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New Instruments and Measurement Methods

Methods of Measuring Small Phase Difference Changes in Interference Devices

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Usp. Fiz. Nauk **106**, 687–721 (April, 1972)

A review is presented of the present status of the technology of interference measurements of small changes of certain physical quantities. The capability limits of measurements with the aid of interference, imposed by diffraction and by noise in the radiation source and receiver, are analyzed. The most effective visual and photometric methods of measuring phase differences of interfering beams are described. The modulation method, in which a phase difference change of $(10^{-5}-10^{-6})\cdot 2\pi$ can be detected, and the phase method used in automatic interferometers are considered in greater detail.

CONTENTS

1. INTRODUCTION

As a rule, optical-interference measurements of physical quantities reduce to a determination of the phase difference between the interfering beams. In most interferometers used in practice, the change in the phase difference, when observed visually, is determined with an error from $2\pi/10$ to $2\pi/20$. Although this represents high accuracy in the measurement of length, angle, wavelength, or refractive index,[1-4] progress in science and technology calls for higher precision.

The accuracy of interference measurements is determined in final analysis by the accuracy with which small changes of the phase difference φ are determined. New measurement methods have been developed recently, and original devices have been designed, in which the sensitivity to very small changes of φ has been increased by several orders of magnitude. Nevertheless, the work in this direction cannot be regarded as complete, since the sensitivity is still insufficient for the solution of certain physical problems, for example the detection of gravitational waves[5].

2. PRINCIPAL LIMITATIONS ON THE ACCURACY OF INTERFERENCE MEASUREMENTS

The possibilities for increasing the sensitivity of interferometric devices are not unlimited. There are two principal limitations stemming from the corpuscular-wave nature of light. Although one always accompanies the other, in each concrete interference instrument the sensitivity is restricted by only one of these limitations, depending on the construction and purpose. With respect to purpose, all devices can be divided into two large groups.

The devices of the first group serve to reveal the phase distributions of the analyzed phase object in the interference field. The second group consists of devices in which information is obtained on the phase difference averaged over the cross section of the light beam. This group includes, for example, instruments used for metrological investigations of changes in the refractive indices of transparent media, or for the measurement of spectral-line contours, displacements, angles, or the optical thicknesses of Fabry-Perot etalons.

Depending on the concrete measurement procedure and on the features of the apparatus, the problems solved by devices of the second group can be subdivided into three types: 1) measurement of very small changes $\Delta\varphi$ of the phase difference φ ($\Delta\varphi \ll 2\pi$). In this case, as a rule, a large relative error (up to 10—30%) is acceptable, but the sensitivity must be very high. 2) Measurement of small changes of φ ($\Delta\varphi \leq 2\pi$). 3) Measurement of phase-difference changes exceeding 2π, which is usually broken up into two stages: measurement of the number of periods (or of the change in the integer number of interference orders) and measurement of the phase shift within 2π (or of the change in the fractional part of the order of interference) corresponding to the second type of problems.

In the case of instruments of the first group, the limitation imposed by diffraction phenomena on phase objects is more important. By phase object we have in mind here any inhomogeneity causing distortion of the front of a plane wave: microscopic roughness of mirror surfaces, inhomogeneity due to inclusions in transparent specimens, unevenness in the phase discontinuities on the interferometer mirrors, gas streams arising, for example, in a wind tunnel, etc. The structure of a phase object is investigated with two-beam or multibeam interference mirdoscopes, phase-contrast microscopes, or Jamin and Zehnder-Mach interferometers.

The first to analyze the diffraction limitation was Ingelstam[6], who investigated the capability limits of multibeam interference microscopy. He established that an increase of the resolution in depth (i.e., in the direction z of wave propagation) is possible only by decreasing the resolution in the plane perpendicular to the wave direction (i.e., in the object plane of the ordinary microscope). Ingelstam's relation between the resolution limits in the object plane and "in depth" was discussed in[6-10] and its mathematical expression, later refined by Rozenberg[8] and Koppelman[9], is

$$(\Delta x)^2 \Delta z \geqslant m\lambda^3/4, \tag{1}$$

where Δx is the smallest resolvable detail in the object plane, Δz is the resolution limit "in depth," i.e., the smallest observable change of thickness in the z direction, m is the order of the interference, and λ is the wavelength of the light. It should be noted that the phase shift $\Delta\varphi$ is connected with Δz by the relation $\Delta\varphi = (4\pi/\lambda)\Delta z$.

Introduction of the phase object into the field of a plane wave changes the phase of the wave in such a way that the phase distribution of the emerging wave front is an exact image of the phase object. This phase distribution is converted into an amplitude distribution in the interference field with the aid of suitably chosen optical systems. If the objects are small, then even a bending of the front, characterizing the phase object, gives rise to distortion due to diffraction effects. In view of a certain connection existing between the radiation field in neighboring points of space, the resultant field experiences the influence of the phase object introduced in it not only behind the object, but also in boundary regions, and this is indeed the cause of the limitation expressed by Ingelstam's uncertainty relation.

Expression (1) characterizes the three-dimensional resolution of an interference instrument. The inequality sign means that in practical devices, for reasons called technical, the resolution becomes worse. Formula (1) can be easily obtained from Heisenberg's uncertainty relation

$$\Delta p_x \, \Delta x \geqslant h, \tag{2}$$

where Δx is the error with which the photon coordinate is determined, Δp_x is the error in the determination of the corresponding photon momentum component, and h is Planck's constant.

The first factor is equal to (see Fig. 1)

$$\Delta p_x = p\,\Delta\psi = (h/\lambda)\,\Delta\psi, \tag{3}$$

where $\Delta\psi \ll 1$ is a small angle characterizing the photon scattering. The photon motion direction z corresponds to the wave-front propagation direction. From the interference maximum condition $2 \cos \psi = m\lambda$ (where l is the distance between the interferometer mirrors, ψ is the angle between the beam and the normal to the mirror surface, and m is the order of the interference) we obtain

$$\Delta\psi = 2\,(\Delta l/m\lambda)^{1/2}; \tag{4}$$

here $\Delta l = \Delta z$. Substituting (3) and (4) and (2) and squaring both sides of the inequality we obtain (1).

Formula (1) makes it possible to determine correctly the order of magnitude of the smallest resolvable details in those cases when the resolution limit is approximately equal to half the width of the apparatus function[11] of the instrument (determined at an intensity level equal to half the maximum intensity). Such a relation between the resolution limit and the width of the apparatus-function contour holds when one uses classical resolution criteria, say those of Rayleigh or Abbe[12], which characterize well the capabilities of optical systems used for visual observation of minute objects. In interferometric devices, however, the minimum observable phase shift can be much smaller. For example, in examinations of distortions or displacements of equal-thickness fringes it is easy to observe shifts equal to one-tenth of the apparent width of the fringe, corresponding to approximately one-tenth of the width of the interferometer apparatus function. A relation more accurately in agreement with the experimental conditions is therefore

$$(\Delta x)^2 \, \Delta z \geqslant \gamma m\lambda^3/4, \tag{1'}$$

where γ is the smallest resolvable displacement in terms of the width of the apparatus-function contour, and depends on the method of observation.

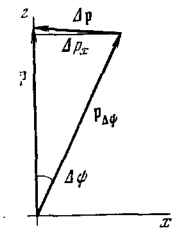

FIG. 1. Change of photon momentum.

In the devices of the second group, the microscopic inhomogeneities are not the objects of the investigations. If the roughnesses of the mirrors are much smaller than the wavelength of the light, they have little effect on the measurement results[13-16]. The diffraction limitation in these measurements is determined by the area of the light beam. Calculation in accordance with (1′) with $m = 10^2$, $\Delta x = 10$ mm, $\gamma = 0.1$, and $\lambda = 0.6$, for example, yields $\Delta z \geq 3 \times 10^{-9}\,\mu$. Obviously, in visual observations with devices of the second group the Ingelstam limitation is of practically no significance. In visual measurements, the smallest resolvable phase shifts are usually 1/2—1/30 of the width of the apparatus function of the instrument, and the quantitative difference between the resolution limit obtained in practice and the classical concepts of the resolving power is not very large. In photoelectric measurements, the smallest observable phase shifts are smaller by several orders of magnitude than the width of the apparatus function, and it becomes obvious that the classical resolution criteria are not suitable for estimates of the capabilities of modern opto-electronic instruments. Attempts have therefore been made recently to introduce new resolution criteria, more relevant to the experimental results in optics in general and in interferometry in particular[11,17].

The development of a new criterion is hindered by the following circumstance: in the general case, interference measurements yield the spatial, temporal, or spectral distribution of some physical quantity. This distribution is the input to the measuring setup. The output signal registered by the instrument and used to reconstruct the initial action is a function of one or several parameters of the interference setup. For example, in Fourier spectroscopy interference instruments, the output signal is the distribution function of the output intensity of the path difference of the beams in the interferometer, from which the spectral distribution of the investigated radiation is determined by means of a Fourier transformation. Investigations[17] have shown that the threshold sensitivity in such measurements depends not so much on the parameters of the instrument as on the form of the input signal.

The problem of choosing a resolution criterion becomes much simpler in the case of interference measurements, when one determines not the distribution but simply the change of a certain physical quantity. This case is the most prevalent in measurement practice, and we therefore confine ourselves to it in this paper. The resolution problem then becomes the problem of observing small changes of the measured quantity, which can be characterized by changes of the phase difference, and the increase in the resolution is limited by fluctuation phenomena in the measurement apparatus.

Generally speaking, the results of objective measurements are influenced by many random disturbing factors, such as changes of the ambient pressure and temperature, vibrations, electric noise and static, variations of the transfer coefficients of individual opto-mechanical and electric elements, and variations of the readings of the electric measuring instrument. But these factors, usually called technical, do not limit the sensitivity in principle, since their disturbing action can be reduced by special means to a sufficiently small tolerable amount. The principal limitation is imposed only by the noise fluctuations of the radiation flux and by the internal noise of the radiant-energy receiver, which cannot be eliminated by any design means. If the receiver is a photographic plate then, in final analysis, the sensitivity at sufficiently high illumination is determined by the internal noise due to the internal structure of the photographic material. In photoelectric measurements, the sensitivity is measured by the shot noise of the photoreceiver, due to the discrete character of the radiation and the discrete character of the carrier emission[18,19]. Additional noise fluctuations due to the correlation between the photons in a light beam from an ordinary source are so insignificant, that they cannot be observed with the aid of one photoreceiver[20]. The optical noise due to plasma oscillations in the gas discharge of a spectral lamp also makes a negligibly small contribution to the shot noise if suitable electric supply circuits are used[21].

The use of lasers in conjunction with interferometers introduces an additional noise due to the spontaneous emission of the atoms of the active medium[19,22,23]. Unlike shot noise, the laser noise is narrow-band. In the case of single-mode lasing, the contour of the noise-power spectral distribution differs little from a Lorentz contour, the distribution maximum corresponding to zero frequency. The spectral noise-power density at frequencies close to zero is inversely proportional to the generation power.

It is obvious that to obtain maximum sensitivity in interference measurements it is most convenient to operate at high generation powers. In these cases the width of the noise line, which is proportional to the generation power, can become appreciable. Thus, at a generation power of 50 μW at the 0.63 μ wavelength of a single-frequency helium-neon laser, the noise line width is approximately 10^5 Hz[22], i.e., it spans practically the entire range of acoustic frequencies, which is most convenient for the processing and investigation of signals in photoelectric devices.

The fluctuations of the intensity of laser emission recorded with a photoreceiver are conveniently characterized by a coefficient equal to the depth of the random modulation of the photocurrent, defined as $M(t) = \sqrt{2}\,\bar{i}(t)/i_0$, where i(t) is the fluctuation of the photocurrent at the instant of time t, and i_0 is the dc component of the photocurrent[22]. In the experiment one usually determines the spectral density (at the frequency f) of this coefficient. According to[22], for a laser power of 50 μW at 0.63 μ, the mean square $\overline{M_f^2}$ of the spectral density of the random modulation depth coefficient on the horizontal section of the noise frequency spectrum is approximately 10^{-12} Hz^{-1} and exceeds $\overline{M_f^2}$ of shot noise (10^{-13} Hz^{-1}) by approximately 10 times.

At low values of the path difference there are no stringent requirements on either the temporal or the spatial coherence of the light entering the interferometer. It is therefore possible to use practically the entire radiation flux from the laser and the greater part of the light from an ordinary source. However, even the brightest non-laser sources, such as a high-pressure mercury discharge lamps, can produce in real interferometer devices photocurrents* amounting only to $10^{-8}-10^{-7}$ A, which is 1/100-th of the maximum photocurrent attained when a laser is used. For shot noise, the modulation depth coefficient is inversely proportional to the dc component of the photocurrent. Therefore the depth of the random photocurrent modulation, which determines the maximum measurement sensitivity, is somewhat larger when ordinary light is used ($\overline{M_f^2} = 6 \times 10^{-12}-6 \times 10^{-11}$ Hz^{-1}) than the modulation depth ($\overline{M_f^2} = 10^{-12}$ Hz^{-1}) produced under favorable conditions by laser noise. Nonetheless, at small path differences it is preferable to use ordinary sources for interferometers with photoelectric registration, because their emission is more stable than that of a laser. Unlike ordinary sources, a laser is characterized by strong variation of the radiation intensity with time, and requires the use of special compensating and stabilizing devices.

When the path difference Δ is increased, the requirements with respect to collimation of the input beam become more stringent in devices in which the fronts of the interfering waves are plane. To satisfy these requirements, a geometric limitation is imposed on the light beams. The mean value of the photocurrent, together with the useful signal, is appreciably reduced. For example, at a path difference of 1 cm it is possible to obtain with a Michelson interferometer a photocurrent not exceeding 10^{-11} A. The depth of the random modulation of the corresponding shot noise

*If a photomultiplier is used, we have in mind the photocathode current, i.e., the primary photocurrent.

($\overline{M_f^2} \approx 10^{-7}$ Hz^{-1}) is then much larger than the values attainable at small Δ.

The laser output radiation is concentrated in a much smaller solid angle than the radiation of ordinary sources. The matching telescopic system used to broaden the laser beam makes this angle even smaller. Therefore even at large path differences (up to 2–3 m) the entire output flux is used, with the exception of the losses in the decoupling elements that are sometimes placed between the laser and the interferometer. Consequently the useful signal and of the noise of the photoreceiver do not differ significantly from those at small Δ. The use of lasers in photoelectric interferometers is therefore preferable at path differences exceeding several millimeters.

In addition to the already noted advantages of lasers, their adoption in interferometry uncovers new possibilities for developing original interference schemes and for new applications of known devices heretofore used with conventional light sources (cf., e.g.,[24]).

The theory of the noise-imposed accuracy limit of interference measurements[25-28] was developed for the most general case of measurements in[28]. It is assumed that the photoreceiver registers the illumination of the central section of a pattern of equal-inclination lines. This method of registration[29] results in an appreciable gain in optical transmission, and also in the sharpness of the fringes in the case of multiple-beam interference, when compared with the use of equal-width fringes.

The setting for maximum intensity, which corresponds to the condition $\varphi_0 = 2\pi n$, where $n = m_0$ is an integer (Fig. 2), is effected in the following manner: during the time $\Delta t/2$ (the first half-cycle), radiation at a phase difference φ_1 is incident on the photoreceiver; then the phase difference is changed jumpwise, with the aid of a device contained in the interferometer, by a fixed amount 2δ, and the photoreceiver registers during a time $\Delta t/2$ radiation at a phase difference $\varphi_2 = \varphi_1 + 2\delta$. The difference between the averaged electric signals of the first and second half-cycles is a measure of the deviation of $\Delta\varphi$ from φ_0.

The maximum possible accuracy of interference measurements, limited only by shot noise, is determined by the following expression[28]:

$$\varphi_0/\Delta\varphi_{\min} = [(\pi^2/2hc)\, D^2\, \Delta t\theta\, (B_e/\Delta k)\, Q/]^{1/2}, \qquad (5)$$

where $\Delta\varphi$ is the rms deviation of the series of measurement, c is the speed of light, D is the diameter of the light beam, Δt is the time of one measurement, θ is the quantum yield of the radiant-energy receiver, B_e is the energy brightness of the source, Δk is the width of the spectral line in wave numbers, and

$$Q = \mu\bar{\tau}\,[I(\varphi_1') - I(\varphi_1'')]^2 \Big/ \int\limits_{\varphi_1'}^{\varphi_1''} I(\varphi)\, d\varphi$$

is a quantity characterizing the dependence of I on φ at the output of the interferometer (see Fig. 2); here μ is the fraction of the order occupied by the line of width Δk, and $I(\varphi)$ is the intensity distribution function normalized by the relation

$$\int\limits_{\varphi_0-\pi}^{\varphi_0+\pi} I(\varphi)\, d\varphi = 1,$$

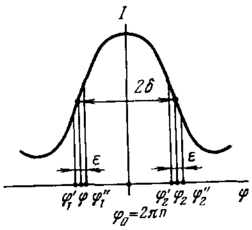

FIG. 2. Output intensity I vs. the phase difference φ in the interferometer.

$\bar{\tau}$ is the transmission of the instrument in the absence of interference, and $\varphi_1' = \varphi_1 - (\epsilon/2)$ and $\varphi_1'' = \varphi_1 + (\epsilon/2)$ are the limits of the phase-difference interval ϵ corresponding to the solid angle within which the flux is registered (the solid angle is governed by the finite dimension of the light source).

In the case of interferometers intended for spectral measurements, the plot of I against φ when an ideally monochromatic source is used for the illumination is the apparatus (transfer) function $I(\nu)$. In the general case, the intensity distribution function is a convolution of the apparatus function and a function characterizing the source emission-line contour. As seen from (5), the maximum accuracy depends both on the properties of the instrument (D, Q) and on the properties of the source (B_e, Δk) and of the receiver (θ, Δt) of the radiation. It is interesting to note that the presence of the term D^2 in (5) is due not to diffraction phenomena (as in Ingelstam's relation) but to the dependence of the output flux on the effective area of the light beam inside the interferometer. On the other hand, in those cases when diffraction phenomena are significant, they are taken into account in the apparatus function (cf., e.g.,[30]), which is implicitly contained in Q.

The use of formula (5) for the orange line of Kr^{86}, the wavelength of which is a primary length standard ($\Delta k = 1.4$ m^{-1}, $B_e = 3 \times 10^{-1}$ W/m^2 sr, $D = 4 \times 10^{-2}$ m, $\theta = 7 \times 10^{-2}$, $\Delta t = 1$ sec), in conjunction with a Fabry-Perot interferometer at an optimal reflection coefficient $\rho = 0.73$ and optimal values of δ, ϵ, and the path difference $\Delta = 0.1$ m (corresponding to $Q = 7.4 \times 10^{-1}$ and $\mu = 0.14$), yields $\varphi_0/\Delta\varphi_{\min} = 2 \times 10^{10}$.

The calculation for a Michelson interferometer under the same conditions and optimal δ; ϵ, and $\Delta = 0.4$ m yields the same value, $\varphi_0/\Delta\varphi_{\min} = 2 \times 10^{10}$. The surprisingly low optimal reflection coefficient of the Fabry-Perot interferometer mirrors and the equality of the maximum accuracies of the Michelson and Fabry-Perot interferometers are attributable to the dependence of the signal/noise ratio on the average radiation flux incident on the photoreceiver.

When laser radiation is used, the ratio $\varphi_0/\Delta\varphi_{\min}$ reaches[31] a value 10^{12}–10^{13}. One might gain the impression that this creates conditions for obtaining an rms relative measurement error 10^{-13}. However, in practical measurements of any quantity, say length or refractive index, the measurement error includes also the error in the wavelength. Since the laser generation wavelength cannot be determined with an accuracy lower than the error in the tabulated value of the primary standard of light wavelength, the Kr^{86} orange line (10^{-8}), the measured quantity cannot be determined with an error lower than 10^{-8}.

There are so many different methods of measuring small changes in phase difference, that their descrip-

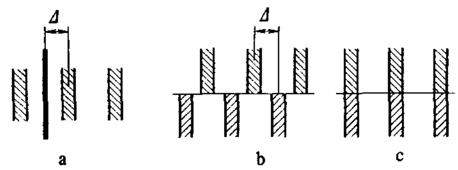

FIG. 3. Field of view in the interferometer.

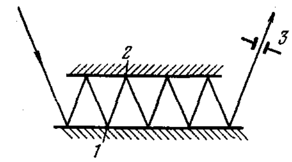

FIG. 4. Simplified diagram of system for multiplying the optical path length of a beam.

tion is impossible in a brief review. We shall consider only the simplest and most effective methods, and their use only with the apparatus in which they give the best results.

3. VISUAL METHODS

a) Fringe-displacement measurement. This method consists of measuring the displacement of the interference fringes in the field of view of the instrument, relative to a stationary sighting line (Fig. 3a) or relative to an identical fringe pattern (Fig. 3b). It is used most frequently in observations of fringes of equal thickness and of equal chromatic order. The measurement accuracy is approximately the same in both cases, but there is some difference in the possibility of using fringes of these types. For example, study the microrelief of highly finished surfaces with the aid of equal-thickness fringes yields a microtopographic map of the surface, whereas fringes of equal chromatic order make it possible to trace the profile of the relief along a chosen line[3].

When estimating the relative positions of the interference fringes and of the sight line, the error in the phase-shift determination is approximately $2\pi/10$. However, the error in the alignment of the centers of the interference fringe and of the fiducial line, and also of the two fringe systems (Fig. 3c; this is the most favorable case for measurements), is much lower. The alignment error is determined by the vernier effect and amounts to approximately 1/10th of the apparent width of the fringe. In view of the nonlinear response of the eye to light, the apparent width of fringes, if the intensity distribution is such that the transitions from one point of the field to the other is smooth, depends on the average brightness of the interference field. Under favorable illumination conditions, good visibility of the fringes and a sinusoidal distribution of the intensities, which are characteristic of two-beam interference, the width of the black fringes is one-third the distance between fringes, and the error in the alignment of the fringes is accordingly $2\pi/30$[2].

The accuracy of the alignment of the fringes (Fig. c) was doubled in[33] by the following device. An image-splitting prism was placed between the output lens of Uverskiĭ's interferometer (of the Michelson type) and the screen on which the lens projected the equal fringes. As a result, two vertically shifted images of the fringes were observed on the screen, one of them rotated in a direction perpendicular to the fringes. When φ is varied, the fringes move in opposite directions, and this doubles the accuracy.

An analogous effect in a multiple-beam wedge interferometer can be obtained by placing between the semi-transparent mirror a crystalline plate cut parallel to the optical axis, and by placing the mirrors in such a way that the angles of the resultant wedges are equal in magnitude and opposite in direction for the ordinary and extraordinary rays.

The measurement accuracy of small changes of certain quantities in a two-beam interferometer can be increased by artificially multiplying these changes via multiple reflections of one of the interfering beams in a system of two or several mirrors (cf., e.g.,[35]). The action of the multiplication system is explained by the simplified diagram shown in Fig. 4. The light beam in the measuring branch of the interferometer is reflected several times between mirrors 1 and 2, is separated with the aid of diaphragm 3, and subsequently interferes with the comparison beam. If the optical thickness of the gap between mirrors 1 and 2 changes, then the changes of the optical path length of the rays is several times larger than the change of the gap thickness. This leads to a corresponding increase of the sensitivity to mirror displacements or to changes of the refractive index of the ambient.

There is an interesting interferometer in which the optical path length is multiplied by repeated reflections between the investigated surface and a glass plate placed near the surface and slightly inclined to it[36,37]. The interferometer can detect surface deformations on the order of 1 nm. This device illustrates well the Ingelstam uncertainty principle, since an increase of "depth" sensitivity inevitably leads to a lowering of the resolution in the investigated plane, as a result of which the interferometer can be used only to determine slowly varying deviations from the plane.

An essential feature of multiplication system is the increased sensitivity due to the increase in the change of the phase difference when the measured quantity is changed, whereas the sensitivity to phase shifts remains unchanged.

Other possibilities of increasing the sensitivity are afforded, in principle, by multiple-beam interferometers. Interference between a large number of beams, produced by dividing the beam amplitude (interference of the Newton type[38]) or by dividing the wave front into partial fronts (interference of the Fresnel type), produces narrow maxima (or minima) on the plot of the radiation output intensity against the phase difference[39]. The narrowing of the contour of the apparatus function of multiple-beam interferometers increases the resolution in comparison with two-beam interferometers. The rapid progress in the techniques for depositing thin metallic and dielectric coatings in the last three decades has made it possible to produce multiple-beam interferometers of the first kind with a large number N_{eff} of interfering beams. These interferometers are a powerful tool for exact spectral measurements, for the study of the microrelief of surfaces, for the measurement of lengths and displacements, for the determination of the refractive index of a transparent medium, for the study of air streams, for plasma research[8,37,40-44], for investigations of the frequency spectrum of laser radiation[45,46], etc.

The width of a Fabry-Perot interferometer fringe corresponds to a phase-difference interval $2\pi/N_{eff}$.[39] The effective number of interfering beams is determined by the reflection coefficient ρ of the mirrors and by the transmission coefficient τ of the layer between mirrors: $N_{eff} = \pi(\tau\rho)^{1/2}(1-\tau\rho)$; for air $\tau \approx 1$.

Since the smallest discernible displacement is $2\pi/10N_{eff}$, an impression may be gained that a suitable increase of ρ can yield an arbitrarily high sensitivity. An increase of the reflection coefficient ρ, however, leads inevitably to an increase in the absorption coefficient α of the reflecting layer, and a corresponding decrease in the transmission of the interferometer at the maximum, expressed by the relation[8] $[1-(\rho+\alpha)]^2/(1-\rho)^2$. Since it is possible to produce multilayer coatings with $\alpha \approx 0.05-0.01$ and $\rho \approx 0.97-0.99$, it is possible to obtain sufficiently narrow fringes at an appreciable transmission.

Other factors that limit the increase of ρ are the imperfection of the surfaces[8,13-16] and non-parallelism[8,13,41,42,47] of the reflecting mirrors. These factors become significant when an "infinitely broad" equal-width fringe or equal-inclination line is observed. For a Fabry-Perot interferometer with flat mirrors, it is meaningless to increase ρ above 0.94, and the realistically obtainable values of N_{eff} do not exceed 30–40. In multiple-beam interferometers with confocal geometry[48], the working dimensions of the mirrors become much smaller, as a result of which large values of N_{eff} up to 100–150 can be obtained. When equal-width fringes are observed, the phase difference is not averaged over the large surfaces of the interferometer mirror. Therefore the use of mirrors with large N_{eff} leads to a corresponding increase of the sharpness of the fringes, this being one of the main advantages of multiple-beam wedge interferometers[39,49]. A large value of ρ, and consequently also of N_{eff}, can be obtained without reflecting coatings by using the effect of disturbed total internal reflection[50]. An interferometer based on the use of this effect made it possible to observe a film-thickness difference of 0.1 nm[51].

A characteristic feature of multiple-beam interferometers is the increased sensitivity to small changes of φ only in a very small interval, amounting to a small fraction of a period. When applied to interferometric control of high-quality surfaces, this shortcoming is eliminated to a considerable degree in the method of multifrequency interferometry[37]. The use of a multifrequency source (or of a radiation source) makes it possible to increase appreciably the amount of information contained in the interference pattern, by superimposing additional fringe systems that overlap the gaps between the fringes of the main system[52].

The contour of the apparatus function of a Fabry-Perot interferometer can be made narrower by placing between its mirrors an active medium whose gain is much lower than the value corresponding to a transition into the lasing regime[53,54]. If the distance between the mirrors is varied uniformly in a range $\pi/2$ (say with the aid of a piezoceramic element) and the changes of the electric signal of the photoreceiver registering the radiation flux passing through the interferometer are simultaneously recorded, then insertion of an amplifying medium between the mirrors makes the interference maximum sharper. The narrowest apparatus-function contour is obtained if a single-frequency gas laser is used as the source, and the active medium is produced by exciting an electric discharge in a gas having the same composition as the laser working medium. The experimentally obtained contour width was[54] $2\pi/250$.

b) Uniform field method. The uniform-field method consists of recording the brightness of a uniformly illuminated field constituting the "infinite-width" fringe produced when the mirrors are made parallel. The change of the phase difference is assessed from the change in the brightness of the field.

A very high measurement sensitivity is obtained by using a half-shadow (stepped-surface) device[55]. The measurement method with the half-shadow arrangement, frequently used in photography, is based on the principle of equating the brightnesses of to contiguous parts of the field of view of the instrument. At the instant when the brightnesses become equalized by a compensator, the boundary between the halves of the field vanishes. The method makes use of the high sensitivity of the eye to contrast, i.e., the ability of the eye to catch a minimal difference ΔB between the brightnesses B_1 and B_2 of the two halves of the field. The contrast sensitivity $\Delta B/B$ is an individual characteristic of the eye and depends on external factors, particularly on the value of B and the angular dimensions of the field. Under optimal observation conditions, $\Delta B/B = 0.02-0.05$.[4] An interferometer with a half-shadow device is constructed in such a way that the phase difference in one half of the field differs by a fixed amount 2δ from the phase difference in the other half. The difference 2δ can be produced by different methods. It is most convenient to coat one half of one of the interferometer mirror with an additional thin film of appropriate thickness (cf., e.g.,[56]).

In two-beam interferometers and in the Fabry-Perot interferometer, in which the apparatus-function contour is symmetrical, the brightnesses of the halves of the field of view are equalized at a phase difference $\varphi_1 = n\pi - \delta$ (n is an integer) in one half and $\varphi_2 = n\pi + \delta$ in the other half of the field. The average phase difference is $\varphi = (\varphi_1 + \varphi_2)/2 = n\pi$. A small change $\Delta\varphi$ in the phase difference in the interferometer (when $\varphi_1 = n\pi + \delta + \Delta\varphi$ and $\varphi_2 = n\pi - \delta + \Delta\varphi$), the brightnesses of the two halves of the field of view are no longer equal, so that small deviations of φ from multiples of π can be seen by the observer.

In two-beam interferometers characterized by a sinusoidal intensity distribution $I = I_0(1 + V\cos\varphi + \eta)$ (where I_0 is the average intensity without allowance for parasitic extraneous illumination, V is the visibility of the interference pattern, and η is the fraction of the parasitic illumination of the interference field), maximum sensitivity is obtained by positioning at the center a black "fringe" corresponding to $\varphi = (2n+1)\pi$. It is easy to prove that when $\Delta B/B = \Delta I/I = 0.05$ and $V = 1$, the minimum observable displacement is given by

$$\Delta\varphi_{min} = 0.025\,[\mathrm{tg}\,(\delta/2) + (\eta/\sin\delta)]. \qquad (6)$$

For values $\eta = 0.05$ and $\delta = 0.3$ we obtain $\Delta\varphi_{min} = 2\pi/800$. Obviously, if there were no parasitic illumination at all, an appreciable increase of sensitivity could be obtained by correspondingly decreasing δ.

In investigation[57] of a visual polarimeter with a half-shadow device, the action of which is similar to that of a two-beam interferometer with an analogous device, a decrease of the half-shadow angle (which corresponds to δ in our case) increases the sensitivity only up to a certain limit. At very small values of δ, the sensitivity is lower because of the decrease in the eye's contrast sensitivity when the field of view has low brightness. The smallest phase-difference change that can be registered in the absence of parasitic illumination is $\Delta\varphi_{min} = 2\pi/10^5$.

For a multiple-beam interferometer of the Fabry-Perot type we have $\Delta\varphi_{min} \approx 2\pi/100N_{eff}$ at $\Delta I/I = 0.05$ and at δ corresponding to the points of maximum slope of the plot of I against φ. If $\rho = 0.92$ ($N_{eff} = 38$), then $\Delta\varphi_{min} = 2\pi/3800$, corresponding to a mirror displacement of 0.07 nm at $\lambda = 0.5\ \mu$. In those cases when the investigated object itself, say the surface of an optical part, is a "step," the smallest observable displacement of the planes is twice as large, i.e., 0.14 nm. Such a resolution is reached under favorable conditions in multiple-beam interference microscopes[43,58].

c) Method of equalizing the brightnesses of neighboring fringes. This method is similar in principle to the half-shadow method and can be used in interference devices in which the change of the phase difference causes a redistribution of intensity in neighboring fringes. This pertains primarily to three-beam interferometers[59-62] and to two-beam interferometers with double passage of the rays[63-68].

The distinguishing features of three-beam interferometers are well illustrated by using as an example the three-slit modification[60] of the Jamin interferometer. In this modification, which was proposed for refractometric use (Fig. 5, where 1—light source, 2—condenser, 3 and 9—slits, 8—beam-splitting plate, 6—cells, 7—compensation chamber, 10—sighting tube). The thick lines show those parts of the external surfaces of plates 4 and 8 which are covered with an opaque and totally reflecting layer of aluminum. The light beam from the illuminator is split in plate 4 into three beams, the dimensions of which are determined by the parallel slits 5. In practice it is convenient to use the two outer beams as comparison beams, and to position the central beam in the working chamber.

The distribution of the intensities I in the fringe system of the three-beam interferometer can be easily obtained by regarding the optical oscillations at any point of the picture as the resultant sum of oscillations obtained in the system of fringes formed by the outer beams, and the oscillations of the coherent background produced by the central beam[60]:

$$I(x, \varphi) = I_0 [3 + 2\cos(2\pi x/a) + 4\cos(\pi x/a)\cos\varphi],$$

where I_0 is the intensity of one beam, x is the coordinate in the direction perpendicular to the fringes, and a is the distance between the centers of neighboring fringes.

Thus, introduction of the middle beam leads to a qualitative change in the intensity distribution. If the central beam is in phase or in counterphase with the external beam, then the resultant effect is a strengthening or weakening of the corresponding fringes (Fig. 6). To the contrary, all the fringes have the same intensity when the phase difference between the central and the external beam is $\varphi = 2\pi n + (\pi/2)$. In the latter case, very small changes of the phase of the central beam lead to a discernible difference in the brightness of the neighboring fringes[59]. Analogous phenomena take place also in other types of three-beam interferometers, those of Rayleigh and Young[61]. The errors in the setting to the position $\varphi = 2\pi n + (\pi/2)$ by equalizing the brightnesses of the fringes was $2\pi/100$—$2\pi/300$ in the experiments of[59,60]. The accuracy with which the null positions can be set can be increased somewhat by using an asymmetrical placement of the slits.

If the beams emerging from a two-beam interferometer are returned to the interferometer[63-66], then fringes similar in form and in their properties to the fringes of a three-beam interferometer are observed. The most widely used in practice[67,68] is the Michelson double interferometer (Fig. 7), where 1—spectral lamp, 2—light filter, 4—diaphragm, 5—semitransparent mirror, 6—objective, and 7—beam-splitting plate). The beams are returned to the interferometer by an optical system consisting of an objective 10 and a mirror 11 in its focal plane. To exclude the rays that fall into the exit diaphragm 13 after a single passage through the interferometer, polarizers 3 and 14 with mutually perpendicular transmission directions are used in conjunction with a quarter-wave plate 9, the principal directions of which make an angle 45° with the directions of the polarizers. Since the polarizers 3 and 14 are crossed, the rays directly reflected from mirrors 8 and 12 do

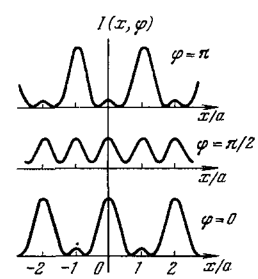

FIG. 6. Distribution of intensity in the field of view of a three-beam interferometer.

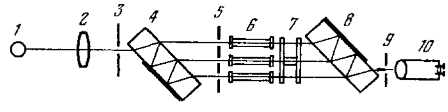

FIG. 5. Three-beam Jamin interferometer.

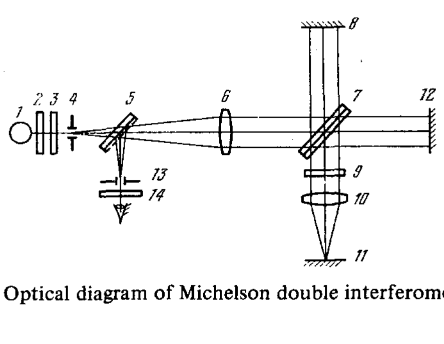

FIG. 7. Optical diagram of Michelson double interferometer.

not pass through the exit diaphragm. At the same time, the double-passage beams cross the quarter-wave plate twice. The direction of the oscillations in these beams is rotated through 90°, as a result of which they pass through the analyzer 14. The observed result is an interference pattern produced by addition of four beams traversing different paths inside the interferometer. Just as in the case of the three-beam interferometer, the positions corresponding to equality of the field illuminations in the neighboring fringes can be set with high accuracy. Theory[63,68] yields an error $\Delta\varphi_{min} = 2\pi/500$ in the setting of these positions. An error close to the theoretical one was obtained with Jamin and Michelson-Twyman-Green double interferometers.

d) Color sensitivity method. It is known that superposition of different colors corresponding to the extreme sections of the visible section results in a purplish-claret hue called the sensitive color. If the purple-claret color is obtained with the aid of an absorbing filter that cuts off the spectrum of the white light incident on it, then the sensitive color changes rapidly from red to violet if the absorption wavelength of the filter is altered insignificantly. Since the eye has different sensitivity to different hues, this change can be used to measure small phase shifts in interferometers. Indeed, if an interferometer is illuminated with white light, then the maximum intensity condition is satisfied for one group of wavelengths, and the minimum condition is satisfied for another. If the phase difference is large, the wavelengths with maximum intensity differ little in magnitude, so that the sum of the group of colors passed by the interferometer is perceived by the eye as white light. On the other hand, if the phase difference is very small, then only several colors are superimposed, and these are seen as a single color, the colors being most saturated at values of φ close to zero. The color perceived by the eye characterizes the phase difference in the interferometer.

Very pure colors are obtained in polarization interferometers. Let us consider a birefringence interferometer (Fig. 8)[69,70] intended for the study of phase objects (N_1 and N_2 are polarizers ($N_1 \parallel N_2$), L is the birefringent system, O_1 is the objective, and O_2 is the eyepiece). We assume that the phase object illuminated by a parallel beam of white light is a glass plate A containing a small region M whose optical thickness differs slightly from that of the remaining part. Prior to passing through the birefringent system L, the incident wave is plane everywhere except in the central part, where it is slightly deformed by the change of the optical thickness of the object in this region. After passing through L, the wave Σ splits into two waves polarized at right angle, OE and EO. The displacement of these two wave surfaces in a direction perpendicular to xx′ is the result of the birefringence in L, and the displacement in the xx′ direction is determined by the path difference produced by L.

Let the system L be such (say, a Savart polariscope) that at normal incidence the waves OE and EO are in phase. If a plate of thickness equal to half the wavelength of yellow light is placed ahead of the analyzer N_2, then the regions A, B, and C (Fig. 9) surrounding the images M'_{OE} and M'_{EO} acquire the sensitive purple color. Indeed, in these regions the waves OE and EO

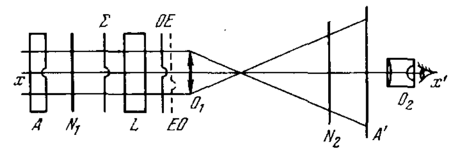

FIG. 8. Diagram of birefringence polarization interferometer.

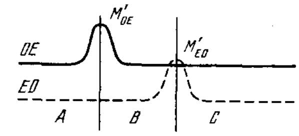

FIG. 9. Shift of wave fronts at the output of a shift interferometer.

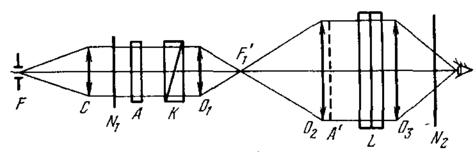

FIG. 10. Diagram of birefringence interference microscope.

are shifted only in the propagation direction of the light, and remain at a distance $\lambda/2$ from each other. In the regions M'_{OE} and M'_{EO} this is no longer the case. The path difference becomes equal to $\lambda/2$ plus or minus the path difference introduced by the object. The sensitive color immediately changes and reveals directly the structure of the object. The sensitive-color method makes it possible to detect phase shifts equal to $2\pi/100-2\pi/300$[71].

In the interferometer of Fig. 8, A can be either the object itself or its image. This circumstance was used by Francon[69,72] to produce a birefringence interference microscope, by replacing the eyepiece of an ordinary polarization microscope with an "interference eyepiece" (see Fig. 10, where F is a slit, C an objective, N_1 and N_2 are polarizers, O_1 is the objective, O_2 a collector, O_3 the eyepiece, and L the birefringent plate. The elements O_2LO_3 constitute the "interference eyepiece"). The use of a compensator K to measure the phase difference between the oscillations in mutually perpendicular planes makes it possible to obtain a quantitative estimate of the phase shifts.

e) Dark field method. This method is widely used to study birefringence in crystals and the optical activity of transparent media. The method is based on extinction of plane-polarized light with a polarizer.

Different methods of light extinction are used in different polarization compensators employed for the analysis of elliptically polarized light[73]. The highest sensitivity is reached in compensators where the elliptically-polarized light is transformed into linearly-polarized light with an oscillation direction that characterizes the phase difference of beams polarized in mutually perpendicular planes. The angle of rotation of the analyzer of polarizer required to produce complete darkening of the field, read on a dial, determines the sought phase shift.

The most widely used compensator of this type (azimuth analyzer in accordance with Jerrard's classification[73]) is the Senarmont compensator. It comprises a quarter-wave plate so mounted that the principal directions of the plate coincide with the semi-axes of the ellipse of the investigated light. The light coming from the plate is linearly polarized and is extinguished by the analyzer. The illumination of the field of view of the instrument when the Senarmont compensator is used is given by

$$E = E_0 \cos^2 [\psi + (\Delta\varphi/2)], \qquad (7)$$

where E_0 is the maximum illumination of the field, ψ is the angle of rotation of the analyzer, and $\Delta\varphi$ is the measured phase shift. At $\Delta\varphi = 0$ we have $\psi_0 = \pi/2$. It follows from (7) that when the analyzer is rotated through an angle ψ the illumination is changed by the same amount as when the phase difference is changed by 2ψ.

The sensitivity of the dark-field method is increased by one or two orders of magnitude if a half-shadow device is used. Instruments for the study of birefringence with half-shadow devices make it possible to observe phase shifts on the order of $2\pi/(2 \times 10^4)$.[73] Polarimeters with half-shadow devices make it possible to determine the angles of the plane of polarization with an error $2\pi/10^4$—$2\pi/10^5$.

Generally speaking, the dark field method is a polarization variant of the uniform field method (when set to minimum interference). Since the dependence of the intensity on the phase difference is a sinusoidal function both in two-beam interferometers and in polarization instruments, the sensitivity to small changes of φ is in principle the same for two-beam interferometers, polarimeters, and instruments for the measurement of birefringence. The difference encountered in practice is due [see formula (6)] to the unequal fractions of the parasitic illumination in the output radiation of the different instruments.

If an interferometer is converted, by suitable devices, into an optical system that is equivalent in its properties to a uniaxial crystalline plate cut parallel to the optical axis, or into a system that rotates the plane of polarization of light just as active media do, then the accuracy attained in the study of birefringence and of the rotation of the plane of polarization can be extended to include also interferometry. Such a possibility of increasing the accuracy of interference measurements was considered by S. I. Pokrovskiĭ back in 1910, and was subsequently studied in greater detail by A. A. Lebedev[75].

Polarization interferometer schemes intended for the measurement of length were investigated recently in[76-78]. The interferometer based on the idea of S. I. Pokrovskiĭ and considered in[76] is based on the Koester comparator scheme. Since this scheme is a more complicated variant of the Michelson interferometer, we confine ourselves for brevity only the principal elements of the Michelson polarization interferometer (see Fig. 11, where 1—polarizer, 4—$\lambda/4$ plate, 7—objective, and 9—diaphragm). Beams reflected from mirrors 3 and 5, meeting on the surface of the beam-splitting plate 2, proceed in the same direction and form elliptically polarized light. The Senarmont compensator 6 ($\lambda/4$ plate), intended for investigations of ellipticity, was oriented at an angle of 45° to the direction of the oscillation planes of the investigated light beams. After passing through plate 6, the light becomes plane-polarized. Its complete extinction is effected by rotating the analyzer 8. The angle of rotation of the analyzer characterizes the measured phase difference of the beams reflected from mirrors 5 and 3.

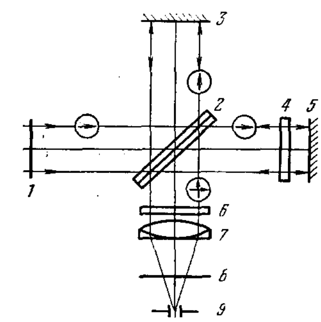

FIG. 11. Diagram of polarization interferometer [69].

In the experimental investigation of the interferometer, the mirror 3 was the working surface of an end gauge. The surface imperfections of the gauge did not make it possible to obtain uniform illumination of view, so that the reproducibility of the measurements was low.

In a relatively recently constructed interferometer[78], the polarization plane is rotated after addition of the optical oscillations. The interferometer is illuminated with circularly-polarized light. It is based on a two-beam interferometer of the Michelson type, in one arm of which the reflector is a flat mirror, and in the other it is trihedral corner reflector. As a result, the beams have oppositely directed circular polarizations after passing through the different arms. When the left- and right-polarized beams from the interferometer are added, plane-polarized light is produced. The angle ψ between the direction of the light oscillations in the beam incident on the interferometer and the direction of the oscillations in the emerging beam depends on the phase difference φ in the interferometer: $\psi = (\varphi - 2n\pi)/2 = \Delta\varphi/2$, where $\Delta\varphi$ is the phase shift between the interfering oscillations, corresponding to the fractional part of the order of the interference. By measuring the angle ψ with the aid of an analyzer provided with a dial, it is possible to determine $\Delta\varphi$. With respect to the character of the physical processes occurring in it, this interferometer can be regarded as an analog of a polarimeter, just as the interference instruments considered above are analogs of devices for the determination of birefringence.

f) The flicker method[79]. It is based on the high sensitivity of the eye to intensity fluctuations of a uniformly illuminated field. To observe the flicker, the phase difference in an interferometer set to produce "an infinitely broad" fringe is varied periodically at a frequency of several oscillations per second. The amplitude of the oscillations is equal to approximately half the width of the intensity distribution function $I(\varphi)$. The strongest flicker of the field is observed if the phase difference corresponds to the slope of the $I(\varphi)$ curve. The flicker becomes minimal and practically disappears at values of φ corresponding to the minimum intensity condition.

The flicker method with setting to maximum intensity was used by Baird[79] to determine the order of the interference in a Fabry-Perot etalon (see Fig. 12, where

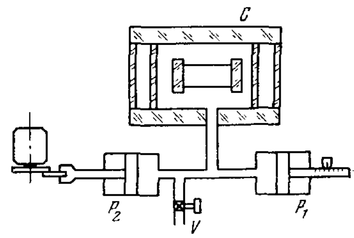

FIG. 12. Diagram of Baird's experimental setup.

C is a chamber with a Fabry-Perot etalon, P_1 is the compensator piston, P_2 is the pulsator piston, and V is a valve for establishing atmospheric pressure in chamber C). The rms error of the setting to maximum position was $2\pi/250$ for an etalon with a distance 100 mm between mirrors and an electrodeless lamp with Hg^{198} as the light source.

In all the foregoing methods, small changes of the phase difference were determined from changes in the intensity. An interesting possibility of determining small changes of φ in a Fabry-Perot interferometer by measuring the phase shift β introduced by the interferometer in a wave passing through it is considered in[8]. When a plane monochromatic wave passes through a Fabry-Perot interferometer with parallel mirrors, the phase of the optical oscillations changes by an amount

$$\beta = 0.5\,[\varphi - (\delta + \delta')] + \operatorname{arctg}\,[\rho \sin \varphi/(1 - \rho \cos \varphi)],$$

where δ and δ' are the phase jumps on the interferometer mirrors.

In the vicinity of the transparency maximum, the error in the determination of the phase difference is given by[8] $\Delta\varphi = \pi\Delta\beta/N_{eff}$, where $\Delta\beta$ is the error in the measurement of the phase shift. The phase shift can be measured, for example, with a three-beam interferometer with an error 1/40, so that the position of the maximum can be determined with an error $\Delta\varphi \approx 2\mu/80N_{eff}$. It should be noted that the method is applicable only in a very small interval of φ near the maximum, when the change of the intensity of the emerging wave is negligibly small.

g) Equidensitometry method. If the light receiver is a photographic plate or a photoreceiver, then the accuracy with which the fringe displacement is measured can be increased by the equidensitometry method.[80] This method reduces to a determination of the equal-density curves on a photographic-plate image of the object of interest to us. If the object is the field of a two-beam interferometer, then separation of thin equal-intensity lines in lieu of blurred fringes with a sinusoidal intensity distribution over the field increases the sensitivity to distortions and shifts of the fringes.

To explain the process whereby the equidensitometric curves are produced, it is useful to carry out an analogy with the shaping of short-duration sharp electric pulses at the points where a sinusoidal voltage goes through zero. To obtain short pulses, rectangular pulses are first produced with the aid of an electronic circuit that carries out a nonlinear transformation of the signal, for example an amplitude discriminator or a Schmitt trigger, after which these rectangular periodic pulses are differentiated. Equidensitometry makes use of an analogous nonlinear-transformation resulting from the peculiarities of the photographic process, followed by subtracting the density distributions of two photographs of the object.

For the subtraction, two photographs of the object, one negative and the other positive, are superimposed in such a way that the contours of the object coincide. The transparent places on the negative then overlap the opaque places of the positive, but the nonlinear dependence of the density on the illumination results in a separation, against the general background of appreciable density, of narrow contour lines corresponding to equal photographic density. The greatest effect is obtained by using high-contrast photographic materials.

To reveal more details which are not observed earlier, the operation of obtaining the equidensitometry curve is repeated with the already obtained narrow lines of equal photographic density, thus enhancing the effect. A tenfold increase of the sensitivity or even more can be obtained with the aid of equidensitometry.

The procedure of obtaining equal-density lines, however, is laborious and time consuming. In addition, for the results to be highly reproducible, careful control of the conditions of the photographic processes, and identity of the photographic materials and developer solutions is necessary. If the equidensitometric curves for two compared interference patterns are obtained under unequal photographic processes conditions, the measurement errors can become appreciable.

The use of electronic equidensitometry is more effective. In this case equal-intensity curves are obtained by scanning over the field and by establishing the points with the extremal values of the intensity or of its rate of change over the field. If the image of the pattern is obtained with the aid of a television receiving and transmitting system, the scanning is effected automatically by the sweep circuits, and to obtain the equidensitometric lines it is necessary merely to carry out the corresponding electronic processing of the video signal[81].

4. PHOTOELECTRIC METHODS

Photoelectric methods of interference measurements have considerable advantages over visual ones. These advantages are higher sensitivity and absence of subjective systematic errors. In addition, the low inertia of the photoreceivers yields information on the phase difference by using the beat signal between the monochromatic components of the emission spectrum of the light source. Such a signal is produced in a photoreceiver that registers the output radiation of a two-beam interferometer set to an "infinitely broad" fringe. A radio-frequency spectrum analyzer, for example a tunable band amplifier, yields the frequency spectrum of the photocurrent.

By regarding the photoreceiver as a square-law detector that averages over a time characterized by its time constant[82], we obtain the following expressions for the dc component i_0' and for the spectral components of the photocurrent[83]:

$$i_0 = (1 + b^2 + 2b \cos \varphi)\, i_0, \qquad (8a)$$

$$[\overline{i^2(\Omega)}]^{1/2} = (1+b^2+2b\cos\varphi)\left[2\Delta\Omega\int_0^\infty G(\omega)\,G(\omega+\Omega)\,d\omega\right]^{1/2}*), \quad (8b)$$

where $[\overline{i^2(\Omega)}]^{1/2}$ is the effective value of the signal separated by the band amplifier, referred to the photoreceiver, Ω is the signal frequency, i_0 is the current produced by the light passing through the reference branch of the interferometer, $b = I_1/I_2$ is the intensity ratio of the light passing through the measuring and reference branches, $\Delta\Omega$ is the amplifier bandwidth, ω is the frequency of the optical oscillations, and $G(\omega)$ is the spectral density of the light intensity, normalized to

$$\int_0^\infty G(\omega)\,d\omega = i_0.$$

It is seen from (8a) and (8b) that both the dc and the ac components of the photocurrent depend on the phase difference in the interferometer. By determining the amplitude modulation of the signal separated by the band amplifier, we can obtain the phase shift between the interfering optical oscillations. It should be noted that a sufficient signal/noise ratio is obtained for such measurements only when lasers are used.

It is possible to obtain in similar fashion information on the phase variation of optical oscillations by using an additional generator (heterodyne) as the source of reference radiation. To attain high sensitivity to small changes of the phase, however, it is preferable to use superheterodyne detection in devices in which the interferometer serves simultaneously also as the resonator of the laser. In this case one determines not the amplitude modulation but the frequency modulation of the photoreceiver. In[85], beams from one laser, which served as the reference generator, and from another laser, whose length was varied by applying a voltage on a piezoelectric element, were incident on a photoreceiver which served as a mixer in this case. The beat-signal frequency was measured with a radio-frequency spectrum analyzer. The setup made it possible to measure an amplitude 10^{-3} nm of oscillations at 5 kHz with an error not exceeding 10%.

Many interference measurements connected with birefringence produced, for example, by electro-optical effects can likewise be performed by the beat method[86]. To this end, a birefringent medium is placed inside the resonator of a single-frequency laser. It is then possible to realize the condition that ensures two-frequency generation, and the frequency difference is determined for the given laser by the parameters of the birefringent medium. By directing the radiation to the surface of the photoreceiver, it is possible to separate a signal at a difference-frequency $\Delta\nu$ (beat signal) equal to[86]

$$\Delta\nu = \nu\,\delta\Delta/\Delta = \nu\,\Delta n\,\Delta_r/\Delta, \quad (9)$$

where ν is the generation frequency, Δ is the beam path difference in the multibeam interferometer, which is the laser resonator in this case**, $\delta\Delta < \lambda/2$ is the difference between the path differences for the two generated waves, Δ_r is that part of the path difference on which birefringence is observed, and Δn is the difference between the refractive indices.

It follows from (9) at Δ = 100 cm, say, the birefringence of an Iceland spar crystal of thickness 2 μ produces a frequency splitting of 200 MHz, and the optical activity of a quartz crystal 100 μ thick leads to a beat frequency on the order of 3 MHz. Consequently, by measuring the beat frequency $\Delta\nu$ we can register quite small values of birefringence. One can use for the measurements either a ring laser*[87] or a linear laser. In investigations of electro-optical effects, the use of a ring laser is preferable, since its design is more suitable for shielding the active medium against the influence of electric and magnetic fields acting on the investigated sample.

Proceeding to the description of photoelectric methods, we note that the most sensitive devices measure the phase shift between interfering oscillations only within a range 2π. This phase shift is equal to $2\pi\Delta m$, where Δm is the fractional part of the order of the interference. In cases when the integer part m of the interference order must also be determined, one can use, performing the measurements for several wavelengths, the method of coincidence of the fractional parts of the orders of the interference[1].

a) Modulation method. Simple registration of the radiation flux at the output of the interferometer (cf., e.g.,[88]) or recording of the changes of the flux during the scanning process[89] still does not result in a noticeable gain in sensitivity. To exploit the maximum capabilities of photoelectric registration it is necessary to increase the gain of the electronic apparatus to such an extent, that the noise amplified together with the useful signal exceeds the sensitivity limit of the output instrument, and to use the well-developed electronic procedures for useful-signal separation, which ensure the maximum signal/noise ratio at the output of the measuring system.

A high sensitivity close to the threshold can be obtained by connecting the photoreceivers that register the illuminations of the phase-shifted sections of the interference field in a differential circuit, and by measuring the amplified difference of the input signals[90]. It is practically impossible, however, to obtain high accuracy by such a method. Unequal time variations of the conversion coefficients of the photoreceivers, the null drift of the dc amplifier, and noise at the amplifier input cause continuous variations of the readings of the output instrument even when the phase difference is constant. For the same reason, the method of level-quantization of the signals offers little promise (cf., e.g.,[91]).

The influence of slowly varying random factors can be eliminated to a considerable degree by using the modulation method developed by Gorelik and Bershteĭn[25,26,92]. It is used successfully both to measure small changes of the phase difference, which are periodic functions of the time, and for the measurements of two constant (in time) phase differences. This

*Formula (8b) is valid for a Gaussian distribution of the phases of the monochromatic radiation components[84] and for the values of $\Delta\Omega$ much smaller than the width of the spectrum $G(\omega)$.

**Obviously, for a ring laser the path difference Δ is equal to the perimeter of the resonator, for a linear laser with flat mirrors it is equal to double the resonator length, etc.

*To determine the birefringence it suffices to register the output flux of the radiation of only one of the two opposing beams propagating in the resonator.

method consists of periodically varying, in a small range, the phase difference (path difference) of the interfering oscillations, and by the same token obtaining modulation of the radiation at the interferometer output. A photoreceiver is used to register the illumination in a small region of the interference field, and a harmonic analysis of the electric signal of the photoreceiver is carried out (for example, with the aid of a resonant filter). It is customary to separate the first harmonic of the signal, and the variation of the phase difference is estimated from its amplitude.

In principle, the sensitivity of the method can be arbitrarily large, for if the filter band is narrow enough, it can separate an arbitrarily small periodic signal against the background of the intrinsic noise of the apparatus. Another advantage of the method is the increased immunity to interference due to random path-difference changes that do not exceed $\lambda/2$. Such changes can result, in particular, from mechanical disturbances. The insensitivity to mechanical disturbances is due to the cutoff of the corresponding noise by the narrow-band filter.

The correct choice of the modulation frequency is very important. If this frequency is too low, the sensitivity can deteriorate, since the filter can start not transmitting not only the useful signal but also low-frequency oscillations due to slow (random) changes of the light-source intensity, and also due to the increased noise (for example, as a result of the flicker effect in the photocells and photomultipliers) of the photoreceiver and to the drift of the gain of the electronic apparatus. In some measurements of one-shot changes $\Delta\varphi$ of the phase difference, the influence of slow drifts is eliminated completely by separating simultaneously the first and second harmonic of the photocurrent and determining $\Delta\varphi$ from the ratio of their amplitudes[93,94].

The first application of the modulation method was in the measurement of small amplitudes of mechanical vibrations[26,95]. Assume that a vibrating object is rigidly coupled to a mirror of a two-beam Michelson interferometer. Then the photoreceiver current is

$$i = i_0 [1 + v \cos (\varphi + \delta \sin \Omega t)], \qquad (10)$$

where i_0 is the dc component of the photocurrent and is due to the light incident on the photoreceiver in the absence of interference (i.e., v = 0), is the mean value of the phase difference, $\delta = 4\pi/\lambda a$ is the phase-modulation amplitude, a is the vibration amplitude, and Ω is the vibration frequency. Fourier expansion of the right-hand side of (10) yields

$$i = i_0 \{1 + vJ_0(\delta) \cos \varphi + v \cos \varphi \sum_{s=1}^{\infty} [1 + (-1)^s] J_s(\delta) \cos s\Omega t - v \sin \varphi \sum_{s=1}^{\infty} [1 - (-1)^s] J_s(\delta) \sin s\Omega t\}, \qquad (11)$$

where $J_s(\delta)$ is a Bessel function of the first kind of order s, and $s > 0$ is an integer.

Let us consider the case when the measured modulation amplitude is very small ($\delta \ll 1$). Using (11), we can show that when $\varphi = n\pi + (\pi/2)$, the very small amplitude is equal to $\delta = 2i_1/(i'_{0\,max} - i'_{0\,min})$, where i_1 is the amplitude of the first harmonic, and i'_0 is the dc component of the signal. Thus, the measurement of δ reduces to relative measurements of i_1 and i'_0. The smallest observable amplitude is given in this case by[26] $\delta_{min} = (10\, e\Delta f/i_0)^{1/2}$, where e is the electron charge and Δf is the bandwidth of the electronic circuitry. Substituting in (13) $i_0 = 2 \times 10^{-8}$ A and $\Delta f = 0.5$ Hz, we obtain $\delta_{min} = 2/(5 \times 10^5)$, corresponding to $a_{min} = 0.0005$ nm for $\lambda = 0.5\ \mu$. Vibration amplitudes of the same order of magnitude were observed in real measurement installations[26,96].

Good results are obtained also in measurements of vibration amplitudes comparable with the wavelength of light. If the measured δ does not satisfy the condition $\delta \ll 1$, it is convenient to represent the reading α of the output instrument that registers the first harmonic of the signal in the form $\alpha = (\alpha_{max}/0.58)J_1(\delta)$, where α_{max} is the first maximum of the reading of the instrument when δ is smoothly increased from zero. From the reading of α and from this formula we can obtain the amplitude δ[95]. The fact that the inverse of $J_1(\delta)$ is not a single-valued function raises no difficulty if the course of α can be traced as δ is varied from very small values to the measured one.

There is also another very effective application of the modulation method. If the range of variations of the vibration amplitude is several microns and it suffices to perform exact measurements only at several points of this range, then the measurement can be confined only to those values of δ at which $J_1(\delta)$ vanishes. In this case the accuracy is determined only by the photoreceiver noise and is practically independent of the slowly-varying random factors such as the instability of the light-source radiation flux or variations of the amplifier gain. When the measurements limit is increased above $(2-3)\lambda$, it is advisable to use the signals of the second, third, etc. harmonics, which are described by Bessel functions of order equal to the number of the harmonic.

In the measurement setup considered above, the modulation was produced by oscillations of the sample, which themselves were the object of the investigation. At the same time, a special modulator must be provided for in instruments for the measurement of constant displacements. When the modulator is in operation, the phase difference becomes a variable (in time) and the measurement yields, in essence, the change $\Delta\varphi$ of the mean value of the phase difference φ. Using the terminology of radio communication, it can be stated that the modulator produces phase modulation, since it modulates the phase difference rather than the amplitude of the optical oscillations. Sometimes, however, amplitude modulation is also used in interferometric devices. In[97], for example, the following modulation method is described: a rotating slotted disk exposes a photoreceiver alternately to a light beam from the interferometer or to a fraction of the comparison beam coming directly from the light source. The measured phase difference is evaluated from the ac component of the photoreceiver signal. The relatively low sensitivity obtained by such a modulation ($\Delta\varphi_{min} = 2\pi/10^3$ when laser illumination is used) shows that intensity modulation is less effective than phase modulation.

Sinusoidal and rectangular phase modulations are the most frequently used in practice[98]. In the case of rectangular modulation, when the phase difference during the second half-cycle differs by a constant amount from the phase difference in the first half-cycle, the photoreceiver output is made up of periodic rectangular pulses. When the phase difference φ changes, the signal changes, but the waveform remains unchanged. In sinusoidal modulation the signal waveform varies continuously with φ. The signal has a simple symmetrical form only in two extreme cases: at the points of the extremum of the distribution $I(\varphi)$, when the signal contains only even harmonics (see formula (11)), and at the inflection points, where the signal waveform duplicates the modulation waveform at small values of δ. The sensitivity of the measurements under optimal values of the modulation amplitude is approximately the same for both modulation methods[98].

In this connection, the choice of the form of the modulation is determined entirely by the degree of complexity of its technical realization. It is easiest to realize rectangular modulation with the aid of a rotating disk that blocks the light in alternate light channels, which differ from each other by a phase shift 2δ. To obtain sinusoidal modulation, there is no need for two comparable optical channels, and this explains its wider use.

Let us consider the case $\Delta\varphi \ll 2\pi$. It follows from (11) that at very small deviations $\Delta\varphi$ of the mean phase difference from a multiple of π, the amplitude of the first harmonic is proportional to the measured phase difference. The first-harmonic phase changes by 180° when the sign of the deviation $\Delta\varphi$ is reversed. The coefficient of proportionality between the readings of the output instrument, which registers the value of the first harmonic, and the changes of the mean value of the phase difference can be determined by using the emission of two spectral lines whose wavelengths are known with high accuracy[31]. The sensitivity threshold[98,99] in the measurement of $\Delta\varphi$ is approximately the same as in the measurement of very small oscillation amplitudes[26].

At small deviations of the mean value of the phase difference ($\Delta\varphi \leq 2\pi$) from multiples of π, the first-harmonic signal is determined, in accord with (11), by the function $\sin(\Delta\varphi)$. In this case, taking the reading α of the output instrument, we can calculate $\Delta\varphi$ from the formula $\Delta\varphi = \sin^{-1}(\alpha/b)$.

It should be noted that to obtain equal accuracies in measurements of δ and $\Delta\varphi$, the design should ensure higher mechanical and temperature stability in the latter case, for in measurements of $\Delta\varphi$ slow random changes of the phase difference enter directly in the measurement result, whereas in the measurements of δ these changes can cause only insignificant changes of the useful signal near the values $\varphi = n\pi + (\pi/2)$ (where the slope of the function $I(\varphi)$ remains almost unchanged).

The limiting capabilities of multiple-beam modulation interferometers were investigated in[98]. An expression for the electric signal of a Fabry-Perot modulation interferometer, which works in transmitted light, is obtained by substituting $\varphi = 2\pi n + \Delta\varphi + \delta \sin \Omega t$ in the Airy formula

$$i = i_{max}/\{1 + F \sin^2 [\pi n + (\Delta\varphi/2) + (\delta/2) \sin \Omega t]\}, \quad (12)$$

where i_{max} is the maximum value of the photocurrent, and $F = 4\tau\rho/(1 - \tau\rho)^2$ is the sharpness coefficient.

It can be shown that the amplitude of the signal first harmonic, determined from (12), is equal to

$$i(\Omega) = -4 i_{max} \{[(1-\tau\rho)/(1+\tau\rho)] \sum_{s=1}^{\infty} (\tau\rho)^s s J_1(s\delta)\} \Delta\varphi. \quad (13)$$

A calculation with the aid of (13), at realistic values $i_{max} = 10^{-8}$ A, $\tau\rho = 0.92$ and $\Delta f = 1$ Hz and at optimal δ yields a deviation $\Delta\varphi_{min} = 2\pi/(5 \times 10^6)$, at which the signal is equal to the photoreceiver shot noise.

Mirror displacements of 0.001 mn were observed experimentally in[98], corresponding to a change of φ by $2\pi/(3 \times 10^5)$. The illumination was by the 546-nm green line of mercury, at a dielectric-coating mirror reflection coefficient was 0.92, light beam diameter 10 mm, distance between mirrors 0.1 mm, and amplifier bandwidth 5 Hz. The results obtained under these conditions agree well with the theory[28].

Technical realizations of the phase modulator can vary. In the apparatus of Baird and Smith[100] for the study of emission-line wavelength changes following changes of the temperature and the spectral-lamp current, the modulation was produced by varying the air pressure at a frequency 15 Hz in a chamber with a Fabry-Perot interferometer, using a pulsator for this purpose. The use of such a modulation method is limited to low frequency, owing to the low inertia with which equilibrium is established in the gas chamber.

The most widely used are electromechanical modulators in which one of the mirrors, or a system made up of several optical elements[94] of the interferometer, is made to oscillate with the aid of an electromechanical converter. Various setups employ electromagnetic, magnetostriction, and piezoelectric converters[93,101].

Interesting possibilities are uncovered by using the polarization of light for the modulation[77,102-104]. In one of the first polarization modulators[105], the lens of objective 2 produced an image 1 of equal-width fringes in the plane of a narrow slit 6 located ahead of photomultiplier 7 (see Fig. 13, where 3 is a half-wave plate used to equalize the average brightnesses of the two images of the interference fringes). The light beam passes through a Wollaston prism 4, where it is divided into two light beams polarized in mutually perpendicular directions. The fringe images produced by these beams are shifted by half a period relative to each other. The radiation flux to the photomultiplier was equal to the sum of the fluxes P_1 and P_2 of both beams passing through the slit 6. When a polaroid 5 rotating with angular velocity Ω is placed in the ray path, the flux to the photomultiplier is $P = 0.5\,(P_1 + P_2) + 0.5\,(P_1 - P_2) \times \cos 2\Omega t$. The ac signal produced by this flux is given by $i_{ac} = i_0 v \sin \epsilon\, (\Delta\varphi \cos 2\Omega t)$, where v is the visibility of the interference pattern, ϵ is the phase-difference interval spanned by the slit, and $\Delta\varphi \ll 2\pi$ is a small deviation of the phase difference from the value $\varphi_0 = (2n + 1)\pi/2$. The device has made it possible to position the null with an rms error $2\pi/400$.

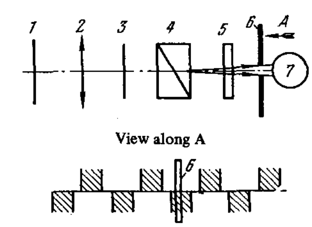

FIG. 13. Polarization modulator.

The imperfections of mechanical modulators containing moving optical parts can cause appreciable systematic errors. Much smaller errors can be expected with modulators based on electro-optical effects. Such modulators, which use the effects of Pockels, Faraday, Zeeman, or Kerr, can be used in polarization interferometers[77,104] and in classical interferometers with polarization modulators[102,103,105].

Intensive work is being done at present towards design of automatic systems for the measurement of changes on the order of interferences (cf., e.g.,[10]). Devices for the determination of changes of the integer part of the order of interference in measurements of length (reversible fringe counters)[106] are sufficiently well developed, but devices for an exact measurement of the fractional part of an interference order (interpolator) are still in the research stage).

b) Phase method. If the intensity at the output of the interferometer varies sinusoidally as a function of φ, then the measurement of the illumination at four points of the interference field, corresponding to phase differences that differ by a known definite amount, makes it possible to calculate the visibility of the interference pattern and the values of φ at each point. This possibility was realized in a setup[107] for the study of the contours of spectral lines by the Fourier transformation method. To accelerate the measurement process, the phase difference was calculated from the photomultiplier signals at four values of φ with a computer.

In the phase method proposed by Obreimov (see[108]) and developed in a number of studies[109-114], the illumination is likewise registered at several points of the field, the values of φ of which differ by a definite amount. The gist of the method is as follows: the signals from photoreceivers that register the illumination of small sections of the field in a two-beam interferometer are regarded as the components of an electric vector **R**. A change in the phase of one of the interfering oscillations by an angle $\Delta\varphi$ causes the vector **R** to rotate through the same angle. The polar angle of the vector **R** is usually measured by phase modulating the sinusoidal signal received by the measuring apparatus, and the phase-shift angle relative to a certain reference signal of the same frequency is subsequently determined with the aid of a radio-frequency phase meter.

All these transformations reduce actually to a solution of a system of trigonometric equations that connect the illuminations of the field segments with the values of the phase shifts. The phase method is therefore sometimes called the trigonometric method. The main advantage of the trigonometric method over the modulation method is that there is no need for a calibrated path-difference compensator when the measurements are made in the interval 2π.

In the first device[108] based on the phase method, a rather narrow section of a fringe is separated from the interference pattern by a slit. A beam-splitting device (wedge) sends light from the left and right sides of this section to two photocells. A third photocell receives attenuated light from the entire interference pattern (background). When the attenuation coefficient is appropriately chosen and the transformation coefficients of the photoreceiver are equal, the photocell signals are connected by the relations[108]

$$U_1 - U_2 = A\sin\varphi = U_{\rm I},\quad U_1 + U_2 - U_3 = A\cos\varphi = U_{\rm II},$$

where U_1, U_2, and U_3 are the signals of the first, second, and third photocells, respectively, A is a constant coefficient, and φ is the phase difference corresponding to the middle of the section separated by the slit.

Since the light source is an ac arc lamp, the light flux is modulated at 100 Hz. In the expressions for the sum and difference of the signals, this can be taken into account by introducing the time factor $\sin\Omega t$, where $\Omega = 2\pi f = 200\pi$ is the circular frequency of the modulation. Then $U_{\rm I} = A\sin\varphi\sin\Omega t$ and $U_{\rm II} = A\cos\varphi\sin\Omega t$. The signals $U_{\rm I}$ and $U_{\rm II}$ are obtained by using electronic addition and subtraction circuits. A phase shifter is used to produce a phase shift of 90° between $U_{\rm I}$ and $U_{\rm II}$, and these signals are then added. The signal resulting from these transformations is equal to $U = A\sin(\Omega t + \varphi)$.

The phase shift φ of the electric signal relative to the reference voltage is measured with a rotating transformer. The rotor of this transformer is turned through an angle equal to the phase shift, which is registered by a photoelectric device with a pulse counter.

A similar realization of the phase method was used in a system with digital readout of the path-difference changes, intended for refractometric purposes[109,110]. The distinguishing features of the system are the production of three signals corresponding to field sections shifted 120° in phase, instead of two signals shifted by 90°, and the use of a selsyn in place of a rotating transformer. The phase shift is read in steps of $2\pi/100$. In both instruments, the changes of φ are determined from the angle of a rotor of an electric machine. Such a construction is logically justified in view of the high accuracy with which angles can be measured. Optical code pickups, for example, can measure angles in a 2π interval with an error of several seconds, corresponding to a phase shift on the order of $2\pi/(2\times 10^5)$. Consequently, the error in the value of the angle has practically no effect on the measurement result.

In the devices indicated above, the changes of φ were converted into angles by an intermediate conversion into a phase shift between electrical oscillations. An instrument without this intermediate conversion operates in the following manner. Signals proportional to $\sin\varphi$ and $\cos\varphi$ are multiplied by $\cos\theta$ and $\sin\theta$ with the aid of sine-cosine potentiometers whose shafts are rigidly coupled (θ is the angle of rotation of the shafts). The difference obtained when the two potentiometers are differentially connected is proportional to $\sin(\theta - \varphi)$ and is monitored on a galvanometer scale. A null reading of the galvanometer is obtained by varying θ. Then θ becomes equal to φ, which is thus determined. If highly accurate sine-cosine potentiometers are used, the phase-shift measurement error is $2\pi/1000$.

A photometric system[112] producing a digital readout of the measurement result with an error $2\pi/100$ was developed at the U.S. National Bureau of Standards. It uses four photoreceivers 1–4 (Fig. 14). The photoreceiver signals are equal to

$$U_1 = U_0(1 + v\cos\varphi),$$
$$U_2 = U_0\{1 + v\cos[\varphi - (\pi/2)]\} = U_0(1 + v\sin\varphi),$$
$$U_3 = U_0[1 + v\cos(\varphi - \pi)] = U_0(1 - v\cos\varphi),$$
$$U_4 = U_0\{1 + v\cos[\varphi - (3\pi/2)]\} = U_0(1 - v\sin\varphi).$$

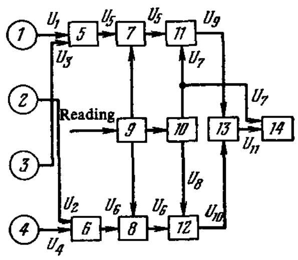

FIG. 14. Block diagram of phase-measuring system [112].

After subtracting U_1 from U_3, and also U_2 from U_4, dc voltages $U_5 = U_{5\,max} \cos\varphi$ and $U_6 = U_{6\,max} \sin\varphi$ are obtained in the subtraction blocks 5 and 6, and are fed to the inputs of electronic switches 7 and 8 (13 is also a subtraction block).

At the instant of reading, the strobe-pulse generator 9 is turned on, and the initial pulse opens switches 7 and 8, so that signals U_5 and U_6 pass to the inputs of the multiplication blocks 11 and 12. The multiplication devices receive simultaneously the voltages $U_7 = U_{7\,max} \cos\Omega t$ and $U_8 = U_{8\,max} \sin\Omega t$ from sine-wave generator 10 with frequency Ω. The outputs of the multiplication devices are the voltages $U_9 = U_{9\,max} \cos\varphi \cos\Omega t$ and $U_{10} = U_{10\,max} \sin\varphi \sin\Omega t$ $(U_{9\,max} = U_{10\,max})$ and are fed to a subtraction circuit, which performs the operation $U_{11} = U_9 - U_{10} = U_{9\,max} \cos(\Omega t + \varphi)$. The phase meter 14 measures the phase shift angle between the voltages U_7 and U_{11}, which is equal to φ.

In our opinion, the most successful phase-measuring device is the original interference instrument described in an article by Crane (see[37]). The electric signal is transformed in it into a phase without the use of analog addition, subtraction, or multiplication devices, so that the errors connected with them are eliminated. The operating principle of the instrument can be explained in the following manner: if one of the mirrors of a Michelson interferometer, positioned to produce an infinitely broad fringe, is moved at a constant velocity, then the radiation output flux will vary sinusoidally. The phase of the periodic light signal depends on the phase difference in the interferometer.* By recording the output flux with the aid of a photoreceiver and by measuring the phase of the electric signal, we can determine φ.

A smooth displacement of the mirror corresponds to a continuous change of the phase of one of the interfering oscillations. Uniform phase variation at a constant oscillation frequency is analogous to a small frequency change. Consequently, phase measurements can be made by using the frequency shift of one of the interfering oscillations, rather than using uniform motion of the mirror, which is difficult to realize.

It is interesting to note that to measure small rapidly-varying phase shifts it was proposed to use an inverse effect, namely the change Δf of the photoreceiver signal frequency with changing φ, given by $d\varphi/dt = 2\pi\Delta f$[83,115]. Using a frequency detector and a storage device, we obtain a voltage proportional to the time variation of the phase, since

$$2\pi \int \Delta f \, dt = \varphi(t).$$

To produce a frequency shift, a special optical device[37] (Fig. 15) was placed in one arm of a Michelson interferometer. In this device light passes twice through a rotating quarter-wave plate, which is thus equivalent in action to a half-wave plate.

The operation of this device is based on the change of the frequency of circularly-polarized light on passing through a half-wave plate. The instrument was used to control the quality of mirror finishes. The reference signal was taken to be the one obtained from the central point of a mirror used as the reflector in the measuring arm of the interferometer. Information concerning the positions of different points was obtained as a result of scanning the entire surface of the mirror. Experiment has shown that the measurement error depends in practice only on the error of the radio-frequency phase meter and can be made equal to one-thousandths of a period.

A frequency shift was effected in[114] by the Doppler effect, with light passing through a diffraction grating formed by an ultrasonic wave. The frequency shift of the optical oscillations can be obtained also by using amplitude modulation. In this case it is necessary to suppress the carrier and to use one of the sideband frequencies of the amplitude-modulated signal spectrum. A single sideband is best modulated by using the Pockels effect in a crystal with a cubic lattice structure. The modulation-induced difference between the directions of the carrier polarization and that of the sideband frequencies makes it possible to suppress the carrier and separate the sidebands[116].

c) Time-conversion method. This method reduces to obtaining and measuring a time interval characterizing the phase difference in the interferometer. To this end, an electromechanical or electro-optical converter is used to alter the phase difference, in accordance with a definite law, by an amount somewhat larger than 2π, i.e., φ is time-modulated. The light flux corresponding to a small section of the interference field is directed to a photoreceiver. The time interval between fixed points (for example, the maxima or the null values) of the photoreceiver signal and of the modulating signal, fed to the electromechanical converter, is measured.

A linear-time converter was used in[117]. This converter is a linear-displacement pickup, for example a piezoelectric element rigidly connected to the interferometer mirror and fed from a generator of a linear electric signal. When the converter is turned on, φ varies linearly with time. The phase shift can be determined by varying the time interval between the pulse that triggers the linear-signal generator and the pulse shaped at a fixed intensity-distribution point.

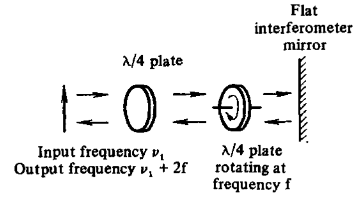

FIG. 15. Optical system for frequency shift.

*Such a method of conversion of an optical frequency into a radio frequency, differs from the heterodyne method and is frequently called the homodyne method.

The phase shift is equal to the ratio $2\pi\tau/T$, where τ is the measured time interval and T is the time required for the converter to change φ by 2π. The time interval between the pulses is usually measured by a scaler device that counts the number of pulses from a stable-frequency generator. By choosing the required ratio of T to the generator frequency, it is possible to obtain a reading of the phase shift $\Delta\varphi$ in digital form. The accuracy with which $\Delta\varphi$ is measured is limited by the imperfections of the electromechanical converters. The use of piezoceramic elements resulted in an rms error $2\pi/30$.[117]

d) Oscillographic methods. In those cases when complete automation of the measurement process is not required, it is possible to obtain sufficiently high accuracy by observing oscillographic figures obtained in a definite manner (cf., e.g.,[68]). In this method a signal equal to b sin φ, where b is a constant coefficient, is applied to the vertical sweep of the oscilloscope, and a signal b cos φ to the horizontal sweep. When the phase difference varies, the spot on the cathode-ray tube screen moves in a circle, one revolution corresponding to a change of φ by 2π. Using an angle scale, phase shifts can be read within the limits of the entire period with an error $2\pi/100$.

Components proportional to sin φ and cos φ were obtained in[93] by using the first and second harmonic of the signal of a two-beam modulation interferometer.

Another method is based on determining the position at which the frequency of an electric signal is equal to double the modulation frequency. To realize this method, φ is varied periodically. As already mentioned, in positions corresponding to an extremum of the function $I(\varphi)$, the signal contains only even harmonics, the second harmonic being predominant. The signal waveform differs little in this case from a sinusoid, and is strongly altered by insignificant changes of φ. The signal picture observed on the cathode-ray screen, and its variations, are outwardly very similar to those shown in Fig. 6. Under favorable conditions the error in the setting of the zero positions is $2\pi/2500$.[118]

An interesting method[119] is one used earlier for positioning against a line in photoelectric microscope[120]. The signal obtained in a modulation interferometer is applied to the vertical sweep of an oscilloscope, and part of the voltage feeding the modulator is applied to the horizontal one. By varying the phase of the horizontal signal with a phase shifter, it is possible to obtain at $\varphi = 2\pi n$ on the screen a symmetrical butterfly-like figure (Fig. 16a). A slight change in the phase difference makes the figure asymmetrical (Figs. 16b and 16c). The rms error in the setting of the zero positions in a Fabry-Perot interferometer was $2\pi/3000$ at $N_{eff} \approx 30$ (Fig. 16d) (a—c are figures used to indicate the position of the interference maximum, d is an oscillogram showing the dependence of the output intensity on the phase difference).

An interesting method of measuring the additional phase shift when light passes through a dispersive medium is considered in[121]. The method is based on an analysis of the waveform of an electric signal obtained by sinusoidal modulation in a two-beam modulation interferometer. If the modulation amplitude δ is equal

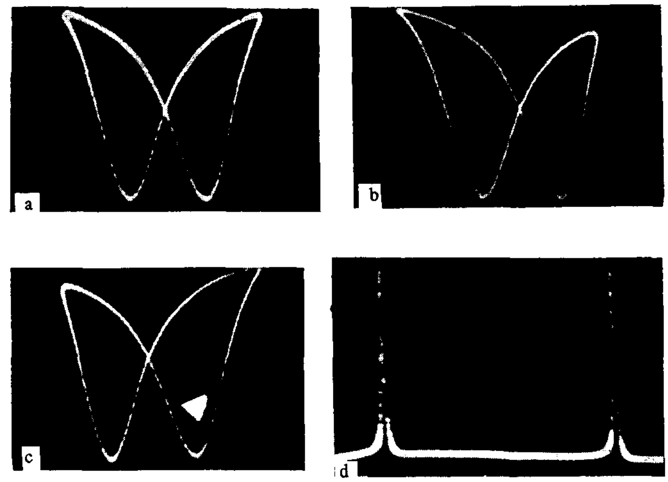

FIG. 16. Oscillographic figures obtained in apparatus with a multiple-beam modulation interferometer.

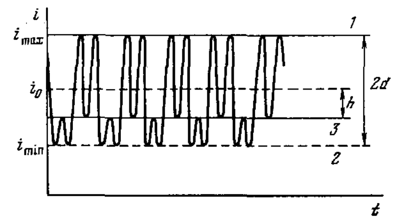

FIG. 17. Time dependence of photoreceiver current at V = 0.55 and $\Delta\varphi = \pi/3$.

to π, then expression (10) for the photoreceiver signal takes the form

$$i = i_0 \left[1 + v \cos\left(2\pi n + \Delta\varphi + \pi \sin \Omega t\right)\right], \quad (14)$$

where n is an integer and $\Delta\varphi$ is the average phase shift between the interfering optical oscillations. A characteristic feature of the plot of (14) (Fig. 17) is that the position of line 3 of the intermediate extrema* relative to the lines 1 and 2 of the maximal and minimal values of t is determined by the value of $\Delta\varphi$. If we use the notation introduced in Fig. 17, then the phase shift between the interfering optical oscillations can be represented in the form

$$\Delta\varphi = (\pi/2) \pm \arcsin(h/d). \quad (15)$$

When the light-source frequency ν is changed by an amount $\Delta\nu \ll \nu$, the phase difference is practically independent of ν and the position of line 3 remains unchanged. If a dispersive medium is placed in one branch of the interferometer, then the phase difference will change appreciably even following a small change of the frequency, and consequently the position of line 3 for different frequencies will be different. A reduction of the oscillograms representing the plot of (14), with allowance (14), makes it possible to obtain the dependence of the phase shift on the radiation frequency. This method was used to determine reliably a change of an average phase-shift difference $2\pi/60$ against the background of random variations of φ in a one-meter Zehnder-Mach interferometer.

*If δ is not equal to π, then it can be readily seen that a doubling of line 3 takes place; this is why it is easy to set to the position $\delta = \pi$.

5. NULL METHOD

Most methods considered above result in an increased sensitivity to $\Delta\varphi$ only near certain values of the phase difference. To determine sufficiently large changes of the phase difference it is therefore advisable to use a null compensation method. It consists of varying φ with the aid of an interference compensator to the nearest null position, and obtaining the phase shift from the compensator reading. The null positions are chosen to be those that can be fixed with the smallest error.

The measurement error in a null system consists of the errors of the setting of the null positions and the error of the compensator reading. The compensators used most widely in interferometry are compensators with glass plates[1,2,122], compensators based on the change of the air pressure in the interferometer (cf., e.g.,[79,89]), and electromechanical compensators[90,101].

The results of certain investigations[83,90,101] show that under favorable conditions the error of compensators in an interval 2π does not exceed $2\pi/10^3-2\pi/10^4$. The procedure of calibrating such compensators with such high accuracy has not yet been sufficiently developed.

In polarization interferometers one uses mainly compensators of two types[73]: direct compensation and azimuth analyzers. The compensators of the first kind, for example the Babinet-Soleil compensator, introduces an additional phase difference between the orthogonal optical oscillations. The error of these compensators is $2\pi/20-2\pi/10^3$. Usually direct compensation is used in shift polarization interferometers using the sensitive-color method. In compensators of the second type, for example, the Senarmon compensator, the errors in measuring the phase shift is approximately $2\pi/10^5$.

From a comparison of the compensator errors with the errors in the setting of the null positions it follows that in all the instruments, with the exception interferometers with azimuth analyzers, the measurement accuracy is determined in practice by the compensator accuracy, which can in principle be increased. It is therefore urgent to work toward improving and certifying path-difference compensators.

In measurements of fast changes of phase differences, φ can be varied automatically. To this end it is necessary to introduce in the interferometer a feedback loop that transmits an electric signal from the output of the interferometer to the device controlling one of the optical elements of the interferometer. If the optical element performs the function of the compensator, then the phase shift can be determined from the control signal in the feedback circuit.

6. CONCLUSION

An examination of the visual methods shows that there are rather many different combinations of a registration method with an interference scheme, making it possible to set the null positions with an error $2\pi/(3\times10^2)-2\pi/10^4$. The smallest errors can be obtained by using the dark-field method in a polarization interferometer $(2\pi/(2\times10^4))$ and the method of uniform field in a multiple-beam interferometer with a half-shadow device $(2\pi/(4\times10^3))$. This does not mean at all that a given combination will give the maximum effect in all cases. The point is that in the analysis of the various methods we did not take into account the influence of ambient temperature and pressure changes, vibrations, and acoustic noise. Very frequently it is precisely these factors (called technical) which determine the measurement accuracy.

In many cases it is therefore advisable to use interferometric installations with lower sensitivity, but stable against external disturbances. For example, three-beam interferometers are very convenient for measurements of the refractive indices of gases and liquids, and polarization shift interferometers are best for the study of the structure of small phase inclusions in glass plates.

The most effective of the photoelectric methods is the modulation method. Its capabilities come into play best in measurements of very small displacements. In all probability, apparatus for the measurement of single displacements amounting to thousandths of a nanometer will be prepared and tested in the near future. It is possible that the temperature stabilization and the shielding against mechanical disturbances will not be satisfactory for such measurements. It will then be necessary to use two modulation interferometers and determine the displacements from the correlation between their signals.

The main trend in the development of interferometry techniques is presently the automatization of the measurement process. The use of the modulation method for the automatization entails great technical difficulties. The phase method will therefore be more rational in the construction of automatic interference measurement devices.

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Translated by J. G. Adashko