, of course, on the spectral composition of the reconstructing beam.

If the reference and object beams are plane (i.e., if the object scatters light only slightly), illumination of a focused-image hologram with a simple (plane or spherical) polychromatic wave produces a spectrally colored fringe which is cut off sharply from the image. When the spectral width of the polychromatic source is reduced, the width of the fringe decreases and in the limit it becomes a single-color point. This phenomenon is due to the diffraction of a plane wave by a grating with a simple spatial structure characterized by a single period. Similar properties are exhibited also by the conventional Fresnel holograms of weakly scattering objects. In these cases the whole of the reconstructed image can be observed by projecting it with an optical system on a diffusely scattering screen.

The complete reconstructed image can be observed without difficulty if a hologram of this type is illuminated with a complex wave produced by an extended source: in this case a grating of period  $d$  diffracts a set of spherical waves of different directions and the resulting pattern is the superposition of many images in a certain range of angles. The spectrally colored image is then characterized by a high purity of colors so that introduction of a narrow-band filter into the illuminating beam reduces strongly the range of angles in which the image is observed.

In the case when one of the interfering beams (reference or object) is plane and the other is scattered, illumination of a focused-image hologram with a simple wave reconstructs the whole image as in the case of illumination with an extended wave. A considerable mixing of the colors is observed: it is due to the diffraction of spherical waves on gratings with different periods in the recorded hologram. Introduction of a filter into the reconstructing beam produces a single-color image and reduces slightly the range of angles in which this image is observed.

Finally, in the case when the object and the reference waves are diffusely scattered, a focused-image hologram records the most complex structure characterized by a large number of periods of the spatial carrier. In this case the spectral colors in the reconstructed image (particularly that obtained using a reconstructing wave produced by an extended source) are mixed so strongly that the images appear black-and-white and the spectral dispersion is partly compensated by the scattering only in the case of large angles.

We have mentioned earlier that focused-image holograms obtained by Stroke<sup>[23]</sup> using an in-line reference beam give rise to black-and-white axial images localized in the hologram plane if the reconstructing beam is white. Such images can be seen along directions inclined slightly with respect to the axis of the illuminating beam (this must be done in order to avoid the masking of the image structure by the strong zeroth-order wave). However, it has been found<sup>[15,27]</sup> that the use of a specially formed in-line reference beam in the

focused-image holography of diffusely scattering objects is not a necessary condition for the reconstruction of axial images. Holograms obtained using an inclined reference beam produce, in addition to an off-axis spectrally colored image, an additional image of the object along the direction of the zeroth diffraction maximum and this image is of the same color as the illuminating beam. The appearance of this image is due to the low-frequency very complex spatial structure recorded in a photographic plate as a result of cross-interference of elementary images carried by all the spatial components of the diffusely scattered object wave. Each such image "sees" all the other images as a complex reference wave and, therefore, the reconstruction is possible only near the hologram plane. In the case of diffraction (scattering) of the reconstructing beam by the recorded structure the reconstructed image is observed along any direction inside a solid angle governed by the relative aperture of the focusing system used in the recording stage. Naturally, the presence of a specially formed inclined reference beam is not necessary for recording of such holograms, i.e., holography is possible with a "local" reference beam<sup>[15]</sup>, which extends the method<sup>[25]</sup> based on the selection of a single spherical wave from the waves scattered by an object.

Such "referenceless" focused-image holograms can be recorded photographically in diffusely scattered coherent light. In this case a focused-image hologram appears to be a negative photographic image of the object but the presence of a complex spatial structure imparts holographic properties to this image. Illumination with an arbitrary source in a paraxial direction produces a positive image with the darkest parts of the negative corresponding to the brightest parts in the reconstructed image (and conversely).

An important feature of such holograms is the absence of any preferential viewing direction, which distinguishes these holograms from those obtained with off-axis reference beams. The reconstructed image is observed for any position of the observer relative to the source-hologram line, provided this direction is within the solid angle governed by the recording conditions. Marked dispersion coloring is observed in the reconstructed image away from the axis of the reconstructing beam. Moreover, reconstructed paraxial images differ from the conventional reconstructions by a considerably wider range of brightness.

Experiments have shown that the diffuse scattering of light by an object is a necessary condition for obtaining paraxial images and the best results are obtained when the specular component is absent from the scattered radiation. Therefore, it is logical to assume that referenceless holograms are recorded with the aid of some extended reference wave which is present in the focused radiation. A comparison of these holograms with the focused-image holograms obtained by Stroke<sup>[23]</sup> by introducing an in-line spherical reference beam shows considerable differences. The Stroke holograms give rise to reconstructed images of considerable depth (because of the absence of dispersion) whereas the referenceless holograms give rise to unblurred images only in the case of very shallow scenes. This confirms the hypothesis that the in-line source, present in the field of the focused wave, is extended.

The possibility of using extended reference waves in focused-image holography is usually analyzed<sup>[4,5]</sup> on

the assumption that the reference wave has a homogeneous transverse amplitude distribution in the hologram plane. However, it is easy to show that when the reference beam in the hologram plane has an inhomogeneous amplitude distribution, the reconstructed image of an object is modulated by the corresponding spatial structure. In other words, two functions of spatial variables, one of which describes the structure of the object and the other the structure of the reference wave in the hologram plane, are multiplied in the optical sense.

We shall use the foregoing discussion as the basis of an elementary theory of photography in diffusely scattered coherent radiation. Let us assume that a focused image of a two-dimensional object, characterized by an amplitude transmission or reflection function  $T(x_0)$ , is formed by a lens in the  $x = \text{const}$  plane (one-dimensional case) with a unit magnification. The diffuse illumination of the object can be represented by a set of plane waves traveling in a certain range of angles near the optic axis, i.e.,

$$I_h(x) = \left| \sum_{n=1}^N T(x) e^{-ikx \sin \theta_n} \right|^2 \\ = N |T(x)|^2 + \sum_n \sum_{m \neq n}^N |T(x) T^*(x) e^{-ikx(\sin \theta_n - \sin \theta_m)} + \text{c.c.}$$

We can rewrite this formula in the form

$$I_h(x) = N |T(x)|^2 + \sum_n \sum_{m \neq n}^N |T(x)|^2 e^{-i2\pi x/d_{nm}} + \text{c.c.};$$

here,  $d_{nm} = 2\pi/k(\sin \theta_n - \sin \theta_m)$  is the period of the interference pattern corresponding to the coherent superposition of two elementary images. The first term describes the negative image being recorded whereas the second and the third terms describe the spatial structure resulting from the interference of all the spatial components of the diffusely scattered wave. The role of an extended reference wave for each elementary image is played by all the other images. This gives rise to cross-modulation of the elementary focused images and, since the lens is tautochronous, all of them are superimposed without displacement.

It follows from the above formulas that each of the "partial" carriers is modulated by the intensity (and not the amplitude) of the object wave without any change in the phase of the envelope.

Illumination of a referenceless focused-image hologram with a plane monochromatic wave produces the following amplitude distribution (the propagation factor is omitted):

$$U_t = N |T(x)|^2 + \sum_n \sum_{m \neq n}^N |T(x)|^2 e^{-ikx \sin \theta_{nm}} + \text{c.c.},$$

where  $\sin \theta_{nm} = 2\pi/kd_{nm}$  corresponds to the diffraction of the reconstructing beam on one of the "partial" carriers. A negative image, described by the first term, is observed strictly in the axial direction and this image can be removed by bleaching the photographic plate. A positive image, in which the amplitude distribution is proportional to the intensity distribution in the original object, is observed along all other directions within a solid angle governed by the geometry of the recording process. A considerable drop in the brightness in the paraxial reconstructed images is due to the fact that the referenceless focused-image holograms represent intensity distributions.

The simplicity of the recording of the in-line focused-image holograms with a local reference beam is not their only advantage; they can be very useful also

when the holographic recording conditions are difficult. Since the object and the reference beams travel along the same optical path, inhomogeneities in this path have practically no influence on the quality of the reconstructed image and this applies also to any mechanical vibrations that the system might experience during the recording stage.

Focused images can also be recorded holographically in the plane of a real image formed by a conventional Fresnel hologram. Focused-image holograms of diffusely scattering objects can be formed using holographic zone plates which ensure that the object is focused and that the scattered reference beam is directed in line with the object. It is shown in<sup>[15]</sup> that the use of standard zone plates, formed by alternating transparent and opaque zones, gives rise to considerable aberrations. Therefore, it is suggested in<sup>[15]</sup> that special aberration-free holographic zone plates should be employed. Such zone plates are formed by recording the results of the interaction between a spherical wave and a reversed reference wave of the same radius of curvature (Fig. 8). This recording process alters the sign of  $Z_R$  in Eq. (11), i.e., the distance from the "holographic lens" to the image becomes

$$Z_R = -m^2 z_0 z_c / (\mu z_c z_r - m^2 z_0 z_r + \mu z_0 z_c).$$

Hence, in the 1:1 focusing case, i.e., when  $m = \mu = 1$ , the point  $z_c$  located at a distance  $z_0$  from the holographic lens is imaged at a distance  $-z_0$  on the other side. An allowance for the change in the sign of  $z_r$  in the Meier expression<sup>[33]</sup> for the spherical aberration

$$S = z_c^3 - (\mu/m^4 z_0^3) + (\mu/m^4 z_r^3) - Z_R^3$$

subject to the condition that  $m = \mu = 1$  and  $z_c = z_0$ , gives  $S = 0$ , i.e., in this case the image is free from the spherical aberration. This also applies to the coma, described by the expression

$$C_x = (x_c/z_c^3) - (\mu x_0/m^3 z_0^3) + (\mu x_r/m^3 z_r^3) - (a_R/Z_R^3),$$

so that a weak astigmatism is the only distortion of the focused image.

It has been found experimentally<sup>[15,18]</sup> that the optimal relationship between the intensities of the object and the reference beams in the focused-image holography differs considerably from the corresponding relationship in the conventional holography. As is known, the optimal conditions in the conventional holography are obtained when the intensity of the reference beam is several (4–5) times higher than the intensity of the object beam. When these intensities are equal, the contrast in the interference pattern is strongest and, consequently, the diffraction efficiency of the hologram is greatest. However, in this case the recording process becomes nonlinear and this has an adverse effect on the quality of the reconstructed image. The consequence of this nonlinearity is the appearance of images in higher diffraction maxima and of a noise halo

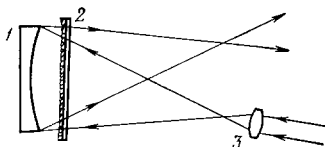


FIG. 8. Arrangement used in the preparation of a holographic zone plate intended for the formation of unit magnification images: 1—spherical mirror; 2—photographic plate with its emulsion facing the mirror; 3—combination of a microscope objective with a point-like aperture.

around the image in the first diffraction maximum as well as false images in the intervals between neighboring maxima.<sup>[36]</sup> In the case of focused-image holograms the reconstructing beam aperture is limited so that only one image can be seen at any one time, i.e., the presence of many images is of little importance. Moreover, the noise halo and the false images are absent because the information about the object is recorded in local form so that the mutual cross-modulation between the image points is minimal. Consequently, the optimal conditions in the focused-image holography are obtained when the intensities of the object and the reference beams are made equal because in this case the intensity (brightness) in the reconstructed image is highest without any adverse effect on the quality of the image.

It must be stressed that only the focused-image holograms can be obtained in higher diffraction orders of the image of an object whereas the conventional holograms form autocorrelations and autoconvolutions of the object in higher diffraction orders.<sup>[36]</sup>

The local recording of information in a focused-image hologram demands the use of a photographic material with a wider dynamic range than that used for the Fresnel holograms in which information is recorded with some redundancy. In principle, a negative image of an object can be obtained at intensities much higher than the intensity of the reference beam.<sup>[15,37]</sup> This is possible because of reduction of the depth of the spatial modulation of the interference fringes with increasing intensity of the object beam. Consequently, the diffraction efficiency of certain parts of a focused-image hologram decreases and the intensity of the corresponding parts of the reconstructed image is minimal, i.e., the brightest parts of the object correspond to the darkest parts of the reconstructed image.

## 7. HOLOGRAPHIC INTERFEROMETRY OF FOCUSED IMAGES

The ability to form high-quality images by white-light reconstruction from focused-image holograms has led to the development of a holographic interferometry method in which a monochromatic source is not required in the reconstruction stage. This method is based on the use of double-exposure focused-image holograms and is described in<sup>[38,39]</sup>. It is known that in recording of holographic interferograms of diffusely scattering objects the interference fringe pattern is localized exactly on the surface of the object only if the object is subjected to "pure" rotation about an axis passing through its surface.<sup>[40]</sup> However, in the majority of the cases encountered in practice the fringes are localized in the direct vicinity of the surface of the object or its holographic copy. This makes useful the property of focused-image holograms which ensures—under certain conditions—a considerable depth of the white-light-reconstructed image so that white light can be used to obtain interferograms representing a great variety of deformations of real objects.

Let us assume that the double-exposure process consists of successive recording—in the same plane  $x$ —of the results of interference between a reference beam  $U_{\text{ref}}(x) = \exp(-k_0 x \sin \theta_0)$  with an amplitude distribution  $T(x)$  characterizing the structure of an object and the same reference beam with a distribution resulting from the rotation of the object through a small angle  $\varphi$  [we shall use the same notation as in Eqs. (1)



and (2)]. The rotation through an angle  $\varphi$  alters the phase of the light wave in the hologram plane. Therefore, the intensity distribution recorded in a doubly-exposed focused-image hologram is

$$I'(x) = |T(x) + e^{-ik_0x \sin \theta_0}|^2 + |T(x) e^{ik_0x \sin \varphi} + e^{-ik_0x \sin \theta_0}|^2 \\ = 2(|T(x)|^2 + 1) + T(x) e^{ik_0x \sin \theta_0} + T^*(x) e^{-ik_0x \sin \theta_0} \\ + T(x) e^{ik_0x \sin \varphi + ik_0x \sin \theta_0} + T^*(x) e^{-ik_0x \sin \varphi - ik_0x \sin \theta_0}.$$

We can easily show that a double-exposure hologram records the results of incoherent superposition of two interference gratings which have different periods and are modulated by the spatial structure of the object:

$$\tau'(x) = 2(|T(x)|^2 + 1) + T(x) e^{i(2\pi/d_1)x} + T^*(x) e^{-i(2\pi/d_0)x} \\ + T(x) e^{i(2\pi/d_1)x} + T^*(x) e^{-i(2\pi/d_1)x}.$$

here  $d_0 = 2\pi/k_0 \sin \theta_0$  and  $d_1 = 2\pi/k_0 (\sin \theta_0 + \sin \varphi)$  are the periods of the gratings. Let us assume that this doubly-exposed focused-image hologram is illuminated normally by a plane-wave monochromatic beam of the same wavelength  $\lambda_0 = 2\pi/k_0$ . The diffraction field in the hologram plane is described by the expression

$$U_n(x) = \tau'(x) e^{-ik_0z} = 2(|T(x)|^2 + 1) e^{-ik_0z} + T(x) e^{-ik_0z - (2\pi/k_0 d_0)x} \\ + T^*(x) e^{-ik_0z + (2\pi/k_0 d_0)x} + T(x) e^{-ik_0z - (2\pi/k_0 d_1)x} + T^*(x) e^{-ik_0z + (2\pi/k_0 d_1)x}.$$

Since the diffraction of a plane wave with a wave number  $k_0$  by gratings with periods  $d_0$  and  $d_1$  gives rise to first-order diffracted waves whose directions are given by the expressions

$$\sin \theta_0 = 2\pi/k_0 d_0, \quad \sin \theta_0 + \sin \varphi = 2\pi/k_0 d_1,$$

we can proceed as in Eq. (3) and—omitting the propagation factors—we obtain

$$U_r(x) = 2(|T(x)|^2 + 1) + T(x) (1 + e^{ik_0x \sin \varphi}) e^{ik_0x \sin \theta_0} + \\ + T^*(x) (1 + e^{-ik_0x \sin \varphi}) e^{-ik_0x \sin \theta_0}. \quad (12)$$

The second and third terms in Eq. (12) describe two symmetric reconstructed waves. We shall consider the term corresponding to the virtual (direct) image

$$U_r^{(v)}(x) = T(x) [1 + e^{ik_0x \sin \varphi}] e^{ik_0x \sin \theta_0} = T(x) (1 + e^{i(2\pi/D)x}) e^{ik_0x \sin \theta_0},$$

where  $D = 2\pi/k_0 \sin \varphi$ . Consequently, an image of the object structure  $T(x)$  with a superimposed parallel fringe pattern is observed in the hologram plane at an angle  $\theta_0$  and the period of the fringe pattern is governed by the angle of rotation of the object  $\varphi$  in the interval between the two exposures.

If the hologram is illuminated with a plane-wave monochromatic beam of arbitrary wavelength  $\lambda \neq \lambda_0$ , the diffraction field in the reconstruction plane becomes (after a similar transformation)

$$U_r^*(x) = 2(|T(x)|^2 + 1) + T(x) e^{ikx \sin \theta_k} + T^*(x) e^{-ikx \sin \theta_k} + \\ + T(x) e^{ikx \sin \theta'_k} + T^*(x) e^{-ikx \sin \theta'_k},$$

here, the index  $k$  indicates that the angle of diffraction of each pair of beams is different from  $\theta_0$  and is governed by the wavelength of the reconstructing beam. Since the directions of the first diffraction maxima are given by the expressions

$$\sin \theta_k = 2\pi/k d_0 = (k_0/k) \sin \theta_0, \\ \sin \theta'_k = 2\pi/k d_1 = (k_0/k) (\sin \theta_0 + \sin \varphi),$$

the expression for the reconstructed wave corresponding to the virtual image becomes

$$U_r^{(v)*}(x) = T(x) e^{-ikx[(k_0/k) \sin \theta_0]} + T(x) e^{-ikx[(k_0/k)(\sin \theta_0 + \sin \varphi)]} = \\ = T(x) (1 + e^{i(2\pi/D)x}) e^{-ikx \sin \theta_k}.$$

This means that, as in the case of single-exposure focused-image holograms, the scale of the image and

the plane of its localization are unaffected by a change in the wavelength of the reconstructing beam and, moreover, this change does not alter the period of the interference pattern. This is due to the fact that the sines of the diffraction angles vary proportionally to the wavelength  $\lambda$ .

Thus, illumination of a doubly-exposed hologram with a plane-wave polychromatic beam

$$U_0(x) = \int_{k_1}^{k_2} e^{-ikz} dk$$

produces the following overall diffraction field in the  $x$  plane

$$U_{r,o}(x) = 2(|T(x)|^2 + 1) \int_{k_1}^{k_2} e^{-ikz} dk \\ + \int_{k_1}^{k_2} T(x) (1 + e^{i(2\pi/D)x}) e^{-ik(z - x \sin \theta_k)} dk \\ + \int_{k_1}^{k_2} T^*(x) (1 + e^{-i(2\pi/D)x}) e^{-ik(z + x \sin \theta_k)} dk.$$

This means that each sinusoidal component of the light incident on a doubly-exposed hologram generates a monochromatic wave of the type

$$T(x) (1 + e^{i(2\pi/D)x}) e^{-ikx \sin \theta_k}$$

and the total reconstructed wave represents a superposition of monochromatic images each of which is observed at a definite angle  $\theta_k$ .

The pattern obtained in this way is simply an achromatic system of interference fringes, formed by the polychromatic source, which is superimposed on the image of the object. The achromatism condition follows from the fact that the sine of the interference angle is proportional to the wavelength of each monochromatic component of the spectrally resolved beam (in this case the interference does not occur in diverging beams).

Reconstruction of a double-exposure focused-image hologram with an extended monochromatic source, which emits an illuminating wave

$$U_0^*(x) = \int_{-\theta \text{ lim}}^{+\theta \text{ lim}} e^{-ikx \sin \theta} d\theta,$$

generates a wave which corresponds to one of the symmetric images:

$$U_r^{(v)*}(x) = T(x) (1 + e^{i(2\pi/D)x}) e^{ikx \sin \theta_k} \int_{-\theta \text{ lim}}^{+\theta \text{ lim}} e^{-ikx \sin \theta} d\theta. \quad (13)$$

It follows from Eq. (13) that the useful image is observed not along one angle  $\theta_k$  but in a range of angles which is governed by the size of the illuminating source (these angles lie within a cone ranging from  $-\theta \text{ lim}$  to  $+\theta \text{ lim}$ ).

Finally, in the case when the reconstruction is performed by a polychromatic beam produced by an extended source, the overall reconstructed wave can be represented in the form

$$U_{r,o}^{(v)*}(x) = T(x) (1 + e^{i(2\pi/D)x}) \int_{k_1}^{k_2} e^{ikx \sin \theta_k} \int_{-\theta \text{ lim}}^{+\theta \text{ lim}} e^{-ikx \sin \theta} d\theta dk,$$

i.e., the range of useful angles becomes wider (because of the dispersion) at right-angles to the direction of the spatial carrier of the hologram.

Experiments involving recording of doubly-exposed holograms followed by reconstruction of interferogram images in white light were reported in<sup>[39]</sup>. In these ex-

periments the object was subjected to pure rotation, which has been considered above theoretically, and to more complex mechanical deformations. The reconstructed images of the interferograms obtained using laser radiation or white light were recorded photographically using a black-and-white film. The photographs were exposed and treated under the same conditions and then subjected to a photometric analysis. Microphotograms obtained in this way made it possible to compare the contrast in the interferograms reconstructed using light of different degrees of monochromaticity. The photometric results indicated that the contrast in the reconstructed interferograms was little affected by the replacement of laser radiation with white light (Fig. 9).

The interference fringe pattern did not appear along the zeroth-order diffraction maximum in the image reconstructed with white light from a doubly-exposed focused-image hologram: only the black-and-white in-line image of the object itself was obtained along this direction. Investigations of such holograms of other forms of deformation of the objects indicated that the interference fringe pattern was not reconstructed in white light (or it was strongly defocused) only in those cases when the object was translated parallel to itself. In the majority of the cases of arbitrary deformation the plane of localization of the interferogram was within the range in which high-quality images were obtained. This region could be widened, as shown earlier in our review, by reducing the width of the spectrum of the reconstructing beam and the aperture in the observation system. The conditions for the observation of white-light-reconstructed interferograms were found to be similar to the conditions for the observation of spectrally colored images formed by single-exposure focused-image holograms and of images of interferograms of weakly scattering objects formed in white light from doubly-exposed Fresnel holograms.<sup>[41]</sup>

New opportunities are opened by the real-time focused-image holographic interferometry.<sup>[42]</sup> This method makes it possible to carry out an interferometric comparison of a reconstructed holographic image with an optical image formed by a focusing system (lens). In this case the interference pattern is localized in the plane of the photographic plate or close to it. Therefore, it is possible to resolve interferograms with closely spaced fringes corresponding to considerable deformations of the object being investigated. Since the interference fringes are localized in the hologram (and

not the object) plane, the eye can be placed quite close to this plane. In this case the angular separation between neighboring fringes becomes greater, which is in contrast to the Fresnel holograms which can be viewed only at distances greater than the separation between the object and the hologram.

The sensitivity in the real-time focused-image holographic interferometry method can be varied by altering the magnification employed in the recording of the hologram. In the interference comparison of a magnified holographic image with an optical image it is possible to obtain a well-resolved interferogram in those cases when the deformation of the object is too weak for the resolution of interference fringes obtained in the case of unit magnification holography.

## 8. CONCLUSIONS

The specific features of the focused-image holography make it possible to apply this method in various scientific and technical situations. The localization of the information in the hologram makes the quality of the reconstructed image practically independent of the wavelength and the direction of the reconstructing beam. This makes it possible to use extended light sources in the recording stage and polychromatic as well as extended sources in the reconstruction stage. The diffraction of the reconstructing beam by the spatial carriers of the hologram produces an image in a wide range of angles and illumination with nonmonochromatic light widens this range, because of the dispersion, at right-angles to the direction of the interference fringes. It has been shown recently<sup>[43]</sup> that focused-image holograms can be recorded in diffusely scattered multimode laser radiation without any loss in the image quality because the use of such radiation simply increases the number of the spatial carriers.

The main factor which limits the usefulness of the focused-image method is the restricted depth of the scene which can be reconstructed with satisfactory quality in white light, particularly when an extended reference source is used. However, since the observation system (eye) has a small aperture, which receives only part of the radiation, the blurring of the image points displaced in depth does not exceed the dimensions of the scattering circle which appears to the eye as a point. However, if an extended reference source is used in the recording stage, the depth of the reconstructed scene does not exceed 1–1.5 cm.

The recent investigations<sup>[44]</sup> have been concerned with the details of the theory of focused-image holography, the quality of the reconstructed (including three-dimensional) images, and the possible practical applications in electron microscopy, recording of double-chamber particle tracks, etc.

Some new applications of the focused-image holographic interferometry method can be expected to result from the localization of the interferograms in the plane of the photographic plate and because these interferograms can be obtained using polychromatic radiation or extended reference sources.

Wider practical applications of the focused-image holography are likely because the requirements which the holographic apparatus and its mechanical stability must satisfy are far less stringent than in the conventional holography. These requirements are also simpler

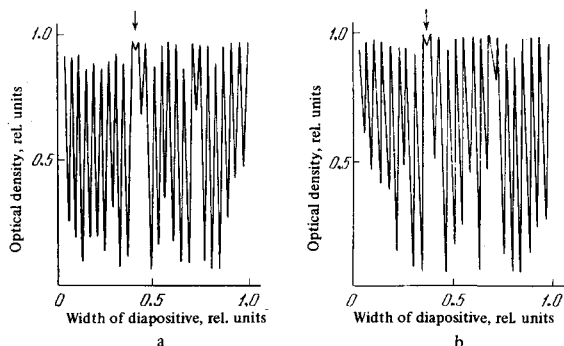


FIG. 9. Microphotograms of interferogram images obtained using laser radiation (a) and white light (b). The arrow indicates the passage of the microphotometer beam across a scribed mark used for focusing purposes.

in respect of the quality of the photographic materials and the treatment of these materials. The methods discussed in the present review may rapidly find various scientific and technological applications.

- <sup>1</sup>D. Gabor, *Nature* **161**, 777 (1948).
- <sup>2</sup>D. Gabor, *Proc. Roy. Soc., London* **A197**, 454 (1949).
- <sup>3</sup>D. Gabor, *Proc. Phys. Soc., London* **B64**, 449 (1951).
- <sup>4</sup>Yu. N. Denisyuk, *Dokl. Akad. Nauk SSSR* **144**, 1275 (1962) [*Sov. Phys.-Dokl.* **7**, 543 (1962)].
- <sup>5</sup>Yu. N. Denisyuk, *Opt. Spektrosk.* **15**, 522 (1963).
- <sup>6</sup>Yu. N. Denisyuk, *Opt. Spektrosk.* **18**, 275 (1965).
- <sup>7</sup>E. N. Leith and J. Upatnieks, *J. Opt. Soc. Amer.* **52**, 1123 (1962).
- <sup>8</sup>E. N. Leith and J. Upatnieks, *J. Opt. Soc. Amer.* **53**, 1377 (1963).
- <sup>9</sup>E. N. Leith and J. Upatnieks, *J. Opt. Soc. Amer.* **54**, 1295 (1964).
- <sup>10</sup>E. N. Leith, J. Upatnieks, and K. A. Haines, *J. Opt. Soc. Amer.* **55**, 981 (1965).
- <sup>11</sup>E. N. Leith and J. Upatnieks, "Photography by laser," *Sci. Amer.* **212**, 24 (June, 1965).
- <sup>12</sup>B. P. Konstantinov, A. N. Zaidel', V. B. Konstantinov, and Yu. I. Ostrovskii, *Zh. Tekh. Fiz.* **36**, 1718 (1966) [*Sov. Phys.-Tech. Phys.* **11**, 1279 (1967)].
- <sup>13</sup>A. W. Lohmann, *J. Opt. Soc. Amer.* **56**, 537 (1966).
- <sup>14</sup>L. H. Tanner, *J. Sci. Instrum.* **43**, 346 (1966).
- <sup>15</sup>G. B. Brandt, *Appl. Opt.* **8**, 1421 (1969).
- <sup>16</sup>L. Rosen, *Proc. IEEE*, **55**, 79 (1967).
- <sup>17</sup>W. E. Kock, L. Rosen, and G. W. Stroke, *Proc. IEEE*, **55**, 80 (1967).
- <sup>18</sup>I. S. Klimenko, E. G. Matinyan, and I. P. Nalimov, *Opt. Spektrosk.* **26**, 1019 (1969).
- <sup>19</sup>G. B. Brandt and A. K. Rigler, *Phys. Lett.* **25A**, 68 (1967).
- <sup>20</sup>K. A. Stetson, *Appl. Phys. Lett.* **11**, 225 (1967).
- <sup>21</sup>J. Hertel, *Optik* **25**, 217 (1967).
- <sup>22</sup>A. P. Komar, B. G. Turukhano, and N. Turukhano, *Dokl. Akad. Nauk SSSR* **186**, 1312 [*Sov. Phys.-Dokl.* **14**, 610 (1969)].
- <sup>23</sup>G. W. Stroke, *Phys. Lett.* **23**, 325 (1966).
- <sup>24</sup>O. Bryngdahl and A. W. Lohmann, *J. Opt. Soc. Amer.* **60**, 281 (1970).
- <sup>25</sup>E. N. Leith and J. Upatnieks, *J. Opt. Soc. Amer.* **57**, 975 (1967).
- <sup>26</sup>G. I. Rukman and Yu. I. Filenko, *ZhETF Pis. Red.* **8**, 538 (1968) [*JETP Lett.* **8**, 328 (1968)].
- <sup>27</sup>I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **28**, 556 (1970).
- <sup>28</sup>E. G. Matinyan, *Avtoreferat kand. dissertatsii* (Author's Abstract of Thesis for Candidate's Degree), Moscow Physicotechnical Institute, 1971.
- <sup>29</sup>L. Rosen, *Appl. Phys. Lett.* **9**, 337 (1966).
- <sup>30</sup>G. W. Stroke, R. Restricker, A. Funkhouser, and D. Brumm, *Appl. Phys. Lett.* **6**, 178 (1965).
- <sup>31</sup>L. Rosen and W. Clark, *Appl. Phys. Lett.* **10**, 140 (1967).
- <sup>32</sup>I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **29**, 1132 (1970).
- <sup>33</sup>R. W. Meier, *J. Opt. Soc. Amer.* **56**, 219 (1966).
- <sup>34</sup>I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **31**, 776 (1971).
- <sup>35</sup>H. J. Caulfield, *Phys. Lett.* **27A**, 319 (1968); *Appl. Phys. Lett.* **16**, 234 (1970); W. T. Cathey, Jr., USA Patent No. 3 415 587, appl. 8 Dec. 1965, publ. 10 Dec. 1968.
- <sup>36</sup>O. Bryngdahl and A. W. Lohmann, *J. Opt. Soc. Amer.* **58**, 1325 (1968).
- <sup>37</sup>N. Nishida, *Appl. Opt.*, **7**, 1862 (1968).
- <sup>38</sup>I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **27**, 367 (1969).
- <sup>39</sup>I. S. Klimenko, E. G. Matinyan, and G. I. Rukman, *Opt. Spektrosk.* **29**, 160 (1970).
- <sup>40</sup>K. A. Haines and B. P. Hildebrand, *Appl. Opt.* **5**, 595 (1966).
- <sup>41</sup>I. S. Klimenko, E. G. Matinyan, and G. I. Rukman, *ZhETF Pis. Red.*, **6**, 535 (1967) [*JETP Lett.* **6**, 57 (1967)].
- <sup>42</sup>I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **32**, 620 (1972).
- <sup>43</sup>I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **31**, 471 (1971).
- <sup>44</sup>R. Vanderwarker and K. Snow, *Appl. Phys. Lett.* **10**, 35 (1967); Sh. D. Kakichashvili, *Ukr. Fiz. Zh.* **14**, 1862 (1969); *Tekh. Kino Telev*, No. 3, 17 (1971); I. Weingärtner, W. Mirande, and E. Menzel, *Optik* **30**, 318 (1969); **31**, 335 (1970); W. Mirande, I. Weingärtner, and E. Menzel, *Opt. Commun.* **1**, 315 (1970); J. Schwider and R. Burow, *J. Opt. Soc. Amer.* **60**, 1421 (1970); J. C. Dainty and W. T. Welford, *Opt. Commun.* **3**, 289 (1971); I. S. Klimenko and E. G. Matinyan, *Opt. Spektrosk.* **31**, 776 (1971); Sh. D. Kakichashvili and V. I. Kakichashvili, *Zh. Tekh. Fiz.* **41**, 1508 (1972) [*Sov. Phys.-Tech. Phys.* **16**, 1184 (1972)].

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