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EXPERIMENTAL STUDIES OF THE $\pi\pi$ INTERACTION

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I. THE SUBJECT OF THE REVIEW

THIS review is devoted to one of the important problems of elementary particle physics—the experimental study of the interaction between π mesons. Knowledge of the quantities characterizing the pion-pion interaction is no less important than knowledge of the πN-interaction constants. The $\pi\pi$-interaction amplitudes are fundamental quantities in the sense that they enter into the description of a large group of very diverse processes. Figure 1 shows some very simple diagrams describing various processes of production and decay of particles, which involve the scattering vertex of two pions. The number of such diagrams can easily be increased.

We will further discuss processes in which the $\pi\pi$ interaction is important, since by analysis of just these processes we can obtain interesting data by an indirect means. It should be noted that as recently as ten years ago we knew practically nothing about the pion-pion interaction, and assumed only that it is a strong interaction. Even our present knowledge of low-energy $\pi\pi$ scattering cannot be said to be exhaustive. However, it is reasonable to say that the principal regularities and characteristics are clear and, what is particularly important, methods of experimental investigation have been developed and tested.

The fact is that direct experiments on the scattering of free pions by free pions are not possible at this time. The π-meson lifetime is approximately 2×10^{-8} sec. Therefore there are no pion targets, and we cannot yet perform experiments on the mutual scattering of pion beams, which cannot conceivably be accumulated in storage rings except in accelerators with energies considerably higher than present-day machines.

Thus, the $\pi\pi$ interaction can be studied only by indirect means. Examples are the study of production of pions by pions, decay of K-mesons and resonances into two or more pions, and production of pion pairs in the annihilation of antiprotons or in collisions of electrons and positrons in colliding beams. There are fundamental difficulties, in that the processes enumerated are not always completely described by the diagrams of Fig. 1, and furthermore, the corresponding diagrams by no means always determine these processes. As an illustration we have shown in Fig. 2 a series of diagrams which can provide, and in a number of cases, as we shall see, do provide a decisive contribution to the production reaction

$$\pi + N \rightarrow \pi + \pi + N. \tag{1}$$

Thus, the question of experimental study of the $\pi\pi$ interaction is a question of separating a definite class of diagram. For just this reason, it will be necessary in the subsequent discussion to dwell in considerable detail on the mechanism of occurrence of certain reactions, particularly reaction (1). These questions are involved in the technique for investigation of the pion-pion interaction.

II. CURRENT TECHNIQUES FOR STUDY OF THE $\pi\pi$ INTERACTION

1. Mechanism of the Reaction $\pi + N \rightarrow \pi + \pi + N$

The characteristics of particles emitted in production of pions by pions from nucleons can be described to a first approximation by phase-space curves in whose calculation the specific nature of the interaction is ignored and only the conservation laws are taken into account. This statistical approach reflected the principal features of the spectra of particles in reaction (1), which were measured at first with large statistical errors; it was used successfully to describe reactions with production of a large number of mesons (the larger the number, the more successfully), predicted the multiplicity of particle production as a function of energy, and was confirmed as the initial approach in analysis of experiments on inelastic scattering. However, even the early com-

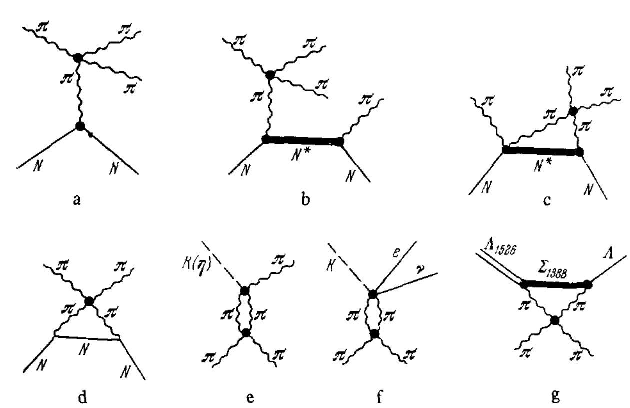

FIG. 1

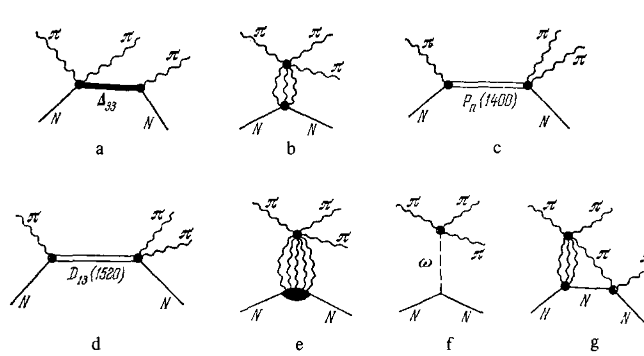

FIG. 2

parisons of more accurate spectra of the pions from reaction (1) with theory indicated the inadequacy of this approach. This is not surprising, since it was clear a priori that the interaction of the hadrons in the final state should have an effect. More surprising is the success of the statistical approach. Surprising and convenient, since up to the present day we still do not know how to take into account the final-state interaction correctly in any case.

Substantial progress was made when various isobar models began to be considered. The essence of these models is that reaction (1) is represented as occurring in two steps:

$$\pi + N \rightarrow \pi + N^* \rightarrow \pi + \pi + N \qquad (2)$$

(see diagram 2a)*, where N*—the excited state of the nucleon—most frequently turns out to be Δ_{33}, the well known isobar with mass 1236, isotopic spin T = 3/2, and spin J = 3/2. The existence of isobars was known from direct experiments on pion-nucleon scattering, from which it also followed that (at least at low relative energies of the pion and nucleon) they determine the nature of πN scattering.

The isobar model for reaction (1) was initially formulated by Lindenbaum and Sternheimer[1] (the LS model) on the assumption of isotropic production and decay of the isobar. The greater accuracy of the model of Bergia, Bonsignori, and Stanghellini[2] (the BBS model) is due to inclusion of interference of the amplitudes corresponding to isobaric states of the first pion with the nucleon and of the second pion with the nucleon.

However, a more important improvement is the inclusion, proposed by Anisovich,[3] of isobar production not only in the S state but also the contribution of other isobaric states and even nonisobaric states (the Yodh-Olsson model[4]) and the decay of the Δ_{33} isobar in the P wave in accordance with its spin. We will not occupy ourselves here with comparison of the experimental data with calculations using the various models—a good review of the models and of other questions related to the reaction (1) mechanism can be found in ref. 5; we will note only that current models claim to give a successful quantitative description of the spectra and angular distributions of the pions, the dependence of the number of events on the relative energy of the pion and nucleon, the ratio of the cross sections for the reaction in different charge states, and so forth. It is important, however, that even in these respects the models require improvement as the experimental errors are reduced and as attempts are made to describe the experimental data more accurately. An example of an improved model is that developed by Anisovich et al.[6], who take into account, in addition to isobar production, diagrams with rescattering of the pions (Fig. 1d). However, this is already an intrusion into another field, to the discussion of which we will progress.

The definite success of isobar models in description of the experimental data indicated the correctness of the chosen method of taking into account the specific nature of the interaction. On the other hand, all of the still existing discrepancies, which are most noticeable in comparison with theory of the low-momentum parts of the spectra of nucleons and the dependence of the number of events on the relative energy of the pions, made it necessary to attempt to include the interaction of the pions in the final state. Here it is appropriate to digress and to introduce some convenient variables for the characteristics of reaction (1), particularly in the case where we are discussing inclusion of the $\pi\pi$ interaction. These are: the pion energy in their center-of-mass system, ω, which is the invariant mass of the dipion system, and p^2—the squared 4-momentum transfer to the nucleon, which at small p^2 is identical with the squared nucleon momentum in the laboratory system.

Distributions of the number of reaction (1) events as a function of ω, or the invariant mass spectra, rather rapidly indicated the existence of features in the $\pi\pi$ system—resonances—whose inclusion was just as necessary as the isobars. By analogy with the isobar models, the scheme for including resonances is as follows:

$$\pi + N \rightarrow N + \omega^* \rightarrow N + \pi + \pi, \qquad (3)$$

where ω^* is any resonance in the $\pi\pi$ system. In part 7 of this chapter we will discuss means of studying resonances, and in part 3 of the fourth chapter the principal data on $\pi\pi$ resonances. However, we still limit ourselves to the question: Is it possible to include the nonresonance $\pi\pi$ interaction, which does not appear as explicitly as the resonances. More accurately, having reversed the problem, we will ask in study of reaction (1), can we obtain information on the nonresonance $\pi\pi$ interaction?

There is no doubt that the possibility exists in principle. It is sufficient if we understand ω^* to mean nonresonance states with definite phase shifts. However, the difficulties are just as certain as the possibility. We have already emphasized the success of isobar models, in which the occurrence of reaction (1) is determined by strong pion-nucleon interactions. It is true that we pointed out the necessity of inclusion of pion-pion scattering in the model.[6] Anisovich et al.[6] have assumed that reaction (1) at an initial pion energy of 350–600 MeV, where appreciable production of the Δ_{33} isobar is observed, is completely determined by isobar diagrams 2a, 2c, 2d, and in addition by the isobar diagram with a pion-pion vertex (see Fig. 1c). With this model they obtained satisfactory agreement with the results of a phase-shift analysis of the elastic and inelastic partial cross sections, adequate agreement with experiment for the total cross sections of all five charge channels of reaction (1), and a description of the pion spectra; they found further that on the assumption of zero scattering length for pions in the state with isospin T = 2 the best agreement with experiment in the pion spectra is obtained for a $\pi\pi$-scattering amplitude which is negative and close to unity in the state with T = 0. However, these conclusions cannot be considered final, since it is not completely clear what will be the effect of reaction (1) channels which have not been included. Turning to the general analysis of reaction (1), we can indicate temporary attempts at phase-shift analysis for relatively low energies,[7-10] where in addition to the strong πN interaction in the final state, in various phase shifts it is necessary to consider in addition the so-called σ meson, which apparently takes into account effectively the $\pi\pi$ interaction. However, such a global approach does not seem promising for the study of pion-pion scattering. In the general case there are too many free parameters, and on the other hand, not enough experimental points. The uncertainty regarding the σ meson, the problem of whose existence we will be forced to discuss later, is apparently a reflection of the arbitrariness which is developing. Therefore, recalling the main problem—the obtaining of information on the $\pi\pi$ interaction, we will subsequently be interested more in special cases of the occurrence of reaction (1).

2. Production of Pions by Pions Near Threshold

About ten years ago Gribov and his co-workers began to develop a rigorous method of considering reactions near the threshold for production of three particles in the final state.[11-14] Without going into the details of the theory, which has been described in detail in review articles and lectures by some of the authors,[15-17] we will note that for a small energy release, when the relative momenta of the particles k_{ij} in the final state are small, the amplitude A_0 of the process can be expanded in a series in powers of the small relative momenta of the particles:

$$A_0 = f_0(1 + ia_{12}k_{12} + ia_{23}k_{23} + ia_{13}k_{13}) + \ldots, \tag{4}$$

where f_0 is a common unknown complex factor, which is identical, however, for all linear terms, and a_{ij} are the rescattering amplitudes of the corresponding particles in the final states.

For the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n \tag{5}$$

there is the possibility in the final state of the charge exchanges

$$\pi^- + \pi^+ \rightarrow \pi^0 + \pi^0, \tag{6}$$

$$\pi^+ + n \rightarrow p + \pi^0, \tag{7}$$

and in Eq. (4) for the amplitude there appear linear terms of the form

$$if_1Ck_{12} \text{ and } if_2Dk_{13}.$$

If the complexity of f_0, f_1, and f_2 is not the same, then the linear terms are preserved also in the expression for the cross section of reaction (5) near threshold, which if we neglect quadratic terms in k_{ij} is

$$d\sigma/d\Gamma = \text{const}\cdot[1 + Ak_{12} + Bk_{13}]. \tag{8}$$

Here $d\sigma/d\Gamma$ is the differential cross section per unit phase space,

$$A = {}^2/_3(a_2 - a_0)\cdot\alpha_{12}, \quad B = (\sqrt{2}/3)(b_{3/2} - b_{1/2})(-\sqrt{2})\,\alpha_{12},$$

$$\alpha_{12} = \frac{3\sin(\delta_{31} - \delta_{11})}{F_{11}\sqrt{10}/F_{31} + F_{31}/\sqrt{10}F_{11} - 2\cos(\delta_{31} - \delta_{11})},$$

where δ_{31} is the phase shift for elastic πN scattering with T = 3/2, J = 3/2 for a pion energy of ≈ 200 MeV at the threshold for reaction (1); δ_{11} is the phase shift for elastic πN scattering with T = 1/2, J = 1/2 at the same energy; F_{11} and F_{31} are the amplitudes of the transitions $\pi N \rightarrow \pi\pi N$ at threshold, when all of the particles produced have zero kinetic energy in the center-of-mass system. $A/B = (a_2 - a_0)/(b_{1/2} - b_{3/2})$, the ratio of the coefficients for the linear terms, is directly expressed in terms of the ratio of charge-exchange amplitudes, which are equal to the differences in the corresponding scattering lengths. $a_2 - a_0$ is the difference in the pion-pion scattering lengths of interest to us in states with isospin T = 2 and T = 0; $b_{1/2} - b_{3/2}$ is the difference in the lengths for scattering of pions by nucleons in isotopic states with T = 1/2 and T = 3/2, which are known from direct, independent experiments.

This method of obtaining information on the pion-pion interaction, which has been called the Ansel'm-Gribov method in the literature, is extremely attractive, since it is exact for $k_{ij} \rightarrow 0$ and in real cases ($k_{ij} \neq 0$) permits its validity to be checked by investigation of how $d\sigma/d\Gamma$ follows Eq. (8); it permits finding out not only the magnitude but also the sign of the difference in the $\pi\pi$ scattering lengths, $a_2 - a_0$, relative to the known sign of the difference $b_{1/2} - b_{3/2}$. On the other hand, the Ansel'm-Gribov method in the form just described is limited. We can hope to obtain information only on the charge exchange amplitude (6) for zero relative pion energy. From Eq. (4) it follows that the amplitudes for rescattering of the particles, in particular, the amplitudes of interest to us for the scattering $\pi^+\pi^- \rightarrow \pi^+\pi^-$ in reaction (5), require for their extraction a knowledge of the quadratic terms in k_{ij} in the expression for the reaction cross section. Generally speaking, for an accidental reason, the smallness of α_{12}, Eq. (8) is valid only for incident pion energies exceeding the threshold by the order of 1 MeV. In other cases (and today this means always), to obtain information on pion-pion charge exchange, it is also necessary to include as a minimum the quadratic terms. In fact, α_{12} is proportional to $\sin(\delta_{31} - \delta_{11})$, and the phase shifts δ_{31} and δ_{11} are of the order of 5°, so that $\sin(\delta_{31} - \delta_{11})$ is of the order of 1/5. Hence A and B are of the order of $(1/5)\mu$ (μ is the pion mass). The omitted terms in Eq. (8) are of order k^2/μ^2. Neglect of these terms is justified if $k/5\mu \gg k^2/\mu^2$. If "much greater" means a factor of three, then $k = 1/15\mu$, which corresponds to a kinetic energy of the order of 1 MeV (in the c.m.s. of the reaction).

Thus, study of the pion-pion interaction by the Ansel'm-Gribov method: involves inclusion of a large number of terms; requires substantially better experimental accuracy, in particular, in elastic πN scattering to obtain values of the higher phase shifts at low pion energy; requires more accurate measurement of the differential cross section for reaction (1) near threshold where, naturally, it is extremely small. The Ansel'm-Gribov method loses its attractiveness. In its simplest form it is cited because, on the one hand, it stimulated a number of the first experiments on study of the pion-pion interaction and, on the other hand, the possibility is not excluded of its successful application to study of the reactions[17]

$$\begin{gathered} \pi^- + p \rightarrow N + 3\pi, \\ p + p \rightarrow 2N + 2\pi, \\ d + d \rightarrow He^4 + \pi^+ + \pi^- \end{gathered} \tag{9}$$

and to obtain information on the $K\pi$ interaction in reactions with production of pions by K mesons.

3. Peripheral Interaction

Let us discuss one of the models of the pion-nucleon interaction which leads to pion production. In this model, pion-pion scattering which is of interest to us is separated in explicit form. We are discussing scattering of the incident pion by a virtual pion from the pion cloud surrounding the nucleon. It seems that, having separated the corresponding part of the reaction, we can obtain information on $\sigma_{\pi\pi}$.

Here, evidently, an analogy will be useful with the method of determining the cross section of the nucleon-nucleon (or πN) interaction from experiments on quasi-elastic scattering of nucleons (pions) by nuclei. Rather old and well known experiments have shown that to a first approximation the scattering of high energy particles by nuclei can be represented as scattering by the individual moving nucleons inside the nucleus. Here we obtain more accurate information on free nucleon-nucleon scattering as the momentum of the nucleon in the nucleus is less and, consequently, as the momentum of the recoil nucleon is less.

By analogy we can expect that in the case of scattering of pions by virtual mesons of the nucleon's meson cloud we will obtain more accurate information on free pion-pion scattering as the momentum of the recoil nucleon becomes smaller. This corresponds to peripheral collisions: scattering of the incident pion by a meson at the periphery of the nucleon.

We can distinguish the role of the peripheral interactions in a somewhat different way. We are interested at the moment in collisions with one pion, but not with a K meson, not with three pions, and so forth. The characteristic radius of separation of one pion from the nucleon is $1/\mu_\pi$, of a K meson—$1/\mu_K$, of three pions—$1/\mu_{3\pi}$, and so forth. The pion is the lightest particle of the strongly acting particles in resonances, so that the spherical shell in the nucleon's meson cloud from radius $1/\mu_\pi$ to radius $1/\mu_{3\pi}$ is filled mainly with single pions. (Here and subsequently $\hbar = c = 1$; we will further express all quantities in units of the pion mass.) Peripheral collisions are just interactions with momentum transfer ~ 1 at distances also of ~ 1.

Remaining within the bounds of somewhat naive geometrical representations and assuming in addition that the probability of interaction is simply proportional to the volume occupied by the virtual particles, some authors, particularly in the first stage of study of the pion-pion interaction, have attempted to extract it from the total cross sections for reaction (1), in particular, comparing the cross sections of different charge states of the reaction. The peripheral model, of course, reflects certain general features of reaction (1), but we already have seen (see part 1) that it is inadequate to describe all cases of the reaction. Special separation of events with small momentum transfer is necessary.

The importance of studying peripheral interactions to obtain information on the scattering of pions by pions was emphasized by Okun' and Pomeranchuk,[18] who proposed to separate phases with large orbital momenta in the scattering. A suitable arrangement for determining the $\pi\pi$-scattering cross section was first proposed by Chew and Low.[19]

4. The Method of Chew and Low

Scattering of the incident pion by a virtual pion from the pion cloud is represented by the diagram shown in Fig. 1a. This is the so-called pole diagram. In fact, the differential cross section for reaction (1) described by the diagram of Fig. 1a has the following form:

$$\frac{d^2\sigma}{dp^2\,d\omega^2} = \alpha\,\frac{f^2}{2\pi}\,\frac{1}{q^2}\,\frac{p^2}{(p^2+1)^2}\,\omega\sqrt{\frac{\omega^2}{4}-1}\;\sigma_{\pi\pi}(\omega). \qquad (10)$$

Here α is a numerical coefficient which is equal to 1 or 2, depending on the charge state of reaction (1) to which formula (10) refers; $f^2 = 0.08$ is the πN-interaction coupling constant (the lower vertex in Fig. 1a); q is the incident pion momentum in the laboratory system; ω is the total energy of the two pions in their center-of-mass system, $\omega_{\min} = 2$; p^2 is the squared 4-momentum transfer, and $\sigma_{\pi\pi}$ is the $\pi\pi$-interaction cross section (in which we are interested) for energy ω (the upper vertex in Fig. 1a). The most characteristic term in Eq. (10) is the propagator $(p^2+1)^{-2}$, which immediately indicates the existence of a pole at $p^2 = -1$, corresponding to scattering by a virtual pion.

Strictly speaking, Eq. (10) is valid only at the point $p^2 = -1$, and the function $\sigma_{\pi\pi}(\omega)$ (or, more accurately, $\sigma_{\pi\pi}(\omega, p^2)$) only at this point actually represents the cross section for scattering of a pion by a real pion. The point $p^2 = -1$ lies outside the physical region and is not experimentally accessible. However, the pole nature of the relation, and particularly the fact that $d^2\sigma/dp^2d\omega^2 \to \infty$ at $p^2 = -1$, permits us to hope that there will be a dominant contribution from the diagram of Fig. 1a and that Eq. (10) will be valid in the immediate vicinity of the pole, including values $p^2 \sim 1$ which are physical for reaction (1). In just the same way we can suppose that $\sigma_{\pi\pi}(\omega, p^2)$ at $p^2 \sim 1$ will not be greatly different from the real $\pi\pi$ cross section. These considerations form the basis for use of Eq. (10) to determine the pion-pion cross section in the physical region. What is being neglected in this approximation? What errors are possible?

We have neglected all other diagrams, about which in the best case we can only say that they do not lead to a pole of second order in the differential cross section at $p^2 = -1$. We can further assume that the nearest pole singularity will be that of the diagram of Fig. 2f or with three virtual pions (Fig. 2b), with a characteristic momentum transfer $p^2 \sim 9$. It is not excluded that, because of interference with the term written out in Eq. (10), it may provide an appreciable contribution at $p^2 \sim 3$.

In any event, these are only rough estimates, and the question of applicability of Eq. (10) in the physical region, i.e., of the contribution of the diagram of Fig. 1a, is at the present time an experimental question, and we will be forced to return to it.

It is possible to attempt to use Eq. (10) in another way, finding $d^2\sigma/dp^2d\omega^2$ at the point $p^2 = -1$ (where Eq. (10) is valid) by extrapolation of the reaction (1) cross-section values from the physical region. This is the so-called extrapolation procedure. It is convenient to perform this extrapolation by means of the auxiliary function:

$$F(p^2, \omega) = \frac{d^2\sigma}{dp^2\,d\omega^2}\left[\alpha\,\frac{f^2}{2\pi}\,\frac{1}{q^2}\,\frac{1}{(p^2+1)^2}\,\omega\sqrt{\frac{\omega^2}{4}-1}\,\right]^{-1}. \qquad (11)$$

It is clear that $F(p^2, \omega) \to \sigma_{\pi\pi}(\omega)$ as $p^2 \to -1$. It is easy to see that

$$F(p^2, \omega) = bp^2, \qquad (12a)$$

if the only diagram contributing to the reaction is the pole diagram. We will make further use of this fact. In all other (more realistic) cases the law of variation of

$F(p^2, \omega)$ is unknown. Furthermore, the law can change along with the change in the contribution of other diagrams, and can depend on p^2, ω, and q. Nevertheless, the most often used method is a linear extrapolation of the form

$$F(p^2, \omega) = a + bp^2, \tag{12b}$$

where the constant term a somehow takes into account the nonpole contribution. Equation (12b) is justified as a first approximation. It would be satisfactory for $p^2 \ll 1$. For $p^2 \gtrsim 1$ its use, generally speaking, is better, but not eminently better, than use of the Chew-Low formula in the physical region. Besides, it is justified only for the condition of a decisive contribution of the pole diagram, Fig. 1a. The search for the conditions of applicability of the pole approximation and the proof of a decisive contribution of the pole under definite conditions constitute one of the important problems of experiments.

5. Necessary Criteria of Applicability of the Pole Approximation

Adequate criteria for applicability of the pole approximation and validity of Eq. (10) are not known to us. Here we will formulate some necessary criteria. Then we will discuss their use and will use the results of analysis of the experimental data on the basis of the necessary criteria to identify the region of a decisive contribution of the pole diagram.

As the first criterion we can name the verification of Eq. (12a). This criterion follows directly from the formula of Chew and Low. It can be formulated as follows: the values of $\sigma_{\pi\pi}$ obtained by extrapolation to $p^2 = -1$ and from Eq. (10) in the physical region must be the same.

The second criterion follows from consideration of the diagram with exchange of one pion and was first expressed by Yang and Treiman.[20] It now bears their name in the literature. Yang and Treiman showed that for one-pion exchange the distribution of the number of reaction (1) events in the angle between the plane of pion emission and the plane of the nucleons should be isotropic. This is a natural consequence of the zero spin of the virtual pion, which cannot transmit information on angular momentum from vertex to vertex. Since the plane of the nucleons cannot be determined in the laboratory system, the criterion is verified in the antilaboratory system, in which the incident meson is at rest.

The third criterion follows naturally from the formulation of the problem itself: $\sigma_{\pi\pi}$ for a given ω, determined from analysis of reaction (1) by the Chew-Low method, must not depend on the incident pion momentum.

In the same way, if $\sigma_{\pi\pi}$ is actually the pion-pion cross section, the same value must be obtained for it—the fourth criterion—from analysis of different reactions, for example, from analysis of the reaction with production of several pions, described by the diagram of Fig. 1b, to which we will return very shortly. Here it is important that the cross section for the reaction

$$\pi + N \rightarrow \pi + \pi + N^* \rightarrow \pi + \pi + \pi + N \tag{13}$$

is given by the formula obtained in the pole approximation and similar to Eq. (10).

Here it is appropriate to recall an attempt to check the applicability of the Chew-Low method in determination of the πN-scattering cross section from experimental data obtained in scattering of nucleons by nucleons.[21] In this case the possibility existed of comparing the value of the πN resulting cross section with free pion-nucleon scattering directly measured experimentally. The inadequate statistics available to the authors compelled them to use for determination of $\sigma_{\pi N}$ a relatively wide interval of momentum transfer. Nevertheless, there is at least a qualitative agreement of the magnitude and behavior of the cross section, found from an equation similar to Eq. (10), with that obtained by direct experiment*).

The fifth necessary criterion is agreement of the values found for $\sigma_{\pi\pi}$ by different methods, for example, by the Chew-Low method, from analysis of τ decays, and so forth (see below).

The pion-pion interaction must be considered isotopically invariant. In the opposite case, isotopic invariance would be destroyed in, say, πN scattering. Hence it follows, and this is the sixth criterion, that definite known relations should be satisfied between the $\sigma_{\pi\pi}$ cross sections in different charge states. In particular, charge symmetry should exist: $\sigma_{\pi^+\pi^+} = \sigma_{\pi^-\pi^-}$, $\sigma_{\pi^-\pi^0} = \sigma_{\pi^+\pi^0}$.

We can formulate further criteria associated with the existence of resonances in the $\pi\pi$ system. For resonances in which the inelastic channels can be neglected, $\sigma_{\pi\pi}$ should reach the unitary limit. The singularities at the $\pi\pi$ vertex should be reflected in the form of singularities in the behavior of the entire reaction.

It is evident that a part of the criteria refer not only to verification of the validity of the Chew-Low method and the pole approximation, but also to the correctness of other methods, for which specific criteria can also be formulated in a number of cases. We will emphasize only once more that the enumerated criteria are only necessary: we can only hope that simultaneous satisfaction of these criteria, particularly such extremely general ones as 3—7, will guarantee to an important degree the separation of the pole diagram and the reliability of the $\sigma_{\pi\pi}$ values.

6. Other Methods of Determination of $\sigma_{\pi\pi}$ in the Peripheral Approximation

We will discuss first the already cited extension of the Chew-Low method to analysis of reaction (13). Formulas for the reaction cross section, corresponding to the diagram of Fig. 1b, have been obtained by a number of workers.[22-26] It is true that they have taken into account in somewhat different ways the lower vertex, which is all that distinguishes diagrams 1a and b. All of

*In 1969 a more accurate study was made (Z. M. Ma et al., Phys. Rev. Lett. 23, 342 (1969)), in which the cross sections for scattering of protons by pions were found by the extrapolation method from the values of a modified momentum transfer which effectively took into account the descent from the mass shell. Comparison of the cross sections found with those measured in direct experiments on free πN scattering in the region of the Δ_{33} isobar indicates agreement within the errors of $\leqslant 10\%$ and is a convincing argument in favor of the applicability of the Chew-Low method.

the formulas obtained have, of course, the identical structure:

$$\frac{d^3\sigma}{dp^2\,d\omega^2\,dW}=\frac{1}{16\pi^3}\left[\frac{1}{q^2}\frac{\omega\sqrt{\frac{\omega^2}{4}-1}}{(p^2+1)^2}\sigma_{\pi\pi}(\omega)\right]\times\left[W\sqrt{\left(\frac{W^2-49}{2W}\right)^2-1}\left(\frac{p_{off}}{p_{on}}\right)^2\left(1+\frac{p^2+1}{200}\right)\sigma_{\pi p}\right][G(p^2)]^2. \quad (14)$$

Here the first brackets contain factors from the Chew-Low formula (10), and the second—terms taking into account the kinematics of isobar production and the probability of production. The different formulas differ in the function $G(p^2)$. In Eq. (14), W is the isobar mass, p_{off} and p_{on} are the momenta of the virtual and scattered meson in the isobar c.m.s. It can be assumed, and this assumption has been made by Key et al.,[26] that $G(p^2) = 1$. It is possible that this is correct, and in any case the numerical difference, say with Selleri's formula,[24] amounts to 14 to 80% for various values of ω. Another cause of the discrepancy of the cross sections found by different workers by means of formula (14) is the different width of the isobar interval used in the various studies for integration over W. Nevertheless, we will see that Eq. (14) permits obtaining valuable information on $\sigma_{\pi\pi}$, especially if we have in mind checking of the fourth necessary criterion for applicability of the pole approximation.

There is a widely used modification of the Chew-Low formula for reaction (1) which is used when it is desired to make use of a wide range of ω and p^2. In this case an additional factor is added to Eq. (10) which has the form of an effective form factor taking into account both the form factors of the vertices and of the propagator and, possibly, also the contribution of some non-one-pion diagrams.

The early studies utilized form factors specified in advance, most often those proposed by Ferrari and Selleri[27]:

$$\Phi(p^2)=\left[0{,}28+0{,}72\Big/1+\frac{p^2+1}{4{,}73}\right]^2, \quad (15)$$

or by Amaldi and Selleri[28]:

$$\Phi(p^2)=\left[0{,}28\Big/1+\left(\frac{p^2+1}{32}\right)^2+0{,}72\Big/1+\frac{p^2+1}{4{,}73}\right]^2. \quad (16)$$

These form factors are not greatly different, especially at small p^2. Both, as should be the case, have been normalized to unity for $p^2 = -1$. Both have a falling nature, and both fall off particularly sharply at small p^2. Form factors (15) and (16) are introduced as universal functions which are independent not only of the initial energy and of ω, but also of the reaction charge channel, and are even identical for reactions (1) and (13). Furthermore, Selleri's form factor (15) was initially used to describe pion production in nucleon-nucleon collisions,[26] and the very form of the dependence goes back to electrodynamics. All of this is somewhat suspicious, but by itself is still in no way bad. More serious is the fact that the only real check of the validity of introducing form factors (15) and (16) was the fact that the $\pi^-\pi^0$-interaction cross section obtained from Eq. (10) with form factor (15), where the constants have been chosen experimentally, reaches the unitary limit at the ρ-meson mass. It appears that the comparison with the unitary limit is incorrect, not only because the use of Eq. (10) in the physical region may not be completely correct, but mainly because it is impossible to ignore the large S-wave phase shift in this region, the possible admixture of other phase shifts, and background events. It is true that Eq. (10) with form factor (15) or, still better, with form factor (16) fitted for this purpose, gives a good description of the dependence of the reaction (1) differential cross sections on p^2 in the region up to $p^2 \sim 50$. However, for large p^2 form factors (15) and (16) do not change greatly. This same circumstance does not permit serious reference to the results of those studies (see, for example, ref. 29) which, with relatively poor statistics in the same region of p^2, reach the conclusion that $\Phi(p^2) \approx 1$ (and for small p^2 directly contradict the experimental data, for example ref. 30, which indicates a weak variation of the form factor).

From (15) and (16) it follows that if $\sigma_{\pi\pi}$ is extracted from the differential cross sections for reaction (1) up to $p^2 \sim 10-20$, then introduction of such form factors changes the values of the cross sections found in comparison with Eq. (10) by approximately three times. This means that the question of the shape of the form factor is extremely important. However, we have seen that this is a purely experimental question. Therefore we will return to it. For the present we will cite ref. 31, where the authors attempt to solve the form factor problem in the following way. Let the formula

$$\frac{d^2\sigma}{dp^2\,d\omega^2}=\alpha\frac{f^2}{2\pi}\frac{p^2}{(p^2+1)^2}\omega\sqrt{\frac{\omega^2}{4}-1}\,\Phi(p^2)\,\sigma_{\pi\pi}(\omega) \quad (17)$$

be valid (this is the weakest point of the discussion); then $\sigma_{\pi\pi}$ can be found by two means. Specifically:

$$\sigma_{\pi\pi}(\omega)=\int_{p^2_{min}}^{p^2_{max}}\frac{d^2\sigma}{dp^2\,d\omega^2}\,d\omega^2\,dp^2\Big/\int_{p^2_{min}}^{p^2_{max}}\alpha\frac{f^2}{2\pi}\times\frac{1}{q^2}\frac{p^2}{(p^2+1)^2}\omega\sqrt{\frac{\omega^2}{4}-1}\,\Phi(p^2)\,d\omega\,dp^2, \quad (18)$$

$$\sigma_{\pi\pi}(\omega)=\int_{p^2_{min}}^{p^2_{max}}\frac{d^2\sigma}{dp^2\,d\omega^2}\Big/\alpha\frac{f^2}{2\pi}\frac{1}{q^2}\frac{p^2}{(p^2+1)^2}\omega\sqrt{\frac{\omega^2}{4}-1}\,\Phi(p^2). \quad (19)$$

Expressions (18) and (19) should be equal, since they represent the same quantity. This is possible when the integrands in (18) or the numerator and denominator in (19) differ for a given ω only by a factor, i.e., when the selected $\Phi(p^2)$ is correct. This comparison of two-integral and one-integral means of determination from the data on differential cross sections for the reaction

$$\pi^-+p\to\pi^-+\pi^0+p \quad (20)$$

refutes form factors (15) and (16), is consistent (within the accuracy of the remark about the unitary limit) with the possibility $\Phi(p^2) = 1$, and has not been checked for other values of $\Phi(p^2)$.

A convenient means of experimentally measuring $\Phi(p^2)$, or more accurately $\Phi'(p^2, \omega) = \Phi(p^2)\sigma_{\pi\pi}(\omega)$, where $\Phi'(p^2, \omega) \to \sigma_{\pi\pi}(\omega)$ as $p^2 \to -1$, was proposed by Baton et al.[32] and has received the name of the pseudoperipheral approximation. In essence, Eq. (17) is postulated. Generally speaking, this is not such a bad approximation, and in fact $\Phi'(p^2, \omega)$ is arbitrary. The only assumption, and apparently one of no small importance, is the omission of additional terms in Eq. (17) not containing the factor p^2 (in the opposite case they can be included in $\Phi'(p^2, \omega)$). However, production of pions (for example, through an isobar) will lead to a constant term in Eq. (17) and will extend beyond the limits of the pseudoperiphery. It is not excluded that the additional terms mentioned can lead in a number of cases to the effective appearance of a function $\Phi'(p^2, \omega)$ which has no relation to the form factor.

Baton et al.[32] have formulated the pseudoperipheral approximation in the following way. Let us form the auxiliary function

$$F'(p^2) = F(p^2)/p^2, \tag{21}$$

where $F(p^2)$ is given by Eq. (11), and extrapolate it to the point $p^2 = -1$, where $F'(p^2, \omega) = \sigma_{\pi\pi}$. The extrapolation law in each range of ω is $\Phi'(p^2, \omega)$. It is obvious that the extrapolation is possible if $F'(p^2)$ does not have a pole at $p^2 = 0$, i.e., for the conditions discussed above and requiring verification each time that they are satisfied.

The pseudoperipheral approximation is identical to the Chew-Low extrapolation procedure when the extrapolation curve passes through zero. A linear extrapolation in the pseudoperipheral approximation corresponds to a quadratic Chew-Low extrapolation, a quadratic to the cubic, and so forth. The values of $\sigma_{\pi\pi}$ obtained are practically identical, but the errors obtained in the pseudoperipheral approximation are, of course, less, since it is postulated that $F(p^2) = 0$ at the point $p^2 = 0$, which, we repeat, requires experimental proof.

In concluding the paragraphs devoted to the pole approximation, it is appropriate to mention the fundamental doubts as to the validity of the pole approximation which have arisen in connection with the development of Regge pole theory and which are hidden under the term conspiracy. The essence of the doubts is the fact that, in addition to the π pole at the point $p^2 = -1$, it is assumed that there is a singularity at the point $p^2 = 0$, which, of course, makes impossible extrapolation to the point $p^2 = -1$ from the physical region of p^2 values and even more the use of Eq. (10) in the physical region. We will limit ourselves here to only this short remark, since there is, obviously, no singularity at $p^2 = 0$. A brief review of experimental studies from this point of view can be found in ref. 33. The facts leading to suggestion of conspiracy are more naturally explained today in terms of the theory of complex angular momenta with inclusion of branching.

We can further note these results of a Regge discussion of reaction (1): the asymptotic formula goes over at low energies in the s-wave region of $\pi\pi$ scattering to Eq. (10); a form factor naturally arises, which, however, approaches unity as the squared total reaction energy approaches zero; in the s-wave region, even inclusion of rescatterings (branchings), i.e., diagrams of the type 1d, does not change the dependence of the differential cross section for the reaction, $\sim p^2$, which is necessary for validity of the pseudoperipheral approximation.

7. Methods of Observing Dipion Resonances

In the preceding discussion we have already repeatedly cited as well known the resonances in the two-pion system. In fact, several years ago there arose a peculiar situation (which to some extent still exists today) in which we knew little of the cross-section values for $\pi\pi$ scattering, but were already convinced of their resonance nature. In study of πN scattering, things were completely the reverse. The first resonances were observed as irregularities in the energy dependence of total and elastic cross sections. In the case of studying the $\pi\pi$ interaction, these methods turned out to be important rather in the negative sense, in concealing certain resonances which are revealed in other ways.

The invariant-mass method consists of searching for singularities in the dependence of the number of events of reactions (1), (13), (9),

$$\tilde{p} + p \rightarrow \pi^+ + \pi^- + k\pi^0 \tag{22}$$

or any other reaction with several pions in the final state, on the invariant mass of the dipion system ω.

The missing-mass method consists of measuring at a definite angle the momentum of the recoil nucleon and calculating from these data the invariant mass of all remaining particles. If reaction (3) occurs, then in the corresponding distribution of the number of events will be observed a maximum corresponding to mass ω, with a width characterizing the lifetime of the resonance. The missing-mass method in the form presented separates all boson resonances. Establishment of the decay scheme and, in particular, identification of the dipion resonances of interest to us at the moment requires additional measurements in this case.

Not every maximum in a distribution of the number of events of a given reaction can be considered a resonance, even if it is statistically certain (extending 4–5 standard deviations from the phase-space curve). It may turn out to be a "ghost"—the reflection of a known resonance. This can easily be illustrated by means of Fig. 3, in which we have drawn the phase-space curve for reaction (1) in coordinates ω^2 and W^2, where W is the mass of the pion-nucleon system. The band in the figure encloses events corresponding to production of a known isobar, for example, Δ_{33}. It is evident that in the distribution of the number of reaction events as a function of ω, i.e., in projection of the diagram in Fig. 3 onto the axis of ordinates, a maximum will arise, since the band in the figure is more densely populated with events corresponding to production of the isobar (but not of a boson resonance!). It happens that both an isobar and a boson resonance are formed. In reaction (13) they can be produced simultaneously. Therefore resonances can conveniently be sought by observing the clustering of points in diagrams like Fig. 3. Note that the location of the phase-space curve relative to the isobar band changes with initial energy, which leads to a change in the location and width of the ghost.

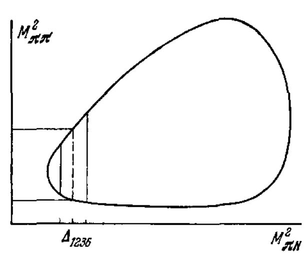

FIG. 3

A maximum in the distribution of the number of reaction (1) events as a function of ω may also have a deeper cause. In refs. 34 and 35, to which we will return again, it was shown that a substantial contribution of a triangle diagram of the type of Fig. 1c leads to a maximum. In this case also the location of the maximum changes in a calculable way with variation of the initial energy.

A maximum corresponding to a resonance must have a Breit-Wigner shape. A resonance must have definite quantum numbers: mass and lifetime (the width and location of the maximum must not change with change of the initial energy), isospin (the number of events in maxima found in different charge channels of reaction (1) should be assigned as the corresponding Clebsch-Gordan coefficients), spin, and parity (there must be definite angular distributions of the pions in their c.m.s.). If a dipion resonance exists with a dominant decay channel into two pions, then the $\pi\pi$-scattering cross section at the resonance ω must reach the unitary limit in the corresponding state, and the phase shift must pass through 90°. If there are several decay channels, then their ratio should not change as the conditions for production of the resonance change. A true resonance found in one reaction will, as a rule, be observed also in other reactions.

It is not always easy to verify the enumerated indications of a resonance. The fact is, for example, that reaction (1) does not occur according to Eq. (3). There are always some nonresonance events (including those associated with a pion-pion interaction in other states), which are usually not quite correctly described by the phase-space curve. There are cases in which the reaction proceeds according to Eq. (2). Nevertheless, the extremely strong resonance nature of the interaction makes possible the successful separation of resonances. This is similar to the way in which the existence of isobars made possible the success of various isobar models in description of reaction (1). Better separation of resonances and determination of their quantum numbers is undoubtedly obtained in the case when the pion-pion interaction is emphasized, i.e., in peripheral interactions. Not without reason is the peripheral nature of the production of dipion resonances confirmed in most studies.

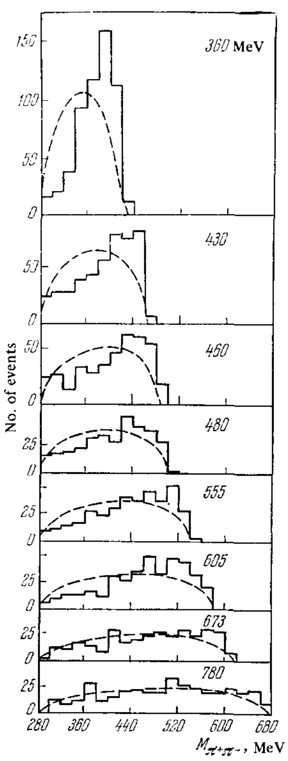

FIG. 4. Distribution in effective mass of the $\pi^+\pi^-$ system for different values of incident pion energy. The dashed curves represent phase space.

If, even on the basis of only a part of the characteristics, we are convinced of the existence of the dipion resonance, and its spin, parity, and the ratio of the decay channels are known, then the pion-pion scattering phase shifts are known in a definite state for a definite ω. And, as occurs in πN scattering, these phase shifts determine to a significant degree the behavior of the entire cross section as a function of energy in the vicinity of the resonance. Partly for this reason is the study of resonances so important and have so many investigations been devoted to the question of the existence of various resonances (which often have not survived to the present day). It can also be said that most studies of the $\pi\pi$ interaction have been stimulated by studies of resonances.

8. Phase-shift Analysis of Pion Center-of-mass Angular Distributions

Studies of the angular distributions of pions are made primarily in the vicinity of resonances in order to establish their spins. While in the early stages it is sufficient to employ rough descriptions of the angular distributions such as isotropic, $\cos^2\theta$, and so forth, accurate studies require phase-shift analysis of the pion angular distributions in their center-of-mass system, similar to the analysis of particles in the center-of-mass system of an elastic reaction. The expressions relating the expansion coefficients B_l of the pion angular distributions in Legendre polynomials with the scattering phase shifts, for example, for $\pi^+\pi^0$ scattering in the region where S and P waves are important, have the form

$$\sin^2\delta_1^1 = (k^2/12\pi)\,B_2, \quad \sin\delta_1^1 \sin\delta_2^0 \cos(\delta_2^0 - \delta_1^1) = (k^2/12\pi)\,B_1, \quad \sin^2\delta_2^0 = (k^2/12\pi)\,(B_0 - {}^1/_2 B_2). \tag{23}$$

Here and subsequently δ_T^J is the phase shift; the lower index is the isotopic spin, and the upper the angular momentum. Pion angular distributions are sometimes described differently. For example, for the $\pi^+\pi^-$ system, where a role is played by all three isotopic spin states with T = 0, 2 in S-wave scattering and T = 1 in P-wave, the differential scattering cross section is proportional to

$$[{}^1/_3 \sin\delta_2^0 - \delta_1^1 \cos(\delta_2^0 - \delta_1^1) + {}^2/_3 \sin\delta_0^0 \cos(\delta_0^0 - \delta_1^1) + 3\cos\theta \sin\delta_1^1]^2 + [{}^1/_3 \sin\delta_2^0 \sin(\delta_1^1 - \delta_2^0) + {}^2/_3 \sin\delta_0^0 \sin(\delta_1^1 - \delta_0^0)]^2. \tag{24}$$

It is important that (23) and (24) contain interference terms sensitive to the relative sign of the phase shifts and which lead to asymmetric angular distributions.

Thus, a phase-shift analysis permits determination of the relative magnitudes and signs of the phase shifts. If one of the phase shifts is known, for example, in the case of the ρ meson $\delta_1^1 = 90°$, then the other phase shifts can be found in the vicinity of the resonance. Many authors[32,36-41] have used this means of determining phase shifts and therefore also $\sigma_{\pi\pi}$ cross sections, relative to a ρ-meson phase shift. Here some have assumed that even at some distance from the ρ-meson region the δ_1^1 phase shift continues to follow a Breit-Wigner formula. This, of course, is incorrect, as was emphasized recently by Bander et al.[42] However, in the vicinity of a resonance this method is satisfactory, and we will later use the corresponding results. It is necessary only to keep in mind the following.

In the first place, we are assuming from the very beginning a dominant contribution of the pion-pion interaction. It has already been repeatedly emphasized that this is far from always the case, and selection of special cases of reaction (1) is required. The selection made is that p^2 be less than a given value p_0^2. The question is whether this selection is sufficient.

In the second place, the results of the phase-shift analysis are apparently unstable against introduction of small added higher waves[24] and are ambiguous even for S and P analysis. All this once more emphasizes the importance of the combined analysis of data by different methods.

9. Obtaining Data on the Pion-pion Interaction from Analysis of Decay Processes

Here we will first discuss the decays $K \to 3\pi$ and $\eta \to 3\pi$. An example of the diagram of a decay in which the $\pi\pi$-interaction vertex is separated in explicit form is shown in Fig. 1e.

Why should we think that pion rescattering is important? A strong qualitative indication of this is the similarity of the pion spectra from τ and η decays, different decays produced in the one case by a weak interaction and in the other case by an electromagnetic interaction. It is reasonable to suppose that the characteristic features of the spectra are associated with rescattering of the pions in the final state.

Attempts were made long ago to utilize the τ decay, for example, in refs. 43 and 44. Recently, the most correct discussion of the decay processes has apparently been made by Anisovich and his colleagues.[45,46] These workers have obtained and compared with experimental data rather massive expressions for pion decay spectra, which are usually referred to unit phase space in coordinates

$$\varepsilon = 1 - k_{12}^2/E \text{ and } Z = 2(k_{13}^2 - k_{23}^2)/\sqrt{3}E. \tag{25}$$

In (25) k_{ij} refers to the relative pion momenta, and the indices 1 and 2 refer to identical pions or $\pi^+\pi^-$ in the decay $K_1^0 \to \pi^+\pi^-\pi^0$, and E is the kinetic energy released in the decay. The basis for derivation of the formulas is the theory already discussed in section 2 of the production of particles near threshold, where, in view of the appreciable energy of ~ 80 MeV released in the $K \to 3\pi$ decay, an expansion was made to terms of order $E^{3/2}$. Inclusion of still higher terms apparently does not change the principal results. Nevertheless, even here it is desirable to have some kind of criteria for checking the correctness of the formulas used. These criteria exist, and are satisfied. The authors correctly predict the different behavior of the pion spectra from the decays $K^+ \to \pi^+ + \pi^+ + \pi^-$ and $K^+ \to \pi^+ + \pi^0 + \pi^0$, $K^0 \to \pi^+ + \pi^- + \pi^0$. In the first of these cases the spectra should follow the laws

$$W^{++-}(\varepsilon) = 1 + \beta E(\varepsilon - 1/2), \quad W^{++-}(Z) = 1 \tag{26}$$

and should be practically independent of k_{ij}, and in the second and third decays the spectra should be similar and should have a characteristic rise at large ϵ, depending just on the magnitudes of the pion-pion scattering amplitudes in states with T = 0 and T = 2. We will make substantial use below of the results obtained by Anisovich et al.[45,46] and will further note only that from analysis of meson decays it is not possible to obtain separately values of a_0 and a_2, but only a relation between their possible values.

We will not discuss in detail the K_{e4} decay (see Fig. 1f). It is necessary only to emphasize that in study of the K_{e4} decay[47] information on $\pi\pi$ scattering at $\omega \sim m_K \sim 500$ MeV can in principle be obtained in the most pure way, since at the end of the reaction there remain only two strongly interacting particles, two pions. Today we do not have available sufficient statistical material on the decay $K \to e + \nu + \pi + \pi$.

The $\pi\pi$-scattering phase shifts for $\omega = m_{K^0}$ enter uniquely into the description of the decay $K_2^0 \to 2\pi$, which in recent years has attracted the close attention of many physicists in connection with the problem of nonconservation of CP parity and has in turn excited a burst of interest in the phase-shift values. The amplitude of decays to two charged pions is $\eta_{+-} = \epsilon + \theta$ and into two neutral pions $\eta_{00} = \epsilon - 2\theta$. Here θ is proportional to $e^{i(\delta_2 - \delta_0)}$, where δ is the phase shift for $\pi\pi$ scattering in the corresponding isospin state. It is not excluded that an intensively carried out detailed study of $K_2^0 \to 2\pi$ not only will require data on $\pi\pi$ phase shifts but will itself permit obtaining such data, provided of course that $\theta \neq 0$.

Finally, there is one additional decay of another class, the cascade decay of a strange resonance

$$\Lambda_{1526} \to \Sigma_{1388} + \pi \to \Lambda + \pi + \pi, \tag{27}$$

whose diagram with a separated $\pi\pi$-scattering vertex is shown in Fig. 1g. This decay has been discussed by Anisovich and co-workers[48] from the point of view of obtaining information on the pion-pion interaction, in the same way as they have discussed the triangle singularity of Fig. 1c in reaction (1).[6] The advantage of considering reaction (27) is that here the quantum numbers of all baryons are exactly fixed and there is no ambiguity from failing to take into account nonresonance background. It is important that analysis of reaction (27) events permits in principle drawing of conclusions as to the sign of a_0.

III. EXPERIMENTAL SITUATION

1. Bubble Chamber Work

Having acquainted ourselves with the methods used to extract information on the pion-pion interaction, let us dwell now on the principal experiments in which the source material was obtained. These consist first of all of numerous and productive studies made with liquid hydrogen bubble chambers in a magnetic field. The possibility of observing all charged reaction products (except very soft recoil protons) and measuring rather accurately their momenta and emission angles has enabled us to obtain complete information on the pion production processes

$$\pi^- + p \to \pi^- + \pi^+ + n, \quad (5)$$

$$\pi^- + p \to \pi^- + \pi^0 + p, \quad (28)$$

$$\pi^+ + p \to \pi^+ + \pi^0 + p, \quad (29)$$

$$\pi^+ + p \to \pi^+ + \pi^+ + n, \quad (30)$$

$$\pi^- + p \to \pi^- + \pi^- + p \quad (31)$$

and on the τ, τ', η, K_{e4} decays and reaction (27). The major limitations of the bubble-chamber technique are practically always imposed on the statistical reliability of the results. In the early work this was true even in the search for resonances. In regard to cross-section measurements, it is necessary to recall that this requires knowledge not of the total cross sections of the reactions listed, which in themselves are small, but of the differential cross sections of the order of microbarns in a limited range of the variables.

Even in 1960 in the work of Bonsignore and Selleri,[49] who used data from analysis of photographs[50] taken with the 72-inch liquid-hydrogen bubble chamber irradiated by 960-MeV pions, the cross section was evaluated on the basis of Eq. (10) for $p^2 \leq 0.3\ M^2$, where M is the nucleon mass. The values found of 20 mb for reaction (5) and 40 mb for reaction (28) are consistent with the qualitative fact found in the same study[50] that the spectrum of recoil nucleons cannot be described statistically or by means of an isobar model, but are in good agreement in the region of relatively low p^2 with the Chew and Low formula.

Similar results were soon obtained also by a number of other investigators.[51,52]

It is necessary to single out the work of Anderson et al.[53] who for the first time attempted to use an extrapolation procedure. Photographs were again taken with the 72-inch hydrogen chamber, irradiated this time by 1.03-BeV/c pions. The statistics used were 1275 reaction (4) events with momentum transfers to the nucleon $p \leq 400$ MeV/c ($p^2 \leq 8$). The entire region of ω^2 investigated, 5.0–27.5, was divided into eight intervals. All extrapolation curves except the first for $\omega^2 = 5.0$–8.2 did not pass through 0, indicating a noticeable contribution of nonpole diagrams. The energy dependence of the cross section gave a clear maximum in the vicinity of the ρ meson. In 1962 Carmony and Van de Walle,[54] with an identical procedure but with initial momentum 1.25 BeV/c and statistics of 1584 reaction (29) events and 411 reaction (28) events, obtained similar results, from which it became even more clear that the cross sections $\sigma_{\pi^+\pi^0}$ and $\sigma_{\pi^-\pi^0}$ obtained are in poor agreement with each other.

Kirz, Schwartz, and Tripp[55] used a hydrogen bubble chamber to study reaction (30) at 357 MeV initial energy. The extrapolation procedure, which as in all bubble-chamber work included a relatively wide range of momentum transfers to the nucleon, led the authors to negative values of the $\sigma_{\pi^+\pi^+}$ cross sections.

Important results were obtained in 1963 with a hydrogen bubble chamber used to study reaction (5) over a wide range of initial energies from 360 to 800 MeV.[56]

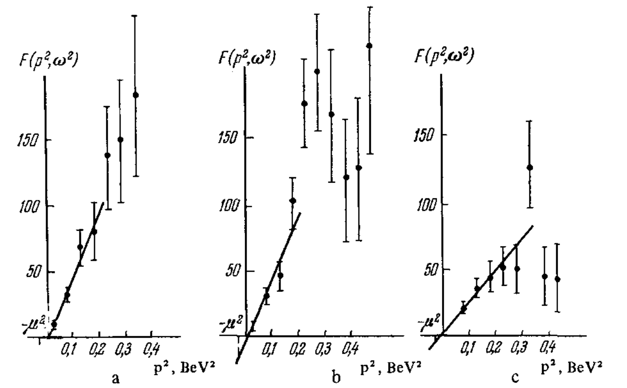

FIG. 5. Linear extrapolation of $F(p^2, \omega^2)$ for different intervals of squared effective mass of the $\pi^+\pi^+$ system. a) $0.07 \leq \omega^2 \leq 0.39$ BeV^2; b) $0.39 \leq \omega^2 \leq 0.59$ BeV^2; c) $0.59 \leq \omega^2 \leq 0.83$ BeV^2.

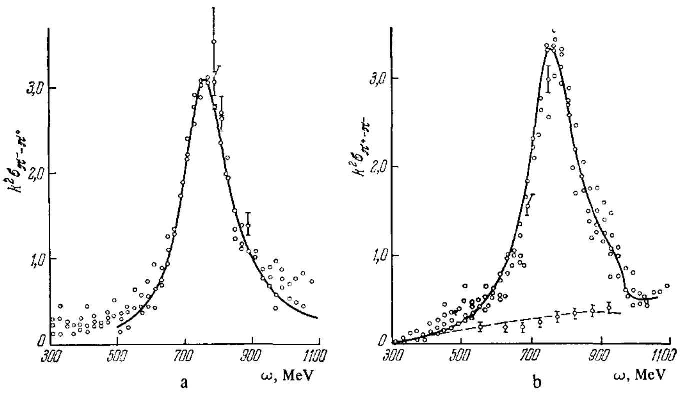

FIG. 6. The quantity $k^2\sigma$ (k is the wave vector of the incident pion in the dipion center-of-mass system) as a function of the effective mass of the $\pi^-\pi^0$ system (a) and of the $\pi^+\pi^-$ system (b). The data have been added from various studies made at different initial pion energies and with different values of momentum transfer.

The authors revealed the existence of a singularity in the spectrum of dipion masses (also observed previously; see, for example, Blokhintsev et al.[57]), which turned out to be a movable singularity (see the spectra of dipion masses in Fig. 4) and was interpreted as the result of contribution of the triangle diagram calculated by Anisovich and Dakhno[34] and Valuev[35]. Application of Eq. (10) to the results in the physical region led the authors for small ω^2 to a $\sigma_{\pi^+\pi^-}$ cross-section value varying from 20 to 120 mb.

In 1965–1966 several articles were published[58,29,59] in which reaction (30) was studied with initial π^+-meson momenta of 2.75, 4.0, and 1.6 BeV/c, respectively. In these articles, no extrapolation was made, and the cross sections were determined from the Chew-Low formula in the physical region with the momentum transfer limited to $p^2 < 15$. The authors note that the distribution in momentum transfer corresponds to the p^2 dependence in Eq. (10), and the distribution in the Treiman-Yang angle, which was obtained, it is true, with low statistical accuracy, remains isotropic up to $p^2 \approx 15$. We must note a recent study made at Dubna[60] in which an extrapolation procedure was used rather successfully to obtain $\sigma_{\pi^+\pi^+}$ from the differential cross sections for reaction

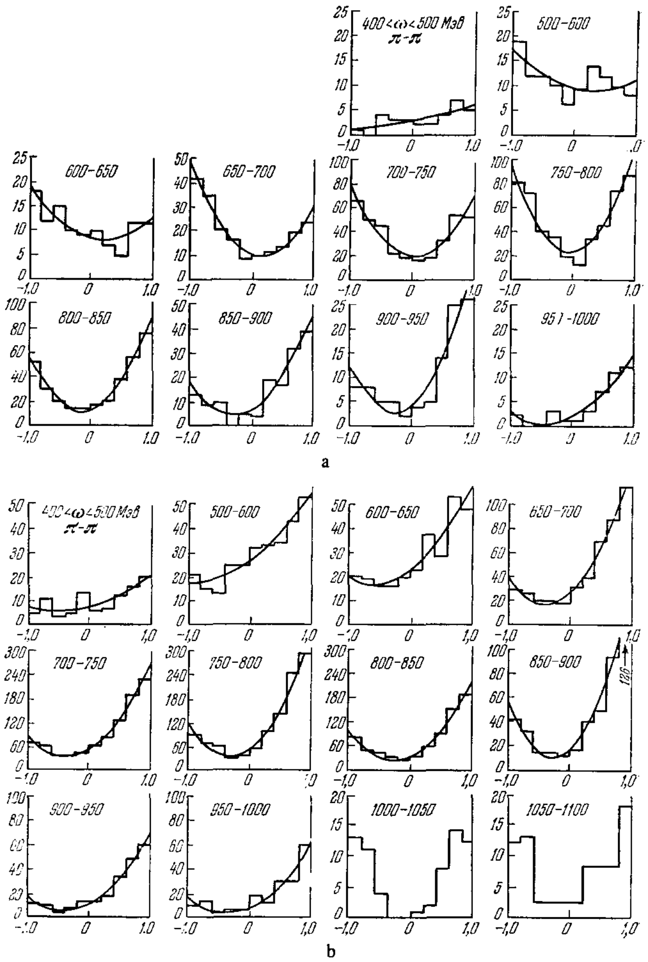

FIG. 7. Angular distributions of pions in the dipion center-of-mass system for different values of ω. a) The $\pi^-\pi^0$ system; the data were obtained with an initial momentum $q \geqslant 2$ BeV/c and a squared momentum transfer $p^2 \leqslant 5$; b) the $\pi^+\pi^-$ system; $q \geqslant 2$ BeV/c, $p^2 \leqslant 4$.

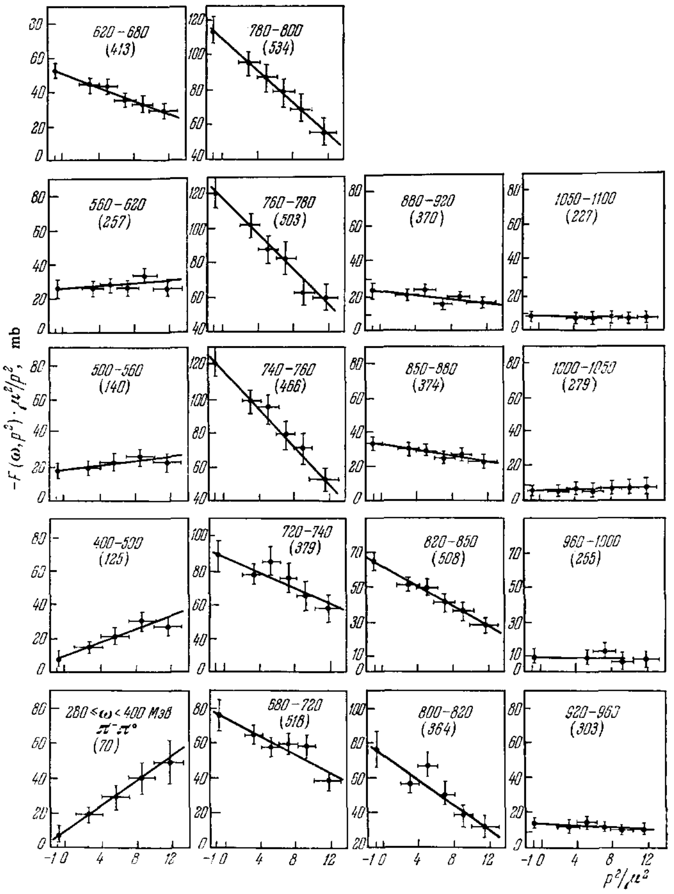

FIG. 8. Linear extrapolation of the function $F'(\omega, p^2) = F(\omega, p^2)/p^2$ for various effective mass intervals of the $\pi^-\pi^0$ system. The numbers of events are shown in parentheses.

(30) at an initial momentum of 2.34 BeV/c. This extrapolation is shown in Fig. 5.

The cross section for $\pi^-\pi^-$ scattering has been obtained a number of times from analysis of reaction (31) on the basis of Eq. (14).[24,61,62] In the work of Ferrari and Selleri,[24] which was performed at the Institute of Theoretical and Experimental Physics at an initial momentum of 3.25 BeV/c, an accurate phase-shift analysis was made of the data obtained.

It is important for what follows to mention the work of Bertanza et al.[63] on study of reaction (5) at an initial π^--momentum of 0.94 BeV/c, which resulted in a $\sigma_{\pi^+\pi^-}$ cross section sharply differing from those obtained in other studies, particularly at small ω. The authors themselves note in this case the violation of the Treiman-Yang criterion.

In regard to other, relatively new bubble-chamber studies of reactions (5) and (28), as a rule they have been performed with initial momenta in the range 2–8 BeV/c, with statistics of 2000–4000 events for p^2 up to 6–15. The data of these studies have been repeatedly combined and used by various authors[36,39-41] to obtain information on the pion-pion interaction by means of phase-shift analyses. The combined data are shown in Figs. 6 and 7.

Figures 6a and b show distributions of the number of events of reactions (28) and (5), respectively, normalized to the unitary limit in the ρ-meson region. Figure 7 shows the angular distributions of $\pi^-\pi^0$ or $\pi^+\pi^0$ and $\pi^+\pi^-$ mesons in their c.m.s. This is the principal experimental material for the phase-shift analysis.

Among recent bubble-chamber studies, we should single out the work of Baton et al.[32] in which the results of a study of reaction (28) at a momentum 2.77 BeV/c were analyzed in the pseudoperipheral approximation. The functions $F'(p^2)$ for various ranges of ω and their extrapolations to the point $p^2 = -1$ can be seen in Fig. 8.

Figure 9 shows one of the results which are important for our discussion: the combined spectrum of pions from the reactions $K^+ \to \pi^0\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$,[64-68] obtained with a bubble chamber, in comparison with the theoretical curve calculated by Anisovich and his coworkers.[46]

The data presented here, and to some extent also data not presented here, which in any event have been discussed in more detail in refs. 33 and 69–71, will be used subsequently in analysis of the applicability of the pole approximation criteria (see the table) and of the behavior of the pion-pion interaction cross sections (see Figs. 20–22 and 24).

All of the experiments cited above were performed

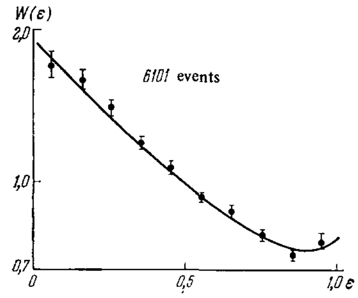

FIG. 9. Energy spectrum of pions from $K^+ \to \pi^0\pi^0\pi^+$ and $K_2^0 \to \pi^+\pi^-\pi^0$ decays. The solid curve is a theoretical calculation.

in liquid-hydrogen bubble chambers. Chambers filled with liquid deuterium permit study of reactions in the neutron in deuterium. Sometimes this is extremely important, since as the result of the reactions, for example,

$$\pi^+ + d \to p_s + p + \eta \to p_s + p + \pi^+ + \pi^- + \pi^0, \qquad (32)$$

$$\pi^+ + d \to p_s + p + \pi^0 + \pi^0, \qquad (33)$$

in addition to the spectator proton, there is formed a recoil proton which is detected and which gives valuable information on the squared momentum transfer in the reaction p^2 and on the ω of the pion system. Reaction (33) has been studied by Braun et al.[72] Valuable information on the interaction of neutral mesons can evidently be obtained by studying the reaction

$$\pi^- + p \to \pi^0 + \pi^0 + n \qquad (34)$$

in a bubble chamber filled with a heavy liquid with a high conversion coefficient, say, in a xenon chamber.

2. Experiments to Investigate the $\pi\pi$ Interaction By Means of Electronics

The possibilities are obvious in principle of studying the pion-pion interaction by means of the methods of nuclear electronics: the substantial improvement in the statistical reliability of the results and the extension in the region of extremely small momentum transfers to the nucleon, into the region of the maximum contribution of the pole diagram. No less obvious are the difficulties, which are associated first of all with the need, in the general case, of studying a many-particle reaction. The latter fact requires construction of complicated hodoscopic systems or design of experiments with the narrowly chosen purpose of measuring quasi-two-particle reactions of type (3), for example, by the missing-mass method.

The first experiment, performed by Abashian, Booth, and Crowe in 1960,[73] was just of this type. These authors investigated the reaction

$$p + d \to \pi^+ + \pi^- + He^3 \qquad (35)$$

and found a singularity in the measured spectrum of He^3 which they interpreted as a resonance in the $\pi^+\pi^-$ system with a mass of about 310 MeV. This resonance received in the literature the name ABC from the first letters of the authors' names. Attempts to find the ABC resonance were undertaken in Dubna[74] by measurement of the spectrum of H^3 from the reaction

$$p + d \to \pi^+ + \pi^0 + H^3. \qquad (36)$$

Observation of the ABC resonance in reaction (36) would

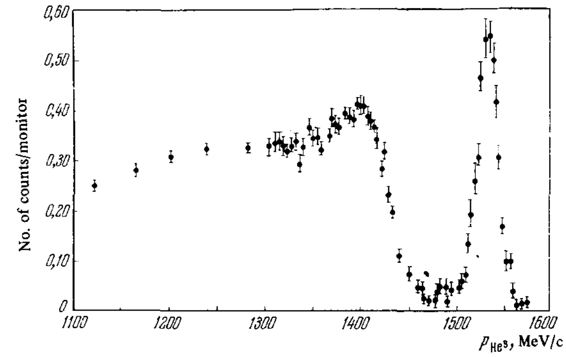

FIG. 10. Momentum spectrum of He^3.

present interest, since it was initially assumed that pions form a resonance in a state with T = 1. The resonance was not observed at Dubna. Later, Crowe et al.[75] made new measurements which confirmed both their previous experimental data and the Dubna data.[74] A typical spectrum of He^3 momenta is shown in Fig. 10. The right-hand peak corresponds to a two-particle reaction with production of one pion, and the maximum in the continuous spectrum at momentum ~1400 MeV/c to the ABC singularity.

It must be said that the existence of the ABC resonance from the very beginning was not completely convincing, since there were contradictory data on the spectrum of dipion masses in reactions (5) and (22) and in the photoproduction of two pions, in which the ABC first did not appear and then did appear, changing its location somewhat, but the extremely convincing hump which can be seen in Fig. 10 forced investigators to return repeatedly to the ABC problem and noticeably stimulated experiments at small values of ω. In particular the careful investigations in 1964–1966 by the CERN group[76] were directed particularly toward study of the region of the ABC resonance: ω = 280–350 MeV. The initial energy of the π mesons was 285 MeV. The experimental equipment consisted of two thin-foil spark chambers, between which was located a liquid hydrogen target. A scintillation counter system, which in the CERN work was not a high-aperture system, separated events with two particles in the final state; one of the π mesons with energy in the interval 50–80 MeV was slowed down, passing through a water Cerenkov counter, and stopped in a thick scintillation counter. By analyzing the pulse height from this counter, the authors could measure the energy of one of the pions (just pions), since the pulse from the Cerenkov counter was taken into account (with an accuracy of ±5 MeV). It is clear that a knowledge of the emission angles of the pions and of the energy of one of them completely determines the kinematics of reaction (5), which was being studied. Use of Eq. (10) gave $\sigma_{\pi^+\pi^-} \approx 25$ mb for $\omega^2 = 5.3$ and $1.5 \le p^2 \le 4$. In a more recent communication, on the basis of the same experimental data the authors reported a $\pi\pi$-scattering differential cross section $d\sigma_{\pi^+\pi^-}/d\Omega|_{\theta=90^\circ}$ = 1.76 mb/sr. Here and subsequently θ is the scattering angle of the pions in their own c.m.s. If we assume that the scattering of the pions in the c.m.s. is isotropic for $\omega^2 = 5.3$, then $\sigma_{\pi^+\pi^-} = 21.2 \pm 4.5$ mb. The energy dependence of the number of reaction (5) events, found in the

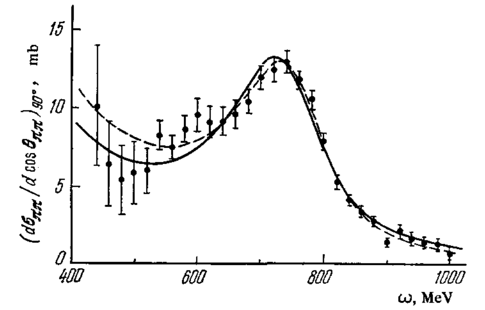

FIG. 11. The differential cross section for $\pi^+\pi^-$ scattering at 90° in the dipion c.m.s. as a function of the effective mass of the system.

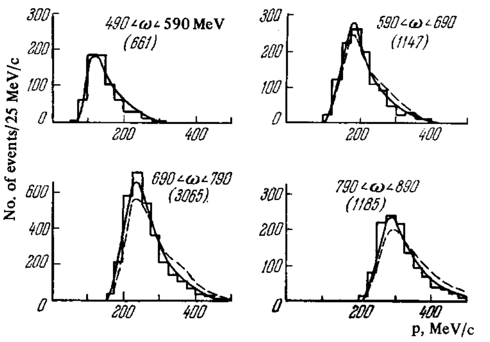

FIG. 12. Number of events of the reaction $\pi^-p \to \pi^-\pi^+n$ as a function of momentum transfer p for various ranges of effective mass of the $\pi^+\pi^-$ system. The solid curves were calculated from the Chew-Low formula; the dashed curves represent phase space.

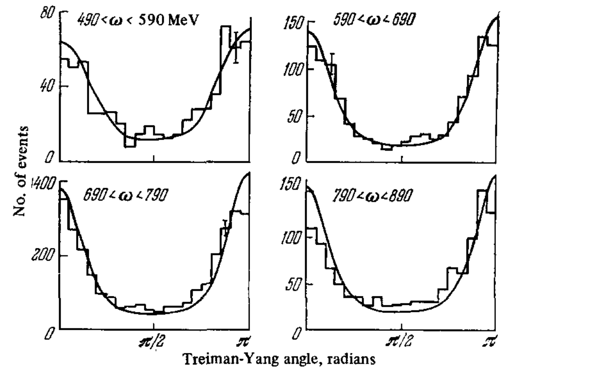

FIG. 13. Distribution of events from the reaction $\pi^-p \to \pi^-\pi^+n$ in the Treiman-Yang angle for four effective-mass intervals of the system. The solid curves were calculated on the assumption of an isotropic distribution with inclusion of the efficiency of the apparatus.

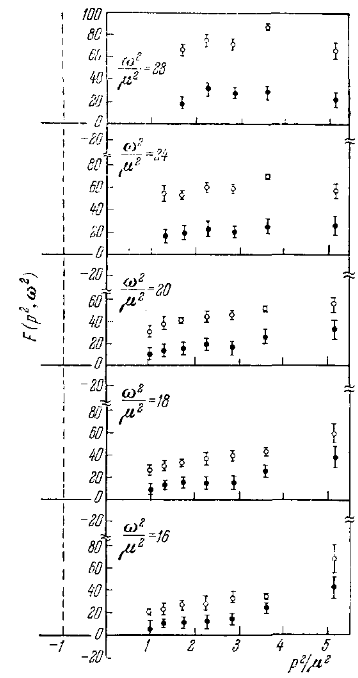

FIG. 14. The function $F(p^2, \omega^2)$ as a function of the squared momentum transfer for various intervals of effective mass of the dipion system. Points: O—$\pi^+\pi^-$ system, ●—$\pi^+\pi^+$ system.

CERN work,[76] definitely indicates the absence of the resonance (ABC) in the entire range of ω studied.

Electronic methods have been used[77,78] to study the problem of existence of another meson, the S_0 or ε, regarding which the situation is not yet clear (we will discuss it in part 3 of Ch. IV). The S_0 meson, first observed in the neutron spectrum from reaction (34) measured by the neutron time-of-flight method, was not found in more accurate experiments in which, in addition to the neutron, the γ rays from π^0 decay were recorded with high efficiency in spark chambers.[77] In 1965 Cronin's group[78] looked for a scalar S_0 meson in the ρ-meson mass region, studying reaction (5) at an initial π^--meson momentum of 1.5 BeV/c. The idea of the experiment was to measure the energy dependence of the differential cross section for $\pi^+\pi^-$ scattering at an angle $\theta = 90°$, where there should be no contribution from the ρ meson with spin 1. This idea also corresponds to an apparatus which is well known to all of us, since, after the experiment described, it was used for an experiment which led to discovery of the nonconservation of CP parity in the decay $K_2^0 \to 2\pi$. This apparatus consists of a spectrometer for two particles of approximately equal momenta emitted in symmetric trajectories in a single magnet. Figures 11—13 show the energy dependence found by Clark et al.[78] of the differential cross section for $\pi^+\pi^-$ scattering at 90°, and the dependence of the number of reaction (5) events on p^2 and on the Treiman-Yang angle in several ranges of ω^2. The solid curves in Fig. 12 were calculated on the basis of Eq. (10) and the dashed curves represent phase space. The solid curve in Fig. 13 corresponds to an isotropic distribution in the Treiman-Yang angle and is calculated with inclusion of the efficiency of the apparatus. We have divided the work of Cronin et al., since the data obtained and which are presented in the figures permit not only conclusions about the S_0 meson but also a number of important conclusions on the mechanism of reaction (5) under the conditions of the experiment and determination of the $\sigma_{\pi^+\pi^-}$ cross section at small ω on the assumption of isotropic scattering of the pions.

The work of Ypsilantis, Wiegand, et al.[79] in 1962 was specially devoted to measurement of $\sigma_{\pi^+\pi^-}$ and $\sigma_{\pi^+\pi^+}$. Reactions (5) and (30) were studied at incident pion momenta of 1.75 BeV/c. A beam of negative or positive pions from the internal target of the Bevatron fell on a 10-cm hydrogen target surrounded by a hodoscopic system of scintillation counters. The front system of 84 counters with scintillators 15 cm thick detected both pions and neutrons delayed relative to the pions. The side system of 48 counters located in the immediate vicinity of the target recorded only pions. In front of all the counters were located lead converters which permitted the recording of even a single γ ray from decay of a π^0 produced in the target. The counter system operated each time that an interaction occurred in the target. The pulses from all counters were recorded on magnetic tape, but were subjected to further analysis by computer only when two pulses and one delayed pulse were recorded simultaneously. The subsequent selection of the necessary events in the computer and calculation of p^2 and ω^2 were carried out on the basis of seven quantities: three pairs of angles—the polar and azimuthal angles for each particle (the angles were measured with an accuracy corresponding to the dimensions of the scintillators in the hodoscope)—and the recoil neutron momentum determined by the time of flight from the target to the front counter system. We can recall that the kinematics of reaction (1) is determined by five variables; the margin naturally improves the accuracy in determination of p^2 and ω^2. The cross sections, which were determined in this work[79] on the basis of Eq. (10) in the physical region, differ from those obtained in other studies, although the dependence of $\sigma_{\pi^+\pi^-}$ on ω has the characteristic maximum in the ρ-meson region. Figure 14 shows values of $F(p^2, \omega^2)$ as a function of p^2 for various ranges of ω^2, beginning with $\omega^2 = 16$. The lower points refer to $\pi^+\pi^+$ scattering, and the upper points to $\pi^+\pi^-$ scattering. The authors do not present extrapolation curves, since, using polynomials of low degree, they cannot obtain positive values for $\sigma_{\pi\pi}$, which are the only ones having a physical meaning. The authors note that the difficulties with the extrapolation may be associated with the strong resonance πN interaction in the final states with $T = 3/2$, $J = 3/2$ and $T = 1/2$, $J = 5/2$, which appear distinctly in the curves of the number of reaction (5) and (30) events as a function of the πN-system invariant mass. The data on verification of the Treiman-Yang criterion rather favor an anisotropic distribution, at least for events with large momentum transfers p^2.

Pion-pion scattering cross sections have been measured in the greatest detail in a series of experiments performed at the Institute of Theoretical and Experimental Physics in 1962–1968.[30,80-83] Reaction (5) was studied at initial π^--meson energies of 290 MeV and 1.17 BeV, and reaction (30) at an initial π^+-meson energy of 720 MeV/c. These studies are distinguished by the fact that events were selected for analysis from reactions (5) and (30) with $p^2 \sim 1$ for small values of relative pion energies $4 < \omega^2 \leq 9$, and in these ranges of p^2 and ω^2 statistics were collected which were several times better than the combined world data; careful justification was made of the applicability of the pole approximation under the conditions of the experiment, and estimates were made of the errors associated with the technique of determining the cross sections. The arrangement in principle of the experiments, which differed in details from each other, was as follows. The pion beam was detected by a beam telescope of scintillation counters and entered a liquid-hydrogen target surrounded by side anticoincidence counters. Particles emitted forward passed through a spark chamber with a small amount of material and were recorded in a hodoscopic system of eight scintillation counter telescopes. Every

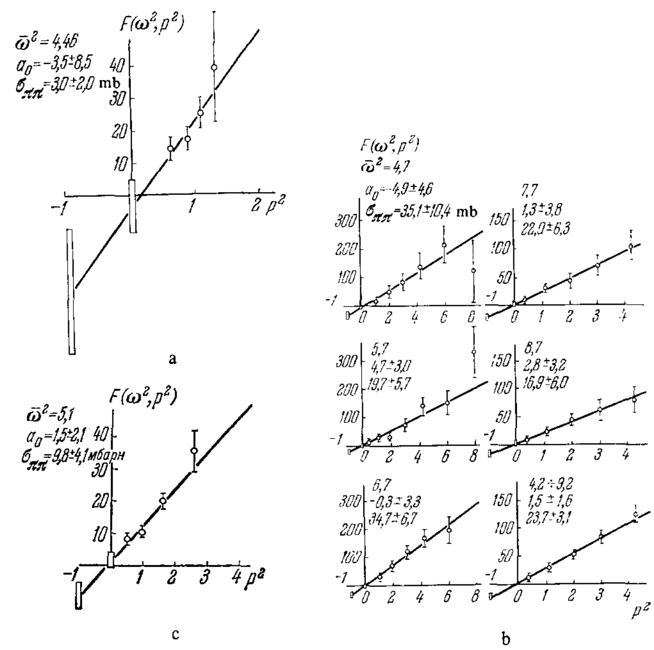

FIG. 15. Linear extrapolation of $F(p^2, \omega^2)$. a) For the reaction $\pi^-p \to \pi^-\pi^+n$ for an initial π^- energy of 290 MeV, b) for the reaction $\pi^-p \to \pi^-\pi^+n$ for initial π^- energy of 1170 MeV; five of the curves refer to various ranges of ω^2 indicated in the figures, and the sixth to the total range, c) for the reaction $\pi^+p \to \pi^+\pi^+n$ for an initial π^+ energy of 590 MeV. Values of $F(p^2, \omega^2)$ for $p^2 = 0$, and values of $\sigma_{\pi\pi}$ obtained by extrapolation, are shown in the figures.

time the hodoscopic target recorded two particles when an interaction occurred in the target, the spark chamber operated. All necessary measurements were made from photographs of the spark chamber. Values of p^2 and ω^2 were determined by a correlation method from the emission angles of the particle on the assumption of an isotropic distribution of the pions in their c.m.s. Additional information on the ranges of particles was obtained from photographs of a heavy spark chamber located beyond the hodoscope. The accuracy achieved can be seen in the results presented below (see also Fig. 20). Figure 15 shows the extrapolation procedures: in Figs. 15a and b for reaction (5) at 290 and 1170 MeV, and in Fig. 15c for reaction (30). Five of the graphs in Fig. 15b refer to different ranges of ω^2, indicated in the figures, and the sixth to the total range. It is evident that in all the plots the experimental points are well described by straight lines passing within the experimental errors through the origin. The deviations from the origin, shown with their errors in Fig. 15b, taken in relation to the quantities found by extrapolation of the $\sigma_{\pi^+\pi^-}$ cross sections, can to a certain degree serve as a measure of the contribution of nonpole diagrams and, consequently, as a measure of the error in the method of determining cross sections. From this, and also from comparison of the cross sections found for different values of q, the systematic (and not statistical) error of the measurements in refs. 30 and 80–83 can be estimated approximately as 15%.

3. Photoemulsion Data on the $\pi\pi$ Interaction

Nuclear emulsion technique is well developed and, so to speak, always ready for new problems. It is no acci-

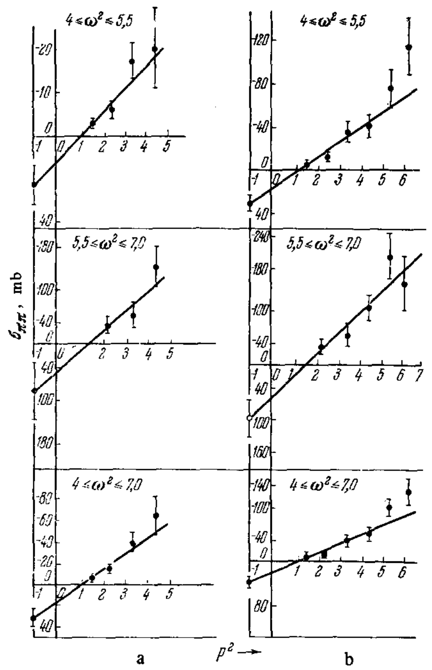

FIG. 16. Linear extrapolation of $F(p^2, \omega^2)$ for different intervals of ω^2. a) $p^2 \leqslant 5$; b) $p^2 \leqslant 7$.

dent that it is one of the first techniques used to study the $\pi\pi$ interaction. This was done at Dubna in Sidorov's group.[84,85] The authors studied reaction (5) with initial meson energies from 210 to 310 MeV. The low initial energies near the reaction threshold[5] were important both for technical reasons, since only in this case could the authors follow the secondary pion track completely and determine its energy, and also in principle, since the main purpose was to extract data on the pion-pion interaction by the method of Ansel'm and Gribov.[12] We have already discussed the difficulties and doubts arising in application of this method. Nevertheless, without presenting detailed data, we must say that the authors found (the final data of ref. 85) that $a_2 - a_0 = (0.25 \pm 0.05)$, having pointed out in their work the need of including quadratic terms. Today it is more interesting to analyze the same data by the Chew-Low extrapolation procedure. Such an analysis, made by Batusov et al.[86] for events with $p^2 < 5$ and $p^2 < 7$, is shown in the two columns of Fig. 16 respectively for $4 < \omega^2 < 5.5$, $5.5 < \omega^2 < 7$, and for the total range of ω^2. The values of $\sigma_{\pi^+\pi^-}$ obtained in these energy ranges are respectively 25 ± 9 mb and 101 ± 58 mb. The errors are statistical; as can be seen, the statistical accuracy is not very great. One is struck by the fact that the extrapolation straight line does not pass through zero, demonstrating a substantial contribution of nonpole diagrams. In addition, the distributions in the Treiman-Yang angle are approximately isotropic; however, in this case this does not argue in favor of one-pion exchange but rather that there is isotropic scattering of the pions for low relative pion energy.[70]

4. Verification of the Necessary Criteria for Applicability of the One-pion Approximation

We will now use the accumulated data to clarify the question of the validity of the idea of one-pion exchange, which serves as the basis of most current methods of obtaining information on the pion-pion interaction. This can be done most economically by means of the table, in which we will compare the results of application of the various criteria, discussed in detail by us in section 5 of the preceding chapter, to the experimental results, which permit at least some of the necessary criteria to be checked. The first column of the table shows the reaction investigated in the work referred to in the last column. The second column gives the initial pion energy, the third and fourth columns give the ranges of p^2 and ω^2, and in the remaining columns under arbitrary designations of the various necessary criteria we have placed plus signs where the results are consistent with the pole nature of the reaction, and minus signs in the opposite case. The asterisks after the signs are explained in the remarks column.

From consideration even of the limited material of the table we can draw the following conclusions.

1. The criteria enumerated in the table are not trivial. There are cases in which they are very noticeably violated, indicating the inapplicability of the one-pion approximation under certain conditions.

2. The criteria are rather sensitive. When the experiment permits verification they always indicate, in agreement with each other, a dominant contribution of the pole diagram or, on the other hand, the substantial role of other diagrams.

3. By no means all data on production of pions by pions can be used to extract information on $\sigma_{\pi\pi}$ on the basis of the hypothesis of peripheral collisions.

4. On the other hand, we can perceive certain rules or, better to say, can formulate such conditions for occurrence of reaction (1) that it will be correct from the current point of view to obtain data on the pion cross sections.

The conditions of a dominant contribution of the pole diagram as the basis for analysis of the applicability criteria have been discussed in detail in refs. 70, 71, and 33. Before turning to these conditions, we will note that the plus and minus signs are to a significant degree arbitrary, and that the comparison is more qualitative than quantitative. The experimental errors, particularly in comparison of the different studies, are still so large that it is still early to make a quantitative analysis of the relative degree of violation of the criteria and to judge the strength and degree of their reliability, although an analysis of this type will certainly provide considerable help not only in rejecting data unsuitable for extraction of $\sigma_{\pi\pi}$ but in estimating and substantially reducing the absolute errors in determination of the cross sections, associated with the technique for obtaining the data.

5. Conditions of Applicability of the One-pion Approximation

First, and above all (this is an old requirement, stipulated already when the Chew-Low method was originated in order that the momentum transfer to the nucleon in reaction (1) should be as small as possible), $p^2 \sim 1$. This condition is important not only in application of Eq. (10) in the physical region, where it expresses a natural desire to approach the pole, but also in

Reaction	q, MeV/c	Upper limit for p^2	Interval of ω^2	$F(p^2) = bp^2$	$\sigma_{\pi\pi}(\omega) \neq f(q)$	Treiman-Yang criterion	Isotopic invariance	Remarks	References
$\pi^-p \to \pi^-\pi^+n$	400	1,5	4—5	+	+		+*	*See Fig. 24	80—82
»	1300	5	4—9	+	+		+*	*See Fig. 24	30
»	1500	4	4—40	+*		+		Checked with poor accuracy	78
»	320—430	7	4—7	—	—	+*		*In the region of S scattering of pions	86
»	470—930	6	4—22*		—			*The upper limit varies with q	63
»	900	6	4—19	—	—	—			79
»	1750	6	16—28	—	—*	—		*The unitary limit is not reached in this region	36
»	> 2000	10	4—60		+*	+**		*See Fig. 6b **Checked with poor accuracy	
$\pi^-p \to \pi^-\pi^0p$ $\pi^+p \to \pi^+\pi^0p$	} 1250	8	4—34	—*			—*	*Outside the errors at large ω	54
$\pi^-p \to \pi^-\pi^0p$	2770	13	4—60	+*	+**			*With inclusion of the form factor **The unitary limit is reached in this region	32
»	⩾ 2000	10	4—60		+*	+**		*See Fig. 6a **Checked with poor accuracy	36
$\pi^+p \to \pi^+\pi^+n$	720	2,6	4—6	+	+*		+**	*See Fig. 21 **See Fig. 24	83
»	460	5	4—7	—					55
»	1600	15	4—60		+*	+*		*Checked with poor accuracy	29. 58 59
»	2340	10	4—60	+	+*	+		*See Fig. 21	60
$\pi^-p \to \pi^-\pi^-\pi^+n$ $\pi^+p \to \pi^+\pi^+n$	} **	15	4—60		+*		+*	*See Fig. 21 **From many studies	

extrapolation procedures (since in the opposite case, other conditions being equal, it is necessary to use polynomials of higher degree), and in the phase-shift analysis of angular distributions, when it is necessary to be convinced of the physical reality of the dipion system. In all current work, an attempt is made to satisfy the condition of small momentum transfer to the nucleon. The limits of acceptable p^2 in each case are apparently one's own. They change as a function of the other conditions of occurrence of the reactions.

Second, reaction (1) should be studied under those initial momenta q at which the triangle diagrams[34,35] do not give a dominant contribution, i.e., not at the isobar production threshold. It is important to emphasize that this condition was violated in the work of Kirz et al.,[56] from which Fig. 4 has been taken.

The third condition it is convenient to explain in Fig. 17, where we have shown the phase space for reaction (1) in coordinates of the squared mass of the πN systems for various values of q, and have marked the bands corresponding to the location of the Δ_{33} isobar. The axis passing through the origin at an angle of 45° to the axis of abscissas is the axis of squared mass of the dipion system—ω^2. In each case ω^2 is calculated from the right-hand point of intersection of the phase-space curve with the axis. It is necessary to select for analysis reaction (1) events lying outside the isobar intersection, which can be achieved by choice of q, ω, θ, or by measurement of the invariant mass $W_{\pi N}$ for each reaction event.

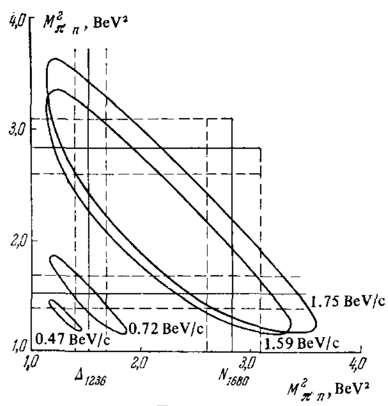

FIG. 17

Figure 17 shows the half-widths of the isobars. If the isobars are produced with large cross sections, then their effect is noticeable even outside the marked zones. Then, and this is a fourth condition, it is necessary to select events with such q that the cross section for production of isobars is minimal. We recall that the cross section for production of isobars (for example, Δ_{33}) changes very strongly, passing through a maximum which, possibly, corresponds to cascade decay of the isobars.

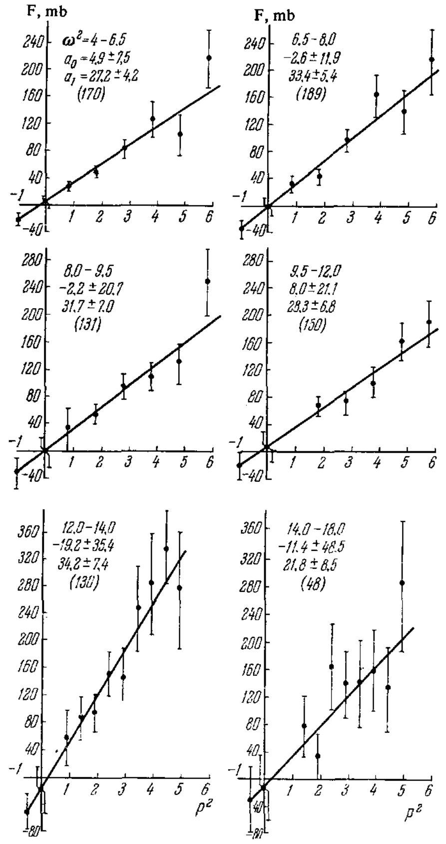

FIG. 18. Linear extrapolation of $F(P^2, \omega^2)$ for various intervals of ω^2. The numbers of events are shown in parentheses.

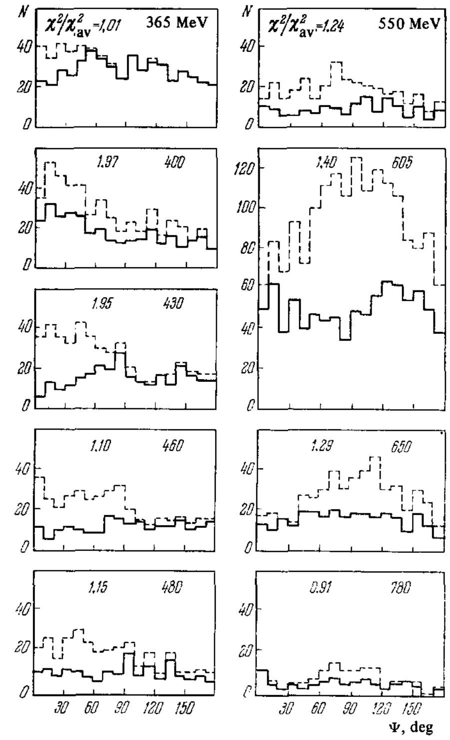

FIG. 19. Distributions of events of the reaction $\pi^- p \to \pi^- \pi^+ n$ as a function of the Treiman-Yang angle for various intervals of ω^2.

Fifth, in order to avoid the principal "enemy" of the pole mechanism, it is necessary to analyze reactions with an isotopically suppressed isobar state at the end of the reaction. Incorrect data on $\sigma_{\pi\pi}$ are obtained in studying reactions with Δ_{33}^{++} in the final state (see, for example, the solid points in Fig. 22).

We note that the generally rather obvious conditions enumerated have appeared here not as the result of speculative although very likely conclusions, but as the consequence of known experimental data. Recently they have found excellent confirmation in the work of Makarov et al.,[87] which was performed at the Physico-technical Institute after their formulation. This work utilized original data on reaction (5) obtained by the photoemulsion method and events measured in a hydrogen bubble chamber irradiated with π^- mesons in the energy range 360—780 MeV, which were provided the authors by Femino, Kirz, and Vittitoe.[88,56,89] These are partly the same data which we have already discussed and which led to varying $\sigma_{\pi^+\pi^-}$ cross sections and to violation of the necessary criteria. At the Physico-technical Institute, for extraction of $\sigma_{\pi^+\pi^-}$, only events outside the isobaric zones and with small $p^2 \leq 6-10$ for various ranges of ω were selected. The $\sigma_{\pi^+\pi^-}$ cross sections obtained are in good agreement with those found previously, for example, in refs. 30 and 82 (see Fig. 22). The extrapolation procedure and its results are well illustrated in Fig. 18, which shows the same quantities as in Fig. 15b. The distributions in the Treiman-Yang angle for the selected reaction (5) events (solid curve) and for all events (dashed curve) are shown for various ranges of ω in Fig. 19. Figures 18 and 19 are rather convincing and, we will hope, after what has been said they will not require additional commentary.

IV. PROPERTIES OF THE PION-PION INTERACTION

1. Cross Sections for Low-energy Pion-pion Scattering

Now that we have discussed the sources of information on the pion-pion interaction, set down conditions when they can be used, and in some degree understand the reliability of the existing results, we will consider the data on $\sigma_{\pi\pi}$, assuming that they are actually the cross sections for scattering of pions by pions. Naturally, we will present and discuss only those data which are satisfactory from the points of view discussed above. A complete, uncritical review of data for the period up to 1966 is given by Barashenkov.[90] Comparison of the corresponding distributions with those presented here shows what contradictory and unfounded conclusions can be reached by using all published results.

Figures 20—22 show measured energy dependences of the cross sections for the different charge states of the dipion system. The studies from which these data have been drawn have been discussed above, and references to them are given in the figures. In all cases where the form factors were not determined within an experiment, they have been set equal to unity for small p^2. All of the errors shown in the figures are statistical. In some cases, in order to keep the figures of reasonable size, for the points of different authors we have shown with horizontal error bars the intervals of ω in

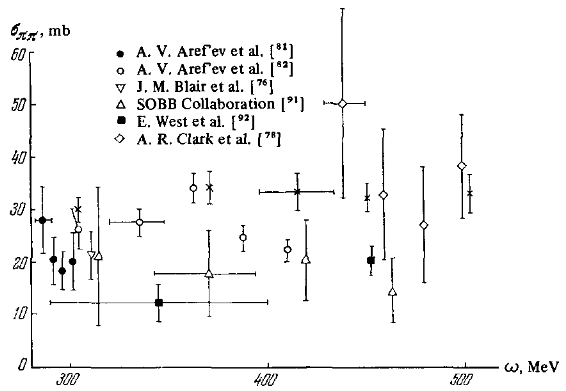

FIG. 20. Energy dependence of the cross section for $\pi^+\pi^-$ scattering.

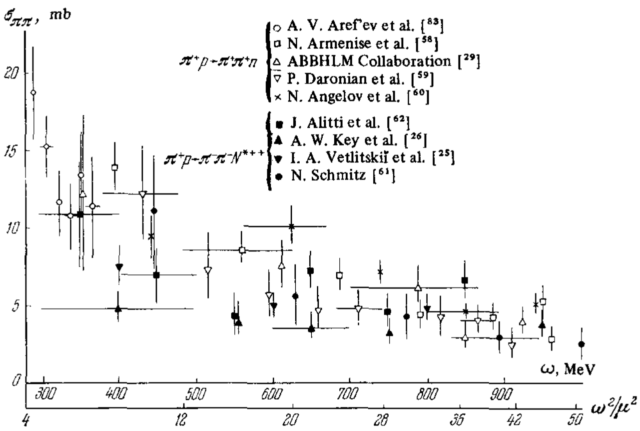

FIG. 21. Energy dependence of the cross section for $\pi^\pm\pi^\pm$ scattering.

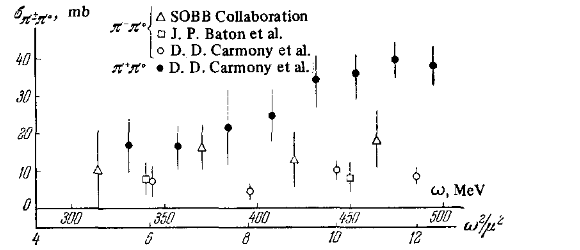

FIG. 22. Energy dependence of the cross sections for $\pi^\pm\pi^0$ scattering.

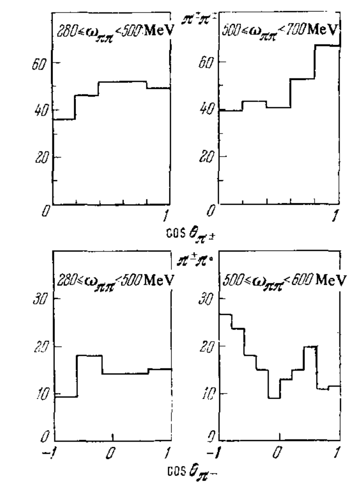

FIG. 23. Angular distributions of pions in the dipion center-of-mass system.

which the cross sections were determined, characteristic of these studies. The hollow points in Fig. 21 refer to the parent reaction (31), and the solid points to reaction (30). We will not need to consider the solid points in Fig. (22), since they are due in significant degree to the resonance πN interaction in the final state, as we have already discussed. The data presented in Fig. 20–22 permit the following conclusions to be drawn:

1. The results of the studies reported are in good agreement with each other, although the data from reaction (31) are scattered, of course, somewhat more strongly. The mutual consistency will be apparent if we take into account the systematic errors not shown in the figures, which in the best cases[30,32,83,87] are on the order of 15%, and in others[54,61,91,92] apparently as high as 30–40%.

2. All the cross sections, as we should expect, have values characteristic for a strong interaction, but nowhere in the ranges of ω shown in the figures do they reach resonance values.

3. The cross sections vary smoothly, not revealing resonance peaks within the resolution in ω. Within the experimental errors, $\sigma_{\pi^+\pi^-}$ and $\sigma_{\pi^-\pi^0}$ remain constant up to $\omega \sim 500$ MeV, and $\sigma_{\pi^+\pi^+} \approx \sigma_{\pi^-\pi^-}$ fall smoothly up to $\omega \sim 900$ MeV.

The angular distributions of pions can be seen in Fig. 7; for the system $\pi^\pm\pi^\pm$ and at small ω for $\pi^\pm\pi^0$ they are further shown, from the sum of all data known to us, in Fig. 23. The data, particularly for small ω, are clearly insufficient. It is difficult to judge about small admixtures of phase shifts, although, as we will see, this can be very important. It is evident only that the S wave is dominant at threshold. This confirms also the relation, valid in the S-wave region, $\sigma_{\pi^\pm\pi^\pm} = 2\sigma_{\pi^-\pi^0}$, which is satisfied at small ω. The rise of the $\sigma_{\pi^-\pi^0}$ cross section relative to the falling $\sigma_{\pi^\pm\pi^\pm}$ cross section is associated with the increasing role of the P wave.

2. Scattering Amplitudes at Threshold

At threshold, for $\omega = 2$, the pion-pion scattering cross sections and amplitudes are related by the expressions:

$$\sigma_{\pi^+\pi^-\to\pi^+\pi^-} = \frac{4\pi}{9}|a_{+-}|^2 = \frac{4\pi}{9}|2a_0 + a_2|^2, \tag{37}$$

$$\sigma_{\pi^+\pi^+} = \sigma_{\pi^-\pi^-} = 8\pi|a_2|^2, \tag{38}$$

$$\sigma_{\pi^+\pi^0} = \sigma_{\pi^-\pi^0} = 4\pi|a_2|^2, \tag{39}$$

$$\sigma_{\pi^+\pi^-\to\pi^0\pi^0} = \frac{8\pi}{9}|a_2 - a_0|^2. \tag{40}$$

Here $a_{+-} = 2a_0 + a_2$ is the amplitude of $\pi^+\pi^-$ elastic scattering in a mixed isotopic state. Extrapolating the data of Figs. 20–22 to $\omega = 2$ and using Eqs. (37)–(39), we find, when systematic errors are included, $|a_{+-}| = 0.95 \pm 0.20$ and $|a_2| = 0.19 \pm 0.02$. Now let us find $|a_0|$. This is conveniently done by means of Fig. 24. Along the abscissa we have plotted a_0, and along the ordinate a_2. The zones parallel to the abscissa denote the regions of probable values of a_2. There are two of them, since we know only $\sigma_{\pi^\pm\pi^\pm} \sim a_2^2$. The width of the bands, as also will be the case later, corresponds to one standard deviation. The parallel inclined bands correspond to

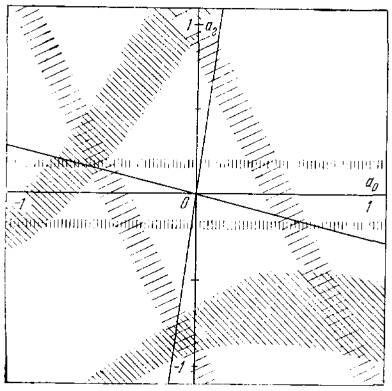

FIG. 24

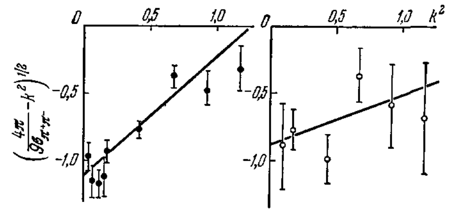

FIG. 25

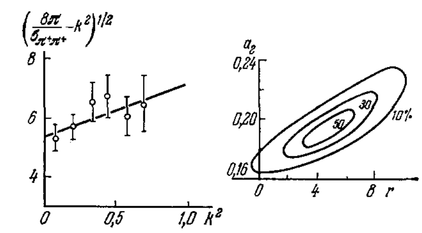

FIG. 26

probable values of a_{+-}, i.e., to Eq. (37). The intersection of the zones gives four regions of possible solutions for a_0 and a_2. In order to choose one of these it is necessary to draw on additional data. The regions bounded by the curves in the second and fourth quadrants show the relation between a_0 and a_2 from analysis of τ decay.[46] We should note first of all the good agreement of the results of Anisovich's analysis[46] with a pair (and particularly with one) of the solutions found. It is important that the pair of separated solutions is such that $|a_0|$ and $|a_2|$ are determined uniquely and that their signs are different. Choice of the solution with $|a_0| \approx 0.6$ is confirmed by the results (which unfortunately are not very accurate) obtained by Braun et al.[72] in study of reaction (33). The lines in Fig. 24 show the center of a certain region of a_0 and a_2 values allowed by these experiments, the limits being rather indefinite. We might expect zones in accordance with Eq. (40), but the authors present their data with respect to $\sigma_{\pi^+\pi^-}$ cross sections obtained under similar conditions for the same q. Thus, they apparently hope to exclude uncertainties associated with the unknown form factors $\Phi'(p^2, \omega)$. This method deserves attention.

In order to choose one of the solutions or, what amounts to the same thing, to determine the sign of the amplitudes, additional data are necessary. The curves relating a_2 and a_0 given by Anisovich and Dakhno[46] are asymmetric, so that there is a hope that improvement of the spectra of pions from τ decay may lead to choice of a single solution. Analysis of Eq. (5) in the region of dominant contribution of the triangle diagram of Fig. 1c[6] and analysis[48] of reaction (27) (Fig. 1g) indicate $a_0 < 0$. We can also attempt to determine the signs of a_{+-} and a_2 in the effective-range approximation. That is, for small relative pion momenta $k = \sqrt{\omega^2/4 - 1} < 1$, i.e., for small ω, there exist the relations

$$\pm\sqrt{\frac{4\pi}{9\sigma_{\pi^+\pi^-}} - k^2} = \frac{1}{a_{+-}} + \frac{k^2 r'}{2}, \tag{41}$$

$$\pm\sqrt{\frac{8\pi}{\sigma_{\pi^+\pi^-}} - k^2} = \frac{1}{a_2} + \frac{k^2 r''}{2} \tag{42}$$

where r is the interaction radius, which we will consider positive, i.e., we will consider the scattering to be potential scattering. Then, provided of course that Eqs. (41) and (42) are applicable, we can find the scattering amplitudes and their signs relative to the sign of the radius. This procedure for a_{+-}, which was carried out for the first time by Aref'ev et al.,[93] is shown in Fig. 25. Figure 26 shows the most correct attempt to determine the sign of a_2 (see also refs. 61 and 32, where the sign of a_2 has been determined by a similar method for $k > 1$). In Figs. 25 and 26 the abscissa represents k^2 and the ordinate the radicals corresponding to Eqs. (41) and (42). It is evident that in these coordinates, as should be the case according to Eqs. (41) and (42), the experimental points lie on straight lines. The two lines in Fig. 25 correspond to the $\sigma_{\pi^+\pi^-}$ cross sections used, which were obtained by extrapolation and in the physical region. The values of $|a_{+-}|$ and $|a_2|$, determined in Figs. 25 and 26 as the free terms of the equation of the straight line, are in good agreement with the values listed above. The sign of the slope of the straight lines gives the sign of the amplitude: $a_{+-} < 0$. The amplitude $a_2 > 0$, but a negative value $a_2 < 0$ is also very likely, as can be seen from the insert in Fig. 26, where we have shown contours of χ^2 as a function of a_2 and r. It is not excluded that $a_0 > 0$, but, adding all data on the amplitude at threshold, it appears that a solution in the second quadrant is preferable. Doubts arise in comparison of the signs of a_0 and a_2 and the signs of the phase shifts δ_0^0 and δ_2^0, found by phase-shift analysis in the ρ-meson region, relative to the sign of the resonance phase shift δ_1^1: it is usually assumed that $\delta_0^0 > 0$, $\delta_2^0 < 0$. Strictly speaking, even the data presented above do not finally exclude the possibility of a solution in the second, third, and fourth quadrants.

3. Resonances in the $\pi\pi$ System

We will first discuss resonances in the low energy region, where we consider the pion-pion interaction cross sections known. We have emphasized the absence of visible irregularities in the energy dependence of the cross sections, and their magnitude, which is far from the unitary limit. However, there have appeared and are appearing in the literature references to resonances in this region. And even though some of these resonances (for example, the ξ with a mass of about 500 MeV) have died a statistical death, as yet not one tenth of the resonances have died, and the others continue to be discussed and deserve attention.

First of these is the ABC resonance or, better to say today, the ABC anomaly at an energy of about 300 MeV. There is no basis to doubt the experimental results of the experiments of Crowe et al. or the existence of a maximum in the spectrum of recoil nuclei in reaction (35). However, what is involved here is apparently not so much the $\pi\pi$ interaction as the type of reaction. Today this statement is adequately justified, since the ABC anomaly is explained by the dominant contribution of the frequently cited triangle diagram.[34,35] However, if we accept this explanation, it is necessary to admit that, in the first place, in a number of studies, for example, which have investigated the spectrum of dipions arising in annihilation of antiprotons, to which the triangle diagram does not apply, the peaks corresponding to the ABC anomaly are statistical; in the second place, the ABC anomaly is an indirect indication of the existence of interesting objects—isonuclei.[94]

The triangle diagram of Fig. 1c is in addition the explanation of singularities which were accepted as a resonance with isospin 0 and zero spin with mass of about 400 MeV, the so-called σ meson. It is sufficient, as an illustration, to look at Fig. 4 from this point of view. However, the question of the σ meson, to which an extensive literature is devoted, is more complicated. The fact is that the σ meson appears in the theoretical analysis of πN and NN collisions,[95-97] and its existence is consistent with data on η and τ decays.[98,99] However, on the one hand, there is no convincing proof of the correctness and, most important, the uniqueness of the theoretical statements, and on the other hand, in theoretical studies themselves recently the σ meson is becoming more and more indistinct. Lovelace et al.[96] give its parameters as mass 430 ± 70 MeV, and width $\Gamma = ^{+400}_{-150}$, which is a strange structure for a Breit-Wigner resonance.

Now, as to the region 700—800 MeV, the region of the generally accepted ρ meson, which even we have considered as well known. The ρ meson is apparently a resonance of Breit-Wigner shape with isospin and spin equal to unity, negative parity, accepted mass and width of the order of 760 and 120 MeV, respectively. (Numerous references to original investigations on observation of the ρ meson and determination of its parameters, as also for other resonances, can easily be found in the well known tables of Rosenfeld.) Here it is necessary to speak of the order of magnitude, in spite of the fact that the statistical accuracies are sufficient. The fact is that in different studies and under different conditions the mass of the ρ meson and particularly its width are extremely different. In some investigations, perhaps, in the "cleanest" conditions, the ρ-meson width approaches 90 MeV. We can recall the "splitting" of the ρ meson, which is not observed at the present time. The angular distributions of pions in decay of neutral and charged ρ mesons are sharply different; the $\pi^+\pi^-$ angular distribution is sharply asymmetric (see Fig. 7). All of this has undoubtedly been responsible for the popularity of the search for an isoscalar scalar meson, the so-called S_0 or ϵ meson with a mass close to the ρ-meson mass, which naturally would explain the features of the ρ meson. In reviewing papers we have already mentioned several original investigations (in particular, the work of Clark et al.[78]). The conclusion of that investigation (where, we recall, $d^3\sigma/dp^2d\omega^2d\theta$ was measured for $\theta = 90°$, where the P-wave contribution should be zero) is as follows: the experimental data are consistent with existence of the S_0 meson, but the corresponding peak in the distribution of the differential cross section as a function of ω can be explained as the consequence of depolarization of the ρ meson. If the pion angular distribution has the form $a + b\cos^2\theta$ then a ratio $a/b = 0.06$ is sufficient to explain the experimental data without S_0. We are alerted also by the closeness of the parameters of the possible S_0 and the ρ. Jones et al.[100] prefer to believe in the existence of the S_0. In addition to the usual analysis of asymmetry of the $\pi^+\pi^-$ angular distribution, they find by extrapolation the asymmetry for $p^2 = -1$ and assert that for $\omega = M_\rho$ it is close to the maximum possible value for interference of the resonance values of the S and P phase shifts. The results of Corbett et al.[77] indicate the absence of the S_0; these authors, with an initial momentum 1.72—2.46 BeV/c, looked for a peak in the mass spectrum of the two neutral pions produced in reaction (34), which is preferable to reaction (5), since the ρ^0 does not decay into $\pi^0\pi^0$. In fact, Corbett et al.[77] state that there is no resonance in the region up to 900 MeV if its width is greater than 30 MeV. Workers who have studied the mass spectrum of the system of two neutral pions in a liquid-deuterium chamber, which employed converter plates in the chamber volume to detect γ rays,[72] do not observe the presence of the S_0. The maximum of the distribution in the region of 700—800 MeV coincides with the maximum of the phase-space curve. Thus, the situation remains uncertain. At present a number of groups are occupied with searches for singularities in the $\pi^0\pi^0$ system. In many investigations a solution of the S_0-meson problem is attempted by means of a phase-shift analysis of $\pi\pi$ data, separating δ_0^0. We will discuss the phase-shift analysis below, and then turn to the question of heavier dipion resonances.

We will pass over the dipion resonances being discussed in the literature today with T = 2 at 850 MeV[24] and T = 0 and J = 0 in the vicinity of 1050 MeV,[101] regarding which we can only make some critical remarks and which need not be considered firmly established. Let us turn to the f_0 meson—an isoscalar resonance with quantum numbers 2^+, mass 1260 ± 10 MeV, and width ~140 MeV. This is a reliable singularity. In the future it will be possible to obtain improved information only on the structure of the resonance and on its quantum numbers. It is important to emphasize that this is a resonance in the D wave (where the δ_0^2 phase shift reaches 90°) with a dominant decay channel to two pions. The still heavier known resonances, whose existence at least as singularities in the missing mass spectra raises no doubt, are less interesting to us from the point of view of the pion-pion interaction, since in their channel the decay into two pions is strongly suppressed.

In conclusion of the paragraph we will draw a somewhat unexpected conclusion: yes, we do not doubt the resonance nature of the pion-pion interaction but, in essence, we know very little about the resonances in the dipion system. New experiments are necessary, even for detailed study of the ρ and f_0 mesons. Universal proof is needed of the existence of resonances with respect to all of the points discussed by us in section 7 of chapter II. It seems extremely important to devote more attention to the mechanism of formation of resonances, and to isolate the conditions under which they appear in pure form.

4. Phase Shifts of $\pi\pi$ Scattering

At low energies, if we assume that only the S wave is important, δ_0^0 and δ_1^0 can easily be found from a_0 and a_2, whose values near threshold have been discussed by us in section 2 of this chapter. We recall that even spins can correspond only to even values of T.

A phase-shift analysis with inclusion of S and P waves (S, P analysis) has been made many times[36-41] on the basis of the angular distributions of Fig. 7. We will not discuss again (see chapter II, section 8) the fundamental questions of the applicability of a phase-shift analysis in this case, but will turn to the results. A strong asymmetry in the $\pi^+\pi^-$ angular distributions in the ρ-meson region, if it is explained by interference of the δ_0^0 and δ_1^0 phase shifts, immediately indicates a large δ_0^0 phase shift. In the opposite case the interference term, which is proportional to $\sin\delta_1^1 \sin\delta_0^0 \cos(\delta_1^1 - \delta_0^0)$, would be close to zero for $\delta_1^1 \approx 90°$. There are discrepancies among different authors as to whether or not the δ_0^0 phase shift passes through 90° in this region. As we can see, we are again discussing the S_0 meson. It appears that for solution of this problem it is necessary first of all to know the sign of the δ_0^0 phase shift.

Ordinarily one concludes from the existence of a positive asymmetry that δ_1^1 and δ_0^0 have the same signs, assuming that $\delta_1^1 > 0$. It is easy to see, however, that the sign of the interference term does not change with a change of sign of δ_0^0, since, in the expression given, both $\sin\delta_0^0$ and $\cos(\delta_1^1 - \delta_0^0)$ change sign. In other words, it is not excluded that the δ_0^0 phase shift is large in absolute magnitude and negative. This possibility is in good agreement with the most probable sign of the δ_0^0 phase shift at threshold. There is an alternative possibility—change of sign of δ_0^0 in the energy interval from threshold up to the ρ meson. In fact, this possibility must be allowed if we are to follow formally all of the asymmetries observed in the $\pi\pi$ system within the bounds of an S, P analysis. However, the variant with change of sign does not seem very attractive, in the first place because it requires an anomalously small $\sigma_{\pi^+\pi^-}$ scattering cross section in the region of change of sign of the phase shift, and in the second place because it is then necessary to acknowledge the accidental agreement of the phase shifts found from angular distributions and from the energy behavior of the $\pi^+\pi^-$ scattering total cross sections. A comparison of the phase shifts has been made in ref. 71. Of course, perhaps $\delta_0^0 > 0$ and $a_0 > 0$ at threshold, but we wish to emphasize that passage of the δ_0^0 phase shift through $-90°$ not only would be an interesting fact, but would also explain a number of features of the observed ρ meson.

Relative clarity and uniqueness exists concerning the phase shift δ_1^1. This energy dependence is shown in Fig. 27, which was taken from the article of Baton et al.[32] Passage of the phase shift through 90° at the

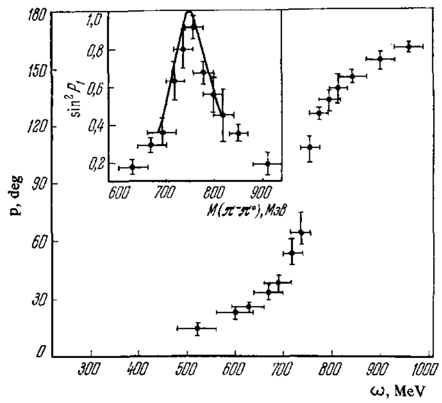

FIG. 27. Energy dependence of the δ_1^1 phase shift of $\pi\pi$ scattering.

ρ-meson mass is easily visible. Particularly obvious is the peak in $\sin^2\delta_1^1$ as a function of ω in the inset.

The moduli of the δ_2^0 phase shifts found both from the angular distributions of the $\pi^-\pi^0$ system and from analysis of angular distributions of pions of the same sign are in good agreement with the values of the $\sigma_{\pi^\pm\pi^\pm}$ cross sections shown in Fig. 21, and can be calculated from them. The δ_2^0 phase shifts are relatively small up to 1 BeV. This is in agreement with the behavior of the asymmetry in the $\pi^\pm\pi^0$ system. Specifically, in the ρ region the asymmetry is practically absent, and $\cos(\delta_1^1 - \delta_2^0)$ is small for $\delta_1^1 = 90°$. The asymmetry arises with decrease and increase of δ_1^1 and changes sign after passage of δ_1^1 through 90°. The sign of the asymmetry indicates that $\delta_2^0 < 0$. Recently a fundamentally important paper[102] has appeared in which the authors attempt to find the sign from observation of interference of the $\pi\pi$-scattering amplitude with the Coulomb amplitude. They also indicate a negative sign of the a_2 amplitude. Unfortunately, the statistics are clearly inadequate.

We have already pointed out some contradictions arising in the course of an S, P analysis, if we believe all of the observed asymmetries. Ferrari and Selleri[24], who analyzed angular distributions in reaction (31), indicate directly the presence of a small δ_2^2 phase shift starting at 600 MeV. There is no doubt that at 700—800 MeV, if not sooner, the δ_0^2 phase shift should also play an important role, passing through a resonance value at 1260 MeV, just as δ_1^1 has an effect up to 400 MeV. It is evident that an S, P analysis is inadequate. However, presence of even a small δ_T^2 phase shift can strongly influence the angular distributions. Therefore, in terms of an S, P, D analysis it is particularly clear how necessary are new accurate data on the angular distribution of the pions in their c.m.s.

Experimentally we know little as yet about the higher phase shifts of $\pi\pi$ scattering and about the behavior of δ_0^0, δ_1^1, and δ_2^0 at energies higher than 1 BeV.

5. Conclusion

The number of papers devoted to the pion-pion interaction is more than several hundred and is increasing, reflecting constant interest in the problem. In this review, which as it is has reached the limits of acceptable size, we could not dwell on all the papers. We have not touched at all on the theoretical papers, and of course these represent the greater number. In these papers various authors, on the basis of the dispersion relations, the hypothesis of partially conserved axial current, crossing symmetry, and other ideas, attempt to predict the $\pi\pi$-scattering amplitudes. It is understandable that strict calculation of the amplitudes is impossible because of the absence of a theory of strong interactions. The existing theoretical predictions can be characterized by noting that the probable values of a_0 and a_2 according to various hypotheses have populated rather densely the square $|a_0| \le 1$, $|a_2| \le 1$. This fact is useful to remember if one is not to be deceived by coincidences of some experimental data with some theory, and must not in any way be interpreted as a call not to be interested in the theoretical studies. A detailed analysis of these studies would unquestionably be useful.

It may be noted that a large number of studies are based on the relation of Chew and Mandelstam[103] $2a_0 = 5a_2$, which is valid in the nonphysical region, at the point where all three of the Mandelstam invariants are equal, $s = u = t$. From this relation it is evident that the signs of a_0 and a_2 would surely be the same. Actually, the relation $2a_0 = 5a_2$ is satisfied by the solution in the third quadrant. However, the situation may change in the physical region, where perhaps[104] we have $2a_0 - 5a_2 = 18a_1$. This returns us to a detailed analysis of the experimental data on the pion-pion interaction and to the urgent problem of their accumulation and improvement.

In this review we have discussed mainly papers published up to January of 1969. We have not endeavored to cite even the experimental papers with extreme completeness. However, we hope that we have been successful in emphasizing the main lines of investigation and the principal properties of the $\pi\pi$ interaction.

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