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DOES THE CHARGE OF THE ELECTRON VARY WITH THE AGE OF THE UNIVERSE?

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INTRODUCTION

DURING the last two years the literature of physics has again contained much discussion of the problem of the possible variation of the atomic constants with the age of the Universe.[1-6] The initiative was taken by G. Gamow, who in his 1967 paper[1] "Electricity, Gravitation, and Cosmology" proposed that the square of the charge of the electron is proportional to the age of the Universe. Immediately after the publication of this paper specific objections were raised. Dyson[2] and Peres[3] deny that there has been an appreciable change of e^2 during the Earth's history, on the basis of existing geological information, and Bahcall and Schmidt[4] give experimental evidence that there is no change of e^2, on the basis of measurements of the spectra of five radio galaxies.

The problem of the variation of the atomic constants with the age of the Universe is of course also a controversial one from a general theoretical point of view. In particular, Ya. B. Zel'dovich[6] calls attention to a contradiction between hypotheses of this sort and the general theory of relativity. However, all of these hypotheses arise from attempts to find a connection between the cosmological parameters and atomic constants, and this problem is of considerable interest. In the present paper we concern ourselves primarily with arguments which have arisen in connection with the suggested change of the charge of the electron.

Let us turn to the history of this question.

More than 30 years ago P. A. M. Dirac showed that the dimensionless ratios of atomic constants (formed from the elementary charge e, the electron mass m, and the speed of light c) to the cosmological constants, namely the Hubble constant H and the density ρ of matter in the Universe—the ratios $Q_1 = mc^3/e^2H$ and $Q_2 = m^4c^6/e^6\rho$—differ only slightly from each other and from the ratio of the electric and gravitational forces acting between the elementary particles, electron and proton: $Q_3 = e^2/GM_pm$, where M_p is the mass of the proton, m is that of the electron, and G is the Newtonian gravitational constant.

On the basis of this empirical connection Dirac proposed[7] that all of the very large ($\sim 10^{40}$) dimensionless universal constants must be regarded as variable parameters characterizing the present state of the Universe.

It follows from this that at least some of the constants must change with the age of the Universe.

I. DIRAC'S PROPOSAL

In the discussion of the universal constants it is convenient to use three independent units of the physical dimensions of velocity, action, and length, which are of fundamental significance in all present theories. They are the speed of light in vacuum, $c = 3 \times 10^{10}$ cm/sec, the "quantum" of action, $\hbar = 1 \times 10^{-27}$ erg sec, and a length characterizing the range of the strong interactions, $\lambda = 3 \times 10^{-13}$ cm, which is numerically equal to the classical electron radius e^2/mc^2. The elementary unit of time is defined as the time required for light to travel a distance of one unit of length. This quantity is often called a "tempon."

The age of the Universe expressed in tempons is

$$\frac{10^{10}\ \text{yrs}}{10^{-23}\ \text{sec}} \sim 3\cdot 10^{40}\,*.$$

This quantity is nearly equal in order of magnitude to the dimensionless ratio of the electric and gravitational forces between proton and an electron:

$$\frac{e^2}{GM_pm} = 2.3\cdot 10^{39}.$$

If we take as unit of time any other quantity composed of atomic constants and having the dimensions of time (e^2/M_pc^3, h/mc^2, h/M_pc^2), then the age of the Universe expressed in such units is again of an order of magnitude comparable with the ratio of the electric and gravitational forces.

If this coincidence is not accidental, but represents a deep connection between cosmology, gravitation, and electricity, then we must suppose that the ratio e^2/GM_pm increases in proportion to the age t of the Universe.

Regarding e^2, M_p, and m as unchanging quantities, Dirac suggested that only the gravitational constant G changes with time, and constructed a cosmological theory in which a basic postulate is the following principle: any two very large dimensionless numbers encountered in nature are connected by a simple mathematical relation in which the coefficients are quantities of the order of unity. In view of the fact that in this theory the gravitational constant changes with time (atomic scale of time), in order to preserve the general theory of relativity Dirac was obliged to introduce new units of distance and time in which the gravitational constant does not depend on the age of the Universe (the so-called "dynamical scale of time").

II. TELLER'S OBJECTION

In 1948 Teller published a short paper[8] in which he analyzed the possibility of a decrease of G with time in paleontological terms. If in fact G decreases with time, then 300 or 400 million years ago G was larger than its present value. On the basis of the thermonuclear theory of the production of solar energy the luminosity L of the Sun is proportional to G^7, and consequently must be decreasing with the age of the Universe ($L \sim 1/t^7$), i.e.,

*Dirac in 1948 took the age of the Universe to be 2×10^9 years.

in the past the Sun shone more brightly. On the other hand, if G decreases with time, in the past the radius R_e of the Earth's orbit must have been smaller ($R_e \sim 1/G$). Using these two facts, Teller showed that in the Cambrian era (~400 million years ago) the Earth's surface would have been boiling, making the existence of trilobites impossible. Indeed, in the Cambrian era there could not have been any oceans at all. In fact

$$T^{\circ}_{\oplus} \sim \left(\frac{L}{R^2}\right)^{1/4} \sim \left(\frac{t^{-7}}{t^2}\right)^{1/4} = t^{-2,25}; \quad t_{\text{comb}} = t_{\text{pres}} - |\Delta t|;$$

$$T^{\circ}_{\oplus\,\text{comb}} = T^{\circ}_{\oplus\,\text{pres}} + \Delta T^{\circ}_{\oplus}; \quad \frac{\Delta T_{\oplus}}{T^{\circ}_{\oplus\,\text{pres}}} = 2,25\,\frac{\Delta t}{t_{\text{pres}}},$$

where $|\Delta t|/t_{\text{pres}} \simeq (4 \times 10^8 \text{ years})/(2 \times 10^9 \text{ years}) = 0.2$. We then get $T^{\circ}_{\oplus\,\text{Camb}} > 370\,^{\circ}\text{K}$.

In this way Teller gave an indirect refutation of the hypothesis that G changes with the time. However, he took the age of the Universe to be 2×10^9 years, instead of the present accepted value 10×10^9 years, and so overestimated the fractional change of G, and consequently also the change of the luminosity of the Sun. If we carry out Teller's calculations in accordance with present-day astronomical data, the "boiling of the oceans" recedes from us in time, making the Cambrian era suitable for marine life.

On the other hand, paleontologists[9] find bacteria and seaweeds whose age is estimated as 3 billion years, so that Teller's arguments remain cogent. If G decreases with time, it is impossible for living organisms to have existed a few billion years ago.

III. GAMOW'S PROPOSAL

Of course, Teller's paper is not the only argument in favor of G's being constant in time. Pochoda and Schwarzschild have shown[10] by computer calculations that if the Sun had shone as brightly two billion years ago as it must have if G changes with time, then the nuclear materials in the Sun could not have sustained such large expenditures over so long a time, and it would by now have burnt up all of its central hydrogen supply and become a red giant star. Gamow,[11] using a different method, came to the same conclusion.

But how can one save Dirac's idea if G does not change with time? The attempt was made. Gamow suggested that we can keep the relation $e^2/G \sim t$ if we assume not $G \sim t^{-1}$ but the alternative hypothesis $e^2 \sim t$. In this case the smaller value of e^2 in the past has no effect on the radius of the Earth's orbit. The problem of the luminosity of the Sun is somewhat more difficult. According to[12] the brightness of the Sun increases as $L \sim k_0^{-1}$, where k_0 is the opacity coefficient of the solar matter. According to the well known Kramers formula this coefficient is proportional to $e^6 = (e^2)^3$, and accordingly must increase as the cube of the time measured from the start of the expansion of the Universe. Then $L \sim t^{-3}$, and the change of the luminosity of the Sun is much weaker than on the assumption $G \sim t^{-1}$. This eliminates the difficulties raised by Teller in his paper[8].

In fact, the temperature of the Earth's surface would vary as $T^{\circ}_{\oplus} \sim L^{1/4} \sim t^{-3/4}$ instead of the law $T^{\circ}_{\oplus} \sim t^{-2.25}$ in the case of change of G. Then (with e^2 changing, not G) even for t equal to one-third of the present age of the Universe the temperature of the Earth's surface would be only somewhere near the boiling point of water. Gamow suggested that the possible increase of e^2 with the age of the Universe can be verified or disproved in two ways:

1. By observing the fine-structure doublets in the spectra of remote galaxies. In fact, both the Doppler effect and the Einstein gravitational shift affect all of the lines of a spectrum in the same way, increasing all the wavelengths by the same factor. On the other hand, the Rydberg constant is proportional to e^4, while the spin-orbit splitting which gives the fine structure of spectral lines is proportional to e^8, which makes it possible to detect a change of e by observing the fine structure in the spectra of remote galaxies.

2. By studying the α and β decay of natural radioactive elements in old rocks. In fact, a change of E e^2 can lead to different changes with time of the probabilities of α decays and of β decays (and, consequently, to different changes of the halflives T_α and T_β). In particular, it affects the branching ratios in the three known radio-active families: ThC, RaC, AcC.

IV. THE OBJECTIONS OF DYSON AND OTHERS

When Gamow's hypothesis $e^2 \sim t$ appeared a number of objections were at once raised.[2,3,4,6]

Dyson,[2] using the data on the distribution of the nuclei ^{187}Re and ^{187}Os on the Earth, concluded that the elementary charge cannot have changed by more than 1/1600 during the entire time the Earth has existed.

In fact, there are definite amounts in the Earth of the isotope ^{187}Os and of the β-active isotope ^{187}Re, which decays into ^{187}Os with a decay half-period 4×10^{10} years and a decay energy $\Delta = 2.6$ keV. According to the semi-empirical mass formula,[20] the energies of the two atoms contain, along with terms not depending on e^2, a Coulomb term $E_C = 0.6\, Z^2 A^{1/3}$, which is proportional to e^2. Furthermore the term E_C is so large that we can neglect all possible nonlinear electromagnetic effects (for example, the nonlinear dependence on e^2 of the mass difference of neutron and proton) and assume that the change of Δ with e^2 is entirely due to the change of E_C.

Accordingly, the difference of the Coulomb energies gives us $(E_C)_{Re} - (E_C)_{Os} = e^2(d\Delta/de^2) = -15.8$ MeV, with a theoretical error of not more than ±10 percent. The decay energy Δ includes, on one hand, this difference of the Coulomb energies, and on the other hand the difference of the energy terms which take into account the symmetry effect in the Weizsäcker formula, and the mass difference of neutron and proton. It is important that Δ is a mass difference between two large quantities, and can be small only for a narrow range of values of e^2. There would be no ^{187}Re left on the Earth if its decay halfperiod had been equal to 2×10^8 years (in an early perion of the Earth's existence, say 3×10^9 years ago). The decay halfperiod for the β transition between these two nuclear states decreases with increase of Δ at least as strongly as $\Delta^{-2.835}$.[13] Consequently, the value of Δ 3×10^9 years ago cannot have been larger than its present value by more than a factor $(200)^{0.353} = 6.5$, where $200 = (4 \times 10^{10} \text{ years}/2 \times 10^8 \text{ years})$ and $0.353 = 1/2.835$. This gives $(d\Delta/dt) \leq 4.75 \times 10^{-9}$

keV/year, and consequently $(1/e^2)(de^2/dt) \leq 3 \times 10^{-13}$ year^{-1}. According to this estimate the increase of e^2 is smaller than that required by Gamow's hypothesis by almost a factor 300.

Dyson points out in conclusion that similar treatments can be given for the isotopes ^{50}V, ^{87}Rb, ^{123}Te, ^{138}La, and ^{176}Lu. In these cases, however, one gets weaker inequalities for $(1/e^2)(de^2/dt)$. In another paper Peres[13] analyzed the possibility of a change of e^2 on the basis of geological data. He called attention to the fact that if Gamow's hypothesis is true the precambrian nuclides would have differed sharply from those of today. The stable heavy elements would have had N/Z ratios closer to 1 (since the deviation of N/Z from unity is due to the electrostatic repulsion between the protons). For example, if e^2 were only a few percent smaller, ^{238}U would be unstable with respect to double β decay to ^{238}Pu, and a further decrease of e^2 would make ^{239}Cm the most stable nuclide with A = 238.

It can, of course, be assumed that our present ^{238}U existed earlier as ^{238}Cm and was produced from it by four β^+ decays (or K captures). The problem of ^{208}Pb is more complicated, however. A billion years ago it would have existed as ^{208}Rn, which is a gas. Our present lead ores would then be uniformly distributed over the whole world, and actually they are not. In a word, the different isotopes of a particular element would have had different chemical histories. Then there would be wide fluctuations in the isotopic composition of elements coming from different sources in the present-day world. It is well known that such variations are extremely small. It can be concluded from this that the electric charge has changed very little or not at all since the time the Earth's core was formed.

V. THE EXPERIMENT OF BAHCALL AND SCHMIDT

Dyson and Peres analyzed the possibility of a change of e^2 during the course of the Earth's history. Still earlier, however, in 1958, Wilkinson,[14] in studying the branching of AcC, came to the conclusion that the charge of the electron is changing by not more than 10^{-12} per year. These are all indirect proofs that the hypothesis $e^2 \sim t$ is without foundation.

Bahcall and Schmidt[4] made a direct test of the Gamow hypothesis. They measured the fine-structure doublets in the O_{III} emission lines in five radio galaxies with $z \approx 0.2$ ($z = \Delta\lambda/\lambda$ is the red shift), and found that $\alpha(z \approx 0.2)/\alpha_{lab} = 1.001 \pm 0.002$, where $\alpha = e^2/\hbar c$ is the fine-structure constant. In this they used unpublished measurements of the wavelengths of the O_{III} multiplet lines at 5007 and 4959 Å in the emission spectra of five radio galaxies.[15]

The travel time of the light from these galaxies is two billion years, i.e., 20 percent of the Hubble expansion. The table gives a list of the observations of the wavelengths of the O_{III} lines of five radio galaxies with sizable red shifts. The formula for the ratio of fine-structure constants is $[\alpha(z)/\alpha_{lab}]^2 = (\delta\lambda/\lambda)_{obs}/(\delta\lambda/\lambda)_{lab}$. Here $\delta\lambda$ is the fine-structure splitting and λ is the average wavelength of the doublet. The hypothesis that α is proportional to the cosmic times t requires $\alpha(z \approx 0.2)/\alpha_{lab} \approx 0.8$, and is thus excluded by the results of Bahcall and Schmidt's observations.

Object	z	λ_{obs}	λ_{obs}	$\alpha(z)/\alpha_{lab}$
In laboratory	0.0	4958.9	5006.8	1
3C219	0.17	5823.1	5880.4	1.009
3C234	0.18	5875.2	5932.3	1.003
3C26	0.21	6003.2	6060.1	0.990
3C171	0.24	6140.6	6200.5	1.005
3C79	0.26	6230.0	6289.7	0.996

VI. DOES A CONNECTION EXIST BETWEEN COSMOLOGY, GRAVITATION, AND ELECTRICITY?

The results given above exclude a variation of the electric charge e not only in proportion to the time t, but also even proportional to ln t, a possibility suggested by Teller in 1948. Accordingly, Dirac's original hypothesis must be rejected, and it is now entirely clear that the surprising number 10^{40}, which seemed so striking to Dirac, has nothing to do with the present age of the Universe.

We cannot, however, easily ignore the coincidences which Dirac pointed out, if we admit the existence of a connection (as yet unknown) between atomic and cosmological constants. In relation to a model of the Universe as a whole these coincidences have an important significance, and they must be preserved in the framework of any cosmological model. Since the recent discovery of residual radiation with temperature $\sim 3°K$[17] evidently confirms the hot model in the expanding-universe theory, Gamow makes an attempt at this, without resorting to the assumption that the electronic charge changes.[18]

If we consider a space filled with a mixture of black-body radiation and matter in the form of baryons in thermal equilibrium, excluding any possible role of neutrinos and other elementary particles at the early stages of the expansion, then the mean total density can be written in the form $\rho = \rho_{rad} + \rho_{mat}$, where ρ_{rad} is the mass density of the radiation and ρ_{mat} is that of the matter; $\rho_{rad} = aT^4/c^2$, where a is the radiation-density constant of Wien's law, T is the radiation temperature in °K, and c is the speed of light. As is well known, as the Universe expands the temperature of the radiation varies in inverse proportion to the radius of curvature of the Universe: $T \sim R^{-1}$, and consequently $\rho_{rad} \sim R^{-4}$, while $\rho_{mat} \sim R^{-3}$. Then the product $\rho_{rad}^{3/4} \cdot \rho_{mat}^{-1}$ is a conserved quantity.

It can be shown[18] (Alpher, Gamov, and Herman) that the ratio of the heat capacities of the radiation and the matter is

$$\frac{c_{rad}}{c_{mat}} = 4{,}945 \cdot 10^4 \rho_{rad}^{3/4} \times \rho_{mat}^{-1} = \text{const } (*).$$

Consequently, c_{rad}/c_{mat} is a characteristic cosmological constant. This quantity does not change during the entire evolution of the Universe. Its numerical value is $c_{rad}/c_{mat} = 4.945 \times 10^4 \rho_{rad}'^{3/4} \times \rho_{mat}'^{-1} \approx 10^{10}$, where ρ_{rad}' and ρ_{mat}' are the respective present densities of radiation and of matter.

This dimensionless characteristic of the cosmological model of the Universe can be regarded as the dimensionless entropy per baryon,

$$\frac{S}{n_b k} = 3.7 \frac{n_\gamma}{n_b} = \frac{1}{2} \frac{c_{rad}}{c_{mat}},$$

where n_γ/n_b is the number of photons per baryon, and

$S/n_b k$ is of the same order of magnitude as $(\hbar c/GM_p^2)^{1/4}$, where k is Boltzmann's constant and $GM_p^2/\hbar c$ is the gravitational analog of the "fine-structure constant" $e^2/\hbar c \sim 1/137$. It must be noted that Zel'dovich and Novikov called attention to this point as early as 1966.[21] They indicated a possible approach to the calculation of the dimensionless entropy as a physical constant characterizing the world: $S^2 \sim (\hbar/M_p c)/(G\hbar/c^3)^{1/2} \sim 10^{19}$.

Let us examine in more detail the role of the cosmological constant c_{rad}/c_{mat} in the evolution of the Universe. It can be shown that this quantity is connected with the characteristic time t* at which the mass density of the radiation becomes equal to that of the matter ($\rho_{rad} = \rho_{mat}$).

In fact, at sufficiently early stages of the expansion of the Universe (small times t), when the density of radiation is much larger than that of matter, $\rho_{rad} \gg \rho_{mat}$, the Friedmann solution for the nonstationary Universe can be written in the form

$$\frac{1}{R}\frac{dR}{dt} = \sqrt{\frac{8\pi G}{3}\rho},$$

where G is the Newtonian gravitational constant, ρ is the mean density, and R is the radius of curvature of space-time. This equation can be integrated if we use $R \sim 1/T$ and $\rho_{rad} = aT^4/c^2$:

$$-\frac{1}{T^3}\frac{dT}{dt} = \sqrt{\frac{8\pi Ga}{3c^2}}, \quad T = \left(\frac{3c^2}{32\pi Ga}\right)^{1/4}\frac{1}{\sqrt{t}} = \frac{1{,}52\cdot 10^{10}}{\sqrt{t}}\ {}^\circ\mathrm{K}.$$

(The constant of integration is chosen so that for $T \to \infty$ we have $t \to 0$). It follows from this that the densities of radiation and matter are

$$\left\{\begin{array}{l}\rho_{rad} = 4{,}42\cdot 10^5 t^{-2}\ \mathrm{g/cm^3},\\ \rho_{mat} = \rho_0 t^{-3/2}\ \mathrm{g/cm^3},\end{array}\right. \qquad (**)$$

where ρ_0 is a constant quantity which can be determined from additional information from observations of the present state of the Universe, namely from data on the residual radiation and the present density of matter ρ'_{mat}. From the relation $\rho_{rad}^{3/4} \times \rho_{mat}^{-1} = \rho_{rad}'^{3/4} \times \rho_{mat}'^{-1}$, taking, for example, t = 1 sec, we get

$$\rho_0 = 1{,}72\cdot 10^4\,\frac{\rho'_{rad}}{\rho_{mat}}.$$

If we take the temperature $\sim 3^\circ$K of the residual radiation and $\rho'_{mat} = 7 \times 10^{-31}$ g/cm^3, then $\rho_0 = 0.090$, and the relations (**) allow us to calculate the characteristic time t*:

$$t^* = (4{,}42\cdot 10^5)^2\cdot \rho_0^{-2} \simeq 3\cdot 10^{13}\ \mathrm{sec}.$$

At the same time we have from the relation (*) $c_{rad}/c_{mat} = 8.5 \times 10^8 \rho_0^{-1}$, and consequently $c_{rad}/c_{mat} = 1.91 \times 10^3 t^{*1/2}$, where t* is in sec.

It is clear from this that the quantity t* is also a characteristic cosmological constant determining the physical conditions (density of matter) of the Universe at any time. This time $t^* \approx 10^6$ years coincides with the time when the hydrogen recombined at a temperature T = 3000–4000°K. Returning to the ratio of the Coulomb and gravitational forces and calculating it for two protons, not a proton and an electron, we get $e^2/GM_p^2 \simeq 1 \cdot 10^{36}$, which is close to t* expressed in tempons:

$$t^* \simeq 10^6 \text{ years} \simeq 3\cdot 10^{13} \text{ sec} = 3\cdot 10^{36} \text{ tempons}$$

If it has a deep physical meaning, the relation $e^2/GM_p^2 = t^*$ is a connection between the atomic constants e and M_p and the cosmological constants c_{rad}/c_{mat} and G. This new connection does not involve any variation of the fundamental constants with time. In fact, t* is actually a constant, just as e^2/GM_p^2 is, and therefore the new relation is preferable to the connection indicated in Dirac's work.[2)] Although this relation e^2/GM_p^2 seems to us a somewhat artificial attempt to preserve the remarkable coincidence pointed out by Dirac, the idea of a dimensionless entropy per baryon (or of c_{rad}/c_{mat}) as a characteristic cosmological constant connected with the atomic constants is itself rather a convincing one.

Accordingly, at present there is no hypothesis (to say nothing of an experiment) claiming to bring out a connection between atomic and cosmological constants which subjects the constancy of the electric charge to any doubt.

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*On the basis of other premises, namely by analyzing the coincidence of the large numbers in terms of the theory containing a cosmological constant, i.e., of the Λ model of the Universe, Ya. B. Zel'dovich [19] suggests the relation $\Lambda^{-1/2}h/M_p c = \hbar/GM_p^2$. This relation is an "eternal" relation between the world constants Λ, G, M_p, $\hbar$, and c and is consistent with the constancy of all these quantities and with the general theory of relativity.

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