539.12

HYPERNUCLEI AND ΛN INTERACTION

B. A. KHRYLIN

Usp. Fiz. Nauk 105, 185—208 (October, 1971)

CONTENTS

1. INTRODUCTION

THE physics of hypernuclei is very young. However, within less than twenty years much information of exceptional interest for elementary-particle physics and nuclear physics has been accumulated. This information is not only of interest but also necessary to supplement our notions concerning baryon-baryon interactions. On the other hand, the Λ hyperon is a unique test body for nuclear physics.

Hypernuclei are bound states of baryons, of which at least one is a hyperon, i.e., has non-zero strangeness. Were the hyperons to decay only as a result of weak interaction, their approximate lifetime in the presence of protons and neutrons would be approximately the same as in the free state ($\sim 10^{-10}$ sec). In the atomic nucleus, however, most hyperons, with the exception of the Λ particle (the hyperon having the smallest mass) vanish in fast reactions with nucleons

$$\Sigma^- + p \rightarrow \Lambda + n, \quad \Sigma^+ + n \rightarrow \Lambda + p, \qquad (1.1)$$
$$\Xi^- + p \rightarrow \Lambda + \Lambda, \quad \Xi^0 + n \rightarrow \Lambda + \Lambda.$$

It is not surprising that the first hypernuclei to be discovered, and the only ones known to this day, are those containing only the Λ particle. We shall therefore call hypernuclei with Λ particles simply hypernuclei. On the nuclear scale, where the time is measured by the period required for light to cover a distance equal to the diameter of the atomic nucleus ($\sim 10^{-23}$ sec), the free Λ particle lives a very long time (Table I). In this sense hypernuclei, which have approximately the same lifetime ($\sim 10^{-10}$ sec), are stable.

Table I. The Λ hyperon

Isotopic spin	$T = 0$
Spin, parity	$J^P = 1/2^+$
Mass, MeV	1115
Lifetime, sec	$2.6 \cdot 10^{-10}$
Decay mode (%)	$p\pi^-$ (68,4)
	$n\pi^0$ (31,6)
	$pe\nu$ ($0{,}9 \cdot 10^{-3}$)
	$p\mu\nu$ ($1{,}5 \cdot 10^{-4}$)
Magnetic moment, nuc. magneton	—0.75

The discovery of hypernuclei was quite unexpected. Although their possible realization was not disputed by anyone, it was also not discussed seriously. In 1952, during a study of cosmic rays with the aid of photographic emulsions, the Polish physicists M. Danysz and E. Pniewski obtained an unusual photograph (Fig. 1). A cosmic particle p collided at the point A with an atomic nucleus of the emulsion, which decayed to form a multiprong star. One of the heavy fragments, f, covered a considerable distance and decayed at the point B into three charged particles and a certain number of neutrons, which, having no charge, left no tracks in the emulsion. The measurements of the tracks have shown that the fragment f had for approximately 10^{-12} sec an energy of 95 MeV (or more, if neutrons were released during its decay). In order to catch one nucleon from the nucleus, it is necessary to consume an approximate energy 8 MeV. Therefore, if the atomic nucleus acquires an energy 95 MeV, it disintegrates within a time much shorter than 10^{-12} sec. *It was the interpretation of this* case that led Danysz and Pniewski to the hypothesis that the fragment f is the system we now call the Λ hypernucleus.

In analogy with the notation used in nuclear physics, the symbol used for hypernuclei is Z^A_Y, where Z is the

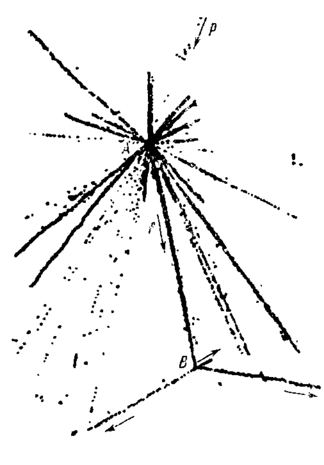

FIG. 1. Emulsion photograph of the first hypernucleus.

charge of the hypernucleus, A the total number of particles, both nucleons and hyperons, and the subscript designates the hyperons contained in the hypernucleus. Such a notation was adopted on historic grounds, when it was assumed during the first stage of the study of the hypernuclei that the Λ particle plays the role of a neutron, but is heavier in mass. It might have been more convenient to use A to denote only the number of nucleons.

In 1954, only eight decays of three types of hypernuclei were known. By 1960, the Λ-particle binding energies were determined with good accuracy for practically all the presently identified hypernuclei. This was followed by considerable improvement in the accuracy of the binding energy, the discovery of the two exotic hypernuclei He^8_Λ and Be^7_Λ (the nuclei He^5 and Be^6 are unstable), and many experiments on Λp scattering were performed.

The physics of hypernuclei can be broken up into two regions: the spectroscopy of hypernuclei (strong interactions) and decays of hypernuclei (weak interactions). In this review we confine ourselves to the spectroscopy of hypernuclei, assuming the particle interaction to be potential. Other aspects of the physics of hypernuclei will be mentioned only when necessary.

The status of the physics of hypernuclei was described systematically in Dalitz's reviews, but these (both the early ones, such as[1], and the later ones[2-4]) have been published in a very small number, and unfortunately are practically unavailable.

2. LOW-ENERGY Λp SCATTERING

A question of primary importance for the physics of hypernuclei is the determination of the characteristics of the ΛN interaction. The type of the nucleon-nucleon (NN) interaction is strongly limited by experiments on NN scattering. Is it possible at present to deduce similar limitations from Λp scattering?

The study of Λp scattering is a difficult experimental task. First, the Λ-particle lifetime, which is long in the nuclear scale, is too short to be able to obtain a beam of Λ particles with an accelerator. The problem is also made complicated by the fact that the Λ particle has no charge. In addition, the Λ particle is a rather rare event terminating a long chain of reactions. To obtain Λ particles it is customary to use a beam of negative K mesons and a hydrogen target. The Λ particles produced in the K^-p reaction are scattered by the target protons. The reaction and the spectrum of the obtained Λ particles are shown in Fig. 2. Seventy per cent of the Λ particles are obtained from the reaction $K^-p \to \Sigma^0\pi^0$, $\Sigma^0 \to \Lambda\gamma$. In all the experiments performed on Λp scattering, neither the beams nor the targets were polarized. This raises a serious difficulty in the interpretation of the results.

The cross section for elastic Λp scattering is similar to the cross section for elastic nucleon scattering, but is smaller by an approximate factor of 5 (Fig. 3). Owing to the scanty statistics, no study of the differential cross section was made. Instead, two ratios in the angular distribution, front-back (F/B) and pole-equator (P/E), were calculated (Fig. 4). At low energies ($p < 250$ MeV/c), $d\sigma/d\Omega$ is isotropic, and at higher energies a certain increase of F/B is observed. The low-energy Λp scattering is described by four parameters, the singlet and triplet scattering lengths (a_s, a_t) and the effective radii (r_s, r_t):

$$\sigma = \frac{\pi}{\left(-\frac{1}{a_s}+\frac{1}{2}r_s k^2\right)^2+k^2} + \frac{3\pi}{\left(-\frac{1}{a_t}+\frac{1}{2}r_t k^2\right)+k^2}; \qquad (2.1)$$

here k is the momentum in the c.m.s. of Λ and p. Expression (2.1) is meaningful if it is assumed that a pure s-wave is produced in Λp scattering. The F/B and P/E ratios show that this is satisfied at least at p = 200–240 MeV/c. In fact, the conditions are somewhat less stringent, for in the case of a large s-wave the small p-wave, by interfering, can produce a large F/B ratio while making a small contribution to the total cross section. To separate the contributions of the sing-

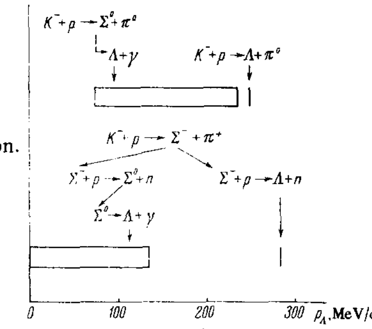

FIG. 2. Spectrum of Λ particles produced in K^-p reaction.

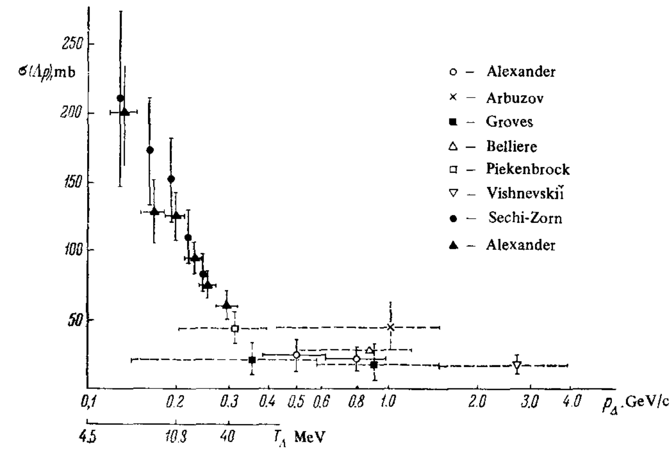

FIG. 3. Cross section for elastic Λp scattering (laboratory frame). The data are taken from [9-14,8,7].

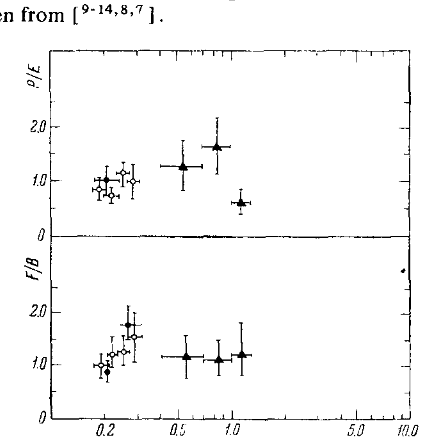

FIG. 4. Ratios F/B and P/E in the angular distribution of elastic Δp scattering.

let and triplet spin states it is necessary to employ additional considerations. To determine the scattering lengths and the effective radii, the χ^2 method is used, i.e., the expression

$$\chi^2 = \sum_{n=1}^{L} \frac{[\sigma_n^T(a_s, a_t, r_s, r_t) - \sigma_n^e]^2}{(\Delta\sigma_n^e)^2}, \qquad (2.2)$$

where $\sigma_n^e \pm \Delta\sigma_n^e$ are the experimental values of $\sigma_{\Lambda p}$ for certain values of the momentum k_n, and σ_n^T denote the values of $\Lambda_{\Lambda p}$ calculated in accordance with formula (2.1) at the same points. Since we know that (2.1) describes well the scattering at low energies, the use of the χ^2 method makes it possible to determine the scattering lengths and the effective radii. However, the results of such a procedure do not agree with one another:

	a_s	a_t	r_{0s}	r_{0t}	
1964 [5]:	−3.6	−0.53			$\times 10^{-13}$ cm
1966 [6]:	−2.46	−2.07	3.87	4.50	
1968 [7]:	−1.8	−1.6	2.8	3.3	
1968 [8]:	−2.0	−2.2	5.0	3.5	

The reasons for this disparity lies in the large experimental errors. Sets of parameters which are only insignificantly worse in the sense of χ^2 fitting, may differ strongly from the "best" values (Table II and Fig. 5). It is necessary to increase greatly the accuracy of the experiment in order to limit to some extent the region of permissible ΛN potentials.

Note the following curious fact. If the old data[5] are discarded, and the remaining results are averaged, then they turn out to fluctuate about the same values $a_s \approx a_t \approx -2.0 \times 10^{-13}$ cm and $r_s \approx r_t \approx 3.8 \times 10^{-13}$ cm.

The only conclusion that can be drawn from the "best" parameters cited by the authors is approximate equality of the singlet and triplet scattering lengths, $a_s/a_t \approx 1$. In other words, it is more likely that the singlet and triplet potentials V_s and V_t of the ΛN interaction are close. But the large errors force us to refrain from final conclusions.

The lack of neutron targets also makes a direct verification of the charge independence of the ΛN forces impossible.

3. THEORY OF ΛN FORCES

The theory of ΛN forces, as in general the meson theory of strong interactions, is far from complete and will therefore not be considered here (a detailed exposition of the present state of the theory of ΛN interactions is contained in a recently published review by Deloff[16]). Certain conclusions of the meson theory, however, have exerted a strong influence on the phenomenological analysis of hypernuclei, and these must be mentioned in order to understand the considerations frequently governing the choice of the parameters of the potentials.

Let us consider the simplest diagrams describing the ΛN interaction. The Λ particle has an isotopic spin T = 0, and the isotopic-spin conservation law prevents it from emitting one pion. The Λ particle is forced to exchange at least three pions with the nucleon (Fig. 6a) or else one heavier K meson (Fig. 6b). Such diagrams make a dominant contribution to the ΛN interaction at distances larger than $\hbar/2m_\pi c \approx 0.7 \times 10^{-13}$ cm (2π-meson radius) and $\hbar/m_K c \approx 0.4 \times 10^{-13}$ cm (K-meson radius). In the hypernucleus, the Λ particle can exchange mesons also with different nucleons (many-particle forces (Fig. 6c)). The diagrams corresponding to exchange of a larger number of mesons determine the interaction at shorter distances.

To find the form of the potential, it is necessary to take into account all the possible diagrams, and not only the indicated ones, something the theory is incapable of at the present time. In practice, therefore, one uses frequently phenomenological potentials that take into ac-

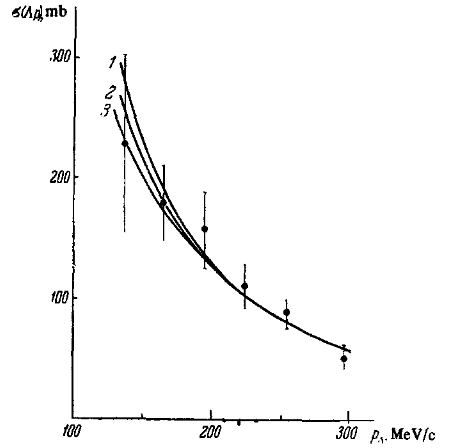

FIG. 5. Cross section for elastic Λp scattering according to Sechi-Zorn et al. [8]. A(2)—best set of parameters in accordance with the χ^2 criterion ($a_s = -2.0 \times 10^{-13}$ cm, $r_{0s} = 5.0 \times 10^{-13}$ cm, $a_t = -2.2 \times 10^{-13}$ cm, $r_{0t} = 3.5 \times 10^{-13}$ cm). The sets B(3) (pure triplet) and F(1) (weak triplet) are given in Table II.

Table II. Sets of parameters of high-energy Λp scattering, having the same χ^2 deviation from the best set[8]. The degree of their agreement with experiment is illustrated in Fig. 5

Parameter	Range of variation	Complete solution (a_s, r_s, a_t, r_t)
a_s	$0.0 > a_s > -15$	B = (0.0 0.0 −2.4 3.0)
		C = (−15.0 11.0 −2.0 3.0)
r_s	$0.0 < r_s < 15$	D = (0.0 0,0 −2.4 3.0)
		E = (0,0 15.0 −2.4 3.0)
a_t	$-0.6 > a_t > -3.2$	F = (−8.0 1.5 −0.6 5.0)
		G = (−1.0 3.0 −3.2 4.5)
r_t	$2.5 < r_t < 15$	H = (−2.0 15.0 −2.0 2.5)
		L = (−1.0 6.0 −0.8 15.0)

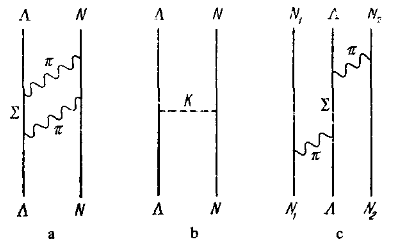

FIG. 6. Simplest diagrams of ΛN interaction.

count certain field-theoretical consideration, for example the radius of the ΛN potential is chosen to be approximately equal to the 2π-meson radius. It is also expected that the two- and three-particle potentials are comparable in magnitude, since their simplest diagrams have coupling constants of the same order of magnitude.

4. BINDING ENERGIES OF HYPERNUCLEI (EXPERIMENT)

Experiments on low-energy Λp scattering yield information that admits of a rather broad interpretation. However, even if the data were much more accurate, the class of potential satisfying the scattering would still remain quite large. Much more sensitive to the form of the potentials are the binding energies. In this connection, the determination of the binding energies of hypernuclei becomes particularly important. A similar situation exists also in nuclear physics, but the experimental material there is much more accurate.

From the point of view of the characteristics of the ΛN interaction, it is convenient to use instead of the binding energy the binding energy of the Λ particle from the hypernucleus, $B_\Lambda = M_\Lambda + M(Z^{A-1}) - M(Z^A_\Lambda)$. Table III lists the separation energies for light hypernuclei as given in[17,18]. The values of B_Λ for hypernuclei, not indicated in Table III, for example Li^{10}_Λ, are less reliable and are shown only in Fig. 7. The hypernucleus Li^6_Λ, reported by Harmsen[19], is not included in the table, since this event has been interpreted within the framework of the known hypernuclei[20]. The first striking fact is the absence of bound states of the systems Λp and ΛN, which agrees with the scattering, namely, potentials in which there are no bound states correspond to negative scattering lengths. The lightest identified hypernucleus is hypertritium H^3_Λ, and the heaviest is C^{13}_Λ.

Table III. Experimental values of the Λ-particle detachment energies B_Λ (data on C^{13}_Λ taken from[26])

Hypernucleus	Decay mode	Gajewski et al. [17]		Bohm et al. [18]		Combined results	
		Number of events	$B_\Lambda \pm \Delta B_\Lambda$, MeV	Number of events	$B_\Lambda \pm \Delta B_\Lambda$, MeV	Number of events	$B_\Lambda \pm \Delta B_\Lambda$, MeV
H^3_Λ	π^-He^3	26	0.13±0.15	86	+0.05±0.08	112	0.07±0.07
	$\pi^-H^1H^2$	6	0.33±0.21	16	−0.11±0.13	22	0.01±0.11
	Total	32	0.2 ±0.12	102	0.01±0.07	134	0.06±0.06
H^4_Λ	π^-He^4	208	2.26±0.07	552	2.29±0.04	760	2.28±0.03
	$\pi^-H^1H^3$	21	1.86±0.10	63	2.08±0.06	84	2.02±0.05
	$\pi^-H^2H^2$	2		7		9	
He^4_Λ	$\pi^-H^1He^3$	48	2.20±0.06	127	2.36±0.04	175	2.31±0.03
	$\pi^-H^1H^1H^2$	1		3		4	
He^5_Λ	$\pi^-H^1H^4$	288		724		1012	
	$\pi^-H^2H^3$	2	3.08±0.03	10	3.08±0.02	12	3.08±0.02
	$\pi^-H^1H^1H^3$			1		1	
He^6_Λ	$\pi^-H^2He^4$	4	4.09±0.27	7	4.38±0.19	11	4.28±0.15
He^7_Λ	π^-Li^7	5					
	$\pi^-H^1He^6$	3	4.67±0.28				
	$\pi^-H^3He^4$	2		1	6.09±0.54		Не усреднено
	$\pi^-H^1H^3H^3$			1	3.75±0.28		
Li^7_Λ	$\pi^-H^1Li^6$	2		1		3	
	$\pi^-He^3He^4$	16	5.46±0.12	32	5.60±0.07	48	5.57±0.06
	$\pi^-H^1H^2He^4$	9		7		16	
Be^7_Λ	$\pi^-H^1H^1H^1He^4$	4	4.81±0.53	7	5.06±0.19	11	4.91±0.16
			4.70±0.52				
			4.24±0.46				
			6.92±0.40				
Li^8_Λ	$\pi^-He^4He^4$	72	6.72±0.08	153		225	
	$\pi^-H^1H^3He^4$	1		2	6.84±0.06	3	6.80±0.05
	$\pi^-H^2Li^6$			1		1	
Be^8_Λ	π^-B^8	4		13		17	
	$\pi^-H^1Be^7$	1	6.67±0.16	2	6.87±0.08	3	6 83±0.07
	$\pi^-H^1He^3He^4$	2		7		9	
Li^9_Λ	π^-B^9	5				5	
	$\pi^-H^1Li^8$	3	8.27±0.18	5	8.23±0.19	8	8.25±0.13
	$\pi^-H^3Li^6$	1				1	
Be^9_Λ	π^-Be^9	33	6.68±0.09	126	6.69±0.05	159	6.69±0.04
	$\pi^-H^1He^4He^4$	6	6.61±0.17	10	6.26±0.11	16	6.37±0.09
	Итого	39	6.66±0.08	136	6.62±0.05	175	6.63±0.04
B^{11}_Λ	π^-C^{11}	4				4	
	$\pi^-H^1H^2He^4He^4$	2	10.30±0.14	1	9.99±0.18	3	10.18±0.11
	$\pi^-He^4Be^7$	2				2	
	$\pi^-He^3He^4He^4$	1		1		2	
B^{12}_Λ	$\pi^-He^4He^4He^4$	11	11.26±0.16	13	10.95±0.16	24	11.10±0.11
C^{13}_Λ	π^-N^{13}					11	11.39±0.15

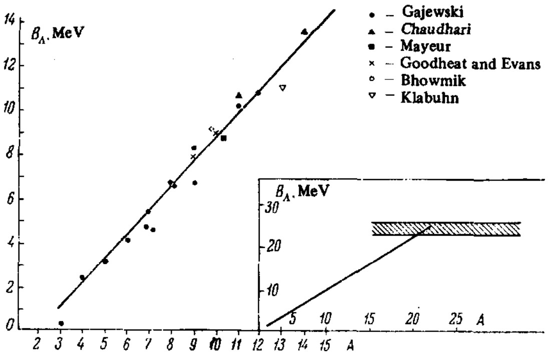

FIG. 7. General course of the dependence of the Λ-particle detachment energy B_Λ on the mass number A. The data are taken from [17,23,22,24-26].

Heavier hypernuclei remind us of a shapeless mass. The only thing that is known concerning them is that they do exist. It is difficult to obtain more detailed information on heavy hypernuclei for the following reasons. The binding energy of a hypernucleus is obtained by determining the kinetic energy of the decay products. The latter is directly connected with the range of the decay particles in the emulsion. But if the nuclear fragments are in excited states, or if the decay is accompanied by a certain number of neutrons, which cannot be noticed in the emulsion, then the excitation energies and the kinetic energy of the neutrons cannot be measured and, dropping out of the energy balance, they make it possible only to obtain the upper bound of the binding energy (or the upper bound of B_Λ). Emission of neutrons also makes it difficult to identify the decaying hypernucleus. Therefore all that can be obtained for heavy hypernuclei so far are rough estimates of the separation energy. According to the data of Bhowmik et al.[21], for example, for hypernuclei with mass number $A \approx 35-80$, the upper bound of B_Λ lies in the region from 22 to 24.5 MeV. Similar values were obtained also in other investigations. This question is considered in detail in Chap. 7.

As already mentioned, a direct study of ΛN interaction is still impossible, and the charge independence of ΛN forces must be assessed from the difference between the values of B_Λ for mirror nuclei, for example for H^4_Λ and He^4_Λ. It is assumed in[17,18,22] that the difference $\Delta B_\Lambda = B_\Lambda(He^4_\Lambda) - B_\Lambda(H^4_\Lambda)$ exceeds the experimental errors and equals $+0.28 \pm 0.07$ MeV. But in[23] it is stated, with the same degree of accuracy, that there is no difference at all, $\Delta B_\Lambda = +0.04 \pm 0.11$ MeV. It is therefore advisable to dwell in detail on the experimental procedure.

From the point of view of rigor and amount of information, particular interest attaches to the work of Gajewski et al.[17], and also its continuation by Bohm et al.[18]. As noted by their authors, the existing disparities in numerous investigations of the binding energies of hypernuclei are due apparently to the use of different methods for calibration, measurement, analysis, and, most importantly, different identification criteria. Since the binding energies are in essence the only source of information on the ΛN interaction, the need for a rigorous selection is obvious.

The results of the joint activity of two groups, the European K^- collaboration and the North-West collaboration of the Enrico Fermi Institute, are reported in[17] and[18]. In the laboratories of both groups, under similar conditions, a study was made of a large number of decays of hypernuclei. After taking into account the differences in the emulsion-stack thickness, the events were processed in accordance with a single program. The momentum balance in the direction of each axis was calculated by least squares, and the obtained kinetic energy was used to determine the binding energies of the hypernuclei. The validity of the method was verified for the two most frequently encountered events, $He^5_\Lambda \to \pi^- + p + \alpha$ and $Li^8_\Lambda \to \pi^- + \alpha + \alpha$. Decays containing neutrons or charged particles with energy below the formation of the visible track in the emulsion, and events in which the hypernuclei decayed in flight, were excluded. Any possible solution involving hypernuclei or decay modes whose existence has not yet been established was discarded. The only two-particle decays included were those for which there was additional information facilitating the identification (β radioactivity of the recoil nucleus, emission of a primary star configuration, etc.). The values of B_Λ given by Gajewski et al.[17] and by Mayeur et al.[22] have a possible systematic error of 0.15 MeV. In a later paper by Bohm et al.[18], the possible systematic error was decreased because of the refinement in the range-energy relation in the emulsion. It is interesting that B_Λ determined for H^4_Λ from two-particle and three-particle decays differ[18] by 0.21 ± 0.07 MeV. It is assumed[18] that this difference is due to unidentified experimental errors and that the difference can be eliminated by a more careful performance of the experiment.

5. s-SHELL HYPERNUCLEI

Among the s-shell hypernuclei are H^3_Λ, H^4_Λ, He^4_Λ, and He^5_Λ, which are obtained by attachment of the Λ particle to H^2, H^3, He^3, and He^4, respectively. Even the simplest hypernucleus, H^3_Λ, consists of three particles, and the calculation of the hypernuclei is essentially a many-body problem. The s-shell hypernuclei, containing the smallest number of particles, are the most amenable to calculation, and have therefore been investigated more thoroughly than other hypernuclei.

Although the number of investigations devoted to the spectroscopy of s-shell hypernuclei is quite large[29-56], there are no firm results. As the experimental values become refined, the parameters and the roles of the individual components of the ΛN interaction become continuously reviewed. It is therefore meaningful to dwell only on the main problems of the spectroscopy of hypernuclei, with illustrations using individual investigations as examples.

5.1. Spins of Hypernuclei

Depending on the direction of the Λ-particle spin, the spin of the hypernucleus can be larger or smaller by 1/2 than the spin of the nucleus. Only for nuclei having zero spin, such as He^4_Λ, it is known beforehand that the spin of the ground state of the corresponding hypernucleus (He^4_Λ) is 1/2. So far no direct experiments on the determination of the spins of hypernuclei have been performed, and when one encounters in the literature the statement that the spin of the ground state of some hypernucleus is known, one has in mind theoretical calculations made by certain persons and leading to such a conclusion. All the calculations of the binding energies of the Λ particles for s-shell hypernuclei are in satisfactory agreement with experiment only if it is assumed that $V_s > V_t$ (the singlet potential exceeds the triplet potential). In other words, the Λ particle settles in the nucleus in such a way that the spin of the hypernucleus is smaller by one-half than the spin of the nucleus. A similar conclusion was arrived at by an investigation of the decays of hypernuclei[27,28]. Although these results are not a rigorous proof, they favor spin values 1/2 and 0 for the hypernuclei H^3_Λ and H^4_Λ (He^4_Λ).

5.2. Paired ΛN Potentials

The simplest paired potentials are described by two parameters, depth and width. The parameters characterizing the depths and widths of potentials with different shapes are not directly comparable. For convenience in comparison, two other problems are chosen, namely the depth parameter s, the number by which it is necessary to divide the potential in order that the binding energy of the interacting particles be equal to zero, and the characteristic action radius b. The form and the parameters of the most widely used potentials are listed in Table IV, which is taken from[29]. By proper choice of parameters, any of them can describe low-energy scattering. Frequently one introduces into the potential an infinite repelling wall of radius r_c, i.e., one more parameter is added. The repelling wall is introduced in the ΛN potentials by analogy with the nucleon potentials. But whereas in nuclear physics this is required by saturation of the nuclear forces and by the course of the scattering phase shift at high energies, there is no such need for hypernuclei. B_Λ ceases to grow because of the finite radius of the ΛN forces. Even the very existence of the hypertritium H^3_Λ can yield an estimate of the depth of the ΛN potentials, $s \geq 0.36$ (see the Appendix).

Table IV. Depth parameter s and characteristic radius b for the most widely used types of potentials

Type of potential	V (r)	Depth parameter s	Characteristic radius of action b
Square well	$-V_0 \quad r<b$; $0 \quad r>b$	$\frac{4}{\pi^2}\frac{MV_0}{\hbar^2}b^2$	
Gaussian potential	$-V_0 e^{-(r/\beta)^2}$	$0.37261\,\frac{MV_0\beta^2}{\hbar^2}$	1.4354β
Exponential potential	$-V_0 e^{-2r/\beta}$	$0.17291\,\frac{MV_0\beta^2}{\hbar^2}$	1.7706β
Yukawa potential	$-V_0\frac{e-r/\beta}{r/\beta}$	$0.59531\,\frac{MV_0\beta^2}{\hbar^2}$	2.1196β

The most complete analysis of the binding energies of Λ particles was performed by Herndon and Tang[30-32]. They investigated the dependence of the cross section of Λp scattering and B_Λ for s-shell hypernuclei on the radius of an infinite repelling wall, on the action radius b, and on the form of the ΛN potentials. The calculation of B_Λ was carried out by a variational method and using potentials of the type

$$V_{s,t}(r)=\begin{cases}\infty, & r<r_c,\\ -U_{s,t}\exp[-\lambda(r-r_c)], & r\geq r_c\end{cases} \tag{5.1}$$

in accordance with the following scheme. The values of U_s and U_t were determined for a certain number of sets of parameters λ and r_c from the hypernuclei H^3_Λ, H^4_Λ, and He^4_Λ, after which $B_\Lambda(He^5_\Lambda)$ and $\sigma_{\Lambda p}$ were calculated. The employed parameters and the results of the calculations are given in Table V. Herndon and Tang's results lead to the following conclusions: If the ΛN interaction can be represented by a central two-particle potential, then its approximate value of b should be 2×10^{-13} cm (at lower values of b we have $a_s/a_t \gtrsim 4$, which contradicts the scattering) and a repelling wall, a hard core of radius of about 0.6×10^{-13} cm at a weak spin dependence $s_t/s_s \approx 0.8$.

The effect of the dependence of B_Λ on the form of the potential was investigated by Dalitz and Downs[33] with a pure attractive potential at values of b from 0.84 $\times 10^{-13}$ cm to 1.48×10^{-13} cm. Using H^3_Λ as an example, they found that a Yukawa potential and a potential of exponential form, with the same value of b, lead to the same value of s and to the same scattering lengths. This result has shown that the role that H^3_Λ plays in hypernuclei is similar to the role of the deuteron in nuclear physics, making it possible to get along with a small number of parameters and assume that $B_\Lambda(H^3_\Lambda)$ is not very sensitive to the form of the ΛN potentials. Herndon and Tang carried out a similar analysis[32] not only for H^3_Λ, but also for H^4_Λ and He^5_Λ, and also reached the conclusion that the choice of the form of the attractive part of the ΛN potential is not critical for B_Λ of s-shell hypernuclei. It is seen from the results of Herndon and Tang that potentials fitted to agree with scattering, and B_Λ for H^3_Λ, H^4_Λ, and He^4_Λ, result in too high values of B_Λ for He^5_Λ. This was the first danger signal. Bodmer[34], to be sure, advanced the next possible explanation. In describing the Λ-He^4 interaction

Table V. ΛN-potential parameters used by Herndon and Tang[30-32] in the analysis of s-shell hypernuclei

Type of potential	b, 10^{-13} cm	r_c, 10^{-13} cm	λ, 10^{13}cm-1	U_s, Mev	U_t, Mev	$-a_s$, 10^{-13} cm	$-a_t$, 10^{-13} cm	r_s, 10^{-13} cm	r_t, 10^{-13} cm	$B_\Lambda(He^5_\Lambda)$, Mev	V_0, Mev
A	1.5	0	2.361	204.0	114.8	1.94	0.74	2.25	3.48	5.58	3.5
B	1.5	0.30	3.935	685.9	529.8	2.07	0.85	2.16	3.42	4.86	6.6
C	1.5	0.45	5.902	1664.5	1450.8	2.07	0.97	2.14	3.02	4.73	10.4
D	1.5	0.60	11.804	7187.2	6751.9	2.16	1.04	2.08	2.87	4.45	21.1
E	2.0	0.45	3.219	451.7	402.1	2.16	1.60	3.15	3.61	5.02	5.5
F	2.0	0.60	4.427	910.6	861.4	2.09	1.84	3.15	3.34	4.82	8.0
G	2.5	0.60	2.727	320.1	319.3	2.33	2.04	4.19	3.75	4.68	4.3

within the framework of the meson theory, the Λ particle, exchanging a pion with He^4 (see Fig. 6a), goes over virtually into a Σ hyperon, and at the same time He^4 should be in a highly excited state. Consequently, owing to the large excitation energy, this interaction channel in He^5_Λ may be suppressed, i.e., the ΛN interaction in He^5_Λ possibly differs from the interaction in other hypernuclei. However, as shown in[35], the potentials chosen to agree with scattering overestimate the binding energy not only of He^5_Λ, but also of the next heavier hypernuclei in the p-shell. Although there are other possibilities of suppressing this effect in the p-shell, to estimate of the real disparity it is more reasonable to choose potentials on the s-shell, and then calculate the Λp-scattering cross section, since the degree of disparity is determined by the degree of agreement with the cross section, and not with the "best" scattering parameters[36]. It may be possible to lower $B_\Lambda(He^5_\Lambda)$ by increasing the radius of the singlet interaction[4,37] (so far it has been assumed in the analysis that $b_s = b_t$).

5.3. Violation of Charge Independence of ΛN Forces

The degree of violation of charge independence in ΛN forces can be assessed from the difference between the values of B_Λ for the mirror nuclei H^4_Λ and He^4_Λ, $\Delta B_\Lambda = B_\Lambda(He^4_\Lambda) - B_\Lambda(H^4_\Lambda) = +0.28 \pm 0.07$ MeV (at $B_\Lambda(He^4_\Lambda) \approx B_\Lambda(H^4_\Lambda) \approx 2$ MeV). We see that the interaction violating the charge independence makes an approximate contribution of 10% to B_Λ. Herndon and Tang[31] propose that such an interaction is of the form

$$U(r) = -\tau_3(\sigma_\Lambda\sigma_N)V_0\exp[-\lambda(r-r_c)], \tag{5.2}$$

where the quantities λ and r_c have the same meaning as in Sec. 5.2, and V_0 is the force parameter. At the sets of potentials used by them to reconstruct ΔB_Λ for the nuclei He^4_Λ and H^4_Λ, the value of V_0 should change from 3.5 MeV to 21.1 MeV (see Table V). Knowing V_0, we can find the proton and neutron potentials separately, using the formulas

$$U_s^p = U_s - 3V_0, \quad U_s^n = U_s + 3V_0, \quad U_t^p = U_t + V_0, \quad U_t^n = U_t - V_0. \tag{5.3}$$

We shall not present the corresponding figures, since the changes are small and do not affect the conclusions of Sec. 5.2.

5.4. Three-particle Forces

An analysis of s-shell hypernuclei with paired central ΛN potentials whose radius was chosen to be 1.5×10^{-13} cm, in accordance with 2π-meson exchange, has led to a strong spin dependence of the ΛN forces ($a_s/a_t \approx 4$). This fact gave rise to the opinion that the data on s-shell hypernuclei contradict Λp scattering ($a_s/a_t \approx 1$). Herndon and Tang[31] have shown that this contradiction can be weakened by increasing the radius of the potential (see Sec. 5.2). Weitzner[38] was the first to indicate the possibility of calculating the binding energies of s-shell hypernuclei with two-particle ΛN potentials without a strong spin dependence, owing to the three-particle ΛNN repulsion. This, however, could not be done for a long time for the following reasons. The long-range three-particle ΛNN potential is due to exchange of p-wave pions[39,40] (the contribution of the s-wave mesons was shown to be negligibly small) and is of the form

$$W_p = -\frac{1}{6}C_p(\tau_1\tau_2)\{[(\sigma_1\sigma_\Lambda) + S_{1\Lambda}(\mu x)T(\mu x)], [(\sigma_2\sigma_\Lambda) + S_{2\Lambda}(\mu y)T(\mu y)]\}\varphi(\mu x)\varphi(\mu y), \tag{5.4}$$

where C_p is a constant, {...} denotes the anticommutator, $x = r_1 - r_\Lambda$, $y = r_2 - r_\Lambda$,

$$S_{1\Lambda}(\mu x) = 3\frac{(\sigma_1 x)(\sigma_\Lambda x)}{x^2} - (\sigma_1\sigma_\Lambda), \quad T(\mu x) = 1 + \frac{3}{\mu x} + \frac{3}{(\mu x)^2}, \quad \varphi(\mu x) = \exp(-\mu x)/(\mu x), \tag{5.5}$$

μ is a parameter characterizing the decrease of the potential. For 2π exchange, $\mu = 1.3992 \times 10^{13}$ cm^{-1}. A more detailed study of the ΛNN potential was carried out by Bhaduri et al.[39] They found that in all the preceding investigations the central part of W_p was ignored without justification. If a cutoff radius d is introduced and it is assumed that $W_p = 0$ at x, y < d, then at $C_p = 1.43$ MeV the value of $B_\Lambda(He^5_\Lambda)$ decreases by 2 and 3.5 MeV for d equal to 1.0×10^{-13} and 0.6×10^{-13} cm, respectively. Such a decrease, as seen from Table V, is quite desirable.

5.5. Tensor ΛN Forces

The tensor ΛN forces do not influence B_Λ of hypernuclei strongly. But whereas the role of the tensor forces in hypernuclei is negligible, the ΛN-scattering parameters are sensitive to the force of the tensor component. This raises the question of whether it is possible to attribute the contradiction between the analysis of the s-shell hypernuclei and scattering experiments to the tensor ΛN forces.

Schrils and Buxton[41] investigated H^3_Λ with ΛN potentials containing a tensor component

$$V^t_{\Lambda N} = -V^t_0\left\{\frac{\exp(-\mu r)}{\mu r} + \delta S_{\Lambda N}\frac{\exp(-\nu r)}{\nu r}\right\}, \quad V^s_{\Lambda N} = -V^s_0\frac{\exp(-\mu r)}{\mu r}, \tag{5.6}$$

where $S_{\Lambda N}$ is defined in (5.5), and δ is a parameter regulating the force of the tensor part. The binding energy of H^3_Λ was assumed to be $E_b = -2.476$ MeV, i.e., $B_\Lambda = 0.251$ MeV. According to the results (Table VI), the short-range tensor component of the ΛN interaction gives a negligibly small contribution to the binding energy of H^3_Λ, in accordance with the earlier statements by Downs and Dalitz[42]. Definite information concerning the radius and depth of the tensor forces cannot be obtained, since different sets can lead to identical results. The tensor component strongly influences the scattering parameters, and this makes it possible to find potentials that agree well with the binding energy of H^3_Λ and the scattering parameters, without introducing an infinite repelling wall into the potentials[41].

6. p-SHELL HYPERNUCLEI

All the remaining identified hypernuclei correspond to nuclei with nucleons in the p-shell. The experimental data on the binding energies of the p-shell hypernuclei are more plentiful but of poorer quality than for the s-shell. In addition, the value of the spin has been es-

Table VI. Influence of tensor component of ΛN interaction on the scattering lengths and on the effective radii[41]. All the potentials give the same binding energy for H^3_Λ. Sets 4 and 5 ($V_0^t > V_0^s$) contradict the proposed value of the ground-state spin (1/2).

№	V_0^t, MeV	V_0^s, MeV	μ, 10^{13} cm^{-1}	δ	ν, 10^{13} cm^{-1}	$-a_t$, 10^{-13} cm	r_t, 10^{-13} cm	$-a_s$, 10^{-13} cm	r_s, 10^{-13} cm
1	27.90	41.76	1.0	3.0	1.5	2.08	3.39	2.76	3.56
2	28.735	41.52	1.0	5.3	2.0	2.08	3.40	2.72	3.59
3	44.605	47.34	1.1	1.55	2.0	2.08	3.40	2.13	3.48
4	49.440	48.41	1.125	0.3	2.0	2.08	3.40	1.97	3.49
5	49.535	48.25	1.125	0		2.08	3.40	1.95	3.50
6	28.735	41.55	1.0	0		1.26	5.31	2.73	3.58

tablished only for one hypernucleus, viz., the spin of Li^8 is equal to 1 (from the decay correlations). Just as in nuclear physics, calculations in this region are the most difficult, since the number of particles is much larger than in s-shell hypernuclei, and at the same time it is not large enough to satisfy the statistical laws. To simplify the calculations it is therefore customary to use definite assumptions concerning the nuclear-core structure.

In most investigations, two types of assumptions are used. The first presupposes that the core has a cluster structure[58,2]: For example, Be^9_Λ is regarded as two α particles plus Λ (2α + Λ), C^{13}_Λ as 3α + Λ, He^6_Λ as α + n + Λ, and Li^7_Λ as α + d + Λ. Thus, the number of bodies considered in the problem is greatly reduced. The potentials describing the interaction between the clusters and the Λ particle are adjusted to fit the binding energies of the lighter nuclei and the hypernucleus, and to fit the scattering of the clusters. In the second type of assumption it is proposed that the Λ particle is located in the self-consistent nucleon field made up of the ΛN potentials and the nucleon density[62-64]:

$$V(\mathbf{r}_\Lambda) = \int \rho_N(\mathbf{r}) V_{\Lambda N}(|\mathbf{r}_\Lambda - \mathbf{r}|)\, d\mathbf{r}; \qquad (6.1)$$

here ρ(r) is the nucleon density of the core nucleus, and $V_{\Lambda N}$ is a certain average ΛN potential independent of the spin-isospin variables of the nucleons.

Both approaches have shortcomings. In the first case the accuracy of the cluster model is not certain. In the second approach we do not know how the lambda-nucleon correlations are to be taken into account in the presence of strong repulsion in the ΛN potentials at short distances.

An analysis of the s-shell hypernuclei makes it possible to determine only the ΛN interaction in the s-state. In heavier hypernuclei, the binding energy depends also on the interaction in a state with higher angular momenta and, for example, calculation of the binding energies of the p-shell hypernuclei can yield information concerning the strength of the ΛN potentials in a state with relative orbital angular momentum l = 1. Brink and Grypeos[65] verified, in the oscillator model, whether it is possible to reconstruct the experimental values of B_Λ by assuming that the potentials acting in the states with l = 0 and l = 1 are equal. Figure 8 shows the results of their calculation with the potential

$$V_{\Lambda N}(r) = \begin{cases} \infty, & r < c, \\ -V_0 \exp[-b(r-c)], & r > c \end{cases}$$

at c = 0.4 × 10^{-13} cm, $V_0 = (3V_t + V_s)/4$ = 330.9 MeV, and

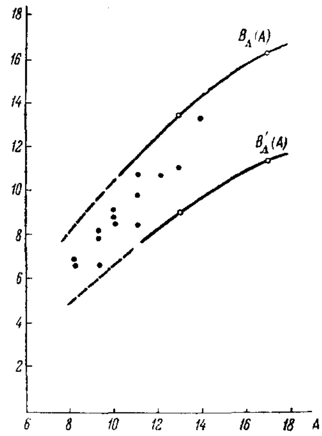

FIG. 8. Dependence of Λ-particle detachment energy in p-shell hypernuclei on the ΛN interaction force in the p-state after Brink and Grypeos [65] (see the text). The black points represent the experimental values of B_Λ.

b = 3.219 × 10^{13} cm, which yields $B_\Lambda(He^5_\Lambda)$. The B_Λ(A) curve describes the detachment energy with $V_{\Lambda N}$ (l = 1) = $V_{\Lambda N}$ (l = 0), while B'_Λ(A) describes it with $V_{\Lambda N}$ (l = 1) = 0. A similar result was obtained also for the ΛN potential with a shorter radius and a larger strength. Thus, to reconstruct the experimental values of B_Λ, it is necessary to suppress the strength of the ΛN interaction in the p-state. The same conclusion was reached by Ho and Volkov[67], but their values of B_Λ were too high even at $V_{\Lambda N}$ (l = 1) = 0. We note that the Ho and Volkov ΛN potentials give too high a value already for He^5_Λ, and their numerical values must therefore be revised.

The identified p-shell hypernuclei are an inconvenient object of such investigations, since an important role may be played in them by tensor and spin-orbit ΛN forces. The most suitable for this purpose would be the hypernucleus O^{17}_Λ, but it has unfortunately not been identified as yet.

The spin orbit ΛN interaction was included in the analysis of B_Λ for the p-shell by Gal[68] (shell model with intermediate coupling). For a best fit the matrix element for this interaction between the $p_{3/2}$ nucleon and the Λ particle in the s state should equal 0.84 MeV. By comparison, the corresponding matrix element of the NN interaction is equal to −1.1 MeV. The necessary values of the sl forces of the ΛN interaction is comparable in magnitude with the sl forces of the NN interaction (but of opposite sign, as can be readily understood using any simple theory of ΛN forces), and represents a sufficiently large energy with respect to deviations of B_Λ from its average A-dependence.

7. HEAVY HYPERNUCLEI

One can hardly expect the Λ particle in a heavy nucleus to be able to cause any appreciable change in the nucleon distribution. One can therefore imagine a Λ particle situated in a Λ-nuclear-core potential of the type (6.1), having a radius $R = r_0A^{1/3}$ ($r_0 \approx 1.2 \times 10^{-13}$ cm according to the usual nuclear estimates) and a depth D_Λ. Then

$$B_\Lambda \approx D_\Lambda - \frac{\pi^2\hbar^2}{2M_\Lambda r_0^2 A^{2/3}}. \quad (7.1)$$

For A ~ 60, the last term in (7.1) is approximately equal to 7 MeV. For a Λ-nucleus potential in the form of a square well it is also easy to write down the succeeding terms of the expansion

$$B_\Lambda = D_\Lambda - \frac{\hbar^2\pi^2}{2M_\Lambda r_0^2 A^{2/3}}\left[1 - \frac{2}{s} + \frac{3}{s^2} + O\left(\frac{1}{s^3}\right)\right], \quad (7.2)$$

where

$$s = [(2M_\Lambda D_\Lambda)/\hbar^2]^{1/2}\, r_0 A^{1/3}. \quad (7.3)$$

Formula (7.2) is frequently used by experimenters, with B_Λ replaced by D_Λ. Since $\lim_{A\to\infty} B_\Lambda = D_\Lambda$, the quantity D_Λ is taken to be the binding energy of the Λ particle in nuclear matter.

The experimental determination of D_Λ is based on a study of the π^-pr modes of the decays of the disintegration hypernuclei produced when K^- mesons interact with silver and bromine nuclei in nuclear emulsion. A value $D_\Lambda = 27 \pm 3$ MeV was obtained in[70], but under an assumption concerning the mass distribution for the disintegration hypernuclei. The succeeding refined value $D_\Lambda = 27 \pm 1.5$ MeV is already free of this assumption[71]. The true value may lower since one determines, strictly speaking, the upper limit of D_Λ (B_Λ), and the statistics in the last experiment is relatively limited. We present also a few other results: $D_\Lambda \leq 33 \pm 2.5$ MeV[72], $D_\Lambda \leq \pm 2$ MeV[73], and the already mentioned $B_\Lambda = 22-24.5$ MeV[21].

On the whole, the picture does not contradict strongly the homogeneous Λ-plus-interaction-nucleus model with a standard distribution of the nucleon density and with the usual parameters of the radius and of the smearing of the surface, which yields $D_\Lambda = 30$ MeV when B_Λ is adjusted to agree with C^{13}_Λ. This, of course, must not be taken too seriously, but nevertheless the homogeneous model, which is the simplest possible, ensures a reasonable interpolation curve between C^{13} and the heavy disintegration hypernuclei with $A \approx 80-100$.

The theoretical situation, connected with the direct calculation of D_Λ from the ΛN (and ΛNN) forces obtained from Λp scattering and B for the s-shell, is still mainly unsatisfactory[74-85]. To obtain D_Λ near the empirical value it is necessary to make rather strong assumptions, motivated mainly by the attempt to obtain such an agreement. It is difficult to obtain a value of D_Λ much lower than 40 MeV on the basis of any ΛN potential that is acceptable in all other respects. Even the assumption that the ΛN interaction in the p state is equal to zero gives with difficulty a value lower than 35 MeV.

8. EXCITED STATES OF HYPERNUCLEI

Excited states of hypernuclei exist without a doubt. For example, a core nucleus in the excited state can retain a Λ particle. Such states, however, which decay via γ-quantum emission, have as a rule a lifetime of 10^{-14} sec, which is much shorter than the Λ-particle lifetime (10^{-10} sec). The hypernucleus is therefore usually emitted, and then decays only from the ground state. The study of excited states with lifetimes much shorter than 10^{-10} sec calls for a direct observation of the γ rays emitted on going to the ground state, and since γ quanta can be emitted also in other processes, for example during the production of the hypernuclei, they must be measured for coincidence with the decay pions. Such an experiment calls for an intense K-meson flux, which is as yet unobtainable.

On the other hand, if the γ-decay time is comparable in magnitude or is longer than the hypernucleus Λ-decay time, then decays of hypernuclei of a given type from the excited state should be observable. This would be manifest experimentally in the appearance of a second peak in the B_Λ distribution. Although for most hypernuclei the statistics are too skimpy to state categorically that such states exist or do not exist, it is more likely that we have two hypernuclei in which the existence of such long-lived isomeric states (i.e., not differing from the ground state with respect to the lifetime) is quite realistic. These are the hypernuclei He^7_Λ and Li^7_Λ (Fig. 9).

The first to indicate the possible existence of an isomeric state of He^7_Λ were Pniewski and Danysz.[86] The isomer He^{7*}_Λ corresponds to a Λ particle joined to the excited state He^{6*} located above the threshold for disintegration into α + 2n. This $J = 2^+$ level in He^7 splits into $J = -5/2^+$ and $J = 3/2^+$, and if the singlet ΛN interaction is stronger than the triplet interaction, the $J = 3/2^+$ level lies lower. Calculations of the electromagnetic-transition probabilities by Elton[87], Law[88], Lodhi[89] (for E2 transition), Rayet (for M1 transition), and Dalitz and Gal[91] show that the lifetimes of such states are longer than in hypernucleus decay (for example, Law and Lodhi give lifetimes $T(E2) > 10^{-9}$ and $\sim 10^{-6}$ sec, respectively).

The B_Λ distribution for Li^7 is likewise not Gaussian, but there is no pronounced second peak, and the deviation can be interpreted in principle as a fluctuation or as an error in the identification of certain events. The isomer state in Li^7_Λ is at present a purely experimental

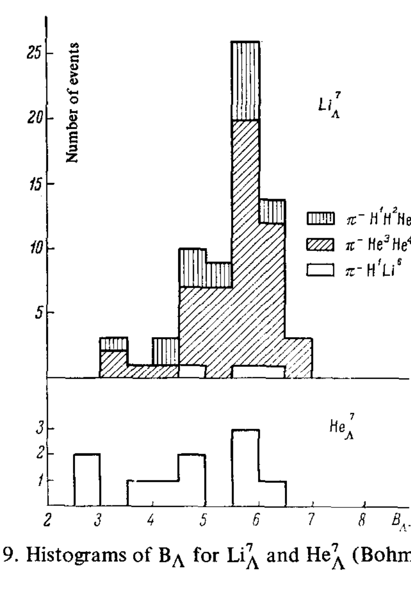

FIG. 9. Histograms of B_Λ for Li^7_Λ and He^7_Λ (Bohm et al. [18]).

question, since the theory requires as yet unavailable information on the ΛN interaction[93].

Great interest is attached to the question of the excited state of the mirror hypernuclei He^4_Λ and H^4_Λ, a state arising as a result of splitting of the level corresponding to the ground state of the core, H^3 or He^3. The magnitude of the splitting is directly connected with the proximity of the singlet and triplet potentials, and its determination yields new reference points in the region most accessible to calculations. All that can be stated at present is that the excitation energy ϵ^* is more likely to lie in the range 0–1 MeV[37]. One can hardly expect this level to be isomeric, since isomerism requires $\epsilon^* \lesssim 0.1$ MeV, and most phenomenological potentials yield much higher values (Table VII). Although the B_Λ distributions are quite smeared (lack of statistics), they are also quite close to Gaussian.

It is expected that H^3_Λ has a $J = 3/2^+$ level lying close to the ground $J = 1/2^+$ level (due to the proximity of the s- and t-ΛN interactions), and having properties analogous to those of the deuteron virtual level[4,37]. The $J = 3/2^+$ level can hardly be located in the discrete region, since $B_\Lambda(H^3_\Lambda)$ is very small, and this requires almost complete equality of the singlet and triplet potentials.

Attempts to find the location of the unstable states of Li^6_Λ and Be^9_Λ in emulsion were unsuccessful[24]. Recently, however, proof was obtained of the possible existence of unstable states of C^{12}_Λ and N^{14}_Λ, produced upon absorption of stopped K^- mesons[93]. The analysis of the reaction products was limited to the configurations B^{11}_Λ or C^{13}_Λ, π^-, and one more track belonging to the proton. Peaks at 156 and 169 MeV were observed in the pion distribution, thus indicating the two-step reactions

$$\begin{aligned} &K^- + C^{12} \to \pi^- + C^{12*}_\Lambda \to \pi^- + H^1 + B^{11}_\Lambda, \\ &K^- + N^{14} \to \pi^- + N^{14*}_\Lambda \to \pi^- + H^1 + C^{13}_\Lambda. \end{aligned} \qquad (8.1)$$

Rough preliminary parameters of these states are given in Table VIII. B^*_Λ denotes the binding energy of the Λ particle in the given state, E^* is the excitation energy relative to the ground state of the hypernucleus; the value of E^* for N^{14}_Λ was taken under the assumption that $B_\Lambda = 13.3$ MeV (this is the result of extrapolation, since the exact value has not yet been established). It is likely that both states are stable against the emission of a Λ particle (this is undoubtedly so in the case of N^{14*}_Λ) the only strong decay mode is proton emission. In the case of N^{14*}_Λ, decay to the excited state C^{13*}_Λ is possible. The existence of the latter is almost certain, with $E^* \approx 4$ MeV[4].

Table VII. Excitation energies of the hypernuclei H^4_Λ and He^4_Λ after Tang[37]. The parameters of the potentials are given in Table V

Type of potential	H^4_Λ B^*_Λ, Mev	H^4_Λ ε^*, Mev	He^4_Λ B^*_Λ, Mev	He^4_Λ ε^*, Mev	Type of potential	H^4_Λ B^*_Λ, Mev	H^4_Λ ε^*, Mev	He^4_Λ B^*_Λ, Mev	He^4_Λ ε^*, Mev
A	0.49	1.42	0.15	2.05	*D*	0.75	1.16	0.39	1.81
B	0.58	1.33	0.24	1.96	*E*	1.50	0.41	1.10	1.10
C	0.75	1.16	0.39	1.81	*F*	1.73	0.18	1.32	0.88

Table VIII. Experimental characteristics of the unstable states of the hypernuclei C^{12}_Λ and N^{14}_Λ

Hypernucleus	B^*_Λ, Mev	E^*, Mev	Γ, Mev	Q, Mev
C^{12*}_Λ	0.4±0.3	10.7	0.9	1.5 for $(p + B^{11}_\Lambda)$
N^{14*}_Λ	2.9±0.3	~10.4	1.8	6.5 for $(p + C^{13}_\Lambda)$

9. DOUBLE HYPERNUCLEI

Two double hypernuclei are known at present. The first, discovered by Danysz et al.[94] in 1963, has not been identified uniquely. Two interpretations are possible, namely $Be^{10}_{\Lambda\Lambda}$ and $Be^{11}_{\Lambda\Lambda}$. A characteristic attribute of a double hypernucleus is a two-step decay due to sequential decay of the Λ particles it contains. The first double hypernucleus is usually interpreted as $Be^{10}_{\Lambda\Lambda}$ because another interpretation would call for the decay $Be^{10}_\Lambda \to 2He^4 + H^2 + \pi^-$, which was never observed. The second double hypernucleus was observed by Prouse[95] and identified uniquely as $He^6_{\Lambda\Lambda}$. The details of the identification of these hypernuclei were described by Filimonov.

The binding energy of the double hypernucleus is characterized by the quantities $B_{\Lambda\Lambda}$ (the energy required to separate the two ΛΛ particles) and $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_\Lambda$, which is the change in the separation energy compared with double the separation energy in the usual hypercharge. The experimental values of $B_{\Lambda\Lambda}$ and $\Delta B_{\Lambda\Lambda}$ are given in Table IX.

The theory of ΛΛ interaction in double hypernuclei has been developed in a number of papers, but no new considerations, compared with the theory of the ΛN interaction and with the calculation of hypernuclei, were advanced either in the mesonic theory of ΛΛ forces[97-101] or in the analysis of the binding energies of double hypernuclei[102-123]. Regardless of the interpretation of the first double hypernucleus, the binding energies of two hypernuclei can be described by any two-parameter singlet ΛΛ potential (both Λ particles, being fermions, are in the s-state with oppositely directed spins) having a more or less reasonable form. But in almost all the calculations the potential is fitted only to $He^6_{\Lambda\Lambda}$, and the characteristic interaction radius $b_{\Lambda\Lambda}$ is specified beforehand; usually $b_{\Lambda\Lambda} \approx 1.5 \times 10^{-13}$ cm, corresponding to exchange of two pions.

The Λ particles are much more weakly attracted than a Λ and a nucleon. For example, if we choose the ΛΛ and ΛN potentials in the form of square wells, then $(UR^2)_{\Lambda N} \approx 3(UR^2)_{\Lambda\Lambda}$, where U is the depth and R is the radius of the corresponding potentials. The region of stability of the double hypernuclei can therefore hardly

Table IX. Energy of separation of the Λ particles of double hypernuclei

Hypernucleus	$B_{\Lambda\Lambda}$, Mev	$\Delta B_{\Lambda\Lambda}$, Mev
$He^6_{\Lambda\Lambda}$	10.8±0.5	4.6±0.5
$Be^{10}_{\Lambda\Lambda}$	17.5±0.4	4.5±0.4

be wider than the region of stability of ordinary hypernuclei. It can be stated with assurance that there is no bound state of the $\Lambda\Lambda N$ system, since it is known that $nn\Lambda^2$ has no bound state. According to Tang and Herndon[106], $H^4_{\Lambda\Lambda}$ is not bound, and $H^5_{\Lambda\Lambda}$ and $He^6_{\Lambda\Lambda}$ are bound (that $H^5_{\Lambda\Lambda}$ and $He^6_{\Lambda\Lambda}$ are bound was predicted even earlier by Nakamura[112] and by Damle and Biswas[113]), with $\Delta B_{\Lambda\Lambda}(H^5_{\Lambda\Lambda}) = 3.5 \pm 0.5$ MeV; this ensures stability against decay into $\Lambda + H^4_\Lambda$. Ananthanarayanan[111] considered double hypernuclei of the p-shell and obtained $\Delta B_{\Lambda\Lambda}(C^{14}_{\Lambda\Lambda}) = 3.75$ MeV and $\Delta B_{\Lambda\Lambda}(O^{18}) = 3.5$ MeV. The foregoing quantities are apparently not very exact, owing to the leeway in the determination of the $\Lambda\Lambda$ potential only on the basis of $He^6_{\Lambda\Lambda}$.

10. PROSPECTS OF THE PHYSICS OF HYPERNUCLEI

As noted by Davis and Sacton[93], it is not at all surprising that some prospects can exist in research with the aid of a technique that has remained essentially unchanged for the last 15 years. Their statement pertains to experiments with nuclear emulsions, but there are still a few questions in this field that can be answered by increasing the statistics. Whereas the emulsion technique did indeed reach the experimental limit in the determination of B_Λ for s-shell hypernuclei, the situation can be considerably improved for most p-shell hypernuclei. Refinement of the data on the p-shell would make it possible to establish the role played by three-particle and tensor forces. Further measurement of B_Λ for isospin doublets other than H^4_Λ and He^4_Λ is needed also for the determination of the possible violation of charge independence in the ΛN interaction.

The greatest hopes in the physics of hypernuclei are pinned on new methods, particularly bubble chambers. Experiments on Λp scattering, at their present-day state, do not make it possible to separate singlet and triplet interaction. The cleanest way to such a separation is connected with polarized Λ particles. The main difficulty is in finding an intense source of strongly polarized Λ particles with low energy, 4—20 MeV. It is possible that reactions suitable for this purpose are

$$\pi^- + p \to \Lambda + K^0 \quad (10.1)$$

or

$$K^- + p \to \Lambda + \pi^+ + \pi^-.$$

Another possibility is to study $\Lambda + H^2$ and $\Lambda + He^4$ scattering. The first reaction is of interest as strong $\Lambda + H^2$ scattering is expected in the J = 3/2 state, owing to the virtual level J = 3/2, T = 0. In addition, since $\Lambda + H^2$ scattering is determined by a different combination of triplet and singlet ΛN potentials than Λ + p scattering, and the three-body problem is now being solved with sufficient accuracy, measurement of $\sigma(\Lambda H^2)$ is most desirable. From the $\Lambda + He^4$ reaction it will be possible to estimate the strength of the spin-orbit ΛN interaction. There are good estimates of the central $\Lambda + He^4$ potential on the basis of $B_\Lambda(He^5_\Lambda)$, and also the shape of He^4 on the basis of electron scattering; the contribution of the *sl* interaction can therefore be separated quite correctly. Preliminary calculations of $\Lambda + He^4$ scattering with allowance for the *sl* forces have already been performed by Alexander et al.[114]

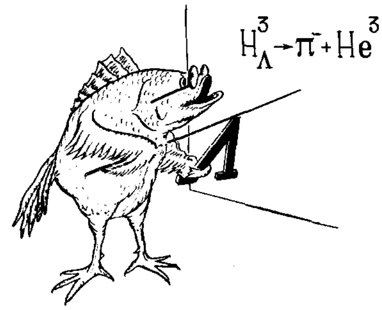

FIG. 10

The next step will apparently be observation and study of Λ-hypernuclear γ rays. An increase in the statistics and γ rays can help ascertain whether isomer states actually exist in He^7_Λ and Li^7_Λ[115]. It has been impossible to observe Λ-hypernuclear γ rays so far[116]. When this happens, it will be possible to study $\gamma\gamma$ correlations between the successive γ rays and $\gamma\pi$ correlations with the pion emitted in the decay of a hypernucleus from the ground state, so that the spins of the hypernuclear states can be determined.

Much can be expected from investigations of hypernuclear states in the continuous spectrum with the aid of two-step reactions of the type

$$K^- + (Z)^A \to \pi^- + (Z)^{A*}_\Lambda \to \pi^- + p + (Z-1)^{A-1}_\Lambda. \quad (10.2)$$

As is well known, about 15% of K mesons coming to rest in emulsion are captured by carbon nuclei, about 3% by N, and 15% by O. Therefore, as soon as we observe C^{12*}_Λ and N^{14*}_Λ, the formation of O^{16*}_Λ with decay into N^{15}_Λ, a hypernucleus not yet identified, is perfectly realistic. A study of the correlations between successively emitted π^- mesons in a reaction of the type (10.2) could lead to a determination of J for excited states of hypernuclei[4].

In conclusion, we note once more the place occupied by the physics of hypernuclei in the physics of strong interaction. This is best stated in a review of the proceedings of the 1969 Argonne conference on the hypernuclear physics: "Hypernuclear physics is in a strange position. It is neither fish nor fowl. High-energy physicists do not look to it for valuable advances in their understanding of the interactions of fundamental particles. Nuclear physicists also view the field as something apart. Its main relevance for the fundamentals is the information it can provide on ΛN and $\Lambda\Lambda$ interactions..." (J. D. Jackson, Science **159**, 1959 (No. 3821, 1346 (1968)).

The author is grateful to A. I. Baz', M. V. Zhukov, and Yu. A. Simonov for reviewing the manuscript and for remarks.

APPENDIX

Let us estimate the lower limit of the ΛN potential from the H^3_Λ binding energy, following Nishijima[117]. The Hamiltonian of H^3_Λ, neglecting the three-particle forces, is

$$H = T_n + T_p + T_\Lambda + V_{np} + V_{\Lambda p} + V_{\Lambda n}, \quad (A.1)$$

where T is the kinetic energy, and the meanings of the subscripts are obvious. If B_d is the deuteron binding energy and B_Λ the Λ-particle separation energy, then

$$\min(H) = -(B_d + B_\Lambda),$$
$$\min(T_n + T_p + V_{np}) = -B_d. \quad \text{(A.2)}$$

min(A) is the lowest eigenvalue of the operator. In the rest system we have $\mathbf{p}_n + \mathbf{p}_p + \mathbf{p}_\Lambda = 0$. We assume that $V_{\Lambda n} = V_{\Lambda p} \equiv V_{\Lambda N}$, and use the inequality min(A + B) ≥ min(A) + min(B), and also the explicit form of the kinetic-energy operator. We then readily obtain

$$-B_\Lambda \geqslant 2 \min\left[\frac{M_\Lambda + 2m}{8mM_\Lambda} p_\Lambda^2 + V_{N\Lambda}\right], \quad \text{(A.3)}$$

or

$$0 > -\frac{2(m + M_\Lambda)}{2m + M_\Lambda} B_\Lambda \geqslant \min\left[\frac{M_\Lambda + m}{2mM_\Lambda} p_\Lambda^2 + \frac{4(M_\Lambda + m)}{2m + M_\Lambda} V_{N\Lambda}\right]. \quad \text{(A.4)}$$

From this we get the limit of the depth parameter of the potential s:

$$s \geqslant \frac{2m + M_\Lambda}{4(m + M_\Lambda)} \approx 0.36.$$

[1] R. H. Dalitz, Preprint EFINS-61-48.
[2] R. H. Dalitz, Proc. Topical Conf. on the Use of Elementary Particles in Nuclear Structure Research, Brussels, 1965, p. 293.
[3] R. H. Dalitz, Hypernuclear Physics, Lectures at the 1966 Varenna Summer School.
[4] R. H. Dalitz, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 708.
[5] B. Sechi-Zorn, R. A. Burnstein, T. B. Day, B. Kehoe, G. A. Snow, G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, A. Fridman, and A. Minguzzi-Ranzi, Proceedings of Conf. on High Energy Physics, Vol. 1, Dubna, 1964, p. 675.
[6] G. Alexander, O. Benary, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth, A. Fridman, and B. Schiby, Phys. Lett. **19**, 715 (1966).
[7] G. Alexander, U. Karshon, A. Shapira, G. Yekutieli, R. Engelmann, H. Filthuth and W. Lughofer, Phys. Rev. **173**, 1452 (1968).
[8] B. Sechi-Zorn, B. Kehoe, J. Twitty, and R. A. Burnstein, Phys. Rev. **175**, 1735 (1968).
[9] G. Alexander, J. A. Anderson, F. S. Crawford, W. Laskar, and L. J. Lloyd, Phys. Rev. Lett. **7**, 348 (1961).
[10] B. A. Arbuzov, E. N. Kladnitsaya, V. N. Penev, and R. N. Faustova, Zh. Eksp. Teor. Fiz. **42**, 979 (1962) [Sov. Phys.-JETP **15**, 676 (1962)].
[11] T. H. Groves, Phys. Rev. **129**, 1372 (1963).
[12] P. Belliere, J. L. Gomez, A. Lloret, A. Rousset, K. Myklebost and J. M. Olsen, Phys. Lett. **12**, 350 (1964).
[13] L. Piekenbrock and F. Oppenheimer, Phys. Rev. Lett. **12**, 625 (1964).
[14] V. F. Vishnevskiĭ, Tu Yuan-ch'ai, V. I. Moroz, A. V. Nikitin, Yu. A. Troyan, Chien Shao-chun, Chang Wen-yu, B. A. Shakhbazyan, and Yen Wu-kuang, Yad. Fiz. **3**, 697 (1966) [Sov. J. Nucl. Phys. **3**, 511 (1966)].
[15] D. Cline, R. March and M. Sheaff, Phys. Lett. **25B**, 446 (1967).
[16] A. Deloff, Fortschr. Phys. **17**, 129 (1969).
[17] W. Gajewski, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. Harmsen, R. Levi Setti, M. Raymund, J. Zakrzewski, D. Stanley, D. H. Davis, E. R. Fletcher, J. E. Allen, V. A. Bull, A. P. Conway, and P. V. March, Nucl. Phys. **B1**, 105 (1967).
[18] G. Bohm, J. Klabuhn, U. Krecker, F. Wysotski, G. Coremans, W. Gajewski, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. O'Sullivan, D. Stanley, D. H. Davis, E. R. Fletcher, S. P. Lovell, N. C. Roy, J. H. Wickens, A. Filipkowski, K. Garbowska-Pniewska, T. Pniewski, E. Skrzypczak, T. Sobczak, J. E. Allen, V. A. Bull, A. P. Conway, A. Fishwick, and P. V. March, Nucl. Phys. **B4**, 511 (1968).
[19] D. M. Harmsen, Phys. Rev. Lett. **19**, 1186, 1409 (1967).
[20] G. Coremans, W. Gajewski, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. H. Davis and J. H. Wickens, Nucl. Phys. **B4**, 580 (1968).
[21] B. Bhowmik, T. Chand, D. V. Chopra, and D. P. Goyal, Phys. Rev. **146**, 703 (1966).
[22] C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. Stanley, P. Allen, D. H. Davis, E. R. Fletcher, D. A. Carbutt, M. A. Shaukat, J. E. Allen, V. A. Bull, A. P. Conway and P. V. March, Nuovo Cimento **43A**, 180 (1966).
[23] K. N. Chaudhari, S. N. Ganguli, N. K. Rao, M. S. Swami, A. Gurtu, J. M. Kohli and M. B. Singh, Proc. Ind. Acad. Sci. **68A**, 228 (1968).
[24] D. T. Goodheat and D. A. Evans, Nucl. Phys. **B2**, 121 (1967).
[25] B. Bhowmik, T. Chand and D. V. Chopra, Nuovo Cimento **59A**, 309 (1969).
[26] J. Klabuhn, U. Krecker, G. Coremans, J. Sacton, P. Vilain, G. Wilquet, D. Stanley, D. H. Davis, J. H. Wickens, A. Filipkowski, W. Gajewski, T. Pniewski, and J. E. Allen, Nucl. Phys. **B12**, 336 (1969).
[27] R. H. Dalitz and S. Liu, Phys. Rev. **116**, 1312 (1959).
[28] D. E. Overseth and R. F. Roth, Phys. Rev. Lett. **19**, 391 (1967).
[29] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, Wiley, 1952 (Russ. transl. IL 1954, p. 50).
[30] R. C. Herndon and Y. C. Tang, Phys. Rev. **153**, 1091 (1967).
[31] R. C. Herndon and Y. C. Tang, Phys. Rev. **159**, 853 (1967).
[32] R. C. Herndon and Y. C. Tang, Phys. Rev. **165**, 1093 (1968).
[33] B. W. Downs and R. H. Dalitz, Phys. Rev. **110**, 958 (1958).
[34] A. R. Bodmer, Phys. Rev. **141**, 1387 (1966).
[35] A. D. Jackson, A. Lande and R. U. Sauer, Phys. Lett. **30B**, 233 (1969).
[36] B. A. Khrylin, Yad. Fiz. **13**, 492 (1971) [Sov. J. Nucl. Phys. **13**, 277 (1971)].
[37] Y. C. Tang, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 276.
[38] H. Weitzner, Phys. Rev. **110**, 593 (1958).
[39] R. K. Bhaduri, B. A. Loiseau and Y. Nogami, Ann. Phys. (N. Y.) **44**, 57 (1967).
[40] Y. Nogami, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 244.
[41] R. Schrils and L. D. Buxton, Nuovo Cimento **54B**, 267 (1968).
[42] B. W. Downs and R. H. Dalitz, Phys. Rev. **114**, 593 (1959).
[43] B. W. Downs and P. J. Phyllips, Nuovo Cimento **41A**, 374 (1966).
[44] A. Gal, Phys. Rev. **152**, 975 (1966).

[45] S. G. Shah and S. P. Pandya, Prog. Theor. Phys. 27, 837 (1962).
[46] T. Kohmura, Prog. Theor. Phys. 34, 956 (1965).
[47] B. A. Loiseau, Nucl. Phys. B9, 169 (1969).
[48] L. D. Buxton and R. Schrils, Nuovo Cimento 59B, 218 (1969).
[49] G. Bhamathi and S. Indumathi, Nucl. Phys. 54, 161 (1964).
[50] G. Bhamathi and I. Seshadri, Nucl. Phys. 76, 625 (1966).
[51] L. H. Schick and A. J. Toepfer, Phys. Rev. 170, 946 (1968).
[52] M. V. Zhukov and B. A. Khrylin, Yad. Fiz. 9, 507 (1969) [Sov. J. Nuc. Phys. 9, 240 (1969)].
[53] A. J. Toepfer, Nuovo Cimento 61A, 761 (1969).
[54] B. F. Gibson, A. Goldberg and M. S. Weiss, Phys. Rev. 181, 1486 (1969).
[55] B. Akhmadzhaev, V. B. Belyaev, and E. Wrzecionko, ZhETF Pis. Red. 10, 557 (1969) [JETP Lett. 10, 365 (1969)].
[56] J. Law, M. R. Gunye and R. K. Bhadhuri, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 333.
[57] P. Vilain, G. Wilquet, J. Sacton, D. M. Harmsen, D. H. Davis, J. H. Wickens and W. Gajewski, Nucl. Phys. B13, 451 (1969).
[58] A. R. Bodmer and S. Ali, Nucl. Phys. 56, 657 (1964).
[59] R. C. Herndon and Y. C. Tang, Phys. Rev. 149, 753 (1966).
[60] S. K. Monga, Phys. Rev. 160, 846 (1967).
[61] G. Willain, Nucl. Phys. 81, 585 (1966).
[62] J. M. Murphy and A. R. Bodmer, Nucl. Phys. 83, 673 (1966).
[63] A. R. Bodmer and J. M. Murphy, Nucl. Phys. 73, 664 (1965).
[64] A. R. Bodmer and J. M. Murphy, Nucl. Phys. 64, 593 (1965).
[65] D. M. Brink and M. E. Grypeos, Nucl. Phys. 80, 681 (1966).
[66] R. Herndon, Y. Tang and E. Schmid, Phys. Rev. 137, 294 (1965).
[67] T. H. Ho and A. B. Volkov, Phys. Lett. 30B, 303 (1969).
[68] A. Gal, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 348.
[69] J. D. Walecka, Nuovo Cimento 16, 342 (1960).
[70] J. Lagnaux, T. Lemonne, J. Sacton, E. Fletcher, D. O'Sullivan, T. Shah, A. Tompson, P. Allen, D. Davis, S. Heenan, A. Montwill, J. Allen, D. Garbutt, V. Bull, P. March, M. Yaseen, T. Pniewski and J. Zakrzewski, Nucl. Phys. 60, 97 (1964).
[71] J. Lemonne, C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. Stanley, P. Allen, D. Davis, E. Fletcher, D. Garbutt, M. Shaukat, J. Allen, V. Bull, A. Conway, and P. March, Phys. Lett. 18, 354 (1965).
[72] Y. Prakash and S. P. Goel, Nuovo Cimento 51A, 340 (1967).
[73] D. P. Goyal, Nucl. Phys. 83, 639 (1966).
[74] B. Ram and W. Williams, Phys. Lett. 31B, 49 (1970).
[75] A. R. Bodmer and S. Sampanthar, Nucl. Phys. 31, 251 (1962).
[76] B. W. Downs and W. E. Ware, Phys. Rev. 133B, 134 (1964).
[77] B. Ram and B. W. Downs, Phys. Rev. 133B, 420 (1964).
[78] B. Ram, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 912.
[79] J. Dabrowski and M. J. M. Hassan, ibid., p. 776.
[80] A. R. Bodmer and D. M. Rote, ibid., p. 521.
[81] J. Dabrowski and H. S. Köhler, Phys. Rev. 136B, 162 (1964).
[82] B. W. Downs and M. E. Grypeos, Nuovo Cimento 44B, 306 (1966).
[83] M. E. Grypeos, L. P. Kok and S. Ali, Nucl. Phys. B4, 335 (1968).
[84] G. Mueller and J. W. Clark, Nucl. Phys. B7, 227 (1968).
[85] G. Mueller and J. W. Clark, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 598.
[86] J. Pniewski and M. Danysz, Phys. Lett. 1, 143 (1962).
[87] L. R. B. Elton, Phys. Lett. 5, 96 (1963).
[88] J. Law, Nuovo Cimento 38, 807 (1965).
[89] M. A. K. Lodhi, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 896.
[90] M. Rayet, reference [4] in[91].
[91] R. H. Dalitz and A. Gal, Nucl. Phys. B1, 1 (1967).
[92] J. Pniewski, Z. Szymanski, D. H. Davis and J. Sacton, Nucl. Phys. B2, 317 (1967).
[93] D. H. Davis and J. Sacton, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 159.
[94] M. Danysz, K. Carbowska, J. Pniewski, T. Pniewski, J. Zakrzewski, E. R. Fletcher, J. Lemonne, R. Renard, J. Sacton, W. T. Toner, D. O'Sullivan, T. P. Shah, A. Thompson, P. Allen, M. J. Beniston, D. H. Davis, D. A. Garbutt, V. A. Bull, R. C. Kumar, P. V. March, Sr. M. Heeran, A. Montwill and J. E. Allen, Phys. Rev. Lett. 11, 29 (1963), Nucl. Phys. 49, 121 (1963).
[95] D. J. Prouse, Phys. Rev. Lett. 17, 782 (1966).
[96] V. A. Filimonov, Usp. Fiz. Nauk 92, 535 (1967).
[97] J. J. de Swart, Phys. Lett. 5, 58 (1963).
[98] A. Deloff and J. Wrzecionko, Nuovo Cimento 34, 1195 (1964).
[99] M. Rimpault and R. Ving Mau, Nuovo Cimento 35, 85 (1965).
[100] B. W. Downs and R. J. Phillips, Nuovo Cimento 36, 120 (1965).
[101] S. Bose and H. C. Khare, Nuovo Cimento 36, 805 (1965).
[102] R. H. Dalitz, Phys. Lett. 5, 53 (1963).
[103] R. H. Dalitz and G. Rajasekaran, Nucl. Phys. 50, 450 (1964).
[104] A. Bodmer and S. Ali, Phys. Rev. 138B, 644 (1965).
[105] Y. C. Tang and R. C. Herndon, Phys. Rev. 138B, 637 (1965).
[106] Y. C. Tang and R. C. Herndon, Nuovo Cimento 46B, 117 (1966).
[107] K. Monga and A. N. Mitra, Nuovo Cimento 42A, 1004 (1966).
[108] S. Ali and A. R. Bodmer, Nuovo Cimento 50A, 511 (1967).
[109] S. Ali and L. P. Kok, Nucl. Phys. B3, 547 (1967).
[110] S. Ali, L. P. Kok and M. E. Grypeos, Phys. Rev. 171, 1203 (1968).
[111] K. Ananthanarayanan, Nuovo Cimento 55B, 419 (1968).

[112] H. Nakamura, Prog. Theor. Phys. **30**, 84 (1963).
[113] P. S. Damle and S. N. Biswas, Nuovo Cimento **32**, 381 (1964).
[114] G. Alexander, A. Gal and A. Gersten, Nucl. Phys. **B2**, 1 (1967).
[115] U. Lynen, Proc. of Int. Conf. on Hypernuclear Physics, Argonne, 1969, p. 193.
[116] G. R. Burleson, ibid., p. 639.
[117] K. Nishijima, Fundamental Particles, Benjamin, 1963.

Translated by J. G. Adashko