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The Vavilov-Cerenkov Effect and the Doppler Effect in the Motion of Sources with Superluminal Velocity in Vacuum*

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It is customary to consider only "subluminal" light sources, or sources moving with a velocity v lower than the velocity of light in vacuum (c). It is assumed in this connection that the Vavilov-Cerenkov effect and the anomalous Doppler effect are possible only in media and waves for which the refractive index $n(\omega) > 1$. For this reason, the phase velocity of the waves is $c_{ph} = [c/n(\omega)] < c$, and these waves can be emitted by a subluminal source if $v > c_{ph}$. Yet, as is well known, there exist also "superluminal" sources, with velocity $v > c$. Examples are light spots produced on a remote screen by a rotating source of light or particles. The spot velocity is $v = \Omega R$, where Ω is the angular velocity of source rotation and R is the distance to the screen. The condition $v > c$ can be realized on earth, and is practically always realized under astronomical conditions for pulsar radiation. It is emphasized in the article that superluminal sources are equivalent in a wide range to subluminal ones, and, concretely, can generate Cerenkov radiation in vacuum and in a medium with $n(\omega) < 1$. The article considers several corresponding possibilities. From this point of view of radiation theory, a major difference between the superluminal and subluminal sources is that the former can not be individual particles (electrons, protons, etc.), since their velocity is always smaller than c. Superluminal sources, which must thus consist of aggregates of particles, must thus have nonzero dimensions, and this leads to a corresponding formation of a spectrum of the radiated frequencies on the short-wave side. Regardless of whether superluminal sources will find interesting applications in physics and astronomy, a study of the radiation of superluminal sources of electromagnetic and gravitational waves (and possibly also neutrinos) is in the authors' opinion of undisputed physical interest.

CONTENTS

1. INTRODUCTION

WHEN a certain "source" moves uniformly in a straight line in a homogeneous medium, radiation is produced only if the source velocity v exceeds the phase velocity c_{ph} of the waves in question in the given medium. The angle θ_0 between the wave vector **k** of the radiated waves and the source velocity **v** is then given by

$$\cos\theta_0 = c_{ph}/v. \tag{1}$$

In acoustics, such radiation from a source faster than sound has been known for a long time (Mach waves); the same can be said of different waves on the surface of a liquid. In electrodynamics, on the other hand, the radiation of a uniformly moving source (say a charge) is known as the Vavilov-Cerenkov effect and was discovered only in 1934. The theory of this effect, developed by Tamm and Frank[1] in 1937, leads naturally to the radiation condition (1), and yields the following expression for the energy emitted by a charge q per unit time:

$$dW/dt = (q^2 v/c^2) \int\limits_{\frac{c}{n(\omega)v} \leq 1} \{1 - [c^2/n^2(\omega)v^2]\}\,\omega\, d\omega, \tag{2}$$

where $n(\omega)$ is the refractive index at the frequency ω for the considered transparent isotropic medium (as is well known, the phase velocity of a wave is $c_{ph} = c/n(\omega)$).

Since the radiation condition (1) is valid for a wave of any kind, it is clear that it has a kinematic (interference) character. Indeed, according to the Huygens principle, each point of the medium in the path of the radiator is a source of secondary waves. The envelope of these waves is a cone, the aperture of which is given by the angle $\theta_0 = \cos^{-1}(c_{ph}/v)$ (see Fig. 1, on which the distance O'O is equal to v, the path covered by the source per unit time; during the same time, the path of the wave front is $c_{ph} = c/n$). It is known that it is precisely the use of the Huygens principle which led to the derivation of condition (1) for Cerenkov radiation[2]. Of course, the corresponding interference condition is automatically taken into account in the electrodynamic calculation in which expressions are used for the radiation field[1]. The radiation condition (1), or, specifically, the Cerenkov radiation condition

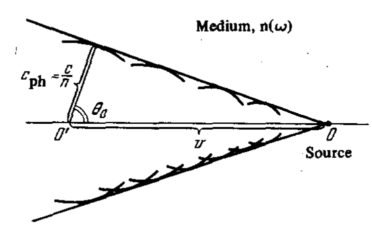

FIG. 1

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$$\cos\theta_0 = c/n(\omega)\,v \tag{3}$$

can be derived also by other means, for example, as the resonance condition $\mathbf{k}\cdot\mathbf{v} = (\omega/c)n(\omega)v\cos\theta = \omega$ between the acting "force" connected with the presence of the source and the oscillators of the field[3], and also from the energy and momentum conservation laws (in the latter case, the quantum formulation is convenient)[4,5]. The condition (1) or (3) remains in force not only in the case of an unbounded medium, but also for sources moving in channels and in slits, or those moving parallel to the interface between two media. In an anisotropic medium, this condition pertains to each of the normal waves separately, and the refractive index $n_e(\omega)$ for the normal wave depends also on the angles between the wave vector $\mathbf{k}$ and, for example, the crystal axes. As to radiation intensity, it can be calculated by different methods[1-5], and, most importantly, depends on the character of the source, with formula (2) pertaining only to the case of a charge moving in an unbounded isotropic medium. A number of expressions for the radiation intensity of dipoles and other multipoles, and also in the presence of boundaries, can be found in the reviews[5-7]. From the radiation condition (3) it is clear that the Cerenkov effect is possible only if

$$v \geqslant c/n(\omega) = c_{\text{ph}}, \tag{4}$$

i.e., as already emphasized, in order for the radiation to be produced it is essential that the source velocity exceed the phase velocity of the light. The same condition is necessary for the appearance of the Doppler effect, for which

$$[vn(\omega)/c]\cos\theta > 1. \tag{5}$$

Properly speaking, the inequality (5) is a definition of the anomalous Doppler effect, wherein waves are radiated inside the Cerenkov cone, i.e., with a wave vector $\mathbf{k}$ making an angle $\theta < \theta_0 = \arccos[c/n(\omega)v]$ with the source velocity $\mathbf{v}$. The foregoing is obvious from the formula for the Doppler effect in a medium[5,8]

$$\omega = \omega_{00}\,[1-(v^2/c^2)]^{1/2}/\,|\,1-(v/c)\,n(\omega)\cos\theta\,|, \tag{6}$$

where ω_{00} is the frequency in the reference frame connected to the source, and the frequency ω and the angle θ pertain to the "laboratory" frame (in which the source has a velocity v). It is usually concluded from the condition (4) that the Cerenkov radiation and the anomalous Doppler effect are possible only in media having a positive refractive index

$$n(\omega) > 1. \tag{7}$$

This limitation is quite important. It suffices to say that in an isotropic plasma, in the widely used approximation, the following condition holds:

$$n(\omega) = [1-(\omega_e^2/\omega^2)]^{1/2} < 1,\quad \omega_e^2 = 4\pi e^2N/m.$$

It is therefore assumed that Cerenkov radiation of transverse waves is impossible in such a plasma [it is precisely for such waves that $c_{\text{ph}} = c/n(\omega) > c$].

The requirement (7), when stated as a condition for the appearance of the Cerenkov effect and of the anomalous Doppler effect, is connected with the assumption that the source velocity is smaller than the velocity of light in vacuum, i.e., that

$$v < c = 3\cdot 10^{10}\ \text{cm/sec} \tag{8}$$

It is precisely this requirement which led in 1904 to Sommerfeld's conclusion, having no connection with reality and then forgotten for many years, that an electron moving in vacuum uniformly but with velocity $v > c$ emits radiation (see[9,10]). Sommerfeld actually considered the Cerenkov effect in a non-dispersing medium, namely vacuum. The corresponding calculation is formally correct, since the equation for the electromagnetic field, and in particular the equation

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\rho\mathbf{v} + \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}, \tag{9}$$

are valid also when $v > c$. Nor is the relativistic invariance of the theory violated here, in spite of the long-held erroneous view. Indeed, as emphasized by Einstein back in 1907 (see[11] and also[12]), the condition $v < c$ for a material "body" or for some "action" is connected not with relativistic invariance but with the causality requirement: no effect should anticipate the cause in any reference frame.

To be sure, it is clear from the relativistic expression for the mass $m = m_0/[1-(v^2/c^2)]^{1/2}$ and from the equation of motion $d m\mathbf{v}/dt = \mathbf{F}$ that no body (particle) can be accelerated to a velocity $v \geq c$. But this in itself still does not negate the possible existence of the hypothetic particles, called tachyons, which move always with velocity $v > c$. Tachyons might also be regarded as particles with imaginary mass $m^* = im$, energy $E = (m^{*2}c^4 + c^2p^2)^{1/2} = (-m^2c^4 + c^2p^2)^{1/2}$, momentum $p = m^*v/[1-(v^2/c^2)]$, and velocity $v = dE/dp = c^2p/E = c^2p/(-m^2c^4 + c^2p^2)^{1/2}$. It is obvious that the momentum p of a tachyon is real if $v > c$ and consequently $p > mc$; the tachyon velocity $v \to c$ as $p \to \infty$, and conversely $v \to \infty$ as $p \to mc$. The quantity $E^2 - c^2p^2 = m^{*2}c^4 \equiv -m^2c^4$ remains invariant under the Lorentz transformation and it is possible[13], in particular, to "forbid" the possible existence of tachyons from the causality condition. Incidentally, not all agree that the existence of tachyons is impossible, although we consider such a conclusion to be sufficiently reliable. We therefore emphasize that tachyons, in any case, have not been observed, and thus the condition $v \leq c$ for all the known particles certainly corresponds to reality.

It is just as undisputed, nevertheless, that sources of electromagnetic (and of any other) waves can move with velocities $v > c$! This pertains, however, not to individual particles (photons, electrons, protons, etc) but to their aggregates or bunches (this will be spelled out more precisely later on). Therefore the requirement $v < c$ [see (8)] as a condition on the source velocity is incorrect, in other words, no such requirement can be advanced. By the same token, the Cerenkov effect and the anomalous Doppler effect can exist also in a medium with a refractive index $n(\omega) < 1$, when $c_{\text{ph}} = c/n > c$. In a certain sense the same can be said also of vacuum, where $c_{\text{ph}} = c$.

Insofar as we know, radiation from sources moving with velocity $v > c$ (if we disregard Sommerfeld's papers[9,10] and Frank's article[8]) has not been considered until recently. Yet the corresponding possibilities[14-16] are quite curious, and their elucidation is the purpose of the present article.

2. SOURCES MOVING WITH SUPERLUMINAL VELOCITY IN VACUUM

It has been long known that velocities exceeding that of light in vacuum are possible and are encountered in physics and astronomy. Disregarding the phase velocity of waves at $n(\omega) > 1$ or the relative velocity of two particles moving away from each other in a given reference frame (this velocity can reach 2c), a velocity higher than c can be possessed by cross sections of wave fronts, and in general by various light spots. Concretely, let us imagine a rotating projector or "beacon." If the angular velocity of the "beacon" is Ω, then the light spot will move on a screen located at a distance R from the source with a velocity (see also below)

$$v = \Omega R. \tag{10}$$

The "beacon" model is presently universally accepted for pulsars (cf., e.g.,[17]), and in this case the velocity of the spot on earth, for all the known pulsars, exceeds the velocity of light c. Concretely, for the pulsar NP 0532 in the Crab Nebula we have $\Omega \approx 200$ and $R \approx 2000$ parsec $\approx 6 \times 10^{21}$ cm, hence $v = \Omega R \approx 1.2 \times 10^{24}$ cm/sec. If a laser or an electron beam is rotated with velocity $\Omega = 10^5$, then $v > c$ already for distances $R > 3 \times 10^5$ cm.

The simplest model or example, in a sense, of motion with superluminal velocity is a light pulse from a plane wave obliquely incident on a certain plane interface (screen)[8]. If the angle of incidence of the wave on the screen is designated ψ (obviously, ψ is the angle between the wave vector **k** in the pulse and the normal to the screen; Fig. 2), then the intersection of the pulse and the screen (i.e., the light spot moves over this screen with a velocity

$$v = c/n_1 \sin\psi, \tag{11}$$

where $n_1 > 1$ is the refractive index in the medium above the screen; for simplicity, this medium is assumed to be non-dispersing (in fact, all that matters to us is that the velocity of the light pulse is assumed equal to c/n_1). It is obvious that the velocity of the light spot (more accurately, strip) can always be made larger than c by decreasing the angle of incidence ψ, and in vacuum this takes place in general at all angles ψ, for in this case

$$v = c/\sin\psi. \tag{12}$$

The role of the light pulse can, of course, be assumed by a beam of electrons moving normally to the front of the beam with velocity $u < c$; in this case

$$v = u/\sin\psi \tag{13}$$

and superluminal velocity of the spot is likewise always

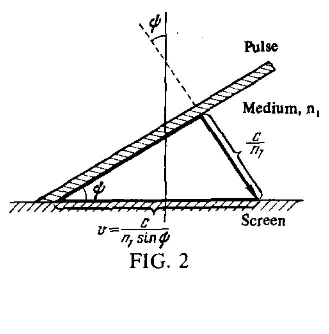

FIG. 2

attainable in principle. Moreover, the velocity v can in all cases be arbitrarily large, for the velocity $v \to \infty$ as normal incidence is approached (as $\psi \to 0$). The latter is perfectly understandable, for at normal incidence the pulse crosses the screen simultaneously over all its surface. The mechanical analog of a pulse incident on a screen are scissors (the role of the spot is played in this case by the point of intersection of the two blades).

For the rotating source mentioned above, the large spot velocity, just as for the case of a pulse crossing a screen, is due to the decrease of the angle between the constant-phase surface (wave front) and the screen. In fact, considering for simplicity a cylindrical source rotating in a vacuum with angular velocity Ω, we write the field in the wave zone in the form*

$$E = \sum_{s=1}^{\infty} E_s r^{-1/2} \exp\{is[(\Omega/c)r + \varphi - \Omega t]\}.$$

The constant-phase surface is determined by the equation

$$(\Omega/c)\, r + \varphi - \Omega t = \text{const},$$

or

$$r(\varphi) = \text{const} + c\,[t - (\varphi/\Omega)]. \tag{14}$$

Equation (14) is that of a spiral. On a remote cylindrical screen of radius R, the equal-phase surface crosses the screen along the generator of a cylinder, for which

$$R = \text{const} + c\,[t - (\varphi_0/\Omega)],$$

and the angle φ_0, which defines the generator in question, has a time variation $d\varphi_0/dt = \Omega$. In other words, the line of intersection (the spot) moves over the screen with velocity

$$v = R\, d\varphi_0/dt = \Omega R.$$

Thus, we have obtained in a more formal manner the obvious (or, in any case, the well known) result (10). It is important that the angle ψ between the equal-phase surface and the screen is equal to (Fig. 3)

$$\operatorname{tg}\psi = -dr/R\, d\varphi = c/\Omega R = c/v.$$

For small angles ψ, of course, $\tan\psi \approx \sin\psi \approx \psi$ and $v = c/\sin\psi$, in accord with (12). In other words, as

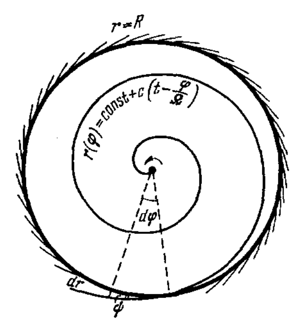

FIG. 3

*This formula gives the solution of the scalar problem. The function E satisfies the wave equation at $r > r_0$ and the boundary condition $E = f(\varphi - \Omega t)$ on the surface of the cylinder $r = r_0$. Thus, the field is static in a coordinate frame that rotates about the z axis with angular velocity Ω.

already noted, the large velocity of the spot is due (for example, when $v \gg c$) to the smallness of the angle ψ between the wave front and the screen.

We have made practically no assumptions above concerning the nature of the field under consideration, and we only assumed (and furthermore solely for the sake of simplicity) that its propagation velocity is equal to c. It is clear therefore that spots with velocities $v > c$ can be obtained not only in the case of electromagnetic waves, but also for gravitational waves. Using the ray treatment, we arrive at the feasibility of obtaining "spots" of arbitrary velocity for neutrinos (velocity c) as well as for all other particles (velocity $u < c$).* There can be no shadow of a doubt that the appearance of spots with velocities $v > c$ does not contradict relativity theory. It suffices to say that this result is obtained for perfectly realistic examples, for example for a beam of light or electrons incident on a screen (see Fig. 2). We note nevertheless, by way of a postscript, that the use of the velocity of light to synchronize clocks, the customary procedure in the exposition of relativity theory, is first, not the only possible method, and second, is the most convenient and advantageous in the majority of cases not because light has the maximum velocity possible, but because this velocity is universal, the same for all inertial reference frames (of course, if identical scales and clocks are chosen in these frames). Finally, when we do refer to the velocity of light in vacuum as the maximum possible, we have in mind the rate of transfer of perturbations, interactions, or "signals." Such a statement is indeed valid (at least within the framework of relativity theory and of all the physics known to us). The light spot mentioned above and its analogs, although capable of moving with velocity $v > c$, does not violate the above statement at all, i.e., it cannot be used to transmit a signal with velocity $v > c$. In fact, let us consider a pulse (of light or electrons) whose intersection with the screen moves on the screen along the x axis with velocity $v > c$ and reaches the points x_1 and x_2 at the instants of time t_1 and t_2, respectively (Fig. 4). Obviously, $x_2 = x_1 + v(t_2 - t_1)$, and when $v = u/\sin\psi > c$ events 1 and 2 are separated by a space-like interval, i.e., $(x_2 - x_1)^2 > c^2(t_2 - t_1)^2$. The perturbation ("notch") which is "imposed" on the moving pulse at the point 1 at the instant t_1 turns up at the point 3 with coordinates $x_3 = x_1 + u\sin\psi(t_2 - t_1)$, $y_3 = u\cos\psi(t_2 - t_1)$ with $(x_3 - x_1)^2 + y_3^2 = u^2(t_2 - t_1)^2 \leq c^2(t_2 - t_1)^2$. But this perturbation does not arrive at the point 1.

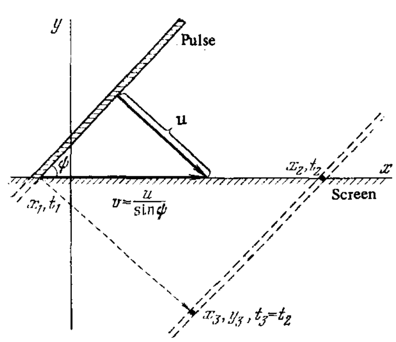

FIG. 4

*When a rotating source emits particles with velocity u, the particle trajectory is $r = r_0 + u(t - t_0), \phi = \Omega t_0$, whence $r = r_0 + u[t - (\varphi/\Omega)]$, where t_0 is the emission time.

The need for distinguishing between the shape and velocity possessed by a moving object in a given reference frame at a given instant of time from the shape and velocity registered at some definite point at the same instant of arrival (but not instant of emission) of light rays coming from the object has been fully clarified and emphasized only relatively recently (cf., e.g.,[18,19]). One important astrophysical consequence of this circumstance is that an object expanding at a velocity u (say the envelope of an exploding galactic core or of a quasar) is seen on the sky, when observed from a remote point, to expand at a rate (for details see[19])

$$u' = u/[1 - (u^2/c^2)]^{1/2}. \quad (15)$$

The discussed effect, like the Doppler effect, is connected with the finite velocity of light, as a result of which light from different parts of the object arrives at the observation point, in general, at different type. The "apparent" (observed at a fixed point) velocity of objects [for example, u′ in (15)] can exceed the velocity of light c. But we wish to emphasize, first, that such a superluminar velocity has a different nature than the superluminal velocity of the spots considered above. Second, allowance for the delay due to the finite speed of light influences strongly also the "behavior" of the spots when they are observed at some point (this was called to our attention by A. A. Lyubushin). As the simplest example, we confine ourselves here to the case of a light spot moving with constant speed v over a plane screen and observed at a point O′ (Fig. 5). By observation we mean here the reception of the light emitted by the spot as a result of the roughness of the screen (i.e., as a result of scattering) or by virtue of the luminescence of the screen. If $v \leq c$, then the spot will be observed in the "usual" manner, as a spot moving downward on the screen. Let us assume now that $v \to \infty$, i.e., the entire track of the spot is traced instantaneously. Then the spot will be first observed at the point O closest to O′ (the line OO′ is perpendicular to the screen). The observer will then see, obviously, two spots moving out of the point O in opposite directions. If $c < v < \infty$, two spots can likewise be observed at a definite time.

3. RADIATION OF SUPERLUMINAL SOURCES

The existence of superluminal velocities and superluminal sources (as we shall henceforth designate sour-

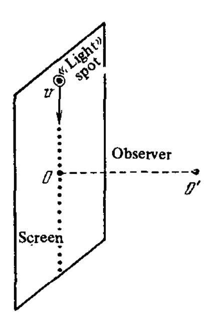

FIG. 5

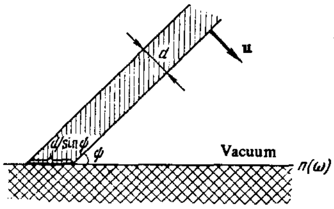

FIG. 6

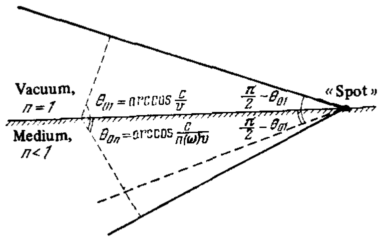

FIG. 7

ces moving with velocity v $> c$)*, as already noted, has been well known for a long time. All that remained obscure was that such sources are "in no way worse" than subluminal sources within the framework of macroscopic theory and the entire macroscopic approach. Macroscopic behavior is understood here in the sense that a superluminal source is not a single point-like (arbitrarily small) particle, but must always be connected with an aggregate of such (macroscopic) particles.** Moreover, in any realistic formulation of the problem, the number of particles responsible for the motion of the superluminal source (spot) turns out to be very large. An adequate theoretical formalism for the analysis of the radiation from superluminal sources is ordinary field theory, in particularly Eq. (9), where the current density $\mathbf{j} = \rho \mathbf{v}$ can, in principle, vary and move at arbitrary frequency and velocity.

We consider a charged filament moving with velocity u at an angle ψ to the boundary of some transparent medium with refractive index $n(\omega)$. In other words, we have a situation shown schematically in Fig. 6 and analogous to that shown in Fig. 2. Prior to crossing the boundary of the medium, the charges making up the filament (say electrons or protons) move uniformly. After crossing the boundary, however, the charges are decelerated, as a result of which a certain current (polarization) appears and travels with velocity $v = u/\sin\psi$, corresponding to the velocity of the intersection of the filament with the boundary of the medium. Such a current appears also if the deceleration of the charges is not taken into account, by virtue of the transition effect (the change of the medium parameters on the path of the charge), which produces transition radiation[20]. We can visualize a situation wherein the charges stop when they reach the medium, and are then, say, neutralized by currents in the medium[14]. As a result, a certain charge q moves on the surface of the medium with a velocity v. We assume for simplicity that the filament has a square cross section (square of side d) and consists of charges with a concentration N. Then the area of the intersection of the filament and the boundary, i.e., the area of the "spot" is $S = d^2/\sin\psi$, and the charge on this area is $q = eNd^3 \cot\psi$ (a charge $eNd^2 v \cos\psi$ crosses the boundary of the medium in a unit time, the charge per unit length along the velocity is $eNd^2 \cos\psi$, and consequently a "spot" length $d/\sin\psi$ corresponds exactly to a charge q). The solution to the problem of the radiation of a charge moving on the boundary between a medium and vacuum is known[6]. The result for the radiated energy can be written in the form

$$dW/dt = (q^2 v/c^2) \int \{1 - [c^2/n^2(\omega) v^2]\} F\omega\, d\omega. \tag{16}$$

It is obvious that at F = 1 this formula goes over into expression (2) for a homogeneous medium. The factor $F(\omega, \ldots)$ takes into account the influence of the boundary, the dimension of the source, etc. It can be assumed from general considerations that the same formula holds also for a superluminal source with $v > c$, with the function $F = F(\omega, \psi, d, \ldots)$ also dependent on the charge in the vacuum.* The factor F can be specified concretely only after an exact calculation and obviously by using a perfectly defined model for the source. This will be done later on. For the present we note that in any case the integration in (16) is over a region of frequencies satisfying the condition (3). For vacuum, of course, it is necessary to put n = 1 (it was assumed above that the medium borders on vacuum). Therefore when $v > c$ in the vacuum (above the medium) radiation occurs always, provided that $F \neq 0$. In practice, however, the factor F must certainly be quite small for a wave of length $\lambda = 2\pi c/\omega$ smaller than the projection of the dimensions of the spot on the direction of the wave vector **k**. In a medium having $n(\omega) < 1$ the situation is the same, but when $n(\omega) > 1$ the role of the cutoff factor can be played also by the condition (4)—radiation in the medium is possible only when this condition is satisfied. In the general case it can also be stated that the radiation is characterized by an angle $\theta_{01} = \cos^{-1}(c/v)$ in the vacuum and an angle $\theta_{02} = \cos^{-1}[c/n(\omega)v]$ in the medium (θ is the angle between **k** and **v**; Fig. 7). Since the velocity of the leading front of the electromagnetic waves is equal to c in any medium, the radiation of a superluminal source in a medium is characterized not only by the angle θ_{02}, but also by the

*Superluminal sources, generally speaking, are defined as sources moving with velocity $v > c_{ph} = c/n$. Such a terminology is reasonable, but we introduce no confusion whatever by defining in the present paper only sources with velocity $v > c$ as superluminal, especially having made this stipulation.

**The macroscopic behavior referred to here is quite relative and is much "weaker" then the conditions connected with the transition to the macroscopic electrodynamics from the equations of microscopic electrodynamics (or, using the older terminology, from the equations of electron theory). In fact, all that follows from the equations of electrodynamics is the continuity equation, and in all other respects the motion of the charges can be specified "from the outside" (whether such a motion is compatible with the equation of motion for the particles is another question). It is clear therefore that even in the framework of the electron theory we can assume without fear of contradiction that the current density $\mathbf{j} = \rho \mathbf{v}$ is arbitrary within wide limits, and assume in particular that $v > c$ (in this sense, Sommerfeld's calculations[9] were perfectly correct).

*It would be more accurate to express the right-hand side of (16) in the form of a sum of two terms

$$\int [1 - (c^2/v^2)]\, F_1 \omega\, d\omega + \int \{1 - [c^2/n^2(\omega) v^2]\}\, F_2 \omega\, d\omega,$$

where the first term corresponds to the radiation power in the vacuum and the second to the radiation power in the medium. However, so long as the factor F is not specified concretely, Eq. (16) is symbolic and can therefore be retained in its present form.

angle $\theta_{01} = \cos^{-1}(c/v)$, which determines in this case the aperture of the cone corresponding to the leading front of the wave. Thus, when $\theta > \theta_{01}$ the field in the medium is equal to zero. In the case of the main part of the radiation and not of the leading front, a similar situation obtains also for the Cerenkov effect in a dispersing medium, where the group velocity $c_{gr} = d\omega/dk = c[d(\omega n)/d\omega]^{-1}$ is smaller than the phase velocity $c_{ph} = c/n$. There is no need here to dwell specially on this aspect of the problem (see[1,21]).

We proceed now to obtain an exact solution of the problem of incidence of a charged filament on an ideally conducting plane[15]. The geometry of the problem is the same as in Fig. 6, but the medium with refractive index $n(\omega)$ is replaced by an ideal conductor. On striking the conductor (on crossing its boundary), the charge vanishes for an external observer, i.e., in the sense of the radiation mechanism we are dealing here with transition radiation. We are interested, however, in the result of interference of such a radiation from a moving filament, it being known beforehand that the resultant radiation will be directed at an angle $\theta_{01} = \cos^{-1}(c/v)$. The field of the filament in the vacuum is a sum of the fields of the filament itself and of its image, i.e., it is generated by a current having a density

$$\mathbf{j} = Q\delta(z)\,[\mathbf{u}_1\delta(\mathbf{s}_1\mathbf{r} - ut) - \mathbf{u}_2\delta(\mathbf{s}_2\mathbf{r} - ut)]; \tag{17}$$

here Q is the charge per unit length of the filament, $\mathbf{u}_1 = u\mathbf{s}_1$ and $\mathbf{u}_2 = u\mathbf{s}_2$ are the velocities of the filament and of its image ($s_1 = s_2 = 1$, $s_{1x} = s_{2x}$, $s_{1y} = -s_{2y}$, $s_{1z} = s_{2z} = 0$; the filament lies in the xy plane and is assumed for simplicity to be infinitesimally thin). In addition it is necessary, of course, to assume that the first term differs from zero in vacuum and the second differs from zero in the metal. The Fourier component is

$$\mathbf{j}_\omega = (1/2\pi)\int \mathbf{j}e^{i\omega t}dt = [Q\delta(z)/2\pi]\,(\mathbf{s}_1 e^{i(\omega/u)\mathbf{s}_1\mathbf{r}} - \mathbf{s}_2 e^{i(\omega/u)\mathbf{s}_2\mathbf{r}}).$$

At large distances from the screen we have for the Fourier component of the vector potential

$$\begin{aligned}\mathbf{A}_\omega &= \frac{e^{ikr}}{cr}\int \mathbf{j}_\omega(\mathbf{r}')\,e^{-i\mathbf{k}\mathbf{r}'}d\mathbf{r}' \\ &= i\frac{Qe^{ikr}}{cr}\left[\frac{\mathbf{s}_1}{(\omega/u)\,s_{1y} - k_y} + \frac{\mathbf{s}_2}{(\omega/u)\,s_{2y} - k_y}\right]\delta\left(\frac{\omega}{u}s_{1x} + k_x\right),\end{aligned} \tag{18}$$

where $\mathbf{k} = (\omega/c)\mathbf{k}_1 = k\mathbf{k}_1$ is the wave vector of the radiated wave (obviously $k_1^2 = 1$ and $k = \omega/c$). Further, it is easy to find the magnetic field $\mathbf{H}_\omega = i\mathbf{k} \times \mathbf{A}_\omega$ and then the integral

$$\begin{aligned}\frac{c}{4\pi}\int_{-\infty}^{+\infty} H^2\,dt &= \frac{c}{4\pi}\int_{-\infty}^{+\infty} dt\int_{-\infty}^{+\infty} d\omega\int_{-\infty}^{+\infty} d\omega' H_\omega H_{\omega'} e^{i(\omega+\omega')t} \\ &= \frac{c}{2}\int_{-\infty}^{\infty} d\omega\int_{-\infty}^{\infty} d\omega' H_\omega H_{\omega'}\delta(\omega+\omega') = \frac{c}{2}\int_{-\infty}^{+\infty} |H_\omega|^2\,d\omega = c\int_0^{\infty} |H_\omega|^2\,d\omega.\end{aligned}$$

We choose the x axis to be the one along which the spot travels to be the polar axis. Let the wave vector of the radiation $\mathbf{k} = (\omega/c)\mathbf{k}_1$ make an angle θ with the polar axis; we denote the azimuthal angle by φ (Fig. 8), with $\pi/2 \leq \varphi \leq \pi/2$ in vacuum.

It is seen from (18) that $\mathbf{A}_\omega$ is proportional to a δ-function of argument $(\omega/u)s_{1x} - k_x$. Obviously, the magnetic field H_ω is also proportional to the δ-function, and the radiation energy is proportional to the square of the δ-function. The integral of the square of a δ-function diverges, indicating an infinite radiation energy. This infinity can be easily explained physically (we assume that the filament takes an infinitely long

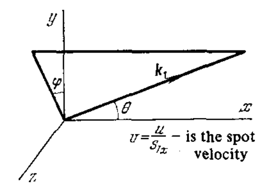

FIG. 8

time to cross the screen). To obtain a finite result, we can consider the motion of a filament during a long but finite time T. Obviously, the radiation energy is proportional to T. The same result is obtained from the following formal procedure. We write

$$\delta^2((\omega/u)\,s_{1x} - k_x) = (u/s_{1x})\,\delta(\omega - (k_x u/s_{1x}))\,\delta((\omega/u)\,s_{1x} - k_x).$$

Now the first factor can be expanded in a Fourier integral

$$\delta^2\left(\frac{\omega}{u}s_{1x} - k_x\right) = \frac{u}{2\pi s_{1x}}\delta\left(\frac{\omega}{u}s_{1x} - k_x\right)\int_{-\infty}^{+\infty} e^{i[\omega-(k_x u/s_{1x})]t}\,dt.$$

Owing to the presence of a δ-function in this product, we can set the exponent under the integral sign equal to zero, by virtue of which we get

$$\delta^2((\omega/u)\,s_{1x} - k_x) = (vT/2\pi)\,\delta((\omega/u)\,s_{1x} - k_x),$$

where T is the total time of motion of the filament and $v = u/s_{1x}$ is the velocity of the source (spot). Proceeding in this manner, we obtain for the energy radiated into a solid angle $d\Omega = \sin\theta\,d\theta\,d\varphi$ in the frequency interval $d\omega$ per unit time the following expression:*

$$\begin{aligned}\frac{dW_{\omega,\theta,\varphi}}{dt} &= \frac{1}{T}c\,|H_\omega|^2 r^2\sin\theta\,d\theta\,d\varphi\,d\omega \\ &= \frac{Q^2 v}{2\pi\omega}\left\{\frac{[\mathbf{k}_1\mathbf{s}_1]}{(c/u)\,s_{1y} - k_{1y}} - \frac{[\mathbf{k}_1\mathbf{s}_2]}{(c/u)\,s_{1y} + k_{1y}}\right\}^2\delta\left(\frac{c}{v} - k_{1x}\right)\sin\theta\,d\theta\,d\varphi\,d\omega.\end{aligned}$$

Owing to the presence of the δ-function, it is therefore clear that the radiation occurs only with a wave vector $\mathbf{k}$ satisfying the condition $k_{1x} = \cos\theta = c/v = \cos\theta_0$, as it should be. After integrating with respect to θ we get

$$\left.\begin{aligned}\frac{dW_{\omega,\varphi}}{dt} &= \frac{Q^2 v}{2\pi\omega}\left\{\frac{[\mathbf{k}_1\mathbf{s}_1]}{(c/u)\,s_{1y} - k_{1y}} - \frac{[\mathbf{k}_1\mathbf{s}_2]}{(c/u)\,s_{1y} + k_{1y}}\right\}^2 d\varphi\,d\omega, \\ \mathbf{k}_1 &= \{\cos\theta_{01},\ \sin\theta_{01}\cos\varphi,\ \sin\theta_{01}\sin\varphi\}, \\ \mathbf{s}_1 &= \{\sin\psi,\ -\cos\psi,\ 0\},\ \ \mathbf{s}_2 = \{\sin\psi,\ \cos\psi,\ 0\}, \\ \cos\theta_{01} &= c/v,\ \ v = u/\sin\psi,\end{aligned}\right\} \tag{19}$$

where ψ is the angle between the particle velocity u and the x axis.

We finally obtain

$$\frac{dW}{dt} = \frac{2Q^2 v}{\pi}\frac{c^2}{u^2} \times \int_0^{\infty}\frac{d\omega}{\omega}\int_{-\pi/2}^{\pi/2} d\varphi\,\frac{[1-(u^2/v^2)] - [1-(c^2/v^2)]\,(1-u(^4/c^2v^2)]\cos^2\varphi\}}{[(c^2/u^2)\cos^2\psi - [1-(c^2/v^2)]\cos^2\varphi]^2}. \tag{20}$$

A charge q moving in a homogeneous medium, as is clear from (2) would radiate in the interval $d\omega\,d\varphi$ with a power

$$dW_{\omega,\varphi}/dt = (q^2 v/2\pi c^2)\,[1 - (c^2/v^2)]\,\omega\,d\varphi\,d\omega,$$

where we put n = 1. Comparing this expression with (19), we see that the filament is equivalent to a charge

$$q = Q\left|\frac{[\mathbf{k}_1\mathbf{s}_1]}{(c/u)\,s_{1y} - k_{1y}} - \frac{[\mathbf{k}_1\mathbf{s}_2]}{(c/u)\,s_{1y} + k_{1y}}\right|\frac{c}{\omega}. \tag{21}$$

Since Q is the charge per unit length of the filament,

*$[\mathbf{k}_1\mathbf{s}_1] \equiv \mathbf{k}_1 \times \mathbf{s}_1$.

the multiplier of Q in (21) is the effective length of the filament responsible for the radiation in the direction of $\mathbf{k}_1$. This length is none other than the length over which the transition radiation is formed in the direction of $\mathbf{k}_1$. The integrals in (19) and (20) diverge as $\omega \to 0$, but this is simply the result of the assumption that the filament is infinite. The radiation decreases with increasing ω, obviously by virtue of the resultant decrease of the length over which the transition radiation is formed. In other problems of this type, the frequency dependence can be different (cf. infra).

As already noted, the mechanism governing the radiation from individual particles or from a filament as a whole on crossing the boundary of a conductor can be assumed to be transition radiation. It can be assumed, however, with equal success (and with the same final results), that bremsstrahlung is produced as a result of the instantaneous stopping of the charges on the boundary (in the case of an ideal conductor, these two possibilities are indistinguishable when it comes to calculate the field in a vacuum[20]). In general, the mechanism of the "elementary act" of radiation, which leads in final analysis to the Cerenkov radiation, is in a certain sense immaterial, for the character of the Cerenkov radiation [we have in mind primarily the radiation condition (3)] is determined by the interference of the waves radiated along the path of the source. The foregoing is, of course, in full accord with the Huygens principle. Thus, the considered radiation from a charged filament incident on a screen is precisely the Cerenkov effect for $v > c$, and furthermore in vacuum (to be sure, the presence of some boundary with a medium is essential here). The radiation intensity and its angular distribution with respect to φ will vary in accordance with the properties of media 1 and 2 (of course, to observe Cerenkov radiation at least one of these two media must be transparent; we have assumed above that medium 1 is the vacuum). For an anisotropic medium under the radiation condition (3), the refractive index $n(\omega)$ must be taken for each normal wave separately, and the value of n depends also on the angles made with the symmetry axes (the crystal axes, the direction of the external magnetic field, etc.).

Special notice should be taken of wave radiation in waveguides[22]*. In general, this raises many problems analogous to those encountered in the theory of Cerenkov radiation when $v < c$ (see[5-7]). It is also obvious that the sources in question (spots) radiate also in the "subluminal" regime, i.e., when $c/n < v < c$. Such sources are of interest also for the excitation, say, of various types of surface waves as the result of the Cerenkov effect or transition radiation on an inhomogeneous surface (in the latter case the requirement $v > c/n$, is of course waived). The foregoing is valid also in the case of non-electromagnetic waves; an example is the possible generation of second sound in He II by a moving source of heat (say the motion of a laser beam over the surface of the helium).

The radiation of a superluminal source is by far not limited to the Cerenkov effect. Thus, even in the case of uniform motion but with "modulation" of the source at a certain frequency ω_0 there will be observed radiation with the Doppler frequency

$$\omega = \omega_0 / | 1 - (v/c)\, n \cos\theta |.$$

This formula differs from (6) only in that the frequency ω_0 is defined in the same laboratory frame as the radiation frequency ω. The modulation can be effected in different ways, such as additional rocking of the beam, varying its density (along the beam), placing a "grid" (periodic inhomogeneities) on the screen, etc. Finally, the distinguishing features of superluminal radiation with $v > c$, just as in the case when $c/n < v < c$, become manifest also when the source moves nonuniformly. Such a case is realized when the particles or photons emitted by the rotating source fall on a spherical or cylindrical screen. A more concrete model is the following[16]: a rotating source (e.g., a pulsar) emits a directed beam of γ rays, which are incident on a "screen" consisting of a more or less dense material (plasma) located a distance R away from the source. Upon striking the screen, the γ rays are scattered by electrons, which produce a certain radial polarization as a result of the recoil. This polarization moves over the screen with a velocity $v = \Omega R$. As a result, a current moves over the screen, with a density

$$\left.\begin{aligned} \mathbf{j} &= \frac{d}{dt}\,[\mathbf{p}(t)\,\delta(\mathbf{r} - \mathbf{R}(t))], \\ \mathbf{p}(t) &= p\,\{\cos\Omega t,\ \sin\Omega t,\ 0\}, \\ \mathbf{R}(t) &= R\,\{\cos\Omega t,\ \sin\Omega t,\ 0\}, \end{aligned}\right\} \qquad (22)$$

where p is the electric dipole moment corresponding to the produced polarization, which in turn is assumed to be point-like; the latter is possible if one considers radiation of waves of wavelength λ much larger than the source dimension l. The resultant radiation at $v = \Omega R > c$ is analogous in its character to synchrotron radiation in a medium under conditions when $v > c/n$ (see[5]); the total radiated power is

$$\frac{dW}{dt} \approx \frac{p^2\,[1 + (v^2/c^2)]}{2v^3} \int\limits_{\Omega \ll \omega \ll c/l} \omega^3\, d\omega. \qquad (23)$$

The integral is cut off at high frequencies because of the finite dimensions of the dipole, a fact not taken into account in (22) and (23); incidentally, in the calculation of[16] the dipole p in (22) was assumed to be directed not along the radius, but along the z axis (i.e., it was assumed that $\mathbf{p} = p\{0, 0, 1\}$), which probably affects only the numerical coefficient in (23). In the pulsar models, a perturbation traveling with a velocity $v > c$ can be produced in the plasma also by magnetodipole radiation or by particle streams emitted from the pulsar.

In connection with the development of laser techniques, particular interest attaches to the possible production of a superluminal source with the aid of light. The use of a rotating beam is not so easy, even with the aid of a laser, if the required field intensity in the spot at $v = \Omega R > c$ is high enough. It is therefore easier to realize incidence of a pulse on a screen (an interface between media), as discussed in Sec. 2 [see Fig. 2 and formulas (11) and (12)]. If the screen is an ideally plane interface between two media, and the problem can be considered in the linear approximation (weak field), then we deal with the usual problem of reflection and

*We note that L. G. Lomize pointed out already a few years ago a possibility similar to that discussed in[22] for the excitation of waves in a waveguide.

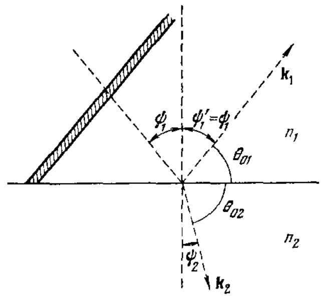

FIG. 9

refraction of light. It is therefore immediately clear (and it follows, of course, from the field equations) that a pulse incident at an angle ψ_1 will also be reflected at an angle $\psi_1' = \psi_1$, and the angle of refraction ψ_2 is determined from the refraction law (Fig. 9)

$$\sin\psi_2/\sin\psi_1 = n_1/n_2, \quad \psi_1' = \psi_1. \tag{24}$$

It is curious, as already noted long ago by Frank[8], that the conditions (24) coincide with the conditions for the appearance of the Cerenkov effect for the pulse under consideration, the intersection of which with the screen moves with velocity $v = c/n_1 \sin\psi_1$ (see (11)). In fact, the Cerenkov angle in medium 1 is determined by the condition $\cos\theta_{01} = c/n_1 v = \sin\psi_1$, whence $\psi_1 = \psi_1' = (\pi/2) - \theta_{01}$, as it should be (see Fig. 9). For medium 2 we have $\cos\theta_{02} = c/n_2 v = (n_1/n_2)\sin\psi_1$, which coincides with (24), since $\cos\theta_{02} = \sin\psi_2$. We can literally state that we did not know for a long time "that we are speaking prose" and that the superluminal Cerenkov condition (and generally the condition for $n_1 > 1$) has already been known for several centuries. The foregoing statements concerning the correspondence between the reflection and refraction laws, on the one hand, and the Cerenkov condition, on the other, are nevertheless natural, since all these relations are obtained from the Huygens principle in the same manner. To obtain some new results it is necessary to consider the problem with allowance for the nonlinearity for different media (particularly for piezoelectrics).

The last remark we wish to make here concerns light spots moving on rough or luminescent screens. In the latter case the radiation coming from the spot is in general incoherent. Practically the same holds for rough screens, since we are dealing usually in this case with rather large light spots (with dimensions considerably larger than the wavelength of the light). If the radiation is incoherent, then interference is impossible and such specific features as a sharp directivity of the Cerenkov radiation are lost.

4. CONCLUSION

The historic fate of research on radiation from sources moving with a velocity higher than the phase velocity of light is quite unique. We have in mind here classical effects, which are qualitatively understandable even within the framework of the simplest optical concepts (the Huygens principle, interference), and are qualitatively described with the aid of Maxwell's equations. We see that the elementary laws of reflection and refraction of light on a plane interface between two media are in fact identical with the condition for Cerenkov radiation from a source traveling along the interface. The Cerenkov condition for a charge—a superluminal source (velocity $v > c$)—was obtained in 1904. Yet the Cerenkov effect was observed experimentally in 1934, and even then only by accident (in the sense that an entirely different question was being investigated), and the development of the theory of this effect called for great and rather prolonged efforts[2]. It is also curious that during the first stage, the potential use of the Cerenkov effect in physics, both for measurement purposes and for the understanding of various phenomena, appeared to be quite modest. Actually, however, the Cerenkov effect and similar phenomena are presently extensively used in all respect, and it can be stated that their study makes up an entire branch in physics, to which a tremendous number of articles and a number of surveys are devoted. It might seem that if the problem has not yet been exhausted, it has been in any case investigated quite fully and comprehensively. But even this is not true, as evidenced by the present article. In fact, there has been a widely held opinion (to which, in particular, we ourselves adhered) that the Cerenkov effect and the anomalous Doppler effect can be observed only for waves corresponding to a refractive index $n(\omega) > 1$ (the condition $c/n < v < c$). Accordingly, the corresponding conditions were considered to be impossible in vacuum. Yet there exist superluminal sources moving with velocity $v > c$. These sources can be considered, in a wide range, on the same basis as the "ordinary" sources moving with velocity $v < c$. Concretely, superluminal sources are capable of generating Cerenkov radiation in any medium, including vacuum or under the condition $n(\omega) < 1$. Superluminal sources of the general type have on the whole the same peculiarities that are known for sources moving with velocity $c/n < v < c$ (anomalous Doppler effect etc). From the point of view of radiation theory, the essential difference between superluminal ($v > c$) and subluminal ($v < c$) sources is that a superluminal source cannot comprise an individual "elementary" particle and has therefore always some size. It is precisely the dimensions of the superluminal source which determine primarily, especially in the case of radiation in vacuum, the short-wave limit of the emitted frequency spectrum. It is therefore unlikely that superluminal sources can be used, for example, for x-ray generation (such a possibility would be enticing since the tendency of the refractive index $n(\omega)$ to unity, which is manifest at high frequencies and prevents the use of the Cerenkov effect for sources with $v < c$ in the x-ray region, does not play such a critical role when $v > c$). We should not be surprised, however, if some interesting applications were to be found in the future for superluminal sources, too. In addition, superluminal sources can be encountered in astronomy. Regardless of such possibilities, the emission of electromagnetic and gravitational waves (and possibly also neutrinos) from superluminal sources, and the entire aggregate of the related problems, are in our opinion of undisputed physical interest.

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