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HOT ELECTRONS IN SEMICONDUCTORS SUBJECTED TO QUANTIZING MAGNETIC FIELDS

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I. INTRODUCTION

STRONG electric fields produce a great variety of effects in semiconductors. They alter basically the quantum states of carriers and their energy spectrum. This gives rise to dependences of the macroscopic properties of semiconductors on the applied field E. Examples of such effects are: the dependence of the complex permittivity on E, resulting from the possibility of fundamental absorption of photons whose energy is less than the forbidden band width (the Franz–Keldysh effect); the tunnel current in a degenerate p-n junction (the Esaki effect), etc.

The application of strong electric fields can give rise to states in semiconductors which are far from thermodynamic equilibrium. Such states appear when sound is amplified or generated by carriers drifting in piezoelectric and nonpiezoelectric semiconductors. These problems have been reviewed by Gurevich[1] and Pustovoĭt.[2]

"Hot" electrons in semiconductors are another example of a nonequilibrium state. The term "hot" electrons has been suggested by Shockley[3] for a nonequilibrium state of carriers in a semiconductor in which their average kinetic energy is increased by an external electric field so that it can be described by an effective temperature $T_e(E)$ which exceeds the lattice temperature T. Such heating of carriers by an electric field alters considerably many physical properties of semiconductors and gives rise to new effects, in particular, to a dependence of the electrical conductivity on the electric field, nonlinear galvanomagnetic effects, S-type and N-type current-voltage characteristics and associated instabilities of various types. The experimental and theoretical investigations of hot electrons were reviewed by Conwell[4] (the Russian translation of this review was supplemented by numerous editorial comments and by references to newer studies). Physical effects which are observed in semiconductors with S- and N-type current-voltage characteristics were reviewed in detail by Volkov and Kogan.[5]

The successful generation of strong magnetic fields has made it possible to investigate experimentally hot electrons under conditions of quantization of their orbital motion. The magnetic fields in which this motion is quantized are usually called the quantizing fields. Such fields alter significantly the energy spectrum of carriers and they are responsible for the appearance of discrete energy levels (the Landau levels). These effects give rise to a magnetic-field dependence of the rates of relaxation in the electron and phonon subsystems of a semiconductor. Thus, an experimenter can now use—in addition to the temperature—a new macroscopic parameter (a quantizing magnetic field) to "control" the characteristic relaxation frequencies in a system by altering the relationships between them.

The most important effect of a quantizing magnetic field is a strong reduction of the ratio of the frequency of electron-electron collisions ν_{ee} to the frequency of electron-phonon collisions ν_{ep}. Consequently, nondegenerate electrons fill preferentially the lowest Landau level, whereas all the other levels contain exponentially small numbers of electrons (it is assumed that the energy of a carrier cyclotron quantum is $\hbar\Omega \gg T_e$).

Under these conditions the frequency of collisions between electrons belonging to different Landau levels becomes exponentially small. Collisions between electrons within the main group located at the lowest Landau level become elastic because of the one-dimensional nature of their motion. This means that such collisions do not alter the state of the system and, naturally, make no contribution to electron-electron relaxation processes.

On the other hand, the relaxation frequency associated with the interaction between electrons and phonons, which is proportional to the product of the volume of the phase space of the momentum of those phonons which interact with electrons and the density of electron states, increases as the square of the magnetic field intensity (if we ignore quantum oscillations). Therefore, in contrast to the classical case, the ratio of the collision frequencies in a quantizing magnetic field is given by

$$\frac{\nu_{ee}}{\nu_{ep}} \sim \left(\frac{T_e}{\hbar\Omega}\right)^2 \exp\left(-\frac{\hbar\Omega}{T_e}\right).$$

The suppression of the electron-electron scattering process by a quantizing magnetic field reduces strongly the efficiency of the redistribution of electrons over their quantum states and this alters considerably the nature of the carrier distribution function.

A quantizing magnetic field has also a strong influence on the relaxation frequency of long-wavelength phonons (τ_{pe}^{-1}) whose momenta are $\hbar q \lesssim \hbar\lambda^{-1} = \hbar q_B < T/s = \hbar q_T$ [$\lambda = (c\hbar/|e|B)^{1/2}$ is the magnetic length or the quantum Larmor radius, s is the velocity of sound] and which interact with electrons. According to the law of conservation of momentum, only these phonons can interact with electrons. The long-wavelength phonons can also relax at a characteristic frequency τ_{pp}^{-1} by interacting with short-wavelength thermal phonons whose momenta are T/s. A quantizing magnetic field can alter the relationship between τ_{pe}^{-1} and τ_{pp}^{-1} and, therefore, such a field can be used to investigate the phonon-phonon relaxation frequency τ_{pp}^{-1}. In fact, when $\tau_{pe}^{-1} \gg \tau_{pp}^{-1}$, the "bottleneck" (whose cross section is proportional to τ_{pp}^{-1}) in the energy relaxation of the hot electrons is the transfer of energy from the long- to the short-wavelength phonons (the thermal reservoir). Finally, in very strong magnetic fields characterized by $q_B > q_T$ the energy of hot electrons may be transferred first to nonequilibrium phonons and from them—by collisions with the boundaries of a sample—to the surrounding medium (the reservoir in which the whole sample is located). These very strong magnetic fields can be used to investigate the energy relaxation frequency of phonons interacting with the boundaries of a sample.

Thus, a quantizing magnetic field affects the relaxation processes and alters considerably the conditions for electron heating and for the appearance of "overheating" instabilities.

The present review discusses the work done on the problems outlined in the preceding paragraphs.*

*We shall not consider the first theoretical investigations [6-9] in which a reduction in the mean free path of carriers due to heating has been predicted. This prediction has been confirmed experimentally in later studies. [10-12] The role of the electron-electron scattering has been investigated in [13-15].

II. HEATING OF ELECTRONS IN CROSSED ELECTRIC AND QUANTIZING MAGNETIC FIELDS

1. Quantitative Estimates of Electron Heating

The principal approximations in the theory of hot electrons can be considered by making qualitative estimates on the basis of the laws of conservation of energy and momentum.

We shall assume that the inequality $\Omega\tau_p \gg 1$ is satisfied (Ω is the cyclotron frequency and τ_r is the momentum relaxation time). An external electric field **E**, applied at right-angles to a magnetic field **B**, does the following work on electrons per unit time: $e^2 n_e \tau_r \mathbf{E}^2/m(\Omega\tau_r)^2$ (n_e is the electron density and m is the effective mass of an electron). The Hall field is, for the time being, assumed to be zero.

The energy balance is of the form

$$\frac{e^2 n_e}{m(\Omega\tau_r)^2}\tau_r E^2 = \frac{(\varepsilon^* - \bar{\varepsilon})\, n_e}{\tau_\varepsilon}, \tag{1.1}$$

where $\bar{\epsilon}$ and ϵ^* are the equilibrium and nonequilibrium values of the mean energy of an electron and τ_ϵ is the energy relaxation time.

It follows from Eq. (1.1) that

$$\varepsilon^* = \bar{\varepsilon}\left[1 + \frac{1}{2}\left(\frac{eER_L}{\bar{\varepsilon}}\right)^2 \delta^{-1}\right], \quad \delta = \frac{\tau_r}{\tau_\varepsilon}. \tag{1.2}$$

In the absence of a magnetic field the expression for the energy of hot electrons is of the same form as Eq. (1.2) but the Larmor radius R_L is replaced by the mean free path.

In a quantizing magnetic field, when $\bar{\epsilon} \ll \hbar\Omega$ (the quantum limit), we have to replace R_L with the magnetic length $\lambda = (c\hbar/|e|B)^{1/2}$ (known also as the quantum Larmor radius). We must also make allowance for the dependence of the inelasticity parameter δ on B. If electrons are scattered quasielastically, i.e., if $\delta^{-1}(B)$ is large, the heating of electrons may be strong even if the parameter $eE\lambda/\bar{\epsilon}$ is small. We shall estimate $\delta(B)$ assuming, for the time being, that the energy and momentum are dissipated in electron-phonon collisions. The order of magnitude of $\delta(B)$ is the same as that of the ratio of the energy transferred by an electron to the lattice in one collision to the characteristic energy of an electron. The probability of emission of a phonon of momentum $\hbar q$ and frequency ω_q, calculated per unit time, is proportional to $(1 + N_q)$ and the probability of absorption is proportional to N_q. The total number of collisions of an electron with phonons is $\sim(1 + 2N_q)$. The energy transferred to the lattice in one collision is $\Delta\epsilon \sim [(1 + N_q) - N_q]\hbar\omega_q(1 + 2N_q)^{-1}$. Electrons interact with phonons whose momentum is $\hbar q \sim \hbar\lambda^{-1}$. At moderately low temperatures $N_q \approx T/\hbar s\lambda^{-1}$ (s is the velocity of sound and T is the lattice temperature). Using these estimates, we find that the inelasticity coefficient is

$$\delta \sim \frac{\Delta\varepsilon}{\varepsilon} = \frac{\hbar s\lambda^{-1}}{T}\,\frac{\hbar s\lambda^{-1}}{\bar{\varepsilon}} = \frac{(\hbar s)^2}{(T\lambda)^2} = \left(\frac{\hbar\tilde{\omega}_q}{T}\right)^2 \ll 1. \tag{1.3}$$

It follows from this formula that the scattering of electrons by phonons remains quasielastic in a wide range of temperatures and magnetic field intensities. Substituting Eq. (1.3) into Eq. (1.2), we obtain the following estimate for the temperature of the hot electrons:

$$T_e = T\left[1+\frac{1}{2}\left(\frac{cE}{sB}\right)^2\right]. \tag{1.4}$$

If the elastic electron-impurity scattering is also important, we find that

$$\delta \sim \left(\frac{\hbar\tilde{\omega}_q}{T}\right)^2\left(\frac{\nu_{ep}}{\nu_{ep}+\nu_{ei}}\right), \tag{1.5}$$

where ν_{ep} and ν_{ei} are the momentum relaxation frequencies in the case of interaction with phonons and impurities, respectively, and Eq. (1.4) becomes:

$$T_e = T\left\{1+\frac{1}{2}\left(\frac{cE}{sB}\right)^2[1+(\nu_{ei}/\nu_{ep})]\right\}. \tag{1.6}$$

Similar expressions for the electron temperature were first obtained by Kazarinov and Skobov.[16]

In the experiments on hot electrons it is usual to employ conditions under which there is no current in the Hall direction ($I_y = 0$) but the Hall field is $E_y \neq 0$. The preceding discussion can be easily extended to this case. For this purpose we shall use the condition $I_y = \sigma_{xx}E_y + \sigma_{yx}E_x = 0$ to express the Joule power of hot electrons I_xE_x in the form

$$I_xE_x = \left[1+\left(\frac{\sigma_{xy}}{\sigma_{xx}(T_e)}\right)^2\right]\sigma_{xx}(T_e)E_x^2. \tag{1.7}$$

It follows from this formula that an allowance for the Hall field in Eq. (1.6) can be made by the simple replacement of E^2 with $[1 + (\sigma_{xy}/\sigma_{xx}(T_e))^2]E_x^2$. We then obtain the following relationship:

$$T_e = T\left\{1+\frac{1}{2}\left(\frac{cE_x}{sB}\right)^2\left[1+\left(\frac{\sigma_{xy}}{\sigma_{xx}(T_e)}\right)^2\right]\left(1+\frac{\nu_{ei}}{\nu_{ep}}\right)\right\}. \tag{1.8}$$

This equation, which gives T_e in terms of T, B, and E, does not always have real positive solutions for T_e. The nature of the solution depends primarily on the carrier relaxation mechanisms which determine the dependence of σ_{xx} on T_e. Thus, for example, if the main contribution to σ_{xx} is made by the elastic scattering on neutral impurities, it follows that $\sigma_{xx} \propto T_e^{-3/2}$. Therefore, it follows from Eq. (1.7) that the Joule power during heating increases proportionally to $T_e^{3/2}$, whereas the power P transferred by electrons to phonons decreases with increasing T_e. Consequently, in fields E_x exceeding a certain critical value E_{cr} the energy balance is no longer possible. The system becomes unstable through heating, which destroys the orbital quantization of electrons by the magnetic field and transfers them to the classical region ($\hbar\Omega < T_e$), where different dependences of I_xE_x and of P on T_e ensure that a stable stationary state is formed. In this situation the current-voltage characteristic is of the S-type.

However, if the Hall field vanishes ($E_y = 0$), the effective electron temperature is given by Eq. (1.6) and can be expressed directly in terms of T, B, and E_x. Substituting this equation into $\sigma_{xx} \propto (T/T_e)^{3/2}$, we can easily show that the current-voltage characteristic is initially (in the ohmic region) linear, rises to a maximum, and then begins to fall. However, in strong electric fields when the heating destroys the quantization ($\hbar\Omega < T_e$), this fall in the current-voltage characteristic changes to a rise. In this way, the characteristic acquires the N-type shape.[16, 17]

In the quantum region we can use the smallness of the parameters $(eE\lambda/\bar{\epsilon})$ and $(\Omega\tau_r)^{-1}$ to establish a relationship between the diagonal $\bar{f}$ and the nondiagonal $\tilde{f}$ elements of the density matrix (in the Landau representation):

$$\tilde{f} \sim \left(\frac{eE\lambda}{\bar{\varepsilon}}\right)\left\{1+\left(\frac{1}{\Omega\tau}\right)+\ldots\right\}\bar{f}.$$

Like the symmetrical part of the distribution function in the classical case, the diagonal element of the density matrix $\bar{f}$ makes no contribution to the transport of charge and energy. Such transport is determined completely by the nondiagonal element $\tilde{f}$, which is the quantum analog of the asymmetrical part of the classical distribution function. Since $\tilde{f} \ll \bar{f}$, the relationship between $\tilde{f}$ and $\bar{f}$ is the same as in the linear form of the quantum transport theory. The principal difference between the theory which makes allowance for the heating of electrons and the linear theory is the dependence of $\bar{f}$ on the electric field through T_e. This is why the Joule power can be represented in the form of Eq. (1.7) with σ_{xx} dependent on T_e. In the linear theory $\bar{f}$ is the thermodynamic equilibrium form of the density matrix.

This analysis of the heating of electrons by a strong electric field is based on the assumption that we can introduce the concept of an effective electron temperature T_e. In view of this, we shall consider the strength of the electron-electron scattering processes. It follows from general considerations that the electron-electron collision frequency increases with increasing electron density and it can exceed the frequency of the collisions between electrons and phonons. The energy acquired by electrons from the external electric field is redistributed rapidly between them because of the high frequency of the electron-electron collisions. The temperature of the electron system rises because the transfer of energy from electrons to phonons (to the lattice) is slow. Since the drift momentum of electrons usually relaxes much faster than their energy, frequent electron-electron collisions under drift conditions ensure that the distribution function remains Maxwellian but the effective electron temperature T_e is, generally speaking, not equal to the lattice temperature.

A consistent analysis of the cases of high and low carrier densities can be carried out if we can establish a quantitative criterion which divides these two cases. This requires an estimate of the effective frequencies of the electron-phonon and electron-electron collisions.

2. Characteristic Frequencies of Electron-electron and Electron-phonon Collisions

In the classical (nonquantum) theory an analysis of the electron-electron and electron-phonon collisions integrals leads to the concept of a critical electron density n_{cr} [18] at which the frequencies of the electron-electron and electron-phonon collisions are equal. A consistent analysis of this problem, based on the Boltzmann equation, was given by Dykman and Tomchuk.[15a] According to these authors,

$$n_{cr} = ms^2T_e^2\left[4\pi e^4 l_{ep} T \ln\left(\frac{r_D}{b}\right)\right]^{-1}, \qquad (2.1)$$

where $l_{ep} = \pi\rho_0\hbar^4 s^2(C_0^2 m^2 T)^{-1}$ is the mean free path of electrons scattered by phonons in the absence of a magnetic field; ρ_0 is the density of the semiconductor under investigation; C_0 is the deformation potential constant; $b \sim e^2/T_e$ is the minimum value of the impact parameter, defined as the distance in which the kinetic energy of the colliding electrons becomes comparable with the energy of interaction between them; $r_D = (T_e/4\pi n e^2)^{1/2}$ is the Debye radius. At electron densities $n > n_{cr}$ the electron-electron collisions predominate over the electron-phonon events and we may introduce the concept of an effective electron temperature.

A quantizing magnetic field reduces the importance of the electron-electron collisions and displaces strongly the value of n_{cr} toward higher electron densities.[19] It follows from Eq. (A.1) that those collisions between electrons at the same Landau level which are not accompanied by transitions to other levels have no influence on the distribution function because the collision integral associated with them vanishes for any dependence of $f_{n\nu}$ on p_z^ν. This happens because the collisions between electrons at the same Landau level ($n_{\nu'} = n_{\mu'} = n_\nu = n_\mu$) are one-dimensional and elastic and, consequently, they do not alter the microscopic state of the electron system. A nonzero contribution to the electron-electron collision integral can be made only by the collisions between electrons belonging to different Landau levels. If $\hbar\Omega \gg \bar{\epsilon}$, only the low Landau levels with $n = 0$ and 1 are important. We can easily demonstrate that the principal contribution to the collision integral $I_\nu^{ee}(f)$ is made only by the terms with $n_\nu = n_{\nu'} = 0$ and $n_\mu = n_{\mu'} = 1$. All the other terms, apart from those just given, make an exponentially small contribution either because of the large values of n (in this case, $n_{\nu'} + n_{\mu'} - n_\nu - n_\mu = 0$, $p_z \sim \sqrt{mT_e}$) or because of the large value of k_z necessary for the transfer of a particle to a higher Landau level (in this case, $n_{\nu'} + n_{\mu'} - n_\nu - n_\mu \neq 0$). The collisions involving a small transfer of the momentum $\hbar k_z$ become important because of the infinite radius of the Coulomb interaction. Therefore, $I_{\nu\nu}^{ee}(f)$ can be expanded as a series in $\hbar k_z/p_z$. The more important terms of this expansion are of the form:[19]

$$I_{\nu\nu}^{ee}(f) \simeq \frac{me^4}{\pi\hbar\lambda^2}\ln\left(\frac{r_D^2}{\gamma_0\lambda^2}\right)\frac{d}{dp_z}\left(f_1\frac{df_0}{dp_z} - f_0\frac{df_1}{dp_z}\right) \qquad (2.2)$$

($\gamma_0 = 1.781$ is the Euler constant).

Equation (2.2) has been derived ignoring the emission or absorption of plasma oscillation quanta (plasmons). The approximation corresponds to a negligible frequency dispersion of the longitudinal permittivity. It has also been assumed that $\lambda/r_D \ll 1$. We shall go over to dimensionless variables in Eq. (2.2) by applying the following transformations:

$$x = \frac{p_z}{\sqrt{2mT_e}}, \quad \Psi_n(x) = \frac{2}{(2\pi\lambda)^2\hbar}\sqrt{2mT_e}\, f_n(p_z)\frac{1}{n_0(n)}, \qquad (2.3)$$
$$n_0(n) = n_0\exp(-n\hbar\Omega/T_e),$$

where n_0 is the density of electrons at the Landau level $n = 0$ which is practically equal—at sufficiently high values of $\hbar\Omega/T_e$—to the electron density n_e; T_e is a formally introduced parameter which becomes identical with the effective electron temperature at electron densities exceeding n_{cr}. The function $\Psi_n(x)$ is subject to the normalization condition

$$\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty} dx\,\Psi_n(x) = 1. \qquad (2.4)$$

In dimensionless variables Eq. (2.2) becomes

$$I_{\nu\nu}^{ee}(f) \simeq \tilde{\nu}_{ee}\frac{d}{dx}\left\{\Psi_1(x)\frac{d\Psi_0(x)}{dx} - \Psi_0(x)\frac{d\Psi_1(x)}{dx}\right\}; \qquad (2.5)$$

here,

$$\tilde{\nu}_{ee} = \frac{2\pi m n_e e^4}{(2mT_e)^{3/2}}\ln\left(\frac{r_D^2}{\gamma_0\lambda^2}\right)\exp\left(-\frac{\hbar\Omega}{T_e}\right). \qquad (2.6)$$

Let us now consider the electron-phonon collision integral $I_{\nu\nu}^{ep}(f)$. At moderately low lattice temperatures, when collisions of electrons with phonons are quasielastic, we encounter a small parameter $\hbar s/\lambda T = (ms^2/T)^{1/2}(\hbar\Omega/T)^{1/2} \ll 1$. The collision integral of Eq. (A.2) can be expanded as a series in terms of this parameter. If the electric field is weak so that it does not affect the energy spectrum of electrons, the expansion becomes

$$I_{np_z}^{ep}(f) = \frac{2\pi^2\hbar^3 s^2}{ml_{ep}}\varepsilon\sum_{n'} g_{n'}(\varepsilon)\left\{\frac{d^2 f_{n'}(\varepsilon)}{d\varepsilon^2} + \left(\frac{1}{T} - \frac{1}{\varepsilon - \hbar\Omega\left(n' + \frac{1}{2}\right)}\right)\frac{df_{n'}(\varepsilon)}{d\varepsilon}\right.$$
$$\left. - \frac{f_{n'}(\varepsilon) + f_n(\varepsilon)}{2T\left[\varepsilon - \hbar\Omega\left(n' + \frac{1}{2}\right)\right]} + \frac{f_{n'}(\varepsilon) - f_n(\varepsilon)}{2ms^2\varepsilon}\right\}, \qquad (2.7)$$

where $\epsilon = \epsilon_{np_z} = \hbar\Omega(n + 1/2) + p_z^2/2m$, $g_n(\epsilon) = 2\sqrt{2m}/(2\pi\lambda)^2\hbar(\epsilon - \hbar\Omega(n + 1/2)]^{-1/2}$ is the density of states of energy ϵ corresponding to the quantum number n.

Going over, as in Eq. (2.2), to the dimensionless variables of Eq. (2.3), we obtain this formula for the quantum limit:

$$I_{\nu\nu}^{ep}(f) = \tilde{\nu}_{ep}\frac{d}{dx}\left\{\frac{1}{x^2}\left[1 + \frac{T}{2T_e}\frac{d}{x\,dx}\right]\Psi_0(x)\right\}, \qquad (2.8)$$

where

$$\tilde{\nu}_{ep} = \frac{s^2}{l_{ep}T}(2mT_e)^{1/2}\left(\frac{\hbar\Omega}{2T_e}\right)^2. \qquad (2.9)$$

The coefficients $\tilde{\nu}_{ee}$ and $\tilde{\nu}_{ep}$ in Eqs. (2.5) and (2.8) are the characteristic frequencies of the electron-electron and electron-phonon collisions. The critical electron density can be found from $\tilde{\nu}_{ee} = \tilde{\nu}_{ep}$. The result is[19]

$$n_{cr} = n_{cr}^{cl}\left(\frac{\hbar\Omega}{T_e}\right)^2\exp\left(\frac{\hbar\Omega}{T_e}\right)\frac{\ln(r_D^2/b^2)}{\ln(r_D^2/\gamma_0\lambda^2)}. \qquad (2.10)$$

3. High Electron Densities

The transport equation which defines the diagonal element of the density matrix is of the form (see Appendix):

$$I_{\varkappa\varkappa}^{ee}(f) + I_{\varkappa\varkappa}^{ep}(f) + I_{\varkappa\varkappa}^{ei}(f) = 0 \qquad (3.1)$$

[$\bar{\kappa} \equiv (n^\kappa, p_z^\kappa)$]. At high electron densities $n_e > n_{cr}$, defined by Eq. (2.10), the electron-electron collisions become dominant and, therefore, the terms $[I_{\bar{\kappa}\bar{\kappa}}^{ep}(f) + I_{\bar{\kappa}\bar{\kappa}}^{ei}(f)]$ in Eq. (3.1) can be ignored compared with $I^{ee}_{\bar{\kappa}\bar{\kappa}}(f)$. An asymptotic solution of Eq. (3.1), corresponding to the spatially uniform distribution, can be represented in the form:

$$f_{\bar{\varkappa}}(T_e)=\exp\left\{(\zeta-E(\bar{\varkappa}))\frac{1}{T_e}\right\}, \tag{3.2}$$

where T_e is the effective temperature of hot electrons. Substituting the solution given by Eq. (3.2) into the rejected part of Eq. (3.1), we obtain the relationship

$$I^{ep}_{\bar{\varkappa}\bar{\varkappa}}[f(T_e)]+I^{ei}_{\bar{\varkappa}\bar{\varkappa}}[f(T_e)]=0, \tag{3.3}$$

which can be used to express T_e in terms of the electric field E. In fact, multiplying Eq. (3.3) by $E(\bar{\kappa})$ and summing over $\bar{\kappa}$, we can find the energy balance equation:[29, 21, 67]

$$I_xE_x=P, \tag{3.4}$$

where

$$P=\frac{2\pi}{\hbar}\sum_{(q,\varkappa,\nu)}\hbar\omega_q|A(q,\bar{\varkappa},\bar{\nu})|^2\,\delta(E(\bar{\nu})-E(\bar{\varkappa})+\hbar\omega_q$$
$$+eE\lambda^2q_y)\{f_{\bar{\varkappa}}(T_e)-f_{\bar{\nu}}(T_e)\}\left\{N_q\left(\frac{\hbar\omega_q+e\lambda^2Eq_y}{T_e}\right)-N_q\left(\frac{\hbar\omega_q}{T}\right)\right\} \tag{3.5}$$

is the power transferred by electrons to the lattice, and the Joule power is

$$(IE)=\frac{2\pi}{\hbar}\sum_{(q,\varkappa,\nu)}-e\lambda^2Eq_y|A(q,\varkappa,\nu)|^2\,\delta\{E(\bar{\varkappa})-E(\bar{\nu})-\hbar\omega_q-eE\lambda^2q_y\}$$
$$\times\{f_{\bar{\varkappa}}(T_e)-f_{\bar{\nu}}(T_e)\}\left\{N_q\left(\frac{\hbar\omega_q+eE\lambda^2q_y}{T_e}\right)-N_q\left(\frac{\hbar\omega_q}{T}\right)\right\}$$
$$+\frac{2\pi}{\hbar}N_i\sum_{(q,\varkappa,\nu)}-\frac{1}{2}e\lambda^2Eq_y|V_i(q,\varkappa,\nu)|^2\,\delta(E(\bar{\varkappa})-E(\bar{\nu})-eE\lambda^2q_y)$$
$$\times[f_{\bar{\varkappa}}(T_e)-f_{\bar{\nu}}(T_e)]; \tag{3.6}$$

here, $N_q(\hbar\omega_q/T)$ is the Planck distribution function and N_i is the impurity concentration. The expression for the electrical conductivity σ_{xx} was deduced from the above equations by Kalashnikov and Pomortsev[21] and later by Calecki,[22] Elesin,[23] and Budd.[24] The expression for the scattering by impurities was given by Adams and Holstein.[25] If $eE\lambda/T_e \ll 1$, the expression for the electrical conductivity reduces to the well-known Titeica formula in the following two cases: 1) an isothermal system with $T_e = T$; 2) quasielastic scattering, $(\hbar s/\lambda T) < 1$, of hot electrons ($T_e \neq T$) by phonons.

It is worth noting that Eqs. (3.5) and (3.6) remain valid also in the case of degenerate carriers but, in this case, $f_\nu(T_e)$ must be assumed to be the Fermi function at all points.

This is a convenient point for considering the role of the term $e\lambda^2Eq_y$ in the argument of the function δ. It is known from the linear transport theory[25] that a logarithmic divergence occurs in σ_{xx} when the Born approximation is applied to the scattering of electrons by impurities. This divergence may be removed by the broadening of the Landau levels or by the inelasticity of the electron scattering. The truncation parameters of this divergence are $(\hbar/\tau\bar{\epsilon})$ and $(\hbar s/\lambda\bar{\epsilon})$ ($\hbar/\tau$ is the width of the Landau level and $\hbar s/\lambda$ is the characteristic frequency of the acoustical phonons interacting with electrons). Another way of removing this divergence is suggested in the nonlinear theory of galvanomagnetic effects.[26] In fact, the argument of the function δ in Eq. (3.6) contains the term $eE\lambda^2q_y$, which acts as the inelasticity parameter in the scattering by impurities and which contributes an additional inelasticity in the case of scattering by acoustical phonons. Therefore, the logarithmic divergence in σ_{xx} is truncated by one of the parameters $(e\lambda E/\bar{\epsilon})$, $(\hbar/\tau\bar{\epsilon})$, and $(\hbar s/\lambda\bar{\epsilon})$.

The energy balance equation (3.4) makes it possible to express the effective electron temperature in terms of the external fields **E**, **B**, and the lattice temperature T. We shall use this circumstance in the calculation of the Joule power. This power depends on the boundary conditions. We shall consider two cases: $E_y = 0$ and $E_y \neq 0$. The case $E_y = 0$ may be realized in infinite systems with the Hall current $I_y = cen_eE_x/B$, or in conductors with the same number of holes and electrons, or in special situations such as that in the Corbino disk. In all other cases, the Hall field is $E_y \neq 0$. However, if $E_y = 0$, it follows that $(IE) = \sigma_{xx}E^2$, whereas if $E_y \neq 0$ and $|\sigma_{xy}| \gg \sigma_{xx}$, we obtain

$$(IE)=\frac{\sigma^2_{xy}}{\sigma_{xx}}E^2_x.$$

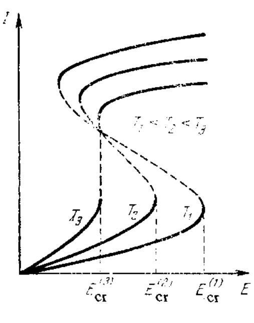
In the absence of the Hall field the solution of the energy balance equation (3.4) in the quantum limit for $Ee\lambda/T_e$ and $\hbar\omega_q \ll T_e$ gives Eq. (1.6) for the effective electron temperature. This expression is identical with that found by Kazarinov and Skobov[16] on the assumption that the electron-electron scattering can be ignored completely. However, if $E_y \neq 0$ the energy balance equation in the quantum limit is of the form[28]

$$\beta_E=x^2(1-x), \tag{3.7}$$

where

$$\beta_E=\frac{1}{4\pi}\left(\frac{cE}{sB}\right)^2\frac{\omega^2_{0e}}{\sigma_{xx}(T,B)\,\nu_{ep}(T,B)},$$
$$\omega^2_{0e}=\frac{4\pi n_ee^2}{m};\quad x=\frac{T}{T_e}.$$

The negative solution of Eq. (3.7) can be ignored because it has no physical meaning. If $\beta_E \ll 1$, which corresponds to small values of E, we can have two positive solutions $x_1 \approx 1-\beta_E$ and $x_2 \approx \sqrt{\beta_E}$. The first solution describes the heating of electrons which occurs when E is increased, whereas the second solution leads to $T_e \to \infty$ as $E \to 0$. The latter solution is a consequence of the reduction in the rate of transfer of heat from hot electrons to the lattice which occurs when T_e is increased and it demonstrates that the energy balance is disturbed if electrons are heated strongly (Fig. 1). When T_e is increased, the Joule power increases as $T_e^{3/2}$ and the power transferred to the lattice increases as $(T_e - T)$ only if the heating is weak, i.e., if $(T_e - T) \ll T$; under strong heating conditions this inequality is not obeyed and P decreases with in-



Current-voltage characteristics for E⊥B after allowance for the Hall field. The characteristics are plotted for three different lattice temperatures ($\hbar\Omega \gg T$). The dashed parts of the curves show the region of transition of the characteristics from the quantum limit to destruction of the qunatization.

creasing electron heating as $T_e^{-1/2}$. It follows that there is a field E_{cr} at which the stationary state of a system is still possible ($P = \text{I}E_{cr}$) but if $E > E_{cr}$ the expression $P = \text{I}E_{cr}$ is no longer obeyed and $\text{I}E > P$. When the field E is increased the two solutions come closer together and they become identical for $\beta_E = \beta_E^{cr} = 4/27$, where $x_{cr} = 2/3$. The critical electric field in which the electron temperature reaches the value $T_e = {}^3\!/\!_2 T$ is given by the formula

$$E_{cr} = \frac{sm}{e}\sqrt{\frac{2}{27}\nu_e(T, B)\,\nu_{ep}(T, B)}. \tag{3.8}$$

In fields $E > E_{cr}$ the energy balance is disturbed and the electron system becomes unstable. In fact, according to Volkov and Kogan,[5] the instability occurs in the region where the differential conductivity $\sigma_d \approx dI/dE$ is negative and the current-voltage characteristic has an S-type bend. The differential conductivity is negative ($\sigma_d < 0$) if the following conditions are satisfied:

$$\frac{\partial\sigma(T_e)}{\partial T_e} > 0, \tag{3.9}$$

$$\frac{\partial \ln \sigma(T_e)}{\partial T_e} > \frac{\partial \ln P(T_e)}{\partial T_e}. \tag{3.10}$$

In our case, $\sigma = \rho_{xx}^{-1}$. The conditions of Eqs. (3.9) and (3.10) are satisfied if $T_e > {}^3\!/\!_2 T$. The possibility of observation of an S-type current-voltage characteristic in a quantizing magnetic field was first pointed out by Kogan.[29] Figure 1 shows the current-voltage characteristics corresponding to different lattice temperatures in the case represented by Eq. (3.7).

Table I gives information on the dependence of E_{cr} on the lattice temperature and on the magnetic field B for various mechanisms of the momentum relaxation of electrons in the case when their energy is transferred to the acoustical phonons. It is evident from Table I that the dependence of E_{cr} on B and T can have a great variety of forms but E_{cr} always decreases with increasing T. This happens because, in all the momentum relaxation mechanisms, the Joule power always increases with T faster than does the power transferred by the electrons to the acoustical phonons. If the electron heating is sufficiently strong so that the quantization condition $T_e > \hbar\Omega$ is violated, the system may revert to a stationary state (Fig. 1). If $\tau_i\tau_\epsilon \propto (T_e)^\rho$ we find that the energy balance equation for $\rho < 1$ has a single-valued solution, whereas for $\rho \geq 1$ the solution is not single-valued and an "overheating" instability may be observed. For example, if the electron momentum and energy are transferred to the acoustical lattice vibrations in the classical (nonquantum) range of strong magnetic fields, the system considered will be in a stationary state and the effective temperature of the strongly heated electrons will be given by

$$T_e = T\frac{eE}{\sqrt{3}\,ms\,\langle\tau_{ak}^{-1}(T)\rangle}, \tag{3.11}$$

where

$$\langle\tau^{-1}(T)\rangle = \frac{4}{3\sqrt{\pi}}\int_0^\infty dx e^{-x}x^{3/2}\tau^{-1}(x);$$

here, $\tau_{a_k}(\epsilon/T)$ is the momentum relaxation time of an electron whose energy is ϵ and which interacts with the acoustical phonons (see, for example,[30]). Table I gives also information on the influence of the temperature on the critical field in various scattering mechanisms.

Table II gives data on the dependences of T_e and of the transverse magnetoresistance on the lattice temperature T and the electric field E in the classical region. The electron energy is transferred to the acoustical or piezoacoustical phonons. The transverse magnetoresistance may increase or decrease with increasing E and may even be independent of E; the actual behavior is determined by the relaxation mechanism of the electron momentum.

Concluding this subsection, we must point out that the results obtained are within the limits of the validity of our theory. In particular, E_{cr} defined by Eq. (3.8) satisfies the inequality $eE\lambda \ll T$. In fact, it follows from Eq. (3.8) that

$$\left(\frac{cE_{cr}}{B}\right)^2\frac{\Omega^2}{\nu_e(T, B)\,\nu_{ep}(T, B)} \sim \frac{1}{10}s^2 < s^2. \tag{3.12}$$

Since the energy of the phonons interacting with electrons satisfies the inequality $\hbar\omega_q \ll T$ and $\hbar\tilde{\omega}_q \sim \hbar s\lambda^{-1}$, it follows that

$$s^2 \ll \frac{T}{m}\frac{T}{\hbar\Omega}. \tag{3.13}$$

It follows from Eqs. (3.12) and (3.13) that

Table I. Dependences of E_r on T and B for various mechanisms of electron momentum relaxation (energy is transferred to acoustical phonons)

Momentum relaxation mechanism	$\nu(T, T_e, B)$	$E_{cr}(T, B)$	$\langle\tau^{-1}(T, T_e)\rangle$
Acoustical phonons	$B^2T^1T_e^{-3/2}$	$B^2T^{-1/2}$	$T^1T_e^{1/2}$
Neutral impurities	$B^2T^0T_e^{-3/2}$	B^2T^{-1}	$T^0T_e^0$
Ionized impurities	$B^0T^0T_e^{-3/2}$	B^1T^{-1}	$T^0T_e^{-3/2}$
Piezoacoustical phonons	$B^1T^1T_e^{-3/2}$	$B^{3/2}T^{-1/2}$	$T^1T_e^{-1/2}$
Optical phonons ($T \ll \theta_D$)	$B^1T^0T_e^{-1}\times$ $\times\exp\left(-\frac{\hbar\omega_0}{T}\right)$	$B^{3/2}T^{-3/4}\times$ $\times\exp\left(-\frac{\hbar\omega_0}{2T}\right)$	$T_e^0T^0\times$ $\times\exp\left(-\frac{\hbar\omega_0}{T}\right)$
Optical phonons ($T \gg \theta_D$)	$B^1T^1T_e^{-3/2}$	$B^{3/2}T^{-1/2}$	$T^1T_e^{-1/2}$

$$(eE_{\mathrm{cr}}\lambda)^2 \frac{\Omega^2}{\nu_e(T, B)\,\nu_{ep}(T, B)} \ll T^2.$$

This inequality means that the work done by the Hall field (and, consequently, by the external field) over a distance equal to the magnetic length λ is small compared with T.

4. Allowance for Phonon Heating[4]

We have assumed that the phonon system is in the state of equilibrium with a reservoir whose temperature is T. However, this assumption is not always justified. The energy acquired by electrons from the electric field is transferred in electron-phonon collisions to the long-wavelength phonons (electrons interact only with these phonons). The long-wavelength phonons then transfer the energy to the short-wavelength (thermal) phonons. We recall that the long-wavelength phonons are those whose momentum is in the range $\hbar q \leq \hbar\lambda^{-1} \equiv \hbar q_B$. The characteristic momentum of the thermal phonons is of the order of $T/s = \hbar q_T$. Therefore, the division of phonons into the long- and short-wavelength groups has meaning only if $q_B \ll q_T$. The behavior of the long-wavelength phonon system is determined by the relationship between the characteristic relaxation times of the interaction of these phonons with the electrons, $\tau_{pe}(q)$, and with the thermal phonons, $\tau_{pp}(q)$. If the long-wavelength phonons collide more frequently with the electrons than with the thermal phonons, the relaxation times obey the inequality $\tau_{pe}^{-1}(q) \gg \tau_{qq}^{-1}(q)$. In this case, the state of the long-wavelength phonons is determined primarily by the electron subsystem and their temperature is equal to the electron temperature T_e. If the opposite inequality $\tau_{pe}^{-1}(q) \ll \tau_{pp}^{-1}(q)$ is satisfied, the state of the long-wavelength phonons is determined by the state of the thermal phonons (the reservoir) and the temperature of the former is equal to the reservoir temperature T. We have assumed implicitly that the second case, i.e., the thermal equilibrium between all phonons, applies. At low temperatures $\tau_{pp}^{-1}(q)$ decreases proportionally to T^4, whereas $\tau_{pe}^{-1}(q)$ increases when T is lowered and when the magnetic field B is increased. Therefore, at sufficiently low temperatures T and in sufficiently strong fields B the first inquality $\tau_{pe}^{-1}(q) \gg \tau_{pp}^{-1}(q)$ may be satisfied. In this case, the heating of electrons gives rise to the heating of the long-wavelength phonons and temperatures of the thermal and long-wavelength phonons are no longer equal. This effect is due to the slowing down of the transfer of the energy, acquired by the long-wavelength phonons from the hot electrons, to the thermal phonons and is known as the "phonon bottleneck."

Table II. Dependences of T_e and of the transverse magnetoresistance on T and E in the classical (nonquantum) region (energy is transferred to acoustical or piezoacoustical phonons)

Momentum relaxation mechanism	$T_e^{(ac)}(T, E)$	$\rho_\perp^{(ac)}(T, E)$	$T_e^{(pac)}(T, E)$	$\rho_\perp^{(pac)}(T, E)$
Acoustical phonons	$T^{-1/2}E^1$	$T^{3/4}E^{1/2}$	$T^{-1}E^2$	$T^{1/2}E^1$
Neutral impurities	$T^0E^{4/3}$	T^0E^0	T^0E^4	T^0E^0
Piezo-acoustical phonons	$T^{-1}E^2$	$T^{3/2}E^{-1}$	—	—

We shall now consider the phonon bottleneck mechanism. We shall introduce the distribution of functions (the diagonal elements of the density matrix) of the long-wavelength phonons N_q and of the short-wavelength phonons N_k (k is the wave vector of a short-wavelength phonon). The electrons will be described, as before, by a Maxwellian distribution function with an effective electron temperature T_e. Since the thermal phonons are in thermodynamic equilibrium with the reservoir at the temperature T, it follows that their distribution is described by the Planckian function. We have to find the distribution function N_q. We shall assume that N_q is isotropic in the phase space of q. This is possible at velocities of ordered electron drift which do not exceed the velocity of sound and under conditions such that the momentum of the long-wavelength phonons is dissipated mainly at the boundaries of a sample and not in the phonon-phonon and phonon-electron collisions, i.e., if the following inequality is obeyed:

$$\tau_b^{-1} \sim \frac{L}{s} \gg \max(\tau_{pp}^{-1}, \tau_{pe}^{-1}),$$

where L is the characteristic dimension of a sample and τ_b is the characteristic relaxation time of the long-wavelength phonons at the boundaries of a sample. Under these conditions the distribution function is of the form[67]

$$N_q = \frac{N_q(T_e)\,\tau_{pe}^{-1}(q, T_e) + N_q(T)\,\tau_{pp}^{-1}(q, T)}{\tau_{pe}^{-1}(q, T_e) + \tau_{pp}^{-1}(q, T)}, \tag{4.1}$$

and in the quantum limit we have

$$\tau_{pe}^{-1}(q, T_e) = \frac{\sqrt{2\pi}\, m^2 s^2 A^{\mathrm{ac}} n_e q^2}{\hbar\,(mT_e)^{3/2}\,|q_z|} \exp\left(-\frac{\lambda^2 q_\perp^2}{2} - \frac{\hbar^2 q_z^2}{8mT_e}\right), \tag{4.2}$$

$$A_{\mathrm{ac}} = \frac{C_0^2 \hbar}{2\rho_0 s V}.$$

The expression for the relaxation frequency of the longitudinally polarized phonons is of the form[33, 34]

$$\tau_{pp}^{-1}(q, T) \simeq \frac{1}{4\pi\rho_0}\left(\frac{T}{\hbar s}\right)^4 \hbar q. \tag{4.3}$$

If the temperature of the reservoir is sufficiently low and the electron heating not too strong, so that

$$10\left(\frac{T}{\theta_D}\right)^4 \ll \left(\frac{n_e}{N_a}\right)\left(\frac{a}{\lambda}\right)\left(\frac{C_0}{T_e}\right)^2 \tag{4.4}$$

(θ_D is the Debye temperature, N_a is the number of lattice atoms per unit volume, and a is the lattice constant), we find that $\tau_{pe}(q, T_e) \ll \tau_{pp}(q, T)$; it then follows from Eq. (4.1) that the temperature of the long-wavelength phonons is equal to T_e and they are described by the Planckian distribution function.

The power transferred by the long-wavelength phonons to the thermal phonons is given by the formula

$$P_{pp}(T, T_e) = \sum_q \hbar\omega_q\,[N_q(T) - N_q(T_e)]\,\tau_{pp}^{-1}(q, T). \tag{4.5}$$

If $\hbar\omega_q \ll T$, we can use Eq. (4.3) to find[31]

$$P_{pp}(T, T_e) \approx \left(1-\frac{T}{T_e}\right) T_e^{3/2} \frac{\sqrt{2m}\,\tau_{pp}^{-1}(q_m)\,q_m^2}{3\pi^2\hbar}. \qquad (4.6)$$

In deriving Eq. (4.6) we have taken account of the fact that the long-wavelength phonons which are in equilibrium with electrons have maximum momenta $(\hbar q_\perp)_m \sim \hbar\lambda^{-1}$ and $(\hbar q_z)_m \sim \sqrt{2mT_e}$ in a plane orthogonal to the magnetic field B and directed along B, respectively, and that $(q_\perp)_m \sim \lambda^{-1} \gg (q_z)_m$. Since the maximum energy of the long-wavelength phonons emitted by electrons is of the order of $\hbar s/\lambda$ and is independent of the electron temperature, it follows that when $\hbar s/\lambda < T$ the thermal phonon subsystem can always be regarded as the thermal reservoir. This is not true in classical (nonquantizing) magnetic fields.[32]

An allowance for the heating of the long-wavelength phonons reduces the rate of rise of the Joule power (in the presence of the Hall field) with increasing electron temperature below the rate of rise of the power transferred to the lattice (it is assumed that the momentum and energy are transferred to the acoustical phonons):

$$(IE) = 2n_e m \left(\frac{cE}{B}\right)^2 \frac{\Omega^2}{\nu_{ep}(T, B)} \left(\frac{T_e}{T}\right)^{1/2}. \qquad (4.7)$$

Therefore, in the quantum limit a stationary state of the electron-phonon system is not disturbed by any field E and we can easily show that

$$T_e = T\,[1 + (E/\tilde{E})^2], \qquad (4.8)$$

where the characteristic electric field $\tilde{E}$ is

$$\tilde{E} = \left[\frac{(mT)^{3/2}\,\nu_{ep}(T, B)\,\tau_{pp}^{-1}(q_m, T)}{3\sqrt{2}\,\pi\hbar e^2\lambda^2 n_e}\right]^{1/2}. \qquad (4.9)$$

However, the energy balance equation may be disturbed before the orbital quantization is destroyed if the scattering of electrons by neutral or ionized impurities is important. In the quantum limit the scattering by neutral and ionized impurities is described by the following expressions:

$$\nu_0 \propto B^2 T_e^{-3/2}, \quad \nu_i \propto B^0 T_e^{-3/2}.$$

The Joule power (IE) increases with the electron temperature as $T_e^{3/2}$. This law is also obeyed by the power transferred to the lattice (thermal phonons), P_{pp}. An "overheating" instability appears when the electric field reaches E_{cr}. We can easily show that E_{cr} is identical with $\tilde{E}$ defined by Eq. (4.9) if we replace $\nu_{ep}(T, B)$ with the momentum relaxation frequency in the case of scattering by impurities. When $E = E_{cr}$ the electron heating is rapid and it may destroy the magnetic quantization of the orbital electron motion if the impurity scattering during heating is not suppressed by the acoustical scattering (this ensures a stationary state of the electron system under quantization conditions). Table III gives information on the dependence of E_{cr} on T and B for various scattering mechanisms. This table gives data on the dependence of E_{cr} on T and B in the case when the energy acquired by the long-wavelength phonons from hot electrons is transferred not to the thermal phonons but directly to the boundaries of a sample. This occurs when $\tau_{pp} \gg \tau_b$ (τ_b is the energy relaxation time at the boundaries of the sample). We can easily show that $(E_b)_{cr}$ is

$$(E_b)_{cr} = \left[\frac{(mT)^{3/2}\,\tau_b^{-1}\nu(T, B)}{2\sqrt{2}\,\lambda^2\hbar e^2\lambda^2 n_e}\right]^{1/2}. \qquad (4.10)$$

We note that when an allowance is made for the heating of the long-wavelength phonons, the value of E_{cr} always increases with increasing T because of the rapid increase in the power transferred to the reservoir but it decreases when the electron density is increased. When no allowance is made for the heating of the long-wavelength by electrons, the value of E_{cr} always decreases with increasing T but is independent of the electron density. Experimental investigations of such dependences should make it possible to determine the role of the phonon bottleneck effect in semiconductors.

The quantization of the cyclotron orbits is destroyed by the strong heating of electrons. Under the phonon bottleneck conditions a stationary state of electrons is established for any momentum relaxation mechanism, whereas in the absence of this effect the scattering by ionized impurities does not ensure that the electron system is in a stationary state. Table IV gives information on the dependence of the electron temperature and of the transverse electrical resistivity $\rho_\perp$ on T, E, and n_e.

The phonon bottleneck effect may occur in semiconductors if the temperature is sufficiently low. For example, when n-type germanium is subjected to a field $B = 10^5$ Oe at T = 15° K, the cyclotron orbits are found to be quantized ($\lambda \approx 0.8 \times 10^{-6}$ cm, $q_T = T/\hbar s \approx 7 \times 10^{-6}\ \text{cm}^{-1}$) and the inequalities necessary for this effect [$q_m < q_T$ and that given by Eq. (4.4)] are satisfied.

Gluzman, Lyubimov, and Tsidil'kovskiĭ [35, 36] measured the electrical resistivity of n-type germanium with $n_e = 8.6 \times 10^{14}\ \text{cm}^{-3}$ at T = 16.6° K in strong electric (up to 50 V/cm) and magnetic (up to 140 kOe) fields. The momentum and energy of the long-wavelength phonons were dissipated at the boundaries of a sample and, therefore, the critical field at which an instability was observed was given by Eq. (4.10). The dependences $\rho_\perp(I)$ obtained in magnetic fields B = 28, 42, and 56 kOe indicated a reduction in the resistivity caused by electric fields E of about 3–4 V/cm, which was evidently due to the destruction of the quantization of the cyclotron orbits. These values of E were close to the val-

Table III. Dependences of E_{cr} on B and T under phonon bottleneck conditions in various momentum relaxation mechanisms (the last column gives the values of E^b_{cr} for energy transfer not to the thermal phonons but directly to the surrounding medium via the boundaries of a sample, $\tau_{pp} > \tau_b$, where τ_b is the energy relaxation time at the boundaries)

Momentum relaxation mechanism	$E_{cr} n_e^{1/2}$	$E^b_{cr} n_e^{1/2}$
Acoustical phonons	$B^{7/4}T^{5/2}$	$B^{3/2}T^{1/2}$
Neutral impurities	$B^{7/4}T^2$	$B^{3/2}T^0$
Ionized impurities	$B^{3/2}T^2$	$B^{1/2}T^0$
Piezoacoustical phonons	$B^{5/4}T^{5/2}$	$B^1T^{1/2}$
Optical phonons	$B^{5/4}T^{9/4}e^{-\frac{\hbar\omega_0}{2T}}$	$B^1T^{1/4}e^{-\frac{\hbar\omega_0}{2T}}$

Table IV. Dependences of the electron temperature T_e and transverse resistivity $\rho_\perp$ on T, E, and the electron density n_e in classical (nonquantum) region

Momentum relaxation mechanism	$\tau_{pp} < \tau_b$		$\tau_{pp} > \tau_b$	
	$T_e(T, E, n_e)$	$\rho_\perp(T, E, n_e)$	$T_e(T, E, n_e)$	$\rho_\perp(T, E, n_e)$
Acoustical phonons	$n_e^{2/9}E^{4/9}T^{-8/9}$	$n_e^{-2/3}E^{2/3}T^{-4/3}$	$n_e^{1/4}E^{1/2}T^0$	$n_e^{1/2}E^{3/4}T^0$
Neutral impurities	$n_e^{1/3}E^{2/3}T^{-4/3}$	$n_e^{-1}E^0T^0$	$n_e^{2/5}E^{4/5}T^0$	$n_e^{-1}E^0T^0$
Ionized impurities	$n_e^{2/3}E^{4/3}T^{-8/3}$	$n_e^{-2}E^{-2}T^4$	$n_e^1E^2T^0$	$n_e^{-5/2}E^{-3}T^0$
Piezoacoustical phonons	$n_e^{2/5}E^{4/5}T^{-2}$	$n_e^{-6/5}E^{-2/5}T^2$	$n_e^{1/2}E^1T^{-1/2}$	$n_e^{-5/4}E^{-1/2}T^{-5/4}$
Optical phonons ($T < \theta_D$)	$n_e^{1/3}E^{2/3}T^{-4/3}\times$ $\times e^{\hbar\omega_0/3T}$	$n_e^{-1}E^0T^0\times$ $\times e^{-\hbar\omega_0/T}$	$n_e^{2/5}E^{4/5}T^0\times$ $\times e^{2\hbar\omega_0/5T}$	$n_e^{-1}E^0T^0\times$ $\times e^{-\hbar\omega_0/T}$

ues of $(E_b)_{cr}$ calculated from Eq. (4.10). In stronger fields E the fall in the resistivity was masked by a strong rise which was attributed to the "transverse breakdown."[37] The same effect was observed in n-type Ge subjected to strong electric fields by Suzuki.[38] It would be desirable to repeat these experiments at different reservoir temperatures because this would give information on the heating of the long-wavelength phonons.

The heating of the long-wavelength phonons under similar conditions but in the absence of the Hall field was studied also by Gurevich and Gasymov.[39]

5. Very Strong Magnetic Fields

When the magnetic field intensity B is increased the maximum momentum of the phonons interacting with electrons increases as $\hbar/\lambda$. When $\hbar s/\lambda \gtrsim T$ practically all the phonons interact with electrons, i.e., all the phonons apparently acquire long wavelengths and the number of the short-wavelength phonons with momenta exceeding $\hbar/\lambda$ becomes exponentially small. The thermal reservoir, which is formed by the short-wavelength phonons when $\lambda^{-1} = q_B < q_T = T/\hbar s$, disappears when the inequality $q_B > q_T$ is obeyed. The medium surrounding a sample then acts as a reservoir. The energy acquired from the hot electrons by the whole phonon system is then transferred across the boundaries of the sample to the surrounding medium (the reservoir). In this case, we encounter a new characteristic (in addition to the relaxation times characterizing the electron-phonon τ_{pe} and phonon-phonon τ_{pp} interactions), which is the relaxation time of the phonons interacting with the boundaries of a sample, τ_b. The analysis of the heating of electrons can be simplified by considering the asymptotic solutions corresponding to different relationships between these three relaxation times.

When τ_b is the shortest of the three characteristic relaxation times, the phonons acquire energy from the hot electrons and transfer it immediately to the boundaries of the sample without coming into collision with other phonons. In this case, we can analyze the weak heating of electrons corresponding to the inequality $q_T < q_B$ and the strong heating when the opposite inequality is satisfied. It is shown in [40] that in the electron temperature approximation, when $\tau_{pp} \gg \tau_{pe}$, we have

$$T_e \propto B^0 (T)^0 E^1, \tag{5.1}$$

if the momentum is dissipated by interaction with ionized impurities, and

$$T_e \propto B^{-1} (T)^0 E^1 \tag{5.2}$$

if the momentum is lost by scattering on neutral impurities.

An "overheating" instability appears in strong fields $E > E_{cr}$. The dependence of E_{cr} on the parameters of the system considered here has been determined by the present authors[40] and can be given by the following formulas:

when $\tau_{pp} \gg \tau_b \ll \tau_{pe}(q_B, T_e)$

$$E_{cr} \approx \frac{4\,[K_0(1)]^{3/2}\,[m\nu_e(T, B)\,\tau_{pe}^{-1}(q_B, T)\,\sqrt{2ms^2T}]^{1/2}}{3\pi\Gamma(5)\,\zeta(5)\,[\lambda q_T]^4\,K_0(q_T/q_B)\,[3n_e e^2\lambda^3]^{1/2}}, \tag{5.3}$$

where $K_0(x)$ is the Macdonald function, $\Gamma(t)$ is the gamma function, $\zeta(x)$ is the Riemann zeta function, and $\nu_e(T, B)$ is the momentum relaxation frequency which occurs in the electrical conductivity σ_{xx}; when $\tau_{pp} \gg \tau_b \gg \tau_{pe}(q_B, T_e)$ we obtain

$$E_{cr} = \left[\frac{(2mT)^{3/2}\,\nu_{ep}(T, B)}{2\,(2\pi)^2\,\hbar n_e e^2\lambda^2}\right]^{1/2}. \tag{5.4}$$

It follows from the above formula that $E_{cr} \propto B^{1/2}(T)^0$ for ionized impurities and $E_{cr} \propto B^{3/2}(T)^0$ for neutral impurities.

We shall now consider the third limiting case, when $\tau_b \gg \tau_{pp} \gg \tau_{pe}$, i.e., when τ_b is the longest relaxation time. Under weak heating conditions ($q_T < q_B$) the phonons emitted by electrons have energies $\hbar s q_B > T$. However, since $\tau_b \gg \tau_{pp}$, these phonons split into two (each of which has an energy $\sim T_e$) before transmitting their energy to the medium surrounding a sample. After a characteristic time[32]

$$\tau_{pp} \approx \frac{60\pi^2\rho_0}{\tilde{\gamma}^2\hbar}\,q^{-5}$$

($\tilde{\gamma}$ is a numerical factor which is ≈ 2 for Si and Ge) these phonons reach equilibrium with other phonons and the hot electrons and then they transfer their energy across the boundaries of the sample to the ambient medium. Under these conditions the energy reaching the boundaries of the sample increases proportionally to T_e^4 and, therefore, the system is stable. Under strong heating conditions ($q_{T_e} > q_B$) the energy of the phonons emitted by the electrons is $\hbar s q_B < T_e$. Therefore, the emitted phonons become thermalized by merging processes because $\tau_{pp} < \tau_b$, i.e., they reach equilibrium with the electrons and the other phonons whose temperature is T_e; next, the energy is transferred by the phonon system to the boundaries of the sample. The characteristic time for the process of merging of two phonons with momentum $\hbar\lambda^{-1}$ can be deduced by means of Eq. (25) given in [34] where the upper limit of the integral must be replaced with λ^{-1}. In this way, we obtain

$$\tau_{pp}^{-1}(q_B, T_e) \approx \frac{\hbar}{4\pi\rho_0} q_B^4 q_{T_e}. \qquad (5.5)$$

The energy balance equation now predicts stationary states of the system. Under strong heating conditions $T_e \propto (T)^0 B^0 E^{4/5}$, if the electron momentum is dissipated on ionized impurities, and $T_e \propto (T)^0 B^{-4/5} E^{4/5}$ if the momentum is dissipated on neutral impurities. The corresponding dependences of the current on the electric and magnetic fields and on the temperature are, respectively,

$$I_x \propto (T)^0 B^0 E^{2,2}, \qquad (5.6)$$

$$I_x \propto (T)^0 B^{-8/2} E^{2,2}. \qquad (5.7)$$

6. Other Mechanisms of Energy Dissipation by Hot Electrons

The energy of hot electrons can be transferred not only to acoustical but also to optical and piezoacoustical phonons. If a semiconductor contains several groups of carriers with strongly differing masses, the energy of the lighter hot carriers may be transferred by collisions to the heavier carriers and then to the lattice.

The dissipation of the energy of hot electrons by interaction with piezoacoustical phonons was first considered by Kogan,[29] who used the electron temperature approximation. It is evident from Table I that the ratio of the frequencies of electron relaxation by acoustical and piezoacoustical phonons is $\nu_{ep}/\nu_{ep}^{(pac)} \propto B$. Therefore, the dependence of $E_{cr}^{(pac)}$ on B and T in the case of scattering of hot electrons by the piezoacoustical phonons is, in accordance with Eq. (3.19), of the form $E_{cr}^{(pac)} \propto E_{cr}^{(ac)}/B$. Dividing the values of $E_{cr}^{(ac)}(T, B)$ of Table I by B, we obtain the dependences $E_{cr}^{(pac)}(T, B)$ applicable to different momentum relaxation mechanisms. The dependence of the electron temperature on the electric field E is then given by an equation similar to Eq. (3.7) in which ν_{ep} is replaced by $\nu_{ep}^{(pac)}$. In the case of interaction with the acoustical and piezoacoustical phonons, the coefficient $\beta_\perp$, which is associated with the quadratic (in the electric field) correction to the transverse electrical conductivity $\sigma_\perp = \rho_{xx}^{-1}$, has the following dependences on B and T:[29]

$$\beta_\perp^{(ac)} \propto T^2 B^{-2}, \quad \beta_\perp^{(pac)} \propto T^2 B^{-1}.$$

The relaxation of the energy of hot electrons by interaction with the optical phonons was considered in the electron temperature approximation by Pomortsev and Kharus.[41, 42] An interesting consequence of their investigations is the prediction of an oscillatory dependence, on the magnetic field, of the power transferred by the electrons to the optical phonons. Every time the parameter $\Delta = \omega_0 - M\Omega$ (ω_0 is the frequency of the optical phonons and M is an integer) is made to vanish by the application of a magnetic field, the power tends to infinity as $\ln(T_e/\Delta)$. Such singularities are due to the contribution of the electrons with zero values of the z component of the momentum. The heating of the optical phonons suppresses the divergence of the power transferred to the lattice as $\Delta \to 0$ and, moreover, such heating destroys the power resonance if the relaxation frequency of the electrons interacting with the optical phonons $\nu_{ep}^{(opt)}$ is considerably higher than the nonelectron relaxation frequency of the optical phonons, $\nu_{pp}^{(opt)}$

If the electrons lose their momentum by interacting with impurities or with the acoustical lattice vibrations, it is found that the electrical resistivity $\rho_\perp$ is a monotonically decreasing function of the electron temperature. Therefore, at the points corresponding to the resonance emission of the optical phonons ($\Delta = 0$), the electron temperature should have minima and $\rho_\perp$ should have maxima.

The electrical resistivity $\rho_\perp$ is an oscillating function of B even in the absence of electron heating[43] but the conditions for the direct observation of this effect are extremely stringent. This is because at low temperatures, necessary to ensure the existence of the Landau levels, the resonance scattering of the electrons by the optical phonons is accompanied by other nonresonance scattering mechanisms (ionized and neutral impurities, acoustical and piezoacoustical phonons) which suppress the resonance scattering effect. In sufficiently strong electric fields which ensure $T_e > T$ the loss of energy by the electrons in the emission of the optical phonons becomes considerable.[44] Therefore, in the nonohmic region we may expect the appearance of oscillations of the magnetoresistance when B is varied.

Stradling and Wood[45] observed such oscillations in n-type GaAs and found that the maxima of $\rho_\perp$ were shifted, relative to the magnetophonon resonance points, toward weaker magnetic fields. Stradling and Wood attributed this effect to electron transitions to impurity levels rather than to the zeroth Landau level.

7. Low Electron Densities

Kazarinov and Skobov[16] were the first to solve the problem of the heating of electrons in crossed (orthogonal) strong electric and quantizing magnetic fields. Kazarinov and Skobov ignored the electron-electron scattering. They showed that, if the electric field is sufficiently weak so that it does not affect the energy spectrum of electrons and if the scattering by phonons is quasielastic ($eE\lambda/\bar{\epsilon} \ll 1$), the energy balance equation for electrons of the ϵ, $\epsilon + d\epsilon$ group obtained in the quantum limit is of the form:

$$\frac{\partial}{\partial\varepsilon} P(\varepsilon) - \frac{\partial}{\partial\varepsilon}(I(\varepsilon)E). \qquad (7.1)$$

The electron distribution function satisfies a first-order differential equation. The effective temperature

T_e can be introduced if the electrons are scattered by impurities and acoustical phonons; in this case, the temperature is given by Eq. (1.6). This is associated with the fact that the ratio ν_{ei}/ν_{ep} is independent of the electron energy ϵ. A reduction in the transverse resistivity due to an increase in the electron temperature has been observed experimentally.[46, 47]

The nonlinear electrical conductivity in the absence of electron-electron scattering was also considered by Calecki.[22] His approach is basically applicable to "warm electrons," i.e., to weak electric fields $E \ll (s/c)B$. Zlobin[48] extended Kazarinov and Skobov's theory of nonlinear galvanomagnetic effects to the case of relaxation of the energy of hot electrons by the interaction with optical vibrations.

In contrast to Pomortsev and Kharus,[41, 42] whose work was considered in the preceding subsection, Zlobin[48] considered the case of low electron densities when the concept of the effective electron temperature could not be introduced a priori.

Collisions of electrons with the optical phonons are strongly inelastic and, therefore, an integral equation for the diagonal element of the electron density matrix cannot be reduced to a Fokker–Planck differential equation. However, at low temperatures $T < \hbar\omega_q$ [$\omega_q = \omega_0(1 - \alpha^2 q^2)$ is the frequency of an optical phonon when the dispersion is allowed for] we can also introduce the concept of quasielastic scattering but only in relation to the double process of absorption-emission of an optical phonon.[9] When the two lowest Landau levels are considered in the quantum limit the equation for the differential energy balance can be represented in the form

$$f(\varepsilon) + T\frac{\partial f}{\partial \varepsilon} + (T\gamma_E)\frac{\partial f}{\partial \varepsilon}\left[\frac{\mathfrak{N}_0'(\varepsilon)}{\sqrt{\varepsilon(\varepsilon + \hbar\omega_0)}} + \frac{\mathfrak{N}_1'(\varepsilon)}{\sqrt{\varepsilon(\varepsilon + \Delta)}}\right] \times \left[\frac{\mathfrak{N}_0(\varepsilon)}{\sqrt{\varepsilon(\varepsilon + \hbar\omega_0)}} + \frac{\mathfrak{N}_1(\varepsilon)}{\sqrt{\varepsilon(\varepsilon + \Delta)}}\right]^{-1} = 0, \qquad (7.2)$$

where

$$\gamma_E = \frac{1}{8}\left(\frac{eE\lambda^3}{\alpha^2\hbar\omega_0}\right)^2, \qquad \Delta = \hbar(\omega_0 - \Omega);$$

$\mathfrak{N}_0'(\epsilon)$, $\mathfrak{N}_1'(\epsilon)$, $\mathfrak{N}_0(\epsilon)$, and $\mathfrak{N}_1(\epsilon)$ are known functions of the energy.[48]

The solution of Eq. (7.2) is of the form

$$f(\varepsilon) = \exp\left\{-\int^{\varepsilon}\frac{d\varepsilon'}{T[1 + g(\varepsilon')]}\right\}, \qquad (7.3)$$

and the electron "temperature" depends on the energy ϵ. However, there are limiting cases when one can still introduce an effective electron temperature which is the same for all electrons, irrespective of their energy.

It is shown in [48] that in the range of energies $\epsilon \ll \hbar\Omega$ the concept of an electron temperature has meaning in the quantum limit ($\Omega > \omega_0 \gg \bar{\epsilon}$) as well as in the magnetophonon resonance region if $\omega_0 - \Omega \lesssim T/\hbar$ and if $\Omega \gg \omega_0 T_e \approx T(1 + 2\gamma_E)$. At the magnetophonon resonance point the electron temperature T_e has a maximum.

When the energy and momentum of carriers are dissipated on the optical phonons at low temperatures ($T \ll \hbar\Omega, \hbar\omega_0$), we have $\sigma_{xx} \propto T_e^{-1}$ and, therefore, the heating of electrons results in a minimum of σ_{xx} at the resonance point $\omega_0 = M\Omega$ (M is an integer). However, the density of the electron states has a singularity at the resonance point and this increases σ_{xx}. This effect is a consequence of the linear theory of transport in weak fields and is not related to heating. Therefore, when carriers are heated the reduction in the conductivity σ_{xx} at the resonance point may be the dominant effect and it may give rise to a minimum, as predicted by the linear transport theory. Aksel'rod et al.[49] observed conductivity minima associated with hot electrons in n-type InSb at temperatures of 16–30°K, in magnetic fields up to 100 kOe, and in electric fields up to 12 V/cm (see also [46]).

Yamada and Kurosawa[50] investigated experimentally and theoretically the behavior of electrons in n-type InSb subjected to crossed fields. They replaced the solution of the transport equation which included an allowance for the scattering on the optical phonons by a simpler approach in which the diffusion equation was solved in the energy space. Their results also yielded a conductivity minimum at the magnetophonon resonance point. However, one should stress that Yamada and Kurosawa[50] did not actually derive the diffusion equation with a collision integral for the interaction between electrons and optical phonons and, therefore, it is difficult to determine what approximations were made.

We shall now consider the limitations of the method in which the diagonal elements of the density matrix are found from the differential energy balance equation $I(\epsilon)E = P(\epsilon)$. This method is based on the assumption that a diagonal element of the density matrix depends on n and p_z only via the energy $E(n, p_z)$. This is strictly true only in the quantum limit when the average energy of electrons is $T_e \ll \hbar\Omega$ and the lowest Landau level with $n = 0$ is practically filled. In this case, we can ignore all the diagonal elements of the density matrix corresponding to $n \neq 0$ since these elements are exponentially small for large values of $\hbar\Omega/T_e$. Therefore, the problem reduces to finding only one diagonal element of the density matrix corresponding to $n = 0$, which depends only on one variable p_z.

If the parameter $T_e/\hbar\Omega$ is not small, it is necessary to make allowance for electrons occupying several Landau levels. In this case, the density matrix can be found only by solving a complex system of differential equations. We can show that the transport equation for the diagonal elements of the density matrix for electrons interacting with the acoustical phonons and with short-range impurities is of the form:

$$\sum_{n'=0}^{\bar{n}}\left\{g_{n'}^{-1}(\varepsilon)\frac{\partial}{\partial\varepsilon}\left[\varepsilon g_{n'}^2(\varepsilon)\left(\frac{f_{n'}(\varepsilon)}{T} + \frac{\partial f_{n'}(\varepsilon)}{\partial\varepsilon}\right)\right]\right\} + \sum_{n'=0}^{\bar{n}} g_{n'}(\varepsilon)[f_{n'}(\varepsilon) - f_n(\varepsilon)]\left[\left(1 + \frac{\nu_{ei}}{\nu_{ep}}\right)\frac{1}{2ms^2} + \frac{\hbar\Omega\left(n' + \frac{1}{2}\right)}{2T\left(\varepsilon - \hbar\Omega\left(n' + \frac{1}{2}\right)\right)}\right]$$
$$+ \left(\frac{cE}{sB}\right)^2\frac{\hbar\Omega}{4}\left(1 + \frac{\nu_{ei}}{\nu_{ep}}\right)\sum_{n'=0}^{\bar{n}}(1 + n + n')\left\{g_{n'}^{-1}(\varepsilon)\frac{\partial}{\partial\varepsilon}\left[g_{n'}^2(\varepsilon)\frac{\partial f_{n'}(\varepsilon)}{\partial\varepsilon}\right] + \frac{3}{4}g_{n'}(\varepsilon)\frac{f_{n'}(\varepsilon) - f_n(\varepsilon)}{\left[\varepsilon - \hbar\Omega\left(n' + \frac{1}{2}\right)\right]^2}\right\} = 0, \qquad (7.4)$$

where $g_n(\epsilon)$ is the density of the electron states corresponding to the Landau level n and located in the energy range ϵ, $\epsilon + d\epsilon$; $\bar{n}$ is the highest Landau level which is still within the range of the energy ϵ. In deriving Eq. (7.4) it has been assumed that the scattering of electrons by the acoustical phonons is almost elastic and the shift of the Landau levels by the electric field is negligible. An equation of this type was derived also by Inoue and Yamashita[51] (their phonon collision integral has two terms less than the integral given above).

In the quantum limit Eq. (7.4) has the form of an equation of continuity in the energy space (7.1) and the distribution of electrons is characterized by an effective temperature T_e defined by Eq. (1.6). It must be stressed that when $T_e/\hbar\Omega$ is not small the transport equation does not reduce to the differential form of Eq. (7.1) and the electron distribution usually becomes oscillatory.

Rigorous allowance for the scattering by the optical phonons complicates the problem even more.

III. HEATING OF ELECTRONS IN PARALLEL STRONG ELECTRIC AND QUANTIZING MAGNETIC FIELDS

In this section we shall assume, as in Sec. II, that the work done by an external field on an electron in the time separating two consecutive collisions is smalll compared with its characteristic energy $\bar{\epsilon}$. When $\mathbf{E} \parallel \mathbf{B}$, the electric field does work eEl in a distance equal to the mean free path l, whereas when $\mathbf{E} \perp \mathbf{B}$ the work done is $eE\lambda$. Therefore, the criterion of weak electric fields in the $\mathbf{E} \parallel \mathbf{B}$ case is of the form $eEl/\bar{\epsilon} \ll 1$. This criterion differs from that for $\mathbf{E} \perp \mathbf{B}$ by the factor $l/\lambda \sim \Omega\tau\sqrt{T/\hbar\Omega}$, which should be greater than unity even in the quantum limit ($\hbar\Omega \gg T$). The effective electron temperature for $\mathbf{E} \parallel \mathbf{B}$ is still given by

$$T_e \approx T\left[1 + \left(\frac{eEl}{\bar{\varepsilon}}\right)^2 \delta^{-1}(T, B)\right],$$

which follows from the energy and momentum balance equations.

A more detailed study of the dependence of T_e on the parameters of a system in the case of different energy and momentum relaxation mechanisms can be made made only if we adopt the microscopic approach to the derivation of the balance equations.

8. High Electron Densities

In this case, we can introduce the concept of an effective electron temperature T_e. In order to find it we shall derive the energy balance equation by equating the Joule power

$$I_z E_z = \sigma_{zz}(T, T_e, B) E_z^2 \propto \tau(T, T_e, B) E_z^2 \tag{8.1}$$

to the power P transferred to the phonons and given by Eq. (3.5). The relaxation time $\tau(T, T_e, B)$ in σ_{zz} can be calculated by means of the transport equation

$$eE_z v_z \frac{\partial f_\nu(T_e)}{\partial E_\nu} = I_{\nu\nu}^{ep}(\varphi) + I_{\nu\nu}^{ei}(\varphi), \tag{8.2}$$

in which the collision integrals are found by means of Eqs. (A.2) and (A.4) with $E_x = 0$, and the distribution function f_ν is selected in the form

$$f_\nu = f_\nu(T_e) + \varphi_\nu, \tag{8.3}$$

where $f_\nu(T_e)$ is the Maxwellian distribution function with an effective electron temperature T_e which depends on T, E_z, and B, and

$$\varphi_\nu = -\frac{\partial f_\nu(T_e)}{\partial E(\nu)} p_z \tilde{\varphi}(E(\nu)). \tag{8.4}$$

Multiplying the transport equation (1.2) by $p_z \delta[E(\nu) - \epsilon]$ and taking a trace over ν, we find that in the elastic scattering approximation

$$\tilde{\varphi}(\varepsilon) = -\frac{eE_z}{m}\tau(\varepsilon), \tag{8.5}$$

where

$$\tau^{-1}(\varepsilon) = \frac{2\pi}{\hbar}\sum_{q\nu\mu} |I_{\nu\mu}^q|^2 (\hbar q_z)^2 \delta[E(\nu) - \varepsilon]\,\delta[E(\mu) - \varepsilon] \times [|C_q|^2 N_q + |G_q|^2 N_i] \left\{\sum_\nu (p_z)^2 \delta[E(\nu) - \varepsilon]\right\}^{-1}. \tag{8.6}$$

Hence, if follows that

$$\sigma_{zz} = -e^2 \sum_\nu (v_z)_\nu^2 \frac{\partial f_\nu(T_e)}{\partial E(\nu)} \tau(E(\nu)) = \frac{n_e e^2}{m}\tau(T, T_e, B) = \rho_\parallel^{-1}. \tag{8.7}$$

Table V gives information on the dependence of $\tau(T, T_e, B)$ on the arguments in the quantum limit. It is evident from this table that the Joule power increases with increasing electron temperature more rapidly than does the power defined by Eq. (3.15) and transmitted to the lattice. Therefore, an "overheating" instability may occur. This result was first derived by Kogan[29] for the case when the electron momentum is dissipated on charged impurities and the electron energy on the acoustical and piezoacoustical phonons. The foregoing still applies when the phonon system is in equilibrium.

The heating of phonons is important and must be allowed for if the frequency of collisions between the long-wavelength phonons and electrons exceeds the frequency of collisions between the long-wavelength and thermal phonons. In this case, the temperature of the long-wavelength phonons is equal to the effective electron temperature T_e (the phonon bottleneck effect) and the "overheating" instability does not appear for any momentum relaxation mechanism with the exception of that involving ionized impurities. Table V gives data on the dependence of T_e and the longitudinal resistivity $\rho_\parallel$ in two cases: when the long-wavelength phonons transfer their energy to the thermal phonons ($\tau_{pp} \ll \tau_b$) and when they transfer their energy to the boundaries of a sample ($\tau_{pp} \gg \tau_b$). The data are given for different momentum relaxation mechanisms. All the results given in Sec. II.2 are applicable also to the case $\mathbf{E} \parallel \mathbf{B}$ if we replace everywhere $\rho_\perp$ with $\rho_\parallel$.

9. Low Electron Densities

Pomortsev[52] was the first to consider the case of low electron densities in the presence of a quantizing magnetic field $\mathbf{B} \parallel \mathbf{E}$ on the assumption of thermodynamic equilibrium in the phonon subsystem. The results obtained by Pomortsev for all the quasielastic scattering mechanisms indicated that the region of quadratic deviation from Ohm's law was extremely narrow. This deviation can be explained by the fact that all the quasielastic scattering mechanisms considered by Pomortsev[52] give rise to the "runaway" of electrons toward

Table V. Dependence of the relaxation time τ of the longitudinal momentum of electrons on T, T_e, and B in the quantum limit. Dependence of the effective electron temperature T_e and of the longitudinal electrical resistivity on T and B

Relaxation mechanism	$\tau(T, T_e, B)$	$\tau_{pp} \ll \tau_b$		$\tau_{pp} \gg \tau_b$	
		$T_e(T, B)$	ρ_L	$T_e(T, B)$	ρ_L
Acoustical phonons	$T^{-1}T_e^{1/2}B^{-1}$	$T^{5}B^{-5/2}$	$T^{3,5}B^{2,25}$	$T^{-1}B^{-2}$	$T^{3/2}B^{2}$
Neutral impurities	$T^{0}T_e^{1/2}B^{-1}$	$T^{-4}B^{-5/2}$	$T^{2}B^{2,25}$	$T^{0}B^{-2}$	$T^{0}B^{2}$
Ionized impurities	$T^{0}T_e^{3/2}B^{0}$	—	—	—	—
Piezoacoustical phonons	$T^{-1}T_e^{1/2}B^{0}$	$T^{-5}B^{-3/2}$	$T^{3,5}B^{3/4}$	$T^{-1}B^{-1}$	$T^{3/2}B^{1/2}$
Optical phonons	$T^{-1}T_e^{1/2}B^{0}$	$T^{-5}B^{-3/2}$	$T^{3,5}B^{3/4}$	$T^{-1}B^{-1}$	$T^{3/2}B^{1/2}$
Optical phonons ($T \ll \theta_D$)	$T^{0}T_e^{0}B^{0}\times$ $\times\exp\left(\frac{\hbar\omega_0}{T}\right)$	$T^{-8/3}B^{-1}\times$ $\times\exp\left(\frac{2\hbar\omega_0}{3T}\right)$	$T^{0}B^{0}\times$ $\times\exp\left(-\frac{\hbar\omega_0}{T}\right)$	$T^{0}B^{-2/3}\times$ $\times\exp\left(\frac{2\hbar\omega_0}{3T}\right)$	$T^{0}B^{0}\times$ $\times\exp\left(-\frac{\hbar\omega_0}{T}\right)$

When $\tau_{pp} \ll \tau_b$ the energy is transferred to the thermal phonons but when $\tau_{pp} \gg \tau_b$ it is transferred directly to the surrounding medium through the boundaries of a sample.

higher values of the energy. At high energies the strongly inelastic collisions become important and they stabilize the distribution function of electrons heated by a strong electric field.

We shall conclude this review by considering the role of the drag effects and of the phonon emission by a supersonic electron flux. This effect was investigated experimentally by Esaki[53] in very pure single crystals of bismuth subjected to crossed fields (a quantizing magnetic field B_z and a strong electric field E_x). At carrier drift velocities $v_{0y} = cE_x/B_z$ lower than the velocity of sound s in Bi the current-voltage characteristics were linear. At the point $v_{0y}^* = s$ there was a sharp kink and in higher fields E_x the current again increased proportionally to E_x but the rate of rise was faster than in the range $E_x < E_x^*$. Thus, the current-voltage characteristics consisted of two linear regions with a kink at the point corresponding to $cE_x^*/B_z = s$. Similar experiments were carried out by Borisov et al.[54] on bismuth in pulsed fields by varying the steepness of the leading edge of the pulses. When the leading edge was very steep, Borisov et al.[54] observed an S-type current-voltage characteristic. When the steepness of the leading edge of the applied pulses was reduced the S-type characteristic transformed into that observed by Esaki.[53]

Unfortunately, there is as yet no self-consistent microscopic theory of these effects, although there have been attempts to develop such a theory.[55-64]

APPENDIX

1. The collision integral of Coulomb particles in a quantizing magnetic field has been investigated by Eleonskiĭ et al.[20] and by Silin.[65] In the case of nondegenerate electrons and in the presence of polarization effects, the collision integral can be written in the form:[20]

$$I_{\nu\nu}^{ee}(f) = \frac{m}{\pi(2\pi\hbar\lambda)^2} \sum_{n'_\nu n_\mu n'_\mu} \int_{-\infty}^{+\infty} dk_z\, dp_z^\mu \Phi(\nu, \nu', \mu, \mu', k_z)$$
$$\times \delta[m\hbar\Omega(n'_\nu + n'_\mu - n_\nu - n_\mu) + (\hbar k_z)^2 + \hbar k_z(p_z^\nu - p_z^\mu)]$$
$$\times [f_{n'_\nu}(p_z^\nu + \hbar k_z) f_{n'_\mu}(p_z^\mu - \hbar k_z) - f_{n_\nu}(p_z^\nu) f_{n_\mu}(p_z^\mu)], \qquad \text{(A.1)}$$

where

$$\Phi(\nu, \nu', \mu, \mu', k_z) = \int_0^\infty dk_\perp k_\perp \left| 4\pi e^2 \left[k^2 \varepsilon\left(\frac{E_{\nu'} - E_\nu}{\hbar} k_z, k_\perp\right)\right]^{-1}\right|^2$$
$$\times \frac{(\bar{n}_\nu!)^2 (\bar{n}_\mu!)}{n_\nu!\, n'_\nu!\, n_\mu!\, n'_\mu!} \left(\lambda^2 \frac{k_\perp^2}{2}\right)^{|n'_\nu - n_\nu| + |n_\mu - n'_\mu|} \exp(-\lambda^2 k_\perp^2)$$
$$\times \left[L_{\bar{n}_\nu}^{|n'_\nu - n_\nu|}\left(\frac{\lambda^2 k_\perp^2}{2}\right) L_{\bar{n}_\mu}^{|n'_\mu - n_\mu|}\left(\frac{\lambda^2 k_\perp^2}{2}\right)\right]^2;$$

$\bar{n}_\nu = \min(n_\nu, n'_\nu)$; $L_n^m(t)$ is a generalized Laguerre polynomial; $k_\perp^2 = k_x^2 + k_y^2$; $\epsilon(\omega, k_\perp, k_z)$ is the longitudinal permittivity of the electron gas.

2. The electron-phonon and phonon-electron collision integrals in a quantizing magnetic field $B = B_z$ perpendicular to a strong electric field $E = E_x$ can be written in the following form (in the Born approximation for the electron-phonon interaction):[66]

$$I_{\nu\nu}^{ep}(f) = \frac{2\pi}{\hbar} \sum_{\varkappa q} (1 - \hat{P}_{\nu\varkappa}) |A(q, \varkappa, \nu)|^2 \delta[E(\nu) - E(\varkappa) + \hbar\omega_q$$
$$+ e\lambda^2 q_y E] \{f_\varkappa(1 - f_\nu)(1 + N_q) - f_\nu(1 - f_\varkappa) N_q\},$$
$$+ I^{pe}(N_q) = \frac{2\pi}{\hbar} \sum_{\varkappa\nu} |A(q, \varkappa, \nu)|^2 \delta[E(\nu) - E(\varkappa) + \hbar\omega_q + e\lambda^2 q_y E) \qquad \text{(A.2)}$$

$$\times \{f_\varkappa(1 - f_\nu)(N_q + 1) - f_\nu(1 - f_\varkappa) N_q\}, \qquad \text{(A.3)}$$

where the operator $\hat{P}_{\nu\kappa}$ is used to make the substitution $\nu \rightleftarrows \kappa$, $A(q, \kappa, \nu) = C_q \langle\kappa|\exp iqr|\nu\rangle$ is the matrix element of the energy of interaction between electrons and phonons, calculated from the wave functions in the Landau approximation ($\nu = n, p_z, x_0$); E is the total field acting on an electron (including the Hall field).

3. The collision integral for electrons interacting with impurities is of the following form (in the linear approximation with respect to the impurity concentration N_i):[27]

$$I_{\nu\nu}^{ei}(f) = N_i \frac{2\pi}{\hbar} \sum_{\varkappa, q} |T(\varkappa, \nu, q)|^2 [f_\varkappa - f_\nu] \delta [E(\varkappa) - E(\nu) - eE\lambda^2 q_y].$$

Here, $T(\nu, \kappa, q) = t(E_\nu)\langle \nu | \exp iqr | \kappa \rangle$ is the scattering amplitude of an electron of energy E_ν interacting with an isolated impurity center. In the Born approximation we have

$$|T(\nu, \varkappa, q)|^2 = |V_q|^2 |\langle \nu | e^{iqr} | \varkappa \rangle|^2,$$

where V_q is the Fourier component of the impurity potential.

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