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From the Current Literature

DOES AN ELECTRON FALL IN A METALLIC PIPE?

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THE question posed in the title may be restated as: With what acceleration does an electron move inside a vertical pipe far from the ends of the pipe or inside any cavity enclosed by a metallic shield if the whole system is situated in a gravitational field? In other words, does any force besides the gravitational force mg act on the electron under these conditions? This question first arose in connection with the setting up of experiments to observe the free fall of elementary particles in the gravitational field (the ultimate aim of such experiments was to verify that antiparticles possess normal gravitational properties). On the electron scale the free fall acceleration $g = 980\ \text{cm/sec}^2$ is very small: the same acceleration is imparted to the electron by a field of magnitude mg/e (m is the mass of the electron and e is the absolute value of its charge), i.e., only 5.6×10^{-13} V/cm. The random electric fields in an instrument for the observation of the free fall of electrons can be by many orders higher. Therefore the trajectory of the falling electron should pass through a region enclosed by a metallic shield, e.g., inside a long metallic pipe. It is important to know whether besides gravity another force is exerted on the charged particle inside the pipe by the electric field which arises because the shield itself is also located in the gravitational field.

This electric field was first calculated by Schiff and Barnhill[1]*. They arrived at the following general conclusion: an electrostatic field equal to mg/e arises in a shielded region of space. It is directed downwards and acts on an electron (negative charge) with a force which balances gravity exactly, so that the acceleration of the electron is equal to zero. A positron should by the same token fall with an acceleration of 2g.

However, Dessler, Michel, Rorschach and Trammel[2] have drawn attention to the fact that Schiff and Barnhill groundlessly neglected in their calculation of the electric field the deformation of the metal under the action of its own weight. They show that this deformation leads to the appearance of a field which is larger than mg/e by several (roughly five) orders of magnitude. In order to understand their idea better, let us first consider a simpler problem: let us find the field inside the metal itself.

Electrons in a metal fill all the energy levels up to the Fermi energy μ. Deformation of the metal changes the magnitude of this energy (see below). But a deformation arising under the action of gravity is non-uniform: it varies with height. For example, if a metallic rod is clamped at the lower end and the upper end is free, then the compression decreases with height, and this leads to the appearance of a vertical gradient in the electron Fermi energy. Electrons overflow from the region of larger Fermi energy to the region of smaller one, as a result of which the upper end of the metal becomes charged relative to the lower end and an electric field appears in the metal and decreases, as it grows, the current generated by the deformation. The field attains just a value that neutralizes the effect of the gradient μ and reduces the electric current to zero.

To understand the possible mechanism underlying the change in the Fermi energy in a deformation and estimate the magnitude of the effect, let us consider the simplest model according to which the metal is a degenerate electron gas in the field of a homogeneous positive charge. The Fermi energy of such a gas is determined by the electron concentration n and is proportional to $n^{2/3}$. Since the relative change in volume associated with a deformation is equal to $dV/V = -dn/n$,* the derivative in this model of the Fermi energy with respect to deformation is found to be equal to $d\mu/(dV/V) = -(2/3)\mu$. In the general case the Fermi energy changes further, owing to the fact that the deformation modifies the intracrystalline field in which the electrons move. However, the characteristic absolute value of the effect remains the same as in the simplest model. If we denote the strain tensor by u_{ik}, then the derivatives $\bar{\lambda}_{ik} = \partial\mu/\partial u_{ik}$ (they are strain potentials averaged over the Fermi surface) are, in absolute value, of the order of the Fermi energy μ, or, which is the same thing, of the order of the atomic energy e^2/a, where a is the interatomic distance, i.e., one–ten electron volts.

Under conditions of thermodynamic equilibrium the electrochemical potential of the electrons which is equal to the sum of their chemical potential (the Fermi energy) μ and their potential energy in the electric and gravitational fields should be constant throughout the metal:

$$\mu(\mathbf{r}) - e\varphi_i(\mathbf{r}) - m\mathbf{g}\mathbf{r} = \text{const}. \quad (1)$$

In this formula $\varphi_i(\mathbf{r})$ is the potential of the macroscopic electric field inside the metal. From the equilibrium condition (1) follows that the field inside the metal is equal to

$$\mathbf{E}_i = -\operatorname{grad}\varphi_i = \frac{1}{e}(m\mathbf{g} - \operatorname{grad}\mu). \quad (2)$$

Let us choose a system of coordinates with the OZ axis directed vertically upwards. The field produced by the deformation is equal to

$$-\frac{1}{e}\frac{d\mu}{dz} = -\frac{1}{e}\bar{\lambda}_{ik}\frac{\partial u_{ik}}{\partial z}. \quad (3)$$

*We must make a qualification here. In [1] the field outside the metal was investigated for the first time; the analogous field arising inside the metal under the influence of acceleration or gravitation has, however, been known for a long time. It was considered in connection with the experiments by Tolman and his co-workers on the observation of the electron-inertial effect. We shall briefly touch upon this question below.

*It must be borne in mind that neutrality is not destroyed inside the metal. Charge can appear only in a thin surface layer, whose thickness in a metal is of the order of the interatomic distance.

Let us determine the derivatives $\partial u_{ik}/\partial z$ with the aid of the equations of elastic equilibrium. As is well known, in an elastically isotropic metal, located in the gravitational field,

$$\frac{\partial u_{zz}}{\partial z}=\frac{\rho g}{E_Y},\quad \frac{\partial u_{xx}}{\partial z}=\frac{\partial u_{yy}}{\partial z}=-\frac{\sigma}{E_Y}\rho g, \tag{4}$$

where ρ is the density of the metal, E_Y Young's modulus, and σ the Poisson coefficient. The density ρ is of the order of M/a^3, where M is the mass of the nucleus of an atom of the metal. Young's modulus is $E_Y \sim 10^{12}$ dynes/cm^2, i.e., of the order of e^2/a^4 (this literal estimate follows from the fact that a strain of the order of unity would give rise to a stress of atomic magnitude). It follows from (3) and (4) and the estimates we have just made that the field produced by the deformation is

$$\frac{1}{g}\,|\,\mathrm{grad}\,\mu\,| \sim \frac{1}{e}\frac{e^2}{a}\frac{(M/a^3)\,g}{(e^2/a^4)} \sim \frac{Mg}{e}. \tag{5}$$

This field is stronger than the one we would obtain if no allowance were made for the deformation of the metal, i.e., stronger than mg/e by roughly a factor of M/m $\sim 10^5$. Notice further that the field is determined not by the strain itself but by its derivative with respect to height and does not therefore depend on where the metal is clamped—at the upper or lower end.

Before proceeding to find the electric field outside the metal, let us recall how the work function for an electron in a metal is related to the Fermi energy in the metal. Let us denote by φ_e the potential of the electric field outside the metal. The difference $\varphi_e - \varphi_i$ is equal to the discontinuity of the potential across the surface double layer. Such a double layer (dipole moment) exists even on a perfectly clean metallic surface owing to the fact that the "center of gravity" of the charge of the electrons in the first (from the surface) primitive cell of a metal does not necessarily lie in the plane passing through the nucleus. The jump in the potential $\varphi_e - \varphi_i$ can vary with adsorption of different molecules, with changes in the occupation of the surface electronic states (e.g., in an oxide film). If the density of the double layer were constant, the change in the work function ΔW would be equal to minus the change in the Fermi energy $-\Delta\mu$. In the general case

$$\Delta W = -\Delta\mu - e\Delta(\varphi_e - \varphi_i). \tag{6}$$

In a state of thermodynamic equilibrium the sum of the work function W and the potential energy of the electron taken with the opposite sign should be constant over the entire surface of the metal:

$$W + e\varphi_e(\mathbf{r}) + mgr = \mathrm{const}. \tag{7}$$

This condition follows from (1) and (6). It follows from this condition that the field outside the metal (outside the double layer) is equal to

$$\mathbf{E}_e = -\,\mathrm{grad}\,\varphi_e = \frac{mg}{e} + \frac{1}{e}\,\mathrm{grad}\,W. \tag{8}$$

The variation of the work function of a metal with height and the corresponding field (grad W)/e (this field was first considered in[2]) are due to the deformation of the metal under the action of gravity. This field is similar to the one that develops in the neighborhood of the surface of contact between two bodies having different work functions. Formally, the only difference between them is that the nonuniformity in the work function, which gravity gives rise to, is much smoother.

It is not possible to estimate the field (grad W)/e outside the metal with the same degree of definiteness as we can the field inside the metal, because it is not known how the surface dipole moment (or the discontinuity in the potential on crossing the surface $\varphi_e(\mathbf{r}) - \varphi_i(\mathbf{r})$) varies with the strain. Naturally, the field outside the metal can attain roughly the same magnitude as the field inside it ($\sim$ Mg/e), but it can also be much smaller. It is known that the contact fields near the interfaces between different bodies and near the edges of crystals whose surfaces have different work functions, are usually compensated by the fields of adsorbed ions, by a redistribution of the electrons in the surface states, etc. The field produced by deformation may be neutralized in exactly the same way.

It should be noted that Schiff and Barnhill[1] calculated the field in a screened region of space by a different method. They proceeded from a general expression they had obtained relating the difference $\Delta_g[\varphi(\mathbf{r}_1) - \varphi(\mathbf{r}_2)]$ between the electrostatic potentials at two points caused by gravity to the change in the mass moment $M_Z = \int z\rho(\mathbf{r})dV$ of the system ($\rho(\mathbf{r})$ is the mass density) when a test charge q moves from one point to the other $(\Delta_q M_Z)_{1\to 2}$. Indeed, if $(\Delta F)_{1\to 2}$ is the change in the free energy of the system when the test charge moves from $\mathbf{r}_1$ to $\mathbf{r}_2$, then

$$\Delta_g[\varphi_e(\mathbf{r}_1) - \varphi_e(\mathbf{r}_2)] = g\left(\frac{\partial[\varphi_e(\mathbf{r}_1) - \varphi_e(\mathbf{r}_2)]}{\partial g}\right)_{g=0} = g\left[\frac{\partial^2(\Delta F)_{1\to 2}}{\partial g\,\partial q}\right]_{g=q=0}.$$

On the other hand, the change in the mass moment linear in q is equal to

$$(\Delta_q M_z)_{1\to 2} = q\left(\frac{\partial(\Delta M_z)_{1\to 2}}{\partial q}\right)_{q=0} = q\left(\frac{\partial^2(\Delta F)_{1\to 2}}{\partial q\,\partial g}\right)_{g=q=0}.$$

Comparing the two equalities, we find that the electric field induced by gravity is equal to

$$E_e = -\frac{g}{q}\frac{\partial}{\partial z}(\Delta_q M_z). \tag{9}$$

The two expressions for the field E_e ((8) and (9)) follow from the general conditions of thermodynamic equilibrium and should therefore follow from each other, as Herring has proved in his very excellent paper[3]. He also very graphically demonstrates in this paper how the linear—in q—deformation of the lattice of the metal arises under the action of a test charge q located near its surface. That part of the change in the mass moment of the metal ΔM_Z when the test charge moves, which is connected with the drift of the deformation produced by the charge (it was neglected in[1]), makes a contribution to the right hand side of (9) exactly equal to (grad W)/e. The other part is connected with the motion of the image charge and makes a contribution equal to mg/e, since this charge is produced by electrons only.

Let us suppose that an electron falls from a height h and that part of its path passes through a metallic pipe. The total increase in its kinetic energy is, of course, equal to mgh, independent of the magnitude of the electrostatic field which is established inside the pipe: the change in the potential in the "interior" part of the pipe is canceled by the changes in the potential near its ends. However, the field influences the time of flight of the electron.

It would be interesting to compare now the conclusions of the theory with experiment. A very important and difficult experiment was performed by Witteborn and Fairbank[4]. They measured the force that acts on an electron moving in a vacuum inside a vertical metallic pipe. The distribution of electrons emitted from a cathode was directly measured in terms of their times of flight through the whole pipe and from it the force accelerating the electrons inside the pipe was found. To increase the accuracy of the experiment a small voltage was applied to the pipe (a weak current was passed through it), which produced an additional field E_a inside it. The total force accelerating the electrons was measured as a function of the auxiliary field E_a, which was, in particular, allowed to go to zero. It turned out that the total force acting on an electron inside the pipe was not greater than 0.09 mg (of course, in the absence of the auxiliary field). This means that within the limits of the accuracy of the experiment the weight of an electron is balanced by the field existing inside the pipe. Thus, to the question posed in the title of the present review, the Witteborn-Fairbank experiment gives the answer: no it does not fall; it moves by inertia.

The result of the experiment is in good agreement with the original conclusion drawn by Schiff and Barnhill[1]. The electric field, which the deformation of the metallic shield gives rise to and which is roughly by five orders of magnitude stronger than mg/e inside the metal, is, for some reason, smaller than 0.09 mg/e outside the metal.

The discrepancy between experiment and the theory of Dessler et al.[2] has stimulated experimenters to set up direct experiments to measure the effect of deformation on the work function of metals. In the instrument constructed by Beams[5] a metallic cross-shaped rotor revolved at 650 rps, so that the acceleration at the periphery of the rotor attained a value of 10^5 g. The potential differences at various points of the revolving rotor were measured with the aid of capacitive probes located above the rotor at different distances from the axis. The idea was that the radial tensile strain caused by the rotation increased with distance from the axis and therefore a contact potential difference should arise between the axis of the rotor and its periphery. Although Beams does not quote quantitative results, he indicates that the observed magnitude of the effect agrees with the theory of Dessler et al.[2].

Craig[6] as well as French and Beams[7] measured (by the vibrating-electrode method) the variation of the work function of a number of metals and metallic alloys as they were uniformly stretched or compressed. In the region of elastic deformations the work function in all the cases decreases during compression and increases during extension by an amount of the order of $10^{-6}-10^{-5}$ eV per kg/cm^2. This means that under the conditions of the experiments[5-7] the change in the work function is of the same order as the change in the Fermi energy inside the metal. It is interesting to note that in the region of elastic deformations the sign of the effect turned out to be as expected on the basis of the simplest model of a metal as a homogeneous degenerate gas.

Thus, the result of the Witteborn-Fairbank experiment, i.e., the fact that no field produced by the vertical gradient of the work function was observed, has not been explained to date. A number of hypotheses has been put forward[6,8,9,14], but not one has yet been validated. Furthermore, attention is drawn in the literature to the differences between experimental setup by Witteborn and Fairbank, on the one hand, and the setup of the subsequent experiments[5-7] on the other. Thus, Harrison[10] thinks it significant that in the first experiment the deformation was nonuniform (in contrast to the experiments[6,7]) and very small (in contrast to the deformation in Beam's experiment[5]). Attention should also be drawn to the fact that under the conditions of the experiment[4], i.e., after cooling the whole apparatus to the temperature of liquid helium, the fields, usually due to junctions of facets of metal crystallites having different work functions (the metal of the pipe was a polycrystal) practically completely vanished on the axis of the metallic pipe in which the beam of moved. But had these fields not been neutralized (as the authors of[4] suppose, by the adsorption of residual gases), they would have been even stronger than the field which gravity could have produced. Evidently, the investigations aimed at the elucidation of the Witteborn-Fairbank experiment will prove to be useful for the physics of metallic surfaces.

There is an obvious analogy between the above-considered problem of the electric field which arises in a conductor under the action of a gravitational field of acceleration g and the problem of the field in an accelerated conductor. As is well known, the acceleration gives rise to a current in the conductor, and this effect or its inverse—the acceleration of the conductor when the current flowing through it is varied—is observed in the so-called electron-inertial experiments. For their analysis an extraneous field, which would produce the same current and acceleration, is introduced. It is well known that the extraneous field (the Tolman-Stewart field) is, irrespective of the type of conductor, equal to $\mathbf{E}_{TS} = (m/e)a$, where a is the acceleration, and m and $-e$ are the mass and charge of the free electron (see, for example,[11,12]). But this expression was derived without making any allowance for the deformation of the conductor which inevitably arises when the conductor is accelerated. The role of the deformation has been investigated by V. L. Ginzburg and the present author[13]. It turns out that although the deformation during acceleration does produce a field $E_d^{(1)} = e^{-1}\nabla\bar{\lambda}_{kl}u_{kl}$, which in the general case exceeds $\mathbf{E}_{TS}$ roughly by a factor of M/m, this field does not, in virtue of its potential character, make any contribution to the emf which arises in a circuit of an accelerated metallic conductor, and does not, therefore, affect the current in the circuit, which is just what is measured in electron-inertial experiments. A current can be excited by only the other part of the "deformation" field, which is, in contrast to $\mathbf{E}_d^{(1)}$, determined by the rate of change of the deformation not only in space, but also in time. It is interesting to note that in those experiments in which the effect is associated with the nonuniform rotation of a circular ring or coil (and only such experiments have, thus far, been done), the contribution of the deformation is either equal to zero (in the original Tolman-Stewart experiment in which they varied the total charge which flowed through the circuit during the whole period of deceleration of the metallic ring), or it is small in comparison with the ordinary effect. It is not inconceivable, however, that an experiment could be set up in which the contribution of the deformation to the observable current would not be small (for details, see[13]).

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