ANGULAR DIVERGENCE OF RADIATION OF SOLID-STATE LASERS

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CONTENTS

INTRODUCTION

THE ability to generate light beams with small angle divergence and high spatial coherence is the most remarkable feature of lasers, and motivates their practical applications. Therefore questions concerning the angle structure of the radiation of lasers and the mechanism of shaping a given structure has been the subject of a very large number of researches.

In the present review we systematize the results of these researches. Although the material employed pertains mainly to solid-state lasers, many of the problems touched upon are of general significance. Principal attention has been paid to papers explaining the most important factors determining the degree of directivity of the radiation. The results of a number of investigations devoted to the influence of readily removable or random factors will not be considered. Such investigations include observations of additional radiation beams resulting from reflection from the polished lateral surface of the active medium,[1,2] the study of the structure of the beam when prisms are used as the elements of the resonator[3] and others.

The results of the theory of ideal empty open resonators will be used in the course of the exposition. Since the main premises of this theory are universally known for resonators with concave or plane mirrors (and their equivalents),[4-6] these will likewise not be discussed here.

The paper is divided in two parts. The first considers the properties of lasers having two-mirror resonators with small diffraction losses, and the second discusses the features of systems using various angular selection devices, including the use of the so-called "unstable" resonators.

I. GENERAL INFORMATION ON THE ANGULAR DIVERGENCE OF RADIATION

1. Results of Experimental Investigations

The very first observations of the generation of solid-state lasers have shown that the angular divergence of their radiation is much larger (usually by 1–2 orders of magnitude) than the expected value corresponding to diffraction by the exit aperture of the generator. To ascertain the causes of the large angular divergence of the radiation, various investigations were undertaken on the dependence of the spatial structure of the generated beam on the homogeneity of the active medium, the pump intensity, etc. Most of these investigations pertain to the case of the resonator with flat mirrors, which was proposed in 1958.[7,8]

Comparison of the experimental data with the results of the theory of open resonators very soon revealed that a regular distribution of the spots in the far-field picture, corresponding to definite modes of the ideal resonator, is observed as a rule only for exceedingly homogeneous samples and at a slight excess above the generation threshold.[9-12] With increasing pump intensity, the number of modes present in the generation increases because of the modes with more complicated structure;[9,10,12,13] to observe them it is necessary to employ, as a rule, high-speed photography, since the far-field picture changes from spike to spike, and it is difficult to interpret the integral distribution.

If active rods with noticeable optical inhomogeneities are used, the identification of individual modes becomes impossible.[14,15,11,12] It was observed in many investigations that small resonator aberrations suffice to produce complete distortion of the form of the field distributions in the lower-order modes. Thus, for example, in one case[16] such a distortion was produced by elastic deformations of the ruby rod, and in another by changing over from samples with optical-length variation $\Delta L \simeq 0.1\lambda$ to samples with $\Delta L \simeq 0.25\lambda$.[11] This has led to a tendency, in the later investigations, to analyze the mechanism of the influence of the properties of the real resonator on the angular divergence of the radiation.

A considerable step forward in the understanding of the gist of this mechanism were the investigations of Evtuhov and Neeland,[10,17] and particularly the work of A. M. Leontovich and A. P. Veduta.[18] It was shown in these investigations that if the inhomogeneous active rod serves as a source of wave aberrations of second order (of the positive-lens type), then a resonator with flat mirrors becomes equivalent to some empty resonator with concave mirrors. The observed field structure corresponds in such cases to the predictions of the theory of generalized confocal resonators. Thus, the concept of the equivalent resonator was used for the first time for the treatment of the experimental data.

The paper of A. M. Leontovich and A. P. Veduta also explained a number of phenomena typical of the use of resonators with "stable" configurations (more accurately, with low diffraction losses). In the case of a small excess above threshold, the only modes excited are the lower modes localized near the axis of the sample, with the smallest diffraction losses. The appearance of generation first in the central zone of the sample is aided by the fact that the pump radiation density, and with it also the inverted population, frequently reaches a maximum precisely on the axis of the sample.[19] With increasing pump power, the threshold for the generation of higher modes is reached. Finally, at large excesses above threshold, the maximum transverse index of the excited modes is determined by the condition for the filling of the entire active sample with the generation radiation.

The picture of the change of the oscillation modes described above was indeed observed in many investigations (for example, [20,21]); the notion of the mechanism determining the maximum index of the transverse modes present in the generation was likewise fully confirmed.[22-24] We note that a "stable" resonator configuration was ensured in different investigations either by using concave mirrors,[22,24] or by introducing positive lenses into a resonator with flat mirrors,[23,24] as well as by the lens character of the sample itself.[25] It is curious also that when modes with higher indices appear in the generation, the lower oscillation modes vanish (for example, [20]).

With decreasing sphericity, as the resonator approaches the planar configuration, the information on the angular divergence becomes less and less systematic. It is very difficult to identify the individual oscillation modes; frequently it is impossible to trace the correlation between the width of the directivity pattern of the radiation and the perfection of the resonator. Moreover, numerous investigations have demonstrated that it is precisely in the case of a planar resonator that the angular divergence of the radiation turns out to be particularly sensitive to wave aberrations occuring inside the resonator, the sources of which are quite varied in the case of solid-state lasers. In spite of this, a careful comparison of the generation characteristics for resonators with flat and spherical mirrors[26] shows that a planar resonator ensures a much larger degree of directivity and brightness of the radiation.*

Inasmuch as for practical applications of lasers an important role is played, as a rule, not by the possibility of identifying the individual modes but by the brightness of the radiation, generators with flat mirrors have for many years been the main type of solid-state laser.

The relatively small angular divergence of the radiation and, at the same time, the variety of factors that influence it, have ensured constant interest in this type of laser on the part of the researchers. We present the main reulsts of numerous experimental investigations devoted to the study (and to attempts at elimination) of the factors that limit the axial strength of a laser with a planar resonator.

The greater part of these investigations pertains to the case of ruby lasers. Being a typical crystalline active medium, ruby has a tremendous number of randomly distributed microscopic and macroscopic inhomogeneities which lead to considerable light scattering. The characteristic magnitude of the loss to small-angle light scattering amounts to $0.01-0.1\ \mathrm{cm}^{-1}$ for this material[31] (as well as for many other crystalline active media[30]).

In the presence of noticeable light scattering, the width of the central core in the angular distribution is usually $5'-20'$. The core is surrounded by a system of relatively intense rings[32-35,11,18] with an angular radius equal to the radius of the rings in the Fabry-Perot interferometer for light with a wavelength equal to the emission wavelength in the central core (for example, [35]). According to certain observations, the widths of these rings are determined by the same relations as in the passive interferometer, and depend accordingly on the quality of the sample.[11] Finally, M. P. Vanyukov et al.[36] proved finally, by introducing an additional scattering element inside the resonator, the connection between the rings in the angular distribution and the light scattering.

Observations of the spatial coherence[32-34] and of the emission spectra[37] have led to the conclusion that the radiation belonging to individual oscillation modes is distributed practically over the entire spot of the far-field picture, and the wave front emerging from the laser differs greatly from a plane one.

None of these facts agree with the notions resulting from the theory of ideal empty resonators. The interpretation of the observed phenomena is made difficult also by the fact that in practically every concrete case there appears the influence of a great variety of types of resonator aberrations. Nevertheless, many workers have succeeded in tracing a distinct correlation between the optical perfection of the active medium and the degree of directivity of the radiation, namely, the angular divergence increases with decreasing homogeneity of the sample, both in the case of amplifiers[38] and the case of optical generators.[38-43] It is curious that the magnitude of a large angular divergence is close to the angular divergence of a well-collimated beam from an external source after passing once through the same sample.[42-44]

The features of the influence of such concrete sources of light scattering as glide surfaces, boundaries between blocks, etc. were considered in detail in [41-43]. It follows from the presented data that the macroscopic inhomogeneities affect mainly the central core of the angular distribution, while scattering by the microscopic inhomogeneities affects the "wings" of the distribution.

The dependence of the angular divergence and of the radiation power on the resonator length was investigated in [45] for the case of a laser using a medium that is macroscopically homogeneous but has a noticeable light scattering by microscopic inhomogeneities ($CaF_2:Sm^{2+}$),

As already indicated, sources of light scattering in crystalline active media are distributed over the volume more or less randomly. The aberrations due to uneven heating of the active rod (the so-called thermal deformations of the resonator) have an entirely different character. Their sources are temperature variations of the refractive index and photoelasticity phenomena due to the presence of thermal stresses.[46,47]

This effect was observed in 1963[48,49] and was investigated in detail in a number of succeeding papers.[50-61] By virtue of its very origin, the thermal deformations,

*Here and throughout we are dealing mainly with solid-state lasers having a sufficiently large value of the parameter $N \equiv a^2/\lambda L_{eff}$ (2a—transverse dimension of the sample, L_{eff}—effective resonator length, see below). At small N it may be more convenient to use concave mirrors with large curvature radii [27], which at the same time ensure approximately the same brightness, plane-spherical resonators with a concave mirror having a radius of curvature somewhat larger than L_{eff} [28,29], or dihedral reflectors with angles somewhat smaller than 180° [169] or 90° [170] between the faces.

unlike light scattering, are equivalent to the presence of a refractive-index gradient that varies slowly over the cross section of the sample.

Even when the laser operates in the single-flash regime, the thermal deformations at the end of the pulse frequently reach noticeable magnitude and greatly influence the angular divergence of the radiation. Thus, M. P. Vanyukov et al.[51] found that the divergence was doubled at the end of the pumping pulse in the case of a neodymium-glass laser, and increased 3–5 times in the case of a ruby amplifier.[62]

The thermal deformations are particularly large in active elements of lasers operating in the periodic[56,59] or the continuous[25] regime. The decrease of the temperature inside such elements with increasing distance from their axis is approximately proportional to the square of the radius, and large aberrations of second order appear (thermal "lens"). As a result, the angular divergence of the radiation of such lasers is quite large and can be significantly decreased by introducing some correcting element into the resonator.[25,63] The usual correcting element is a lens or a spherical mirror replacing a flat one.

We note that compensation of the "sphericity" of the resonator due to the inhomogeneity of the sample is sometimes carried out in the regime of single flashes and naturally exerts the same influence on the angular distribution of the radiation.[64,65] In particular, in [64] the use of a convex mirror of suitable radius of curvature has led to a decrease of the angular divergence by a factor of 4.

Thermal deformations of the resonator are the main and most important source of aberrations in lasers using such highly homogeneous active media as neodymium glass.* The possibilities for their complete cancellation are quite limited, especially in generators without Q switching.

Indeed, in the case of Q switching, the generation pulse is so short that the thermal deformation does not have time to change noticeably during the generation time, and can be compensated beforehand by a corrector (a master generator in the system with high radiation brightness has been constructed on the basis of this principle[66]). In the case of a laser without Q switching, the thermal deformations, as already indicated, change appreciably during the generation pulse, and their dynamic compensation is a very complicated problem which has not yet found an acceptable solution. All the more curious is the possibility of complete annihilation of thermal deformations for emission of one of the polarizations by using an active element of flat shape and by suitably choosing the thermooptical constants of the active medium. For the case of neodymium glass, such a selection is perfectly feasible;[61,168] in accordance with the data on the properties of lasers with birefringent active media,[67] the generation of polarized radiation in such elements does not involve energy loss.

Besides the already considered factors, the spatial structure of the beam is influenced by many others. Thus, an important role is played by the accuracy of the adjustment of the flat mirrors: if they are not parallel, the spot in the far zone becomes elongated in the misalignment direction, and in the case of misalignment on the order of several minutes it breaks up into a number of individual spots.[68-71] Particularly sensitive to nonparallelism of the mirrors are resonators with a homogeneous active medium and accordingly with a small radiation divergence.[71]

It has also been noted that the angular divergence of the radiation depends on the character of the distribution of the inverted population over the cross section of the sample,[72,73] etc.

The aggregate of many factors causes in final analysis the angular divergence to amount usually to several minutes of angle and to exceed greatly the diffraction limit even in the case of lasers using highly homogeneous active media.

From the point of view of the structure of the beam itself, the large magnitude of its divergence may be a manifestation of both the multimode character of the generation and of the fact that in systems with considerable aberrations the radiation wave front pertaining to the oscillation modes with the highest Q may differ greatly from a plane front. The latter phenomenon is frequently called mode deformation.*

From the data given in many papers[11-15,32-34,37,68-71] we can draw the qualitative conclusion that in the case of lasers with flat mirrors, mode deformations are particularly important in the presence of noticeable aberrations. Attending a decrease of the aberrations is an increased role of multimode generation (for example [121]); finally, if the experiment is performed with particular care and the active medium is homogeneous, then the angular divergence of the radiation is due mainly to the multimode generation.[45]

Although the indicated general tendencies are quite clearly manifest, it is very difficult to distinguish experimentally between the influence of the multimode generation and oscillation-mode deformation. The overall picture is greatly complicated by the unique kinetics of the solid-state lasers; as already mentioned, it is also difficult to separate the influence of individual types of aberration. All this makes it practically impossible to systematize the concrete data on the angular divergence of radiation of solid-state lasers. We shall henceforth pay principal attention to the results of a theoretical analysis which makes it possible to understand at least qualitatively the majority of the phenomena listed above. Experimental information will serve where necessary as criteria for the validity (or the advantage of utilization) of one particular theoretical model or another.

We stop first of all to discuss the very possibility of using concepts developed by the theory of empty open resonators to describe the properties of real lasers.

2. Model of Open Resonator with Active Medium in the Quasistationary Approximation

To explain the experimentally observed regularities, extensive use is made of the results of the theory of empty open resonators. When the resonator is filled with an active medium, its properties, generally speak-

*The initial inhomogeneity of the refractive index and the light scattering in neodymium glass are quite small [30].

*Although this term is not quite exact, it is used in many papers and is convenient.

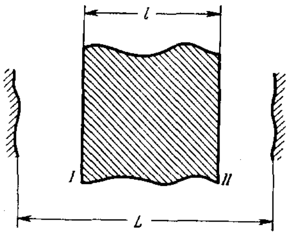

FIG. 1. Active layer in open resonator. $\mathbf{r}_1$ lies in plane I, $\mathbf{r}_2$ in plane II, and r on the surface of one of the mirrors.

ing, are significantly altered. These changes must be carefully taken into account.

To ascertain the connection between the spatial distribution of the radiation in real lasers and in empty resonators, let us consider an idealized case of an infinite plane-parallel active layer of thickness l between two mirrors of finite dimensions that are separated by a distance L (Fig. 1). The refractive index of the medium of the layer, n, will be assumed to be constant and different from unity, the inverted population is assumed uniformly distributed over the volume, the surfaces are assumed nonreflecting, and the mirror reflection coefficients (in terms of intensity) are 1 and R′. The steady-state distribution of the field will be sought by the method of Fox and Li,[74] and we confine ourselves, as usual, to the scalar formulation of the Huygens-Fresnel principle and assume that the transverse dimensions of the region in which the radiation is concentrated are small compared with its length (see, for example, [29]).

We consider first the passage of a light wave through the active layer. The field distribution $u(\mathbf{r}_2)$ on the second surface of the layer can be expressed in terms of the distribution in the first surface $u(\mathbf{r}_1)$ with the aid of the standard relation

$$u(\mathbf{r}_2) = -\frac{i}{\lambda' l}\int\int u(\mathbf{r}_1)\, e^{\left(i\frac{2\pi}{\lambda'}+\frac{k'}{2}\right)\left(l+\frac{|\mathbf{r}_2-\mathbf{r}_1|^2}{2l}\right)}\, dS_1,$$

where $\lambda' = \lambda/n$ is the wavelength inside the layer, k' is the Bouguer coefficient of light intensity amplification, and the integration is carried out over the first surface.*

Changing over to the wavelength in free space λ and neglecting the term $(k'/2)|\mathbf{r}_2 - \mathbf{r}_1|^2/2l$ in the argument of the exponential, we obtain

$$u(\mathbf{r}_2) = e^{\frac{k'}{2}l}\,\frac{-ie^{i\frac{2\pi}{\lambda}nl}}{\lambda\frac{l}{n}}\int\int u(\mathbf{r}_2)\, e^{i\frac{2\pi}{\lambda}\frac{|\mathbf{r}_2-\mathbf{r}_1|^2}{2l/n}}\, dS_1. \tag{1}$$

From (1) we see that the form of the wave front after passing through the active layer of thickness l is transformed in the same manner as after passing through a distance l/n in free space; the optical thickness of the layer nl determines only the total phase advance; the amplitude of the field is multiplied by $\exp(k'l/2)$.

It is now easy to trace the passage of the light wave through the resonator in the forward and backward directions and to set up the corresponding integral equation. Obviously, it differs only by a constant factor in the right-hand side from the equation for the equivalent empty resonator consisting of two mirrors having the same configuration, but with total reflection, and separated by a distance $L_{eq} = L - l + l/n$. We write it in the form

$$\gamma u(\mathbf{r}) = e^{k'l}\sqrt{R'}\, e^{i\frac{2\pi}{\lambda}2L_0}\,\hat{P}(L_{eq})\,u(\mathbf{r}); \tag{2}$$

Here $L_0 = L - l + nl$ is the optical length of the resonator, and $\hat{P}(L_{eq})$ is the integral operator that transforms, in accordance with the Huygens-Fresnel principle, the scalar field distribution function $u(\mathbf{r})$ on one of the mirrors of the equivalent empty resonator into the field distribution function on the same mirror after complete passage of the wave through the resonator; all that is excluded from this operator is the constant common phase factor $\exp[i(2\pi/\lambda)2L_{eq}]$.

To determine the spectrum of the eigenvalues of (2) we use information concerning the properties of the empty equivalent resonator. When a wave with a field distribution $u_m(\mathbf{r})$ corresponding to a mode with a transverse index m passes through the equivalent resonator, an additional phase shift $-4\pi p'$ is produced, and the amplitude is decreased by a factor $\exp(-4\pi p'')$ $(1 - \exp(-8\pi p''))$ is the diffraction loss; we use the notation of [6]; the argument of the exponential is twice as large as in the corresponding expressions of [6] because the calculation is for a double passage through the resonator).

Thus,

$$\hat{P}u_m(\mathbf{r}) = \alpha_m u_m(\mathbf{r}), \quad \alpha_m \equiv e^{-4\pi i(p'-ip'')}. \tag{3}$$

Substituting (3) in (2), we obtain

$$\gamma_m = e^{k'l}\sqrt{R'}\, e^{-4\pi p''}\, e^{i\left(\frac{2\pi}{\lambda}2L_0-4\pi p'\right)}.$$

The connection between the foregoing analysis and the determination of the stationary distribution of the field with complex frequency $\omega = \omega' - i\omega''$ [6] is established in the following manner. At the resonant frequency $\omega'_m = 2\pi c/\lambda_m$ the quantity γ_m should be real, and the total phase advance $(\omega'/c)2L_0 - 4\pi p'$ should equal $2\pi q$, where q is an integer (the axial index). The field amplitude changes by a factor γ_m after a time $2L_0/c$, and $\gamma_m = \exp(-2\pi L_0\omega''/c)$.

Thus,

$$\omega'_{qm} = \frac{c}{2L_0}(2\pi q + 4\pi p'_m),$$
$$\omega''_m = \frac{c}{2L_0}\left(4\pi p''_m + \ln\frac{1}{\sqrt{R'}} - k'l\right). \tag{4}$$

Relations (4) determine the spectrum of the eigenvalues of the resonator with the active medium. It is remarkable that the active medium does not enter in any way as some external source of excitation of the oscillations. Its presence causes mainly a realignment of the spectrum of the natural frequencies of the system.* This leads, in particular, to a conclusion concerning the possibility of separating individual oscillation modes in resonators with large losses.

Indeed, the condition for the overlap of the resonator curves corresponding to two neighboring natural frequencies of the system ω_1 and ω_2 can be written in the form[6]

$$|\omega'_1 - \omega'_2| < \frac{\omega''_1 + \omega''_2}{2}. \tag{5}$$

*The signs in front of i have been reversed compared with [29], since we assume, in accord with the notation of [6], a field ~ exp $(-i\omega t)$ (and not $\exp(i\omega t)$ as in [29]).

*For an exact calculation of ω'_{qm} it is necessary also to take into account the dependence of n and k′ on λ ("frequency pulling"; see, for example, [30]), but this will not be done here.

In the absence of pumping ($k' \leq 0$), this condition is almost always satisfied, since the losses in the resonators of solid-state lasers are quite large (their main source is most frequently not diffraction effects, but inactive absorption and the diversion of energy through the semitransparent mirror). Thus, when such a system is excited by an external source of electromagnetic oscillations, its resonant properties are insufficient for the separation of the individual oscillation modes.

The situation is different in the presence of inverted population. The values of ω'' decrease in accordance with (4); on approaching the generation threshold, some of them tend to zero. As a result, the resonant properties of the system can become fully manifest even in the regime of regenerative amplification, in spite of satisfaction of the condition (5) in the absence of excitation of the active medium. The resonant properties of the system become even more clearly pronounced during the time of generation, when one or several of values of ω''_m are equal to zero.*

Analyzing the foregoing example, we can formulate the conditions satisfaction of which makes it possible to use directly the results of the theory of open empty resonators for the description of the structure of the field in real lasers. The most important of these conditions are as follows:

1. It is necessary to have high optical homogeneity of the active medium. The passage of the wave front through an inhomogeneous medium cannot be reduced to the passage over a corresponding path in empty space; the choice of the equivalent empty resonator becomes more complicated and in many cases utterly impossible.

2. In the case of a resonator with external mirrors it is necessary to exclude the influence of the light reflected from the active-element surfaces facing the mirrors. In the presence of antireflection coatings, it can be usually assumed that this condition is satisfied. In some cases (see Sec. II.2) such measures turn out to be insufficient, and to eliminate side effects it is necessary to incline the interfaces considerably to the direction of propagation of the radiation.

3. In the derivation of (2) we used the model of an infinite homogeneous layer. In order for this model to be valid it is necessary that both the refractive index and the inverted population be constant not only immediately between the mirrors, but also in the entire region where the radiation field has a noticeable intensity; there should be no influence of the side walls of the active element.

In real lasers this condition is hardly ever satisfied. Moreover, in the case of flat mirrors of the resonator, the role of the aperture diaphragms limiting the generation zone is frequently performed not by the mirrors but by the lateral surfaces of the active element. To be sure, in order to exclude the appearance of the additional light beams described in [1,2] and caused by total internal reflection from the lateral surfaces, these surfaces are usually ground dull or the rod is placed in an immersion medium. Such measures make the well known model of an ideal dielectric rod between mirrors of finite dimensions ([29], p. 149 and others) perfectly unacceptable, but do not eliminate partial scattering of the diffracted radiation.

Since the decrease of the role of the light scattering by the side surfaces is accompanied by an increased degree of directivity of the radiation, it is reasonable nevertheless to use the model of the open resonator in estimates of the limiting characteristics of the system.

4. In order for the distribution of the laser radiation field to be describable by the eigensolutions of the stationary integral equation, it is necessary that the generation conditions remain unchanged during a sufficiently long time. This requirement again is far from always satisfied. Thus, in a number of theoretical[75,76] and experimental[77-79] papers it was shown that in the case of the single-pulse regime of a laser with a planar resonator and relatively homogeneous medium, the spatial distribution of the inverted population and of the radiation field change exceedingly rapidly (within the time 10^{-9}–10^{-8} sec). In such a situation it is meaningless to consider the spatial structure of the radiation and disregard its connection with the kinetics of the generation. This connection was investigated in detail by A. F. Suchkov and V. S. Letokhov.[80,81]

The operating regime of solid-state lasers is nonstationary also in the absence of Q-switching of the resonator (we have in mind primarily the presence of characteristic radiation spikes). A detailed theoretical description of the space-time structure of the radiation is possible only in some of the simplest cases (for example, [80]), which are never realized in practice. Therefore in the overwhelming majority of the studies use is made of a quasistationary approximation, which does not take into account the features of the generation kinetics. The character of the errors incurred thereby becomes clear if it is recognized that in real lasers the role of the "primer" in the formation of individual pulses (spikes) of generation is practically always played by the spontaneous noise radiation.* A narrowly-directed light beam is produced only in the course of the nonlinear amplification of the "primer." Therefore deviations from the quasistationary regime lead, as a rule, to a certain decrease of the directivity of the radiation.

In spite of this, the quasistationary approximation is reasonable in many cases.

If the aberrations (and with them the angular divergence in the quasistationary approximation) are large, then the features of the kinetics, for obvious reasons, no longer have a strong influence on the resultant width of the directivity pattern. In particular, the angular distribution of the radiation becomes approximately the same in the monopulse and in the ordinary generation regimes.

For the case of the ordinary regime of free generation, the quasistationary multimode approximation can be used to describe satisfactorily the properties of even lasers with highly homogeneous media ([45]; Sec. I.4).

Having made all the necessary stipulations, we proceed to an exposition of the results of the most impor-

**If this circumstance is disregarded, it is concluded erroneously that the generation is "nonresonant" in systems with very large losses. The size of the loss itself is important, but affects not so much the resonant properties of the system as the character of the kinetics of the generation and the mode competition (Sec. I.4).

*For the case of the random-spike regime, the validity of this statement follows from the fact that the spectral composition of the radiation is not the same in succeeding spikes (for example, [82,9,83]).

tant theoretical papers concerning the spatial structure of the field of a laser in the stationary generation regime. Most of these papers can be broken up into two main groups. One includes papers in which the influence of different perturbations on the field configuration and individual oscillation modes is analyzed, and the other is devoted to analysis of the mode competition in ideal resonators. The mechanism of mode competition in resonators with aberrations is too complicated and is not discussed in the literature.

3. Influence of Aberrations on the Field Configuration of Individual Oscillation Modes

Most reasonable results concerning the calculation of the field configuration in the presence of aberrations were obtained either by the numerical method of Fox and Li, or by directly solving the differential equations for the electromagnetic field, or else, finally, by series expansion in the eigenfunctions of the ideal resonator. The latter method is quite lucid, especially in the form proposed by V. V. Lyubimov.[84] Since we shall explain the main qualitative laws precisely with the aid of this method, we shall stop to discuss it in greater detail.

The possibility of series expansion in terms of a system of eigenfunctions u_m of the operator of the ideal resonator $\hat{P}$ requires that this system be complete. For open resonators with mirrors of finite dimensions, this condition, generally speaking, is not satisfied.[6] Moreover, for modes of higher order, the scalar formulation of the Huygens-Fresnel principle becomes invalid. However, the eigenfunctions corresponding to modes with the lowest transverse indices practically coincide in the case of slightly concave mirrors with the functions of the Hermitian operator of an infinite confocal resonator,[85] and in the case of flat mirrors they are quite close to the eigenfunctions of a closed resonator with flat mirrors, which form a complete set.[86] It must also be borne in mind that in real generators the rays propagating at large angles to the resonator axis and corresponding to modes of higher order are usually absent. Therefore, in all cases of practical importance we can confine ourselves to the first few terms of the expansion.[85]

For the reasons given above, the series expansion in the eigenfunctions of the resonators with both concave and flat mirrors turns out to be possible and leads to results that coincide in practice with the data of the exact calculations. In the case of small aberrations, perturbation-theory methods can be used.

In the first approximation of perturbation theory, the solutions of the equation $(\hat{P} + \hat{P}')u'_m = \alpha'_m u'_m$, corresponding to a resonator with aberrations, are described by the formulas

$$u'_m = u_m + \sum_k a_{mk} u_k, \quad a_{mk} = \frac{P'_{km}}{\alpha_m - \alpha_k}, \qquad \alpha'_m = \alpha_m + P'_{mm}. \tag{6}$$

Relations (6) have a very simple meaning. The matrix elements P'_{km} of the perturbation operator are equal to the relative amplitudes of the light wave scattered as a result of the perturbation from one of the oscillation modes of the ideal resonator into another (this will be clearly seen in the analysis of the concrete forms of the perturbation operator). The quantities a_{mk} are the

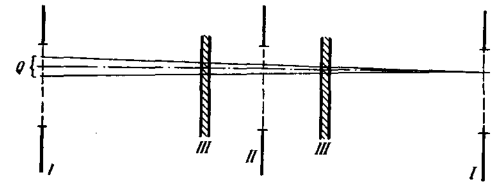

FIG. 2. Equivalent diagram of resonator with flat mirrors. I, II—mirrors, III—perturbation zone.

amplitude of the induced oscillations; naturally, they are inversely proportional to the frequency differences between the driving force and the free oscillations, $\alpha_m - \alpha_k$. In the case of resonators with concave mirrors, these differences are approximately proportional to the curvature of the mirrors and are usually many times larger than the differences for the case of flat mirrors.[6]

This circumstance is the main cause of the relatively weak dependence of the form of the field distribution in resonators of "stable" configuration not only on the misalignment of the mirrors (which reduces in first approximation to a shift of the resonator axis), but also of aberrations of other types. As a result, the angular divergence of the radiation of a laser with concave mirrors is determined as a rule not by the influence of the aberrations but by the index of the oscillation modes present in the generation (see Sec. I.4). In the present section we shall therefore not discuss further the properties of resonators of "stable" configuration with aberrations,* and confine ourselves to resonators with flat mirrors.

In the case of flat mirrors with a large number of Fresnel zones, the action of the operator $\hat{P}$ reduces to a considerable degree to parallel transfer of the wave front. The diffraction "mixing" of the radiation is small; the source of the field at an arbitrary point (see the equivalent diagram of Fig. 2) is in essence the field of the initial wave in a small region Q about the same point. This circumstance greatly facilitates the calculation of the matrix elements of the perturbation operator.

Indeed, assume that the perturbation source is concentrated in a narrow zone (Fig. 2). When the wave passes, its amplitude becomes multiplied by a factor f(r) that varies slowly over the resonator cross section (in the general case, this factor includes both amplitude and phase corrections). If the changes of f(r) over the dimensions of the region Q are small, then the distribution of the field of the wave passing through the entire resonator in the forward and backward directions turns out to be multiplied by $f^2(r)$ regardless of where the perturbation zone is located on the length of the resonator. Therefore all such aberration sources (including a nonuniformly-excited active medium) can be regarded as concentrated in narrow zones near the mirrors, as is done in many papers.[89-93] *

Thus,

$$\hat{P} + \hat{P}' = F\hat{P},$$
$$P'_{km} \approx \int\int u_k(\mathbf{r})\, u_m(\mathbf{r})\, [F(\mathbf{r}) - 1]\, dS \tag{7}$$

---

*These properties are considered in detail in [85], and also in [87,27,88] and elsewhere.

*It is easily seen that this approximation is equivalent to the calculation of the corresponding integral by the constant-phase method.

(We have used $\alpha_m \approx 1$; F is a factor describing the summary influence of all the aberration sources; information on the normalization of u_k can be found in [85]).

In the calculation of the expansion coefficients a_{mk} by means of formulas (6) and (7) it is convenient to introduce one more simplification, replacing both the eigenfunctions and the eigenvalues by the corresponding quantities for the closed resonator. In fact, the fact that the eigenfunctions of the open (u_m) and closed (u_m^0) resonators with flat mirrors of large dimensions are close has already been mentioned. In particular, for a two-dimensional resonator with mirrors of width 2a we have[6]

$$u_m(x) \approx \binom{\cos}{\sin} \frac{\pi m x}{2a\left(1+\beta\frac{1+i}{M}\right)}, \quad u_m^0(x) = \binom{\cos}{\sin}\frac{\pi m x}{2a},$$

$$m = \begin{cases} 1, 3, 5, \ldots, \\ 2, 4, 6, \ldots, \end{cases}$$

where $-a \leq x \leq a$, $\beta/M \approx 0.16/\sqrt{N}$; usually $N = a^2/\lambda L_{eq} \gg 1$, and β/M is a small parameter.

The difference of the eigenvalues for the open resonator, accurate to terms of the same order of smallness, is determined by the value of the phase corrections $4\pi p'_m$.[6] The phase corrections, unlike the diffraction losses, are practically independent of small parallel shifts or of the inequality of the sizes of the mirrors, and coincide with the corrections for the closed resonator. In the two-dimensional case

$$4\pi p'_m \approx m^2 \frac{\pi}{8N}, \quad \alpha_m - \alpha_k \approx (k^2 - m^2)\frac{\pi i}{8N}.$$

It follows therefore that the result of the influence of the perturbations on the form of the field distribution depends little on random factors. Therefore information obtained with the aid of the foregoing approximation can serve as an objective characteristic of the radiation field of real lasers; the calculation of the diffraction losses calls for a more complicated analysis (for example, [94]).

Let us proceed to consider types of perturbations, starting with stationary phase aberrations.

The most detailed study was made of the influence of wave aberrations of the first order (violation of the parallelism of the mirrors). In this case we have for a two-dimensional resonator

$$F(x) - 1 = e^{i\frac{2\pi}{\lambda}2\varepsilon x} - 1 \simeq i \cdot \frac{2\pi}{\lambda} \cdot 2\varepsilon x$$

(ϵ is the misalignment angle). The perturbation is antisymmetrical; consequently, only a_{mk} with odd $|m - k|$ do not vanish. A simple analysis shows that with increasing misalignment angle, the center of gravity of the field distribution shifts monotonically towards the farther edges of the mirrors.* In particular, the expression for the eigenfunction of the lowest mode is of the form $u'_1 \approx u_1 + 4(a\epsilon/\lambda)Nu_2$ ([86]; Fig. 3a). In accordance with this expression, the fundamental mode turns out to be noticeably deformed even at extremely small misalignment angles ϵ. When ϵ reaches the value $\lambda/4aN$, the angular divergence of the radiation of the fundamental mode is approximately doubled;[84] at the same time, perturbation theory itself ceases to be valid.

*The opposite conclusion, drawn in [6], from which it was taken by the authors of [71], is based on an error in the reasoning.

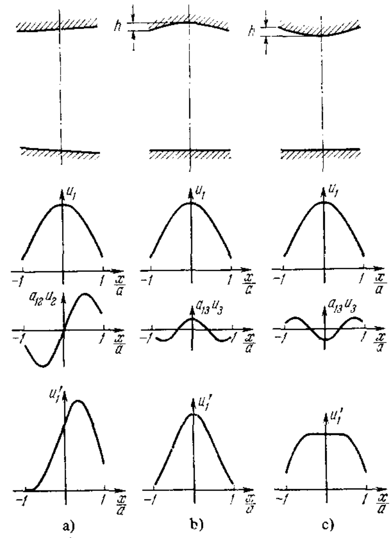

FIG. 3. Influence of phase aberrations on the distribution of the field of the lowest mode. a) Misalignment of mirrors, b) concave mirrors, c) convex mirrors.

Since $p'_m \sim m^2$, the mode deformations decrease rapidly with increasing transverse index. Therefore in the usual multimode regime the total angular divergence turns out to be much less sensitive to misalignments than the configuration of the field of the fundamental mode.

More detailed information on the structure of the field and diffraction losses in resonators with flat misaligned mirrors are given in [70,71,88,94–97]. In the last of these references, the corresponding results were obtained in analytic form. It is quite remarkable that, in accordance with the data of [94], the diffraction losses of the lowest-order modes increase rapidly with increasing misalignment angle. As a result, the plots of the losses of the different modes against the misalignment angle intersect.

A similar intersection of the curves was observed also in [98] in calculations (by an iteration method) for a planar resonator with large irregular aberrations, due to thermal deformations of the active element in imperfect illuminating systems. This fact shows that in the case of resonators with large diffraction losses attempts to classify modes by the size of their losses can lead to misunderstandings (see, for example, [171,159,172]).

Approximately the same volume of calculations as for systems with flat nonparallel mirrors has been performed for resonators with small second-order aberrations (convex and concave mirrors [95,94,88] etc.). In the case of a two-dimensional resonator we have

$$F(x) - 1 \approx i \cdot \frac{2\pi}{\lambda} \cdot 2h\left(\frac{x}{a}\right)^2,$$

where h is the difference of the distances between the mirrors on the edge and at the center of the resonator (Figs. 3b and 3c). The perturbation is symmetrical, $u'_1 \approx u_1 - 0.6(h/\lambda)Nu_3$.[86]

When $h < 0$ (concave mirrors) the field is naturally concentrated near the resonator axis ($a_{13} > 0$, Fig. 3b). The diffraction losses then decrease.[94,95] At deflections $|h| \gtrsim \lambda/\pi^2 N$ [18] one can determine the distribution of the field in the fundamental mode by using the results of the theory of generalized confocal resonators.

When $h > 0$ (convex mirrors) the field distribution over the resonator cross section becomes more uniform (Fig. 3c). With increasing curvature of the mirrors one even observes a tendency to the formation of a "dip" of the intensity on the resonator axis.[80] The diffraction losses increase sharply.* We shall discuss in greater detail the properties of resonators with convex mirrors in Sec. II.2.

The case of a planar resonator with randomly distributed inhomogeneities, which is very important for solid-state lasers, was considered in a number of papers. Attempts to represent the light scattering simply as diffusion of the generation radiation in a direction perpendicular to the resonator axis[100] are hardly justified, since the modes in the lasers are determined principally by interference effects, which are not taken into account in the diffusion equation.

M. S. Soskin and V. I. Kravchenko[101,71] succeeded in explaining a number of phenomena observed near the generation threshold by using a model of a resonator with "stepwise" mirrors. According to this model, the field is concentrated on individual small sections of the resonator cross section, between which there is a weak diffraction coupling. However, at a large excess above the generation threshold, the field intensity distribution inevitably becomes more uniform (owing to the nonlinearity of the active medium). The experimental data of [33,34,37] also indicate that in the case of intense pumping, the radiation of the individual oscillation modes is distributed practically uniformly over the entire cross section of the resonator (a negligible fraction of the total radiation flux is concentrated in the "filaments" that are sometimes observed). Therefore the indicated model is hardly suitable for an estimate of the angular divergence of the radiation in the usual generation regime.

V. V. Lyubimov[102] proposed apparently the most successful method of taking into account the influence of the randomly located inhomogeneities. His method is based on an estimate of the spectral width of the region in which locking of a group of modes of an ideal resonator is possible. This width turned out to be proportional to $\rho_{\text{scat}}^{1/2}$, where ρ_{scat} is the loss to light scattering per single pass of light through the sample.

The frequency locking results in the complicated complexes that are the oscillation modes of the real system. The angular aperture θ of the radiation of such complexes can be estimated from the formula $\theta = \rho_{\text{scat}}^{1/4} \times \sqrt{\lambda/L_{\text{eq}}}$,[103] which is in satisfactory agreement with the experimental data.[45,103]

If the source of the light scattering is random variations of the optical length of the resonator (for example, due to inaccuracies in the manufacture of the mirrors), then the angular aperture of individual oscillation modes can be estimated from the formula $\theta \approx \sqrt{\Delta L/L_{\text{eq}}}$, where ΔL is the rms deviation of the length.[102] Both this and the preceding formula are applicable only if the angular divergence of the radiation greatly exceeds the diffraction limit.

In accordance with the concepts developed in [102,103], rings in the angular distributions are due to the interaction between the low modes and the modes that differ both in their transverse and axial indices, but have close frequencies.

With this, we can conclude the analysis of the influence of stationary phase perturbations on the properties of empty resonators. We proceed now to an exposition of the results of investigations in which account is also taken of the nonuniformity of the distribution of the gain over the resonator cross section.

The first calculations of this type were performed by T. I. Kuznetsova;[104] the distribution of the radiation was obtained by solving the differential equations for the electromagnetic field at a complex value of the dielectric constant of the medium. It was shown that in the case of nonuniform inversion there occurs a diffraction "feeding" of zones with small inversion (or with large losses) by radiation from the region in which the gain prevails over the losses. As a result, the equal-phase surface ceases to coincide with the surface of the mirrors, and the front of the radiation emerging from the resonator may differ strongly from a plane front even for the lowest mode.

The concrete calculations were performed by T. I. Kuznetsova for the case of infinite flat mirrors, when the field is localized in a finite region only because of the presence there of a jump of the inverted population. For solid-state lasers, such a model can be used primarily near the generation threshold. It was possible to explain with the aid of this model,[106] in particular, the singularities, described in [63,105], of the energy characteristics of lasers with nonuniform pumping.

The deformations of the lower modes of a two-dimensional open resonator with flat mirrors, due to the nonuniform inversion, were calculated for individual particular cases, using numerical methods, by Tang and Statz[89,93] and also by Li and Skinner.[90] It was observed that these deformations reduce principally to changes not of the amplitude but of the phase of the field.

In addition to nonuniform distribution of the gain, in [93] account was taken also of the presence of anomalous dispersion of the active medium. As a result of its influence, the form of the field distribution becomes dependent on the location of the generation frequency relative to the frequency corresponding to the center of the line of the working transition; at lower frequencies, the field "contracts" to the resonator axis, and at higher frequencies its distribution becomes somewhat broader.

An examination of the effects connected with the nonuniformity of the distribution of the inverted population is also the subject of [107], but the results there are in error (the complex character of the field amplitude was not taken into account in the derivation of the initial equations).

The mechanism of the influence of the amplitude aberrations as well as of the phase aberrations can be understood with the aid of perturbation theory. The quantity $F - 1$, which determines the matrix elements of the perturbation operator, is equal in the general case to

*In [99] a unique model was used to perform a number of calculations for a planar resonator with aberrations. For the case of convex mirrors, the paradoxical conclusion that there are no losses for certain modes was reached. Therefore the very possibility of using such a model requires apparently a more thorough analysis than in [99].

$$e^{k'l}(1-\rho)e^{i\left(\eta k'l+\frac{4\pi}{\lambda}\Delta L\right)}-1\approx k'l-\rho+i\left(\eta k'l+\frac{4\pi}{\lambda}\Delta L\right),$$

where ρ is the summary loss (including the loss for transmission of the mirrors), ΔL the variation of the optical length of the resonator due to stationary phase distortions, and $\eta k'l$ the phase distortions due to the presence of anomalous dispersion of the active medium.* In the case of uniform Lorentz broadening $\eta = 2(\nu - \nu_0)/\Delta\nu_l$, where ν and ν_0 are the working frequency and the frequency at the center of the luminescence line, respectively, and $\Delta\nu_l$ is the half-width of the luminescence line (see, for example, [107]).

If there are no phase aberrations and the only source of the perturbation is the nonuniformity of the distribution of k' and ρ over the resonator cross section ($\eta = 0$), then the matrix elements of the perturbation operator are real. The coefficients a_{mk} turn out to be, like $\alpha_m - \alpha_k$, pure imaginary quantities (see (6)). It is precisely for this reason that the amplitude aberrations lead primarily to phase distortions of the eigenfunctions.

Within the framework of the first approximation of perturbation theory, it is also easy to find self-consistent solutions for the single-mode generation regime.[86] The reason for the nonuniformity of the distribution of the inverted population is in this case the very presence of the inhomogeneous generation field. The results agree sufficiently well with data of exact numerical calculations (Fig. 4).

The corresponding analysis shows that small phase aberrations of the negative-lens type (convex mirrors) contribute to attainment of single-mode generation.[86] Questions concerning the number of modes present in the generation will be considered in greater detail in the next section.

Summarizing the materials pertaining to the influence of aberrations on the structure of the fields of the individual oscillation modes, we must emphasize that the quantity of published information is quite large; we have presented here only a tentative systematization of this information.

4. Competition of Transverse Modes of Oscillation

The problem of multimode generation arose initially in connection with the difficulty of interpreting the experimental data concerning the spectral composition of the radiation: according to the notions of the elementary theory, the generation should occur at a single frequency close to the center of the line of the working transition; actually this is far from being the case.

T. I. Kuznetsova and S. G. Rautian[110] have shown that in spite of the predictions of the elementary theory, the single-mode generation regime is unstable. On the basis of similar representations, Tang and Statz[111,112] succeeded in finding the stationary distribution of the intensity of radiation of generation between individual oscillation modes for the case of a resonator with flat mirrors. The model was based on allowance for the saturation of the inverted population and for the different spatial distributions of the fields of the individual modes. The interaction of the radiation and the matter was described with the aid of an equation of the balance type; the broadening of the working-transition line was assumed to be homogeneous, and the excess above generation threshold to be small.

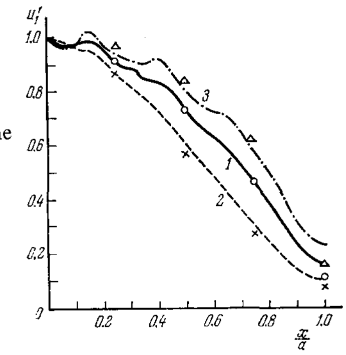

FIG. 4. Distribution of the field of the lowest mode in a two-dimensional resonator with allowance for the saturation of the gain and the anomalous dispersion of the active medium. N = 10, ρ = 0.1, twofold excess over the generation threshold. Points—calculation by perturbation theory, solid curves—exact numerical calculation [93]. 1—η = 0, 2—η = −1, 3—η = +1.

In spite of the limited applicability of the obtained relations, the results of Tang and Statz were of great importance for the correct understanding of the mechanism of multimode generation. The validity of the main notion, namely that the differences of the spatial distributions of the individual modes are important, was confirmed by results of a number of investigations of the spectral characteristics of lasers.[113-115,37]

To be sure, the establishment of the stationary equilibrium distribution of intensity among the modes, which was predicted by the calculations, was never observed in practice (owing to the "spike" character of the kinetic generation regime). Nonetheless, following the publication of [110-112], articles in which the selective capabilities of the resonator were estimated only from the point of view of the magnitudes of the diffraction losses, without allowance for the features of the spatial structure of the field (for example, [16]), hardly ever appeared.

It was also shown in [112] that in the case of a planar resonator, at least at slight excess over threshold, the calculations of the spectral and angular characteristics can be carried out independently (a similar result for large excesses above the generation threshold was obtained in [92]). This has made it possible to limit subsequent work to an analysis of papers devoted to the competition of modes with different transverse indices.

Worthy of interest among such papers is the already mentioned article by Fox and Li.[91] The traditional iteration method was used to investigate the features of generation for both flat and concave mirrors with round apertures. The calculations took into account the presence of a nonlinear active medium and accordingly the nonuniform distribution of the gain, due to the nonuniformity of the field of the generated radiation. The greater part of the calculations were performed for the case of relatively small mirror dimensions (N = 5).

In the case of concave mirrors, in full accord with the experimental data (see Sec. I.1), with increasing generation threshold one observes the process of "crowding out" of the lower modes by modes with higher transverse indices. The same result was obtained with the aid of analogous calculations in [117,24].

*Generally speaking, the refractive index depends on the degree of excitation of the active medium not only because of the anomalous dispersion. In particular, for neodymium glass it is more important that the atoms have different polarizabilities in the ground and excited states [118]; the same effect for the case of ruby is described in [109].

It is explained by the fact that in the case of concave mirrors, the radiation of the higher-order modes is distributed in a larger volume than the radiation of the lower modes. Owing to the resultant nonuniformity of the distribution of the inverted population, the average gain for the lower mode turns out to be much smaller.

The situation is different in lasers with flat mirrors. The volumes of the modes with different transverse indices are approximately the same, and in the case of uniform pump distribution the lower mode cannot be "crowded out."

This is how it turned out in the paper of Fox and Li. Moreover, even for a 20-fold excess over the threshold, the calculations predicted generation at only one lower mode. The authors considered this fact to contradict the model of Tang and Statz, but in [92] the latter was extended to the case of a large excess above threshold and it was shown that the number of modes is determined not so much by the pump intensity as much by the ratio of the diffraction losses to the nonselective losses (i.e., those common to all modes). In the calculations of Fox and Li many modes could not appear apparently because of the too low value of the nonselective losses, which was entirely untypical of solid-state lasers, namely 1% (usually the nonselective losses in solid state lasers amount to more than 10%).

The competition of transverse modes was considered in [92] under conditions when the pump not only has arbitrary intensity but, like the loss sources, is nonuniformly distributed over the cross section of the resonator. It was shown that the number of modes q' in a two-dimensional resonator, and consequently the magnitude of the angular divergence of the radiation $\theta \approx \lambda q'/2a$, saturate very rapidly with increasing pump intensity above threshold ($q' \sim [(\kappa - 1)/\kappa]^{1/3}$, where κ is a parameter equal, in the case of a four-level medium, to the ratio of the pump intensity to the threshold value). If a sufficiently intense pump is uniformly distributed and $q' \gtrsim 3$, the following simple formulas hold:

$$q' \approx 1{,}5\sqrt[3]{\rho}\,\frac{a}{\sqrt{\lambda L_{\mathrm{eq}}}}, \quad \theta \approx 0{,}7\rho^{1/3}\sqrt{\frac{\lambda}{L_{\mathrm{eq}}}}. \tag{8}$$

We see from them that inasmuch as $q' \sim a$, the angular divergence of the radiation does not decrease with increasing cross section of the active elements, and moves farther and farther away from the diffraction limit. The same regularities should also hold in a three-dimensional resonator,[92] as was confirmed by direct experiment for the case of a neodymium-glass laser.[45]

The data given in [92] also make it possible to draw a number of conclusions concerning the operation of a laser with nonuniform distribution of the pump intensity I and of the losses ρ over the cross section of the resonator. If I/ρ can be approximately represented in the form of the sum of the intensities of the lower modes, then the generation will be realized just at these modes. In this case both the mode deformation and the summary angular divergence are small. A simple analysis shows that such a situation should be observed most frequently if I/ρ decreases smoothly from the center towards the edge of the resonator; the intensity of the lowest mode increases noticeably compared with the case of uniform pump distribution.*

When the ratio I/ρ is minimal at the center of the resonator, this should lead as a rule to a large angular divergence, not only because of the presence of many modes in the generation, but also because of the considerable mode deformation. Finally, in the case of a sharply asymmetrical distribution of the pump, the angular divergence of the radiation should be especially large.

All the foregoing regularities are actually observed in laser investigations if measures are taken to reduce the phase aberrations of the resonator to a minimum. Thus, in [73] a large concentration of the pump on the axis of a cylindrical rod caused a decrease of the angular divergence of the radiation. Interesting results were also obtained by comparing the cases of symmetrical and asymmetrical distribution of the pump over the cross section of the resonator;[72] they can serve as a good illustration of the presented considerations (Fig. 5). In particular, the sharp asymmetry of the angular distribution in the case of asymmetrical pump distribution offers evidence of large mode deformation, since the modes of the ideal resonator have a symmetrical far-field picture.

Summarizing the materials concerning multimode generation of solid-state lasers, it should be noted that most of the calculations[111,112,74,92,24] were performed in an approximation in which the intensities of the modes present in the generation are additive, and not their amplitudes.† The interference terms are not considered, for even in the case of two modes their consistent calculation is quite complicated (for example, [120]). For a number of reasons it can be assumed that the use of such a very simple approximation is reasonable.

Indeed, the most characteristic generation regime of solid-state lasers is the regime of irregular spikes of radiation. It is known that the energy characteristics of the generator, when averaged over a sufficiently large number of spikes, correspond to the data of the probabilistic calculations in the stationary approximation. The same considerations can also be advanced in favor of the validity of the discussed model, if there is no additional mechanism that makes simultaneous generation on many modes energetically "convenient."

The presence of such a mechanism leads, as a rule, to the phenomenon of the so-called mode locking. Mode locking is most frequently observed in monopulse lasers with passive shutters: the smallest shutter losses are obtained in the case when the radiation is concentrated in a spatially short train with large energy density (see, for example [121]). The length of the train, in turn, is inversely proportional to the number of locked modes, as a result of which simultaneous generation on many modes turns out to be energetically convenient. Natu-

*In real lasers such effects can appear only in the absence of thermal deformations.

†According to the explanation of [118], the unclearly described calculations of A. M. Ratner [119] were made in the same approximation. Since Ratner calculated only the summary intensity of the fields, whereas an evaluation of the divergence calls for knowledge of the distribution of this intensity among the individual modes, we shall not stop to discuss the results of [118,119] in detail.

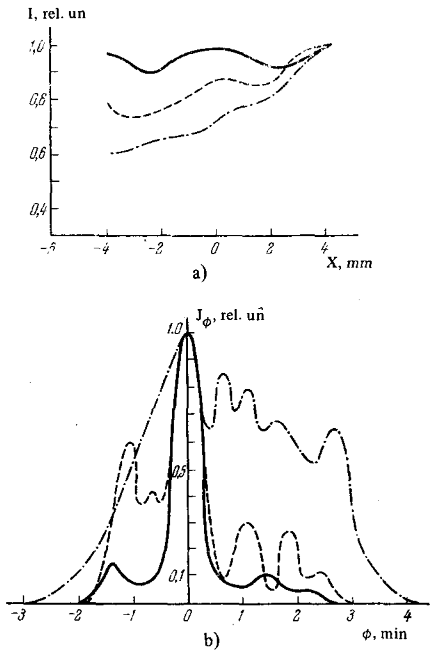

FIG. 5. Angular distribution of laser radiation in the case of pumping that is not uniform in one of the directions, in accordance with the pump distribution of [72]. a) Pump distribution (thickness of active element 8 mm), b) angular distribution of the radiation.

rally, one cannot obtain estimates for such a regime by using the simplest balance equations.

In the case of solid active media one also observes in individual spikes simultaneous generation on several transverse modes (predominantly in the case of a ruby laser and quite rarely for neodymium-glass lasers[122]). The possibility of locking of transverse modes was discussed theoretically in [123]. Finally, the phenomenon of transverse-mode locking was indeed observed recently in investigations of lasers with high uniformity of the pump-radiation distribution.[124] In spite of this, the locking of transverse modes is not as clearly favored energywise as in the example discussed above, and cannot change significantly the number of modes present in the generation.

By virtue of this, a multimode approximation of the probability theory is convenient for an estimate of the angular characteristics of the radiation averaged over a large number of generation spikes. The influence of the singularities of the kinetics and of the inevitable aberrations can only increase the angular divergence compared with the value predicted by (8) (as was indeed observed for large cross sections of the active element [72]). It is clear that in the case of a large number N of Fresnel zones special measures are necessary in order to bring the width of the directivity pattern close to the diffraction limit. Part II of the paper is devoted to an examination of these measures.

II. METHODS OF ANGULAR SELECTION OF RADIATION

1. Angular Selection in Lasers with Planar Resonators

Various procedures, unified under the common name of angular selection, are used in order to reduce the angular divergence of the radiation. In evaluating them, the following must be borne in mind.

Angular selection in the general case causes a decrease of both the number of non-synchronized modes present in the generation and of their deformations. In the case of an ideal active medium, the mode deformations are small,[92] and the number of modes depends principally on the ratio of the diffraction to the non-selective losses. Therefore for angular selection with an ideal medium it is necessary to increase the differences of the diffraction losses.

In the presence of aberrations of any type, to decrease the mode deformations, in accordance with Sec. I.3, it is necessary to increase the differences of the eigenvalues of the operator $\hat{P}$ (see (6)), including the phase corrections.

Having made this general remark, let us proceed to consider individual methods of angular selection.

The simplest and most natural method of angular selection is to decrease N by increasing the resonator length[10,125-128,45] (such measures as the use of diaphragms of small cross section greatly reduce the efficiency of the laser and will not be considered here). This increases both the diffraction losses and the phase corrections. As a result, if the aberrations are small, the angular divergence of the radiation decreases like $L_{eq}^{-1/2}$ up to the diffraction limit without an appreciable decrease of the generation power.[45] In the case of large resonator aberrations, the "forced" decrease of the mode deformations is connected with a sharp increase of the diffraction losses; as a result, it is impossible to reach the diffraction angle without loss of generation power.[127] This is precisely why the axial strength of the light passes through a maximum with increasing resonator length long before the angular divergence of the radiation reaches the diffraction limit.[127] With increasing cross section of the active element, the required distance increases rapidly ($\sim a^2$), and its magnitude becomes unreasonable already at $2a > 1$ cm. In this case the desired effect can be reached by introducing into the resonator with small L special additional elements called angular selectors. Historically, the first type of angular selector was a system used in the case of a planar resonator and consisting of two confocal lenses on a small-aperture diaphragm placed in their common focus.[129-131] A planar resonator with such a selector (Fig. 6a) is perfectly identical to a concentric resonator with a diaphragm in the central plane [132,66] (Fig. 6b). In [133], the role of the diaphragm was played by a passive shutter.

The action of a selector based on a Fabry-Perot interferometer[134,135] (Fig. 6c) is based on the fact that the transmission of the interferometer depends not only on the wavelength but also on the radiation propagation direction. Since this dependence becomes stronger for oblique incidence of the beam, the interferometer is mounted at a certain angle to the resonator axis. To realize angular selection in both directions it is necessary to use two interferometers.

Perhaps the most widely used selection method is based on the use of the dependence of the reflection coefficient at the boundary of two media on the angle of incidence near the critical angle of total internal reflection.[136] To intensify the selection, it is possible

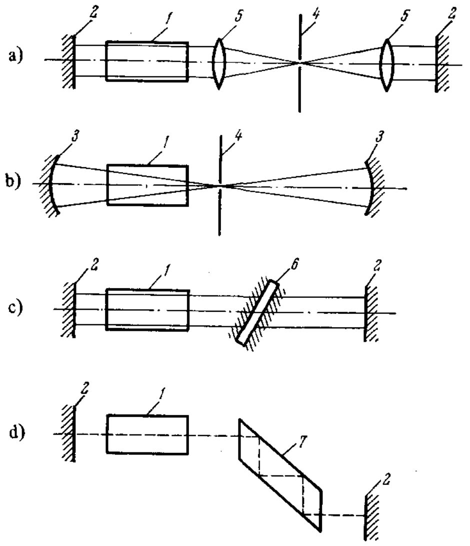

FIG. 6. Diagrams of angular selectors. 1—Active sample, 2—flat mirror, 3—spherical mirror, 4—diaphragm with aperture, 5—lens, 6—Fabry-Perot interferometer, 7—plane-parallel plate.

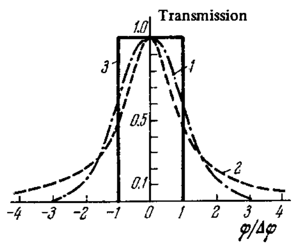

FIG. 7. Form of the transmission bands of different angular selectors. 1—Gaussian selector, 2—selector based on Fabry-Perot interferometer, 3—"ideal" selector.

to make the light experience numerous reflections (see Fig. 6d). Various modifications of selectors of this type were proposed ([137–143] and others).

Let us examine the mechanism of the influence of the selectors on the angular divergence. It is obvious from quite general considerations that the presence of a selector whose transmission depends on the direction of propagation of the radiation affects primarily the magnitude of the losses. The phase corrections are determined by the phase velocity possessed by the wave front with the corresponding structure, and should change insignificantly in the presence of the selector. The results of rigorous calculations[144] confirm this conclusion.

We present data for the idealized case of a Gaussian selector whose transmission-band shape is intermediate between those of the real selectors shown in Fig. 6, and described by the formula $g^2(\varphi) = \exp[-(\varphi/\Delta\varphi)^2]$ (Fig. 7; see [144]), where φ is the angle between the direction of propagation of the radiation and the resonator axis, $\Delta\varphi$ the bandwidth, and g^2 the intensity transmission. If it is recognized that transverse modes with index m correspond to values $\varphi = \pm m\theta_{\text{dif}}/2$ ([8]; $\theta_{\text{dif}} = \lambda/2a$), we get from this directly the value of the loss introduced by the selector $\Delta(4\pi p''_m) = m^2(\theta_{\text{dif}}/2\Delta\varphi)^2$ (the same result was obtained in [144] by a more rigorous method).

We now trace the manner in which the angular divergence of the radiation should change with the bandwidth of the selector.

In the absence of aberrations, the role of the selector reduces to a change of the conditions for mode competition (Sec. I.4) by increasing the loss differences. The losses introduced by a Gaussian selector are larger than the diffraction losses in an ideal empty resonator at $2\Delta\varphi/\theta_{\text{dif}} < (a^2/\lambda L_{\text{eq}})^{3/4}$. Since $a^2/\lambda L_{\text{eq}} = N$ is usually large for solid-state lasers, the angular divergence can decrease strongly even at a relatively large selector bandwidth. An estimate shows that to reach the single-mode regime with an ideal active medium it suffices to use a selector with $\Delta\varphi$ larger than $\theta_{\text{dif}}/2$ by several times.

Such a possibility is based on the fact that the losses introduced by this type of selector are noticeable already at $\varphi \ll \Delta\varphi$ (see Fig. 7). From among the real selectors, such a (quadratic) form of the $g(\varphi)$ dependence at small φ is possessed only by the selector based on the Fabry-Perot interferometer (Fig. 6c). Selectors with apertures (Figs. 6a and 6b) and those using total internal reflection (Fig. 6d) have transmission bands with a form close to rectangular ("ideal" selectors [144,145]). In this case, as expected, the presence of a selector influences the losses noticeably only for those modes whose indices are close to the value $2\Delta\varphi/\theta_{\text{dif}}$,[144] and therefore selectors of these types can decrease the angular divergence only to a value approximately equal to $\Delta\varphi$.

In the case of large aberrations, the main function of the selector should be to decrease the deformation of the modes with the highest Q. We recall that the mode deformations themselves can be treated as a result of the presence of induced oscillations in other modes (see Sec. I.3, and also [120]). Starting from this, it is easy to verify that to decrease the mode deformations, the loss introduced by the selector should turn out to be larger in magnitude not only than the diffraction losses, but also than the phase corrections in the empty resonator. It follows therefore that in the case of noticeable aberrations only selectors with relatively small bandwidths can be useful (regardless of the shape of the band).

When selectors are used, just as in the case of angular selection by increasing the resonator length, interest attaches not only to the angular but also to the energy characteristics of the radiation. The possibility of decreasing the width of the directivity pattern without an appreciable loss of radiation power is connected with the magnitude and character of the aberration. This connection is most clearly manifest when the form of the scattering indicatrix is considered.

Figure 8 shows schematically two directivity patterns of a light beam after a single pass through an active sample (the wave front is plane on entering the sample). These diagrams characterize directly the angular divergence of the radiation on leaving the laser system, using the given sample as the final amplifier (provided, of course, that the beam is well collimated at its input).

The first of them (Fig. 8a) pertains to the case of weak light scattering by microinhomogeneities. The greater part of the energy of the light passing through the sample (of the order of $1 - \rho_{\text{scat}}$; we neglect the presence of secondary diffraction maxima) is concentrated in the central core having the diffraction width.

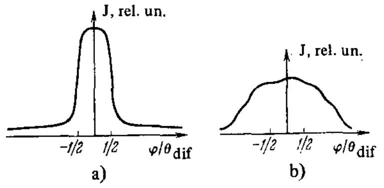

FIG. 8. Directivity pattern of radiation for a single pass through the active sample. a) Light scattering by microinhomogeneities, b) presence of macroinhomogeneities.

The remaining part of the radiation is distributed in a relatively wide range of angles.

Figure 8b corresponds to the presence of wave aberrations of low order (light scattering by macroinhomogeneities). What is broadened principally is the central core of the diagram itself, and the axial strength of the light decreases appreciably. It follows from the Rayleigh criterion, incidentally, that the width of the core begins to exceed the diffraction limit noticeably when the aberrations exceed $\lambda/4$.

It is obvious that even for equally large initial values of the angular divergence of the laser, attempts to decrease it will lead to essentially different results in the cases of Fig. 8a and 8b. The light scattering by microinhomogeneities does not prevent the diffraction limit from being reached; the power of the generated radiation decreases negligibly in this case (the effective losses increase approximately by ρ_{scat}[45]). In the case of Fig. 8b, an attempt to narrow down the angular divergence to a value smaller than the width of the central core of the scattering indicatrix entails inevitable and considerable energy losses.

With this, we can conclude the analysis of the mechanism of the influence of angular selectors on laser characteristics. When all the foregoing devices are used, as well as some others (for example [146]), a decrease of the angular divergence and an increase of the radiation brightness are observed. Nevertheless, angular selectors are not widely used, owing to the stringent requirements with respect to manufacturing and adjustment accuracy, as well as many concrete shortcomings inherent in each type of selector.

The main shortcoming of a system with a diaphragm (Fig. 6a or 6b) is the undesirable concentration of the radiation in a small part of the cross section. Even in lasers of relatively low power this leads to destruction of the diaphragm or to electric breakdown near its surface.[66] When a Fabry-Perot interferometer is used, it is necessary to cope with the presence of a large number of transmission maxima. Finally, selectors based on total internal reflection are especially complicated to construct and adjust.

All this led to numerous and repeated attempts to attain high directivity of radiation with the aid of simpler means. Such attempts led at best to a slight increase of the brightness, and were of no great significance.

Thus, in [147] the diffraction losses were increased by providing the output mirror of a planar resonator with a semitransparent reflecting coating only on a section having an area smaller than the cross section of the active rod. The same device with slight modifications was used in [148]. This selection method found its logical culmination in a laser described by Yu. A. Kalinin, A. A. Mak, et al., in which the resonator mirrors had variable transmission over the cross section.[149] In all cases the degree of directivity was increased and the misalignment sensitivity of the mirrors was decreased, but in the case of intense pumping the angular divergence exceeded the diffraction limit appreciably.

We note that the generators with mirror transmissions that are variable over the cross section are a clear-cut example of systems in which the mode structure depends extremely strongly on the excitation conditions. If the pump is uniformly distributed and the generation threshold is slightly exceeded, the field configuration of the individual modes is close to the configuration predicted by the theory of corresponding empty resonators.[150] If the excess above threshold is large, then as a result of mode competition the distribution of the gain over the resonator cross section approaches the distribution of the losses, and the structure of the individual modes becomes similar to the structure in lasers with ordinary flat mirrors. To estimate the angular divergence at a large excess above the generation threshold, and for an ideally active medium, it is possible to use the results of [92].

In addition to the already mentioned papers, many articles have been devoted to angular selection but contain little useful information. These include, for example, the series of papers by N. E. Korneev et al.[151-156] They use resonators of a great variety of types: convex-concave with a geometry close to semi-concentric,[151] confocal,[153,155] with convex mirrors,[152,154] and finally a system equivalent to a planar composite resonator.[156] Judging from the presented concrete data, the angular divergence turned out to be approximately the same in all cases and exceeded the diffraction limit by 5–10 times. This contradicts the statement made by the authors that the diffraction limit had been reached, and the treatment of the results of the papers does not correspond to modern theoretical notions.

In conclusion, mention should be made of one more method of constructing highly efficient laser systems with small angular divergences. It is universal and consists of using a driver generator and a number of stages of succeeding amplification. The driver generator can be of low power and have low efficiency, so that it is easy to obtain single-mode operation under these conditions. The generator radiation is usually fed to a telescopic system, which increases the diameter of the light beam, and then to amplification stages which are in turn separated by telescopic systems (Fig. 9).

Since small aberrations lead in light amplifiers to much smaller distortions of the wave front than in generators with flat mirrors (Sec. 1.3), such systems indeed ensure a minimum angular divergence of the radiation, but they are very complicated. It has been shown recently that similar output characteristics can be obtained from simpler generators with "unstable" resonators. The next section is devoted to their discussion.

2. Lasers With "Unstable" Resonators

In 1962 Boyd and Kogelnik[157] compiled a classification of open resonators made up of two spherical mir-

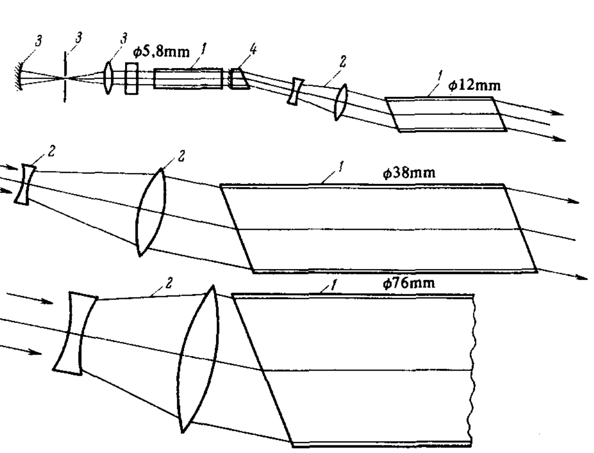

FIG. 9. Example of a multistage laser [66]. 1—Active rod, 2—telescopic system, 3—selector unit with diaphragm and total-reflection mirror, 4—semitransparent mirror on a Brewster-angle prism, 5—shutter.

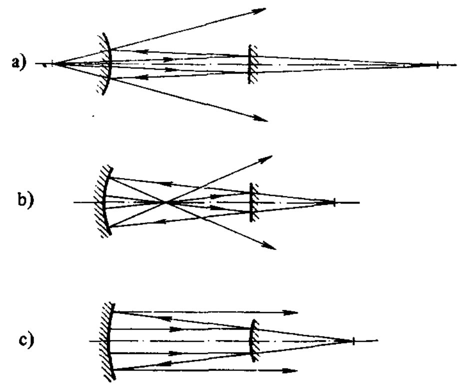

FIG. 10. Different types of unstable resonators.

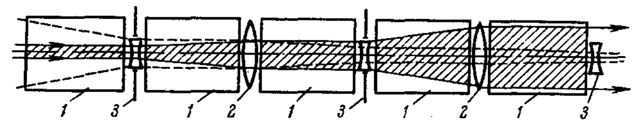

FIG. 11. Equivalent diagram of a laser with a telescopic resonator. 1—Active sample, 2—gathering lens equivalent to a concave mirror, 3—scattering lens equivalent to a convex mirror.

rors with arbitrary curvature radii R_1 and R_2. They showed that in the case when the product $(1 - L/R_1) \times (1 - L/R_2)$ becomes smaller than zero or larger than unity (L is the distance between the mirrors, and the radii of convex mirrors are assumed to be negative), the diffraction losses increase exceedingly sharply. Since stable generation is impossible in the presence of large diffraction losses, resonators with such parameters were called unstable and for a long time were neglected by the specialists.

Interest in them was revived only after the publication in 1965 of a paper by A. Siegman.[158] He succeeded not only in correctly understanding certain features of unstable resonators, but also in arriving at the conclusion that their use may be promising. Nonetheless, as indicated by Siegman himself, the main problem—establishing the possibility of the selection of transverse modes—was not solved by him either theoretically or experimentally. Many important features of lasers with unstable resonators were revealed only by subsequent investigations. The advantages of such systems were completely realized relatively recently.[159,160]

We shall first discuss briefly the properties of empty unstable resonators: this question has not yet been discussed systematically in the literature.

A very interesting feature of unstable resonators is that their main properties can be described within the framework of the simplest geometrical approximation. In this approximation the fundamental mode of an ideal unstable resonator consists of spherical waves traveling in both directions and having virtual or real centers. The positions of these centers are conjugate with respect to each of the mirrors (Fig. 10). Satisfaction of this condition ensures reproducibility of the form of the wave front after its passage through the resonator.[158] Starting from this, it is easy to find with the aid of geometrical optics both the locations of the centers themselves and the coefficient of magnification M of the transverse dimensions of the spherical wave after passage through the resonator (see Fig. 10).

We note that the curvature of a "converging" wave having the opposite propagation direction (with the directions of the arrows in the figure reversed), is also reproducible, but the dimension of its cross section does not increase, but decreases by a factor M. Inasmuch as the cross section continues to decrease on further passing through the resonator, such a wave does not correspond to a steady-state solution. At the same time, the cross section of a wave with the ray travel as shown in the figure turns out to be limited because of the finite dimensions of the mirrors. Some of the radiation leaves the resonator, and this determines the large magnitude of the losses. In accordance with the fraction of the radiation remaining in the resonator, the loss is equal to $1 - 1/M$ in a two-dimensional resonator and $1 - 1/M^2$ in a three-dimensional one; it is determined only by the ratios R_1/L and R_2/L and does not depend on either the parameters of the mirrors or on which of them (or in part both) limits the cross section of the beam.

For the case of unstable resonators, the influence of the wave aberrations of lower order can frequently be estimated within the framework of the geometrical approximation and turns out to be small. Indeed, let us consider in accordance with [160] the passage of a light wave through the equivalent scheme of the laser with the so-called telescopic resonator proposed in [159] (Fig. 11). This resonator consists of confocal convex and concave mirrors (Fig. 10c); the generation radiation propagating towards the convex mirror is a parallel beam, thereby ensuring a number of practical advantages.

As seen from Fig. 11, the radiation filling the entire cross section of the resonator is first "spread" during several passes from the central part of the cross section. The dimension of this part decreases with increasing number of passes in geometric progression, and rapidly becomes small enough to permit regarding the steady-state front as plane within its limits.

If M and the number of the Fresnel zones N are sufficiently large, the process of "spreading" of the radiation is described by the geometrical approximation. We see that the influence of the aberrations accumulates essentially only within a small number of passes.* The expression for the steady-state field distribution can then be represented in the form[161]

$$u(r) = \prod_{m=0}^{\infty} \frac{\Phi\left(\frac{r}{M^m}\right)}{\Phi(0)}, \tag{9}$$

where r is the distance to the resonator axis, $\Phi(r)$ is a factor describing the influence of the amplitude and phase aberrations for passage of the wave through the resonator in both directions (this factor is similar to the factor F introduced in Sec. I.3, but in the general case it is not equal to it); in the absence of aberrations $\Phi(r) \equiv 1$.

It is easily seen that (9) is a solution of the equation[171,161]

$$\gamma u(r) = \frac{\Phi(r)}{M^k} u\left(\frac{r}{M}\right)$$

(For a two-dimensional resonator $k = \frac{1}{2}$, and for a three-dimensional one $k = 1$).

Estimates made with the aid of (9) show that the general deformations of the wave front naturally decrease with increasing M, and even at M = 2 they slightly exceed the magnitude of the aberrations per pass that are inevitable in any laser scheme.

These are the most important properties of unstable resonators that can be found with the aid of the geometrical approximation. In the diffraction approximation, the picture would be more complicated. Computer calculations have shown that an increase of the transverse dimensions of the mirrors is accompanied by a successive alternation of the modes having the highest Q;[159,162,172] the alternation occurs at mirror dimensions corresponding to integer values of the parameters $N_{eq} \equiv \Delta\varphi/\pi$, where $\Delta\varphi$ is the phase advance over the mirror for the geometrical-approximation wave (in the case of a symmetrical resonator using convex mirrors we have $N_{eq} = (N/2)(M - 1/M)$;[171,159-162] for a telescopic resonator $N_{eq} = 2h/\lambda$, where h is the height of the convex mirror); the modes at the points of alternation are doubly degenerate in the losses; finally, the distributions of the fields of the highest-Q modes differ somewhat from the predictions of the geometrical approximation.

Siegman and Miller,[172] analyzing these effects, reached the conclusion that it is advisable to use resonators with $N_{eq} = \frac{1}{2}$, corresponding to convex mirrors with heights $h < \lambda/4$.† Actually, however, resonators with small N_{eq} (and $M \approx 1$) are of no interest whatever, since the field distribution in them depends on the inhomogeneity of the medium almost as strongly as in the case of flat mirrors.

We have already seen that in the case of large N_{eq} (and M) the situation is different. In addition, in real systems of large size, the phenomena revealed by computer calculations should not occur: as shown in [161], their occurrence is connected only with the use of the little-justified assumption that the mirror edge is ideally sharp and exactly outlined. Even a slight "smoothing" or unevenness of the edge, such as almost always occurs, lifts the loss degeneracy of the modes, the eigenvalues approach the analogous quantities for a resonator with infinite mirrors,[175,176] and the field distribution of the fundamental mode begins to be splendidly described by the geometrical-approximation formula (9). To lift the degeneracy, it suffices, for example, for the reflection coefficient of the mirrors to decrease to zero not abruptly but over a zone of width $\delta \gtrsim a/2N_{eq}$.[161] In necessary cases (principally at relatively small N_{eq}) the "smoothing" can be realized purposely by such devices as cutting a chamfer, using a diaphragm with an uneven contour, etc.[6,161]

We note that the character of the edge of the mirrors is important because after a single passage through an unstable resonator, the external part of the beam, which is strongly perturbed by the diffraction, is taken to the outside, and far from the boundary of the geometrical shadow the influence of the diffraction is noticeable only in the case of an ideally sharp and exactly delineated edge.

It also follows from the results of [161] that resonators with a "smooth" edge should reliably ensure generation on one transverse mode even for an arbitrary but not too uneven distribution of the inverted population of the resonator cross section.

Thus, in spite of the notions of [172], it is precisely the unstable resonators with large N_{eq} and M which satisfy all the requirements that can be imposed on resonators for lasers with high spatial coherence of the radiation.* Their advantages becomes obvious if attention is called to the fact that a laser with a telescopic resonator corresponds to a system consisting of a driving generator and an amplifier with a matching telescope between them. The role of the generator is carried out in this case by the central zone of the sample cross section, and the role of the amplifier by the peripheral zone. Unlike the scheme shown in Fig. 9, the amplification of the beam occurs here also during the stage of its expansion, which replaces, as it were, the intermediate amplification stages.

Let us dwell now on questions concerning the choice of optimal parameters of unstable resonators.

We have seen that the "sensitivity" of the resonator to aberrations decreases with increasing M. But M cannot be chosen arbitrarily large, since the energy characteristics of the lasers depend on the radiation losses.

It was shown in [160] experimentally and in [163] by calculation that the energy characteristics of lasers with planar and telescopic resonators are approximately the same if the radiation losses for these two types of resonator are equal. In the case of a planar resonator, the magnitude of the radiation losses is determined by the transmission coefficient of the output mirror. In the case of unstable resonators and large-dimension active elements, both mirrors are usually made totally reflecting;[159,160,164] the radiation is extracted from the laser through an annular zone around the mirror bound-

*In this approximation, for the case of a planar resonator the distortions of the wave front would accumulate without limit, and it is impossible to determine the steady-state field distribution without allowance for the diffraction effects.

†Such systems belong in essence to the "transition region" recommended in [173,174].

*The foregoing considerations are fully confirmed by the results of the experimental paper [177], namely, generation at a single transverse mode with an angular divergence of the radiation ~2″ was obtained in the case of a neodymium-glass laser with $N_{eq} \approx 700$ and M = 2.

ing the cross section of the beam (Fig. 10, see also [158]); the radiation loss is equal to $1 - 1/M^2$.

It follows therefore that in the case of a telescopic resonator with total-reflecting mirrors, all the known formulas of the probability theory[30] remain in force, except that the reflection coefficient of the output mirror of the planar resonator R' is replaced by $1/M^2$. At the optimal value of M (which is equal to $1/\sqrt{R'_{opt}}$), the telescopic resonator results in approximately the same maximum efficiency as in a generator with flat mirrors. The use of unstable resonators of other types leads to a decrease of the efficiency, owing to the poor filling of the active element by the generation radiation (by a factor 1.5–2 when $M \gtrsim 2$[159,160,164]).

If the gain of the radiation after passing through the active element is small and the radiation loss cannot be made sufficiently large, then the cross section of the annular zone through which the radiation emerges becomes exceedingly small. This leads to a considerable increase of the angular divergence of the radiation even in the absence of aberrations.[165,159] In such cases it may therefore be more convenient to use a semitransparent mirror in conjunction with a still smaller M (so that the total value of the losses to radiation remains optimal). Incidentally, at small M a plano-convex system of mirrors (Fig. 10a) ensures almost the same output parameters of the lasers as a telescopic resonator. It is necessary only to compensate for sphericity of the wave emerging from the plano-convex resonator.[166,159]

The foregoing considerations allow us to conclude that the use of unstable resonators is particularly convenient if the active elements have large dimensions and the medium is sufficiently homogeneous. In this case replacement of the planar resonator by a telescopic one should cause a sharp narrowing of the directivity pattern without an appreciable lowering of the efficiency.

This conclusion was confirmed by results of investigations of a neodymium-glass laser.[159,160,164] For an active element of 45 mm diameter and 600 mm length, the angular divergence of the radiation was ~20″ (instead of the usual several minutes, although M was relatively small (~2). One additional condition, satisfaction of which turned out to be necessary to attain the maximum radiation brightness, was revealed: namely, there should be no sources of light scattering leading to the occurrence of a "converging" wave (see the commentaries to Fig. 10) even with negligibly small initial intensity.[160] In the case of a telescopic resonator the "converging" wave (shown dashed in Fig. 11) arises upon reflection of the "normal" wave from the flat separation surfaces perpendicular to the system axis, and therefore, in spite of the presence of antireflection coatings, the ends of the active rods should be inclined at an appreciable angle (2–3°).*

In conclusion, let us mention a certain properties of lasers with unstable resonators.

From the point of view of the spectral and temporal characteristics, this class of generators differs little from generators with flat mirrors. Only the radiation spikes have a shorter duration and the time interval between them increases somewhat. This is caused by the exceedingly rapid establishment of the oscillations over the entire cross section of the resonator, owing to the presence of a mechanism of forced "spreading" of the radiation ([166,159], see also [75]). In the monopulse regime, the same mechanism leads to a very appreciable change of the output parameters of the laser (thus, in [164] the laser pulse decreased from 40 to ~15 nsec on changing over to the unstable resonator). We note that Q switching can be realized with the aid of small-area shutters (relative to the dimension of the convex mirror).

The parameters of lasers with unstable resonators, as expected, depend little on the accuracy of the adjustment of the mirrors. In the case of small transverse displacements or rotations of the mirrors, there is only a certain change in the direction of the radiation; the curvature of the wave front varies with the distance between mirrors. The magnitudes of these effects correspond to the estimates carried out within the framework of geometrical optics.[159]*

A particularly important role in unstable resonators is played by the section adjacent to the axis, from which radiation "spreads out" over the entire cross section. The requirements imposed on the optical homogeneity of the medium and on the quality of the mirrors in this section are particularly stringent. At the same time, by introducing radiation from an external source into it, it is apparently possible to realize effective control of the radiation of the entire laser.[160]

With this, we conclude the examination of the properties of lasers with unstable resonators. The appearance of this class of generators has made it possible to solve one of the problems of quantum electronics, namely, the production of simple laser systems ensuring high brightness of radiation at relatively large values of a highly-homogeneous active medium. If the medium is strongly inhomogeneous, no tricks with resonators can greatly increase the directivity of the radiation without a loss of its power. Indeed, we have already mentioned (Sec. I.1) that in the case of a strongly inhomogeneous medium the angular divergence of the radiation θ in a laser with flat mirrors approaches the angular divergence θ_1 of a well-collimated beam after a single passage through the active sample. Unstable resonators can ensure in the best case (at $M \gg 1$) an angular divergence of radiation approaching θ_1, and therefore their use is advantageous only if $\theta \gg \theta_1$.

In order to obtain the minimal angular divergence with the aid of unstable resonators, it is necessary to impose definite requirements not only on the optical homogeneity of the medium, but also on the uniformity of its excitation. Thus, if the gain per pass of the resonator is less than M^2 near the system axis and is large on the periphery, the generator can still be above the self-excitation threshold, in spite of the predictions of the geometrical approximation. The steady-state field distribution will then be determined principally by diffraction effects (an analogous situation takes place in a laser with flat misaligned mirrors). The directivity

*In accordance with [178,161], it is precisely the "converging" wave resulting from the diffraction by the sharp edge which is responsible for the effects of mode degeneracy etc. observed in computer calculations [171,162,172].

*Results of similar observations for a CO_2 laser were recently published [167].

of the radiation in such cases becomes much worse.

Thus, to form powerful beams with high directivity of radiation in the general case of an inhomogeneous active medium it is necessary to have other devices, perhaps fundamentally new ones.

In conclusion the author is grateful to N. A. Sventsitskaya, V. E. Sherstobitov, and M. P. Vanyukov for a number of critical remarks.

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