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# LINEAR TRANSFORMATION AND ABSORPTION OF WAVES IN A PLASMA

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CONTENTS



## INTRODUCTION

As is well known, there exist two linear mechanisms of collisionless absorption of waves in a plasma—cyclotron damping and Landau damping. Landau damping is important for waves whose phase velocity is close to the thermal velocity of the charged particles. Under ordinary conditions of the experiments on the interaction of high-frequency waves with a plasma, this criterion is satisfied for the so-called plasma waves, the propagation of which is possible only at nonzero temperature (when the temperature tends to zero, their refractive indices become infinite). Therefore only plasma waves can be effectively absorbed outside the cyclotron-resonance region. In a homogeneous plasma, the electromagnetic waves excited from the outside are not coupled to the plasma waves and propagate independently of them. However, in a real inhomogeneous plasma, a coupling may arise between these waves if there exists in the plasma a region where the refractive indices of the waves are close to each other. Under such conditions, the electromagnetic wave incident on the plasma from the outside becomes transformed into a plasma wave, which is effectively absorbed. Thus, collisionless absorption of high-frequency waves in a plasma in a wide range of conditions (outside the region of cyclotron resonances) is connected with wave transformation. The transformation also strongly influences the plasma radiation, since the plasma waves cannot be radiated directly.

Interest in linear transformation of waves in a plasma was first stimulated in connection with ionospheric and astrophysical research. Recently, this interest was again revived in connection with experiments on collisionless absorption and emission of waves by a laboratory plasma and by the proposed use of collisionless absorption to heat a plasma in a magnetic trap. The investigations of the most recent years led to considerable development in the theory of linear transformation of waves and to establishment of the correspondence between this theory and experiments on wave absorption by a plasma. The present article is devoted to an exposition of the results of these investigations.

## 1. THEORY OF LINEAR WAVE TRANSFORMATION IN A PLASMA

The main features of the wave-transformation process were elucidated in the first theoretical papers devoted to the propagation of radio waves in the ionosphere and to the problem of radio emission from the sun and from the planets. These investigations yielded a number of quantitative results pertaining to a weakly-inhomogeneous isotropic plasma[1-4], and also to a magnetoactive plasma with the magnetic field perpendicular to the concentration gradient[5-7]. A review of this group of investigations is contained in the book[8] (see also[9]). A more detailed investigation of the problem of transformation of waves in an isotropic plasma is given in subsequent papers[10-13].

A significant step towards understanding of the problem of transformation was made in[14], and particularly in[15]. In these papers, the connection between this problem and the theory of asymptotic solutions of differential equations with a small parameter preceding the highest-order derivative was analyzed in these papers[16]. The conditions under which complete wave transformation takes place were also determined in[15]. Finally, a recent paper[17] contains a more general analysis of the transformation problem for a plasma layer at an arbitrary direction of the magnetic field and an arbitrary wave incidence angle.

Besides general investigations, a number of concrete calculations were performed recently to determine the transformation efficiency in different cases[18-25]. Some results were obtained in connection with the so-called "volume" resonances of plasma waves[26-29].

We present below the theory of linear transformation of electromagnetic waves into plasma waves for the case when the plasma concentration depends on one coordinate, and give a summary of the numerical results obtained to date*.

### 1.1. Electromagnetic and Plasma Waves

As is well known, the propagation of a monochromatic wave $\mathbf{E}(\mathbf{r})\exp(-i\omega t)$ in an arbitrary medium is described by the wave equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E}-\frac{\omega^2}{c^2}\mathbf{D}=0, \tag{1}$$

where $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{j}/c$ is the induction vector and $\mathbf{j}$ is the density of the current produced by the wave. The thermal motion of the particles causes the connection

*There is at present no transformation theory for a plasma whose parameters depend on two or three coordinates. The first investigation pertaining to a plasma with two-dimensional inhomogeneity has reported only recently [113,114].

between **D** and **E** to be nonlocal, namely, the current at a given point **r** depends on the value of the field in a certain region around this point. The nonlocal connection between **D** and **E** is frequently referred to as spatial dispersion.

We consider a plasma whose concentration n depends on one coordinate x. The plasma is situated in a homogeneous external magnetic field **H** lying in the (X, Z) plane and making an arbitrary angle $\alpha$ with the X axis. In such a medium there can propagate waves of the type $\mathbf{E}(\mathbf{r}) = \mathbf{E}(x)\exp(ik_y y + ik_z z)$, where $k_y$ and $k_z$ are constant quantities (in contrast to a homogeneous medium), the angle $\theta$ between the wave vector and the magnetic field depends in this case on the coordinates). The induction vector $\mathbf{D}(\mathbf{r})$ also takes the form $\mathbf{D}(x)\exp(ik_y y + ik_z z)$; the most general expression for the linear nonlocal connection between $\mathbf{D}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})$ is the integral relation

$$D_\alpha(\mathbf{r}) = \int \varepsilon_{\alpha\beta}(x, \mathbf{r}-\mathbf{r}')\, E_\beta(\mathbf{r}')\, d\mathbf{r}', \tag{2}$$

where the form of the tensor $\epsilon_{\alpha\beta}(x, \mathbf{r}-\mathbf{r}')$ is determined by the properties of the plasma. Owing to the complicated character of the wave equation, it cannot be solved exactly, and we consider approximate solutions, assuming first that the plasma inhomogeneity is weak. In this case we can use the geometrical-optics approximation, according to which

$$E_\beta(x) = E_{0\beta}(x)\exp\left\{i\int^x k_x(x')\,dx'\right\}, \tag{3}$$

where $E_{0\beta}$ and $k_x$ are slowly varying functions. Substituting (3) in (2) and (1), we find that in first approximation the quantities $E_{0\beta}$ satisfy the equations

$$\left[k^2\delta_{\alpha\beta} - k_\alpha k_\beta - \frac{\omega^2}{c^2}\varepsilon_{\alpha\beta}(x, \mathbf{k})\right]E_{0\beta} = 0, \tag{4}$$

where

$$\varepsilon_{\alpha\beta}(x, \mathbf{k}) = \int \varepsilon_{\alpha\beta}(x, \mathbf{r}-\mathbf{r}')\, e^{-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}\, d\mathbf{r}' \tag{5}$$

and **k** is a vector with components $k_x$, $k_y$, and $k_z$.

In a homogeneous medium, Eqs. (4) are exact, and the tensor $\epsilon_{\alpha\beta}$ (5) represents the usual dielectric tensor calculated with allowance for the spatial dispersion. The system of homogeneous equations (4) has a nonzero solution only when its determinant $\Delta$ vanishes. The equation $\Delta = 0$, the so-called dispersion equation, determines the value of $k_x$ for given values of $\omega$, $k_y$ and $k_z$. The explicit form of the dispersion equation is that of a complete equation of fourth degree (formally because its coefficients depend on k via $\epsilon_{\alpha\beta}$):

$$\sum_{n=1}^{4} b_n k_x^n = 0; \tag{6}$$

here

$$b_4 = \varepsilon_{xx}, \quad b_3 = [k_y(\varepsilon_{xy} + \varepsilon_{yx}) + k_z(\varepsilon_{zx} + \varepsilon_{xz})],$$

and the remaining coefficients will not be presented here, since they are quite complicated. The dependence of $\epsilon_{\alpha\beta}$ on $k_x$ is connected with the thermal motion and vanishes at T = 0*. Accordingly, Eq. (6) with $T \neq 0$ is transcendental and has, generally speaking, an infinite number of solutions. On the other hand, if T = 0, then there remain only four solutions, $k_x = k_i$, i = 1, 2, 3, 4. It follows therefore that different solutions of (6) behave differently as $T \to 0$, namely, four of them tend to the "cold" limit $k_i$, and the remaining ones become meaningless; an analysis shows that they become infinite. The waves of the former type will be arbitrarily called electromagnetic or "cold" (these are precisely the waves excited from the outside, and those of the latter type will be called plasma waves. Two of the four electromagnetic waves are ordinary and the other two extraordinary, in accordance with the classification adopted for a homogeneous plasma. Each of these waves, however, corresponds to its own value of the angle $\theta$ between **k** and **H** ($\theta = \theta_i(x)$). As is well known, the approximation based on neglecting the spatial dispersion is usually a good one for electromagnetic waves also at $T \neq 0$. The corresponding condition is that the thermal corrections to the dielectric tensor be small. These corrections can be obtained by expanding $\epsilon_{\alpha\beta}$ in powers of k; it should be borne in mind here that in wave-transformation problems the components of the wave vector $k_y$ and $k_z$ are specified by the boundary condition and are usually of the order of $\omega/c$, whereas $k_x$ varies in the plasma and can reach large values. It can therefore be assumed that the tensor $\epsilon_{\alpha\beta}$ depends effectively only on $k_x$. In the case of a homogeneous medium, the expansion of $\epsilon_{\alpha\beta}$ contains only even powers of $k_x$ and, say for the component $\epsilon_{xx}$, can be written in the form

$$\varepsilon_{xx}(k_x) = \varepsilon_{xx} + \beta N_x^2, \tag{7}$$

where

$$\varepsilon_{xx} \equiv \varepsilon_{xx}(k_x)_{k_x=0}, \quad N_x = \frac{k_x c}{\omega}. \tag{8}$$

The parameter $\beta$ is proportional to $v_{Te}^2/c^2$, where $v_{Te}^2$ is the mean squared thermal velocity of the electron (see p. 418). With the exception of values of $\omega$ close to the cyclotron frequency or its second harmonic, the condition

$$\beta N_x^2 \ll 1 \tag{9}$$

is usually well satisfied when $v_{Te}^2 \ll c^2$ for all "cold" waves.

The corrections to the other components of the tensor $\epsilon_{\alpha\beta}$ have the same order of magnitude. In an inhomogeneous medium there appear in the expansion (7) also terms with odd powers of $k_x$. They are, however, small and do not influence the obtained estimates*.

The inequality (9) means that the field of the wave changes little within the region in which a connection between **E** and **D** appears. In this case, the field $E_\beta$ in

*We speak of temperature only for brevity, since we shall not use henceforth the assumption that the particles have a Maxwellian velocity distribution. The expression T = 0 denotes complete absence of thermal motion of the plasma particles.

*The following remark should be made concerning such estimates. We call the term $\beta N_x^2$ the "thermal" correction, implying by the same token that the first term of the expansion $\epsilon_{xx}(x)$ is the limit as $T \to 0$ of the tensor $\epsilon_{\alpha\beta}(x, k_x)$. At the same time, the equality $\epsilon_{\alpha\beta}|_{T=0} = \epsilon_{\alpha\beta}|_{k=0}$ holds only in the case of a homogeneous plasma. In an inhomogeneous medium, these two quantities do not coincide, but the difference is significant only in an inhomogeneous magnetic field near the cyclotron-resonance points. We shall not consider this case in what follows, and accordingly will not distinguish between $\epsilon_{\alpha\beta}$ at T = 0 and $\epsilon_{\alpha\beta}$ at k = 0.

(2) can be taken outside the integral sign at the point $\mathbf{r}' = \mathbf{r}$, after which it turns out to satisfy the "cold" wave equation

$$(\mathrm{rot\,rot}\,\mathbf{E})_\alpha - \frac{\omega^2}{c^2}\varepsilon_{\alpha\beta}(x)E_\beta(x) = 0, \tag{10}$$

where

$$\varepsilon_{\alpha\beta}(x) = \int \varepsilon_{\alpha\beta}(x, \mathbf{r}-\mathbf{r}')\,d\mathbf{r}' = \varepsilon_{\alpha\beta}(x, k_x)_{k_x=0}.$$

Thus, solutions for which the spatial dispersion plays no role can also exist in the case when the geometrical-optics approximation is not valid (i.e., for a strongly inhomogeneous plasma). The condition for this is, as before, the inequality (9), where $N_x$ should be taken to mean the "effective refractive index"

$$N_{\mathrm{eff}} = \frac{c}{\omega\lambda_0}; \tag{11}$$

$\lambda_0$ is the characteristic spatial scale of field variation. The system (10) is equivalent to a fourth-order equation, so that in the general case, at given values of $\omega$, $k_y$, and $k_z$, there exist four independent electromagnetic waves $\mathbf{E}_c$. These waves do not exhaust all the solutions of the exact wave equation (1)—(2). The remaining solutions, the plasma waves $\mathbf{E}_p$, have a very large refractive index, and their field can be regarded as potential*. The wave equation for them is the Poisson equation

$$\mathrm{div}\,\mathbf{D} = 0, \quad \mathbf{E} = -\nabla\varphi. \tag{12}$$

Thus, the general solution of the wave equation (1)—(2) is a superposition

$$\mathbf{E} = \mathbf{E}_c + \mathbf{E}_p \tag{13}$$

of solutions of the simple equations (10) and (12).

The refractive indices of the electromagnetic waves are functions of x. Typical $N_{xi}(x)$ plots, the so-called dispersion curves) are shown in Fig. 1. We see that at T = 0 one of the refractive indices $N_{x4}$ becomes infinite at a certain point $x = x_0$. The position of this point can easily be found by noting that if $k_x \to \infty$, then only the first two terms play a role in (6). Therefore $k_4 = -b_3/\epsilon_{xx}$ and the position of the poles of the refractive index is determined by the condition

$$\varepsilon_{xx}(x) = 0. \tag{14}$$

In the vicinity of the root $x_0$ of this equation, the condition (9) is violated, i.e., spatial dispersion becomes significant. Since the dispersion curves of Fig. 1a were obtained without taking the spatial dispersion into account at all, it is natural to take into account, as the next step, the thermal corrections, which are assumed to be small, i.e., the expansion (7) is used. With such an approach, it suffices to take into account the corrections only in the component $\epsilon_{xx}$, since here the principal term of the expansion is small. In a small vicinity of the point $x_0$ we can approximate $\epsilon_{xx}(x)$ by the linear relation

$$\varepsilon_{xx}(x) = \frac{x_0 - x}{l}, \quad \frac{1}{l} = -\left.\frac{\partial\varepsilon_{xx}}{\partial x}\right|_{x=x_0} = \left(\frac{1}{n}\frac{dn}{dx}\right)_{x=x_0}, \tag{15}$$

*The potential nature of waves with large refractive index follows directly from the wave equation. The term $(\omega^2/c^2)$ curl curl E, which contains the second derivatives of the field with respect to the coordinates, is of the same order of magnitude as $N^2E$, whereas $D \approx N_0^2 E$, where $N_0$ is the "cold" refractive index. If $N \gg N_0$, equality is possible only if curl E is anomalously small.

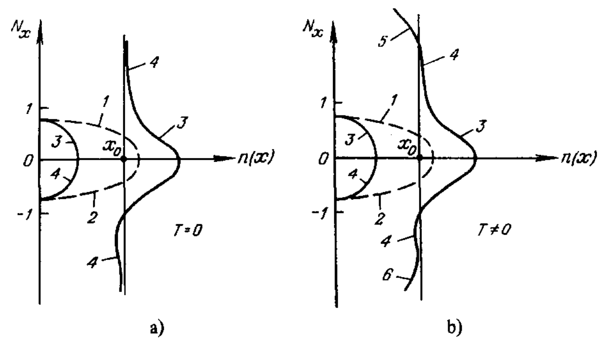


FIG. 1. Dispersion curves $N_x(n)$ at $\alpha \neq 0, \pi/2$. Dashed curves 1 and 2 correspond to ordinary waves, and continuous curves 3 and 4 for extraordinary waves, while curves 5 and 6 correspond to plasma waves.

and the remaining components of the tensor $\epsilon_{\alpha\beta}$ can be regarded as constant. Substituting (7) and (15) in the dispersion equation, we see that it now constitutes a sixth-order equation with a small coefficient at the highest power of $k_x$. The three roots of this equation having the smallest absolute values are close to the three first roots of the "cold" equation, and the remaining roots can be obtained by retaining the first three terms in the equation. The dependence of these roots on x (with $\beta > 0$) is shown in Fig. 1b. At sufficiently large $|x - x_0|$ we have

$$k_4 = \frac{\sigma}{x - x_0}, \quad k_{5,6} = \pm\frac{\omega}{c}\left(\frac{x_0 - x}{\beta l}\right)^{1/2} + \frac{1}{2}\frac{\sigma}{x_0 - x}, \quad \sigma = lb_3(x_0). \tag{16}$$

The root $k_4$ coincides with the corresponding root of the "cold" equation and the roots $k_5$ and $k_6$ correspond to plasma waves (they tend to infinity as $T \to 0$). As seen from the figures, the "cold branch" goes over continuously into plasma branches. This means that in the inhomogeneous medium, the oscillations of different types are coupled to one another: the electromagnetic wave traveling towards the point $x_0$ is partly reflected in the form of a plasma wave (and vice versa). It is this phenomenon that constitutes the linear transformation of the waves.

An analysis shows[16] that the connection between the electromagnetic and plasma waves in the vicinity of the point $x_0$, determined by Eq. (14), arises also in the case when the electromagnetic waves cannot be described in the geometrical-optics approximation, and therefore the concept of the refractive index $N_x(x)$ becomes meaningless. Namely, the point $x_0$ is a singular point of the "cold" wave equation (10), at which the field component $E_x$ becomes infinite (see (19)), so that the condition (11) is violated also in the general case.

Thus, representation of the solution in the form of a superposition of electromagnetic and plasma waves (13) is valid everywhere with the exception of the vicinity of the point $x_0$. In this vicinity, the separation into modes is impossible, and we have a certain single solution. For this reason, the vicinity of the point $x_0$ can be called the region of mode interaction or the transformation region. In order to obtain the complete solution of the wave equation, it is necessary to match expression (13) to the solution in the interaction region. The matching condition results in a definite connection between the amplitudes of the electromagnetic and the plasma waves outside the interaction region, and this indeed solves the transformation problem.

From the mathematical point of view, expression (13) is an asymptotic representation of the exact solutions outside the interaction region. Such representations have, generally speaking, different forms to the right and to the left of the point $x_0$ (the Stokes phenomenon). In our case this means that if, for example, $\mathbf{E}_c$ in (13) represents a certain linear combination of the four electromagnetic waves considered above, then the coefficients in this combination can be different in the regions $x < x_0$ and $x > x_0$. Owing to the large number of unknown constants, their direct determination from the matching conditions is a complicated problem. The results can be obtained in a much simpler and clearer manner by considering the behavior of the solutions not only for real x, but also in the entire complex plane.

### 1.2. Coupling Between Electromagnetic and Plasma Waves

To fulfill the program undertaken, it is necessary first to determine the forms of the solutions $\mathbf{E}_p$ and $\mathbf{E}_c$ near the transformation region. For a plasma wave this problem has in fact already been solved, since for this wave the geometrical-optics approximation is valid outside the transformation region (owing to the large refractive index), and the solution of the dispersion equation is given by formulas (3) and (16). Introducing in place of x the natural variable

$$\xi = \frac{x - x_0}{\gamma l}, \quad \gamma = \left[\frac{c^2\beta}{\omega^2 l^2}\right]^{1/3}, \tag{17}$$

and choosing the normalization in such a way that the energy flux transported by the wave is equal to unity, we have in the transparency region for the plasma waves ($x < x_0$, $\xi < 0$)

$$E_{px}^{\pm} = \left(\frac{8\pi}{\omega l}\right)^{1/2} \frac{i}{\gamma^{1-i\sigma}} (-\xi)^{-1/4} \exp\left\{\pm i\,\frac{2}{3}(-\xi)^{3/2} - \frac{i\sigma}{2}\ln(-\xi)\right\}. \tag{18}$$

The plus sign corresponds here to the root $k_6$, and the minus sign to the root $k_5$ of Eq. (16).

In order to find the limiting form (as $x \to x_0$) of the solutions $\mathbf{E}_c$, it is necessary to solve the "cold" wave equation in the vicinity of the point $x_0$. It is natural to expect here that the "effective refractive index" will be very large in this region, and accordingly to assume that the field is potential, $\mathbf{E}_c = -\nabla\varphi$, $\varphi = \Phi(x)\exp[ik_y y + ik_z z]$. In the Poisson equation (12), as in the investigation of the dispersion equation in Sec. 1.1, we can use expression (15) for $\epsilon_{xx}$, and regard the remaining components $\epsilon_{\alpha\beta}$ as constant. Retaining only the terms containing derivatives of the rapidly varying function $\Phi$, we have

$$(x - x_0)\Phi'' + (1 - i\sigma)\Phi' = 0,$$

whence $\Phi \sim (x - x_0)^{i\sigma}$. The expression obtained pertains to a wave of the fourth type; the remaining electromagnetic waves have no singularities at the point $x_0$, and their field can be assumed constant in a small vicinity of this point. Thus, as $x \to x_0$

$$\left.\begin{aligned} E_{cx} &\to B\left(\frac{l}{x_0 - x}\right) e^{-i\sigma \ln \frac{l}{x_0 - x}} + \mathscr{E}_x, \\ E_{cy} &\to B\,\frac{k_y l}{\sigma}\, e^{-i\sigma \ln \frac{l}{x_0 - x}} + \mathscr{E}_y, \\ E_{cz} &\to B\,\frac{k_z l}{\sigma}\, e^{-i\sigma \ln \frac{l}{x_0 - x}} + \mathscr{E}_z, \end{aligned}\right\} \tag{19}$$

where B and $\mathscr{E}_\alpha$ are constants determined by the boundary conditions. This result was first obtained in[30]*. It follows from (19) that the point $x_0$ is a singular point of the "cold" wave equation regardless of the applicability of the geometrical-optics approximation. It constitutes a branch point of the functions $E_{c\alpha}$. It turns out as a result that the solution of the "cold" Equation (10) is not determined uniquely by the boundary conditions, which in problems involving the reflection of waves from a plasma layer specify the amplitudes and polarizations of the incident waves. To verify this, we assume that rare collisions with frequency $\nu \ll \omega$ occur in the plasma, and we calculate the energy absorbed in this case in the cold-plasma layer. Taking the collisions into account, we obtain for $\epsilon_{xx}$ an imaginary increment $\epsilon''_{xx} \sim i\nu/\omega$, and accordingly $x_0 = x_0' + ix_0''$, where $x_0'' \sim \nu l/\omega$. The power absorbed in one $cm^2$ of the surface of the layer is given by

$$Q = \frac{\omega}{8\pi}\int \epsilon''_{\alpha\beta} E_\alpha E_\beta^* \, dx,$$

where the integration is over the entire thickness of the layer and $\epsilon''_{\alpha\beta}$ is the antihermitian part of the dielectric tensor. If $\nu$ tends to zero, the main contribution is made by the term $\epsilon''_{xx}|E_x|^2$, since $E_x$ has the strongest singularity. Substituting the expression (19) for $E_x$ and extending the integration limits to infinity, we find

$$Q = \frac{\omega l}{8\pi}|B|^2 \frac{1 - \exp(-2\pi\sigma)}{2\sigma}. \tag{20}$$

The absorbed power turns out to be independent of $\nu$, and we can therefore assume that there is finite absorption also at $\nu = 0$. This result is well known[8] and is intuitively explained by the fact that the decrease of absorption as $\nu \to 0$ is offset by the increase of the maximum value of $E_x$.

If we consider now the solution that satisfies the same boundary condition, but assume that $\nu < 0$, then we can obtain in perfectly similar fashion

$$Q = -\frac{\omega l}{8\pi}|B'|^2 \frac{\exp(2\pi\sigma) - 1}{2\sigma}. \tag{21}$$

Expressions (20) and (21) differ in sign, so that formula (21) describes generation of energy. We see thus that in a cold collisionless plasma, under the same boundary conditions, it is possible to obtain two perfectly different solutions, depending on the method of going to the limit $\nu = 0$. In essence, these two solutions differ in the method of going around the singular point $x_0$: in the case corresponding to (20), the singular point was shifted upward from the real axis and, by continuing the solution along the real axis, we circuit if from below; in the second case the circuiting is from above.

As already indicated, we shall need to consider in what follows the behavior of the solutions at complex values of the argument. To eliminate ambiguity, it is necessary to draw in this case a cut on the complex x plane from the point $x_0$ to infinity. If the cut is drawn in the upper half-plane, then it is possible to circuit the singular point only from below, and the corresponding solution describes energy absorption; if the cut is in the lower half-plane, we have "generation" of energy. The

*The singularity of the electric field of the wave was first investigated for particular cases in [31,32].

solutions satisfying the same boundary conditions and differing only in the position of the cut will be designated $E^u$ and $E^l$ (the superscript u corresponds to a cut in the upper half-plane). It is obvious that the solution $E^u$ is continuous in the lower half-plane, and $E^l$ is continuous in the upper x half-plane.

We now proceed to determine the field in the transformation region and match it to the asymptotic expressions (18) and (19).

If we assume, as in Sec. 1.1, that the weak-dispersion approximation is applicable in the transformation region, then it suffices to take into account the thermal corrections only to the component $\epsilon_{xx}$. Without assuming applicability of the geometrical-optics approximation, we should in this case replace $k_x^2$ in (7) by $-d^2/dx^2$, and we consequently have

$$D_x = \varepsilon_{x\alpha}E_\alpha - \frac{c^2\beta}{\omega^2}E_x'', \quad D_y = \varepsilon_{y\alpha}E_\alpha, \quad D_z = \varepsilon_{z\alpha}E_\alpha.$$

Substituting this expression in (12), using (15), and retaining only the terms with the derivatives of $\Phi$, we obtain

$$\frac{\beta l c^2}{\omega^2}\frac{d^3W}{dx^3} - (x - x_0)\frac{dW}{dx} - (1 - i\sigma)W = 0, \tag{22}$$

where $W = d\Phi/dx$, so that $E_x = C_1W + C_2$, where $C_1$ and $C_2$ are constants.

Introducing a new variable $\xi$ in accordance with (17), we can rewrite (22) in the form

$$W''' - \xi W' - (1 - i\sigma)W = 0, \tag{23}$$

where the prime denotes differentiation with respect to $\xi$. The three independent solutions, $W_k(\xi)$, $k = 1, 2, 3$, of Eq. (23) can be obtained by the Laplace-transform method; they are given by

$$W_k(\xi) \sim \int\limits_{C_k} t^{-i\sigma} \exp\left\{\xi t - \frac{t^3}{3}\right\} dt, \tag{24}$$

where the integration contour $C_k$ emerges from the origin and goes off to infinity along the line $\arg t = 2\pi k/3$. (In the t plane, a cut is drawn along the line $\arg t = \pi/3$.) The behavior of the functions $W_k(\xi)$ at $|\xi| \gg 1$ was investigated in[16]. For one of the solutions, $W_2$, this behavior turns out to be as follows. There is a sector P on the complex $\xi$ plane (Fig. 2), in which the asymptotic form of $W_2$ contains a term

$$\frac{i\sqrt{\pi}}{\xi^{1/4}}\exp\left\{-\frac{2}{3}\xi^{3/2} - \frac{i\sigma}{2}\ln\xi\right\},$$

that coincides, apart from a constant factor, with the plasma wave $E_p^-$ (18). At real negative $\xi$, Eq. (24) describes a wave traveling away from the point $x_0$.

In addition to the sector P, there is on the complex

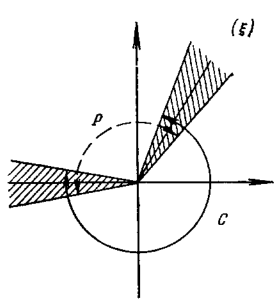


FIG. 2. Asymptotic behavior of the function $W_2(\xi)$ at $|\xi| \gg 1$. The shaded areas are those in which the asymptotic form of $W_2$ is a superposition of the "cold" and plasma waves.

plane a sector C (Fig. 2) in which the asymptotic form of $W_2$ contains a term proportional to $\xi^{-1+i\sigma}$, i.e., having the same dependence on the x coordinate as the "cold" wave (24). It is important that this "cold" part of the asymptotic form is continuous in the entire sector C, which includes both the positive and negative parts of the real axis.

In the narrow regions where the sectors P and C overlap, the asymptotic expression for $W_2$ is a superposition of both terms with perfectly defined coefficients.

The asymptotic behavior of the two other solutions $W_1$ and $W_3$ is similar in character, but differs in the placements of the sectors P and C, namely, for the solution $W_1$ the sector P is turned through an angle $2\pi/3$ compared with Fig. 2, and for $W_3$ it is turned through an angle $-2\pi/3$. Accordingly, $W_1$ contains at large negative values of $\xi$ a plasma wave propagating towards the point $x_0$, and the "cold" part of its asymptotic form is continuous in the upper $\xi$ half-plane. The solution $W_3$ increases exponentially at real positive $\xi$.

We see from the foregoing that the transformation of the electromagnetic wave into a plasma wave is described by the solution $W_2$, which does not contain an incident plasma wave. Since the "cold" part of $W_2$ is continuous in the entire lower $\xi$ half-plane, this function can be matched only with a "cold" solution of the type $E^u$, having the same property (a solution of the type $E^l$ experiences a discontinuity on the cut in the lower half-plane).

Comparing the asymptotic form of $W_2$ with formulas (18) and (19), we can obtain final expressions for the solution of the wave equation describing the transformation of electromagnetic waves into plasma waves. These expressions have the following form[17]: at $x < x_0$ in the region of transparency for the plasma wave

$$E_x = E^u - \left(\frac{l\omega}{8}\right)^{1/2}\frac{B^u \exp\left(-\frac{\pi\sigma}{2}\right)}{\Gamma(1 - i\sigma)}E_p^-, \tag{25}$$

at $x > x_0$

$$E_x = E^u, \tag{26}$$

in the interaction region we have

$$E_x = -\frac{B^u \exp(-\pi\sigma)}{\Gamma(1 - i\sigma)\gamma^{1-i\sigma}}W_2(\xi) + \mathscr{E}_x^u; \tag{27}$$

and the remaining components can be obtained from the condition that the field is potential; they turn out to be small quantities of the order of $\gamma$ compared with $E_x$. In formulas (25) and (26), $E^u$ is the same solution of the "cold" equation (10).

The expressions presented pertain to the case $\beta > 0$. When $\beta < 0$, the transparency region for the plasma waves lies to the left of the point $x_0$; formulas (25) and (26) remain in force also in this case, provided the substitution $\gamma \to |\gamma|$ is made.

The obtained formulas lead to the following conclusion. Since plasma waves cannot propagate in vacuum, the form of the field outside the plasma and, in particular, the amplitudes and polarizations of the reflected waves are determined completely by the "cold" solution $E^u$ and can be obtained consequently without taking spatial dispersion into account at all. In other words, if a solution is obtained for the reflection of waves from a certain layer of cold plasma (with account taken of the rule of circuiting around the singular point), then this solution is also applicable to a hot plasma (provided only the conditions (29) and (30) below are satisfied). The solution $\mathbf{E}^u$ describes, as we have seen, absorption of energy in the plasma. When account is taken of the spatial dispersion, however, the energy is not absorbed at the singular point, and is carried away completely by the plasma wave. This is seen directly from formula (25), since the plasma-wave energy is exactly equal to Q (20) (it is necessary to use in the comparison the formula $|\Gamma(1-i\sigma)|^2 = \pi\sigma/\sinh\pi\sigma)$*. If we introduce in place of Q the dimensionless absorption coefficient $A_0$, equal to the ratio of Q to the energy flux of the incident wave, and define the energy transformation coefficient $A_T$ analogously, then we get the equality

$$A_T = A_0. \tag{28}$$

The quantity $A_0$, which is proportional to the square of the modulus of the residue at the singular point $x_0$, depends on the polarization of the incident wave. For each incidence direction there exist two independent polarizations. One can be chosen such that $B^u = 0$. The corresponding wave experiences no transformation, while the wave polarized orthogonally to it is transformed to a maximum degree.

A solution describing the transformation of a plasma wave into an electromagnetic wave can be constructed in perfectly similar fashion, using the function $W_1$ in the interaction region; this function is matched to the "cold" solution of the type $\mathbf{E}^l$. We note that the coefficient of transformation of a plasma wave into an electromagnetic wave can be obtained by the reciprocity theorem—it is equal to the coefficient of transformation of the electromagnetic wave into a plasma wave, calculated for the opposite direction of the magnetic field. A situation is possible wherein the plasma wave produced upon transformation of an electromagnetic wave is not absorbed in the plasma but reaches a second singular point or is reflected and is then retransformed into an electromagnetic wave. This gives rise to a large phase shift (much larger than $2\pi$) between the wave directly reflected from the layer and the wave resulting from the transformation. This shift depends very strongly on the frequency, leading to a sharp (resonant) frequency dependence of the wave-scattering amplitude. Such a phenomenon is observed experimentally in small-dimension plasmas. These resonances will be considered in greater detail in Sec. 2.1.

Formulas (25)–(28) are also valid in the presence of collisions, provided only the ratio $\nu/\omega$ is so small that we can neglect the "ordinary" collisional absorption, i.e., the one not connected with the transformation. Then the asymptotic form of the solutions in vacuum does not depend on the ratio of the small parameters $\gamma$ and $\nu/\omega$, which can be arbitrary (both larger and smaller than unity). However, the form of the field inside the layer depends very significantly on this ratio. When $\nu/\omega \ll 1$, allowance for the collisions reduces to assuming that $x_0$ is the complex quantity, so that real x correspond to complex $\xi$ lying in the lower half-plane:

$$\xi = \xi' + i\xi'', \quad \xi' = \frac{x - x_0'}{\gamma l}, \quad \xi'' = -\frac{x_0''}{\gamma l} \approx -\frac{\nu}{\gamma\omega}.$$

If $\nu/\omega \gg \gamma$, then $\xi'' \gg 1$ and we have $|\xi| \gg 1$ on the entire real axis, with the entire axis lying outside the sector P. In this case only the "cold" part of the asymptotic expression remains in the expression for $W_2$ on the real axis, and the field has the form $\mathbf{E} = \mathbf{E}^u$ for all real x. Thus, in the case when $\nu/\omega \gg \gamma$ it is not important to take spatial dispersion into account.

It follows therefore that at an arbitrarily small $\nu \neq 0$ and $T \to 0$ the solution containing no incident plasma waves tends to the limit $\mathbf{E}^u$. We can therefore state that energy absorption in the pole of the refractive index is the "cold" limit of the transformation process. A solution containing the plasma wave $\mathbf{E}_p^+$ has no "cold" limit.

In the opposite limiting case $\nu/\omega \ll \gamma$ we can neglect the influence of the collisions in the transformation region and assume that $\xi$ in (27) is real. Outside the interaction region, so long as the weak-dispersion approximations are valid, the wave amplitude decreases in proportion to the factor $\exp(-\nu\sqrt{\xi'}/\omega\gamma)$ (we have expanded $(\xi' + i\xi'')^{3/2}$ in the exponent of (18), assuming that $\xi'' \ll \xi'$). When the wave propagates from the point $x_0$, its refractive index increases rapidly and the weak-dispersion approximation becomes invalid. In the case of strong dispersion the plasma waves are rapidly damped (by the Landau mechanism), with the exception of the case $\alpha = \pi/2$.

In order for all the results obtained in this section to be valid, it is essential to satisfy the condition

$$\gamma = \left(\frac{c^2\beta}{\omega^2 l^2}\right)^{1/3} \ll 1. \tag{29}$$

This inequality ensures validity of the weak-dispersion approximation in the mode-interaction region; it also guarantees satisfaction of the inequality (9) outside this region. One more condition results from a comparison of the dimensions of the interaction region with those of the region in which the "cold" solutions have the simple form (19):

$$\gamma \frac{l}{\lambda_0} \ll 1, \tag{30}$$

where $\lambda_0$ is the characteristic scale of variation of the field far from the singular point. The parameter $\gamma$ in (29) and (30) is connected with the quantity $\beta$, which is defined by the expression

$$\beta = \sum_{k=e,i} \frac{v_{T_k}^2 \omega_{p_k}^2(x_0)}{c^2} \left[\frac{3\omega^2}{(\omega^2-\omega_{H_k}^2)(\omega^2-4\omega_{H_k}^2)} \sin^4\alpha + \frac{6\omega^4 - 3\omega^2\omega_{H_k}^2 + \omega_{H_k}^4}{(\omega^2-\omega_{H_k}^2)^3} \sin^2\alpha\cos^2\alpha + \frac{3}{\omega^2}\cos^4\alpha\right],$$

where the summation is carried out over the types of charges, $\omega_{pk} = (4\pi ne^2/m_k)^{1/2}$ is the plasma frequency, and $\omega_{H_k} = eH/m_k c$ is the cyclotron frequency, with $\alpha$ the angle between the concentration gradient and the magnetic field.

In the foregoing analysis we disregarded nonlinear effects. We can estimate the conditions under which such an approach is justified. Since the transformation is not connected with any subtle details of the distribu-

*A similar result for an isotropic plasma of small dimensions was first obtained in [10].

tion function, the influence of the nonlinearity can be regarded as small if

$$v \ll v_T,$$

where v is the translational velocity in the wave field. Assuming as an estimate $v = eE/m_e\omega$, $E = E_0/\gamma$, and $\gamma = (v_{Te}/\omega l)^{2/3}$ ($E_0$ is the field intensity in vacuum), we obtain

$$\frac{eE_0}{mc\omega} \ll \frac{\omega l}{c}\gamma^{5/2}. \tag{31}$$

When $\gamma \ll \nu/\omega$ the quantity $\gamma$ in (31) must be replaced by $\nu/\omega$.

It should be noted that besides the direct influence of the nonlinearity on the wave interaction, there can also occur "slow" nonlinear effects connected with the change of the plasma parameters (concentration, temperature, concentration distribution). These effects can be due, for example, to plasma heating, which results in additional ionization, to the occurrence of plasma instabilities, to the action of the pressure of the high-frequency field of the wave[3], etc. It is hardly possible to take into account the aggregate of such effects in the general case. However, even in the presence of "slow" nonlinearities, the foregoing results remain valid if realistically obtained plasma parameters are employed.

### 1.3. Transformation Conditions

Let us consider now the conditions under which transformation is possible in a given plasma layer. These conditions state that at least one point $x = x_0$, at which $\epsilon_{xx} = 0$ [Eq. (14)], must be contained inside the layer.

In the employed coordinate system (the x axis directed along the concentration gradient, the magnetic field makes an angle $\alpha$ with the x axis) we have

$$\varepsilon_{xx} = \varepsilon \sin^2\alpha + \eta\cos^2\alpha,$$
$$\varepsilon = 1 - \sum_{e,\,i} \frac{\omega_p^2}{\omega^2-\omega_H^2}, \quad \eta = 1 - \sum_{e,\,i}\frac{\omega_p^2}{\omega^2}, \tag{32}$$

where $\epsilon$ and $\eta$ are the diagonal components of the dielectric tensor $\epsilon^0_{\alpha\beta}$, calculated in a coordinate system with the z axis directed along the magnetic field. When account is taken of (32), the equation $\epsilon_{xx} = 0$ can be written in the form

$$\operatorname{tg}^2\alpha = -\frac{\eta(x_0)}{\varepsilon(x_0)}. \tag{33}$$

It coincides with the condition that the refractive index of a wave propagating in a homogeneous plasma with

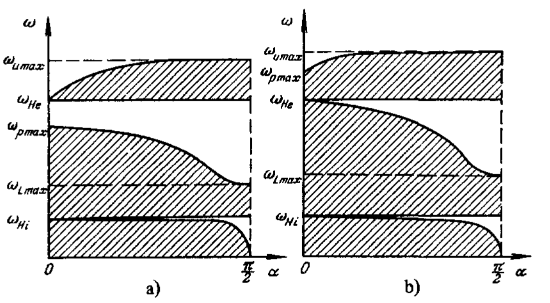


FIG. 3. Transformation bands as functions of the angle $\alpha$. a) $\omega_{pmax} < \omega_{He}$; b) $\omega_{pmax} > \omega_{He}$.

concentration $n(x_0)$ at an angle $\alpha$ to the magnetic field becomes infinite[34]. Such a result is perfectly understandable: in a plasma with one-dimensional inhomogeneity, the components of the wave vector $k_y$, $k_z$ are fixed, and therefore at a point where $k_x$ becomes infinite the wave propagates along the x axis, i.e., precisely at an angle $\alpha$ to the magnetic field.

As is well known[35], Eq. (33) is satisfied at any point with concentration n at three values of the wave frequency $\omega = \omega_i(n, \alpha)$. At $n = 0$ we have $\omega_1 = \omega_{He}$, $\omega_2 = \omega_{Hi}$, $\omega_3 = 0$ and all the frequencies $\omega_i$ increase monotonically with increasing n. Therefore, if the concentration in the layer lies in the range $0 \le n \le n_{max}$, transformation is possible in three frequency bands:

$$\omega_{1\,max} \gg \omega \gg \omega_{He}, \quad \omega_{2\,max} \gg \omega \gg \omega_{Hi}, \quad \omega_{3\,max} \gg \omega \gg 0, \tag{34}$$

where $\omega_{i\,max} = \omega_i(n_{max})$. These bands are shown schematically in Fig. 3. We shall not consider the lowest-frequency band. The positions of the upper and middle frequency bands are determined by the relations

$$\omega_1^2 = \frac{1}{2}\left[\omega_U^2 + \sqrt{\omega_U^4 - 4\omega_p^2\omega_{He}^2\cos^2\alpha}\right], \tag{35}$$
$$\omega_U^2 = \omega_{He}^2 + \omega_p^2;$$
$$\omega_2^2 = \frac{1}{2}\left[\omega_U^2 - \sqrt{\omega_U^4 - 4\omega_p^2\omega_{He}^2\cos^2\alpha}\right] \tag{36}$$

when $\cos^2\alpha \gg m_e/m_i$ and

$$\omega_2^2 = \omega_L^2\left(1 + \frac{m_i}{m_e}\cos^2\alpha\right), \quad \omega_L^2 = \frac{\omega_{He}\omega_{Hi}\omega_p^2}{\omega_p^2 + \omega_{He}^2} \tag{37}$$

when $\cos^2\alpha \ll 1$, $\omega_p^2 \gg \omega_{He}\omega_{Hi}$.

Here $\omega_U$ and $\omega_L$ are respectively the upper and lower hybrid frequencies; $\omega_p \equiv \omega_{pe}$. When $\alpha \ne 0$, the upper and middle bands are separated by a certain frequency interval in which wave transformation is impossible. The limits of the transformation frequency bands are determined by the distribution of the plasma concentration and do not depend on the method of wave excitation. Strictly speaking, this result pertains only to the one-dimensional case. Usually the plasma parameters depend on at least two coordinates, but frequently the dependence on one of them is much weaker than on the other. It can be assumed that the results for a one-dimensional layer are suitable also in this case*.

The question of the transformation efficiency, i.e., the value of the coefficient $A_0$ or $A_T$, becomes much more complicated and we shall discuss it in this section only qualitatively. According to (20) and (28), the transformation efficiency depends on two quantities: the parameter $l$, which determines the relative concentration gradient at the point $x_0$, and the coefficient $B^u$, which at a given incident-wave amplitude characterizes the degree of penetration of the field into the plasma. The roles of both factors depend essentially on the dimensions L of the plasma. If these dimensions are small compared with the average wavelength of the electromagnetic oscillations in the layer

$$k_0 L \ll 1$$

($k_0$ can be defined as the ratio of $\omega/c$ to the value of the

*See [114] concerning the transformation conditions in a plasma with two-dimensional inhomogeneity.

refractive index calculated at typical values of the layer concentration, but far from the singular point $x_0$), then the wave penetrates into the plasma without opposition. The transformation in this case, however, is generally speaking small, owing to the smallness of the parameter $l \sim L$. The only exception is the case when the point $x_0$ lies near the concentration maximum, where $dn/dx \ll n/L$ and, consequently, $l \gg L$. This case is realized only in planar geometry in a narrow frequency band near the upper limit of the transformation regions, and in this case $A_T \sim 1$.

In the opposite limiting case the parameter $l$ is usually large and the transformation efficiency is determined exclusively by the wave penetration (by the barrier effect). If the magnetic field is homogeneous, then there always exists between the boundary of the layer and the point $x_0$ a region in which the transformed electromagnetic wave cannot propagate (see Fig. 1). This leads to the appearance of a "barrier" multiplier $\exp(-\delta)$ in $A_T$, where

$$\delta = \frac{c}{\omega} \int \operatorname{Im} k_x \, dx.$$

The integration is carried out over the entire region where $\operatorname{Im} k_x \neq 0$. It follows therefore that when $k_0L \gg 1$ the transformation efficiency is generally speaking small. The most favorable transformation conditions are those in the intermediate region $k_0L \sim 1$, when the plasma dimensions are of the order of the length of the "cold" wave. In this case the barrier attenuation is relatively small and at the same time the parameter $l$ is not small. One must therefore expect $A_T$ to have a value close to unity in the entire frequency band in which the transformation is possible.

One can indicate cases where the barrier attenuation turns out to be small also in a weakly inhomogeneous plasma at $k_0L \gg 1$.

1. At frequencies close to $\omega_{He}$ or $\omega_{Hi}$, the point $x_0$ tends to the boundary of the plasma, where $n = 0$; accordingly the width of the opacity region also tends to zero.

2. In an isotropic plasma, as is well known, the point $x_0$ is determined by the condition $\omega^2 = \omega_p^2(x_0)$, and the cutoff point, at which $k_x$ vanishes, is determined by the condition $\omega^2 = \omega_p^2(x_0) - \omega^2 k_z^2/c^2$, so that the width of the opacity region lying between them decreases with decreasing $k_z$. However, at $k_z = 0$ there is no transformation, for in this case $E_x \equiv 0$. Consequently, there is a region of optimal values $k_z \ll \omega/c$ (see Fig. 6). An analogous situation takes place also in a magnetoactive plasma at $\omega_{p\max}^2 \gg \omega_{He}^2$ and $\omega^2 \gg \omega_{He}^2$.

3. In the upper frequency band, at $k_z = (\omega/c)\,[\omega_{He}/(\omega_{He} + \omega)]^{1/2} \sin\alpha$ and $k_y = 0$, the dispersion curves of the ordinary and extraordinary waves have a tangency point (Fig. 4), leading to a coupling between these waves and to elimination of the barrier attenuation. Indeed, as seen from Fig. 4, the wave of type I reaches the tangency point without opposition and is partially transformed there into a wave of type II, which is transformed into a plasma wave at the point $x_0$. The coupling between the electromagnetic waves of different types remains also in a certain region of values of $k_z$ and $k_y$ close to the indicated values. This effect was first observed in radio sounding of the ionosphere, where it became manifest in the form of the so-called "signal tripling"[8].

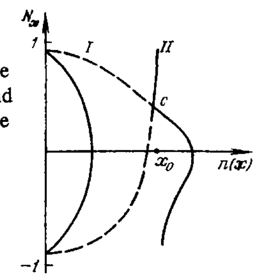


FIG. 4. Vanishing of the barrier in the case of coupling between the ordinary and extraordinary waves. The dispersion curve of the ordinary wave (dashed line) is tangent to the curve of the extraordinary wave (solid line) at the point c.

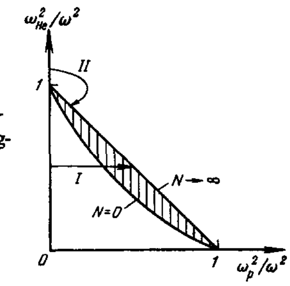


FIG. 5. Accessibility of the transformation region in an inhomogeneous magnetic field.

4. In an inhomogeneous magnetic field, in the upper frequency band, the singular point $x_0$ may turn out to be accessible (the opacity region vanishes) if the magnetic field decreases along the wave propagation direction[36]. This possibility is illustrated in Fig. 5, which shows the lines $N = 0$ and $N \to \infty$ on the $(\omega_{He}^2/\omega^2, \omega_p^2/\omega^2)$ plane for the case of perpendicular propagation of the wave and $\alpha = \pi/2$. The opacity region lying between them is shaded. In a homogeneous magnetic field, the wave "propagates" parallel to the abscissa axis and must cross the opacity region (curve I). In an inhomogeneous magnetic field the wave can reach the point $x_0$ after bypassing the opacity region (curve II). This is possible if the quantity $\omega_U^2 = \omega_p^2 + \omega_{He}^2$ decreases from the plasma boundary towards the point $x_0$.

5. In the middle frequency band at $\alpha = \pi/2$, the point $x_0$ is accessible[34] if

$$k_z^2 > 2\frac{\omega^2}{c^2}\left[1 + \frac{\omega_p^2(x)}{\omega_{He}^2}\right].$$

Such a wave can be produced with the aid of a slow-wave system with spatial period $2\pi/k_z$. In the region between the surface of the slow-wave system and the plasma boundary (more accurately, extending to the surface with the concentration at which $\omega_p = \omega$), the wave field decreases like $\exp[-(k_z^2 - \omega^2/c^2)^{1/2}x]$. If the required slowing-down is small, $k_z^2 \approx 2\omega^2/c^2$, then the characteristic length over which the decrease takes place is relatively large, on the order of the wavelength in vacuum.

In the foregoing cases, the decrease of the barrier "attenuation" leads to transformation with an efficiency on the order of unity.

### 1.4. Results of Calculations of the Transformation Efficiency

Quantitative formulas for $A_T$ were obtained only in a few cases. These are principally cases when the system of "cold" wave equations reduces to a single second-

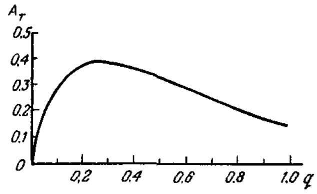


FIG. 6. Transformation coefficient $A_T$ for a weakly inhomogeneous isotropic plasma [11].

order equation, i.e., for an isotropic plasma and for normal incidence ($k_z = k_y = 0$) on a layer of magnetoactive plasma at $\alpha = \pi/2$.

In the case of a weakly inhomogeneous isotropic plasma, the transformation coefficient depends on a single parameter $q = (\omega l/c)^{2/3} \sin^2\theta_0$, where $\theta_0$ is the angle of incidence and $\sin^2\theta_0 = c^2(k_y^2 + k_z^2)/\omega^2$. The function $A_T(q)$ was investigated in several papers. An expression for $A_T$ was obtained in[4] for $q \gg 1$ and in[12] for $q \ll 1$; an expression suitable for all q was obtained in[11] (Fig. 6). A maximum value $A_T = 0.4$ is reached at $q \approx 0.25$.

At normal incidence on a layer of a magnetoactive plasma, the problem reduces to the solution of the equation (see (32))

$$\frac{d^2E}{dx^2} + \frac{\omega^2}{c^2}\left(\frac{\varepsilon^2 - g^2}{\varepsilon}\right)E = 0, \qquad g = i\varepsilon^0_{xy} = \sum_{e,i}\frac{\omega_H\omega_p^2}{\omega(\omega^2 - \omega_H^2)}. \tag{38}$$

If the concentration is linear in x, the solutions of this equation are not expressed in terms of known functions. An approximate solution by the phase-integral method leads to the expression (in the upper frequency band)[7]

$$A_T = 4e^{-\delta_0}(1 - e^{-\delta_0})\sin^2 S,$$
$$\delta_0 = \frac{4\sqrt{2}}{3}\left(\frac{\omega l}{c}\right)u^{3/2}\left[(1+u)K\left(\sqrt{\frac{1-u}{2}}\right) - 2uE\left(\sqrt{\frac{1-u}{2}}\right)\right],$$
$$S = \frac{2\sqrt{2}}{3}\left(\frac{\omega l}{c}\right)u^{3/2}\left[(1-u)K\left(\sqrt{\frac{1-u}{2}}\right) + 2uE\left(\sqrt{\frac{1+u}{2}}\right)\right];$$

here $u = \omega_{He}/\omega$, and K and E are complete elliptic integrals. Generally speaking, the phase-integral method is valid when $\omega l/c \gg 1$. In this case $A_T$ may not be small, provided only that $u \ll 1$. For such conditions we have $S = \delta_0/2 = 1.75\,\omega l u^{3/2}/c$. The dependence of $A_T$ on $\delta_0$ in this approximation is shown in Fig. 7a.

The exact solution of the wave equation (38) can be obtained by approximating $(\epsilon^2 - g^2)/\epsilon$ by some simple function. In[22] the following approximation was used:

$$\frac{\varepsilon^2 - g^2}{\varepsilon} = \begin{cases} 1 + C_0\dfrac{c}{\omega(x - x_0)} & \text{for } x < x_0, \\ C_1^2 + C_0\dfrac{c}{\omega(x - x_0)} & \text{for } x > x_0 \end{cases} \tag{39}$$

and the coefficient $A_T$ was calculated as a function of $C_0$ and $C_1$ (see Fig. 7b)*. At $C_1 = 1$, Eq. (38) was investigated in[37] (see also[8]). The approximation (39) imitates a dependence of the concentration on x such that n increases smoothly from zero at $x \to -\infty$ to a constant value $n_\infty$ at $x \to \infty$, with $n_\infty$ such that there is no second zero (at $x > x_0$) of the function $\epsilon^2 - g^2$. Such an approximation is sufficiently realistic in the upper band; in this case one must put

$$C_0 = \left(\frac{\omega_{He}}{\omega}\right)^2\frac{\omega l}{c},$$
$$C_1^2 = \frac{\omega^2(\omega^2 - \omega_{He}^2 - 2\omega_{p\infty}^2) + \omega_{p\infty}^4}{\omega^2(\omega^2 - \omega_{He}^2 - \omega_{p\infty}^2)},$$
$$\omega_{p\infty} = \omega_p(x) \text{ for } x \to \infty.$$

The case when the point $x_0$ is accessible in a weakly inhomogeneous plasma was investigated in[15,18]. The transformation in this case turned out to be complete, $A_T = 1$ (there is complete absorption in the corresponding "cold" equation[37]). It should be pointed out here that this result is valid independently of the condition (30).

In the case when the "cold" equations do not reduce to a single second-order equation, the coefficient $A_T$ was calculated only for a strongly inhomogeneous plasma. This analysis was carried out in[12] for the case $\alpha = \pi/2$ and $k_y = 0$, and in[17] for arbitrary $\alpha$ and $k_y$. For a plane layer bordering on vacuum on both sides, the transformation coefficient $A_T$ has, as a function of the frequency, a narrow maximum near the upper boundary of the transformation region. This dependence is illustrated in Fig. 8 (see also[38]). Calculations for a cylindrical plasma of small dimensions in a waveguide were carried out in[25]. Figure 9 shows the results of the calculations of the damping constant (damping over a distance equal to the wavelength in the waveguide), obtained assuming complete absorption of the plasma wave resulting from the transformation.

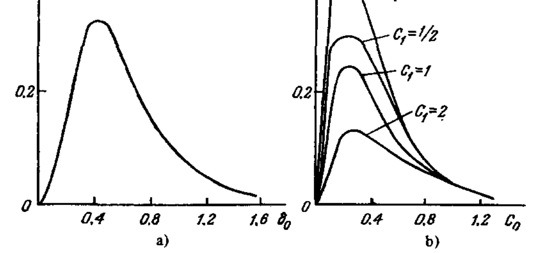


FIG. 7. Transformation coefficient for normal incidence of the waves on a plasma layer. The wave is polarized perpendicular to the magnetic field, $\alpha = \pi/2$. a) Approximate solution for $\omega l/c \gg 1$, $\omega_{He} \ll \omega$ [7]; b) exact solution for the case described by formula (39)[22].

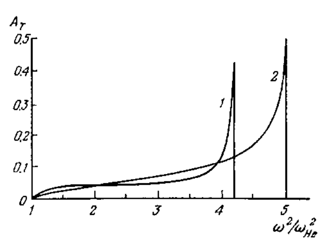


FIG. 8. Transformation coefficient for a plasma layer of small thickness [17].

$$n(x) = n_{\max}\left(1 - \frac{x^2}{a^2}\right), \qquad \frac{\omega_{He}a}{c} = 0.1, \qquad \frac{\omega_{p\,\max}^2}{\omega_{He}^2} = 4.$$

Normal incidence of waves; the polarization is perpendicular to **H** and $\Delta$n.

1—α=30°; 2—α=90°.

*The analysis in [22] pertained to the frequency region $\omega \approx 2\,\omega_{He}$. Actually, however, the results are valid in the entire upper frequency band.

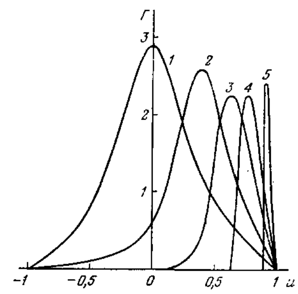


FIG. 9. Plots used to calculate the transformation of a wave propagating in a waveguide having on its axis a plasma cylinder with parabolic concentration distribution [25]. For a round waveguide, the damping constant $\kappa$ (connected with the transformation) is determined by the relation $\varkappa = \frac{\pi a^2}{b^2}\left(1 - \frac{\omega^2 b^2}{c^2}\right)\Gamma\left(\pm\frac{\omega_{He}}{\omega}, \frac{\omega_{p\,\max}}{\omega}\right)$, where a is the plasma radius, b is the waveguide radius ($a \ll b$), and the signs ± correspond to two waves with circular polarization. For a rectangular waveguide $\varkappa = \frac{\pi a^2}{b_1 b_2}\left(1 - \frac{\omega^2 b_1 b_2}{c^2}\right)\left\{\Gamma\left(\frac{\omega_{He}}{\omega}, \frac{\omega_{p\,\max}}{\omega}\right) + \Gamma\left(-\frac{\omega_{He}}{\omega}, \frac{\omega_{p\,\max}}{\omega}\right)\right\}$, where $b_1$ and $b_2$ are the longer and shorter sides of the waveguide. The figure shows the function $\Gamma(u, v)$ at the following values of $v^2$: 1–4.0, 2–2.0, 3–1.0, 4–0.6, 5–0.2.

As already mentioned, some numerical calculations were made also in connection with an investigation of resonances in small-dimension plasmas. A review of the results of these calculations can be found in[29] (see also[28]).

In conclusion we note that the equality $A_0 = A_T$ is valid accurate to small terms of order $\gamma$. If, however, $A_0 = 0$, then it is precisely these terms which describe the remaining weak effect. The transformation picture will then differ qualitatively from that described above. The same pertains also to the case when the condition $\gamma \ll 1$ is not satisfied, which is possible, for example, in a very strongly inhomogeneous plasma, when the concentration changes noticeably over distances on the order of the plasma-oscillation wavelength. In such cases the transformation should not be connected with the singular point and can be "distributed" over the volume. An example of such a transformation as applied to plasma-wave resonances is considered in[39]. Another example of a transformation connected with strong inhomogeneity is the frequently discussed excitation of plasma waves following the incidence of an electromagnetic wave on a plasma with an abrupt boundary.

## 2. EXPERIMENTAL INVESTIGATIONS OF THE ABSORPTION OF HIGH-FREQUENCY WAVES BY A PLASMA

In this chapter we describe the experimental investigations of absorption of high-frequency waves by a plasma at a low collision frequency when the usual collision absorption is not significant. Under such conditions, the absorption outside the region of cyclotron resonances can be connected with transformation of the incident wave into a slow plasma wave, which attenuates effectively in the plasma. We shall therefore compare the experimental absorption data with the theory of linear transformation of waves in the inhomogeneous one-dimensional plasma considered in the preceding chapter. The main qualitative theoretical conclusions used in the comparison can be formulated as follows.

1. Linear transformation of the waves occurs in the plasma region where the "cold" refractive index of the wave propagating in the direction of variation of the plasma parameters becomes infinite. Accordingly, there should exist three frequency bands within which transformation is possible (see Fig. 3). The positions of the two high-frequency bands (upper and middle) are determined by relations (34)–(37). It is important to note that it depends on the angle between the concentration gradient and the magnetic field, but does not depend on the direction of wave incidence.

2. The efficiency of the linear transformation is determined primarily by the "accessibility" of the transformation region to the incident wave and by the value of the concentration gradient. In the presence of an opacity region, the transformation efficiency is maximal when the characteristic dimensions are of the order of the average wavelength, and decreases with decreasing and increasing dimensions. The maximum efficiency depends on the method of feeding the wave and reaches several dozen percent under typical conditions.

3. To ensure efficient transformation at plasma dimensions much greater than the average wavelength, it is necessary to eliminate the opacity region in which the wave field decreases exponentially. In the upper band of transformation frequencies, this can be attained in an inhomogeneous magnetic field by introducing the waves into the plasma in regions where the magnetic field exceeds the cyclotron field. In the middle band, the dimensions of the opacity region can be greatly decreased by introducing into the plasma waves that have been slowed down to the limit.

4. The efficiency of linear transformation of the waves depends on the collision frequency and on the temperature of the plasma charged particles in wide ranges of their variation. These parameters influence only the absorption length of the plasma waves.

The manifestations of wave transformation can vary depending on the ratio of the absorption lengths of the slow plasma waves to the plasma dimensions. In most cases the absorption length of the plasma waves is much smaller than the plasma dimensions. The plasma wave produced as a result of the transformation is then completely absorbed and the absorption efficiency is equal to the transformation efficiency. In cases when the absorption length of the plasma waves is small compared with the plasma dimensions, transformation can lead to excitation of volume resonances of plasma waves.

Observation of volume resonances of plasma waves in a plasma of small dimensions was temporarily the first experimental evidence of wave transformation. By now, many papers on plasma resonances have been published. A brief review of the results of these papers is given in Sec. 2.1. In Secs. 2.2 and 2.3 we describe and discuss experimental investigations of the absorption of high-frequency waves by a plasma under conditions when linear transformation can be significant. Accordingly, we consider only experiments in which the collision frequency of the charged particles is much smaller than the field frequency. We consider neither cyclotron absorption nor experiments in which nonlinear interaction mechanisms were explicitly manifest.

### 2.1. Excitation of Plasma-wave Resonances

Resonance effects in the interaction between waves in the plasma in the absence of a magnetic field were first observed already in an early investigation by Tonks[40]. These effects were investigated later in[41-45,27] and in many other places. An extensive bibliography on plasma resonances can be found in the monographs[29,46].

In experiments on the excitation of plasma resonances, a cylindrical gas-discharge plasma having a diameter small compared with the wavelength was sounded with microwaves. The sounding was with the aid of antennas, resonators, and waveguides placed in such a way that the direction of propagation of the wave and the high-frequency field were perpendicular to the axis of the tube with the plasma (Fig. 10). Scattering, reflection, and transmission of electromagnetic waves were investigated, and in some experiments also the thermal radiation of the plasma. The dependence of the transfer coefficients, of the reflection, and of the radiation power on the discharge current (i.e., on the electron concentration) is characterized by the presence of a series of relatively narrow maxima or minima, the positions of which depend on the experimental conditions (they are usually called Tonks-Dattner resonances). A typical plot of a signal passing through a waveguide with a plasma is shown in Fig. 11. The largest peak (called the principal peak) is observed at the maximum electron concentration, which exceeds by several times the critical concentration $n_c$ at which the plasma frequency is equal to the field frequency. The remaining peaks correspond to lower concentrations, and their amplitude decreases with decreasing concentration. The last of the series of peaks is usually observed at a maximum concentration close to critical. Only in some experiments were weak maxima of the reflected signal observed at concentrations somewhat lower than critical; their amplitude was much lower than that of the principal one.

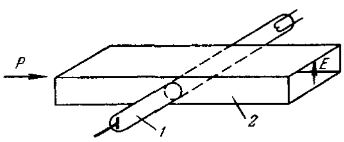


FIG. 10. Scheme for plasma sounding in plasma-resonance experiments (waveguide variant). 1–Gas-discharge tube, 2–waveguide. The arrows show the direction of the high-frequency electric field (E) and the sounding direction (P).

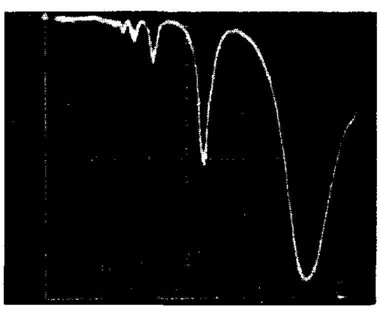

FIG. 11. Wave attenuation in a waveguide with a plasma as a function of the discharge current [42]. Discharge in mercury vapor, p = 2 × $10^{-3}$ mm Hg, f ≈ 3 GHz; ordinate–amplitude of wave passing through waveguide.

The principal reflection maximum, as established already in[40,41], corresponds to the resonance of wave scattering by a cylinder of cold plasma. In the case when the plasma dimensions are much smaller than the wavelength in vacuum, the condition for such a resonance is determined by the well known relation

$$\varepsilon + \varepsilon_b = 0, \quad \frac{\omega_p^2}{\omega^2} = 1 + \varepsilon_b, \tag{40}$$

where $\epsilon = 1 - \omega_p^2/\omega^2$ is the effective dielectric constant of the plasma and $\epsilon_b$ is the dielectric constant of the medium surrounding the plasma (in the case of a thin cylindrical tube, $\epsilon_b$ is intermediate between the values of $\epsilon$ of the tube material and of vacuum). When condition (40) is satisfied, the field inside the plasma and the scattered field increase strongly.

The nature of the remaining peaks on the curves of the type of Fig. 11 was established much later (see[47,27,29]). They could be attributed to the excitation of volume resonances of longitudinal plasma waves. The propagation of longitudinal waves in the absence of a magnetic field is described approximately by the dispersion equation

$$\omega^2 = \omega_p^2 + 3k^2 v_{Te}^2, \tag{41}$$

which is valid when $\omega^2 \gg k^2 v_{Te}^2$. It is seen from the equation that the longitudinal waves can propagate in the region $\omega_p < \omega$ or $n < n_c$. The region of wave propagation is bounded also on the low-concentration side, namely, when the phase velocity approaches the thermal velocity of the electrons, the collisionless damping of the waves increases and their propagation becomes impossible. Under conditions of experiments on plasma resonances, the region of propagation of the plasma waves at $n_{max} > n_c$ extends from the surface $n(r) = n_c$ almost to the plasma boundaries. This is connected with the fact that in low-pressure discharges, the electron concentration in the main part of the tube usually changes in a limited range, from $n_{max}$ to $(0.3-0.5)n_{max}$, and decreases sharply only in a layer of thickness on the order of the Debye radius next to the wall. Inasmuch as the layer thickness is much smaller than the wavelength ($kr_D \ll 1$), it can be assumed that the plasma waves are reflected from this layer without appreciable attenuation. As a result, a radial standing wave should be produced between the plasma boundary and the surface $n(r) = n_c$ ($\omega_p = \omega$). In the case of low losses, when multiple reflections are possible, the wave excitation becomes resonant. The resonance condition can be represented in the geometrical-optics approximation in the form[27]

$$\int_{r_c}^{a} \left[k^2(r) - \frac{s^2}{r^2}\right]^{1/2} dr = m\pi + \varphi, \tag{42}$$

and when $a \gg r_d$

$$\int_{r_c}^{a} k(r)\, dr = m\pi + \varphi,$$

where m is the number of nodes of the standing wave between the plasma boundary (r = a) and the surface

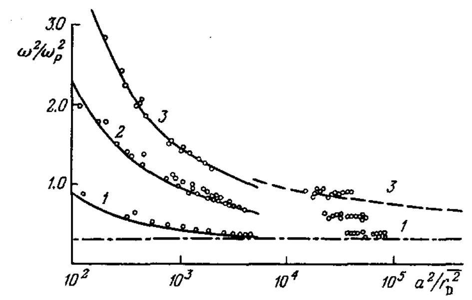


FIG. 12. Comparison of the experimental and calculated data on the positions of the plasma resonances [27]. The experimental data (points) were obtained for a discharge in mercury vapor with dipole excitation of the resonances. a = 0.5 cm, $\epsilon_b$ = 2.1, f = 200–3000 MHz. Solid curves–results of numerical integration of the wave equation; dashed line–results of calculation by formula (42); dash-dot line–formula (40). 1–Principal resonance; 2–m = 1, 3–m = 2.

$n = n_c$ ($r = r_c$), $\varphi$ is the total change of phase following reflections from these surfaces, and s depends on the method of excitation: in dipole excitation s = 1, in quadrupole excitation s = 2, etc.

The calculation of the resonance conditions under the assumption that the longitudinal waves are reflected from the plasma boundaries (from the walls of the tube) leads to results that agree well with the experimental data. This is seen in Fig. 12, where the results of one of a series of experiments are compared with the calculation results.

The amplitude of the resonant effects is determined by the damping of the longitudinal plasma waves. The experimentally observed decrease in the amplitude with increasing number of the resonance (see Fig. 11) is due to the broadening of the region of propagation of the plasma waves, which leads to an increase in their damping. A particularly strong increase should occur in the collisionless damping near the boundary, since the decrease of the boundary concentration leads to a corresponding decrease of the phase velocity of the wave [see (41)][47]*.

The agreement between the experimental data on the position of the Tonks-Dattner resonances and the calculation of the conditions of the radial resonances of the plasma waves confirms the interpretation presented above. Recently, other confirmations of the occurrence of plasma waves upon excitation of resonances were also obtained.

In[49], Raman scattering of electromagnetic waves from plasma oscillations produced upon excitation of resonances was observed experimentally. Signals of two frequencies were fed to a waveguide into which a tube with plasma was introduced. One frequency was close to the plasma frequency for resonance excitation, and the other was several times larger than the plasma frequency. It was demonstrated that a noticeable difference-frequency signal is registered only under conditions at which plasma resonances are excited. The amplitude of this signal agrees in order of magnitude with the expected value.

In[50] waves were excited in a plasma under conditions corresponding to the first resonances with the aid of very short microwave pulses (of duration ~1 nsec). Reflected pulses with a considerable delay were observed and could be related to the excitation of plasma waves. The delay time of the first of these reflected pulses corresponded to the propagation time of the slow plasma waves from the place of generation to the plasma boundary and back. The remaining weaker pulses corresponded to multiple reflections of the plasma wave.

Finally, measurements of the radial distribution of high-frequency electric fields in a plasma were measured in[51] following the excitation of the first Tonks-Dattner resonances. Measurements performed with the aid of a movable post probe have revealed the occurrence of a radial structure with a scale smaller than the vacuum wavelength by a factor of several times ten. This structure is apparently connected with a standing plasma wave.

Thus, experiments on plasma resonances offer evidence of excitation of slow longitudinal waves in the plasma under the influence of electromagnetic waves incident from the outside, i.e., of wave transformation.

It is significant that the main resonant peaks are observed only in those cases when the maximum electron concentration exceeds the critical value, and in the volume of the plasma there is a surface $\omega_p = \omega$ near which the wave transformation connected with the plasma inhomogeneity takes place.

We now proceed to describe investigations of the excitation of plasma-wave resonances in the magnetic field near the harmonics of the electron cyclotron frequency. These investigations were initiated in[52], where intense radiation at the harmonics of the electron cyclotron frequency up to very high numbers (up to 45) was observed, and also in experiments[53] that revealed the presence of singularities of the absorption near the cyclotron harmonics. Following these experiments, many investigations have been devoted to the emission, absorption, and scattering of waves near cyclotron harmonics (see[46]). In most investigations, the experiments were performed with a plasma in a cylindrical tube of small diameter, along the axis of which a magnetic field was directed. Schemes for plasma sounding were analogous to those mentioned above. The singularities near the harmonics of the electron cyclotron frequency were manifest in all the experiments only in the case when the maximum value of the upper hybrid frequency in the plasma volume exceeded the field frequency ($\omega_{max} > \omega$). Figure 13 shows typical plots of microwave absorption against the magnetic field, obtained in[54]. The curves clearly show a fine structure of the singularities near the second harmonic of the cyclotron frequency. A series of absorption peaks are observed in this region, and the distance between peaks depends on the electron concentration. Similar singularities were observed in[55,56] in the emission spectrum of a plasma.

In[54] the observed absorption peaks near the second cyclotron harmonic were compared with the resonances of weakly-damped plasma waves propagating across the magnetic field. At not too high a plasma temperature,

*The conclusion that the damping of plasma waves increases with decreasing boundary concentration, or upon "smearing" of the boundary region where the concentration drops to zero, is confirmed by experiments on the scattering of radio waves from meteor trails. In these experiments it is usually possible to observe only one principal resonance (see, for example, [48]).

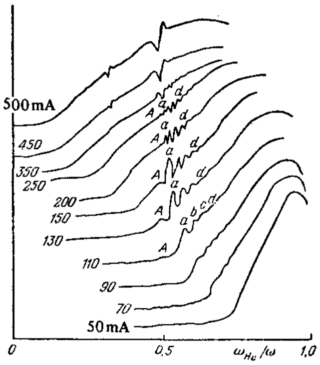


FIG. 13. Absorption of waves by a plasma as a function of the magnetic field [54]. Helium, p = 0.3 mm Hg, a = 0.4 cm, f ≈ 5 GHz. The curves obtained at different discharge currents are shifted in a vertical direction; the values of the currents are indicated on the figure.

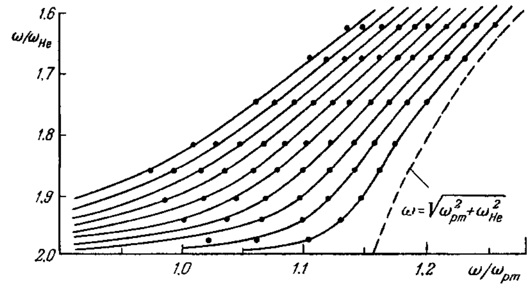


FIG. 15. Comparison of the experimental and calculated data on the positions of the plasma-wave resonances [57]. Points—experimental data; the solid lines were calculated with the aid of formula (45).

the dispersion equation for the plasma waves propagating across the magnetic field has near the second harmonic the form

$$(4\omega_{He}^2 - \omega^2)(\omega_{He}^2 + \omega_p^2 - \omega^2) = 3\omega_p^2 k^2 v_{Te}^2 \tag{43}$$

(we have omitted here terms proportional to $(kv_{Te})^4$ and to higher powers). It determines the wave propagation conditions:

$$\begin{aligned} &\text{for } \omega < 2\omega_{He} \quad \sqrt{\omega_p^2+\omega_{He}^2} > \omega, \\ &\text{for } \omega > 2\omega_{He} \quad \sqrt{\omega_p^2+\omega_{He}^2} < \omega. \end{aligned} \tag{44}$$

The experiments of[54] registered absorption peaks at $\omega \lesssim 2\omega_{He}$. The plasma waves can propagate in this case inside the region bounded by the inequality (44). It is natural to assume that the absorption peaks correspond to radial "internal" resonances. The condition for such resonances, in the geometrical-optics approximation, can be written in a form analogous to (42):

$$\int_{r_1}^{r_2} \left[k^2(r) - \frac{s^2}{r^2}\right]^{1/2} dr = \left(m + \frac{1}{2}\right)\pi, \tag{45}$$

where $r_1$ is the radius at which the field frequency is equal to the upper hybrid frequency, $r_2$ is the turning point near the axis, and m is the number of nodes of the standing wave in the region from $r_1$ to $r_2$. The results of the calculation of the resonance conditions in accordance with formula (45) agree well with the experimental data.

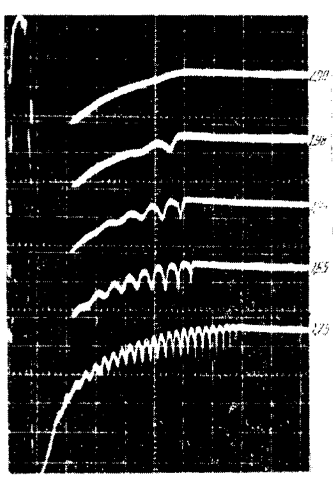

FIG. 14. Reflection of waves from waveguide with a plasma during the course of its decay [57]. Neon, p = 0.02 mm Hg, f ≈ 400 MHz. Different curves were obtained at different values of the magnetic field; the corresponding values of $\omega/\omega_{He}$ are shown on the right.

A detailed comparison of the experimental data on the positions of the absorption peaks with the calculation data is given in[57]. Figure 14 shows plots of wave reflection from a strip waveguide into which a tube with a plasma was introduced; these plots were obtained by varying the concentration during the plasma decay. It is seen that the number of observed resonances increases on deviating from the second harmonic. This corresponds to a decrease of the wavelength of the longitudinal oscillations (see (43)). As seen from Fig. 15, the experimental data on the positions of the resonant peaks agrees well with the calculation in accord with (45). In[57] absorption resonances at $\omega > 2\omega_{He}$ were observed and investigated. The plasma waves can propagate in this case from the region $\omega_U(r) = \omega$ towards smaller concentrations (see (44)), and the resonances are the result of reflection of the waves from this region and from the plasma boundaries. With decreasing magnetic field, these resonances go over into the Tonks-Dattner resonances considered above.

As shown in[58], plasma-wave resonances at $\omega > 3\omega_{He}$ can be suppressed under certain conditions. The effect of "suppression" of the resonances was registered in tubes of relatively large diameter, under conditions when there was practically no concentration jump on the plasma boundary. It is apparently connected with the damping of the plasma waves in a region in which the refractive index is large (near the plasma boundaries).

Thus, the data available on the excitation of plasma resonances in a magnetic field confirm the existence of efficient transformation of electromagnetic waves introduced from the outside into longitudinal waves propagating across the magnetic field. It can be noted that resonances are observed only when the maximum value of the upper hybrid frequency is larger than the field frequency, i.e., when there exists in the volume of a plasma a region of transformation connected with the field singularity [see formulas (34), (35) and Fig. 3].

In a number of experiments, direct proof of excitation of plasma waves at frequencies close to the harmonics of the electron frequency was obtained and was based on measurements of the distribution of the high-frequency field in the plasma, (see, for example[59-63]). In these experiments the waves were excited in a plasma by means of a coupling element of the capacitive or probe type, and the wavelength outside the plasma was

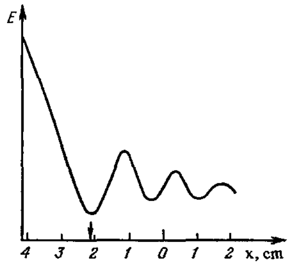


FIG. 16. Distribution of high-frequency field in a plasma upon excitation of waves with the aid of a probe [62]. Helium, $p = 5 \times 10^{-4}$ mm Hg, a = 5 cm, $n \approx 10^9$ cm$^{-3}$, $H \approx 100$ Oe, f = 500 MHz. The arrow marks the point at which $\omega_U(r) = \omega$.

much larger than the plasma dimensions. The radial distribution of the field in the volume of the plasma was determined with the aid of a movable high-frequency probe. The characteristic distribution obtained in[62] is shown in Fig. 16. The distribution constitutes a super-position of the high-frequency field produced by the coupling element (this field decreases in the interior of the plasma) and plasma waves propagating in the regions $\omega_Q > \omega$. The phase velocity of the plasma waves, determined from their wavelength, is close to the calculated value.

Experiments were undertaken recently on the excitation of plasma waves under conditions corresponding to volume resonances in a magnetic field, with the aid of short microwave pulses[64]. In these experiments (analogous to the already mentioned experiments[15]) on the delay of signals reflected from the plasma, it was possible to determine the time of propagation of the plasma waves from the transformation region to the axis of the plasma column, and to establish by the same token the localization of the transformation region. In the range of the experimental conditions, this region coincided with the surface $\omega = \omega_U(r)$. In addition, from the amplitude of the reflected pulses it was possible to estimate the transformation coefficient. It turned out to be on the scale of several tenths, in accordance with the theoretical estimates.

So far we have spoken of excitation of plasma-wave resonances as a result of "localized" transformation connected with the singularity of the field. Attention should be called, however, to a possible influence of another mechanism of wave transformation "distributed" over the volume of an inhomogeneous plasma, a mechanism that becomes significant under conditions when the characteristic dimensions of the inhomogeneity are comparable with the wavelength of the plasma oscillations (see p. 422). The presence of such a mechanism is evidenced apparently by the results of[61], where measurements of the distributions of the amplitude and phase of the field of the plasma waves excited with the aid of a capacitive-type coupling element have shown that a traveling plasma wave was produced in the plasma, with a group velocity directed away from the axis towards the periphery of the plasma. The authors propose that under the conditions of their experiment the decisive role was played by the "distributed" transformation in the central part of the plasma. This mechanism was all the more important, since with the wave-excitation method employed there the resultant wave polarization was such that the efficiency of the transformation due to the field singularity was relatively small (see Sec. 1.2). The "distributed" transformation could exert a strong influence on the excitation of the plasma oscillations also in some other experiments—in those cases when their wavelength was comparable with the characteristic dimensions of the inhomogeneity.

### 2.2. Absorption in the Band Between the Electron Cyclotron Frequency and the Upper Hybrid Frequency

Absorption in the frequency region corresponding to the highest transformation band ($\omega_{He} < \omega < \omega_{U\,max}$) was noted in many investigations. The first data on the dependence of the efficiency of absorption of microwaves by plasma on the magnetic field were obtained in[65,53] at small transverse plasma dimensions (compared with the wavelength). These investigations were devoted to the radiation and absorption of waves by a gas-discharge plasma produced in a tube with small diameter, along the axis of which a magnetic field was directed. It was shown that at sufficiently high electron concentration, the magnetic field corresponding to the absorption maximum turns out to be smaller than the cyclotron field, and that the shift increases with increasing concentration. Judging from the conclusions drawn by the authors, the maximum of the absorption corresponds to equality of the field frequency to a certain mean value of the upper hybrid frequency. Incidentally, the electron-concentration data needed for a quantitative analysis are given only in[65] for a very limited range of conditions. It should also be noted that measurements of the absorption efficiency[53] were carried out at relatively high gas pressures, at which the ratio of the electron collision frequency to the field frequency is not very small. In this case a definite role should be played by the usual collision-governed absorption of the incident wave.

Investigations of wave absorption in a plasma having small transverse dimensions at a collision frequency much lower than the field frequency were reported in recent papers[66-68]. We shall stop to discuss here the experiments described in[66,67], since they cover a wider range of plasma parameters. The experiments were performed with an argon plasma produced by an electrode discharge in a glass tube of 1 cm diameter and 25 cm length. The magnetic field was directed along the axis. The electron concentration on the axis was measured with the aid of a probe. The tube was placed in a waveguide for the 10-cm band at a small angle to the axis, to ensure matching (see the scheme in Fig. 17). This waveguide was used to measure the reflection coefficient and the absorption efficiency. Figure 18 shows typical results of the measurements of the attenuation of a signal passing through the waveguide with the plasma at a low level of the supplied microwave power. Notice should be taken of the weak depen-

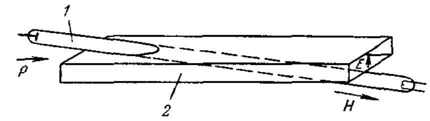


FIG. 17. Scheme for plasma sounding in the experiments of [66,67]. 1—Tube with plasma, 2—waveguide. The arrows show the directions of the magnetic field (H), of the high-frequency electric field (E), and of the sounding (P).

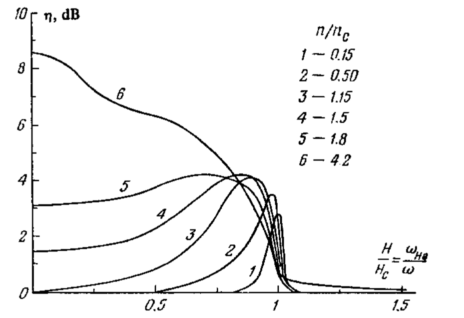


FIG. 18. Attenuation of waves in a waveguide with a plasma [66]. Argon, p = 0.03 mm Hg, a = 0.5 cm, f = 3.15 GHz.

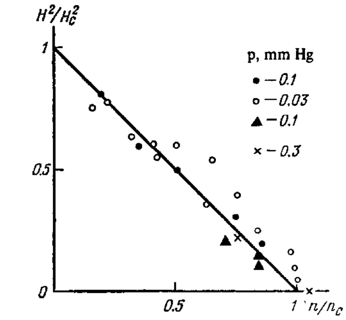


FIG. 19. Minimum magnetic-field values limiting the region of wave absorption [66]. The solid line corresponds to the condition $\omega_{U\max} = \omega$.

dence of the results on the gas pressure, i.e., on the collision frequency of the electrons in the range of the experimental conditions ($\nu/\omega = 10^{-2}$–$10^{-3}$).

In the examination of the experimental results of[66-68] it should be borne in mind that under the conditions of these experiments the concentration gradient in the region of interaction between the waves in the plasma was directed perpendicular to the magnetic field (the magnetic field along the axis, the gradient along the radius). Therefore the wave transformation should occur in the plasma region at which the field frequency is equal to the upper hybrid frequency $\omega = \omega_U = [\omega_p^2(r) + \omega_{He}^2]^{1/2}$ [see formula (35)]. At a fixed frequency, the range of magnetic fields within which a transformation region occurs in the interior of the plasma is determined by the range of variation of the concentration inside the plasma. If the concentration changes from zero to $n_{max}$, then the transformation turns out to be possible in the magnetic-field region bounded by the inequality

$$\omega_{He}^2 + \omega_{p\max}^2 > \omega^2 > \omega_{He}^2$$

or

$$1 - \frac{\omega_{p\max}^2}{\omega^2} < \frac{\omega_{He}^2}{\omega^2} < 1,$$

$$1 - \frac{n_{\max}}{n_c} < \left(\frac{H}{H_c}\right)^2 < 1.$$

Accordingly, at a maximum electron concentration lower than the critical value ($n_{max} < n_c$, $\omega_{pmax} < \omega$) absorption was observed in the magnetic-field range from the cyclotron field $H_c$, at which $\omega_{He} = \omega$, to the field at which $\omega_{Umax} = \omega$. The absence of noticeable absorption in magnetic fields exceeding the cyclotron field can be seen from Fig. 18. The agreement between the minimum values of the field and the condition $\omega_{Umax} = \omega$ is illustrated in Fig. 19. At $n_{max} > n_c$ ($\omega_{pmax} > \omega$), absorption was observed in the entire range of magnetic fields from zero to the cyclotron field.

The general form of the dependence of the absorption effectiveness on the magnetic field, shown in Fig. 18, is due to the shift of the surface of efficient transformation $\omega_U(r) = \omega$ from the axis to the periphery when the field changes from $H_{min}$ to $H_c$. The initial growth of the absorption is connected with the increase of this surface. As this surface shifts towards the plasma boundaries, the absorption efficiency again decreases because of the increase in the relative concentration gradient in the transformation region. Data on the absorption efficiency at electron concentrations close to threshold can be compared with the results of the calculation of the efficiency of linear transformation performed for conditions corresponding to the described experiment[25] (a parabolic radial distribution of electrons was assumed in the calculation). Such a comparison is shown in Fig. 20, which gives the results of the calculation together with the experimental data on the dependence of the absorption on the concentration in the absence of a magnetic field and at a fixed magnetic field[69]. It is seen that the results agree satisfactorily.

It should be noted that in some experiments the plots of the absorption against the magnetic field reveal weak maxima near the electron cyclotron frequency harmonics (see[53,66]). These singularities are apparently connected with the plasma-wave resonances considered in the preceding chapter.

Systematic investigations of the absorption of microwaves by a plasma with transverse dimensions on the order of the wavelength are reported in[70-73]. These studies were devoted to the formation of a hydrogen plasma under the influence of microwaves in a homogeneous magnetic field and in magnetic traps with mirror field configuration. The plasma was produced by means of stationary and pulsed wave sources operating in the 3 cm band. The microwave energy was fed to the plasma with the aid of a cylindrical waveguide in which a $TE_{11}$ mode was excited with an electric field directed perpendicular to the axis. The plasma was produced in

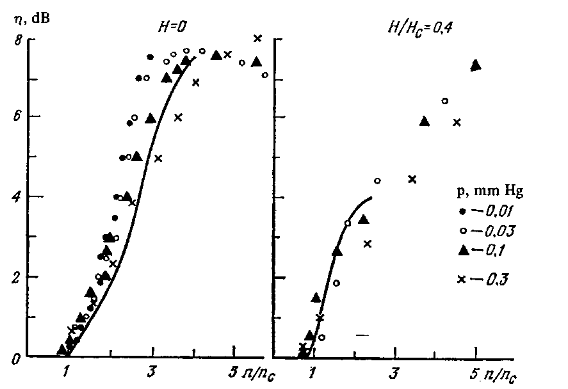


FIG. 20. Comparison of results of wave-absorption measurements with the results of calculation of the transformation efficiency [69]. The experimental points correspond to the pressures indicated on the figure; the solid lines represent the results of the calculation.

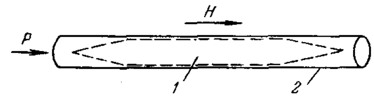


FIG. 21. Diagram of experiments in which plasma was produced under the influence of microwave power [70-73]. 1–Tube with plasma, 2–waveguide.

a dielectric tube inserted into the waveguide (see Fig. 21). The ends of the tubes were made conical to reduce reflections.

The plasma was formed in a magnetic field corresponding to the electron cyclotron resonance. The magnetic field was then varied and its values limiting the parameter region within which the plasma could be maintained under the influence of the microwave power (the region of "existence" of the plasma) were determined.* The concentration and the average energy of the electrons were determined during the experiment with the aid of microwave methods and diamagnetic probes. The efficiency of wave absorption in the waveguide with the plasma was also measured. The range of the parameter values in the described experiments is given in the table.

The results of the determination of the region of existence of the plasma are shown in Fig. 22. In this figure, the ordinates represent the magnetic field values limiting the region of plasma existence in different experiments, and the abscissas represent the electron concentration averaged over the plasma cross section. The experiments showed that within the limits of the plasma existence region, the absorption efficiency changes from 50 to 90% in almost the entire range of variation of the parameters. Only when the boundaries of the region are approached (starting with magnetic fields differing from the limiting values by 1–2%) did the absorption efficiency decrease. This shows that the region of existence of the plasma produced by microwave power is limited by the conditions for efficient absorption of the waves, i.e., that the plasma existence region is simultaneously the efficient-absorption region.

It is seen from Fig. 22 that the maximum magnetic field at which effective absorption was observed was only a few percent larger than the cyclotron field* (the field at which $\omega_{He} = \omega$). The minimum magnetic field at electron concentrations lower than the critical value is close to the field at which the maximum value of the upper hybrid frequency is equal to the field frequency ($\omega_{Umax} = \omega$). This follows from a comparison of the experimental data with the $\omega = \omega_U(\bar{n})$ curve on Fig. 22. When these are compared, it must be borne in mind that the maximum electron concentration in the region of interaction between the waves in the plasma exceeds somewhat the average concentration (the largest difference may be by a factor 1.5–1.7). At electron concentrations larger than critical, the minimum magnetic field was smaller by a factor 20–30 than the cyclotron field, and at lower fields it was impossible to maintain the plasma because of the strong transverse diffusion.

Thus, the established boundaries of the region of effective absorption are determined by the inequality $\omega_{He} < \omega < \omega_{Umax}$. They correspond to the limits of the high-frequency wave-transformation band at an angle $\alpha = 90°$ between the magnetic field and the concentration gradient [see formulas (34) and (35)]. In the described experiments, the angle varied along the tube with the plasma, and approached 90° (the gradient direction was close to radial) in the central part of the tube. The fact that high absorption efficiency is observed under conditions at which transformation is possible is a natural consequence of the fact that the characteristic transverse dimensions of the plasma are of the order of the wavelength. In this case the efficiency of transformation and absorption, for a plane wave incident on

Characteristics of power input

| $f=\frac{\omega}{2\pi}$, Hz | w, W/cm³ |
|---|---|
| $10^{10}$ | 0.01–200 |

w-power input per unit volume of the plasma.

Plasma parameters

| a, cm | H, Oe | p, mm Hg | $n_{max}$, cm⁻³ | $U_e$, eV |
|---|---|---|---|---|
| 2–4 | 0–6000 | $10^{-4}$–$10^{-1}$ | $10^{9}$–$10^{14}$ | 1–$10^{3}$ |

a–plasma radius, p–hydrogen pressure, $U_e$–average electron energy.

Dimensionless parameters

| $a/\lambda$ | $\omega_{He}/\omega$ | $\nu/\omega$ | $\omega_p/\omega$ |
|---|---|---|---|
| 0.5–1.5 | 0–2 | $10^{-2}$–$10^{-5}$ | $3\cdot10^{-2}$–10 |

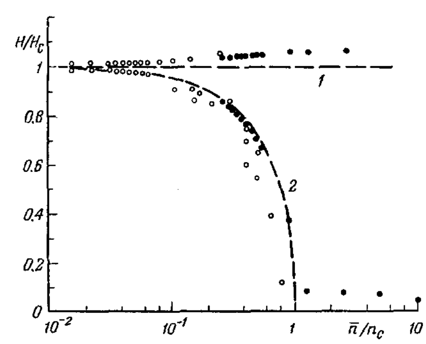


FIG. 22. Region of existence of plasma under the influence of microwave power [70-73]. The experimental data were obtained for hydrogen pressures $10^{-2}$–$10^{-4}$ mm Hg; the dashed lines correspond to the theoretical boundaries of the upper transformation band: $\omega = \omega_{He}$ (1) and $\omega = \omega_U$ (2).

*In determining the region of existence of the plasma in the pulsed regime, the magnetic field was varied within a time shorter than the duration of the microwave pulse.

*In certain experiments, at relatively high collision frequency and long wave-plasma interaction lengths, a broadening of the region of existence was observed in the direction of larger magnetic fields. We do not consider these experiments here, since the usual collision absorption played an important role in them (see p. 430).

the plasma, can reach several times 10% (see Secs. 1.3 and 1.4). In the waveguide, the absorption efficiency may be much higher, since reflections from the metallic walls produce conditions for multiple interaction between the wave and the plasma. Some influence on the absorption efficiency can be exerted also by the coupling between electromagnetic waves of different polarization, which occurs in an inhomogeneous plasma under conditions when the refractive indices of these waves are close (see Fig. 4). A possible influence of this effect is indicated in[74].

In the described experiments, as well as in those considered earlier, singularities of wave absorption were observed near the electron cyclotron frequency harmonics from the second to the 20th[70,71]. They were manifest in a decrease of the reflection coefficient of the waves from the waveguide with the plasma near the harmonics; accordingly, the absorption efficiency increased somewhat (by 10–20%). It can be assumed that, just as in other cases, the singularities near the harmonics are connected with the conditions for the propagation of the longitudinal waves across the magnetic field.

When examining the results of[70-73], attention should be called to the fact that, in accord with the theory of linear transformation, the boundaries of the wave-absorption region and the absorption efficiency change little following considerable variations of the collision frequency and of the average electron energy. The experimental data on the boundaries of the efficient-absorption region, obtained at different gas pressures, agree well with a common curve (see Fig. 22). The range of variation of the collision frequency for which these data were obtained constitutes three orders of magnitude ($\nu/\omega = 10^{-2}-10^{-5}$). The transverse electron energy was altered in the experiments by application of a short microwave pulse whose duration was shorter than the energy loss time. An increase of the energy from several electron volts to one keV did not lead to a considerable change in the absorption efficiency.

Finally, one must emphasize the weak dependence of the absorption efficiency on the high-frequency field intensity. Absorption data obtained at different high-frequency powers differ little, although the power variation, at a fixed electron concentration, reached two orders of magnitude (in the pulsed regime). This can be regarded as an indication that the absorption mechanism in the described experiments was linear. Such a result agrees with the estimate of the conditions for the influence of nonlinear effects on wave transformation, given in Sec. 1.2. The minimum intensity of the high-frequency electric field outside the plasma in the experiments ($\sim$100 V/cm) satisfies, as can be readily verified, the inequality (31).

The connection between absorption and transformation of waves in a plasma having transverse dimensions on the order of the wavelength was recently confirmed in[75], where measurements were made of the localization of the absorption region. In this experiment, the plasma was produced in a toroidal metallic chamber, along the axis of which a magnetic field was directed, under the influence of microwave power in the 3-cm band. At sufficiently low neutral-gas pressure in the chamber ($p \approx 10^{-3}$ mm Hg in hydrogen), the plasma was homogeneous along the torus. A movable electrostatic probe was used to determine the transverse electron distribution. The same probe was used to determine the location of the absorption of the microwave power, which was revealed by the sharp increase of the fast-electron current (the current to the probe was registered at a negative bias). Typical measurement results are shown in Fig. 23. In all cases, the absorption region turned out to be localized between the surface in the plasma on which the upper hybrid frequency is equal to the field frequency ($\omega_U = \omega$) and a surface corresponding to much higher concentrations, with the maximum absorption always shifted relative to the surface $\omega_U = \omega$ (this is seen also from Fig. 23). It follows therefore that the absorption cannot be connected with "cold" electromagnetic waves, since the absorption maximum for these waves would occur on the surface $\omega_U = \omega$, and these waves cannot propagate at all in the region of much higher concentrations. At the same time, the plasma waves should attenuate precisely in that region where absorption was observed, namely, they are formed on the surface $\omega_U = \omega$ and propagate towards higher concentrations. Calculations show that the collision-damping length of the plasma waves under the conditions of the performed experiments is close to the experimentally determined dimensions of the absorption region.

The experimental data on wave absorption in a plasma with dimensions much larger than the wavelength, under conditions when linear transformation is possible, are reported in[76-78]. In these investigations, the absorption efficiency was determined from measurements of the intensity of the thermal radiation of the plasma (as is well known, the relation between these characteristics is uniquely determined by the electron temperature). In[76,77] are described experiments with a cesium plasma using apparatus with surface thermal ionization (the so-called Q machine). The thermal ionization was effected in the apparatus on end plates, and the magnetic field was directed along the axis; the concentration gradient was perpendicular to the axis. In the experiment of[78], a low-frequency discharge plasma was produced in a tube of rectangular form. The measurements of the plasma radiation intensity (in the 3-cm and 0.8-cm bands) were made with the aid of antennas placed near the plasma boundaries so as to receive radiation with a high-frequency electric field polarized perpendicular to the main magnetic field.

In experiments performed in a homogeneous magnetic field, the radiation in the region between the electron

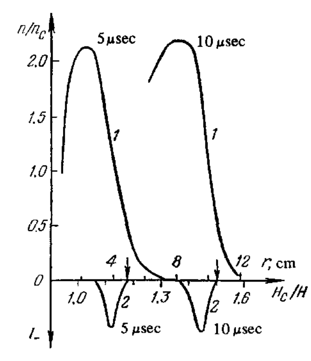


FIG. 23. Localization of the region where waves are absorbed by a plasma [75]. $H_2$, $p = 10^{-3}$ mm Hg, f = 9.45 GHz. 1—Electron concentration, 2—probe current at negative bias; the location of the surface $\omega_U(r) = \omega$ is marked by arrows (the data are given for two instants of time after the start of the microwave pulse).

cyclotron frequency and the lower hybrid frequency turned out to be quite weak. Its intensity was much lower than the black-body radiation intensity calculated from the electron temperature. Figure 24 shows, by way of example, the results of the measurements of[78], under conditions when a definite role was played by collision absorption ($\nu/\omega \approx 10^{-2}$). The curve shows two maxima. The first has been demonstrated to correspond to the appearance of cutoff of the electromagnetic wave (the wave ceases to pass through the plasma), and the second to cyclotron resonance beyond which the cutoff disappears. The absorption intensity decreases greatly between the maxima. It is natural to attribute this dip to the influence of the opacity "barrier" that makes the transformation region inaccessible to the wave incident on the plasma when the plasma dimension greatly exceeds the wavelength (see p. 420). As noted earlier, the opacity region between the plasma boundary and the transformation region can be eliminated in an inhomogeneous magnetic field if the field outside the plasma exceeds the cyclotron value and decreases from the periphery towards the axis (see Fig. 5). To verify this effect, a transverse inhomogeneity of the magnetic field was produced in the experiments of[76]. Under conditions when the magnetic field near the receiving antenna was larger than the cyclotron field and a region in which $\omega_U = \omega$ existed in the interior of the plasma, the radiation intensity turned out to be close to that of a black body (Fig. 25). This result shows that the absorption efficiency is close to 100%. It agrees with the theoretical conclusion that linear transformation has high efficiency in the absence of an opacity region.

Thus, the experimental data obtained in[66-78] concerning the absorption of waves by a plasma agree with the linear-transformation theory. Absorption in the frequency region $\omega_{He} < \omega < \omega_{U\max}$ was also observed in other investigations devoted to the effects produced at the harmonics of the electron cyclotron frequency, to microwave discharge, and to the formation and heating of plasma in magnetic traps under the influence of microwave power (see, for example,[79-85]). However, we shall not consider these investigations, since they do not deal with the absorption conditions and mechanism, and contain no information on the absorption efficiency. We note only that the assumption that the observed absorption is connected with wave transformation does not contradict the experimental data.

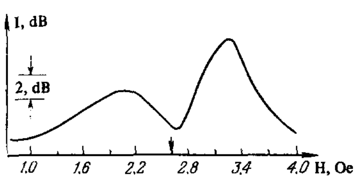


FIG. 24. Radiation intensity of decaying high-frequency discharge plasma against the magnetic field [78]. Helium, p = 0.28 mm Hg, $l$ = 5 cm, n = 4 × $10^{11}$ cm$^{-3}$, f = 9.15 GHz ($2l$ is the dimension of the plasma in the sounding direction). The arrow marks the field at which $\omega_{U\max} = \omega$.

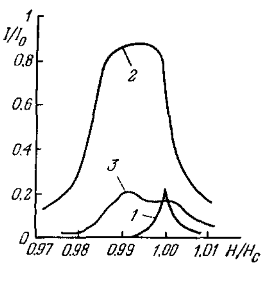


FIG. 25. Radiation intensity of a plasma in a setup with thermal ionization [76]. Cs, a = 1.5 cm, n = 2 × $10^{11}$ cm$^{-3}$, T ≈ 2300°K. 1—Homogeneous magnetic field, 2—inhomogeneous field increasing from the plasma axis towards the antenna, 3—inhomogeneous field decreasing from the plasma axis towards the antenna. The ordinates represent the ratio of the measured radiation intensity to the calculated black body radiation intensity at T = 2300°K ($I_0$).

## 2.3. Absorption in the Band Between the Electron Cyclotron Frequency and the Lower Hybrid Frequency

In this section we consider experimental investigations in which data were obtained on the absorption in the middle band of transformation frequencies. The position of this band and its boundaries are determined by formulas (34), (36), and (37) (see also Fig. 3). The corresponding range of plasma parameters, within which transformation is possible at a fixed frequency, is illustrated by Fig. 26. The upper and middle transformation bands in the figure correspond to magnetic fields smaller and larger than the cyclotron field. We see that at concentrations higher than critical, the transformation is possible at magnetic fields both smaller and larger than the cyclotron field. Between the parameter ranges corresponding to the two transformation bands, however, there is a "gap," a region in which linear transformation is impossible. The width of this gap depends on the angle $\alpha$ between the concentration gradient and the magnetic field. At small angles the gap is small ($H_c < H < H_c/\cos\alpha$). It increases with increasing angle and at $\alpha = \pi/2$ it covers the region of fields between the cyclotron field and the magnetic field at which $\omega_{L\max} = \omega(H_c < H \lesssim H_c\sqrt{m_i/m_e})$.

Let us consider first the experimental data on the absorption of microwaves at magnetic fields stronger than the cyclotron field, and let us compare the absorption conditions with the wave-transformation conditions. Attention should be called first to the fact that in the experiments described in the preceding section, no noticeable absorption was observed at magnetic fields exceeding the cyclotron field by more than several per-

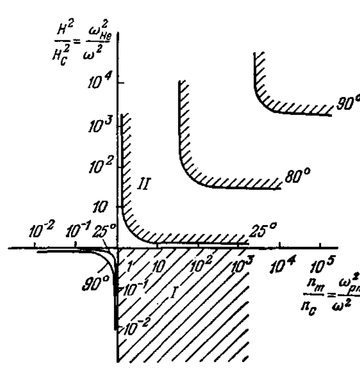


FIG. 26. Ranges of plasma parameters within which linear transformation is possible. I.—Range corresponding to upper transformation band, II—range corresponding to middle transformation band. The different curves correspond to the high-frequency band boundaries at the values of the angle $\alpha$ indicated on the figure.

cent. This result agrees with the presence of a gap between the parameter regions corresponding to the wave-transformation bands. In the experiments of[56-68], in which plasma was investigated in long tubes of small diameter, the angle between the concentration gradient and the magnetic field was close to 90° and the width of the gap between the high-frequency transformation regions, as noted above, was very large. Therefore, naturally, there was no noticeable absorption in the range of magnetic fields from the cyclotron value to a field 1.5—2 times larger than the cyclotron field (see Fig. 18). In[70-73], in which the region of existence of a plasma under the influence of microwave power was determined, the plasma produced at a cyclotron magnetic field vanished under the conditions of most experiments when the field rose to 3—5% above the cyclotron value (see Fig. 22). This indicates that the parameter region in which collisionless absorption is impossible has been reached. Only in certain experiments, at an electron concentration higher than critical, and at a relatively high electron collision frequency and a large interaction length ($L \approx 20\,\lambda$), when there was noticeable collision absorption, did the region of plasma existence broaden to include stronger magnetic fields[71]. In this case the absorption efficiency decreased in magnetic fields slightly exceeding the cyclotron field, but with further increase of the field (to values double the cyclotron field) the efficiency again increased. It can be assumed that the observed decrease of the absorption efficiency is connected with the passage through the gap between the transformation regions. Such a conclusion, however, cannot be regarded as proved.

Experimental evidence of the existence of collisionless absorption at magnetic fields greatly exceeding the cyclotron value was first obtained in[86]. In this experiment the plasma was produced in a cylindrical tube of 2.5 cm diameter, placed in a homogeneous magnetic field parallel to the axis. The tube was filled with hydrogen. The plasma was produced by microwaves from a three-centimeter stationary generator in the electron cyclotron resonance regime. The absorption of waves of lower frequency, with $\omega_{He}/\omega \approx 3$, in such a plasma was determined. To this end, the tube with the plasma was inserted in a rectangular waveguide (Fig. 27). Power was fed to the waveguide from a low-power 10-cm source. A probe was used to determine the attenuation of the signal passing through the waveguide with the plasma. The reflection coefficient from the waveguide was also measured. The average electron concentration and the electron longitudinal distribution were detemrined with the aid of a resonator that moved longitudinally. The results of the measurements of the wave absorption in the waveguide with the plasma at different hydrogen pressures ($p = 2 \times 10^{-4}$—$10^{-3}$ mm Hg, $\nu/\omega = 5 \times 10^{-5}$—$2.5 \times 10^{-4}$) are shown in Fig. 28. As seen

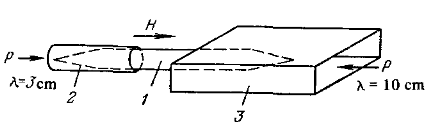


FIG. 27. Scheme for feeding microwave energy to a plasma in the experiments of [86,87]. 1—Tube with plasma, 2—waveguide for 3-cm band, 3—waveguide for 10-cm band.

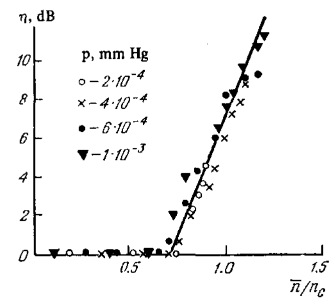


FIG. 28. Absorption of microwaves by a plasma at a magnetic field exceeding the cyclotron value ($H/H_c \approx 3$) [86]. The experimental data were obtained at different argon pressures, as indicated in the figure.

from the figure, strong absorption occurs starting with a concentration close to critical. The dependence on the concentration is threshold-like and abrupt. The absorption efficiency increases by almost one order of magnitude when the concentration is decreased within 40%. Attention should be called to the fact that the absorption efficiency near the threshold is practically independent of the gas pressure, i.e., of the electron collision frequency: the data obtained at different pressures fit on one curve. The observed absorption threshold at $\bar{n} \approx 0.75\, n_c$ ($n_{max} = (1-2)n_c$) corresponds to the boundary of the middle transformation calculated from formula (36) at an angle $\alpha < 45°$ between the concentration gradient and the magnetic field. In fact, under the experimental conditions, the angle $\alpha$ varies in the region of plasma-wave interaction from small values near the end of the tube with the plasma to values close to 90° in the midsection, and one can speak only of a certain value of $\alpha$ averaged over the interaction region. The independence of the experimental result of the gas pressure is apparently connected with the fact that the concentration distribution remains practically unchanged (meaning also the value of $\bar{\alpha}$)—at low pressures, when the charged-particle mean free path is larger than the tube dimension, this distribution is determined by the shape of the tube.

The data on the boundaries of the wave-absorption band in magnetic fields larger than the cyclotron value were obtained in[87] by determining the region of existence of the microwave-induced plasma*. In these experiments, unlike the analogous experiments in magnetic fields weaker than the cyclotron field (described in the preceding section), it was impossible to use the same microwave source to produce the plasma and to maintain its stationary state in the existence region. This is due to the presence of the already mentioned gap between the cyclotron magnetic field in which plasma can be produced at low concentrations and the absorption region, which corresponds to the middle transformation band (Fig. 26). Therefore a separate generator was used in[87] to form the plasma, with a frequency much higher than the frequency of the main source. The plasma was produced in an argon-filled quartz tube of 2.5 cm diameter. The stationary magnetic field was directed along the axis. On one side of the tube, power was fed through a waveguide from a stationary 10-cm source, and from the other side power was fed from a pulsed 3-cm source (see Fig. 27). The electron concentration was measured with a high-frequency

*Further development of the experiments of [87] is described in [115].

interferometer. The pulsed source was used to obtain plasma in the electron-cyclotron-resonance regime. After the pulsed source stopped operating, the plasma was kept up at sufficiently high concentrations by absorption of energy from the main stationary generator (at one third the cyclotron frequency). By varying the magnetic field and the supplied power (i.e., the concentration) it was possible to determine the plasma-existence region boundaries corresponding to the boundaries of the region of efficient absorption.

Figure 29 shows the measurement results at two argon pressures ($p = 1.2-3.6 \times 10^{-2}$ mm Hg, $\nu/\omega = 10^{-3}-10^{-2}$). The experiment has clearly demonstrated the gap between the regions of efficient absorption as the magnetic field is decreased from its initial value (three times the cyclotron value). The plasma vanished at the field values designated in Fig. 29. The discharge was restored at a magnetic field close to the cyclotron value, and existed down to small values of the field. The experimental data on the boundaries of the region of efficient absorption at two pressures, as seen from Fig. 29, agree with the theoretical boundaries of the transformation region as calculated from formula (36) at angles $\alpha$ equal to 40 and 60°. These angles should be compared with the mean values of the angles between the concentration gradient and the magnetic field in the region of plasma-wave interaction. Under the conditions of the described experiment, the distribution of the charged-particle concentrations should be determined by the ratio of the longitudinal and transverse diffusion coefficients (unlike in the preceding investigation, the mean free path of the ions was much shorter than the length of the tube with the plasma). Order-of-magnitude estimates of the average angles between the concentration gradient and the magnetic field, based on the known values of the diffusion coefficients, $\tan \bar{\alpha} = (D_{\parallel}/D_{\perp})^{1/2}$, agree with the values given in Fig. 29. The increase of $\alpha$ with decreasing pressure is attributed to the growth of the longitudinal-diffusion coefficient.

Analogous experiments on the determination of the region of existence of plasma induced by high-frequency power were performed recently[88] in another frequency band (100—300 MHz) using a different method of supplying the high-frequency energy (the energy was fed to a tube with independently-produced plasma with the aid of a single-turn system). In these experiments, too, the determined boundaries of the existence region turned out to be in good agreement with (36).

Collisionless wave absorption by a plasma under a wide range of conditions, corresponding to the middle transformation band (in magnetic fields exceeding the cyclotron value, and at concentrations much larger than critical), was registered also in[89], in which the region of existence of the plasma was determined with the microwave power supplied through a slow-wave system. Qualitatively the results of this investigation agree with the results of[87], which were described above, and the authors of[89] assume that they admit of an analogous interpretation. Incidentally, a definite role in the experiment of[89] could be played also by direct absorption of slow waves excited from the outside by the Landau mechanism.

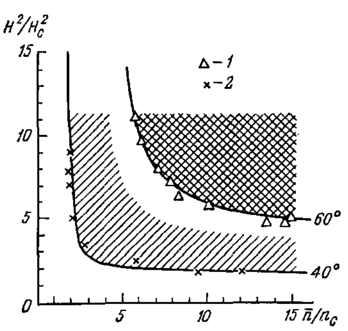


FIG. 29. Region of existence of a plasma induced by microwave power at magnetic fields exceeding the cyclotron field [87]. Experimental points: 1—p = 1.2 × 10⁻², 2—p = 3.6 × 10⁻² mm Hg (Ar); the solid line corresponds to the theoretical boundaries of the middle transformation band at $\alpha$ = 40 and 60°.

We proceed now to investigate wave absorption by a plasma at frequencies corresponding to the lower hybrid frequency band

$$\omega_L = \left[\omega_{He}\omega_{Hi}\frac{\omega_p^2}{\omega_p^2+\omega_{He}^2}\right]^{1/2}$$

or at frequencies close to it. Absorption in this frequency region was observed in a number of investigations (see, for example,[90-101]). The first of them were aimed at studying volume resonances of magnetosonic waves[90-92]. These investigations have demonstrated effective penetration of the waves into the plasma and their strong absorption under conditions of magnetosonic resonances. At sufficiently high source power, considerable heating of the ions under the influence of the magnetosonic waves was observed[93,94]. The measured absorption and heating efficiencies could not be attributed in many cases to collisions[92-94]. It is possible that an appreciable role was played in these cases by absorption connected with linear wave transformation.

Investigations of absorption in a high-frequency discharge plasma of low pressure in a magnetic field are described in[95,96]. In different gases, the authors observed a broad absorption maximum at frequencies close to the lower hybrid frequency ($\omega = (1-5)\omega_{L\max}$). A decisive influence on the absorption was exerted in these experiments by collisions, since the effective collision frequency was of the order of the field frequency ($\nu/\omega = 0.1-5$). The role of the transformation could therefore not be large.

Experiments on the interaction of high-frequency waves with a plasma at lower collision frequencies, at which the influence of the collision absorption is small, are reported in[97-101]. In[97] there is a brief description of an experiment on plasma heating by high-frequency power at frequencies corresponding to the lower hybrid frequency band. The authors indicate that, according to preliminary data, the efficiency of energy input to the plasma turned out to be much higher than the efficiency of the ordinary collision absorption.

More detailed information on the conditions of effective wave absorption by a plasma are given in[98], which describes investigations of a plasma produced in a magnetic trap of the mirror type under the influence of high-frequency power ($f = 130-150$ MHz). The high-frequency field was introduced into the plasma from the ends with the aid of two coaxial waveguides (see Fig. 30). The plasma was produced in a chamber 1 m long filled with hydrogen or helium at pressures $10^{-3}-10^{-4}$ mm Hg. The charged-particle concentration was $10^{11}-10^{14}$ cm$^{-3}$, and the magnetic field was 5—10 kOe. Under such conditions, the boundaries of the plasma column practically

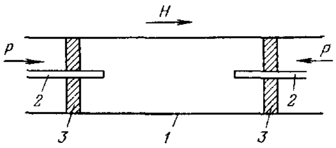


FIG. 30. Scheme for supplying high-frequency power to the plasma in the experiments of [98]. 1—Metal chamber, 2—coaxial leads, 3—ceramic insulators.

coincided with the force lines of the magnetic field, i.e., the concentration gradient was perpendicular to the magnetic field. The efficiency of absorption of high-frequency power by the plasma could be assessed by measuring the glow of the spectral lines. At $p \approx 10^{-4}$ mm Hg and $n \approx 10^{12}$ cm$^{-3}$, a strong dependence of the glow intensity on the constant magnetic field was established (see the characteristic curve on Fig. 31). A decrease in the magnetic field to values corresponding to $\omega_{L\max} \approx \omega$ led to a sharp decrease of the glow intensity, and at the lowest pressures also to termination of the discharge owing to the decreased absorption efficiency. It is natural to assume that this effect is connected with leaving the limits of the wave transformation region—with a concentration gradient perpendicular to the magnetic field, the high-frequency boundary of the middle transformation band corresponds exactly to the condition $\omega_{L\max} = \omega$ [see (34), (37), and Fig. 26]. At high concentrations ($n = 10^{13}$–$10^{14}$ cm$^{-3}$), the absorption efficiency turned out to be smaller and its dependence on the magnetic field weaker. The authors attribute this change to the appearance of an opacity barrier that makes the region $\omega = \omega_L$ inaccessible. Indeed, the value of $k_z$, determined by the distance between the electrodes, turns out at large concentrations to be insufficient to satisfy the accessibility condition (see p. 420).

The influence of the method of wave excitation and of the plasma characteristics on the maximum-absorption condition is described in[99,100]. These experiments were devoted to a study of the absorption of a weak high-frequency signal. The plasma was produced independently with the aid of a microwave discharge in a tube of 5 cm diameter filled with hydrogen at pressures $5 \times 10^{-4}$–$10^{-2}$ mm Hg. The magnetic field had a mirror configuration, but in the central part it was practically homogeneous. In this part, a signal from a high-frequency generator operating at 120–140 MHz was applied to the tube with the aid of a turn 6–12 cm long (see Fig. 32). The longitudinal concentration distribution was determined with the aid of a microwave interferometer and a diamagnetic probe, both of which could be moved longitudinally. At low pressures ($p < 10^{-3}$ mm Hg), the longitudinal change of the concentration in the region of the high-frequency turn did not exceed 10%, i.e., the concentration gradient was perpendicular to the magnetic field. At higher pressures, the concentration decreased noticeably away from the discharge region towards the opposite end of the tube, owing to transverse diffusion. The maximum concentration reached $2 \times 10^{12}$ cm$^{-3}$.

When the magnetic field varied, a broad absorption maximum was observed under different conditions. At low gas pressures, the absorption maximum was observed at a magnetic field at which the largest value of the lower hybrid frequency was close to the field frequency. A change of the dimensions of the exciting turn and its rotation through 10° relative to the tube produced no change whatever in the magnetic field corresponding to the maximum absorption. On the other hand, an increase of the pressure from $5 \times 10^{-4}$ to $10^{-2}$ mm Hg led to an increase of the maximum-absorption frequency by 1.5 times. The results agree with the assumption that the absorption is due to the linear wave transformation. Indeed, the limiting frequency of the transformation band does not depend on the method of excitation and is determined by the angle between the concentration gradient and the magnetic field. With increasing pressure, the average value of this angle, as indicated, deviates from 90° and the limiting frequency of the middle transformation band increases [in accord with (37), even a small deviation of $\alpha$ from 90° should greatly alter the limiting frequency]. This may be the reason for the observed increase in the maximum-absorption frequency.

The same apparatus was also used to perform experiments on plasma heating in a waveguide at a pressure $5 \times 10^{-4}$ mm Hg and at charged-particle concentrations $5 \times 10^{11}$–$5 \times 10^{12}$ cm$^{-3}$[101]. The method of producing the plasma and the scheme for supplying the high-frequency energy in these experiments were analogous to those used in the preceding experiments (see Fig. 32). Heating was with a pulsed generator of frequency $\approx 120$ MHz and 3 kW power at a pulse duration 2 $\mu$sec. The change of the transverse charged-particle energy due to heating was determined from the increase of the diamagnetic signal (the concentration remained unchanged during the time of the pulse, since there was no time for ionization to develop). A typical plot of the heating against the magnetic field is shown in Fig. 33. We see that the heating efficiency decreases sharply when the magnetic field decreases to values corresponding to $\omega_{L\max} < \omega$, i.e., on leaving the limits of the

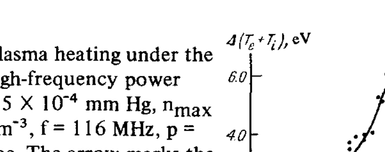


FIG. 32. Diagram showing the supply of high-frequency energy to the plasma in the experiments of [99-101]. 1—Tube with plasma, 2—waveguide for the 3-cm waves used to form the plasma, 3—turn to feed the high-frequency signal in the 120–140 MHz range.




FIG. 31. Glow intensity of a plasma produced by high-frequency power [98]. 1–$H_2$, $p \approx 10^{-4}$ mm Hg, $n \approx 10^{12}$ cm$^{-3}$, line $H_\beta$; 2–He, $p \approx 10^{-4}$ mm Hg, $n \approx 10^{12}$ cm$^{-3}$; 3–He, $p \approx 10^{-4}$ mm Hg, $n \approx 10^{12}$ cm$^{-3}$. The arrows mark the values of H corresponding to the condition $\omega_L = \omega$.

FIG. 33. Plasma heating under the influence of high-frequency power [101]. $H_2$, $p \approx 5 \times 10^{-4}$ mm Hg, $n_{\max} = 1.4 \times 10^{12}$ cm$^{-3}$, f = 116 MHz, p = 3 kW, $\tau = 2$ $\mu$sec. The arrow marks the value of H corresponding to the condition $\omega_{L\max} = \omega$.

transformation region. The maximum plasma heating efficiency reached 20%. Measurements of the longitudinal distribution of the diamagnetic signal have shown that a considerable fraction of the energy input was used to heat the ions (the diamagnetic signal decreased appreciably when the region of energy input was moved away to a distance close to the ion mean free path). It should be noted that the energy-input efficiency decreased at maximal electron concentrations ($n > 10^{12}$ cm$^{-3}$). This decrease, just as in the experiments of[98], could be connected with the influence of the opacity barrier.

## CONCLUSION

The review of the research on absorption of high-frequency waves by a plasma, presented in the present article, shows that the absorption is due in a wide range of conditions to transformation of the incident wave into a slow plasma wave—a transformation connected with the inhomogeneity of the plasma. We note the main results evidencing that the experimental data on the absorption agree with the conclusions of the theory of linear wave transformation.

1. In the band between the lower hybrid frequency and the upper hybrid frequency there are two regions of collisionless absorption. Their boundaries practically coincide with the theoretical boundaries of the wave-transformation regions.

2. The positions of the efficient absorption regions are not connected with the method of wave excitation, but depend essentially on the angle between the concentration gradient and the magnetic field.

3. In the upper transformation-frequency band, at plasma dimensions on the order of the wavelength, a high efficiency of collisionless absorption is obtained, in agreement with the predictions of the theory. For the case of a plasma with dimensions smaller than the wavelength, for which numerical calculations of the transformation efficiency have been made, the results of the calculations agree quantitatively with the absorption-efficiency measurements.

4. At plasma dimensions greatly exceeding the wavelength, high absorption efficiency (in the upper band of the transformation frequencies) can be obtained only in an inhomogeneous magnetic field, under conditions when the region of opacity between the plasma boundary and the transformation surface is eliminated.

5. Within the limits of the transformation bands, the wave-absorption efficiency depends little on the collision frequency and on the charged-particle temperature when these parameters are varied in a wide range. The absorption efficiency also changes little when the high-frequency field intensity is varied over a considerable range.

6. Measurements of the absorption localization, performed for the upper band of the transformation frequencies, have shown that absorption takes place in the region of propagation of the plasma waves produced as a result of the transformation.

We have considered in this article data on the influence of the transformation on the absorption of waves by a plasma. The transformation can also determine the inverse process, radiation, under conditions when plasma waves are excited in the plasma. However, the interpretation of the radiation experiments is in many cases ambiguous, since there are no reliable data on the characteristics of the waves excited in the plasma. Without stopping to describe these experiments, we mention here only some of them.

We have already discussed experiments on thermal radiation from a plasma under conditions when the influence of the transformation is significant[65,68,76,77]. In essence, what was determined in them was the absorption coefficient, since at a given electron temperature the thermal radiation is uniquely connected with this coefficient.

Strong above-thermal radiation of high-frequency waves was observed in many experiments with a plasma having a nonequilibrium electron velocity, particularly in experiments with a plasma pierced by an electron beam (see, for example,[102-108]). Since plasma waves which cannot be radiated directly are excited in such a plasma, the observed radiation is connected with wave transformation. The nature of the transformation could be different in different experiments—in addition to linear transformation connected with the plasma inhomogeneity, an important role could be played by transformations connected with strong inhomogeneity of the plasma-wave field, and also nonlinear transformations of different types. Apparently, however, in some experiments where radiation was observed at frequencies higher than the electron cyclotron frequency (it was particularly strong near cyclotron harmonics), the linear transformation connected with the inhomogeneities of the plasma played the decisive role. This is evidenced by the fact that intense radiation was observed only under conditions when the maximum value of the upper hybrid frequency was higher than the radiation reception frequency[82,106,107], i.e., when a transformation region existed in the volume of the plasma. The conclusions of the theory of linear transformation agree with the observed decrease of the radiation intensity with increasing transverse dimensions of the plasma, when they exceed the wavelength, and also with the decrease of the radiation intensity at appreciable gas pressures, when the collision absorption increases[82,107]. We should furthermore call attention to the narrow maximum of the high-frequency field observed in[108] in a plasma produced by an electron beam. This maximum was located in the plasma region in which the upper hybrid frequency was close to the field frequency, and it can be assumed that it determines the behavior of the field in the transformation region.

Linear wave transformation apparently becomes manifest also in the effects of induced radiation and plasma echo (see[109-111]). These effects are usually connected with excitation of plasma waves. Since, however, they become manifest in the form of radiation, a transformation of the plasma waves into electromagnetic waves connected with radiation should occur. It should be noticed, incidentally, that the need for transformation can lead to a delay of the induced radiation and of the echo—the propagation of plasma waves from the excitation region to the radiation region can last a relatively long time, since the group velocity of these waves is low.

Thus, under suitable conditions, linear wave transformation connected with plasma inhomogeneity can lead, on the one hand, to efficient absorption of the high-frequency waves by the plasma, and on the other hand to radiation of plasma waves excited by an electron beam or by other agents. Both effects can find practical application. Of particular importance is the possibility of using collisionless absorption connected with wave transformation to heat a plasma to high temperatures, in experiments aimed at realizing controlled thermonuclear fusion[112]. An essential advantage of such a method of heating compared with others used at the present time is that the energy-input efficiency is independent of the temperature, of the collision frequency of the charged particles, and, within certain limits, of the value of energy input.


1 N. G. Denisov, Zh. Eksp. Teor. Fiz. **31**, 609 (1956) [Sov. Phys.-JETP **4**, 544 (1957)].
2 V. V. Zheleznyakov, Radiotekhnika i élektronika **1**, 840 (1956).
3 D. A. Tidman, Phys. Rev. **117**, 366 (1960).
4 V. V. Zheleznyakov and E. Ya. Zlotnik, Izv. Vuzov (Radiofizika) **5**, 644 (1962).
5 N. G. Denisov, Radiotekhnika i élektronika **1**, 732 (1956).
6 N. G. Denisov, Zh. Eksp. Teor. Fiz. **34**, 528 (1958) [Sov. Phys.-JETP **7**, 364 (1958)].
7 N. G. Denisov, Radiotekhnika i Élektronika **4**, 388 (1959).
8 V. L. Ginzburg, Rasprostranenie Élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Nauka, 1967.
9 K. G. Budden, Radio Waves in the Ionosphere, Cambridge (1961).
10 V. B. Gil'denburg, Zh. Eksp. Teor. Fiz. **45**, 1978 (1963) [Sov. Phys.-JETP **18**, 1359 (1964)].
11 A. D. Piliya, Zh. Tekh. Fiz. **36**, 818 (1966) [Sov. Phys.-Tech. Phys. **11**, 609 (1966)].
12 O. Ya. Omel'chenko and K. N. Stepanov, Ukr. Fiz. Zh. **12**, 1445 (1967).
13 I. G. Kondrat'ev and M. A. Miller, Izv. Vuzov (Radiofizika) **11**, 885 (1968).
14 G. M. Zaslavskiĭ, S. S. Moiseev and R. Z. Sagdeev, Dokl. Akad. Nauk SSSR **158**, 1295 (1964) [Sov. Phys.-Doklady **9**, 863 (1965)].
15 S. S. Moiseev, Proceeding of the Seventh International Conference on Phenomena in Ionized Gases, Vol. II, p. 645, Belgrade, 1966.
16 W. Wasow, Annals of Mathematics **52**, 350 (1950).
17 A. D. Piliya and V. I. Fedorov, Zh. Eksp. Teor. Fiz. **57**, 1198 (1969) [Sov. Phys.-JETP **30**, 653 (1970)].
18 T. H. Stix, Phys. Rev. Lett. **15**, 878 (1965).
19 C. W. Horton, Phys. Fluids **9**, 815 (1966).
20 P. Gorman, Phys. Fluids **9**, 1262 (1966).
21 K. E. Longren, A. Sjolund and P. Weissglass, Plasma Phys. **8**, 657 (1966).
22 H. H. Kuehl, Phys. Rev. **154**, 124 (1967).
23 V. Kopecky and J. Preinhafelter, Plasma Phys. **11**, 333 (1969).
24 V. Kopecky, J. Preinhafelter and S. Vaclavik, J. Plasma Phys. **3**, part 2, 179 (1969).
25 V. I. Fedorov, Zh. Tekh. Fiz. **41**, 680 (1971) [Sov. Phys.-Tech. Phys. **16**, 533 (1971)].
26 J. A. Fejer, Phys. Fluids **7**, 439 (1964).
27 J. V. Parker, J. C. Nickel and R. W. Gould, Phys. Fluids **7**, 1489 (1964).
28 V. B. Gil'denburg, Yu. M. Zhidko, I. G. Kondrat'ev and M. A. Miller, Izv. Vuzov (Radiofizika) **10**, 1358 (1967).
29 P. Vandenplas, Electron Waves and Resonances in Bounded Plasma, Amsterdam, 1969.
30 V. V. Dolgopolov, Zh. Tekh. Fiz. **36**, 273 (1966) [Sov. Phys.-Tech. Phys. **11**, 198 (1966)].
31 L. A. Zhekulin, Zh. Eksp. Teor. Fiz. **4**, 76 (1934).
32 K. Forsterling, Arch. electr. Übertag **3**, 115 (1949); **5**, 209 (1950).
33 V. B. Gil'denburg, Zh. Eksp. Teor. Fiz. **46**, 2156 (1964) [Sov. Phys.-JETP **19**, 1456 (1964)].
34 T. Stix, Theory of Plasma Waves, McGraw, 1962.
35 V. D. Shafranov, Voprosy teorii plazmy (Problems of Plasma Theory), No. 3, Atomizdat, 1963, p. 3.
36 T. H. Stix, Phys. Fluids **3**, 19 (1960).
37 K. G. Budden, Physics of the Ionosphere, Reports on the Physical Soc. Conf., 320 (1955).
38 V. Agnello, P. Fieffe-Prevot, G. Ychtchenko, G. Kulinski and P. Lallia, Ninth International Conference on Phenomena in Ionized Gases, 1969, Contributed Papers, Bucharest, 1969, p. 519.
39 S. Gruber, Phys. Fluids **11**, 858 (1968).
40 L. Tonks, Phys. Rev. **37**, 1458; **38**, 1219 (1931).
41 D. Romell, Nature **167**, 243 (1951).
42 A. Dattner, Ericsson Technics **13**, 309 (1957); **19**, 3 (1963).
43 A. M. Messian and P. E. Vandenplas, Physica **28**, 537 (1962).
44 F. W. Crowford, J. Appl. Phys. **35**, 1365 (1964).
45 H. J. Schmitt, Appl. Phys. Lett. **4**, 111 (1964).
46 G. Bekefi, Radiation Processes in Plasmas, Wiley, 1966.
47 V. B. Gil'denburg, Zh. Tekh. Fiz. **34**, 372 (1964) [Sov. Phys.-Tech. Phys. **9**, 292 (1964)].
48 R. L. Gloss, Y. A. Clegg and T. B. Kaiser, Phil. Mag. **44**, 313 (1953).
49 R. Stern and N. Tzoar, Phys. Rev. Lett. **15**, 485 (1965).
50 S. Aksornkitti, H. Hsuan and K. Lunnsren, Phys. Rev. **186**, 160 (1969).
51 B. Gregory and K. Parbhakar, Ninth International Conference on Phenomena in Ionized Gases 1969, Contributed Papers, Bucharest, 1969, p. 444.
52 G. Landauer, Plasma Phys. **4**, 395 (1962).
53 G. Bekefi, Y. D. Coccoli, E. B. Hooper and S. Y. Buchsbaum, Phys. Rev. Lett. **9**, 6 (1962).
54 S. Y. Buchsbaum and A. Hasegava, Phys. Rev. Lett. **12**, 685 (1964); Phys. Rev. **143**, 303 (1966).
55 K. Mitani, H. Kubo and S. Tanaka, J. Phys. Soc. Jap. **19**, 211, 221 (1964).
56 S. Tanaka, H. Kubo and K. Mitani, J. Phys. Soc. Jap. **20**, 462 (1965).
57 H. J. Schmitt, G. Meltz and P. J. Freiheit, Phys. Rev. **139**, A1432 (1965).
58 V. B. Gil'denburg and G. A. Markov, Izv. Vuzov (Radiofizika) **11**, 781 (1968).
59 F. W. Crawford, G. S. Kino and H. H. Weiss, Phys. Rev. Lett. **13**, 229 (1964).
60 R. S. Harp, Appl. Phys. Lett. **6**, 51 (1965).
61 S. Gruber and G. Bekefi, Phys. Fluids **11**, 122 (1968).

[62] S. Hiroe and H. Ikegami, Ninth International Conference on Phenomena in Ionized Gases 1969, Contributed Papers, Bucharest, 1969, p. 450.
[63] F. Leuterer, Ninth International Conference on Phenomena in Ionized Gases 1969, Contributed Papers, Bucharest, 1969, p. 449, 535.
[64] F. Leuterer, Fourth European Conference on Controlled Fusion and Plasma Physics, Contributions, Rome, 1970, p. 133.
[65] J. L. Hirshfield and S. C. Brown, Phys. Rev. 122, 719 (1961).
[66] M. V. Krivosheev, Zh. Tekh. Fiz. 39, 816 (1969) [Sov. Phys.-Tech. Phys. 14, 613 (1969)].
[67] V. E. Golant and M. V. Krivosheev, ibid. 39, 958 (1969) [14, 719 (1969)].
[68] F. Blum, L. Bauer, R. Gould and R. Stenzel, Phys. Fluids 12, 1018 (1969).
[69] V. E. Golant, M. V. Krivosheev and V. I. Fedorov, Zh. Tekh. Fiz. 40, 382 (1970) [Sov. Phys.-Tech. Phys. 15, 282 (1970)].
[70] A. I. Anisimov, V. I. Vinogradov, V. E. Golant and L. P. Pakhomov, ibid. 37, 680, 202 (1967) [12, 486, 141 (1967)].
[71] V. N. Budnikov, N. I. Vinogradov, V. E. Golant and A. A. Obukhov, ibid. 37, 861 (1967); 38, 28 (1968) [12, 617 (1967); 13, 19 (1968)].
[72] A. I. Anisimov, N. I. Vinogradov, V. E. Golant, S. I. Nanobashvili and L. P. Pakhomov, ibid. 38, 1902, 1908 (1968) [13, 1529, 1534 (1969)].
[73] A. I. Anisimov, V. N. Budnikov, N. I. Vinogradov, V. E. Golant, S. I. Nanobashvili, A. A. Obukhov, L. P. Pakhomov, A. D. Piliya and V. I. Fedorov, Plasma Physics and Controlled Nuclear Fusion Research, Vol. 1, Vienna, 1969, p. 399.
[74] J. Musil and F. Zacek, Ninth International Conference on Phenomena in Ionized Gases 1969, Contributed Papers, Bucharest, 1969, p. 527.
[75] A. I. Anisimov, N. I. Vinogradov, V. E. Golant and L. P. Pakhomov, Zh. Tekh. Fiz. 41, 696 (1971) [Sov. Phys.-Tech. Phys. 16, 546 (1971)].
[76] A. Y. Wong and A. F. Kuckes, Phys. Rev. Lett. 13, 306 (1964).
[77] A. F. Kuckes and A. Y. Wong, Phys. Fluids 8, 1161 (1965).
[78] S. J. Tetenbaum and H. N. Bailey, Phys. Rev. Lett. 19, 12 (1967).
[79] S. Takeda, Journ. Phys. Soc. Jap. 20, 287 (1965).
[80] V. V. Alikaev, V. M. Glagolev and S. A. Morozov, Plasma Physics and Controlled Nuclear Fusion Research, Vol. II, Vienna, 1969, p. 877.
[81] G. Lisitano, Proceeding of the Seventh International Conference on Phenomena in Ionized Gases, vol. 1, Belgrade, 1966, p. 464.
[82] S. J. Tetenbaum, Phys. Fluids 10, 1577 (1967).
[83] J. Datlov, J. Musil and F. Zacek, Proceeding of the 4th Czechoslovak Conference on Electronic and Vacuum Physics, Prague, 1968, p. 62.
[84] A. F. Kuckes and R. B. Turner, Plasma Physics and Controlled Nuclear Fusion Research, Vol. 1, Vienna, 1969, p. 427.
[85] O. A. Anderson, D. H. Birdsall, G. W. Hortuan, E. J. Laner and H. P. Furth, Plasma Physics and Controlled Nuclear Fusion Research, Vol. 1, Vienna, 1969, p. 443.
[86] V. N. Budnikov, V. E. Golant and A. A. Obukhov, Zh. Tekh. Fiz. 38, 576 (1968) [Sov. Phys.-Tech. Phys. 13, 427 (1968)].
[87] V. N. Budnikov, V. E. Golant and A. A. Obukhov, ibid. 40, 138 (1970) [15, 97 (1970)]; Phys. Lett. 31A, 76 (1970).
[88] B. V. Galaktionov, V. E. Golant, V. V. D'yachenko and O. N. Shcherbinin, Zh. Tekh. Fiz. 40, 2322 (1970) [Sov. Phys.-Tech. Phys. 15, 1813 (1971)].
[89] G. Lisitano, P. Caldirola, M. Fontanesi and E. Sindoni, International Conference on Closed Systems, Dubna, 1969.
[90] A. P. Akhmatov, P. I. Blinov, V. F. Bolotin, A. V. Borodin, P. P. Gavrin, E. K. Zavoĭskiĭ, I. A. Kovan, M. N. Oganov, B. I. Patrushev, E. V. Piskarev, V. D. Rusanov, G. E. Smolkin, A. R. Striganov, D. A. Frank-Kamenetskiĭ, G. A. Cheremnykh and R. V. Chikin, Zh. Eksp. Teor. Fiz. 39, 536 (1970) [Sov. Phys.-JETP 12, 376 (1961)].
[91] I. A. Kovan, B. I. Patrushev, V. D. Rusanov, G. N. Tilinin and D. A. Frank-Kamenetskiĭ, ibid. 43, 16 (1962) [16, 10 (1963)].
[92] A. V. Bartov, E. K. Zavoĭskiĭ, I. A. Kovan, B. I. Patrushev, V. D. Rusanov and D. A. Frank-Kamenetskiĭ, Nuclear Fusion Supplement, pt. 3, p. 1067 (1962).
[93] I. A. Kovan, L. L. Kozorovitskiĭ, I. M. Podgornyĭ, V. D. Rusanov, V. P. Smirnov, A. M. Spektor and D. A. Frank-Kamenetskiĭ, Atomnaya Énergiya 25, 503 (1968).
[94] I. A. Kovan and A. M. Spektor, Zh. Eksp. Teor. Fiz. 53, 1278 (1968) [Sov. Phys.-JETP 26, 747 (1969)].
[95] H. Schlüter and C. J. Ransom, Ann. of Phys. 33, 360 (1965).
[96] G. S. Mills, M. E. Oakes and H. Schlüter, Phys. Lett. 21, 45 (1966).
[97] V. V. Alikaev, Yu. I. Arsen'ev, V. M. Bogachev, V. M. Glagolev, S. A. Morozov, I. N. Khramkov and N. S. Cheverev, Plasma Physics and Controlled Nuclear Fusion Research, Vienna, Vol. II, 1969, p. 381.
[98] O. M. Shvets, S. M. Sotnikov, V. F. Tarasenko and S. S. Ovchinnikov, Zh. Tekh. Fiz. 39, 610 (1969) [Sov. Phys.-Tech. Phys. 14, 459 (1969)].
[99] B. V. Galaktionov, V. E. Golant, A. D. Piliya and O. N. Shcherbinin, ibid. 39, 960 (1969) [14, 721 (1969)].
[100] B. V. Galaktionov, V. E. Golant and O. N. Shcherbinin, Ninth International Conference on Phenomena in Ionized Gases 1969, Contributed Papers, Bucharest, 1969, p. 516.
[101] B. V. Galaktionov, V. V. D'yachenko and O. N. Shcherbinin, Zh. Tekh. Fiz. 40, 2317 (1970) [Sov. Phys.-Tech. Phys. 15, 1809 (1971)].
[102] R. A. Demirkhanov, A. K. Gevorkov, and A. F. Popov, ibid. 30, 315 (1960) [5, 290 (1960)].
[103] V. A. Suprunenko, Ya. B. Faĭnberg, V. T. Tolok, E. A. Sukhomlin, N. I. Reva, P. Ya. Burchenko, N. I. Rudnev and E. V. Volkov, Atomnaya énergiya 14, 349 (1963).
[104] I. F. Kharchenko, Ya. B. Faĭnberg, E. A. Kornilov and N. S. Pedenko, Zh. Tekh. Fiz. 34, 1031 (1964) [Sov. Phys.-Tech. Phys. 9, 798 (1964)].
[105] O. S. Pavlichenko, L. A. Dushin, Yu. K. Kuznetsov and I. Yu. Adamov, ibid. 35, 1394 (1965) [10, 1082 (1966)].
[106] L. A. Dushin, Yu. K. Kuznetsov and O. S. Pavlichenko, ibid. 38, 1251 (1968) [13, 1030 (1969)].
[107] M. I. Kogan and I. E. Sakharov, ibid. 37, 2173 (1967)

[12, 1599 (1968)].
[108] I. E. Sakharov, ibid. 39, 278 (1969) [14, 198 (1969)].
[109] R. M. Hill, D. E. Kaplan and S. K. Ichiki, Phys. Rev. Lett. 19, 154 (1967).
[110] D. E. Baldwin, D. M. Henderson and I. L. Hirshfield, Phys. Rev. Letters 20, 314 (1968).
[111] L. O. Bauer, F. A. Blum and R. W. Gould, Phys. Rev. Lett. 20, 435 (1968).
[112] V. E. Golant and A. D. Piliya, Paper at Mezhdunarodnoi (International Conference on Closed Systems), Dubna, 1969.
[113] N. S. Erokhin and S. S. Moiseev, Preprint, Physicotech. Inst. Ukr. Acad. Sci. No. 70-20, Khar'kov, 1969.
[114] A. D. Piliya and V. I. Fedorov, Zh. Eksp. Teor. Fiz. 60, 389 (1971) [Sov. Phys.-JETP 33, 210 (1971)].
[115] V. I. Arkhipenko, V. N. Budnikov and A. A. Obukhov, Phys. Lett. 31A, 76 (1970).

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