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INSTABILITY OF BEAMS OF CHARGED PARTICLES IN A PLASMA

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1. INTRODUCTION

THE instability of beams of charged particles in the plasma has already been the subject of exhaustive reviews by Faĭnberg[3-5] and Briggs[6]. During the time elapsed since the writing of this review, new results were obtained, both experimental and theoretical, concerning the instabilities that were previously either unknown or not subjected to a systematic experimental investigation. A distinguishing feature of these "new" instabilities is the decisive role played by the ions in the dynamics of the developing oscillations, and, consequently such clearly pronounced effects as the interruption of the beam current and the acceleration (heating) of the plasma ions to high energies. These instabilities are characterized also by other occurrence conditions and by a different spatial structure. They constitute material which is of interest for applications and is convenient for an experimental verification of the modern theory. We present below a review of the theoretical and experimental data on these instabilities. We start with a short introduction.

It is known that in a plasma there can exist slow electrostatic waves whose phase velocities are much smaller than the velocity of light. Therefore, if a beam of charged particles propagates in the plasma, then it is relatively easy to create a situation wherein the beam-particle velocity exceeds the phase velocities of the plasma waves. At the same time, these waves are excited by the beam particles via the Cerenkov effect or the anomalous Doppler effect. Under certain conditions this excitation has a coherent character, meaning instability of the beam. Intensive investigations of such beam instabilities were initiated about 20 years ago, following the fundamental theoretical papers of Akhiezer and Faĭnberg[1] and of Bohm and Gross[2]. We can mention a number of causes stimulating these investigations.

1. Of greatest interest in plasma physics are the collective properties of the plasma, due to the motion of a large number of plasma particles. A study of these properties, generally speaking, entails great difficulties, since a plasma, as is well known, has a very large number of degrees of freedom. Great interest attaches in this connection to the particular case when the plasma includes a sufficiently intense group of particles moving with a definite velocity (beam). In this case it is possible to "assign" to the plasma particles a relatively narrow band of degrees of freedom, which can be relatively easily readjusted at the experimenter's will. This produces very favorable possibilities for the investigation of a large class of collective phenomena in a plasma.

2. It turns out that the passage of a beam of charged particles through a plasma leads, under certain conditions, to heating of the electrons and ions of the plasma to high temperatures. This phenomenon is of great interest in three respects: first, from the fundamental point of view; second, it may have a bearing on the acceleration of cosmic-ray particles; third, it can be used as a method of heating the plasma in magnetic traps in order to perform research on controlled thermonuclear reactions and high-temperature plasma.

3. The instability of beams of charged particles in a plasma is of great interest for a large group of applications. In particular, this phenomenon determines the limiting intensity (density) of beams of charged particles, the possibility of accelerating charged particles of a plasma by external magnetic fields, etc.

4. Two-stream instabilities must be taken into consideration in the development of new (coherent) methods of accelerating charged particles to high energies[59]. Finally, it should be noted that interest in problems of the stability of beams of charged particles in a plasma has greatly increased recently in connection with the uncovering of prospects for using relativistic beams of ultrahigh intensity[60].

At the present time we know of a great variety of two-stream instabilities. All of them, however, have certain common properties. One such property is that any two-stream instability is produced by (resonant) interactions of (at least) two components of charged particles. One of these components is made up of the beam particles, and the other of the particle of the medium through which the beam propagates*. The instability consists of a progressive growth in time of the initial fluctuations that occur accidentally in the beam. It can be regarded as a result of positive feedback realized by the particles of the medium. Such particles may be either the plasma electrons or the plasma ions, or else electrons from an external circuit that bounds the space in which the beam propagates. In accordance with the three indicated methods of realizing positive feedback between the beam and the medium particles, one distinguishes between three types of instability of a beam of charged particles. For example, in the case of an electron, these are the electron-electron, electron-ion and the so-called Pierce instabilities. Electron-electron instabilities were exhaustively considered in the reviews[3-7]. The present article is devoted mainly to a systematic analysis of electron-ion instabilities.

The review consists of three parts (Ch. II–IV). In Ch. II we present the theoretical data on the instabili-

*Somewhat in a class by itself is only the so-called slipping-stream instability of a beam of particles with a strongly inhomogeneous velocity profile. A review of theoretical investigations of this instability is contained in a recent monograph by Timofeev [61] (see also [52,53]). This instability has not yet been investigated experimentally.

ties in question. In Ch. III we present an analysis of the experimental data and compare them with the theory. Chapter IV contains a description and an analysis of the results of the investigation of a number of the aforementioned sharply pronounced nonlinear effects observed in experiments with intense (unstable) electron beams in a plasma.

II. THEORY OF INSTABILITY OF BEAMS OF CHARGED PARTICLES IN A PLASMA

We consider first electron-ion beam instabilities. They can be divided into two groups: 1) instabilities of a homogeneous plasma and 2) instabilities of an inhomogeneous plasma. The latter are related by their physical nature, on the one hand, to the former and on the other hand to the so-called universal (drift) instability of an inhomogeneous plasma (see below). We shall use the electrostatic approximation, since the correction for the non-potential character of the oscillations in question becomes important only when their phase velocities approaches the velocity of light. The electrostatic approximation is sufficient in practice even for the analysis of electron-ion oscillations in relativistic beams.

1. Instability of a Spatially-homogeneous Electron Beam

Let us consider first the simplest case, when a monoenergetic electron beam moves through a "background" of slow positive ions that cancel out its space charge: the ion density n_+ is equal to the beam electron density n_1 (quasineutral electron beam). We shall investigate the stability of such a system using the dispersion equation, which expresses the connection between the frequency of the possible oscillations of the charged particles and the wave vector $\mathbf{k}$ of the oscillations. To derive the dispersion equation, we reason as follows: Assume that an electric field $\mathbf{E} = -\operatorname{grad}\psi$, varying with a frequency ω was produced accidentally in the system in question. For simplicity we consider first a case when the charged particles oscillate only along $\mathbf{E}$. In this case $\mathbf{E}$ produces oscillations of the ions, with frequency ω and with velocity $\sim eE/-i\omega M$ (M is the ion mass), which corresponds to a variable density of the ion current $j_+ = e^2 n_+ E/-i\omega M$ (we assume that all the variables have a time variation $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$. According to the continuity equation $\partial\rho_+/\partial t = -\operatorname{div} j_+$, the variable density ρ_+ of the ion space charge will equal $e^2 n + \operatorname{div} E/\omega^2 M$. The influence of the field $\mathbf{E}$ on the beam electrons will be in principle analogous, but owing to the Doppler effect the electrons (moving with velocity u) will experience oscillations with frequency $\omega' = \omega - \mathbf{k}\cdot\mathbf{u}$ (we neglect relativistic effects for the time being); therefore the variable density of the space charge of the beam electrons will be

$$\rho_- = \frac{e^2 n_1 \operatorname{div} \mathbf{E}}{(\omega - \mathbf{ku})^2 m} = \frac{e^2 n_1 \operatorname{div} \mathbf{E}}{\omega_1'^2 m}.$$

Consequently, in accordance with the Poisson equation $\operatorname{div}\mathbf{E} = 4\pi(\rho_+ + \rho_-)$, we obtain

$$\left.\begin{aligned}&[\omega_1^2/(\omega - \mathbf{ku})^2] + (\omega_+^2/\omega^2) = 1,\\ \text{or}\quad &(\omega_1^2/\omega'^2) + (\omega_+^2/\omega^2) = 1,\end{aligned}\right\} \tag{1}$$

where $\omega' = \omega - \mathbf{ku}$; $\omega_1 = (4\pi n_1 e^2/m)^{1/2}$ and $\omega_+ = (4\pi n_+ e^2/M)^{1/2}$ are the Langmuir electron and ion plasma frequencies, and $n_1 = n_+$.

A dispersion equation of the type (1) is encountered in many theoretical papers (see, for example,[8-12]). Let us take the relativistic effects into account. Since the oscillation frequency in the frame of reference of the beam, is

$$\omega' = (\omega - \mathbf{ku})\gamma_0,$$

where $\gamma_0 = [1 - (u^2/c^2)]^{-1/2}$, the dispersion equation (1) takes the form

$$[\omega_1'^2/\gamma_0^2(\omega - \mathbf{ku})^2] + (\omega_+^2/\omega^2) = 1, \tag{1$'$}$$

where ω_1' is the Langmuir frequency of the beam, expressed in terms of its density (n_1') and the electron mass (m) in the reference frame of the beam (i.e., m is the rest mass). Since the beam density in the laboratory system is $n_1 = \gamma_0 n_1'$, Eq. (1′) can be rewritten[9,14,62] in the form

$$[\omega_1^2/\gamma_0^3(\omega - \mathbf{ku})^2] + (\omega_+^2/\omega^2) = 1, \tag{1$''$}$$

where ω_1 is the Langmuir frequency of the beam expressed in terms of n_1 (which equals n_+) and the rest mass of the electron $\omega_1^2 = \gamma_0\omega_1'^2 = M\omega_+^2/m$).

We have considered a case when the oscillations of all the charged particles are directed along the electric field of the oscillations $\mathbf{E}$. This case is realized either if the problem has planar geometry, or in the absence of an external magnetic field. In the more general case when the direction of the oscillations of the particles makes a certain angle θ with the direction of $\mathbf{E}$ (this angle can be different for the ions than for the electrons), the terms in the left-hand side of (1), as can be readily seen from the foregoing derivation, must be multiplied by $\cos^2\theta$. Under the conditions of the experiment there frequently rises a situation whereby the electron oscillations are along the external magnetic field $H = H_z$, directed along the velocity of the beam electrons and making an angle θ with the field $\mathbf{E}$, while the ions oscillate along the field $\mathbf{E}$. This occurs if

$$\omega_{Hi} \ll \omega \ll \omega_{He},$$

where $\omega_{Hi} = eH/Mc$ and $\omega_{He} = eH/mc$ are the ion and electron Larmor frequencies (i.e., when the electrons are "magnetized" and the ions are not). In this case

$$\cos\theta = \begin{cases} k_z/k & \text{for electrons,} \\ 1 & \text{for ions,} \end{cases}$$

where k is the absolute value of the projection of the wave vector of the oscillations $\mathbf{k}$ on the direction of the beam velocity, $k = |\bar{\mathbf{k}}|$; in the nonrelativistic case the dispersion equation of the oscillations takes the form

$$[\omega_1^2/(\omega - k_z u)^2](k_z^2/k^2) + (\omega_+^2/\omega^2) = 1, \tag{2}$$

and in the relativistic case

$$[\omega_1^2/\gamma_0^3(\omega - k_z u)^2](k_z^2/k^2) + (\omega_+^2/\omega^2) = 1. \tag{2$'$}$$

The solution of the dispersion equation (2), which is of fourth degree in ω, is best obtained out by the grapho-analytic method described in[11]. Denoting the left-hand sides of (1) and (2) by $F(\omega)$, let us draw the

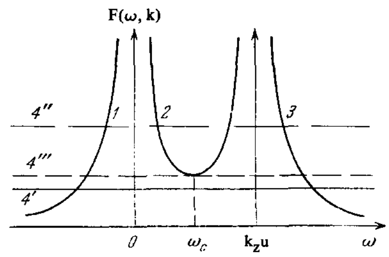

FIG. 1. Dispersion curves: 1–3) Branches of the function $F(\omega, k)$ at k = const, 4) $F(\omega, k) \equiv 1$, 4′) instability, 4″) stability, 4‴) critical regime.

branches of the function $F(\omega)$ (Fig. 1). We see that, depending on the parameters in the dispersion equation, the branches of the function $F(\omega)$ may intersect the line $F(\omega) \equiv 1$ either in four or in two points. In the former case all four roots of the dispersion equation are real and the system is stable. In the latter case only two roots are real, and one of the remaining two (complex conjugate) roots has a positive imaginary part, $\omega = \omega_r + i\gamma$, $\gamma > 0$; in this case instability takes place, and the oscillations increase with time with an increment γ. In the critical regime corresponding to the instability threshold, the line $F(\omega) \equiv 1$ is tangent to the central branch of the function $F(\omega)$, and in this case $\partial F/\partial\omega = 0$. This regime corresponds, in the case described by Eq. (2), to an oscillation frequency

$$\omega = k_z u \{1 + [(M/m)(k_z^2/k^2)]^{1/3}\}^{-1}. \tag{3}$$

Substituting the value of ω from (3) in (2), we obtain an expression for the threshold (critical) beam density n_{1cr}, starting with which instability sets in:

$$\omega_{1cr}^2 \equiv 4\pi n_{1cr} e^2/m = k^2u^2 \{1 + [(m/M)(k^2/k_z^2)]^{1/3}\}^{-3}, \tag{4}$$

and for the threshold current of the beam

$$I_{cr} \equiv \pi a^2 e n_{1cr} u = (ma^2/4e) k^2 u^3 \{1 + [(m/M)(k^2/k_z^2)]^{1/3}\}^{-3}, \tag{5}$$

where a is the radius of the beam.

Expressions (3)–(5) were obtained for the nonrelativistic case. In the relativistic case, as can be readily seen,

$$I_{cr} \equiv (ma^2/4e)\,\omega_{1cr}^2 u = (ma^2/4e) k^2 u^3 \gamma_0^3 \{1 + [(m/M)(k^2/k_z^2)\gamma_0^3]^{1/3}\}^{-3}. \tag{5'}$$

It is interesting to note that if the second term in the curly brackets of the denominator of (5′) is much larger than the first, then as $u \to c$ the current I_{cr} ceases to depend on the beam-electron energy:

$$I_{cr} \to (Ma^2/4e) k_z^2 c^3. \tag{5''}$$

For example, for argon ions, a = 1 cm, $k_z \approx \pi/L \approx 3 \times 10^{-2}\ \text{cm}^{-1}$ (beam length L = 100 cm), we get from (5″) $I_{cr} \approx 3 \times 10^5$ A.

The instability increment γ can be estimated from the dispersion equation (2), by assuming (in accordance with (3)) that $\omega \ll k_z u$. At a sufficiently large excess above the critical value (beam current $I \gg I_{cr}$), this yields

$$\gamma \approx (m/M)^{1/2} k u \gg \omega_+. \tag{6}$$

In the case described by the dispersion equation (1) or (1′) (planar geometry or H = 0), the expressions for the quantities ω, ω_{1cr}, and I_{cr} are obtained from (3)–(5) by substituting $k = k_z$.

From the point of view of experiment, greatest interest attaches to the case when the beam of the radius a propagates along the axis of a metallic cylinder of radius $R_0 \gg a$ (beam length $L \gg R_0$). In this case the instability threshold current is determined by the quantity $k^2 = k_z^2 + k_r^2$, where[13]

$$k_r^2 \approx 2/a^2 \ln(R_0/a), \tag{7}$$

and the minimal value of k_z can be assumed to equal π/L (see below).

In another case, when $a = R_0$ we have $k_r = 2.4/a$. The criterion for the electron-ion beam instability in the case of an arbitrary ratio of a, R_0, and L (assuming magnetization of both the electrons and of the ions) was obtained in the theoretical paper[14].

Expression (4) is equivalent to the instability condition

$$(\omega_+^2 k^2/k_z^2)^{1/3} + (\omega_1^2)^{1/3} > (k^2u^2)^{1/3}. \tag{8}$$

The oscillations considered by us are volume oscillations and constitute the oscillations of the space charges of the particles of both polarities inside the beam; accordingly we have div $\mathbf{E} \neq 0$ inside the beam. Unlike such oscillations, there exists one surface mode of oscillations, wherein the particle charges are produced only on the surface of the beam, and accordingly we have div $\mathbf{E} = 0$ inside the beam. Naturally, such a mode is possible only when the beam radius is smaller than the radius of the metallic cylinder ($a < R_0$). Excitation of such a mode was considered in[16] (see also[15]). It has been shown that the dispersion equation for this mode differs from Eq. (2) in that the ion term enters with the same factor k_z^2/k^2 as the electron term (the quantity k^2 is determined as before by expression (7)). Therefore the critical current, unlike (5′), contains in the curly brackets of the denominator the term $(m\gamma_0^3/M)^{1/3}$, without the factor $(k^2/k_z^2)^{1/3}$. We see that this critical current is larger than the excitation current of the volume mode considered by us, namely, the surface mode is more difficult to excite. We shall therefore consider only the volume modes.

It is important to note that the expression sometimes cited in the literature for the frequency of the considered electron-ion oscillations is the one derived by Buneman[10]:

$$\omega \approx \omega_1^{1/3}\omega_+^{2/3} \approx \omega_1 (m/M)^{1/3}. \tag{3a}$$

This expression is valid only for the one-dimensional case when $k = k_z$. In fact, in this case we obtain from (3) $\omega \approx ku(m/M)^{1/3}$, from which expression (3a) follows on the basis of (5). The instability increment is in this case

$$\gamma_{max} \approx (3^{1/2}/2^{4/3})(m/M)^{1/3}\omega_1 \approx 0.7\omega. \tag{3b}$$

If $k_z \ll k$, expressions (3a) and (3b), naturally, no longer hold.

We have thus considered the electron-ion instability of a homogeneous quasineutral monoenergetic electron beam. In the case of the electron-electron instability, the entire analysis remains essentially the same. For example, for "magnetized" electrons ($\omega_{He} \gg \omega$) the dispersion equation, as can be readily seen from the foregoing derivation, has in the nonrelativistic regime the form

$$[\omega_1^2/(\omega-k_z u)^2](k_z^2/k^2)+(\omega_2^2/\omega^2)(k_z^2/k^2)=1, \quad (9)$$

from which follows the instability condition

$$(\omega_1^2)^{1/3}+(\omega_2^2)^{1/3}>(k^2u^2)^{1/3}, \quad (10)$$

where ω_2 is the Langmuir frequency of the plasma electrons ($\omega_2^2 = 4\pi n_2 e^2/m$) and expressions for ω and I_{cr}:

$$\omega=k_z u\,[1+(n_1/n_2)^{1/3}]^{-1}, \quad (11)$$

$$I_{cr}=(ma^2/4e)\,k^2u^3\,[1+(n_2/n_1)^{1/3}]^{-3}. \quad (12)$$

In the relativistic regime, obviously,

$$I_{cr}=(ma^2/4e)\,k^2u^3\gamma_0^3\,\{1+[(n_2/n_1)\,\gamma_0^3]^{1/3}\}^{-3}. \quad (12')$$

From condition (10) follows the interesting conclusion that when $\omega_2 > ku$ the instability sets in at any beam density or, in other words, there is no instability threshold with respect to the beam density. This corresponds to the fact that when $\omega_2 > ku$ the middle branch of the function $F(\omega)$ in Fig. 1 does not cross the horizontal line $F(\omega) \equiv 1$ at any beam density. On the other hand, if the Langmuir frequency of the plasma electrons ω_2 is smaller than ku, then according to (10) the instability sets in, but only starting with a certain threshold beam density. Two cases are then possible. In the first case, when ω_2 is slightly smaller than ku, the phase velocity ω/k_z of the developing oscillations, as seen from Fig. 1, is very close to the velocity of the beam electrons. This means that at a sufficiently small ratio n_1/n_2 (rarefied beam and dense plasma) the instability is based on the Cerenkov effect. In the other case, when the ratio n_1/n_2 is sufficiently large (dense beam), the phase velocity ω/k_z of the growing oscillations is much smaller than the velocity of the beam electrons; in this case the instability is based on the anomalous Doppler effect [17].

The physical meaning of this effect can be explained in this case as follows. The beam particles can, in principle, execute collective (Langmuir) oscillations with different amplitudes. Therefore the beam can be regarded as a system of oscillators capable of being at different levels of vibrational energy. In transitions between these levels, the beam emits oscillations that propagate in the plasma with a certain phase velocity. If this velocity exceeds the beam-particle velocity ($\omega/k_z < u$), then the radiation of the oscillations by the beam is accompanied not by a decrease of the vibrational energy of the beam (as is the case for $\omega/k_z > u$) but by an increase. The characteristic feature of the anomalous Doppler effect consists indeed of a transition of the radiating system to a higher energy level[17]. This transition, denoting buildup of oscillations in the system, is effected, naturally, at the expense of the energy of the longitudinal motion of the beam.

The buildup of the electron-ion oscillations, considered above for a quasineutral beam, is also connected with the anomalous Doppler effect. A classification of beam instabilities from the point of view of the elementary processes on which they are based is given in the review [3].

We have previously distinguished between electron-ion and electron-electron types beam instability in a plasma into depending on which of the two plasma components (ions or electrons) plays the principal role in the buildup of the oscillation. The type of instability actually realized is determined by the ratio of the ion and electron (plasma) terms in the dispersion equation; in the case of a three-component system consisting of magnetized electrons and nonmagnetized ions, this equation is

$$[\omega_1^2/(\omega-k_z u)^2](k_z^2/k^2)+(\omega_+^2/\omega^2)+(\omega_2^2/\omega^2)(k_z^2/k^2)=1. \quad (13)$$

The foregoing distinction between the instability branches has a physical meaning if the second and third terms of the left side of (13) and accordingly the thresholds (5) and (12) are significantly different. Otherwise the instability is due to the interaction of all three components of the system.

2. Instability of a Spatially-inhomogeneous Electron Beam in a Plasma

In the preceding section we considered the excitation of axially symmetrical oscillations, i.e., oscillations in which the electric field **E** and the wave vector **k** have only two components: longitudinal E_z and k_z, and radial E_r and k_r (accordingly, $k^2 = k_z^2 + k_r^2$). The azimuthal components E_φ and k_φ were assumed to be equal to zero. Such a limitation of the degrees of freedom of the resultant oscillations was connected with the fact that in the case considered above, that of homogeneous distribution of the cross section of the beam, the oscillations that have no axial symmetry, i.e., for which $E_\varphi \neq 0$ and $k^2 = k_z^2 + k_r^2 + k_\varphi^2$, require a higher threshold current, proportional to k^2, for their occurrence.

The situation, however, changes fundamentally if the distribution of the current density in the cross section of the beam is essentially inhomogeneous ($\nabla n_1 \neq 0$). In this case a new instability mechanism arises, connected with the drift motion of the charged particles in the crossed fields—azimuthal electric field E_φ of the oscillations and longitudinal magnetic field H_z. This is essentially the same mechanism causing the drift or gradient (sometimes called universal) instability of an inhomogeneous plasma—the same instability that is of greatest interest in investigations on control thermonuclear fusion (see, for example,[18] concerning this instability). The difference is connected only with the electron velocity distribution function, which in this case is not Maxwellian, and includes a particle beam. Thus, the instability accompanying the buildup of axially-asymmetrical oscillations of a strongly inhomogeneous beam in a plasma combines the feature of the "ordinary" two-stream instability of a homogeneous plasma and the drift instability of an inhomogeneous plasma. We shall call this drift-beam instability. The theory of this instability was developed by Mikhaĭlovskiĭ [19] and by Rukhadze and co-workers[20].

In order to consider this theory, we turn to the derivation of the dispersion equation (1), (2), (13) for axially-symmetrical electron-ion oscillations and see how this equation must be modified in order for it to describe also oscillations that have no axial symmetry. We shall consider as before the case when the plasma ions are not magnetized, and the electrons are magnetized ($\omega_{Hi} \ll \omega \ll \omega_{He}$). In this case the only new effect due to the presence of the azimuthal (perturbed) field E_φ and the radial charged-particle density gradient is the radial drift of the beam and plasma electrons with velocity $\sim cE_\varphi/H$. As a result of this drift, radial particle currents are produced: $j_\perp \sim necE_\varphi/H \approx ineck_\varphi\psi/H$, where n is the particle density and ψ is the perturbed potential. These currents change the amplitude of the space-charge density oscillations of the particles in accordance with the continuity equation

$$\frac{\partial\rho}{\partial t} = -\operatorname{div}(\mathbf{j}_\parallel + \mathbf{j}_\perp),$$

where $\mathbf{j}_\parallel$ is the longitudinal particle current (along H). For example, for the plasma electrons

$$\operatorname{div}\mathbf{j}_\parallel = -e^2 n_2 \frac{\partial E_z}{\partial z} \Big/ i\omega m = -n_2 e^2 k_z^2 \psi / i\omega m,$$

$$\operatorname{div}\mathbf{j}_\perp \simeq \frac{ceE_\varphi}{H}\frac{\partial n_2}{\partial r} \approx \frac{n_2 e^2 k_\varphi}{i\omega_{He} m}\frac{\psi}{R} \simeq \frac{n_2 e^2 s\psi}{i\omega_{He} m R^2},$$

where R is the characteristic transverse dimension ("radius") of the plasma, and s is the number of the azimuthal mode, i.e., the number of azimuthal waves subtended by the perimeter of the plasma. Consequently, the alternating density of the space charge of the plasma electrons is

$$\rho_2 \approx (e^2 n_2 k_z^2/m\omega^2)\,\psi - (e^2 n_2 s/R^2 m\omega\omega_{He})\,\psi.$$

The expression for the alternating density ρ_1 of the space charge of the beam electrons will be perfectly analogous, apart from the substitutions $n_2 \to n_1$, $\omega \to (\omega - k_z u)$, $R^2 \to a^2$, where a is the "radius" of the beam (we are considering for the time being the nonrelativistic regime). Finally, the alternating density of the space charge of the ions ρ_+ will be described by the same expression as before (the ion oscillations are as before along the resultant electric field $E = -\operatorname{grad}\psi$). Substituting the expressions for ρ_1, ρ_2, and ρ_+ in the Poisson equation $\nabla^2\psi = -k^2\psi = -4\pi(\rho_+ + \rho_1 + \rho_2)$, we obtain a dispersion equation of the type $F(\omega, k) = 1$ with five terms in the left-hand side—one ionic and two electronic:

$$\frac{\omega_1^2 k_z^2/k^2}{(\omega - k_z u)^2} - \frac{2s\omega_1^2}{a^2 k^2 \omega_{He}(\omega - k_z u)} + \frac{\omega_2^2 k_z^2/k^2}{\omega^2} - \frac{2s\omega_2^2}{R^2 k^2 \omega_{He}\omega} + \frac{\omega_+^2}{\omega^2} = 1. \qquad (14)$$

This dispersion equation differs from (13) in the presence of two "drift" terms (second and fourth), which are connected with the drift motion of the beam at plasma electrons in the crossed fields: the perturbed electric field E_φ and the main magnetic field H_z. These additional terms, which differ from zero only if the conditions $k_\varphi \neq 0$ and $\operatorname{grad}(n_1, n_2) \neq 0$ are simultaneously satisfied, greatly changes the stability criterion of the system, as will be shown subsequently.*

Let us proceed to analyze Eq. (14). Turning to Fig. 1, we can readily see that, other conditions being equal, the beam-drift term (which, according to (3′), has a positive sign), contributes to a "detachment" of the central branch of the function $F(\omega)$ from the horizontal straight line $F(\omega) \equiv 1$. This should lead to a decrease of the critical current at which the instability occurs, provided only that the indicated effect is not cancelled by another effect mentioned above, namely the raising of the instability threshold as a result of the increase of k^2. It is likewise easy to see that the plasma drift term (having a negative sign) contributes to stabilization of the instability.

The presence of drift terms of first order in ω in the dispersion equation excludes the possibility of an exact analytic determination of the critical current of the instability by the method used above. We shall therefore proceed in a different manner: we shall assume that the inequality $\omega < k_z u$ is sufficiently strong. Neglecting in (14) the value of ω compared with $k_z u$ and turning to the case of a quasineutral electron beam ($n_2 \ll n_1$, $n_+ \approx n_1$), we obtain

$$\omega_{1\mathrm{cr}}^2 \approx k^2 u^2 [1 + (2su/a^2\omega_{He}k_z)]^{-1} \qquad (15)$$

(when $\omega_1 > \omega_{1\mathrm{cr}}$, the quantity ω^2 defined by (14) becomes negative, corresponding to the buildup of oscillations). Accordingly, the instability threshold current is

$$I_{\mathrm{cr}} \approx (ma^2/4e)\,k^2u^3 [1 + (2su/a^2\omega_{He}k_z)]^{-1}. \qquad (16)$$

If the second term in the denominators of (15) and (16) is large compared with unity

$$su/a^2\omega_{He}k_z > 1 \qquad (17)$$

(strongly inhomogeneous beam of small radius a in weak magnetic fields), then the considered drift effects greatly lower the instability threshold.

The instability increment at a beam current much larger than critical ($I \gg I_{\mathrm{cr}}$) is of the order of

$$\gamma \approx (\omega_{Hi}k_z u/s)^{1/2}. \qquad (18)$$

In the relativistic regime, as can be easily shown (taking into account the Lorentz transformations of ω, ρ, and E_φ), the only change occurs in the first term of (14), in whose denominator there appears the familiar factor γ_0^2 appears. Accordingly the threshold current will be expressed by the relation[15]

$$I_{\mathrm{cr}} \approx (ma^2/4e)\,k^2u^3\gamma_0^3 [1 + (2su\gamma_0^3/a^2\omega_{He}k_z)]^{-1}, \qquad (16')$$

where $\omega_{He} = eH/mc$.

It is seen from (16′) that in the relativistic regime the drift effects strongly influence the two-stream instability already if the following condition is satisfied:

$$su\gamma_0^3/a^2\omega_{He}k_z > 1. \qquad (17')$$

If the second term in the denominator of (16′) is much larger than unity, and if γ_0 is sufficiently large ($u \approx c$), then the critical current is independent of the energy of the beam electrons

$$I_{\mathrm{cr}} \approx (mc^2/4e)\,\pi a^2\omega_{He}/L, \qquad (16'')$$

where we assume $k_z = \pi/L$, $s = 1$ and $k^2 \approx k_r^2 + k_\varphi^2 \approx 2/a^2$. For example, at $H = 10^4$ Oe, a = 1 cm, and L = 100 cm formula (16″) yields $I_{\mathrm{cr}} \approx 300$ A.

It is of interest to compare the threshold (16″) with

*The coefficient 2 in the drift terms is connected with a more detailed account of the radial distribution of the beam and plasma density. We assume the parabolic distribution [19] $n_1(r) = n_1(0)\,[1-(r^2/a^2)]$, $n^2(r) = n_2(0)[1-(r^2/R^2)]$.

the critical current (5″) for the excitation of axially-symmetrical oscillations. We see that the threshold (16″) is lower than the threshold (5″) under the condition

$$eH/Mc \equiv \omega_{Hi} < \pi c/L.$$

This condition can be violated only in giant magnetic fields, for example in the case of argon and $L = 10^2$ cm when $H > 10^7$ Oe (!).

As already indicated, expressions (15) and (16) are valid only for a quasineutral beam, i.e., for a system consisting of two components, fast electrons and slow ions. Thus an additional plasma is introduced in such a beam, then the instability threshold rises. At sufficiently large plasma density, when the drift plasma term is much larger than the drift beam term, the instability vanishes. Therefore the condition

$$\frac{n_2}{n_1} > R^2/a^2, \tag{19}$$

as seen from the dispersion equation (14), is apparently the sufficient condition for stability. A condition close to (19) can be readily obtained by neglecting in the dispersion equation (14) the homogeneous beam term compared with the drift term, and by assuming for simplicity $\omega \ll k_z u$ $(k_z^2/k^2)\,\omega_2^2 \gg (m/M)(\omega_1^2 + \omega_2^2)$, $\omega_2^2 s^2 \gg k^2 k_z^2 R^4 \omega_H^2$. Then the stability condition (the imaginary part of the complex frequency ω is negative) takes the form

$$\frac{n_2}{n_1} > (R^2/a^2)\, R^2\omega_{He}k_z/su\,^*). \tag{19'}$$

If $R^2\omega_{He}k_z/su \approx 1$ (as is the case in the experiments described below, then the conditions (19) and (19′) are practically equivalent (the derivation of (19′) is due to V. V. Arsenin[82]).

We now proceed to a more detailed comparison of the considered drift-beam instability with the "universal" instability of a spatially-inhomogeneous plasma, in which the electrons have a Maxwellian velocity distribution. Unlike the drift-beam instability in which the phase velocity of the oscillations ω/k_z is much smaller than the beam-electron velocity u, the universal instability is caused by resonant electrons moving along the magnetic field ($H = H_z$) with velocities v_z close to the phase velocity ω/k_z of the (drift) wave excited by them. If the plasma were spatially-inhomogeneous, then buildup of the wave by the resonant electron would be impossible at a Maxwellian electron velocity distribution, owing to the well known "Landau damping"[21]. However, the plasma inhomogeneity gives rise to a new effect, namely, the drift of the charged particles in the crossed fields (the perturbed E_φ and the external H_z) causes the number of the resonant particles that lag the wave and absorb its energy to decrease, whereas the number of resonant particles leading the wave and transferring energy to it increases. Total cancellation of the Landau damping (meaning instability) sets in at the condition[18,19]

$$k_\varphi \frac{\partial f_0}{\partial r}\frac{v_z}{\omega_{He}\omega} > \frac{\partial f_0}{\partial v_z}\Big|_{v_z=\omega/k_z} \tag{20}$$

and the additional condition

$$v_{Ti} \ll \omega/k_z \ll v_{Te},$$

where f_0 is the distribution function of the plasma electrons with respect to the velocities and the coordinates, T_e, T_i, $v_{Te} = (T_e/m)^{1/2}$, $v_{Ti} = (T_i/M)^{1/2}$ are the temperatures and the thermal velocities of the plasma electrons and ions. In the case of a Maxwellian electron velocity distribution, condition (20) takes the form

$$k_\varphi v_z/a\omega_{He}k_z > 1, \quad sv_z/a^2\omega_{He}k_z > 1 \tag{21}$$

or

$$\omega \leqslant \omega^* = cT_e k_\varphi/eHa, \tag{21a}$$

where a is the "radius" of the plasma, i.e., the characteristic dimension of the transverse (relative to the direction of the magnetic field) inhomogeneity of the plasma density, $s = k_\varphi a$ is the number of the azimuthal mode of the instability, $v_z = \omega/k_z$ is the velocity of the resonant electrons, and ω^* is the so-called drift frequency.

We see that when the substitution $v_z \to u$ is made the criterion (21) goes over into the criterion (17). This is an illustration of the fact that the "universal" and the drift-beam instabilities are based on the same physical mechanism. At the same time, the difference in the form of the electron velocity distribution function lies in the fact that the drift-beam instability has a threshold (16) with respect to the electron density, whereas the universal instability has no such threshold. This circumstance makes it possible to simulate effectively the universal instability in experiments with electron beams. We shall return to this question in the next chapter.

3. The Pierce Instability[22]

In this section we continue the analysis of the stability of a quasineutral electron beam consisting of a stream of fast electrons with concentration n_1 and a "background" of compensating ions with concentration $n_+ = n_1$ (the concentration of the plasma electrons is $n_2 = 0$). We have seen above that such a system of charged particles reveals an instability that is connected in fundamental fashion with the character of the motion of the "background" ions and vanishes if the ions become immobile. In fact, if the ion mass tends to infinity, then the increment of the considered instability tends to zero. This is seen from expressions (6), (3b), and (18), as well as directly from the dispersion equation (1), (13), and (14), which gives purely real values of the oscillation frequency ω as $M \to \infty$ ($n_2 = 0$). This is natural, since infinitely heavy (immobile) cannot realize the positive feedback which is necessary for the occurrence of instability.

Nonetheless, as first shown by Pierce[22], a quasineutral electron beam with infinitely heavy ions can be unstable if the space in which it propagates is bounded on all sides by metallic walls interconnected by an external circuit with infinite conductivity. In this case the function of the positive feedback, which leads to the buildup of the instability, is assumed by the electrons of the external circuit, which maintain the beam-limiting walls at an equal potential.

*See [82] concerning the control of the instability in question by means of feedback.

Following Pierce, let us consider a plane quasineutral electron beam compensated by infinitely heavy ions. The beam propagates between two ideal grids, the distance L between which is much smaller than the transverse dimensions of the beam. The grids are interconnected by an ideally conducting external circuit, the electrons of which "see to it" that the potential difference between the grids at any instant of time is strictly equal to zero.

Assume that oscillations with frequency ω have occurred accidentally in the beam. Let us see under which conditions these oscillations can grow in time. We assume first that the grids do not influence the behavior of the beam. Then the oscillations are described by a dispersion equation (1), which gives two possible values of the wave number of the oscillations

$$k_1=(\omega+\omega_1)/u, \quad k_2=(\omega-\omega_1)/u. \qquad (22)$$

These two values of the wave number correspond to two space-charge waves: a slow one, with a phase velocity ω/k_1, and a fast one with a phase velocity ω/k_2. The total oscillation of the potential (satisfying, naturally, the Poisson equation $k^2\psi = 4\pi\rho$) will be described by the equation

$$\psi=[A\exp(ik_1z)+B\exp(ik_2z)]\exp(-i\omega t).$$

Both waves have an amplitude that is constant in time, since the increment of the oscillations, as seen from the dispersion equation (1), is equal to zero. We now take into account the influence of the grids. Under the influence of the electric field of the fast and slow waves there will be induced in the grids charges whose "task" is to satisfy the condition that the grids be equipotential. (If such charges were not induced, then the indicated condition might be violated since the phase difference of the potential oscillations at the locations of the grids, generally speaking, could be arbitrary). In the space between the grids, the potential of the electric field of the induced charges satisfies the Laplace equation $\nabla^2\psi = 0$, i.e., $\psi = Cz + D$. Thus, the general solution of the Poisson equation for the potential (with allowance for the field of the charges induced on the grid) is

$$\psi\exp(i\omega t)=A\exp(ik_1z)+B\exp(ik_2z)+Cz+D. \qquad (23)$$

To determine the constants A, B, C, and D, Pierce formulated the following boundary condition: in the plane of the first grid ($z = 0$) all the variable quantities (the potential ψ, the velocity v, and the density ρ) are equal to zero. Expressing v and ρ in terms of ψ with the aid of the Poisson equation, the continuity equation, and the equation of motion (in analogy with the procedure used at the beginning of Ch. II), and using the indicated boundary condition, we obtain three equations, from which we determine the three quantities B/A, C/A, and D/A. Substitution of these quantities in (23) yields

$$\psi\exp(i\omega t)\sim\frac{\omega^2}{u}z+\frac{i}{2}\omega_1\left\{\frac{\omega-\omega_1}{\omega+\omega_1}\left(\exp\left[i(\omega+\omega_1)\frac{z}{u}\right]-1\right)+\frac{\omega+\omega_1}{\omega-\omega_1}\left(\exp\left[i(\omega-\omega_1)\frac{z}{u}\right]-1\right)\right\}. \qquad (23a)$$

In order to derive the dispersion equation from the thus-obtained expression for the potential $\psi(\omega, \omega_1, z)$, we use one more boundary condition, namely $\psi = 0$ when $z = L$. We then obtain

$$\omega^2+\frac{i}{2}\frac{\omega_1u}{L}\left\{\frac{\omega_1-\omega}{\omega_1+\omega}\left(\exp\left[i(\omega+\omega_1)\frac{L}{u}\right]-1\right)+\frac{\omega_1+\omega}{\omega_1-\omega}\left(\exp\left[i(\omega-\omega_1)\frac{L}{u}\right]-1\right)\right\}=0. \qquad (24)$$

Solution to the dispersion equation gives the following result. If ω_1 is sufficiently small ($\omega_1 < \pi u/L$), then all the roots of (24) have negative imaginary parts, meaning damping of the oscillations, i.e., stability of the beam. When $\omega_1 \gg \pi/L$, Eq. (24) yields

$$\omega\approx(i\pi/4)[\omega_1-(\pi u/L)], \qquad (25)$$

Corresponding to aperiodic instability (the real part of the oscillation frequency is equal to zero). The critical (threshold) regime separating the instability from the instability is one in which

$$\omega_1=\omega_{1P}\equiv\pi u/L \qquad (26)$$

and $\omega = \gamma = 0$ (ω_{1P} is the Pierce threshold). In this regime, as seen from (23) and (23a), the constants A, B, C, and D, which determine the function $\psi(z)$ of (23), have values $C = D = 0$ and $B = -A$, and the distribution of the potential oscillation amplitude along the beam is given by

$$\psi\sim A\sin(\pi z/L). \qquad (27)$$

When $\omega_1 \gtrsim u/L$, we have aperiodic instability with an increment

$$\gamma=(\pi/4)[\omega_1-(\pi u/L)]=(\pi/4)\alpha/\tau\approx(\pi^2/8)\tau^{-1}\Delta j/j_P, \qquad (28)$$

where $\tau = L/u$ is the electron time of flight, $\alpha = (\omega_1 L/u) - \pi$ is the supercriticality parameter,

$$j_P=(m/4\pi e)\omega_{1P}^2u=(\pi/4)(m/e)u^3/L^2 \qquad (29)$$

is the threshold beam current instability above which the Pierce instability sets in, $\Delta j = j - j_P$, and j is the beam current density. (The right-hand inequality in (28) is valid when $\Delta j < j_P$.) We see from (28) that at not too low a supercriticality ($\Delta j/j_P$) the increment γ is of the order of the reciprocal time of flight of the electron.

In the unstable regime, all the constants A, B, C, and D turn out to be different from zero and the distribution of the potential-oscillation amplitude along the beam is no longer represented by half a sinusoid, but acquires a much more complicated character[39]; when $\alpha \ll 1$

$$\psi\exp(-\gamma t)\sim A\left\{\sin\left(\frac{\pi+\alpha}{L}z\right)-\frac{\alpha}{4}\left[2\left(\cos\frac{\pi z}{L}-1\right)+\frac{\pi z}{L}\sin\frac{\pi z}{L}\right]\right\}. \qquad (26a)$$

It is seen from this expression that when $\alpha > 0$ ($j > j_P$) the extremum shifts from the plane $z = L/2$ (where it is situated when $\alpha = 0$) towards smaller z.

We have thus considered the Pierce instability in a planar geometry. From the experimental point of view, greatest interest attaches, naturally, to cylindrical geometry, when a beam of radius a and of limited length propagates along the axis of a metallic cylinder of radius $R_2 \gtrsim a$ in a strong external magnetic field. As applied to this case, the problem of the Pierce instability has been solved so far only numerically with the aid of a computer[23]. It has been shown that in the cylindrical geometry the Pierce instability exists and the instability threshold is given by

$$\omega_{1P}^2 \equiv (4e/ma^2u)\, I_P = k^2u^2, \tag{29a}$$

where $I_p = \pi a^2 j_p$ is the Pierce threshold current, $k^2 = (\pi/L)^2 + k_r^2$, and k_r^2 is determined by the transverse dimensions of the beam; when $L \gg R_0 \gg a$ the value of k_r^2 is determined by (7). In accordance with (29a) and (7), in the case of a sufficiently long and thin beam ($L \gg R_0 \gg a$) we have

$$I_P \approx 66 \cdot 10^{-6} V_0^{3/2} / \ln (R_0/a) \; [\text{Ampere}], \tag{29b}$$

where $V = W_1/e$ is the energy of the beam electrons (in volts). For example, at $W_1 = 1$ keV and $R_0/a = 5-6$ we have $I_P \approx 1$ A, at $R_0 = a$ we have $k \approx 2.4/a$, and

$$I_P = 190 \cdot 10^{-6} V_0^{3/2} \; [\text{Ampere}]. \tag{29c}$$

Thus, the Pierce instability can be defined as an electrostatic instability induced in a quasilinear electron beam by positive feedback via an external circuit. In this property it recalls one more electrostatic instability with which the maximum current in a beam of charged particles of the same sign, bounded by the space charge is connected. The problem of the limiting current in a beam of particles of the same sign (for example, in an electron beam in the absence of compensating ions) was first solved by Bursian (see[24] as applied to a plane beam geometry (the same geometry as in the Pierce problem). The main result of this paper can be illustrated with the aid of a plot of the static potential V in the midpoint of the interelectrode space ($z = L/2$) against the beam current density (Fig. 2). In the case of a small beam current, the potential V of the plane $z = L/2$ is close to the grid potential: $V \approx V_0 = W_1/e$ (W_1 is the beam electron energy). With increasing current, the space charge of the beam increases (there are no compensating ions), and the beam potential decreases. So long as the beam current density j is such that $V > V_{min} = V_0/4$, the decrease of V with increasing j is continuous. However, when $V = V_{min}$, the increase of j leads to a jumplike decrease of V to zero. Then a virtual cathode is produced in the beam and reflects an appreciable part of the electrons in the direction of the source, and the current of the electrons reaching the second grid decreases jumpwise. Thus, at $V = V_{min}$ an electrostatic instability sets in and limits the maximum possible beam current. The limiting beam current density, above which the instability sets in,

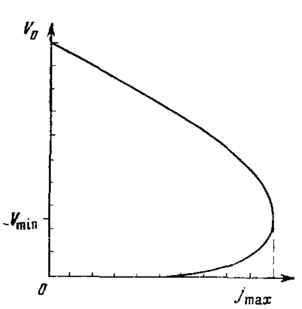

FIG. 2. Potential of midpoint of interelectrode space vs. beam current density.

can be expressed by the relation

$$j_{\max} \equiv (m/4\pi e)\, \omega_{1\max}^2 u \approx (\pi m/4e)\, u^3/L^2, \tag{30}$$

or

$$\omega_{1\max} = (\pi/L)\, u, \tag{30a}$$

where u is the velocity of the beam electrons near the potential minimum.

Attention is called to the fact that expressions (30) and (29) for j_{max} and the Pierce threshold j_P have a perfectly identical structure, and j_{max} differs from j_P only because the beam potential V differs from $V_0 = W_1/e$ (owing to the absence of compensating ions).

Following Bursian[24], the problem of the limiting current in the beam of particles of the same sign was solved by other authors, particularly by Smith and Hartman[25], who considered the case of a cylindrical beam geometry. In the case of a this long beam in a spacious shell ($a \ll R_0 \ll L$), the entire distortion of the potential by the space charge is concentrated outside the beam, $V_{min} = V_0/3$; all the beam electrons (at points distant from the ends) have approximately the same velocities $u \approx (2eV_{min}/m)^{1/2} = (2W_1/3m)^{1/2}$. In this case the maximum beam current differs from the Pierce current by a factor $3^{3/2}$ [23]:

$$I_{\max} = 3^{-3/2}{}_{P}\, I_{\Pi}, \tag{31}$$

where I_P is determined by (29b). Another common property of the Pierce and Bursian instabilities is the fact that both are characterized by a hard excitation regime: the energy of the perturbed electric field increases within a time on the order of the electron time of flight to a value equal to the energy of the beam electrons. Therefore the Pierce instability and the Bursian instability should lead to the formation of a virtual cathode in the beam. (See the theoretical papers[26] concerning this question.)

In the case of the relativistic regime, the problem of the Pierce instability has a very limited meaning. In fact, as seen from the dispersion equation (2′), the beam electrons "become heavier" (in the oscillations) by a factor γ_0^3 and when $\gamma_0 \approx (M/m)^{1/3}$ neglect of the part played by the ions in the oscillations is no longer correct. At smaller γ_0, the current I_P can be determined from (5) by putting in it $M = \infty$:

$$I_P \approx (ma^2/4e)\, k^2u^3\gamma_0^3. \tag{29′}$$

The critical current of the Bursian instability in the relativistic regime ($u \approx c$, regardless of the distortion of the potential) follows from (30):

$$I_{\max} \approx (ma^2/4e)\, k^2c^3\gamma_0. \tag{30′}$$

The difference between (29′) and (30′) is determined by the relativistic Doppler effect.

In the cylindrical case, in the presence of a strong external longitudinal magnetic field,

$$I_{\max} = mc^3\gamma_0/e\, [1 + 2 \ln (R_0/a)]. \tag{30″}$$

It should be noted that if an intense relativistic electron beam propagates through the "background" of the positive ions that compensate for its space charge in the absence of an external magnetic field, then, according to the Alfven theory[63], it should be subject to an instability connected with the "compression'' of the beam by its own magnetic field. This instability occurs if the energy of the beam's own magnetic field is comparable with its kinetic energy and is characterized by a threshold (Alfven current)

$$I_A = (mc^3/e)\,\gamma_0\beta = 1{,}7\cdot 10^4\gamma_0\beta \text{ [ampere]}, \qquad (30''')$$

where $\beta = u/c$.

As seen from a comparison of (30''') and (30''), the Alfven current I_A exceeds the Bursian current I_{max} very little, only by a factor $1 + 2\ln(R_0/a)$.

4. Instability of Ion Beams in a Plasma

In considering the instability of an ion beam in a plasma, we must take into account one more factor, which so far has been disregarded, namely the thermal motion of the plasma electrons. The dispersion equation analogous to (13) then takes the form

$$\frac{\omega_1^2}{(\omega-k_z u)^2}+\frac{\omega_+^2}{\omega^2}+\frac{\omega_2^2 k_z^2/k^2}{\omega^2-k_z^2 v_2^2}=1, \qquad (32)$$

where $\omega_1^2 = 4\pi n_1 e^2/M_1$, $\omega_+^2 = 4\pi n_+ e^2/M_+$, $\omega_2^2 = 4\pi n_2 e^2/m$ — are the Langmuir frequencies of the beam ions, the plasma ions, and the plasma electrons, $n_2 = n_1 + n_+$, $v_2 = (T_e/m)^{1/2}$, and T_e are the thermal velocity and the temperature of the plasma electrons. (Equation (32), like (13), pertains to the case when the electrons are strongly magnetized and the ions are not magnetized.)

We shall consider slow electrostatic waves, in which the inertia of the plasma electrons can be neglected:

$$\omega/k_z \ll v_2;$$

Eq. (32) then takes the form

$$[\omega_1^2/(\omega-k_z u)^2]+(\omega_+^2/\omega^2)=1+(\omega_2^2/k^2 v_2^2). \qquad (32a)$$

With the aid of simple steps perfectly analogous to those used in Ch. II, we obtain the beam threshold density n_{1cr}, starting with which the instability sets in

$$\omega_{1cr}^2 = 4\pi n_{1cr} e^2/M_1 = k_z^2 u^2\,[1+(\omega_2^2/k^2 v_2^2)] \times \{1+[(M_1/M_+)\times(n_+/n_1)]^{1/3}\}^{-3}, \qquad (33)$$

and the frequency of the oscillations that build up in the case when $n_1 \gtrsim n_{1cr}$:

$$\omega \approx k_z u\,\{1+[(M_+/M_1)\,(n_1/n_+)]^{1/3}\}^{-1} \qquad (34)$$

(The oscillation increment is of the order of the frequency ω). Neglecting unity in the brackets of the numerator of (33) and using simple transformations, we can represent the condition for the instability of the ion beam in the form

$$W_1/T_e < (k^2/2k_z^2)\,(n_1/n_2)\,\{1+[(M_1/M_+)\,(n_+/n_1)]^{1/3}\}^3, \qquad (35)$$

where W_1 is the beam ion energy*.

It is interesting to note that relation (35) contains a factor k^2/k_z^2. Under the experimental conditions, this factor can be quite large. For example, if standing waves are produced (the beam length L spans an integer number of half-waves), when

$$k_z = m\pi/L \qquad (m = 1, 2, 3, \ldots); \qquad (36)$$

*It is easy to note that the form of relations (32a) and (35) does not depend on the presence or on the absence of a longitudinal magnetic field (we always assume that the ions are not magnetized).

the ratio k^2/k_z^2 can be of the order of the square of the ratio of the beam length to its diameter (independently of the value of H—see the last footnote). In this case the relation (34) describes oscillations at a frequency approximately equal to an integer multiple of the reciprocal time of flight of the beam ions.

We have considered the case $\omega/k_z \ll v_2$. In the opposite case, the dispersion equation (32) takes the form

$$\frac{\omega_1^2}{(\omega-k_z u)^2}+\frac{\omega_+^2}{\omega^2}+\frac{\omega_2^2 k_z^2/k^2}{\omega^2}=1. \qquad (32b)$$

In the particular case of a quasineutral ion beam (the space charge of the beam is compensated by electrons, $n_1 = n_2$, and there are no slow ions, $n_+ = 0$), the threshold of the instability is given by

$$\omega_{1cr}^2=\frac{k_z^2 u^2}{\{1+[(\omega_2^2/\omega_1^2)\,(k_z^2/k^2)]^{1/3}\}^3}=\frac{k^2 u^2}{[(k^2/k_z^2)^{1/3}+(M/m)^{1/3}]^3}, \qquad (33a)$$

and the oscillations (when $\omega_1 > \omega_{1cr}$) have a frequency

$$\omega \approx k_z u\,\{1+[(m/M)\,(k^2/k_z^2)]^{1/3}\}^{-1}. \qquad (34a)$$

In conclusion it should be noted that a quasineutral ion beam can experience not only an oscillatory but also an aperiodic instability[27] analogous to the Pierce instability of a quasineutral electron beam. The difference between these instabilities lies in the fact that in the case of an ion beam the particles of the "background"—the electrons—can be much more mobile than the beam particles. Connected with this difference is the fact that the threshold current for an ion-electron Pierce instability (in addition to the factor $(e/M)^{1/2}$) contains a factor W_1/T_e. In the case of a plane ion beam of length L, the threshold current density is[27]

$$j_{cr} \simeq (W_1/T_e)\,(4/\pi)\,(2e/M)^{1/2}\,(W_1/e)^{3/2}/L^2. \qquad (37)$$

When $W_1 \gg T_e$, the threshold beam current may lie beyond the capabilities of practical realization.

Thus, we have represented in this section the theory of stability of monoenergetic beams of charged particles. This theory is valid also for beams with a small thermal velocity scatter, provided the value of the scatter Δu satisfies the condition

$$\Delta u < \gamma/k_z, \qquad (38)$$

where γ is the instability increment. In particular, for the electron-ion instability with dispersion properties (3), the condition (38) means

$$\Delta u/u < [(k^2/k_z^2)\,(m/M)]^{1/2}; \qquad (38a)$$

for example, under the conditions of the experiments described below

$$\Delta u/u < 0{,}2. \qquad (38b)$$

III. EXPERIMENTAL DATA

1. Electron-ion Oscillations in Electron Beams

Before describing the experimental data, let us make a few general remarks concerning the possible approaches to an experimental verification of the theory of beam instabilities.

1. From the relations given above we see that a verification of the theory can follow two lines, either by investigating the dispersion properties (the fre-

quency characteristics and the spatial structure) of the observed oscillations, or by measuring the thresholds (critical currents) of the instabilities. In the study of the dispersion properties of the oscillations, the following difficulty arises: In order to be able to compare experiment with the linear theory described above, it is necessary to work with oscillations of sufficiently small amplitude, but such oscillations are difficult to separate from the background of the plasma noise. On the other hand, if the oscillation amplitude is large enough, then their dispersion properties, in principle, may be distorted by the resultant nonlinear effects. These shortcomings do not appear in a method where the theory is verified by using the (experimentally measured) instability thresholds. This second method also involves measurement of the dispersion properties of the oscillations, but this measurement plays a subsidiary role and is carried out to make sure of the fact that a study of "what is required" is made.

2. The verification of the theory presented above can yield the most definite results if the experimentally investigated system of charged particles consists of only two components, fast beam electrons and (an equal amount of) slow ions, i.e., it is quasineutral electron beam. On the other hand, if in addition to these two components there is also an "excess" plasma (of sufficiently high density) then, as shown above, the dispersion properties and the thresholds of the instabilities turn out to be essentially different and are much more difficult to interpret. Therefore we shall first report on the experimental data on electron-ion instabilities of a quasineutral electron beam.

3. One of the methods of obtaining the considered two-component system (and apparently the most convenient one) is to pass a sufficiently intense electron beam through a radified gas. In this case the slow electrons produced when the gas is ionized by the beam are pushed out by the negative space charge of the beam and rapidly go out of the system, while the ions are retained in the beam for a longer time and cancel out the space charge of the beam. Let us consider this phenomenon in somewhat greater detail.

Let u be the velocity of the beam electrons, n_1 their concentration, n_0 the concentration of the neutral-gas molecules, σ the cross section for their ionization by the beam electrons, n_+ the ion concentration, and v_+ their average velocity. The beam is in the form of a cylindrical rod (length L, radius a) and propagates along a strong magnetic field, on the axis of an equipotential volume with metallic walls. The equilibrium potential φ of the beam (relative to the walls) is determined both in magnitude and in sign by the relation between the rate of ion production in the beam, $n_1 n_0 \sigma u \pi a^2 L$, and the free flux of the ions along the magnetic field, $2(n_+ v_+/4)\pi a^2$. If the former of these quantities is smaller than the latter, then the beam potential is negative and the ions can accumulate in the beam until quasineutrality $n_+ \approx n_1$ is established. Consequently, the condition that the potential of the quasineutral beam be negative is given by

$$n_0 \sigma u L < v_+/2, \tag{39}$$

or

$$1/n_0 \sigma u > 2L/v_+, \tag{39a}$$

meaning that

$$\tau_i > \tau_+, \tag{39b}$$

where $\tau_i = 1/n_0\sigma u$ is the average time of ionization of the gas by the beam electrons, and $\tau_+ = 2L/v_+$ is the average time of flight of the ion along the beam. If, for example, the ions and the atoms of the neutral gas belong to hydrogen, $L = 10^2$ cm, $v_+ = 2.5 \times 10^6$ cm/sec (the average ion energy is ~ 3 eV), $u = 10^9$ cm/sec, and $\sigma \approx 10^{-16}$ cm^{-2}, then condition (39) is satisfied for $n_0 < 1.25 \times 10^{11}$ cm^{-3}, i.e., at gas pressures

$$p_0 < 2{,}5 \cdot 10^{-6} \text{ mm Hg}.$$

This condition is readily satisfied in experiments; in addition, it is important to recognize that under real conditions the area through which the ions escape from the system is several times larger than the cross section area of the electron beam πa^2 (the cause of this phenomenon will be discussed later). Therefore at $p_0 \lesssim (1-2) \times 10^{-6}$ mm Hg the beam potential will be negative even for heavy gases (nitrogen, argon) with larger ionization cross sections.

If condition (39) is not satisfied, the equilibrium ion density turns out to be larger than the beam-electron density; then the potential reverses sign, the slow electrons remain in the beam, and our system of charged particles no longer consists of two components, but of three. At sufficiently large gas pressures, the density of the "excessive" plasma will be much larger than the density of the beam electrons.

We now proceed to describe the experimental data on beam electron-ion instabilities. As to the Pierce and Bursian instabilities, the question of their experimental observation is considered in Sec. 1 of Ch. IV, in connection with a discussion of the mechanism of beam limitation in quasineutral electron beams. An investigation of the thresholds of the electron-ion instabilities and of their dispersion properties was carried out by the author and his co-workers[28,29]. These experiments were carried out with a two-component system comprising a quasineutral beam propagating along a magnetic field in a gas at $p_0 \approx 10^{-6}$ mm Hg along the axis of an equipotential metallic cylinder. The beam electron energy W_1 amounted to hundreds of electron volts, the current amounted to tens and hundreds of milliamperes, the beam length was ~ 100 cm, the beam diameter ~ 1 cm, the cylinder diameter 30 cm, and the magnetic field intensity amounted to several hundred or several thousand oersted.

Two types of oscillations are observed in the experiments. A common property of these oscillations is their electron-ion character: The part played by the ions in the oscillations is expressed, in particular, by their acceleration to high energies (on the order of the beam-electron energies) and by a strong increase of the radius of the ionic "background." Both types of oscillation are long-wave, the beam length subtending usually half the oscillation wavelength ($\lambda_z \approx 2L$, $k_z = 2\pi/\lambda_z \approx \pi/L$). In the cross section, however, the spatial structure of the oscillations is essentially different, the oscillations of one of the indicated types having axial symmetry $k^2 = k_r^2 + k_z^2$, $k_\varphi = 0$), and the oscillations of the other type being axially-asymmetri-

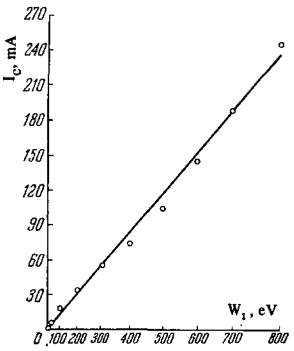

FIG. 3. Dependence of the critical current I_c of the excitation of axially-symmetrical electron-ion oscillations on the beam-electron velocity cubed. H = 7800 Oe, L = 90 cm, p = 1.4 × 10^{-6} mm Hg.

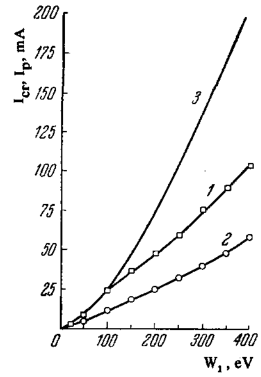

FIG. 5. Critical excitation current of axially-asymmetrical electron-ion oscillations vs beam electron energy. $2R_0$ = 10 cm, 2a = 1 cm. 1) H = 4000 Oe, 2) H = 600 Oe, 3) I = I_p—Pierce current.

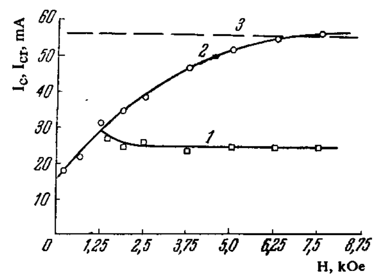

FIG. 4. Thresholds of electron-ion instabilities as functions of the magnetic field intensity. 1) I = I_c—critical current for the excitation of oscillations having axial symmetry, 2) I = I_{cr}—critical current for the excitation of oscillations having no axial symmetry, 3) I = I_p—Pierce instability excitation current. W_1 = 200 eV, $2R_0$ = 30 cm, 2a = 1 cm.

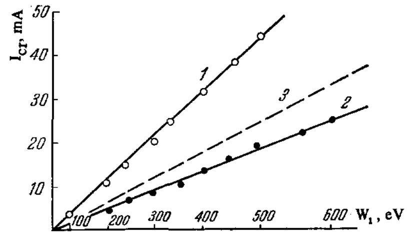

FIG. 6. Critical excitation currents of electron-electron oscillations vs beam-electron velocity cubed. 1—$n_2 < n_1$, 2—$n_2 > n_1$, 3—theoretical value of the threshold (12) at $n_2 = n_1$.

cal. The latter propagate azimuthally (around the beam) in a direction corresponding to the direction of the Larmor rotation of the electron. The perimeter of the beam ($2\pi a$) is usually equal to one azimuthal wavelength of the oscillations, meaning that $k_\varphi \approx 1/a$. The frequencies of the oscillations lie in the electron-ion band described by the theory developed in Ch. II.

A characteristic property of the oscillations of both types is the presence of a distinct threshold (critical current) of excitation, which is quite sensitive to the beam parameters. Experimental data on the thresholds of the considered instabilities, as functions of the magnetic field intensity and of the electron beam energy, are given in Figs. 3—5.*

These data can be summarized as follows:

1) The excitation threshold of the electron-ion oscillations that have no axial symmetry (I_{cr}).	1a) independent of H, proportional to u^3.
2) Excitation threshold of electron-ion oscillations that have no axial symmetry (I_{cr}).	2a) *small and large H*: proportional to u^3 and independent of H; *large u and small H*: approximately proportional to the product u^2H.

3) The ratio of the thresholds depends on the magnetic-field intensity:

$$I_c/I_{cr}: \begin{matrix} >1 \text{ at small } H, \\ <1 \text{ at large } H. \end{matrix} \qquad (42)$$

4) The excitation threshold of the axially-symmetrical oscillations is smaller by a factor 2—3 than the theoretical Pierce current calculated from formula (29b) and represented by the straight line 3 in Fig. 3. The excitation threshold of the oscillations having no axial symmetry is much smaller than the Pierce current at small H, and is quite close to it, or even coincides with it, at large H. The Pierce current, being dependent on a rather small number of parameters and amenable to a quite accurate calculation, is shown in Figs. 4 and 5 as a "standard" for estimating the absolute value of the investigated thresholds.

Finally, in order to compare the experiments with theory, we note that under the conditions corresponding to Fig. 4 the denominator of formula (5) amounts to approximately 2—3, while the quantity $2su/a^2\omega_{He}k_z$, which enters in (16) amounts to approximately 4 at H = 1500 Oe and W = 200 eV.

The presented experimental data show that there is good agreement between the observed threshold I_c and the theoretical relation (5), on the one hand, and between the observed threshold I_{cr} and the theoretical relation (16) on the other.

These data, together with the results obtained by studying the spatial structure and the frequency spectra of the oscillations[28,29], allow us to conclude that the electron-ion oscillations described here are direct consequences of the theoretically-predicted electron-ion beam and drift-beam instabilities.

The theory considered above admits of one more experimental verification: if we change from a quasi-neutral (two-component) beam to a three-component system, to which the density n_2 of the plasma electrons, for example, is larger than or equal to the density of the beam electrons n_1, then an electron-electron instability should be observed, with a threshold (12) and an oscillation frequency (11). An experiment[30] whose results are shown in Fig. 6 has confirmed this theoretical prediction, namely, the observed oscilla-

*The curve 2 of Fig. 4 and curves 1 and 2 of Fig. 5 actually show the values of the limiting currents exceeding I_{cr} by not more than 10–15% (see [29]).

tions are electronic (there is no ion acceleration), their excitation threshold is proportional to u^3, does not depend on H, decreases with increasing ratio n_2/n_1, and, finally, is smaller than the Pierce current by approximately the "required" number of times. Moreover, the dispersion properties of the oscillations are well described by the linear theory (which takes into account the formation of standing waves along the beam). The latter is not surprising, since in the case of electron-electron oscillations the nonlinear effects are much less pronounced than in the case of ion-ion oscillations: for example, the change of the beam radius under the influence of the oscillations (even if it were to occur), does not influence directly the oscillation frequency, in accordance with relation (11), unlike the case (3), where the oscillation frequency is sensitive to the change of $k \sim 1/a$. It is appropriate to note in this connection that the frequency characteristics of electron-electron oscillations agree with the theory only as a rough approximation (more accurately speaking, they do not contradict it, if reasonable assumptions are made regarding the nonlinear effects[29].

It is important to mention one more effect observed on going over from a two-component system to a three-component system, namely, the increase of the threshold I_c an appearance of a beam-electron velocity scatter, which makes it difficult to satisfy the conditions (38); these stop the excitation of the axially-symmetrical electron-ion oscillations[29].

The excitation of electron-ion oscillations that have no axial symmetry continues so long as a condition inverse to (19) is satisfied[29].

It is now of interest to examine the developed theory and the presented experimental data from an entirely different point of view. To this end, we disregard for the time being the two-stream instabilities and recall that the theory predicts a so-called universal (drift) instability of a plasma in magnetic fields, due to the spatial inhomogeneity of the latter (see above). This instability is of fundamental interest in connection with the development of the problem of controlled thermonuclear reactions and a high-temperature (inevitably inhomogeneous) plasma. Its experimental identification, however, encounters great difficulties. These are connected with the fact that in the case of a Maxwellian velocity distribution of the plasma electrons this instability has no threshold with respect to the particle density, and from the point of view of its dispersion properties it is very difficult to distinguish it from instabilities of an entirely different nature (see, for example,[31]). We have seen at the same time that if we change the electron distribution function, i.e., if we change over from a Maxwellian distribution to a δ-function (monoenergetic quasineutral electron beam), then a threshold, expressed by relation (16), appears for the instability in question (which we shall call not drift instability but drift-beam instability). This threshold is expressed by relation (16). Owing to the presence of this threshold, the experimental identification of the instability, as shown above, is quite easy. Thus, the experiments described by us are of interest also from a different point of view, since they successfully stimulate the universal instability of a plasma (for more details see[32]).

2. Ion Oscillations in Electron Beams

It has been long known (see, for example,[33,34]) that when an electron beam passes through a "background" of positive ions (in a rarefied gas), electrostatic oscillations are observed with a frequency close to the Langmuir ion frequency: $\omega \approx \omega_+ = (4\pi n_+ e^2/M^{1/2}$. We shall discuss here the mechanism whereby these oscillations are excited. It is easy to see that this mechanism differs from the mechanism considered above for the buildup of electron-ion oscillations. We arrive at this conclusion on the basis of the fact that the phenomenon in question is observed already at relatively small beam current, much smaller than the electron-instability thresholds described above. These instabilities are excited, so to say, hydrodynamically: all the beam electrons have velocities greatly exceeding the phase velocity of the electron-ion oscillations, and the excitation of the instability corresponds to the anomalous Doppler effect; this mechanism differs in that has a distinct excitation threshold [see (5) and (16)]. In addition to this mechanism, another mechanism of oscillation buildup is possible in accordance with the theory, and is connected with the Cerenkov effect. It is realized when the electron beam has a large velocity spread and the oscillations are excited not by all the electrons but only a small fraction of them, the so-called resonant electrons whose velocities of which are equal to (or slightly larger than) the phase velocity of the excited waves. According to the theory, there is no excitation threshold current in such a (kinetic) mechanism of oscillation buildup. The beam current governs in this case not the excitation of the oscillations, but their increment (amplitude).

In real experiments, kinetic excitation of the oscillations can be effected by secondary-emission electrons knocked out by the fast (primary) electrons from the beam collector. Indeed, the role of the secondary electrons in the buildup of ion Langmuir oscillations turns out to be quite appreciable[29,30].

It is important to note that the considered "ionic" oscillations are observed not only in a quasineutral (two-component) beam, but also in a three-component system, in which the density of the "extra" plasma is of the order of (or even larger than) the density of the beam electrons. The general case of excitation of these oscillations was investigated experimentally in[29], where it was shown that the oscillation frequencies agree well with the theoretical formula[37]

$$\omega^2 = \omega_+^2/(1 + k^{-2}d^{-2}), \tag{43}$$

where k is the total wave number, $d = (T_e/4\pi n_2 e^2)^{1/2}$ is the electron Debye radius, T_e is the temperature of the plasma electrons, and n_2 is their density. At sufficiently low plasma density (when $k^2d^2 \gg 1$) we have $\omega \approx \omega_+$. This case was investigated in numerous papers, for example in[33,34]. At sufficiently high plasma density, when $k^2d^2 \ll 1$, the ion-Langmuir oscillations are transformed into ion-sound oscillations;

$$\omega \approx kv_s. \tag{43a}$$

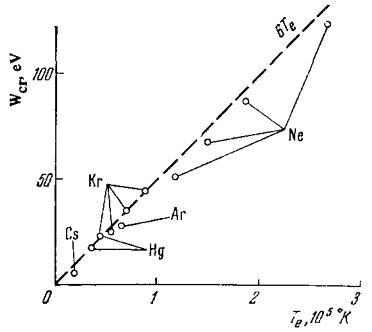

FIG. 7. Dependence of the energy of excitation of ion-ion oscillations by an ion beam in a gas of the same material on the plasma electron temperature T_e.

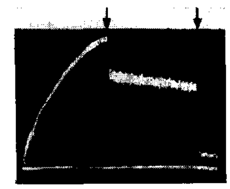

FIG. 8. Oscillogram of beam current in the case of monotonic increase of the current emerging from the source [28]. The current growth duration is much larger than the time of neutralization of the space charge of the beam τ_i. The left arrow indicates the instant of interruption of the current, and the right arrow the instant of termination of the beam pulse. Sweep 10 msec: $W_1 = 600$ eV, $H = 4000$ Oe, $L = 100$ cm, $p = 1 \times 10^{-6}$ mm Hg, $2R_0 = 30$ cm, $2a = 1$ cm. Limiting current 180 mA.

where $v_s = (T_e/M)^{1/2}$ is the velocity of the ion sound; in this case (which was investigated in[29], the oscillation frequency ceases to depend on the density of the charged particles.

3. Instability of Ion Beams in a Plasma

The instability of ion beams in a plasma was observed in the experiments of Gabovich and co-workers (see[35], and also the recent paper[73]). The main results of these experiments are as follows: 1) there is a certain threshold beam-ion energy W_{cr}, above which the beam is stable, 2) the threshold energy of the ions is proportional to the plasma electron temperature T_e. These results are shown in Fig. 7, taken from[35]. It is easy to see that they are in good qualitative agreement with the theoretical relations (33) and (35).

It is of interest to compare the measured and theoretical values of the coefficient β of the proportionality of the threshold value W_{cr} to the quantity T_e. According to formula (35), the sought coefficient under the experimental conditions in[35] (at $n_+ \approx n_1$, $k_z \approx \pi/L \approx 2$ cm^{-1}, $k \approx 1/a \approx 4$ cm^{-1}, $M_2 = M_1$) turns out to be equal to 16, whereas experiment yields $\beta = 6$. Since the experimental conditions in[35], mainly the ratio n_+/n_1, are not very accurately known, we can state only that theory is in qualitative agreement with experiment and there are no large quantitative discrepancies.

Chernov and co-workers[72] investigated experimentally the convective instability of the ion beam (protons with energy of several dozen keV), compensated by electrons. It was shown that the threshold of this instability and its dispersion properties agree with the theory presented above (see Eqs. (32)). The authors of[72] used this instability to develop a device which can be called, in a certain sense, a traveling-wave tube using an ion beam.

So far, there has been no experimental verification of the "ionic" Pierce instability[27]. The instability of ionic beams in a transverse magnetic field was investigated in[36].

IV. EFFECTS DUE TO TWO-STREAM INSTABILITIES

1. Limiting Currents in Quasineutral Electron Beams

If the beam consists of particles of the same sign and propagates in an ideal vacuum, then the beam current cannot be increased above a certain limit, whose magnitude is determined by the threshold of the electrostatic instability of Bursian (see Sec. 3 of Ch. II). This is the well known phenomenon wherein the beam current is limited by space charge, and has been investigated theoretically[24,25,23] and experimentally[38,40]; there is good quantative agreement between theory and experiment. If the space charge of the beam were to be cancelled by particles of opposite sign, then it might seem that the limiting currents in the beams could be appreciably increased. It turns out, however, that this is not so, and the limiting currents in quasineutral electron beams remain in principle at approximately the same level as before, and after this level is reached the current is interrupted. This phenomenon was noted already by Pierce[22,25]. It is demonstrated in Fig. 8.*

An experimental investigation of the mechanism of limitation (interruption) of the current in a quasineutral electron beam was carried out by Volosov[39], Atkinson[40], and the author with his co-workers[28,29]. It was shown in[28,29] that, in a wide range of experimental conditions, the reason for the current interruption is the drift-beam electron-ion instability; the limiting current of the beam exceeds very slightly (by ~10–15%) the threshold (16) for the occurrence of this instability. In particular, if the beam length and the electron velocity are sufficiently large, and the intensity of the (longitudinal) magnetic field, to the contrary, is not too large (for example, $L \approx 10^2$ cm, $u \approx (1-2) \times 10^9$ cm/sec, $H \lesssim (1-2) \times 10^3$ Oe), then the limiting current of the beam is appreciably small (by several times) than the Pierce current (29b). Under other conditions (small u, large H) the threshold of the drift-beam instability and the limiting current of the beam are close to the Pierce threshold. Finally, it is possible to choose experimental conditions such that the drift-beam instability becomes stabilized. Then the beam current, according to the theory developed above, should be determined by the Pierce instability. The indicated conditions are easiest to realize in two cases: 1) if the magnetic field intensity

*When speaking of interruption of the current, we have in mind a phenomenon wherein a "virtual cathode" (region with negative potential equal to the beam electron energy) is produced in the electron beam and reflects an appreciable fraction of electrons. The formation of the virtual cathode in the case of interruption of the current in a quasineutral electron beam was demonstrated in the experiments of [28].

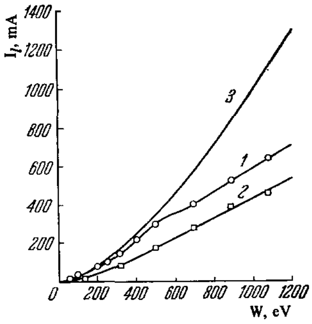

FIG. 9. Dependence of the limiting current I_l on the beam electron energy [28]. 2a = 1 cm, $2R_0$ = 6 cm, L = 10 cm, 1–H = 5200 Oe, 2–H = 1200 Oe, 3–Pierce current.

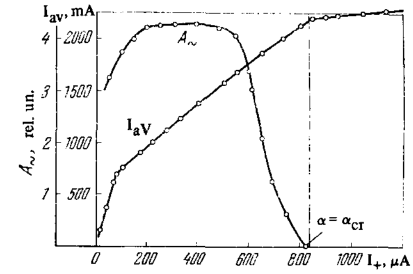

FIG. 10. Current of a beam passing through a plasma (I_{av}) and oscillation amplitude A against the plasma concentration [41]. Abscissas—values of I_+ of the indicator of the plasma density n_2. With increasing I_+, n_2 increases monotonically; α_{cr} = 30–40. The beam electron energy is $W_1 \approx 100$ eV, the beam current at the entrance to the plasma is $I_1 = 2A \approx 10^2$ Ip.

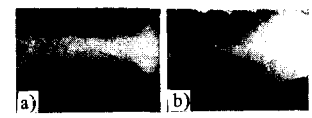

FIG. 11. Propagation of relativistic electron beam in a gas (air, pressure, 2×10^{-1} mm Hg). The beam moves from right to left, the length of the shown beam section is 40 cm. a) Beam electron energy W_1 = 2.5 MeV, current I = 20 kA, beam passes freely; b) W_1 = 1.5 MeV, I = 40 A, beam is blocked.

is sufficiently large and the beam length $L \sim 1/k_z$ is sufficiently small, for example L = 10 cm, as in Volosov's experiment[39]; 2) if the beam diameter coincides with the diameter of the shell (R_0 = a), and the electron energy is sufficiently small, as in Atkinson's experiment [40], where $W_1 \lesssim 60$ eV. In both cases, the drift-beam instability of the lowest spatial mode cannot develop: in the former case as because the criterion (17) is not satisfied, and in the latter because of the vanishing of the field E_φ on the beam boundary. As to the higher modes of the drift-beam instability, they can likewise not be excited, since their threshold (as a result of the large value of k^2) exceed under the indicated conditions the Pierce threshold (29). Indeed, in both cases[39,40] the experimentally measured limiting currents in a quasineutral electron beam turn out to be quite close to the Pierce current[22,23], in good agreement with the theory developed here.

It should be indicated that, for the reasons mentioned above, even at a small beam length (L = 10 cm) the increase of the electron velocity and the decrease of the magnetic-field intensity lead to an appreciable deviation of the limiting beam current from the Pierce current (Fig. 9)[28]. It was indicated in error in[39] that the limiting current does not depend on the magnetic field at $H \geq 20$ Oe. Thus, the limitation (break) of the current in a quasineutral electron beam is due to two causes, either drift-beam instability or Pierce instability, depending on which threshold (16) or (29a) is the lower.

It must be emphasized, however, that the indicated correspondence between the limiting beam current I_l and the threshold of the drift-beam instability I_{cr} is observed only when $I_{cr} > I_{max}$, where I_{max} is the limiting beam current, limited by the space charge in vacuum (the Bursian threshold)[28]. This is natural, since the blocking of the beam (the break in the current) occurs only at $I > I_{max}$. On going to a three-component system, the limiting stable current of the electron beam increases greatly[29], and at a sufficiently large density of the "extra" plasma ($n_2 \gg n_1$) it can exceed the Pierce current by several orders of magnitude. Thus, at certain conditions[41,44] the limiting current density of the stable beam amounts to

$$j_l \approx n_2 v_2 / 4, \tag{44}$$

where v_2 is the thermal velocity of the plasma electron; in the experiments of[41], the current j_l determined by relation (44) exceeded the Pierce current by approximately two orders of magnitude. This was illustrated by Fig. 10, taken from[41], which shows the dependence of the time-averaged beam current I_{av} passing through a plasma on the plasma density at a constant value of the current entering the plasma (I_1). We see that with increasing plasma density the current of the beam passing through it first increases sharply, and then practically reaches saturation. The latter means that the entire current of the electron beam (I_1) passes through the plasma. The plasma density corresponding to saturation was determined by relation (44). Under the conditions of Fig. 10, the limiting stable beam current (saturation current) exceeds the Pierce current by approximately two orders of magnitude. Fig. 10 shows also the influence of the concentration of the plasma on the amplitude A_∞ of those oscillations in the beam-plasma system, which are responsible for the limitation of the current of the (transmitted) beam. Under current saturation conditions, when the ratio α of the electron plasma density n_2 to the beam density n_1 reaches the critical value $\alpha_{cr} \approx 30–40$, the oscillations in question vanish and the beam current becomes stable (in the indicated sense). We note that in such a "stable" beam there is an intense buildup of Langmuir electron oscillations, causing a strong scatter of the beam electron velocities, but exerting no influence on the current passing through the plasma[41]. As to the instabilities of the intense relativistic beams, their thresholds have not yet been investigated experimentally, although experiments with such beams are being diligently carried out at present[64,68,76]. One of the results of these experiments is the establishment of the very fact that the current in the beams is limited to values close to I_{max} and I_A, determined by formulas (30″) and (30‴). This fact is demonstrated in Fig. 11 (from[64]), which shows the glow of a gas through which a relativistic

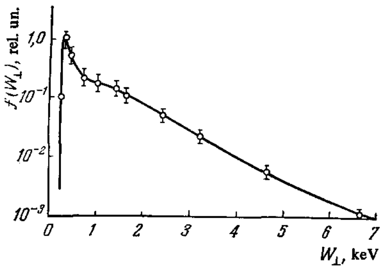

FIG. 12. Energy spectrum of the ions in a magnetic trap [32]. $W_\perp$—energy of ion motion perpendicular to the magnetic field. The average ion energy $W_{av} \approx 1.2$ keV, the beam current is I = 10 A, W_1 = 1 keV, H = 3000 Oe.

beam with electron energy 1.5–2.5 MeV ($\gamma_0 = 4-6$) propagates. The concentration of the compensating ions is apparently close to the concentration of the beam electron; in case a) the beam current is 20 kA, and in case b) it is 40 kA; we see that in the latter case the beam is blocked.

It should be noted that if the parameters of the beam-plasma system are not uniform along the beam, then the dynamics of the instability may be greatly altered; this effect was considered theoretically in[70,71] with electron-electron instability as an example.

2. Acceleration (Heating) of Plasma Ions by Electron Beams

It was observed in the experiments of the author and co-workers[42,77] that during the course of the development of electron-ion beam instabilities the plasma ions are accelerated (heated) to rather high energies, of the order of the energies of the beam electrons and higher. This nonlinear phenomenon is natural, although it cannot be quantitatively reconciled with the existing theory, in view of its linearity. Comparison with theory is possible only for those conditions in which ion acceleration is observed in the experiment.

This phenomenon becomes more strongly pronounced in the case of a three-component system, when the electron beam passes through a column of a much denser (initially cold) plasma; the ratio of the plasma density n_2 to the beam density n_1 amounts, for example, to several times ten[42]*; in absolute magnitude n_2 is of the order of several times 10^{12} cm^{-3} and the beam current is $I \approx 20$ A[42]. Under definite conditions, practically all the plasma-column ions become heated to a temperature of about 1 keV; the ions are accelerated mainly perpendicularly to the direction of the beam velocity (i.e., perpendicular to H). The latter circumstance favors the use of this phenomenon for the accumulation of a plasma with hot ions in a trap with magnetic mirrors. Figure 12 shows the energy spectrum of the protons, measured following the passage of an unstable plasma beam along the axis of such a trap (the spectrum is taken from[32]). We see that the spectrum contains an appreciable number of ions with energies of several keV, i.e., exceeding by several times the energy W_1 of the primary beam electrons, which equals 1 keV*. The total density of the fast ions amounts to $\sim 10^{11}$ cm^{-3} in a plasma volume $(5-10) \times 10^3$ cm^3 (the average plasma diameter in the trap is ~15 cm and the average length ~50 cm), and the average energy (temperature) of the ions is 1–1.5 keV. Under these conditions, the flux of fast ions from a plasma beam 100 cm long in the trap is 1–2 A (for details see[42]).

The described phenomenon can be called turbulent heating of plasma ions by an electron beam. This phenomenon is used, in particular, in the well known experiments of Ioffe and his co-workers for the investigation of the behavior of a plasma with hot ions in traps with combined magnetic field[43]. In these experiments the plasma filling the trap is generated in an unstable plasma beam.

The main instability, which is most responsible for the effective acceleration of the ions to the plasma beam, was identified by the author and co-workers[29,44]. This instability turned out to be the above-described drift-beam instability. The possibility of occurrence of this instability at an initial ratio $n_2/n_1 \approx 20-40$ seemed at first glance improbable, since, as was shown above, the instability in question, in accordance with relation (19), vanishes already at $n_2/n_1 \approx 5-6$. Experiments[41,42] have shown, however, that if the beam current exceeds the limiting current (44), i.e., if

$$n_1 v_1 > n_2 v_2 / 4, \quad (45)$$

then the drift-beam instability is preceded by two other ("extraneous") instabilities. During the course of development of these instabilities, the ratio R^2/a^2 increases significantly (R—plasma radius, a—beam radius), and the ratio n_2/n_1 decreases accordingly.

At the start of the drift-beam instability, a ratio inverse to (19) is approximately satisfied. The indicated "extranous" instabilities and their nature are described in[44].

The gist of the matter, in short, reduces to the following, As shown by experiment[45], the plasma beam, generally speaking, can be characterized in several discrete states by essentially different degrees of turbulence. The relatively "quiet" state (in which, however, Langmuir electron oscillations develop to a full extent and the beam-electron distribution function assumes the form of a plateau) is a state with a sufficiently large ratio n_2/n_1 of the plasma electron density to the beam electron density. For example, $n_2/n_1 \gtrsim 30$. No significant heating of the plasma ions takes place in this state, but an instability is observed at frequencies from several kHz to several dozen kHz. It is manifest in the formation of a plasma "flare," which is homogeneous along the magnetic field and rotates around the beam axis, in the same direction as the Larmor rotation of the ion, and with a velocity $\sim cE_r/H$, where $E_r \approx T_e/eR$ is the radial electric field due to the equilibrium potential of the plasma $\varphi \approx T_e/e$. The charges are separated azimuthally in

*We shall henceforth call such a system a plasma beam.

*We note that the energy spectrum includes also ions with energies $W_\perp$ = 20–30 keV, but their concentration is relatively small, on the order of $10^{-4}-10^{-3}$ of the total plasma density (see also [78]).

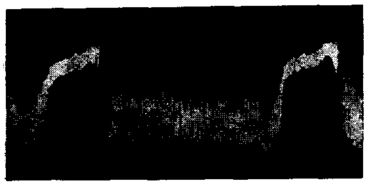

FIG. 13. Oscillogram of the current of an electron beam passing through a plasma column in the relaxation regime of instability [41]. An upward deflection corresponds to an increase of the electron current. Long period–500 μsec, the swing of the oscillations is almost equal to the beam current in the smooth sections of the oscillogram.

FIG. 14. Oscillograms of the current of an electron beam passing through a plasma column (top) and of the flux of accelerated ions emitted from the column in the relaxation regime of instability [41,42]. In the lower oscillogram, upward deflection from the horizontal line corresponds to an increase of the ion current from zero. The conditions are approximately the same as in Fig. 13.

the flare: there is an excess of ions on the leading front of the flare, and on an excess of electrons the trailing edge. The azimuthal electric field due to this charge separation leads to a radial growth of the flare. Such a spatial structure is characteristic of centrifugal flute instability of a plasma rotating in crossed radial electric (E_r) and longitudinal magnetic fields[11]. The conditions for the excitation of the observed instability also correspond to the theoretical criterion for the buildup of a centrifugal instability. The considered instability leads to an increase of the plasma radius R and to a decrease of density n_2, and these changes are the stronger the smaller the initial value of n_2. If the plasma density is decreased (by regulating the flow of gas into the plasma source), then an instant is reached in which the ratio R^2/a^2 increases appreciably jumpwise and the plasma beam goes over into another state. In this new state there is already observed an appreciable heating of the plasma ions, as well as phenomena evidencing the buildup of another instability, at frequencies close to the ion-acoustic frequency (see relation (43a)). This instability leads in turn to an additional appreciable increase of the ratio R^2/a^2 and to a decrease of n_2/n_1. By the same token, it directly prepares the conditions for the occurrence of a drift-beam instability, with which the transition of the plasma beam to a third state, with the highest turbulence level and with the most intense heating of the plasma ions, is connected. Interruption of the beam current (formation of a virtual cathode in the stream of primary electrons) is observed in this state[41]. Thus, the drift-beam instability turns out to be responsible not only for the acceleration of the ions to high energies, but also for the interruption of the current in the plasma beam.

The interrelation between the break of the current of the electron beam and the heating of the ions of the plasma through which this beam passes is demonstrated by the oscillograms in Figs. 13 and 14, obtained under conditions when the density of the plasma column n_2 drops periodically below the instability threshold (44). We see that in this case the instability has the character of "bursts" characterized, first, by a sharp decrease of the beam current passing through the plasma: (by a factor of several times or even by one order of magnitude) and, second, by emission of large fluxes of accelerated ions from the currents. In those time intervals when there is no instability (smooth sections on the oscillograms of the beam current in Figs. 13 and 14), the beam passes freely through the plasma column and there is no acceleration (heating) of the ions.

As to the very process of acceleration of the ions to high energies, it is apparently stochastic: the turbulent state of the plasma beam is characterized by a continuous spectrum of oscillations in the frequency range from several kHz to several MHz, including the Larmor frequency of the ions; the assumption of the stochastic character of the acceleration of the ions corresponds to the fact that the accelerated ions have a continuous energy spectrum (see Fig. 12).

Acceleration of the ions to high energies by passage of an electron beam through a plasma, and the transitions of the plasma beam between different discrete states, were observed also in the experiments of Neidigh, Alexeff, Fumelli, and their co-workers[46,78]. The nature and mechanism of these phenomena are apparently analogous to those described above.

It is important to note that a fraction of the ions ($\sim 10\%$) in the unstable plasma beam considered here is accelerated to kilovolt energies along the direction of propagation of the beam (i.e., along H)[42]. Longitudinal acceleration of ions to the indicated energies is observed also in a stable beam, but in this case it occurs only during those few dozen microseconds (from the instant of turning on the beam source) during which the plasma beam propagates (with ionic velocity) from the source to the beam collector[42]. The acceleration of the ions occurs in the latter case occurs in the deep potential well (virtual cathode) which is always present on the leading front of the propagating beam. It must be emphasized that the energies of the accelerated ions exceeds the energies of the beam electrons. It is possible that this mechanism has a bearing on the phenomenon of very effective longitudinal acceleration of the ions in powerful electron beams, observed in the experiments of Plyutto and his co-workers[65], and also the longitudinal acceleration of ions in the experiments[67] and in the plasma guns of Post's laboratory[66]. The indicated mechanism was recently postulated also by Rostoker in order to explain the acceleration of ions to energies of several MeV, observed in intense relativistic electron beams[68,76]; in the latter case the current of the accelerated ions amounted to hundreds of amperes, and the maximum ion energy (just as in[65]) is 10 times larger than the energy of the beam electrons. This phenomenon is of great interest as one of the possible methods of collective acceleration of charged particles to high energies. Close to this phenomenon is the acceleration of ions in the case of the escape of a (Maxwellian) plasma into vacuum, which was investigated theoretically in[79].

In concluding this section, we make one remark concerning the anomalous diffusion of the plasma across a strong magnetic field, caused by the electron-ion two-stream instabilities. Anomalous diffusion of plasma particles across a magnetic field was observed by Bohm[47] who obtained, for certain conditions, the well known diffusion coefficient

$$D_B = cT_e/16eH.$$

Diffusion with such a coefficient is considered to be very large; for example, such a diffusion might complicate the possibility of producing a quasistationary thermonuclear reactor with positive yield[48].

It thus turns out that in the development of a drift-beam instability of a plasma beam there occurs so fast a diffusion of the charged particles across the magnetic field, that the diffusion coefficient exceeds D_B, even if one substitutes for T_e the energy of the primary beam electrons (for details see[45]).

To compare the experiments with the theory, it is necessary to consider the theory in the nonlinear region.

3. Interruption of the Plasma Electron Acceleration in an Electric Field

There exists one more interesting phenomenon, which has a direct bearing on the discussed electron-ion instabilities of beams in a plasma. Namely, an external solenoidal electric field applied to the plasma cannot accelerate the plasma electrons to any appreciable energy; the acceleration process stops at an electron energy $\sim 50-70$ keV. Unfortunately, there are not enough experimental studies of this phenomenon to permit a full interpretation. We therefore confine ourselves here only to a comparison of the conditions under which this phenomenon is observed with the theory presented above.

The neatest experiments on plasma electron acceleration in toroidal setups were performed by Stefanovskiĭ[49]. In these experiments, the strong inhomogeneity of the accelerating electric field along the orbit of the electrons was first eliminated, and as a result it was possible to increase appreciably the number and energy of the accelerated electrons. Stefanovskiĭ believes[49] that all the plasma electrons were accelerated in his experiments. In the language of the present article, this means that the system of charged particles constituted a (two-component) quasineutral electron beam. But such a beam, in principle, is subject to the electron-ion instabilities considered above. In order to estimate their possible role in this case we compare the theoretical values of the thresholds and increments of these instabilities with the circumstances under which the acceleration of the electrons was interrupted in the experiments of[49].

The conditions of the experiments of[49] were as follows: accelerated-electron current $I_{max} \approx 1200$ A, accelerated-electron energy $W_1 = 50-70$ keV, lifetime of beam $T \approx 10^{-7}$ sec, intensity of longitudinal magnetic field $H = 1.2 \times 10^3$ Oe, length of electron orbit $L \approx 100$ cm, radius of plasma column $a \approx 2.7$ cm.

Argon ions were used. Putting $(k_z)_{min} = \pi/L$, k_φ, $k_r \approx 1/a$, and taking into account the (small) relativistic correction, we obtain for these conditions the following values of the electron-ion instability thresholds (5) and (16):

for two-stream instability: $I_{cr} \approx 600\ a$,
for drift-beam instability: $I_{cr} \approx 250\ a$.

We see that these thresholds are smaller than the maximum current of the accelerated electrons in the experiments of[49], i.e., electron-ion instabilities are possible.

We now must see whether these instabilities "have time" to interrupt the beam current within a time $T \approx 10^{-7}$ sec. Let us estimate their increments. From relation (6) we find for the increment of the two-stream electron-ion instability a value $\gamma \approx 2 \times 10^7$, i.e., $2\gamma T \approx 4$. As to the increment of the drift-instability (18), which is proportional to $(k_z)^{1/2}$, we choose for its estimate the maximum possible k_z (larger than π/L) at which the condition (17) is satisfied, i.e., $k_z \approx 2u/a^2\omega_{He}$. Then, according to (18), we obtain for argon

$$\gamma \approx (\omega_{Hi}/\omega_{He})^{1/2}(2u^2/a^2)^{1/2} \approx 3\cdot 10^7, \tag{18a}$$

i.e., $2\gamma T \approx 6$. Now let us find the time of development of the instability. We define this as the time interval during which the energy density of the oscillations has a change to grow from a level of thermal noise to a value nW_1 (since, according to[49], all the electrons are accelerated, we have $n_1 = n$). The energy density of the thermal noise is

$$W_T = T_e(4\pi d^3/3)^{-1}, \tag{46}$$

where r is the Debye radius

$$d = (T_e/4\pi ne^2)^{1/2}.$$

Consequently

$$nW_1/W_T = W_1 T_e^{1/2}/3\,(4\pi ne^6)^{1/2}. \tag{47}$$

For the conditions of[49], i.e., at $n = 10^{10}$ cm^{-3}, $T_e \approx 5$ eV, and $W_1 = 60$ keV, the ratio in (47) is approximately 3×10^9. On the other hand, the oscillation energy (the square of the amplitude) increases like $\exp(2\gamma t)$. Therefore the sought time is

$$\tau = \ln(nW_1/W_T)/2\gamma, \tag{48}$$

i.e.,

$$\tau \approx 10/\gamma.$$

In the case of drift-beam instability the obtained value of τ, expressed in units of γ^{-1}, exceeds the duration of the current pulse of the accelerated electrons by only approximately three times. (In the case of the "ordinary" two-stream electron-instability, this difference is larger, about five.) In addition, the following must be borne in mind: 1) The initial level of the noise in the plasma can greatly exceed the thermal level (46) (by virtue of various factors connected, in particular, with the method of producing the plasma). 2) Owing to the well known skin effect of the accelerating electric field[49], the characteristic dimension of the transverse current-density gradient (which in the theory is assumed approximately equal to the column radius a) can be much smaller than a. Consequently, the increment of the drift-beam instability can exceed the value (18) assumed above. 3) The nonlinear increment γ need not necessarily have the value (18) that follows from the linear theory. In other words, it can be stated that some difference between the experimentally observed time of instability development and the theoretical time of development of the beam-drift instability can hardly be regarded as exceeding the accuracy of the calculations.

Thus, out of the two considered electron-ion instabilities, the drift-beam instability (which has a smaller threshold and a larger increment) is the more probable cause of the interruption of the acceleration of the electrons by an external electric field in the experiments of[49]*. This instability can be stabilized in principle by increasing the magnetic field intensity. To this end, however, under the conditions of the experiments of[49], the field would have to be of the order of tens of thousands of Oersteds in the nonrelativistic case (see the condition (17) and much stronger in the relativistic case (see (17′)).

It should be noted that, in the opinion of authors of theoretical papers[52,53], the reason for the interruption of the acceleration of the electrons in the experiments of[49] could be the drift slipping-stream instability, mentioned in the introduction, and caused by the strong electron-velocity gradient in the beam. This instability, like the drift-beam instability, develops under conditions that are close to (17) (if the characteristic dimension of the transverse beam-electron velocity gradient is close to the beam radius), i.e., is also stabilized in principle by sufficiently strong magnetic fields. A different point of view in the theoretical paper,[54] according to which the reason of the considered phenomenon may be the so-called "negative mass" instability, which is characteristic of cyclic accelerators with soft focusing.

Thus, for a unique identification of the cause of the interruption of acceleration of electrons by an external electric field it is necessary to have more complete experimental data on the dispersion properties, spatial structure, thresholds, and possibility of stabilization of the instabilities under conditions of the type of[49]; see also[75] in this connection.

We note finally that the appearance of two-stream electron-ion instability is indicated also in experimental papers[55,74] in connection with the investigation of the mechanism of the anomalously low electric conductivity of a plasma in strong electric fields[55] and the mechanism of plasma heating in a strong-current gas discharge[74]. These questions are considered also in[50,51].

Stabilization of the instabilities upon acceleration of plasma electrons by an external electric field was successfully realized with a linear plasma betatron[80], where an appreciable increase of the accelerated-electron current was attained. This result was obtained by foregoing attempts to accelerate all the plasma electrons, i.e., by changing over to a three-component system, in which the density of the accelerated-electron beam is small compared with the total plasma density. It is interesting to note that this result is in full accord with the fact described in Sec. 1, namely that the limiting current of the beam in a plasma is strongly increased when the plasma density is increased.

*Stefanovskiĭ also believes that the discussed cause is not connected with the "ordinary" two-stream electron-ion instability (see [49], comments to Fig. 14).

V. CONCLUSION

Thus, the experimental data on the thresholds of electron-ion two-stream instabilities confirm well the existing linear theory. Two remarks should be made concerning this theory. First, everywhere in this review, just as in[19,20], in the derivation and in the analysis of the dispersion equations we have used, for the sake of simplicity and clarity, the so-called quasiclassical approximation. In this approximation, which is sufficiently rigorously valid when $k_r a \gg 1$, the perturbation of the electric potential is of the form $\Psi = \Phi(r)\exp(is\varphi + ik_z a - i\omega t)$, where $\Phi(r) \approx \exp(ik_r r)$. We, however, used this notation in in[28-30,32] also for large-scale perturbations ($k_r a \approx 1$), which play the most important role in the experiment: it is just these perturbations which determine the instability thresholds and the limiting beam currents. The indicated lack of rigor is perfectly justified, since the result of a comparison of all the experimental data with the more refined theory[13-16] is in essence the same. In other words, by foregoing the quasiclassical approximation and using a somewhat more rigorous (but much more cumbersome) theory we obtain only slight quantitative corrections, which in this review would be patently insufficient compensation for the loss of such important advantages of the theory as simplicity and clarity.

Much more important is another circumstance. The existing theory of drift-beam instability (both the approximate and the more refined one[15]) is linear, and therefore, while well explaining the conditions for the occurrence of such nonlinear phenomena as heating of the plasma ion and the interruption of the current in the beams, it is incapable of explaining the dynamics of these phenomena. This circumstance emphasizes once more the importance of further development of the nonlinear theory of electron-ion two-stream instabilities. From the point of view of applications, this need is connected with the task of increasing the efficiency of turbulent heating of the plasma ions by beams of charged particles, and with the development of the electrons and physics of accelerators and relativistic beams of ultrahigh intensity.

The present status of the nonlinear theory of the phenomena in question can be found in the monographs[18,56] and in the articles[10,57,58,81].

In conclusion we recall that throughout this review, when speaking of beams, we had in mind a situation wherein the directed velocity of the particles of the beam is larger than the thermal velocity (or at least is comparable with it). We therefore did not discuss at all the instability of the current in a plasma when the directed (current) velocity of the electrons is much smaller than the thermal velocity. This question and the associated phenomenon of turbulent heating of a plasma by a current flowing through it is the subject of work by Zavoĭskiĭ, Rudakov and their co-workers (see, for example, the review[69] and the literature cited there), and also of Suprunenko and co-workers (see, for example,[74] and the work cited there).

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