# Interaction of high-energy particles with deuterons


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The purpose of the review is to compare the existing experimental data on total cross sections and also the differential cross sections for elastic and inelastic scattering of fast particles by deuterons with theoretical predictions based on a diffraction model of the interaction. A detailed discussion is given of corrections to the total cross sections arising as the result of inelastic screening, i.e., taking into account the possibility of diffraction excitation of the incident particle by the first nucleon, as a result of which a shower is formed which subsequently is absorbed in the second nucleon. A summary is presented of values of the parameter $\langle R^{-2}\rangle$ obtained from the total cross sections for $\pi$d and $\bar{p}$d scattering at energies from 2 to 60 BeV. It is shown that the comparison of theoretical angular distributions with the experimental data on $\pi$d, pd, and $\bar{p}$d scattering indicates that the theory correctly reproduces the main characteristics of elastic scattering. Discrepancies are observed only for very large momentum transfers (greater than 1 BeV/c). A satisfactory description is also given of pd scattering with breakup of the deuteron, in which a characteristic qualitative feature is observed—a peak corresponding to double scattering of the incident particle by the nucleons of the deuteron. In the concluding chapter the main premises of the Glauber theory are discussed, the diagram approach is briefly described, and on its basis an estimate is made of the accuracy of the theory. (539.171)


## CONTENTS



## 1. INTRODUCTION

Strong interactions are studied mainly by bombardment of various targets by beams of fast particles. The purest experiments from the point of view of elementary-particle physics are those performed in hydrogen (proton) targets. However, more complex targets are frequently used for technical reasons or for the purpose of studying nuclear interactions. In the theoretical interpretation of data on the interaction of particles with nuclei, difficulties associated with the need of taking into account nuclear effects are necessarily encountered.

A special place is occupied by experiments with a deuterium target. In the first place, interactions with deuterons can, as a rule, be studied in apparatus intended for measurements with an ordinary hydrogen target. In the second place, the proton and neutron in the deuteron are weakly bound and the nuclear corrections are small (for example, of the order of 10% in the total cross sections). In the third place, the nuclear effects in scattering of fast particles can be reasonably well calculated by means of the Glauber theory[1-4]. This theory is based on the fact that the interaction between particles at high energies has a diffraction nature. As a consequence of this, scattering by a deuteron can be represented as the superposition of direct scattering by one of the nucleons (the impulse approximation) and a process in which the incident particle successively interacts with the two nucleons (two-fold rescattering). Effects associated with rescattering depend substantially on the structure of the deuteron and most of all on the average distance between the proton and neutron.

The purpose of the present review is to compare the existing experimental data on the interaction of fast particles with deuterons with theoretical predictions based on the assumption of a diffraction nature of the elementary interactions. We first discuss the Glauber correction to the total cross section, which arises as the result of mutual screening of nucleons and which is determined by the structure of the deuteron. Here information is used on the deuteron form factor obtained from elastic electron scattering. Then a comparison of theory with experiment is made for the angular distributions of elastic pd, $\pi$d, and $\bar{p}$d scattering, and also for pd scattering with breakup of the deuteron. We discuss corrections to the Glauber theory which arise at high energies as the result of the virtual production of particles (inelastic screening). In conclusion a brief review is given of the principal assumptions of the Glauber theory, and the accuracy of the theoretical predictions is discussed.

## 2. TOTAL CROSS SECTIONS AND THE GLAUBER CORRECTION

The total cross sections for interaction of particles with deuterons are usually measured with high accuracy. The Glauber theory permits calculation of the correction for screening, so that the theoretical error in the total cross section is $\sim 1\%$. As a result it has been possible to find the cross sections for interaction of particles with a neutron with satisfactory accuracy from data on deuteron cross sections. As experiments show, the cross sections for interaction of particles with protons and neutrons are very similar at high energies. This circumstance is naturally explained by the theory of complex angular momenta. In this connection it is interesting to know the value of the difference in the total cross section $\sigma_p - \sigma_n$, and particularly its behavior with energy.

The total cross section for interaction of a particle with a deuteron can be represented in the following form:

$$\sigma_d = \sigma_p + \sigma_n - \Delta, \tag{1}$$

where $\Delta$ is the cross-section defect, a quantity which is positive for sufficiently high energies, which corresponds to the idea of nucleon screening. Under certain assumptions, which although rather crude are qualitatively correct, the quantity $\Delta$ is represented in the form[1]

$$\Delta = (4\pi)^{-1} \langle R^{-2} \rangle \sigma_p \sigma_n, \tag{2}$$

where R is the distance between the proton and neutron in a deuteron, and the angular brackets denote averaging over the deuteron bound state. This result was obtained with the following conditions:

1) The radius of strong interactions is much smaller than the size of the deuteron. This is a rather crude assumption, and as a matter of fact for this reason the coefficient $\langle R^{-2} \rangle$ in Eq. (2) not only is determined by the deuteron structure but also depends on the nature of the elementary interactions. However, this dependence is weak, and theoretically the coefficient $\langle R^{-2} \rangle$ in Eq. (2) is almost constant. A detailed analysis is given in Chap. 3.

2) The cross section for scattering with charge exchange of target nucleons is much smaller than the elastic cross section. Here it follows from isotopic invariance that in Eq. (2) we can set $\sigma_p = \sigma_n$.

3) The amplitude of forward elastic scattering by a nucleon is pure imaginary. The validity of the last two assumptions is discussed in the Appendix (page 61). Analysis shows that the corresponding corrections do not exceed $(0.1-0.2)\Delta$ in the energy region above 1 BeV.

Equation (2) permits calculation of the nuclear correction in the deuteron by means of one parameter which does not depend on energy and which is common for all incident particles: $\langle R^{-2} \rangle$. This quantity can in principle be found from the deuteron wave function. However, it depends substantially on the behavior of the wave function at small distances, and therefore it is difficult to find with high accuracy (see Chap. 3 for more detail).

In two cases the quantity $\Delta$ can be found experimentally:

a) Scattering of $\pi$ mesons. The value of the cross-section defect is found from comparison of the total cross sections $\sigma(\pi^- p)$, $\sigma(\pi^+ p) = \sigma(\pi^- n)$ and $\sigma(\pi^+ d) = \sigma(\pi^- d)$ (the equalities are accurate to the radiation corrections). All three cross sections are measured with high accuracy.

b) Nucleon scattering. It is necessary to compare $\sigma(pp)$, $\sigma(pd)$, and $\sigma(pn)$. The cross section $\sigma(pn)$ is determined by bombardment of a proton target by a beam of neutrons; the accuracy in measurement of this quantity is somewhat poorer than for the other cross sections. For energies $\gtrsim 10$ BeV the difference $\sigma(pp) - \sigma(pn)$ lies within the experimental error. Therefore in

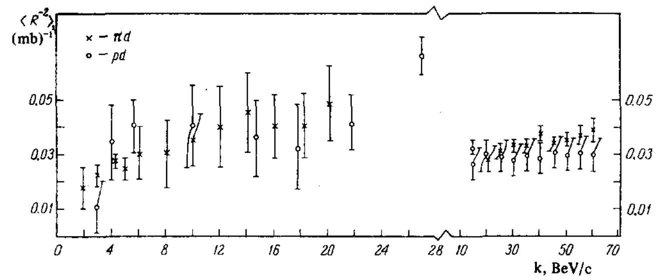



FIG. 1. The quantity $\langle R^{-2} \rangle$ determined by Eq. (2) from data on scattering of $\pi$ mesons and nucleons by deuterons [5,6]. The data obtained on the basis of experiments at Serpukhov [5b,6b] are shown separately at the right; here in determination of $\langle R^{-2} \rangle$ it was assumed that $\sigma(pp) = \sigma(pn)$.

estimating the value of $\Delta$ at high energies we can neglect this difference[1)].

Figure 1 shows the value of $\langle R^{-2} \rangle$ calculated from Eq. (2) by means of the data existing in the literature[5a,6a]. (The behavior of the quantities $\Delta$ and $\langle R^{-2} \rangle$ for pd scattering has been discussed in ref. 7.) Within the rather large experimental errors the values of $\langle R^{-2} \rangle$ are weak functions of energy and agree in measurements with $\pi$ mesons and nucleons. It is not excluded that there is some rise in $\langle R^{-2} \rangle$ with increasing energy for $E > 20$ BeV. This effect can be produced by inelastic screening (see Chap. 6).

## 3. DEUTERON FORM FACTOR

The Glauber theory[1] gives a result which is more accurate than Eq. (2) and which takes into account the finite value of the strong-interaction radius:

$$\Delta = 2 \int \rho(4\tau)(d\sigma_{el}/d\tau)\, d\tau, \tag{3}$$

where $d\sigma_{el}/d\tau$ is the elastic differential cross section for scattering of the incident particle by a nucleon (the difference in the cross sections for protons and neutrons is small), and $\tau$ is the squared momentum transfer ($\tau > 0$), $\rho(\tau)$ is the deuteron form factor:

$$\rho(\tau) = \int |\varphi_d(\mathbf{r})|^2 \exp(i\mathbf{q}\mathbf{r}/2)\, d^3r, \quad \tau = \mathbf{q}^2, \quad \rho(0) = 1 \tag{4}$$

(with accuracy to certain complications due to the nucleon and deuteron spins).

For small momentum transfers and for the condition that the scattering amplitude is pure imaginary (this assumption is necessary to obtain Eq. (3)), the strong-interaction cross section is represented in the form

$$d\sigma_{el}/d\tau = (16\pi)^{-1} \sigma_{tot}^2 e^{-A\tau}. \tag{5}$$

If we assume that $\rho(4\tau)$ changes significantly more rapidly than the elastic cross section, we can take the cross section for $\tau = 0$ outside the integral sign and obtain Eq. (2), where by definition

$$\langle R^{-2} \rangle \equiv (1/2) \int \rho(4\tau)\, d\tau. \tag{6}$$

The form factor is measured in elastic scattering of electrons by deuterons:

$$d\sigma(ed)/d\tau = [d\sigma_0(ed)/d\tau]\,[\rho(\tau)\, G_s(\tau)]^2,$$

where $d\sigma_0/d\tau$ is the cross section for a point particle, and $G_S(\tau)$ is the isoscalar form factor of the nucleon. The elastic cross section for ed scattering has been measured[8] up to $\tau = 35\ \mathrm{F}^{-2} = 1.37\ (\mathrm{BeV/c})^2$. The experiment was carried out for small electron-scattering angles, and therefore only the charge portion of the cross section was determined, i.e., some combination of the charge and quadrupole form factors. The data obtained have been compared with various theoretical predictions. In particular, satisfactory agreement of the experiment has been observed with the Hamada-Johnston model[9] ($\chi^2 = 67$ for 62 degrees of freedom). This model uses for calculation of the wave function of the deuteron a nucleon-nucleon potential with a large number of parameters which are determined from data on scattering at small angles. At small distances there is a repulsion with radius $r_c = 0.485$ F ($r_c^{-2} = 0.16$ (BeV/c)$^2$). The model takes into account nucleon spins and the D wave in the deuteron. The form factor of interest to us is expressed in terms of the radial wave functions of the S and D waves, u(r) and v(r), as follows:

$$\rho(\tau) = \int_0^\infty [u^2(r) + v^2(r)]\, j_0(r\sqrt{\tau}/2)\, dr,$$

where $j_0$ is a spherical Bessel function. The result of the calculation with the Hamada-Johnston model is shown in Fig. 2. Integration of Eq. (6) over the region $0 < \tau < 20$ F$^{-2}$ leads to the value $\langle R^{-2}\rangle = 0.039$ mb$^{-1}$, which is consistent with the data shown in Fig. 1. We must of course keep in mind that the uncertainty in the value of the integral can be significantly greater than the uncertainty in the form factor for finite $\tau$ known to us from ed scattering.

In the region $0 < \tau < 20$ F$^{-2}$ the form factor $\rho(\tau)$ calculated from the Hamada-Johnston model is approximated with an accuracy of $\sim 0.01$ over the entire interval by the equation

$$\rho(\tau) = be^{-a_1\tau} + (1 - b)\, e^{-a_2\tau}, \qquad (7)$$

where $a_1 = (1.15 \pm 0.01)$ F$^{-2}$, $a_2 = (0.22 \pm 0.01)$ F$^{-2}$, $b = 0.40 \pm 0.01$. The first exponential corresponds to the size of the deuteron, and the second to the repulsive core.

The form factor in the Hamada-Johnston model changes sign for $\tau \sim 22$ F$^{-2}$, and this of course is not described by Eq. (7). However, for $\tau \gtrsim 20$ F$^{-2}$ the very concepts of a nonrelativistic wave function and the ordinary form factor (4) lose their meaning. In this region not only the relativistic corrections but also corrections due to meson currents become important. In other words, on close approach of the nucleons in the deuteron, we can observe with appreciable probability mesons emitted by one nucleon and absorbed by the other. These mesons affect the charge distribution in the interior of the deuteron. Estimates of the effect of exchange currents have been made many times (for example, see the references given by Elias et al.[8]). In the recent work of Blankenbecler and Gunion[10] the correction for the form factor was calculated on the basis of the hypothesis of vector meson dominance. It was shown that this correction is not only very important for $\tau \gtrsim 20$ F$^{-2}$ but also makes appreciable contribution to the deuteron magnetic moment. In what follows we will not consider the observed deviations from the ordinary nonrelativistic model of the deuteron, which have little effect on the total cross sections and also on the differential cross sections for small momentum transfers.

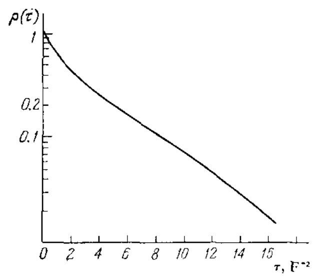



FIG. 2. The deuteron form factor in the Hamada-Johnston model.

Slopes of elastic-scattering differential cross sections

| Process | $\pi^+p$ | | | $\pi^-p$ | | | $K^+p$ | | $K^-p$ | | $pp$ | | | $\bar{p}p$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| k, BeV/c | 2 | 10 | 20 | 2 | 10 | 20 | 2 | 10 | 2 | 10 | 2 | 10 | 20 | 2 | 10 |
| A*), (BeV)$^{-2}$ | 6 | 8 | 10 | 8 | 10 | 10 | 3 | 6 | 7 | 8 | 6 | 9 | 10 | 15 | 13 |

*In all cases the errors in A are less than 1 (BeV)$^{-2}$.

The role of the finite size of the diffraction cone in scattering by nucleons[2)] ($A \neq 0$) can be estimated in the following way. We will substitute Eqs. (5) and (7) into Eq. (3). Then instead of the definition (6) we obtain

$$\langle R^{-2}\rangle = \frac{1}{2}\left(\frac{b}{4a_1 + A} + \frac{1-b}{4a_2 + A}\right).$$

For $\pi$ mesons at energies $\gtrsim 10$ BeV, we have $A = 10$ (BeV/c)$^{-2}$ = 0.39 F$^2$. Using the parameters of Eq. (7), we obtain for $\pi$d scattering $\langle R^{-2}\rangle = (0.028 \pm 0.001)$ mb$^{-1}$. Thus, the measured value of the cross-section defect $\Delta$ agrees quite satisfactorily with the Hamada-Johnston model for the deuteron wave function. As a rule, it is sufficient to use the simplified formula for the form factor $\rho(\tau) = \exp(-a\tau)$ (this form factor corresponds to a wave function with a Gaussian shape[12]). Then the correction for screening has the form

$$\Delta = (4\pi)^{-1}[2(4a + A)]^{-1}\sigma_{tot}^2.$$

Here we can take $4a = 1.36$ F$^2$ = 35 (BeV/c)$^{-2}$. Thus, the effect of the finite radius of the strong interaction amounts to ~ 20–30% of the correction for screening (the effect is particularly noticeable in the case of $\bar{p}$d scattering), and its inclusion evidently improves the agreement of the data on the value of $\langle R^{-2}\rangle$ obtained from scattering of $\pi$ mesons and nucleons with the data on ed scattering with large momentum transfers. We note that, since the slopes of the differential cross sections A are of the same order for different particles and change slowly (logarithmically) with energy, the effective value of the parameter $\langle R^{-2}\rangle$ in the correction for screening can be assumed constant up to an energy of the order 20 BeV.

## 4. ANGULAR DISTRIBUTION OF ELASTIC pd AND πd SCATTERING

In investigating the question of how accurate is the correction given by the Glauber theory to the total cross section, it is useful to compare the predictions of this theory for angular distributions with the experimental data. The study of angular distributions permits separation of the contribution of double scattering. The point is that, while the Glauber correction amounts to only several percent for the total cross sections, for momentum transfers greater than 0.6–0.7 BeV/c the differential cross section is completely determined by double scattering.

According to the Glauber theory, the amplitude for elastic scattering of a particle with momentum **k** by a deuteron, F(**q**), where **q** is the momentum transfer, is expressed in terms of the amplitude for scattering by a

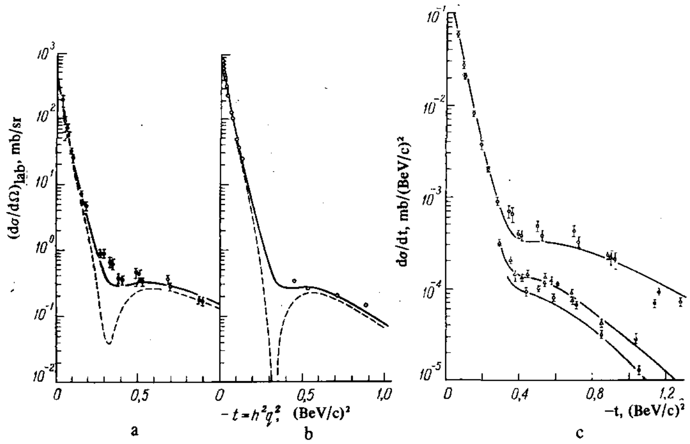



FIG. 3. Differential cross sections for pd scattering. a) k = 1.7 BeV/c [15]; b) k = 2.8 BeV/c [16] (theory—in ref. 14; the dashed curves have been plotted without inclusion of the D wave); c) k = 1.7, 4.54, and 6.37 BeV/c [17].

proton $f_p(\mathbf{q})$ and a neutron $f_n(\mathbf{q})$ in the following way:

$$F(\mathbf{q}) = [f_p(\mathbf{q}) + f_n(\mathbf{q})]\,\rho(q^2) + \frac{i}{2\pi k}\int \rho(4q'^2)\, f_n\left(\frac{1}{2}\mathbf{q}+\mathbf{q}'\right) f_p\left(\frac{1}{2}\mathbf{q}-\mathbf{q}'\right) d^2q'; \qquad (8)$$

here $\rho$ is the deuteron form factor (4), and the integration is carried out over the momenta $\mathbf{q}'$ lying in a plane perpendicular to the incident-particle momentum. The amplitudes are normalized as follows:

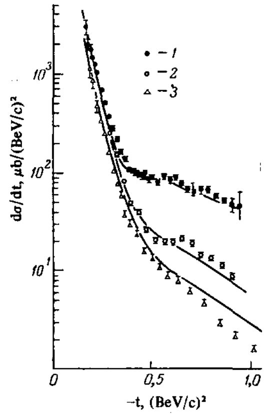
$$d\sigma/d\Omega = |F(\mathbf{q})|^2, \quad \sigma_{tot} = (4\pi/k)\,\mathrm{Im}\,F(0),$$

where $\sigma$ is the cross section in the laboratory system. In particular, the latter equality, in the case where $f_p(\mathbf{q}) = f_n(\mathbf{q})$ is a pure imaginary quantity in the region of small $\mathbf{q}$, leads to representation of the cross-section defect $\Delta$ in the form of Eq. (3).

The angular distribution obtained from Eq. (8) for elastic scattering has qualitatively the following form. The terms corresponding to single scattering are dominant for small q, but fall off rapidly with increasing q as the result of the factor $\rho(q^2)$. The term corresponding to double scattering, which is small for small q, drops much more slowly: the scale of falloff is determined by the behavior of the differential cross section for scattering by an individual nucleon. The fact is that the main contribution to the integral in Eq. (8) is from the region of small $\mathbf{q}'$, where $\rho(4q'^2) \sim 1$, and therefore the drop associated with "friability" of the deuteron is absent. In other words, double scattering in which a momentum $\mathbf{q}$ is transferred to the deuteron occurs preferentially in such a way that the particle sequentially transfers to each of the nuclear nucleons a momentum $\mathbf{q}/2$, but the relative momentum of the nucleons in this case remains small.

If the amplitudes for scattering by the neutron and proton are pure imaginary, the terms corresponding to single and double scattering in Eq. (8) have opposite signs. Thus, if we take into account only the S state of the deuteron, the amplitude should pass through zero at a certain momentum and correspondingly the differential cross section should go to zero. Inclusion of the real parts of the amplitudes $f_p$ and $f_n$ leads to the result that the amplitude for scattering in the deuteron does not go to zero but has a deep minimum which is not observed experimentally. The situation is corrected by taking into account the D wave in the deuteron, which leads to filling in of the interference minimum[4,13,14].

Figure 3 shows a comparison of the experimental data on elastic pd scattering with the theoretical calculation on the basis of Eq. (8). The deuteron D wave was



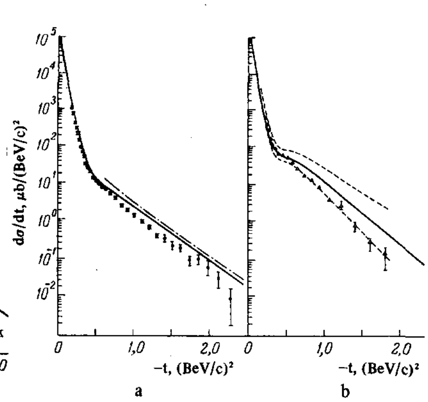




FIG. 4 FIG. 5

FIG. 4. Differential cross sections for elastic $\pi^-$d scattering for three initial momenta: 1—0.9 BeV/c [18], 2—3.75 BeV/c [19], 3—9.0 BeV/c [20] (the theoretical curves have been taken from ref. 21).

FIG. 5. Differential cross sections for elastic $\pi^-$d scattering at 9.0 BeV/c (a) and pd scattering for 12.8 BeV/c (b) for large momentum transfers. The experimental data have been taken from refs. 20 and 23; the theoretical curves have been taken from refs. 21 and 22. The dot-dash curve in part a) corresponds to inclusion only of double scattering, and the dashed curves in part b) were obtained by varying the pN-scattering parameters. Good agreement is obtained for those parameter values which are inconsistent with the experimental data on pp and pn scattering.

taken into account in the calculations. The agreement with experiment is quite satisfactory. The dashed lines in Figs. 3a and b show the results of the calculation not taking into account the D-wave admixture. It is quite apparent that the D-wave admixture has little effect on the result both in the single-scattering region (small angles) and in the double-scattering region (large angles), but is extremely important in the region of the interference minimum.

A similar situation is observed also in $\pi$d scattering. In Fig. 4 the differential cross sections for elastic $\pi^-$d scattering for three energies[18-20] are compared with theoretical calculations from Alberi and Bertocchi[21]. In scattering with large momentum transfers, as we could expect from the very beginning, the situation is somewhat poorer. The theoretical curves for both $\pi^-$d and pd scattering lie substantially above the experimental points (see Fig. 5, taken from the work of Bradamante et al.[22]; the experimental data on the $\pi^-$d scattering are from ref. 20, and those for pd scattering from ref. 23). The cause apparently lies in the fact that for large momentum transfers ($q^2 \sim 1.5-2.0$ (BeV/c)$^2$) the main assumptions of the theory are invalid. For example, large corrections are made which are associated with inclusion of nucleon recoil and virtual rescattering of nucleons (chapter 7). Moreover, as was noted in chapter 3, we do not have reliable information on the deuteron wave function at small distances, since in analysis of data on ed scattering the relativistic corrections, exchange currents, double scattering, and other effects become important. In order to obtain agreement of theory and experiment at large momentum transfers in terms of the ordinary Glauber theory, it would be necessary to assume that $\langle R^{-2} \rangle = 0.02$ mb$^{-1}$, i.e., much less than the presently accepted value (see Chap. 2 and Fig. 1).

It is interesting to note that a preliminary calculation carried out by Franko and Glauber[4] in which the $\pi$N and pN scattering amplitudes were taken in the exponential form (i.e., the cross sections in the form of Eq. (5)), the exponent being assumed constant and independent of momentum transfer, gave results which are in agreement with experiment up to momentum transfers $q^2 = 2$ (BeV/c)$^2$. However, more accurate calculations taking into account the change in slope of the cone in $\pi$N and pN scattering[21,22] have shown that this agreement was only apparent.

Quite recently experimental studies have been made of elastic $\bar{p}$d scattering at momenta from 1.6 to 2.0 BeV/c.[24] The differential cross sections obtained in that work for four different initial momenta are shown in Fig. 6, together with the results of calculations using the Glauber theory. The solid curve which agrees best with the experimental data was obtained on the assumption that the ratio of the real part of the $\bar{p}$p and $\bar{p}$n scattering amplitudes to the imaginary part is 0.4.

In all of the figures shown the theoretical predictions are in good agreement with the experimental data for small momentum transfers, agreement being observed even where the main premises of the theory are clearly no longer valid (for example, at low energies). The same good agreement is observed also in analysis of data on $\pi$d and pd scattering at other energies[4].

It should be noted that the question of the causes of the filling of the interference minimum can nevertheless hardly be considered solved. It is not excluded that, in addition to the D-wave contribution, other reasons are also important here (inelastic processes in the intermediate state, the contribution of terms usually discarded, which will be discussed at the end of chapter 7, and so forth). An experiment with polarized particles or with detection of the polarization of the final particles would be of assistance in solving this problem[14,25]. The corresponding studies are only beginning (see the article by Booth et al.[26], who have measured the polarization of protons in pd scattering at 1 BeV/c, and the work of Bunce et al.[27a], who have determined the polarization and quadrupolarization of deuterons in dp scattering for a deuteron momentum of 3.6 BeV/c, and also ref. 27b, where the asymmetry and polarization have been determined at 600 MeV. The results of preliminary calculations of polarization effects[27c] only qualitatively agree with the experimental data. They turn out to be very sensitive to the possible change of the ratio Re f/Im f for the NN-scattering amplitude with momentum transfer).

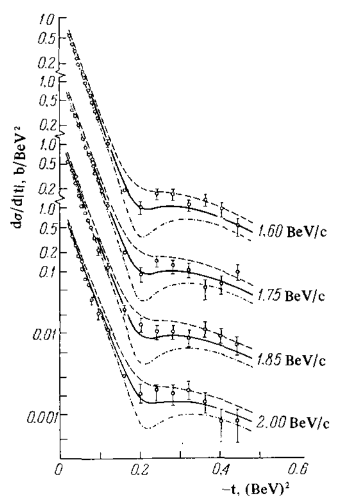



FIG. 6. Differential cross sections for elastic antiproton-deuteron scattering [24]. Curves—calculation according to the Glauber theory with use of various values of $\alpha$ = Re f/Im f; the dot-dash, solid, and dashed curves correspond to $\alpha_p = \alpha_n = 0$, 0.4, and 0.6.

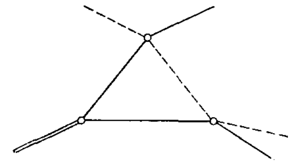


FIG. 7. Feynman diagram corresponding to inelastic double scattering by a deuteron.

On the whole the comparison of theoretical angular distributions with experimental data indicates that the theory correctly transmits the basic behavior of elastic scattering.

## 5. INELASTIC SCATTERING BY DEUTERONS

An additional possibility of studying the scattering of high-energy particles by neutrons appears in study of the process accompanied by breakup of the deuteron. Here information can be obtained not only on the total cross section but also on the behavior of the differential cross section for small momentum transfers.

The point is that if we measure the spectrum of particles elastically scattered by deuterons at a fixed angle, under certain conditions a peak will appear corresponding to double scattering of the particle by the nucleons of the deuteron (see the diagram of Fig. 7). This peak has been observed in the momentum spectrum of protons with initial momentum 19.2 BeV/c which have been scattered by deuterons at angles from 40 to 65 mrad[28] (see Fig. 8, where the left-hand peaks are single, i.e., quasi-elastic, scattering; the right-hand peaks arise from double scattering). The location of the peak corresponds to elastic pd scattering, but the area under it is much greater than that which would be given by the elastic process[28,29a].

The appearance of the peak can be understood qualitatively in the following way. Imagine that successive scattering of the incident proton occurs by two stationary nucleons, the first scattering angle being $\theta_1$ and the second $\theta_2$. If we consider for simplicity the case in which the two momenta lie in the same plane, we then have the condition

$$\theta_1 + \theta_2 = \theta, \qquad (9)$$

where $\theta$ is the angle at which the secondary-proton spectrum is recorded. We will assume to begin with that the differential cross sections for scattering by protons and neutrons are identical. The pp-scattering cross section is well described by the equation

$$d\sigma/d\Omega \sim \exp(-Ak^2\theta^2),$$

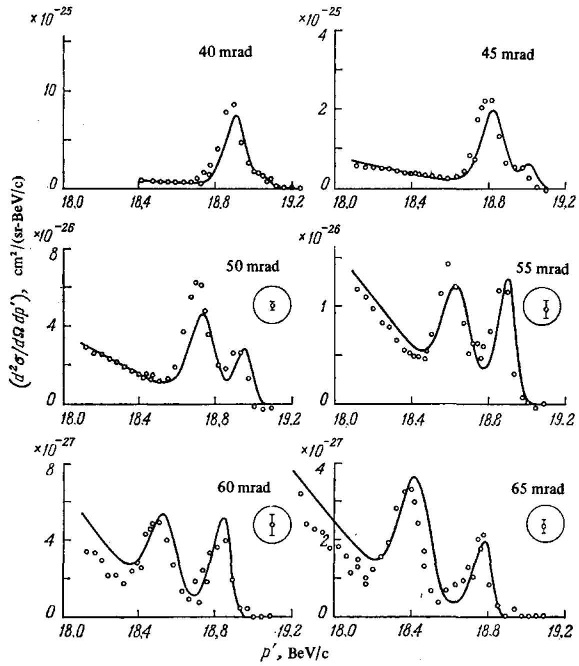



FIG. 8. Momentum spectra of protons from inelastic pd scattering at angles from 40 to 65 mrad for an initial momentum of 19.2 BeV/c. The experimental data have been taken from ref. 28, and the theoretical curves from ref. 29a.

where k is the initial momentum, $\theta$ is the scattering angle, and $A \approx 10\ (\mathrm{BeV/c})^{-2}$ for energies in the vicinity of 19 BeV. Therefore the cross section for double scattering will contain a factor

$$d^2\sigma/d\Omega\, dk' \sim \exp\left[-Ak^2(\theta_1^2+\theta_2^2)\right]. \tag{10}$$

For the condition (9), Eq. (10) will have a maximum for $\theta_1 = \theta_2 = \theta/2$. This corresponds to a difference of the initial and final momenta of the fast proton $\Delta k = k^2\theta^2/4m$. Exactly the same value of $\Delta k$ occurs in elastic pd scattering at an angle $\theta$. The spectrum of protons after double scattering is cut off at $k' = k - (k^2\theta^2/4m)$, since $\Delta k = k^2(\theta_1^2 + \theta_2^2)/2m$ cannot, if we take into account Eq. (9), be less than $k^2\theta^2/4m$, and a peak exists at this value k'. For smaller values of the momentum k', Eq. (10) can be written in the form

$$d^2\sigma/d\Omega\, dk' \sim \exp(-2Am\, \Delta k).$$

The width of the peak corresponding to double scattering is of the order $(2Am)^{-1} \approx 50$ MeV/c. Of course, the nucleon motion inside the deuteron and the experimental resolution result in the peak being smeared out and its right-hand boundary being no longer sharp.

A detailed calculation of the shape of the scattered-proton spectrum has been carried out by Glauber et al.[29a]. The results are shown in Fig. 8 by the solid lines. A Gaussian wave function was used for the deuteron[12]. The cross sections for pp and np scattering were assumed identical. As can be seen from Fig. 8, good agreement of the theoretical and experimental distributions was obtained.

The size of the right-hand peak in Fig. 8, and also the location and magnitude of the minimum between the peaks, are sensitive to the parameters of the amplitude for scattering by the neutron and could show its difference from the pp-scattering amplitude. One of the conclusions which are drawn by Glauber et al.[29a] is just that the pp and pn scattering amplitudes at 19.2 BeV/c agree within the experimental accuracy[28].

An important point in discussion of inelastic scattering is the choice of the complete system of functions describing the neutron and proton produced in breakup of the deuteron. Glauber et al.[29a] chose two systems: plane waves (which corresponds exactly to the diagram of Fig. 7) and functions in an oscillator well whose parameters were chosen so that the Gaussian wave function of the deuteron described the ground state in this well. The results agree with high accuracy, which evidently indicates a low sensitivity to the choice of the system of functions of the final slow proton and neutron and permits us to hope that the theoretical calculation is reliable.

Among the other conclusions of Glauber et al.[29a] we note the following: within the experimental accuracy there is no need to consider that the ratio Re f/Im f for the pp- and pn-scattering amplitudes changes with momentum transfer; for the energies and momentum transfers considered, the contribution of processes with intermediate inelastic channels amounts to no more than 20%.

Improvement of the experimental data will permit more accurate conclusions to be drawn regarding the pn scattering amplitude and the angular dependence of the ratio of the real and imaginary parts of the amplitude.

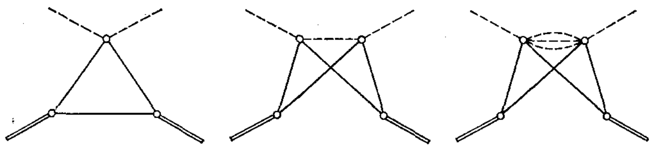
Recently a paper by Amaldi et al.[29b] has appeared in which the spectra have been measured of protons scattered by deuterons at angles from 13 to 107 mrad (the initial momentum was 24.0 BeV/c). Quasielastic (single) and double scattering peaks were clearly observed. The cross section for elastic pd scattering was measured separately for values of $|t|$ from 0.6 to 1.8 $(\mathrm{BeV/c})^2$. It agrees beautifully with the Glauber theory and gives about 1/3 of the integral of the cross section under the right-hand peak in the scattered-proton spectrum (the locations of the double-scattering peak and the peak from elastic pd scattering coincide kinematically). The authors extract from the set of data the differential cross section for elastic pn scattering for $|t|$ from 0.1 to 5.8 $(\mathrm{BeV/c})^2$. Within experimental error it agrees with the differential cross section for elastic pp scattering.

## 6. INELASTIC SCREENING

The Glauber theory directly takes into account only elastic rescattering by nucleons, it being assumed that if the incident particle goes into any excited state (for example, a many-particle state), it is knocked out of the beam. However, at high energies inelastic rescattering is possible: diffraction excitation of the particle occurs in the first nucleon and a shower is produced which then is absorbed by the second nucleon (Fig. 9). Inelastic rescatterings have been discussed by several authors[30-33]. This effect can be observed only at rather high energies. One nucleon cannot absorb a momentum greater than the average momentum in the deuteron state $q^2 \sim \langle R^{-2}\rangle$, since in this case the probability that the deuteron not break up is small (in other words, the form factor drops sharply for $q^2 \gtrsim \langle R^{-2}\rangle$). On the other hand, the minimum momentum which the incident particle must transfer in order to be excited to a state with mass w is

$$\tau_w = (w^2 - \mu^2)^2/4E^2,$$

where $\mu$ and E are the mass and energy of the incident particle in the laboratory system. Therefore inelastic



a b c

FIG. 9. Feynman diagrams describing elastic scattering by a deuteron: a) single scattering, b) elastic double rescattering, c) rescattering with formation of particles in an intermediate state.

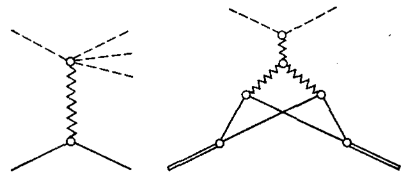


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FIG. 10. a) Reggeon mechanism of particle production at high energies; b) reggeon diagram for interaction of a particle with a deuteron at ultrahigh energies [41].

screening associated with the virtual formation of a particle with mass w is important for

$$E \geqslant (1/2)(w^2-\mu^2)[\langle R^{-2}\rangle]^{-1/2}.$$

For example, if the incident particle is a $\pi$ meson and the virtual particle is a $\rho$ meson, then inelastic screening can in principle begin to be felt already at energies of several BeV. For a quantitative estimate of inelastic screening, data are necessary on the inelastic interactions with nucleons, in particular, on the average phase shift of the amplitude for production of virtual particles. If we assume that, as in the elastic case, this amplitude is pure imaginary, we obtain the simple result[33]:

$$\Delta = 2\int \rho(4\tau)(d\sigma/d\tau)\,d\tau; \qquad (11)$$

here, in contrast to Eq. (3), $d\sigma/d\tau$ is the total cross section for all interactions of the incident particle with transfer to the nucleon of momentum $\mathbf{q}$, $\mathbf{q}^2 = \tau$:

Recently the idea has been advanced (see, for example, the work of Ter-Martirosyan[34]) that inelastic processes at high energies can be separated into two main groups. The first group consists of processes in which each of the colliding particles transfers with small momentum transfer to a state with a small mass (in comparison with the energy). These processes, often called quasielastic, are in many respects analogous to elastic scattering, especially in cases where excitation of the particles occurs without change of the quantum numbers and can be due to exchange of a vacuum Regge pole. Here the differential cross section has a diffraction nature with a clearly expressed exponential peak at $\tau = \tau_W$. It is natural to suggest that the amplitude of this diffraction excitation is pure imaginary, although direct measurement of the amplitude phase shift is impossible. However, the main contribution to the total inelastic cross section is apparently from processes belonging to the second group, in which the momenta of the particles produced are distributed uniformly (processes of the "comb" type). Nevertheless the main contribution to screening of nucleons is from elastic and inelastic diffraction processes, since, in contrast to comb processes, they are concentrated almost completely in the region of small $\tau$.

In order to estimate the contribution of diffraction inelastic processes to the cross-section defect, we will substitute into Eq. (11) the form factor in exponential form and the cross section in the form

$$d\sigma/d\tau = \int_\mu dw\,A(w)\,\sigma(w)\exp[-A(w)(\tau-\tau_w)], \qquad (12)$$

where $\int dw\,\sigma(w) = \sigma_{dif}$ is the combined cross section for all diffraction processes of interaction of the incident particle with a nucleon in which the nucleon is not excited. As a result,

$$\Delta = 2\int_\mu dw\,\sigma(w)\frac{A(w)}{4a+A(w)}\exp(-4a\tau_w) = 2\sigma_{dif}\frac{\overline{A}}{4a+\overline{A}},$$

where $\overline{A}$ is the average value of the slope in the inelastic cross section. The exponential under the integral sign, naturally, cuts off the integral over w.

The effect of inelastic screening in scattering by deuterons has been evaluated numerically in several papers[32,35,36] (for pd scattering) and by Gurvits and Marinov[37] (for $\pi$d scattering). Pumplin and Ross[32] directly utilized data[38] obtained by detection of protons in the reaction p + p → p + X. If the inelastic reaction for small $\tau$ is completely described by the diffraction mechanism, the contribution due to it to $\Delta$ at an energy of 30 BeV can reach 30% of the elastic contribution. The effect of inelastic screening in pd scattering was calculated subsequently by Anisovich et al.[35] These authors used another method of extrapolating the data on the inelastic cross section to the region of large w, and also used newer experimental data. Their result is that inelastic screening amounts to 25—30% of the elastic screening in the energy interval 15—30 BeV, and changes little in this interval. Kaĭdalov and Kondratyuk[36], in calculation of inelastic screening, utilized the quantitative evaluations of the contributions of various mechanisms to the total cross section obtained by Kaĭdalov[39] on the basis of analysis of the experimental data[38] by means of the theory of complex angular momenta. If we assume that diffraction excitation is described by exchange of a vacuum pole (Fig. 10a), we can extrapolate the data on the reaction p + p → p + X for small $\tau$ and $w \lesssim 2$ BeV to the high-energy region. As a result for k = 20 BeV/c for proton scattering the contribution of diffraction excitation to the cross-section defect $\Delta_1$ amounts to ~ 12% of the elastic contribution $\Delta_{el}$. This ratio slowly rises with increasing energy, as a result of decrease in $\tau_W$ for fixed w, reaching ~ 16% at k ~ 100 BeV/c. We note again that when data on the probability of inelastic processes are used to estimate the value of $\Delta_1$, the amplitude of the reaction p + p → p + X is assumed to be pure imaginary.

In regard to processes of the comb type, there is as yet no way in which the average phase of the production amplitude can be predicted. The corresponding contribution to the screening $\Delta_2$ can be negative if the average phase shift is small and can be maximal if the amplitude is pure imaginary (this, however, is very doubtful). In the latter case $\Delta_2$ can be of the order of $\Delta_{el}$. Kaĭdalov and Kondratyuk[36] noted also that $\Delta_2 \sim \langle R^{-3}\rangle$, and not $\langle R^{-2}\rangle$ as for the Glauber term, and also for $\Delta_1$. This is explained by the fact that the cross section for a process of the comb type is distributed almost uniformly over the entire region of secondary masses w, and therefore the cutoff of the deuteron form factor is important both in the transverse and longitudinal components of the momentum transfer.

Gurvits and Marinov[37] used data on the $\pi^-$p interaction obtained with the large hydrogen bubble chamber at

CERN[40] to estimate the effect of inelastic screening in $\pi d$ scattering. For k = 16 BeV/c these authors found partial cross sections for processes of the form $\pi^- p \to p\pi^- + m\pi$ (m = 0, ..., 5) and distributions in momentum transfer to the proton $d\sigma_m/d\tau$ which have a clearly expressed exponential peak forward. The integral in Eq. (12) was replaced by a summation over the number of $\pi$ mesons m. The process $\pi^- p \to p\pi^-\pi^-\pi^+$ can occur by a diffraction mechanism, has a large value of the product of the slope by the cross section, and gives the principal contribution to inelastic screening. Altogether at 16 BeV/c $\Delta = \Delta_{el} + \Delta_1 \approx 2$ mb ($\Delta_{el} = 1.4$ mb). The necessary data on inelastic $\pi p$ interactions at higher energies do not yet exist, and therefore there is at present no possibility of interpreting the data obtained at Serpukhov on the value of $\Delta$ for energies up to 60 BeV.

Some arguments relating to the energy dependence of $\Delta$ have been given by Kancheli and Matinyan[41]. These authors suggested that the amplitude for interaction of the incident particle with two nucleons in the deuteron is described by a diagram of the form of Fig. 10b. If the duality hypothesis is valid, the scattering of a reggeon by a particle on the average is satisfactorily described by single reggeon exchange also for not very large energies w. Since only the integral over w enters into the result, in considering the various reggeons we can investigate the asymptote of $\Delta$. The authors reached the conclusion that there is a logarithmic variation of $\Delta$ which possibly reaches a maximum at an energy of the order of several tens of BeV.

In conclusion we should note that the rather accurate experimental determination of the value of $\Delta$ can become a valuable source of information on the inelastic interaction of the incident particle with a nucleon, and in particular on the relation of the different mechanisms for particle production and on the average phase shift of the production amplitudes.

## 7. BASIC ASSUMPTIONS AND ACCURACY OF THE GLAUBER THEORY

In conclusion we will dwell briefly on the basic assumptions and initial relations of the Glauber theory. In this way we will obtain an idea of the theoretical accuracy of the results and also of what kind of questions can be answered by more accurate measurements.

The Glauber theory of interaction of high-energy particles with nuclei[1-4] is based on an analogy with the propagation of light in a nonuniform medium. It is assumed that the energy of the particle is so large that its wavelength is small in comparison with the radius of interaction of the particle with a nucleon, which will subsequently be denoted by $r_N$ ($r_N^2 \sim A$). Here a large number of partial waves contribute to the amplitude of interaction with the nucleon, and scattering by the nucleon occurs mainly at small angles. Only processes with small energy and momentum transfer are discussed: $\Delta k/k \ll 1$, $\Delta E/E \ll 1$.

The principal assumptions of the theory are as follows:

a) It is assumed that during the time of interaction of the incident high-energy particle with the nuclear system, the nucleons in it are at rest (the nucleons are frozen), i.e., $v_N/v \ll 1$, where v is the velocity of the particle and $v_N$ is the characteristic velocity of the nucleons in the nucleus.

b) Deviations from geometrical optics are neglected, i.e., it is assumed that the particle is propagated through the nucleus along a straight line. This requires that the condition $kr_N^2 \gg R$ be satisfied, where k is the incident-particle momentum and R is the nuclear radius.

c) The transfer of longitudinal momentum is neglected, which is satisfactory only in scattering by very small angles. This assumption is not fundamental. Equations can be obtained which take into account also the longitudinal momentum transfer, but they are unwieldy and it is difficult to work with them.

d) A collision between the incident particle and an intranuclear nucleon is assumed to be the same as if the nucleon were free.

e) The phase additivity rule is assumed valid: the phase shift of a wave describing a particle passing through the nucleus is equal simply to the sum of the phase shifts at the individual nucleons. This rule is based on analogy with optics and quasiclassical theory and clearly cannot be exact. It is violated, for example, in the inclusion of inelastic processes in the intermediate state (see Chapter 6) and of three-particle forces.

If the incident-particle energy is not very high (up to several BeV) and we can neglect inelastic processes in the intermediate state, then Eq. (1) is obtained also from the relativistically covariant formalism in discussion of the Feynman diagrams of Fig. 9b[30,31,33]. For small momentum transfers the integration over the zero components of the momenta in the expression for the diagram of Fig. 9b is easily carried out, after which we obtain the equation (for simplicity it is written out for forward scattering)

$$M^{(2)} \sim \int \frac{\Phi(\mathbf{q})\,\Phi(\mathbf{q}_1+\mathbf{q})\,f_n f_p}{-2\mathbf{kq}+q^2+(E/m)(2q_1^2+q^2+2\mathbf{qq}_1)-i\eta}\,d\mathbf{q}\,d\mathbf{q}_1; \qquad (13)$$

here E is the total energy of the incident particle, $\mathbf{q}$ is the momentum of the nucleon in which the first collision occurs in the system in which the deuteron is at rest, $\mathbf{q}_1$ is the momentum transferred in the first collision, m is the nucleon mass, $\Phi(\mathbf{q})$ is the deuteron wave function in the momentum representation, $f_n$ and $f_p$ are the amplitudes for scattering by the neutron and proton, which are, generally speaking, taken off the mass shell. (We note that Eq. (13) is derived particularly simply if the nuclear diagram technique is used[42].)

The Glauber result is obtained if the propagator of the fast particle between collisions, which occurs in the denominator of the integrand of Eq. (13), is reduced to an eikonal form. For this purpose it is necessary to neglect small terms $(E/m)(2q_1^2 + q^2 + 2\mathbf{qq}_1)$ and $q^2$ in comparison with $2\mathbf{kq}$. The first of these terms corresponds to the nucleon recoil in virtual scattering, and its rejection is equivalent to the assumption of frozen nucleons. Discarding the $q^2$ term corresponds to the transition to geometrical optics in discussing the motion of the particle inside the nucleus. The remaining eikonal propagator of the form $(2\mathbf{kq} + i\eta)^{-1}$ allows the fast particle to move only in a straight line. If we further neglect the dependence on the virtual amplitudes $f_n$ and $f_p$ off the mass shell and the contribution to Eq. (13) of the integral in the sense of the principal value (i.e., if we limit ourselves to subtraction in the pole of the propagator), a result identical with Eq. (8) is obtained.

Estimates of the accuracy of Eq. (8) show the following. If we use the eikonal form of the fast-particle propagator, there are firm bases for assuming that the

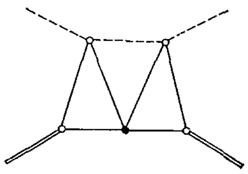

FIG. 11. Feynman diagram corresponding to the process with virtual rescattering of the nucleons.

effects of going off the energy shell and treating the integral in the sense of a principal value are compensated by the contribution of multiple scattering[43-45]. Taking into account the nonstraightness of the particle path leads to corrections of the order 10—20% in the double-scattering term for energies in the vicinity of 1 BeV[46-48]. The correction rapidly goes to zero with increasing energy. Effects due to inclusion of nucleon recoil and virtual rescattering of nucleons (see the diagram of Fig. 11) have been discussed in detail by us previously[48,49]. Inclusion of nucleon recoil leads to a correction of the order 10—20% (in the same direction as the deviation from geometrical optics), which does not decrease with increasing energy. It is interesting that the contribution from recoil effects is related in a simple way to the contribution from nucleon-rescattering effects. Furthermore, if we leave only those diagrams of the type shown in Fig. 11 in which scattering of the incident particle occurs by different particles (i.e., first by a neutron and then by a proton or the reverse), the corrections from recoil and rescattering effects are completely compensated for zero momentum transfer. In this case there remain only the corrections due to double scattering of the incident particle by the same particle (i.e., twice by the neutron or twice by the proton). For πd and pd scattering at small angles in the region 500—800 MeV, all of the corrections indicated above compensate each other practically completely. This apparently explains the success of the Glauber theory even at these low energies.

As follows from the results of Kolybasov and Kondratyuk[49], the corrections to the double-scattering amplitude lead to corrections of the following order in the observed quantities:

1) for the total cross sections the corrections are $\sim \alpha/kR$, $\alpha/mR$, $1/(kR)^2$, $1/(mR)^2$, where k is the incident-particle momentum, R is the deuteron radius, and $\alpha$ is the ratio of the real part of the amplitude for scattering of the incident particle by the nucleon to the imaginary part;

2) in addition, for the elastic-scattering differential cross sections for momentum transfers $\tau \lesssim R^{-2}$, the corrections are also of the order $(\tau R/16m)^2$, $(\tau R/8k)^2$, $(\tau/16m^2)^2$ (see also ref. 50);

3) the most appreciable corrections, $\sim 1/kR$, $1/mR$, must be expected in the polarization effects which are determined by the interference of the real and imaginary parts of the amplitude.

In conclusion the authors express their gratitude to I. S. Shapiro for discussion of the material of the review.

## APPENDIX

## THE GLAUBER FORMULA WITH INCLUSION OF CHARGE EXCHANGE AND THE REAL PART OF THE ELASTIC SCATTERING AMPLITUDE

We present here equations for the cross-section defect $\Delta$ which replace Eq. (2) in the case where the difference in the proton and neutron cross sections, the effect of charge exchange, and also the departure of the elastic-scattering amplitude phase shift from 90° must be taken into account. The charge-exchange effect was first discussed by Wilkin[51]. The real part of the amplitude is easy to take into account by proceeding from Eq. (8); this result is contained in the first paper by Glauber[1].

1. The incident particle has isospin 1/2 (nucleon, antinucleon, K meson). The charge-exchange amplitude is expressed in terms of the difference of the elastic amplitudes, for example:

$$f(K^-p \to \bar{K}^0 n) = f(K^-n \to K^-n) - f(K^-p \to K^-p). \quad (A.1)$$

Substituting this expression into the term corresponding to charge exchange in an equation similar to Eq. (8), we obtain

$$\Delta = (1/4\pi)\langle R^{-2}\rangle \{2\sigma_p\sigma_n(1-\alpha_p\alpha_n) - (1/2)[\sigma_p^2(1-\alpha_p^2)+\sigma_n^2(1-\alpha_n^2)]\}, \quad (A.2)$$

where $\sigma_p$ and $\sigma_n$ are the total cross sections for interaction of the incident particle with the proton and neutron, $\alpha_{p,n} = \mathrm{Re}\, f_{p,n}/\mathrm{Im}\, f_{p,n}$ for $\tau = 0$.

2. The incident particle has isospin 1 (π meson). Instead of Eqs. (A.1) and (A.2) we have

$$f(\pi^-p \to \pi^0 n) = (1/\sqrt{2})[f(\pi^-n \to \pi^-n) - f(\pi^-p \to \pi^-p)],$$
$$\Delta = 1/4\pi\langle R^{-2}\rangle \{(3/2)\sigma_p\sigma_n(1-\alpha_p\alpha_n) - (1/4)[\sigma_p^2(1-\alpha_p^2)+\sigma_n^2(1-\alpha_n^2)]\}.$$

The corrections to Eq. (2) due to the charge-exchange effect are of the order $\rho^2$, where $\rho = (\sigma_p - \sigma_n)/\sigma_p$ (if we neglect the unknown quantity $\alpha_p - \alpha_n$). The value of $\rho$ can be found comparatively accurately for $\pi^-$ mesons (since $\sigma(\pi^-n) = \sigma(\pi^+p)$), and also for neutrons from the data of neutron experiments. For other particles it is necessary in calculation of $\rho$ to use the theoretical formula for the Glauber correction

$$\rho \approx (\sigma_d/\sigma_p) - 2 + (4\pi)^{-1}\langle R^{-2}\rangle \sigma_p (1-\rho). \quad (A.3)$$

For $\pi^-$ mesons $\rho \approx 0.3$ at k = 5 BeV/c and falls off slowly with increasing k, so that for k > 10 BeV/c $\rho < 0.1$. For protons $|\rho| < 0.05$ at k > 4 BeV/c. An evaluation by means of Eq. (A.3) gives $|\rho| < 0.1$ for K mesons for momenta exceeding several BeV/c.

The corrections due to the quantity $\alpha$ are roughly of the same scale. For protons the value of $\alpha_p^2$ is maximal for k ~ 5 BeV/c and then falls off slowly with increasing energy. For high energies $\alpha_p^2$ is in general small for all particles. This quantity takes on its largest value for $K^+p$ scattering: for k ~ 10 BeV/c $\alpha_p^2 \approx 0.16$ (see, for example, ref. 52 and also the report by Giacomelli[53]).

[1]It should be noted that the experimental accuracy of the inequality $\sigma(pp) = \sigma(pn)$ for energies up to 30 BeV where it has been measured is 1—1.5 mb (the value of $\Delta$ for pd scattering is 3—3.5 mb). The errors in the quantity $\langle R^{-2}\rangle$ in Fig. 1 do not take into acocunt the possible difference $\sigma(pp)-\sigma(pn)$ for k > 30 BeV/c.

[2]The value of the parameter A in Eq. (5) is given in the table for various processes (for example, see the review article by Lasinski et al. [11]).

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