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Wave Scattering by Statistically Uneven Surfaces

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CONTENTS

INTRODUCTION

SCATTERING of waves by uneven surfaces is encountered in many practical problems in acoustics, radiophysics, and optics. The propagation of acoustic and electromagnetic waves over a rough terrain or over a wavy water surface, and the determination of the surface structure and properties of dry land, sea, planets, the ocean bottom and others are but a few of a long list of such problems. Both when the properties of the scattered field are determined from known surface characteristics and when the inverse problem is solved, it is necessary to know the connection between the properties of the scattering surface and the characteristics of the field scattered by it. The main task of the theory is to determine this connection.

If the structure of the surface roughnesses is sufficiently simple, then various models can be used in the calculations (sinusoidal or sawtooth roughnesses, plane on which hemispheres or half-cylinders are randomly disposed, etc.). The most general approach, covering mainly the case of practical importance of surfaces whose roughness is due to natural causes, is the one using a statistical description of the surface itself as well as of the waves scattered from it via random fields. This review is devoted to researches that employ just this approach.

Among the surveys of this question, we point out the reviews[1-6] and the monographs[7-9]. In this review, in addition with the methods long employed to solve the problem (Kirchhoff's method and the perturbation method), we describe recently developed methods that make it possible to take into account multiple scattering by the surface (the integral-equation method and the Green's function method). In addition, we report a number of results obtained by traditional methods but not covered in the reviews and monographs cited above.

In spite of the large number of publications on the scattering of waves by statistically rough surfaces, the theory of the problem is still far from completely developed. This is due both to the abundance of factors that must be taken into account in the construction of a theory satisfying the needs of modern experiments, and to the need for refining the approximate solutions that are already available.

1. KIRCHHOFF'S METHOD

The gist of the method lies in the assumption that the field reflected from an uneven surface S can be calculated by geometrical optics, i.e., as in the case of an infinite plane tangent to the given point of a surface S. This assumption is justified if the radius of curvature of the roughnesses is large when expressed in terms of the wavelength.

In particular, we can speak here of surfaces having continuous irregularities (without breaks) or surfaces consisting of flat areas (faces) of sufficiently large size. In the latter case, Kirchhoff's hypothesis does not hold on the edges.

From the point of view of the theory, the former case is of great interest, and we shall pay principal attention just to surfaces with continuous irregularities. Furthermore, in many applications of the theory (for example, in investigations sea waves by acoustic or radiophysical methods) the representation of the surface S in the form of aggregate of flat areas is only a rough model. To be sure, if we deal in optics, for example, with reflection from ground glass, then we can encounter the second case.

The local condition for the applicability of the Kirchhoff approximation is[10,11]

$$k\rho\cos^3\theta \gg 1,$$

where $k = \omega/c$ is the wave number, ρ is the radius of curvature of the irregularity, and θ is the local angle of incidence. In the presence of surface waves on the interface S, it is necessary, in addition, to stipulate that the field due to these waves is small[12].

a) Assume that an acoustic wave

$$\Phi(\mathbf{r}) = A(\mathbf{r})\, e^{i\psi(\mathbf{r})}, \tag{1.1}$$

described in the geometrical optics approximation, particularly a plane wave or a guided spherical wave, is incident on a surface $z = \zeta(x, y)$. In accordance with Kirchhoff's principle, we have for the reflected field and for its normal derivative on the interface S

$$\varphi(\mathbf{r}) = V\Phi(\mathbf{r}), \quad \frac{\partial\varphi(\mathbf{r})}{\partial n} = -V\frac{\partial\Phi(\mathbf{r})}{\partial n}, \tag{1.2}$$

where V is the local Fresnel reflection coefficient. The reflected field at the point of observation **R** is connected with $\varphi(\mathbf{r})$ and $\partial\varphi(\mathbf{r})/\partial n$ by Green's formula

$$\varphi(\mathbf{R})=\frac{1}{4\pi}\int_S\left[\varphi(\mathbf{r})\frac{\partial}{\partial n}\frac{e^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|}-\frac{\partial\varphi(\mathbf{r})}{\partial n}\frac{e^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|}\right]dS. \quad (1.3)$$

Assuming satisfaction of the inequality

$$z-\zeta(x,y)\geqslant 0, \quad (1.4)$$

we use Weyl's expansion of the scalar Green's function in plane waves

$$\frac{e^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|}=\frac{i}{2\pi}\int_{-\infty}^{\infty}e^{i\varkappa(\mathbf{R}-\mathbf{r})}\frac{d^2\varkappa_\perp}{\varkappa_z}, \quad (1.5)$$

where $\kappa = (\kappa_\perp, \kappa_z)$, $\kappa_z = (k^2-\kappa_\perp^2)^{1/2}$.

We now substitute (1.5) and (1.2), with allowance for (1.1), into (1.3) and change to integration over the underlying surface S_0, obtaining an expression for the scattered field. In the case of an absolutely rigid surface (V = 1), this expression is[13]

$$\varphi(\mathbf{R})=-\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\int_{S_0}\frac{A_0q^2}{q_z\varkappa_z}e^{i\psi_0+i\varkappa(\mathbf{R}-\mathbf{r}_\perp)+iq_z\zeta}d^2\mathbf{r}_\perp d^2\varkappa_\perp, \quad (1.6)$$

where $A_0 = A(\mathbf{r}_\perp)$, $\psi_0 = \psi(\mathbf{r}_\perp)$, and $\mathbf{q} = \nabla\psi_0 - \boldsymbol{\kappa}$ is the scattering vector.

Expression (1.6) describes the field scattered by either a bounded or an infinite surface, with the rather weak condition (1.4) imposed on the position of the point of observation over the irregularities. The source of the primary wave should be located in the Fraunhofer zone relative to the height σ of the irregularities:

$$k\sigma^2 \ll L,$$

where L is the characteristic scale of variation of the amplitude $A(\mathbf{r})$ and of the gradient of the phase $\nabla\psi(\mathbf{r})$ of the primary wave (1.1).

If the point of observation is located at a distance Z from the irregularities, such that $Z \gg \sigma$ and $Z \gg \lambda$, then the integral with respect to $\boldsymbol{\kappa}_\perp$ can be calculated by the stationary-phase method, and formula (1.6) becomes simpler:

$$\varphi(\mathbf{R})=\frac{i}{4\pi}\int_{S_0}\frac{A_0q^2}{R_1q_z}e^{ikR_1+i\psi_0+iq_z\zeta}d^2\mathbf{r}_\perp, \quad (1.7)$$

where $\mathbf{q} = \nabla\psi_0 - k(\mathbf{R}_1/R_1)$, $\mathbf{R}_1 = \mathbf{R} - \mathbf{r}_\perp$.

By calculating the integral in the averaged expression (1.7) by the stationary phase method, we obtain the mean value of the scattered field

$$\langle\varphi(\mathbf{R})\rangle = f_{1\zeta}(q_{zc})\,\varphi^{(0)}(\mathbf{R}), \quad (1.8)$$

where $f_{1\zeta}(q_{zc})$ is the characteristic function of the uneven surface, q_{zc} is the value of the z-component of the scattering vector at the stationary point, which coincides with the point of geometric reflection of the wave (1.1) from the plane S_0, while $\varphi^{(0)}(\mathbf{R})$ is a field regularly reflected from S_0 and calculated in the ray approximation. Formula (1.8) is valid when the distances from the source of the primary wave and from the observation point to the uneven surfaces are such that the following conditions are satisfied for $R_m = \min(R, L)$:

$$kR_m \gg \{(k\sigma)^2,\ (k\sigma)^4\sin^2 2\theta_c\}, \quad (1.9)$$

where θ_c is the angle of incidence of the primary wave at the point of specular reflection from the plane S_0, i.e., at the stationary point.

If the incident wave is plane and the surface S is infinite, then expression (1.8) is valid under the condition (1.4), which is less stringent than (1.9). If the surface is not plane in the mean, formula (1.8) also remains in force in the case of gently-sloping irregularities[14]. The quantity $f_{1\zeta}(q_{zc})$ in (1.8) has the meaning of the effective reflection coefficient of the mean field.

With the aid of the dynamic relations (1.6) and (1.7) we can easily obtain expressions for the correlation function $\Psi(\mathbf{R}, \mathbf{R}') = \langle\varphi(\mathbf{R})\,\varphi^*(\mathbf{R}')\rangle - \langle\varphi(\mathbf{R})\rangle\langle\varphi^*(\mathbf{R}')\rangle$ and for the average fluctuation intensity $I = \Psi(\mathbf{R}, \mathbf{R})$ of the scattered field. It turns out that when the distances R and R_0 from the point of observation and from the source to the uneven surface are such that

$$\min(R, R_0) \gg l_f\sqrt{kl_f}, \quad l_\Phi=\begin{cases} l \ \text{ for } (k\sigma)^2 \ll 1, \\ l/2k\sigma\cos\theta_c \ \text{ for } (k\sigma)^2 \gg 1, \end{cases} \quad (1.10)$$

incoherent addition of the intensities of the waves reflected by individual elements of the uneven surface takes place:

$$I=\int_{S_0}A_0^2(\xi)\,J(\xi)\,d^2\xi; \quad (1.11)$$

here J is the average intensity of the field scattered from a unit area of the roughness into the Fraunhofer zone (relative to the dimensions of this roughness) when a plane wave of unit amplitude is incident.

A method for calculating the average intensity (1.11) was proposed by Isakovich[15]. He was also the first to obtain an expression for J, the form of which for statistically homogeneous roughnesses is[15,16]

$$J=\frac{q^4}{16\pi^2R_2^2q_z^2}\int_{-\infty}^{\infty}e^{i\mathbf{q}_\perp\boldsymbol{\rho}}\left[f_{2\zeta}(q_z, -q_z, \boldsymbol{\rho})-f_{1\zeta}(q_z)f_{1\zeta}(-q_z)\right]d^2\rho, \quad (1.12)$$

where $\mathbf{R}_2 = \mathbf{R} - \xi$, $\mathbf{q} = \nabla\psi_0(\xi) - k(\mathbf{R}_2/R_2)$, $f_{2\zeta}(q_z, -q_z, \boldsymbol{\rho})$ is the two-dimensional characteristic function of the uneven surface, and $f_{1\zeta}(q_z)$ is, as before, its one-dimensional characteristic function.

It can be shown[13,17,18] that expression (1.12) describes the average scattering intensity not only in the far zone relative to the dimension L_S of the rough area S, but also at shorter distances satisfying the inequality

$$\min(R, R_0) \gg kl_fL_S. \quad (1.13)$$

The condition (1.13) means physically that the angles at which the area S is seen from the source and from the observation point should be small compared with the scattering diagram of an individual inhomogeneity.

Using the correlation function $\Psi(\mathbf{R}, \mathbf{R}')$ obtained with the aid of (1.7), we can estimate, in the case of a Gaussian distribution of the heights of the irregularities with one correlation scale l, the radii $r''_\perp$ and $r''_\parallel$ of the transverse and longitudinal correlations of the scattered field in the far zone of an individual irregularity, and express them in terms of the effective dimension l_f of the irregularity. If the uneven surface contains the entire area essential for the scattering, and if the change of the slowly-varying factors in this region is sufficiently small, the indicated estimates take the form

$$\begin{aligned} r''_\perp &\approx l_f\,[(R''+R_0)/R_0]\cos\theta_c, \\ r''_\parallel &\approx kl_f^2\,[(R''+R_0)/R_0]^2\cos^2\theta_c, \end{aligned} \quad (1.14)$$

where $\mathbf{R}'' = (\mathbf{R} + \mathbf{R}')/2$.

Formulas (1.14) coincide with analogous estimates for a phase screen with small phase dispersion. The analogy with the problem of diffraction by an inhomogeneous phase screen was noted by Tamoĭkin and Fraĭman[19] and used by them to investigate the correlation of amplitude and phase fluctuations of a field reflected from a surface having two irregularity scales, on which a plane wave is incident.

The problem of scattering of electromagnetic waves differs from the scalar case only in that polarization must be taken into account. For a primary wave specified in the geometrical-optics approximation, the derivation of the dynamic relations does not differ in principle from the scalar case. However, the calculations are somewhat more cumbersome since, first, it is necessary to use the vector Green's formula in place of (1.3), and second, when the electric and magnetic vectors of the reflected field are calculated by Kirchhoff's method it is necessary to take into account the difference between the Fresnel reflection coefficients for vertically and horizontally polarized components of the incident wave (relative to the local plane of incidence). In the case of an ideally conducting surface, the final expressions for the electric vector of the scattered wave are given by formulas (1.6) and (1.7), in which the scalar amplitude factor $A_0 q^2$ must be replaced by the vector factor $\{(\mathbf{e}_0 \cdot \mathbf{q})\mathbf{q} - \mathbf{q} \times [\mathbf{e}_0 \times \mathbf{q}]\}$[20]. For the average field we can obtain a vector analog of (1.8) with the same *effective reflection coefficient*.

At the distances (1.10) from the uneven surface, the average energy characteristics of the scattered electromagnetic fields (such as the average intensity, the average Poynting vector, the mean-squared component of the electric vector along an arbitrary direction, the elements of the polarization matrix) can be calculated with the aid of incoherent addition. In this case the quantity $A_0^2(\xi)$ in (1.11) is replaced by more complicated functions, which also vary slowly as functions of ξ. If the source of the primary wave is an elementary electric dipole, then in the case of Gaussian statistics of the irregularities it is possible to obtain formulas in closed form for the average energy characteristics of the back-scattered field[20]*.

In the case of back scattering to a point source, it is possible to calculate also the average intensity of fluctuations of an acoustic field[13,24].

It is important that all the expressions for the average energy characteristics contain as an integrand the quantity J, defined by formula (1.12). This quantity, as already indicated, is the mean intensity of the field scattered from a unit surface area at distances (1.13) from S, and particularly in the Fraunhofer zone relative to S upon incidence of a plane wave.

b) The scattering of a plane wave in the far zone relative to S has been the subject of a rather large number of papers. In this formulation, the problem was first solved by Isakovich[14,15] and was subsequently treated also by others[25-33]. The case of back scattering was considered in[34-36], the scattered field due to incidence of a circularly polarized wave on a surface was calculated in[37], the possibility of determining the surface characteristics from wave-scattering experiments was studied in[38-40], attempts to take into account the dependence of the Fresnel reflection coefficients on the slopes of the irregularities were made in[41], and attempts in general to carry out averaging with allowance for the statistics of the random slopes were made in[42-45].

As a rule, the edge effects due to diffraction of the primary wave by the boundary of a finite rough area were neglected in all the calculations. This neglect, however, is not always permissible. As shown in[46], the edge effect makes a noticeable contribution to the scattered field in scattering of a vertically polarized wave and at glancing angles $< 30°$. We note that there is no edge effect when the scattering is by a section of an unbounded surface.

The approach developed in[47] is based on representing the field at the observation point in the form of a sum of fields coming from reflecting spots on a rough surface. In this approximation, the scattering problem reduces to finding the geometrical characteristics of the surface. Thus, for example, the average cross section for scattering from a unit area is proportional to the number of reflecting spots on the surface, and also to the quantity $\langle|\rho_1\rho_2|\rangle$, where ρ_1 and ρ_2 are the principal curvature radii of the irregularities at the locations of the reflecting spots. In the case of a normal distribution of the heights of the irregularities, with a Gaussian correlation function, the average scattering cross section can be expressed in explicit form in terms of the parameters of the problem[48] and the result agrees with the expressions obtained by calculating the integral (1.12) in the ray approximation.

As shown in[49], the same result is obtained for the average cross section if the calculation is based on the hypothesis that the rough surface can be broken up into flat elementary areas with random slope distributions, and the reflected field is calculated in accord with the laws of geometrical optics directly in space (and not only on the surface as in Kirchhoff's method). This hypothesis, first advanced by Bouguer[50], is widely used in optical calculations of the reflection and transmission of light[51,52] when the diffraction effects due to breaks in the interface are negligible. The corrections for the diffraction can in this case be approximated by assuming in the calculation the scattering patterns of the elementary flat areas.

The results of calculations performed within the framework of this approximation are used in practice to determine the heights of the irregularities and the distribution functions of the elementary areas, principally from experiments on the scattering of light by ground glass and by surfaces of ground metals[54-62]. This approach was used in[63] to calculate a number of characteristics of the light field scattered by a wavy sea surface.

Finally, there are studies (cf., e.g.,[64,65]) in which the ray approximation was used to interpret certain experimental data on the reflection of radar signals from the surfaces of the moon and of planets.

We note that in the far zone of a bounded surface the depolarization component of the scattered field in the incidence plane (especially in back scattering) turns out to be equal to zero already in the dynamic part of the problem. In the near wave zone, however, particularly

*If the surface S is not ideally conducting, than the calculations become much more complicated[21-23], for in this case it is necessary to take into account the statistics of the random slopes of the uneven surface, on which the Fresnel reflection coefficients depend.

in scattering by an unbounded surface, the mean square of this component differs from zero[20,66]. This is due to the fact that in the former case all the reflecting spots responsible for the scattering lie on surface elements that are inclined at one and the same angle, whereas in the second case we have a set of reflecting spots from elements having all possible inclinations.

We turn now to evaluation of the integral (1.12), which plays an important role in the calculation of the energy characteristics of the field, both for scattering from an area of finite dimensions and for an unbounded rough surface. This integral can be calculated exactly only in a few particular cases, and it is therefore usually estimated by approximate methods.

For the case of irregularities that are high compared with the wavelength ($q_z^2\sigma^2 \gg 1$), a method of calculating (1.12), suitable for an arbitrary distribution function of the irregularities and using a three-point probability density of the latter, was proposed in[67]. The result of the calculation by this method can be represented in the form of a series in the small parameter $(q_z\sigma)^{-2}$, the first term of which corresponds to the ray approximation. In this approximation, the result is[67,68]

$$J = \frac{q^4}{4R_2^2 q_z^4} w_s\left(-\frac{q_x}{q_z}, -\frac{q_y}{q_z}\right), \tag{1.15}$$

where $w_s(\zeta_x', \zeta_y')$ is the joint distribution density of the slopes of the surface.

In the opposite case, of irregularities that are small in comparison with the wavelength ($q_z^2\sigma^2 \ll 1$), the integral (1.12) is the Fourier transform of the correlation function, i.e., it is proportional to the spectral density of the uneven surface $W(q_x, q_y)$[26]:

$$J = (\sigma^2 q^4/4R_2^2)\, W(q_x, q_y).$$

If the heights of the irregularities have a normal distribution with a single correlation scale, then formula (1.15) yields the same result as obtained in[15], where (1.12) was calculated by the saddle-point method. The saddle-point method can be used also in the case when it is necessary to take into account several irregularity correlation scales. For example, for one-dimensional irregularities with a correlation function of the type

$$K(x) = e^{-x^2/l^2}\cos\Omega x, \tag{1.16}$$

there is a whole number of correlation scales that are significant when $l\Omega \gg q_z\sigma$. The saddle-point method then yields[15]

$$J = (L_y q^4/8\sqrt{2}\,\pi^{3/2} R_2^2 \Omega\sigma q_z^3)\exp(-q_x^2/2\Omega^2\sigma^2 q_z^2)\times$$
$$\times\sum_n \exp[-i\cdot 2\pi n(q_x/\Omega) - 4\pi^2 n^2 (q_z^2\sigma^2/\Omega^2 l^2)], \tag{1.17}$$

where L_y is the length of the surface in the y direction, and the summation is over the saddle points. In the case of integer values of $N = q_x/\Omega$, expression (1.17) has maxima analogous to the spectra for scattering by a periodic surface. Unlike the latter, however, we are dealing here with intensity rather than amplitude spectra. We note that the correlation function (1.16) describes sufficiently well not too large a section of a wavy sea surface[15,69,70].

In the case of a normal distribution of the irregularities with a Gaussian correlation coefficient, calculation of the integral (1.12) at an arbitrary height of the irregularity leads to an infinite series[8]

$$J = \frac{q^4 l^2}{16\pi R_2^2 q_z^2} e^{-q_z^2\sigma^2}\sum_{n=1}^{\infty}\frac{(q_z\sigma)^{2n}}{n!\,n} e^{-q_\perp^2 l^2/4n}.$$

For the specular-reflection directions ($q_\perp = 0$) this series is expressed in terms of tabulated functions:

$$J = (q^4 l^2/16\pi R_2^2 q_z^2)\, e^{-q_z^2\sigma^2}[\mathrm{Ei}(q_z^2\sigma^2) - C - 2\ln q_z\sigma],$$

where Ei(t) is the integral exponential function and C is Euler's constant. The representation of J in the form of an infinite series is possible also in the case of anisotropic irregularities[71].

A model with an exponential correlation results from a description of the surface with the aid of a Markov process[7]. However, the calculations of J for such a surface model[73-76] are hardly justified[77,78], since the conditions for the applicability of the Kirchhoff method are violated in this case. The use of the correlation coefficient proposed in[79]

$$K(\rho) = \exp\left\{-\frac{|\rho|}{L} + \frac{|\rho|}{L}\exp\left(-\frac{|\rho|}{l}\right)\right\},$$

which behaves like an exponential one at $|\boldsymbol{\rho}| \gg l$ but, unlike the latter, is twice continuously differentiable at zero, makes it possible to calculate the variance of the slopes in the usual manner, $\sigma_s^2 = -K''(0)\sigma^2$.

From the experimental point of view, it is of interest to calculate the fluctuations of the amplitude and phase of the scattered field. In the case of small values of the Rayleigh parameter ($2k\sigma\cos\theta \ll 1$), when the scattered field component is small in comparison with the specularly reflected one, we can put

$$\varphi^{(0)} + \varphi^{(1)} = (A_0 + \delta A)\, e^{i(\varphi_0 + \delta\varphi)} \approx A_0 e^{i\varphi_0}(1 + \delta A\cdot A_0^{-1} + i\delta\varphi).$$

Thus, by separating the real and imaginary parts of the ratio $\varphi^{(1)}/\varphi^{(0)}$, we can obtain the fluctuations of the amplitude and of the phase; by subsequently averaging the bilinear combinations of δA and $\delta\varphi$ we can then obtain their autocorrelation and mutual correlation functions[80,81].

If the number of irregularities participating in the scattering of the wave by the surface is large, then, by virtue of the central limit theorem, the scattered field will have a normal distribution[8,82]. Of course, the foregoing does not pertain to the regularly reflected field component, which is significant at small values of the Rayleigh parameter near the specular-reflection direction.

A Gaussian distribution of the scattered field was used in[83,84] in an investigation of the properties of the random field of the intensity of laser radiation scattered by a moving diffuse surface.

c) To describe wave reflection from a statistically uneven surface that varies in time, the quasistationary formulation of the problem suffices in most cases. This approximation is justified if the response of the receiving apparatus to temporal variations of the surface is quasistationary. In the far zone of a surface S with a normal distribution of the irregularities, the temporal correlation function of the scattered field takes in this approximation the form

$$\Psi(\tau) = \frac{S_0 q^4 e^{-i\omega_0\tau}}{16\pi^2 R^2 q_z^2}\int_{-\infty}^{\infty} e^{i\mathbf{q}_\perp\boldsymbol{\rho}}\{e^{-q_z^2\sigma^2[1-K(\boldsymbol{\rho},\tau)]} - e^{-q_z^2\sigma^2}\}\, d^2\rho,$$

where $K(\boldsymbol{\rho}, \tau)$ is the space-time correlation coefficient to the surface. The inverse Fourier transform of $\Psi(\tau)$ yields the spectral density of the reflected field.

If the dynamic equations describing the temporal variations of the surface are linear, then $K(\boldsymbol{\rho}, \tau)$ can be represented in the form of a superposition of traveling plane waves

$$K(\rho, \tau) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} \widetilde{K}(\varkappa_\perp, \theta)\, e^{i[\varkappa_\perp\rho - \Omega(\varkappa_\perp)\tau]}\, d^2\varkappa_\perp\, d\theta$$

with a dispersion law $\Omega(\boldsymbol{\kappa}_\perp)$. It was established in[85] that in the far zone of S, in the case $q_z^2\sigma^2 \ll 1$, the scattered field has a line spectrum in first order in $(q_z\sigma)^2$. The broadening of the spectral lines, which is connected with the nonlinearity of the dispersion law, is observed only in second order. Irregularities of large height ($q_z^2\sigma^2 \gg 1$) produce in the case of a nonlinear dispersion law a continuous spectrum. The linear dispersion law $\Omega = \boldsymbol{\kappa}_\perp \cdot \mathbf{v}$ leads to a Doppler frequency shift that depends both on the direction of the wave vector of the incident wave and on the observation direction.

The results of the calculations of the spectral density of the pressure of an acoustic wave reflected from sections of sea surface with large and small waves[86] agree with these conclusions. Complete coherence of the field takes place only for reflection from a weakly wavy surface in the direction of the specular ray[87].

In the case of uniform motion of an unbounded surface, different sections of the surface yield different values of the Doppler shift. The frequency spectrum of the reflected field has in this case a finite width[88], unlike the case of scattering in the far zone of a bounded rough area.

d) The Kirchhoff approximation can be generalized in natural fashion to include the case of an anharmonic incident field. By virtue of the superposition principle, the average field of a reflected regular sound signal with a spectrum $S(\omega)$ is given by

$$\langle\varphi\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)\langle\varphi(\omega, t)\rangle\, d\omega,$$

where $\langle\varphi(\omega, t)\rangle$ is the average field of the reflected harmonic wave with frequency ω. For the space-time correlation coefficient of the scattered field we can easily obtain the expression

$$\Psi(\mathbf{R}, \mathbf{R}', \tau) = \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} S(\omega) S^*(\omega') K(\omega, \mathbf{R}, \omega', \mathbf{R}', \tau)\, d\omega\, d\omega', \quad (1.18)$$

where

$$K(\omega, \mathbf{R}, \omega', \mathbf{R}', \tau) = \frac{\langle\varphi(\omega, \mathbf{R}, t)\varphi^*(\omega', \mathbf{R}', t')\rangle - \langle\varphi(\omega, \mathbf{R}, t)\rangle\langle\varphi^*(\omega', \mathbf{R}', t')\rangle}{\langle|\varphi(\omega, \mathbf{R}, t) - \langle\varphi(\omega, \mathbf{R}, t)\rangle|^2\rangle}$$

is the coefficient of the mutual space-time correlation of harmonic fields with different frequencies.

Knowing $K(\omega, \mathbf{R}, \omega', \mathbf{R}', \tau)$ we can estimate the radius of the frequency correlation of the scattered field, i.e., the width of the transmission band of the communication channel with scattering of the wave by the rough surface. The corresponding estimates were made for both the case of scattering in the far zone of a limited area S[89] and the case of scattering of a directed spherical wave by an unbounded rough surface[90].

In the scattering of noise signals it is necessary to carry out the averaging also over the ensemble of realizations of the primary field. In the case of stationary noise, averaging of (1.18) yields

$$\Psi(\mathbf{R}, \mathbf{R}', \tau) = \int_{-\infty}^{\infty} G(\omega) K(\omega, \mathbf{R}, \mathbf{R}', \tau)\, d\omega, \quad (1.19)$$

where $G(\omega)$ is the energy spectrum of the primary noise signal.

Calculation of the average field and of its spatial-temporal correlation function for a high-frequency pulse $A(t) = A_0\exp[i\omega_0 t - (t^2/\tau_0^2)]$ and noise with an energy spectrum $G(\omega) = G_0\exp[-(\omega - \omega_0)^2/\Omega^2]$[91] makes it possible to estimate quantitatively effects such as the additional damping of the scattered pulse, its spreading, and the change of the central frequency of the spectrum. Knowing how each harmonic component of the signal is scattered, and assuming that the scattered sense is stationary in the broad sense, we can easily obtain the energy spectrum of an anharmonic signal[92].

e) At large incidence and observation angles, the scattering is strongly influenced by the blocking of both the incident and the reflected waves by individual sections of the irregular surface[93,94]. The first attempt to take shadowing into account in Kirchhoff's method was made in[95], where use was made of the solution of the problem of wiggles of a random function over a specified determined function. Another approach, based from the very outset on geometrical representations, was proposed in[96], but the results obtained there turned out to be in error[97-98]. On the basis of the method proposed in[96], the theory of the problem was subsequently developed further first under the assumption that the heights of the irregularities have a normal distribution[99-101], and then also without this limitation[102-104].

We note also a paper[105] in which an original method is used to calculate the shadowing function with the aid of a simple geometrical identity.

The problem of taking into account the shadowing relative to the incident field consists of determining, from the known probability density $w(\zeta, \zeta_x')$ of the heights and slopes of the entire uneven surface, the probability density of the heights and slopes of its illuminated part of $w_{eff}(\zeta, \zeta_x') = w(\zeta, \zeta_x')P_1(\zeta, \zeta_x', \psi)$, where P_1 is the probability that a ray drawn at an angle ψ from a point of the surface with height ζ and slope ζ_x' never crosses the irregularities. The value of P_1 is expressed in terms of the probability density $g_\tau(A|B)$ of intersection of the beam with the surface at a distance τ from a chosen point (event A) under the condition that there are no intersections in the interval $(0, \tau)$ (event B), in the following manner:

$$P_1(\zeta, \zeta_x', \psi) = \theta(\operatorname{tg}\psi - \zeta_x')\exp\left[-\int_0^\infty g_\tau(A|B)\, d\tau\right],$$

where $\theta(x)$ is the Heaviside function.

An exact expression for $g_\tau(A|B)$ can be derived in principle, but it is too complicated; $g_\tau(A|B)$ is therefore usually calculated approximately under the following assumptions:

1) there is no correlation between the height and the slopes of the irregularities in the case of weak and strong shadowing;

2) multiple intersections of the beam and the surface is neglected in the intermediate region of grazing angles ψ.

Under the indicated assumptions, we can calculate $w_{eff}(\zeta, \zeta'_x, \psi)$ and the shadowing function $P(\psi)$, which yields the probability that an arbitrary point of the surface is illuminated (regardless of its height and slope). The shadowing of the reflected beams is taken into account in exactly the same manner as that of the incident ones. The averaged intensities of the sound field scattered in the far zone of surface S, over the distribution w_{eff}, yields[103]

$$I = I_0 Q(\psi, \chi),$$

where I_0 is the average scattering intensity in the far zone without allowance for the shadowing, ψ is the grazing angle of the incident wave, and χ is the observation angle. In the case of weak and strong shadowing

$$Q(\psi, \chi) = [1 - e^{-\Lambda(\psi) - \Lambda(\chi)}]/[\Lambda(\psi) + \Lambda(\chi)],$$

and in the intermediate region

$$Q(\psi, \chi) = 1/[1 + \Lambda(\psi) + \Lambda(\chi)],$$

where

$$\Lambda(t) = \operatorname{ctg} t \int_{\operatorname{tg} t}^{\infty} (\zeta'_x - \operatorname{tg} t)\, w(\zeta'_x)\, d\zeta'_x.$$

2. PERTURBATION METHOD

In practice one frequently deals with scattering of waves by surfaces on which the irregularities, whose height, while small in comparison with the wavelength, varies noticeably already over distances much smaller than λ. Such a situation arises, for example, in optics in the scattering of light by thermal fluctuations of a liquid surface, in radiophysics and acoustics when waves are reflected from slightly rough surfaces such a sea ripples. In this case the Kirchhoff approximation is no longer suitable and it is natural to use the perturbation method to solve the scattering problem. The gist of the method consists of expanding both the boundary conditions and the sought solution in powers of the small parameter $|k\zeta| \sim |\nabla\zeta| \ll 1$ and calculating the successive approximations for the scattered field. The use of the perturbation method presupposes that the irregularities are small ($|k\zeta| \ll 1$) and are gently sloping ($|\nabla^2\zeta| \ll 1$), but does not call for the irregularities to be smooth functions over distances of the order of the wavelength. We describe here the three modifications of the perturbation methods most frequently encountered in the literature on waves scattering by statistically rough surfaces.

a) The first method consists of applying Green's formula to the calculation of the scattered field. We represent the total sound field in the upper medium in the form of a perturbation-theory series

$$\varphi(\mathbf{R}) = \Phi(\mathbf{R}) + \sum_{n=0}^{\infty} \varphi^{(n)}(\mathbf{R}), \tag{2.1}$$

where $\Phi(\mathbf{R})$ is the field of the primary wave, and the n-th term of the series is of the order of $|k\zeta|^n \sim |\nabla\zeta|^n$. Assume, for simplicity, that the surface is absolutely soft. Substituting then (2.1) in the boundary condition expanded in powers of ζ:

$$\varphi|_S = \varphi_0 + \left(\frac{\partial\varphi}{\partial z}\right)_0 \zeta + \left(\frac{\partial^2\varphi}{\partial z^2}\right)_0 \frac{\zeta^2}{2} + \ldots = 0,$$

we obtain the boundary conditions on the plane z = 0 for the successive approximations

$$\left.\begin{aligned} &\varphi_0^{(0)} + \Phi_0 = 0, \\ &\varphi_0^{(1)} + \left[\frac{\partial(\varphi^{(0)} + \Phi)}{\partial z}\right]_0 \zeta = 0, \\ &\varphi_0^{2} + \left(\frac{\partial\varphi^{(1)}}{\partial z}\right)_0 \zeta + \left[\frac{\partial^2(\varphi^{(0)} + \Phi)}{\partial z^2}\right]_0 \frac{\zeta^2}{2} = 0, \\ &\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{2.2}$$

From the boundary conditions (2.2) we can easily obtain, using Green's formula, also the scattered field itself. In the first approximation we have

$$\varphi^{(1)}(\mathbf{R}) = \frac{1}{2\pi} \int_{S_0} \left[\frac{\partial(\varphi^{(0)} + \Phi)}{\partial z}\right]_0 \zeta(x, y) \frac{\partial}{\partial z} \frac{e^{ikR_1}}{R_1} d^2\mathbf{r}_\perp,$$

where $\varphi^{(0)}$ is the field regularly reflected by the flat area S_0 and $\mathbf{R}_1 = \mathbf{R} - \mathbf{r}_\perp$.

In the case of scattering by an absolutely rigid surface, the first-approximation field depends already not only on ζ but also on $\nabla\zeta$, and is given by[9]

$$\varphi^{(1)}(\mathbf{R}) = \frac{1}{2\pi} \int_{S_0} \left\{ \frac{\partial(\varphi_0^{(0)} + \Phi_0)}{\partial x} \frac{\partial\zeta}{\partial x} + \frac{\partial(\varphi_0^{(0)} + \Phi_0)}{\partial y} \frac{\partial\zeta}{\partial y} - \left[\frac{\partial^2(\varphi^{(0)} + \Phi)}{\partial z^2}\right]_0 \zeta \right\} \frac{e^{ikR_1}}{R_1} d\mathbf{r}^2_\perp.$$

If a plane wave $\Phi = \exp[-i(\mathbf{k}_{0\perp} \cdot \mathbf{r}_\perp) + ik_{0z}z]$ is incident on an absolutely rigid area, then the regularly reflected field component is $\varphi^{(0)} = \exp[-i(\mathbf{k}_{0\perp} \cdot \mathbf{r}_\perp) - ik_{0z}z]$, and the average intensity of the scattered field in the far zone of the area S_0 is equal to

$$I = \frac{4S_0}{R^2} [k^2 - (\mathbf{k}_{0\perp}\varkappa_\perp)^2]\, W(\varkappa_x - k_{0x}, \varkappa_y - k_{0y}),$$

where $\varkappa$ is the wave vector in the observation direction, and W(u, v) is the spectral density of the uneven surface.

In scattering of electromagnetic waves, it is of interest to calculate the dispersion tensor

$$\mathscr{P}_{ik} = \langle (E_i - \langle E_i \rangle)(E_k - \langle E_k \rangle)^* \rangle \qquad (i, k = x, y, z),$$

which makes it possible to deduce the polarization properties of the scattered field. In first order perturbation theory, the components of the dispersion tensor are also proportional to the spectral density of the uneven surface[106]. This points to a selective mechanism of wave scattering in the far zone of an area with small irregularities[5], since the scattering in a given direction is determined by a single spectral component of the surface.

Within the framework of the first approximation it is easy to obtain an expression for the correlation function and to estimate the radii of the transverse and longitudinal correlations of the scattered field in the far zone of an individual surface irregularity[107]. Under the assumptions used in the derivation of (1.14), these estimates coincide with (1.14) at $(k\sigma)^2 \ll 1$. We note that they were initially obtained precisely by the perturbation method.

In some cases it is possible to solve the more general problem of reconstructing the correlation function of a rough surface from the correlation function of the scattered field[107]. As applied to a wavy water surface, an integral relation was derived in[108] for the description of the joint frequency-space-time correlation of the scattered field, and the corresponding correlation intervals were estimated[108,109].

Since the smallness of the scattered field in comparison with the specularly reflected one is ensured as soon as the conditions of applicability of perturbation theory are satisfied, the question of the correlation of the field amplitude and phase fluctuations is solved in the manner indicated in Sec. 1[110-111]. We can also consider the correlation of a narrow-band noise signal scattered from the surface[112].

We note, finally, that when the problem is solved by the perturbation method it is also possible to take into account such factors as the fluctuations of the surface impedance, the sphericity of the earth, the directivities of the radiator and of the receiver, and also the waveform of the incident pulse[113].

b) The second modification of the perturbation method is the Rayleigh method*, which is not connected with the use of Green's formula. According to Rayleigh, the scattered field is represented in the form of a superposition of plane traveling and inhomogeneous waves, each of which is a solution of the Helmholtz equation. Such a representation of the scattered field on an interface S is possible when this interface deviates little from a plane. In the case of a periodic interface S, one can indicate a numerical criterion for the validity of Rayleigh's assumption[114]. No such criterion was derived for statistically uneven surfaces.

This method was first used by Rayleigh[115] in the problem of scattering of a plane acoustic wave by a sinusoidal undulating surface. Using the representation of the uneven surface in the form of a Fourier series with random coefficients, Mandel'shtam[116], and later Andronov and Leontovich[117], developed on the basis of Rayleigh's method the theory of scattering of light by thermal fluctuations of a liquid surface. Later on the solution of the problem by the Rayleigh method was improved both in its dynamic part (representation of the uneven surface by a Fourier integral, calculation of the scattered field in the second approximation in the small parameter $|k\zeta|$) and in its statistical part (determination of the statistical characteristics of the reflected field independently of the nature of the irregularities). These generalizations were made in[118-122].

When a plane sound wave is incident on a surface, the total field in the upper and lower media, according to Rayleigh's hypothesis, is written in the form (the z axis is assumed to be directed downward)

$$\varphi(\mathbf{R}) = e^{-ikX\sin\theta}\left[e^{ikZ\cos\theta} + Ve^{-ikZ\cos\theta}\right] + \int_{-\infty}^{\infty} A(\varkappa_\perp)\, e^{i(\varkappa_\perp \mathbf{R}_\perp) - i\varkappa_z Z}\, d^2\varkappa_\perp, \tag{2.3}$$

$$\varphi'(\mathbf{R}') = V' e^{-ik'(X\sin\theta' - Z\cos\theta')} + \int_{-\infty}^{\infty} A'(\varkappa_\perp)\, e^{i(\varkappa_\perp \mathbf{R}'_\perp) + i\varkappa'_z Z'}\, d^2\varkappa_\perp, \tag{2.4}$$

where k and k′ are the wave numbers of the upper and lower media, respectively, V and V′ are the Fresnel coefficients of reflection and transmission, θ and θ' are the angles of incidence and reflection in the absence of surface irregularities, $\kappa_z = (k^2 - \kappa_\perp^2)^{1/2}$, $\kappa'_z = (k'^2 - \kappa_\perp^2)^{1/2}$.

The solution for a plane interface is given by formulas (2.3) and (2.4) if we put in them $A(\kappa_\perp) = A'(\kappa_\perp) = 0$. In the presence of irregularities, the problem reduces to a calculation of the unknown amplitudes $A(\kappa_\perp)$ and $A'(\kappa_\perp)$ from the boundary conditions

$$\rho\varphi = \rho'\varphi', \quad \frac{\partial\varphi}{\partial n} = \frac{\partial\varphi'}{\partial n} \quad \text{на } S, \tag{2.5}$$

where ρ and ρ' are the densities of the upper and lower media. This calculation is usually carried out by the perturbation method. We represent the unknown amplitudes in the form

$$\begin{aligned} A &= A_1 + A_2 + \dots, \\ A' &= A'_1 + A'_2 + \dots, \end{aligned} \tag{2.6}$$

where the n-th term is of the order of $|k\zeta|^n$. Assuming the quantities $\kappa_z\zeta$, $\kappa'_z\zeta$, and $\nabla\zeta$ to be small, we expand (2.3)—(2.5), taking (2.6) into account, in powers of these small parameters. Substitution of (2.3) and (2.4) in (2.5) leads then to a system of algebraic equation relative to the successive approximations of the unknown amplitudes, the solution of which yields the scattered field in the approximation of interest to us.

We note that since the representation (2.3) describes waves propagating only upwards from the surface, the Rayleigh method does not make it possible to take into account multiple scattering by the irregularities. Therefore attempts to obtain on its basis an exact solution of the problem[123-125] are apparently inconsistent.

In the far zone of a rough area, the first approximation of the Rayleigh method yields the selective scattering mechanism referred to above. Calculation in the second approximation[123] reveals depolarization of the electromagnetic wave in the plane of incidence.

The Rayleigh method is convenient for the solution of problems on scattering of an acoustic wave by an interface between a liquid and a solid, when transverse (shear) waves can exist in the solid in addition to the longitudinal waves[126-130]. The boundary conditions then become more complicated, since they express the continuity of the normal components of the displacement and of the stress, and also the vanishing of the tangential stress components.

c) The third modification of the perturbation method is connected with the use of nonlocal boundary conditions on the plane, first introduced by Bass.[131] It can be shown that in the first approximation the surface irregularities are equivalent to the presence on the plane of effective surface currents*

$$\mathbf{j}_e = (c/4\pi)(1-\varepsilon)\varepsilon^{-1}[\mathbf{n}, \nabla_\perp E_{0z}\zeta], \quad \mathbf{j}_m = (c/4\pi)\, ik(1-\varepsilon)\mathbf{E}_{0\perp}\zeta,$$

where $\mathbf{E}_0$ is the zeroth-approximation field and ϵ is the dielectric constant of the lower medium. The nonlocal boundary condition for the electromagnetic field in the presence of surface currents takes the form[132]

$$\begin{aligned} &[\mathbf{nE}(\mathbf{r})] + \frac{1}{2\pi}\int d^2\mathbf{r}'\, \frac{e^{ik\sqrt{\varepsilon}|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\left\{ik[\mathbf{n}[\mathbf{nH}]] - \frac{1}{\varepsilon}[\mathbf{n}\nabla E_z]\right\} \\ &\qquad = \frac{4\pi}{c}\left\{\mathbf{j}_e(\mathbf{r}) + \frac{ik}{2\pi}\int d^2\mathbf{r}'\, \frac{e^{ik\sqrt{\varepsilon}|\mathbf{r}-\mathbf{r}|'}}{|\mathbf{r}-\mathbf{r}'|}\left[\mathbf{n},\ \mathbf{j}_m + \frac{1}{k^2\varepsilon}\nabla(\nabla\mathbf{j}_m)\right]\right\}. \end{aligned}$$

This boundary condition forms, together with Green's vector formula, a system of integral equations which is solved by the Fourier-transformation method.

Using the solution obtained in this manner, Bass[132] investigated in the quasistatic approximation the frequency spectrum of a field scattered by gravitational-capillary waves on a surface of a heavy incompressible liquid. The calculations have shown that in this case the spectrum consists of two frequencies symmetrically located relative to the incident-signal frequency. In general, it is shown in the cited paper that in the case of a determined surface whose shape satisfies the equation

*Sometimes called Rice's method in the literature.

*$[\mathbf{n}, \nabla_\perp \mathbf{E}_{0z}\zeta] \equiv \mathbf{n} \times \nabla_\perp \mathbf{E}_{0z}\zeta$.

$$\hat{L}\left(\frac{\partial}{\partial t},\ \nabla_{\perp}\right)\zeta(\mathbf{r}_{\perp},\ t)=0,$$

where $\hat{L}$ is an arbitrary linear operator, the frequency spectrum consists of l harmonics with frequencies $\omega_0 + \Omega_l$, where Ω_l are the roots of the dispersion equation

$$\hat{L}\left[-i\Omega,\ -i(\mathbf{k}_{0\perp}-\varkappa_{\perp})\right]=0.$$

In the presence of dissipative processes, the spectral-line broadening investigated in[133] is observed.

When waves are scattered by a surface with random irregularities, the nonlocal boundary conditions can be derived also directly for the moments of the scattered fields. This obviates the need for solving the dynamic part of the problem. If the Leontovich boundary conditions are valid at each point of the surface, then we have for the mean value of the scattered field the following boundary condition[134]:

$$\langle E_{x,y}\rangle=\frac{\langle\zeta^2\rangle}{2\pi}\int dS'\left\{\frac{e^{ik\rho}}{\rho}\left[\Delta_\rho\frac{\partial K(\rho)}{\partial\rho_{x,y}}\langle E_z\rangle-\nabla_\rho\frac{\partial K(\rho)}{\partial\rho_{x,y}}\nabla_{\mathbf{r}'_\perp}\langle E_z\rangle+\frac{\partial K(\rho)}{\partial\rho_{x,y}}\nabla_{\mathbf{r}'_\perp}\frac{\partial\langle\mathbf{E}_\perp\rangle}{\partial z}-\frac{\partial\langle\mathbf{E}_\perp\rangle}{\partial z}\nabla_\rho\frac{\partial K(\rho)}{\partial\rho_{x,y}}\right]+V_z''\left[\frac{\partial K(\rho)}{\partial\rho_{x,y}}\langle E_z\rangle-\frac{\partial\langle E_{x,y}\rangle}{\partial z}K(\rho)\right]\right\}\mp\eta\langle H_{y,x}\rangle, \tag{2.7}$$

where $K(\rho)$ is the correlation coefficient of the irregularities, η is the surface impedance, and

$$V_z''=\lim_{z\to 0}\frac{\partial^2}{\partial z^2}\frac{e^{ik(\rho^2+z^2)^{1/2}}}{(\rho^2+z^2)^{1/2}}.$$

Bass[135] calculated with the aid of (2.7) the reflection coefficients of the average field from a statistically uneven surface following the incidence of an acoustic as well as an electromagnetic wave. In particular, for the reflection coefficient of an acoustic wave from an absolutely soft surface the formula is

$$V=-1+2k\langle\zeta^2\rangle\left\{k-i\int_0^\infty\frac{e^{ik\rho}}{\rho}\frac{d}{d\rho}\left[K(\rho)I_0(k_{0x}\rho)\right]d\rho,\right. \tag{2.8}$$

and becomes much simpler under the conditions $kl \gg 1$, $(\pi/2)-\theta \gg (2/kl)^{1/2}$, namely,

$$V=-1+2k\langle\zeta^2\rangle\cos^2\theta.$$

Let us list, finally, a few more problems solved by the method of small perturbations. These include the problem of wave reflection from a dielectric layer with statistically uneven boundaries[136,137], the calculation of the intensity of a sound field passing through an air-water interface[138,139], the calculations of the characteristics of a surface wave produced in scattering from the boundary of a liquid[140], and the transformation of a surface wave into a three-dimensional one when propagating along a rough surface[141].

d) The real surfaces produced by natural causes have as a rule a complicated structure. They are characterized by several correlation scales, and sometimes by sets of them. If the conditions for the use of the perturbation method hold for such irregularities, then the dynamic part of the scattering problem remains unchanged, and the complex structure of the irregularities is taken into account in the second, statistical part of the problem. Thus, for example, in the calculation of the average intensity of the scattered field in first order of perturbation theory, the structure of the irregularities is accounted for in the explicit form of the spectral density of the surface.

Real irregularities, however, can frequently be regarded as large-scale formations on which a slight ripple is superimposed. Such a model can be used, for example, to describe the wavy surface of the sea. In this case it is necessary to modify the solution of the scattering problem even in its dynamic part.

The method proposed by Kur'yanov[142] for calculating the scattered field consists of regarding the slight ripples as a small perturbation. The scattered field is then calculated by the perturbation method, using as the first approximation the field scattered by the smooth large-scale irregularities, which is calculated by the Kirchhoff method. This method was used in[143-146]* to calculate a number of statistical characteristics of a scattered electromagnetic field under the assumption that both types of irregularity are statistically independent.

A somewhat different calculation method, based on incoherent addition of fields scattered by small-scale irregularities, was proposed by Semenov[148-149]. If the intensity of a wave incident on an element dS of an unperturbed surface is denoted by I_0 = const, then the intensity of the incoherently scattered field in the far zone is written in the form

$$I(\mathbf{R})=\frac{I_0}{R^2}\int_S F(\chi,\ \varphi,\ \theta)\,dS, \tag{2.9}$$

where $F(\chi, \varphi, \theta)$ is the indicatrix for wave scattering from an elementary area Ds containing a slight ripple, and depends on the local incidence angle θ and observation angles χ and φ.

Formula (2.9) is valid at not too large inclination angles of the large-scale irregularities, when the mutual influence of the neighboring sections of the surface S can be neglected. As to the intensity of the coherently reflected waves, its calculation calls for knowledge of the scattered field itself. The latter can be obtained by the Kirchhoff method, using instead of the Fresnel reflection coefficients the average-field coefficients of the type (2.8), calculated with the aid of the nonlocal boundary conditions.

3. METHODS ACCOUNTING FOR MULTIPLE SCATTERING BY THE SURFACE

We describe here two methods which in our opinion are the most promising ones, in which account can be taken of multiple scattering from an uneven surface. These are the integral-equation method and the Green's function method.

a) The first of them was proposed by Lysanov[150] in connection with the solution of the problem of scattering of an acoustic wave by a surface with one-dimensional periodic irregularities. The method consists of an approximate calculation of the field or of its normal derivative on the surface S from Green's integral formula

$$\varphi(\mathbf{R})=\Phi(\mathbf{R})-\frac{1}{4\pi}\int_S\left\{\varphi(\mathbf{r})\frac{\partial}{\partial n}\frac{e^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|}-\frac{\partial\varphi(\mathbf{r})}{\partial n}\frac{e^{ik|\mathbf{R}-\mathbf{r}|}}{|\mathbf{R}-\mathbf{r}|}\right\}dS, \tag{3.1}$$

where φ is the total acoustic field, $\mathbf{n}$ is the outward normal to S, and Φ is the field of the primary wave. The field at an arbitrary point $\mathbf{R}$ over the surface is then ob-

*See also the discussion of[146] in[147].

tained by substituting the obtained values of $\varphi(\mathbf{r})$ and $\partial\varphi(\mathbf{r})/\partial n$ in (3.1).

Let the scattering surface be absolutely soft; then $\varphi(\mathbf{r}) = 0$ on the interface S. Making the observation point identical with the point $\mathbf{r}'$ on S, we obtain from (3.1) an integral equation with respect to $\partial\varphi/\partial n$ on S

$$\Phi(\mathbf{r}') = -\frac{1}{4\pi}\int_S \frac{\partial\varphi(\mathbf{r})}{\partial n}\frac{e^{ik|\mathbf{r}'-\mathbf{r}|}}{|\mathbf{r}'-\mathbf{r}|}dS. \tag{3.2}$$

If the inequalities

$$\frac{1}{2}\left|\frac{\partial\zeta}{\partial x}\right|^2_{\max} \approx \frac{1}{2}\left(\frac{\sigma}{l}\right)^2 \ll 1,\quad k\sigma\left|\frac{\partial\zeta}{\partial x}\right|_{\max} \approx \frac{(k\sigma)^2}{kl} \ll 1 \tag{3.3}$$

are satisfied, the kernel of (3.2) reduces to a difference kernel. Solving this equation in the case of a plane primary wave with the aid of the Fourier transformation and substituting the value of $\partial\varphi(\mathbf{r})/\partial n$ obtained in this manner in (3.1), we obtain[151]

$$\begin{gathered}\varphi(\mathbf{R}) = -\frac{1}{4\pi^2}\int d^2\mathbf{q}_\perp \frac{e^{i\left(\mathbf{q}_\perp\mathbf{R}_\perp + \sqrt{k^2-q_\perp^2}\,z\right)}}{\sqrt{k^2-q_\perp^2}}\int d^2\varkappa_\perp\sqrt{k^2-\varkappa_\perp^2}\,\cdot\\ \tilde{F}_{k_{0z}}(\varkappa_\perp-\mathbf{k}_{0\perp})\,\tilde{F}_{\sqrt{k^2-q_\perp^2}}(\mathbf{q}_\perp-\varkappa_\perp),\end{gathered} \tag{3.4}$$

where

$$\tilde{F}_{k_{0z}}(\varkappa_\perp-\mathbf{k}_{0\perp}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ik_{0z}\zeta - i(\varkappa_\perp-\mathbf{k}_{0\perp})\mathbf{r}_\perp}d^2\mathbf{r}_\perp,$$

$\mathbf{k}_0 = (\mathbf{k}_{0\perp}, -k_{0z})$ is the wave vector of the incident wave.

Averaging (3.4) over the ensemble of surfaces, we obtain for statistically isotropic irregularities the reflection coefficient of the average field

$$V = -e^{-(k_z\sigma)^2} + \frac{1}{k_{0z}}\int_0^\infty \varkappa_\perp d\varkappa_\perp (k^2-\varkappa_\perp^2)^{1/2}\int_0^\infty \rho\, d\rho\, I_0(k_{0\perp}\rho)\, I_0(\varkappa_\perp\rho)\, f_1(\rho), \tag{3.5}$$

where $f_1(\rho) = e^{-(k_z\sigma)^2} - f_{2\zeta}(-k_{0z}, -k_{0z}, \rho)$ and $f_{2\zeta}$ is the two-dimensional characteristic function of the irregularities.

In the limiting case of small $k_z\sigma$ (in which case, according to (3.3), $kl \gg 1$), there follows from (3.5) the expression (2.8) cited above and obtained with the aid of the nonlocal boundary conditions. In the case of nonglancing incidence of a plane wave on the surface, at large $k_z\sigma$ (in which case we must have $kl \gg 1$) we obtain from (3.5) the same average-field reflection coefficient as in Kirchhoff's method (see (1.8)).

For the average intensity of the total field in the Fraunhofer zone of a rough area S we obtain

$$\begin{aligned}I = \frac{S_0}{R^2}\iint_{-\infty}^{\infty} d^2\mathbf{q}_{1\perp}\, d^2\mathbf{q}_{2\perp}\,\tilde{f}(\mathbf{v}_\perp-\mathbf{q}_{1\perp},\ \mathbf{k}_{0\perp}-\mathbf{q}_{2\perp},\ \mathbf{q}_{2\perp}-\mathbf{v}_\perp)\\ \times\sqrt{k^2-q_{1\perp}^2}\left(\sqrt{k^2-q_{2\perp}^2}\right)^*,\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}\tilde{f} = \frac{1}{(2\pi)^6}\iiint_{-\infty}^{\infty} d^2\mathbf{r}_{1\perp}\, d^2\mathbf{r}_{2\perp}\, d^2\mathbf{r}_{3\perp} f_{4\zeta}(-k_{0z},\ -\nu_z,\ k_{0z},\ \nu_z,\ \mathbf{r}_{1\perp},\ \mathbf{r}_{2\perp},\ \mathbf{r}_{3\perp})\\ \times e^{i(\mathbf{v}_\perp-\mathbf{q}_{1\perp})\mathbf{r}_{1\perp} - i(\mathbf{q}_{2\perp}-\mathbf{k}_{0\perp})\mathbf{r}_{2\perp} - i(\mathbf{v}_\perp-\mathbf{q}_{2\perp})\mathbf{r}_{3\perp}},\end{aligned}$$

$\boldsymbol{\nu}$ is the wave vector in the observation direction.

In cases when the conditions for the applicability of the Kirchhoff method or of the perturbation method are satisfied, formula (3.6) goes over into the expressions obtained by these methods for the average intensity.

Following the procedure described here, we can obtain, under the same conditions (3.3), an expression for the correlation function of the scattered field, and also consider the case of a spherical primary wave[152].

Thus, even if we assume that the inequalities (3.3) are satisfied, the integral-equation method gives more exact expressions for the first two moments of the scattered field than the Kirchoff method and the method of small perturbations.

b) The Green's-function method is based on finding and solving the equations directly for the statistical characteristics of the scattered field. It is not necessary here to solve the dynamic part of the problem, i.e., to find the field scattered by each of the realizations of the random surface. This method, initially developed in quantum field theory, was subsequently used to find the statistical characteristic of a field passing through a randomly-inhomogeneous medium[153,154].

It was first used to calculate the moments of a field scattered by an uneven interface by Bass, Freĭlikher, and Fuks, who considered both scattering by a rough plane[155] and propagation in a waveguide with rough walls[156-158].

Let $\mathbf{R}_\Sigma = \mathbf{r} + \mathbf{n}(\mathbf{r})\,\zeta(\mathbf{r})$ be the radius vector of an uneven surface Σ, consisting of a regular surface S with a radius vector $\mathbf{r}$ and a normal $\mathbf{n}(\mathbf{r})$, and random irregularities $\zeta(\mathbf{r})$. Assuming the surface Σ to be absolutely soft, we denote by $G(\mathbf{R}, \mathbf{R}_0)$ the Helmholtz-equation Green's function satisfying the boundary condition $G(\mathbf{R}_\Sigma, \mathbf{R}_0) = 0$. If we confine ourselves to terms linear in ζ, this boundary condition takes the form

$$G(\mathbf{r}, \mathbf{R}_0) + \zeta(\mathbf{r})\frac{\partial}{\partial n}G(\mathbf{r}, \mathbf{R}_0) = 0. \tag{3.7}$$

From Green's theorem with allowance for (3.7) we get an integral equation for $G(\mathbf{R}, \mathbf{R}_0)$

$$G(\mathbf{R}, \mathbf{R}_0) = G_0(\mathbf{R}, \mathbf{R}_0) - \frac{1}{4\pi}\int_S \frac{\partial}{\partial n}G_0(\mathbf{R}, \mathbf{r})\,\zeta(\mathbf{r})\frac{\partial}{\partial n}G(\mathbf{r}, \mathbf{R}_0)\,dS, \tag{3.8}$$

where $G_0(\mathbf{R}, \mathbf{R}_0)$ is the Green's function of the Helmholtz equation with boundary condition $G_0(\mathbf{r}, \mathbf{R}_0) = 0$ on S; this function is assumed known. The surface S is assumed plane.

Assuming the distribution function of ζ to be Gaussian, iterating in (3.8), and using a Feynman-diagram technique[153,154], it is possible to obtain a closed Dyson equation for the average Green's function $\langle G(\mathbf{R}, \mathbf{R}_0)\rangle$ and the Bethe-Salpeter equation for the second moments of the scattered field. Representing $\langle G(\mathbf{R}, \mathbf{R}_0)\rangle$ in the form of a superposition of plane waves, each of which is reflected from the surface and has its own reflection coefficient $V(\boldsymbol{\kappa}_\perp)$, we can find approximate expressions for $V(\boldsymbol{\kappa}_\perp)$ in which, however, multiple reradiation of the primary field by the irregularities are taken into account.

In the Bourret approximation, the reflection coefficient $V(\boldsymbol{\kappa}_\perp)$ coincides with that obtained in[135] with the aid of the nonlocal boundary conditions for the average field. A more exact value of $V(\boldsymbol{\kappa}_\perp)$ is obtained by separating in the mass operator a certain infinitely-summable subsequence of diagrams. One such possibility was indeed considered in the cited paper[155]. For $V(\boldsymbol{\kappa}_\perp)$ we have in this case the expression

$$V(\varkappa_\perp) = \frac{1-(\varkappa_z/4\pi^2)\,\gamma(\varkappa_\perp)}{1+(\varkappa_z/4\pi^2)\,\gamma(\varkappa_\perp)},$$

where

$$\gamma(\varkappa_\perp) = \int\frac{(k^2-p_\perp^2)^{1/2}\,\tilde{K}(\varkappa_\perp-\mathbf{p}_\perp)}{1+(p_z/4\pi^2)\int (k^2-q_\perp^2)^{1/2}\,\tilde{K}(\mathbf{p}_\perp-\mathbf{q}_\perp)\,d^2\mathbf{q}_\perp}\,d^2\mathbf{p}_\perp,$$

$\tilde{K}(\kappa_\perp)$ is the spectral density of the uneven surface, and $\kappa = (\kappa_\perp, \kappa_z)$ is the wave vector of the plane wave reflected from the surface.

To find $V(\kappa_\perp)$ we can also use the Dyson nonlinear equation obtained in the simple-vertex approximation.

In the problem of wave propagation in a waveguide having statistically rough walls, the Green's function method makes it possible to calculate the change of the spectrum of the normal waves[156], to calculate the attenuation of the average field both at the critical frequency[157] and at subcritical frequencies, and finally to derive the radiation transport equation from the Bethe-Salpeter equation.[158]

4. CONCLUSION

The theory of wave scattering by statistically uneven surfaces is treated in the literature in two ways corresponding to two methods of solving the dynamic part of the problem, namely the Kirchhoff approximation and the perturbation method. These methods solve the problem of finding the statistical characteristics of the scattered field for either smooth irregularities or irregularities that are small compared with the wavelength, and also in the case when slight ripple is superimposed on large-scale irregularities. Formulas connecting the statistical characteristics of the rough surface with the statistical properties of the field scattered by were obtained for irregularities of these types. The results of the theory have been confirmed by a number of experiments, both in laboratories and under natural conditions. In this review we hardly touched upon the experimental studies, for their description could be the subject of a separate review.

In addition to the traditional methods for solving the problem, which were cited above, more rigorous methods have been successfully developed recently, namely the integral-equation method and the Green's function method. These make it possible to take into account multiple scattering of waves from a surface. The results obtained by these methods are of undoubted interest from the theoretical point of view, and possibly will become of practical value when the experimental accuracy reaches the appropriate level.

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