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SUPERNOISELESS CYCLOTRON-WAVE AMPLIFIERS

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1. INTRODUCTION

INTEREST in instruments whose operation is based on the transverse (cyclotron) waves of an electron beam did not arise suddenly. Back in 1947, Smith and Shulman[1] calculated the energy exchange between the transverse field of a parallel-plate capacitor and an electron beam drifting in a longitudinal homogeneous magnetic field. Somewhat later (1949), Cuccia[2] proved experimentally the possibility of using planar devices for coupling with the transverse field in order to modulate and demodulate an electron beam.

The revival of parametric ideas in 1957–1959 suggested the application of the principle of parametric amplification in electronics. This principle is in itself "noiseless," since the energy is drawn in this case from the pump source, and not from the electron beam that is subject to noise from the cathode. In addition, parametric amplification is accompanied by acceleration of the electrons and therefore makes it possible to choose as the working wave the fast wave of the electron beam, from which the noise can be removed beforehand in a usual passive coupling device. These two basic premises gave grounds for hoping that extremely low intrinsic noise levels can be obtained in electron-beam parametric amplifiers (EPA).

The first attempts at constructing such instruments were connected with the use of the then well known longitudinal modulation of the electron beam, usually described with the aid of a fast (+) and slow (–) wave of space charge, the phase velocities of which are equal to

$$v_{\mathrm{ph}\pm} = v_0/[1 \mp \omega_q(\omega)\,\omega^{-1}], \qquad (1.1)$$

where v_0 is the average unperturbed velocity of the electron beam, $\omega_q(\omega)$ is the reduced plasma frequency, and ω is the signal frequency.

Several types of EPA for space-charge waves were proposed and investigated in detail (see, for example[3,11]). These differed in the construction of the individual elements, in the type of the interaction, in the method of introducing the signal, in the method of removing the noise, etc.

It turned out, however, that owing to the low dispersion of the phase velocities of the space-charge waves, combination frequencies of the type $|m\omega_p \pm n\omega|$ (ω_p—pump frequency, m, n—numbers of the natural series) play an important role in the amplification region. This causes a decrease of the gain and an appreciable increase in the noise coefficient of the instrument as a result of the transfer of the noise from these waves.

A low noise level in EPA with longitudinal modulation could not be obtained experimentally. This was apparently connected also with the fact that for the low-noise beams the phase velocities of the fast and slow waves of the space charge differ little in magnitude, making it very difficult to remove the noise from the fast wave beforehand by using a passive coupling device.

At the same time, attempts to construct EPA with transverse modulation of the electron beam were quite successful.

An electron beam drifting freely in a longitudinal electric field $H_0 = H_z$ is described in the transverse direction by two independent coordinates, and can therefore contain twice as many normal modes as in the longitudinal case. Two of them are fast, two are slow, two are cyclotron, and two are synchronous. The cyclotron waves describe the transverse electron velocities and the synchronous waves their transverse displacements. The phase velocities of the fast (+) and slow (–) cyclotron waves are determined by the expressions

$$v^{c}_{\mathrm{ph}\pm} = v_0/(1 \mp \omega_c\omega^{-1}), \qquad (1.2)$$

($\omega_c = |e|B_0/m$ is the cyclotron frequency) and can differ greatly from each other. The two synchronous waves have the same phase velocity, equal to the velocity v_0 of the longitudinal motion of the beam.

In the region of cyclotron resonance, the dispersion of the phase velocity of the fast cyclotron wave is maxima, and its magnitude

$$v^{c}_{\mathrm{ph}+}(\omega \to \omega_c) \to \infty, \qquad (1.3)$$

whereas for the slow and synchronous waves the phase velocities remain finite and relatively small in magnitude*:

$$v^{c}_{\mathrm{ph}-}(\omega \to \omega_c) \to v_0/2, \quad v^{c}_{\mathrm{ph}\pm} \equiv v_0. \qquad (1.4)$$

This makes it possible, on the one hand, to ensure selective interaction with only the fast cyclotron wave of the electron beam, and on the other hand to expect a weak coupling with the higher combination frequencies in the case of parametric amplification.

Although historically the interest in transverse waves of the electron beam arose in connection with the adoption of parametric ideas by electronics, and led to the appearance of supernoiseless EPA (with equivalent two-channel temperature of the intrinsic noise $< 100^\circ$K), by now there are many known parametric instruments operating on cyclotron or synchronous waves[11].

The most interesting among them is the electrostatic amplifier (diftron), which makes it possible to realize an equivalent single-intrinsic-noise temperature of $\sim 140^\circ$K and below[12,13].

Having rather low intrinsic-noise level, cyclotron-wave amplifiers compete successfully with such devices as supernoiseless traveling wave tubes, masers, and

*Since the potential of the beam does not exceed 10 – 15 V when ransverse waves are used.

solid-state parametric amplifiers, and are even superior to them in some respects (operating stability, high directivity, broad dynamic band, ability to withstand considerable electric overloads, freedom from the need of the use of cryogenics, etc.).

This paper supplements and expands the previously published review[14], pointing out at the same time the physically most important results obtained in recent years. For this reason, many problems discussed in sufficient detail in earlier works[10,11,14] are not treated here.

2. METHODS OF ANALYSIS OF TRANSVERSE OSCILLATIONS IN ELECTRON BEAMS

The analytic description of transverse oscillations of an electron beam drifting in a longitudinal homogeneous magnetic field B_0 is based usually on the simplest disc model of the electron beam[15].

We represent the beam as an aggregate of flat discs of infinitesimally small thickness, moving along the z axis with constant longitudinal velocity v_0. If the transverse dimensions of the disc and its displacements from the unperturbed position are small compared with the wavelength, then the coupling between the discs, due to the longitudinal space charge, is also insignificant and can be neglected. The problem reduces to an analysis of the transverse oscillations of the individual disc. Let the behavior of the individual (i-th) electron be characterized by a complex coordinate $\mathbf{r}_i = x_i + jy_i$, and let the coordinates of the mass center of the disc be $\mathbf{R} = N^{-1}\sum_{i=1}^{N}\mathbf{r}_i$, where N is the number of electrons in the disc. Then the orbits of the individual electrons relative to the mass center of the disc are described by the coordinates $\rho_i = \mathbf{r}_i - \mathbf{R}$ (Fig. 1).

The equation of the transverse motion in terms of the variables $\mathbf{r}_i$, $\mathbf{R}$, and ρ_i assume, in complex notation, the form

$$\frac{d^2\mathbf{r}_i}{dt^2} - j\omega_c\frac{d\mathbf{r}_i}{dt} = \frac{e^2}{m}\sum_{k=1}^{N}\frac{\mathbf{r}_i-\mathbf{r}_k}{|\mathbf{r}_i-\mathbf{r}_k|^3} + \frac{\mathbf{F}_i}{m}, \quad (2.1)$$

$$\frac{d^2\mathbf{R}}{dt^2} - j\omega_c\frac{d\mathbf{R}}{dt} = \frac{1}{mN}\sum_{i=1}^{N}\mathbf{F}_i, \quad (2.2)$$

$$\frac{d^2\rho_i}{dt^2} - j\omega_c\frac{d\rho_i}{dt} = \frac{e^2}{m}\sum_{k=1}^{N}\frac{\rho_i-\rho_k}{|\rho_i-\rho_k|^3} + \frac{1}{m}\mathbf{F}_i - \frac{1}{mN}\sum_{k=1}^{N}\mathbf{F}_k, \quad (2.3)$$

where $\omega_c = e'B_0$, $e' = |e|/m$ is the specific charge of the electron, and $\mathbf{F}_i$ is the external transverse force acting on the electron numbered i.

Principal interest attaches to the last two equations. The first characterizes the motion of the mass center of the cross section of the beam, and consequently, describes the behavior of the signal and of the noise of the beam, while the second is important in our investigations of the internal structure of the beam (expansion of thermal orbits, their balancing, etc).

In those cases when the function $\mathbf{F}_i$ depends linearly on the transverse coordinates and their derivatives, or else is entirely independent of them, the equation of motion for the mass center of the disc coincides with the equation of motion of one electron without allowance for the Coulomb interaction.

The solution of the problem with the Coulomb sum in Eqs. (2.1) and (2.3), entails considerable difficulties.

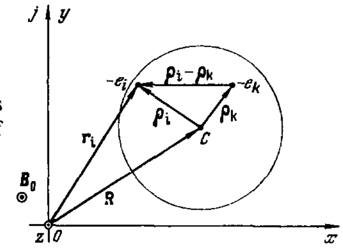

FIG. 1. Individual beam cross section (disc). C – mass center of the disc.

Appreciable simplification can be obtained for a model of a round uniformly charged cylindrical beam, if one uses the approximation

$$e^2m^{-1}\sum_{k=1}^{N}(\rho_i-\rho_k)|\rho_i-\rho_k|^{-3} \to -\omega_q^2\rho_i/2 \quad (2.4)$$

($\omega_q^2 = e^2N/m$ is the plasma frequency, which in a number of cases makes it possible to describe correctly the actual physical processes in the system[16])*.

The problem of transverse beam oscillations can be analyzed by directly solving Eqs. (2.2) and (2.3) (kinematic approach) or else by using information concerning the longitudinal structure of the beam (coupled-wave method). The former method is simpler, physically more lucid, and is preferred in the analysis of problems in the given-field approximation, while the latter is convenient in the calculation of self-consistent problems involving the energy exchange between the beam and lumped or distributed coupling devices.

2.1. Kinematic Analysis

For many problems of the theory of cyclotron-wave accelerators, the structure of the external forces $\mathbf{F}_i$ is such that Eqs. (2.2) and (2.3) with allowance for (2.4) can be reduced to standard form:

$$\ddot{\mathbf{R}} - j\dot{\mathbf{R}} = \varepsilon[\mathbf{f}(\mathbf{R}, \mathbf{R}^*, \theta) - j\Delta\dot{\mathbf{R}}], \quad (2.5)$$

$$\ddot{\rho}_i - j\dot{\rho}_i = \varepsilon[\psi(\rho_i, \rho_i^*, \theta) - j\Delta\dot{\rho}_i + q\rho_i], \quad (2.6)$$

where the superior dot denotes the total derivative with respect to the transit angle $\theta = \omega_{0e}(t - t_0)$, ω_{0e} is the frequency of the resonant harmonic or the subharmonic of the external force, $\Delta = (1 - \omega_c\omega_{0e}^{-1})/\epsilon$ is the relative difference between the natural frequency of the system ω_c and ω_{0e}, the value of the parameter ϵ is chosen such that it characterizes the intensity of the external forces, and $q = \omega_q^2/2\epsilon\omega_{0e}^2$ is the space-charge parameter.

Equation (2.6) has the same structure as (2.5), since the term $q\rho_i$ can always be introduced in the function $\psi(\rho_i, \rho_i^*, \theta)$.

To solve (2.5) it is advantageous to use the method of variation of the constants, i.e., to assume that, unlike in the unperturbed case (ϵ = 0), the synchronous (α) and cyclotron (β) radii depend on the transit angle θ, and the oscillations occur at the frequency of the resonant harmonic of the external force*, i.e., that

*For a more rigorous model, the transverse motion of the beam in a drift tube with infinitely conducting walls was analyzed by Carrol [16] by a numerical method.

*Principal interest usually attaches to the resonant part $|1 - \omega_c\omega_{0e}^{-1}| \ll 1$, where the oscillations of the system are best considered at the frequency of the resonant harmonic of the driving force.

$$\mathbf{R}(\theta)=\boldsymbol{\alpha}(\theta)+\boldsymbol{\beta}(\theta)e^{j\theta}. \quad (2.7)$$

The problem reduces to finding $\boldsymbol{\alpha}(\theta)$ and $\boldsymbol{\beta}(\theta)$. The introduction of three new unknown functions in place of one makes it possible to impose an additional connection between $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. The latter, in analogy with[17], is best chosen in the form

$$\dot{\boldsymbol{\alpha}}(\theta)+\dot{\boldsymbol{\beta}}(\theta)e^{j\theta}=0. \quad (2.8)$$

Then, substituting (2.7) in (2.5) and using (2.8), we get

$$\dot{\boldsymbol{\alpha}}=j\varepsilon\mathbf{f}(\boldsymbol{\alpha}+\boldsymbol{\beta}e^{j\theta},\ \boldsymbol{\alpha}^*+\boldsymbol{\beta}^*e^{-j\theta},\ \theta)+\varepsilon\Delta\boldsymbol{\beta}e^{j\theta}, \quad (2.9)$$

$$\dot{\boldsymbol{\beta}}=-j\varepsilon\mathbf{f}(\boldsymbol{\alpha}+\boldsymbol{\beta}e^{j\theta},\ \boldsymbol{\alpha}^*+\boldsymbol{\beta}^*e^{-j\theta},\ \theta)e^{-j\theta}-\varepsilon\Delta\boldsymbol{\beta}. \quad (2.10)$$

Although a certain simplification has been attained and the order of the initial equation (2.5) has been lowered, nevertheless the system (2.9) and (2.10) cannot be integrated in general form for an arbitrary function $\mathbf{f}(\mathbf{R}, \mathbf{R}^*, \theta)$. However, in two limiting cases it is usually possible to obtain an approximate analytic solution, at the expense of further simplifications.

A situation is possible in which the external forces act on the system only in a narrow region of values of θ (short electromagnetic lenses, $\epsilon \neq 0$, when $\theta \in \delta\theta$, $|\delta\theta| \ll 1$). This makes it possible to regard the oscillating factors $e^{\pm jn\theta}$ ($n = 1, 2, \ldots$) in (2.9) and (2.10) as constants during the course of integration, thus greatly simplifying the solution of the problem.

Finally, in supernoiseless amplifiers the character of the action of the field on the beam is most frequently adiabatic in practice, i.e., the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ change little under the influence of this field within the period of the fundamental motion. This is mathematically reflected in the smallness of the parameter ϵ. In the case of a resonant adiabatic field ($\epsilon \ll 1$, $\epsilon|\Delta| \ll 1$) it is possible to apply to the system (2.9) and (2.10) Kapitza's averaging procedure[17]. The approximate smoothed-out equations (2.9) and (2.10) take the form

$$\dot{\boldsymbol{\alpha}}=j\varepsilon\frac{1}{2\pi}\int_0^{2\pi}\mathbf{f}(\boldsymbol{\alpha}+\boldsymbol{\beta}e^{j\theta},\ \boldsymbol{\alpha}^*+\boldsymbol{\beta}^*e^{-j\theta},\ \theta)\,d\theta, \quad (2.9')$$

$$\dot{\boldsymbol{\beta}}=-j\varepsilon\frac{1}{2\pi}\int_0^{2\pi}\mathbf{f}(\boldsymbol{\alpha}+\boldsymbol{\beta}e^{j\theta},\ \boldsymbol{\alpha}^*+\boldsymbol{\beta}^*e^{-j\theta},\ \theta)e^{-j\theta}\,d\theta-j\Delta\boldsymbol{\beta}, \quad (2.10')$$

and the quantities $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in the right side are regarded as constants during the course of the averaging.

The solution obtained from the smoothed equations (2.9′) and (2.10′) takes into account only effects of first order of smallness in ϵ. In those cases when this does not suffice, it is necessary to apply to Eq. (2.5) the asymptotic methods of the theory of nonlinear oscillations[18], and a solution can be sought for the n-th approximation, unlike (2.7), in the form

$$\mathbf{R}(\theta)=\boldsymbol{\alpha}(\theta)+\boldsymbol{\beta}(\theta)e^{j\theta}+\sum_{k=1}^{n-1}\varepsilon^k\mathbf{u}_k(\boldsymbol{\alpha},\ \boldsymbol{\beta},\ \boldsymbol{\alpha}^*,\ \boldsymbol{\beta}^*,\ \theta), \quad (2.11)$$

where the functions $\mathbf{u}_1, \mathbf{u}_2, \ldots$ take into account all arbitrary harmonics in θ, with the exception of the zeroth and the first (the latter requirement is essentially a supplementary condition similar to (2.8)).

The abbreviated equations are sought in this case also in the form of an expansion in powers of the small parameter ϵ:

$$\dot{\boldsymbol{\alpha}}=\sum_{k=1}^{n}\varepsilon^k\mathbf{A}_k(\boldsymbol{\alpha},\ \boldsymbol{\beta},\ \boldsymbol{\alpha}^*,\ \boldsymbol{\beta}^*),\quad \dot{\boldsymbol{\beta}}=\sum_{k=1}^{n}\varepsilon^k\mathbf{B}_k(\boldsymbol{\alpha},\ \boldsymbol{\beta},\ \boldsymbol{\alpha}^*,\ \boldsymbol{\beta}^*). \quad (2.12)$$

Substituting (2.11) and (2.12) in the initial equation (2.5) and equating the terms at equal powers of ϵ in the right and left sides of (2.5), we obtain for the first two approximations

$$\mathbf{A}_1=j\mathbf{f}_{0,0},\quad \mathbf{B}_1=-j\mathbf{f}_{0,1}-j\Delta\boldsymbol{\beta}, \quad (2.13)$$

where

$$\mathbf{f}_{0,k}=\frac{1}{2\pi}\int_0^{2\pi}\mathbf{f}_0e^{-jk\theta}\,d\theta,\quad \mathbf{f}_0=\mathbf{f}(\mathbf{R}_0,\ \mathbf{R}_0^*,\ \theta),\ \mathbf{R}_0=(\mathbf{R})_{\varepsilon=0}. \quad (2.14)$$

Accordingly

$$\mathbf{A}_2=j\mathbf{f}_{1,0}+\Delta\mathbf{A}_1-j\hat{\sigma}_1\mathbf{A}_1, \quad (2.15)$$

$$\mathbf{B}_2=-j\mathbf{f}_{1,1}-\Delta\mathbf{B}_1+j\hat{\sigma}_2\mathbf{B}_1, \quad (2.16)$$

where

$$\mathbf{u}_1(\boldsymbol{\alpha},\ \boldsymbol{\beta},\ \boldsymbol{\alpha}^*,\ \boldsymbol{\beta}^*,\ \theta)=\sum_{k(1-k)\neq 0}\frac{\mathbf{f}_{0,k}}{k(1-k)}e^{jk\theta}, \quad (2.17)$$

$$\mathbf{f}_1=\left(\frac{\partial\mathbf{f}}{\partial\mathbf{R}}\right)_{\varepsilon=0}\mathbf{u}_1+\left(\frac{\partial\mathbf{f}}{\partial\mathbf{R}^*}\right)_{\varepsilon=0}\mathbf{u}_1^*-\hat{\sigma}_2\mathbf{u}_1, \quad (2.18)$$

$$\mathbf{f}_{1,k}=\frac{1}{2\pi}\int_0^{2\pi}\mathbf{f}_1e^{-jk\theta}\,d\theta, \quad (2.19)$$

$$\hat{\sigma}_1=\mathbf{A}_1\frac{\partial}{\partial\boldsymbol{\alpha}}+\mathbf{B}_1\frac{\partial}{\partial\boldsymbol{\beta}}+\mathbf{A}_1^*\frac{\partial}{\partial\boldsymbol{\alpha}^*}+\mathbf{B}_1^*\frac{\partial}{\partial\boldsymbol{\beta}^*}, \quad (2.20)$$

$$\hat{\sigma}_2=j\Delta\frac{\partial}{\partial\theta}+\mathbf{A}_1\left(2\frac{\partial^2}{\partial\theta\,\partial\boldsymbol{\alpha}}-j\frac{\partial}{\partial\boldsymbol{\alpha}}\right)+\mathbf{B}_1\left(2\frac{\partial^2}{\partial\theta\,\partial\boldsymbol{\beta}}-\frac{\partial}{\partial\boldsymbol{\beta}}\right)+\mathbf{A}_1^*\left(2\frac{\partial^2}{\partial\theta\,\partial\boldsymbol{\alpha}^*}-j\frac{\partial}{\partial\boldsymbol{\alpha}^*}\right)+\mathbf{B}_1^*\left(2\frac{\partial^2}{\partial\theta\,\partial\boldsymbol{\beta}^*}-\frac{\partial}{\partial\boldsymbol{\beta}^*}\right). \quad (2.21)$$

The first approximation of the asymptotic method does not differ from the Kapitza averaging method. The second and succeeding approximations refine the first and make it possible to take into account successively in the solution (2.11) the small high-frequency vibrations (the harmonics of the signal and pump, combination frequencies, etc.), which appear against the background of the fundamental motion of the system.

2.2. Coupled Waves in the Beam. Method of Coupled Waves

We represent the radius vector $\mathbf{R}$, which enters in Eq. (2.2), in the form of a sum of two circularly polarized quantities[19],

$$\mathbf{R}(z,\ t)=\mathbf{R}_+(z)e^{j\omega(t-t_0)}+\mathbf{R}_-(z)e^{-j\omega(t-t_0)}, \quad (2.22)$$

where ω is the frequency at which the transverse oscillations of the beam are considered.

In perfect analogy we can put in a number of cases also

$$(Nm)^{-1}\sum_{i=1}^{N}\mathbf{F}_i=\mathbf{F}_+(z)e^{j\omega(t-t_0)}+\mathbf{F}_-(z)e^{-j\omega(t-t_0)}. \quad (2.23)$$

Approximately assuming further that

$$\frac{d}{dt}\approx\frac{\partial}{\partial t}+v_0\frac{\partial}{\partial z}=\pm j\omega+v_0\frac{\partial}{\partial z}, \quad (2.24)$$

for the right (+) and left (−) circularly polarized quantities we obtain from (2.2)

$$\frac{d^2\mathbf{R}_\pm}{dz^2}\pm j(2\beta_e\mp\beta_c)\frac{d\mathbf{R}_\pm}{dz}-\beta_e(\beta_e\mp\beta_c)\mathbf{R}_\pm=\frac{1}{v_0^2}\mathbf{F}_\pm(z), \quad (2.25)$$

where $\omega/v_0 = \beta_C$ and $\omega_C/v_0 = \beta_C$.

In the beam drift region ($\mathbf{F}_\pm \equiv 0$), the solution of (2.25) takes the form

$$\mathbf{R}_+(z)=\mathbf{R}_{1+}e^{-j(\beta_e-\beta_c)z}+\mathbf{R}_{2+}e^{-j\beta_e z}, \quad (2.26)$$

$$\mathbf{R}_-(z) = \mathbf{R}_{1-}e^{j(\beta_e+\beta_c)z} + \mathbf{R}_{2-}e^{j\beta_e z}, \tag{2.27}$$

where $\mathbf{R}_{1\pm}$ and $\mathbf{R}_{2\pm}$ are arbitrary integration constants, having the meaning of amplitudes of the waves of the beam.

Thus, in a beam that drifts freely in a longitudinal magnetic field, there can exist, independently of each other, four waves: fast cyclotron $\mathbf{R}_{1+}$, slow cyclotron $\mathbf{R}_{1-}$, and two sychronous waves $\mathbf{R}_{2\pm}$ with equal phase velocities*. In other words, at a fixed frequency ω the beam has four degrees of freedom in the transverse direction.

It is easy to establish a connection between the kinematic and wave approaches, by changing over to a coordinate system that moves along the b axis with velocity v_0, and putting for this purpose $z = v_0(t - t_0)$ in (2.26) and (2.27). We then obtain from (2.22), (2.26), and (2.27)

$$\mathbf{R}(z = v_0(t-t_0),\ t) = \mathbf{R}_{2+} + \mathbf{R}_{2-} + (\mathbf{R}_{1+} + \mathbf{R}_{1-})\, e^{j\omega_c(t-t_0)}. \tag{2.28}$$

Comparing (2.28) with (2.7) (at $\epsilon = 0$), we get

$$\alpha = \mathbf{R}_{2+} + \mathbf{R}_{2-}, \quad \beta = \mathbf{R}_{1+} + \mathbf{R}_{1-}. \tag{2.29}$$

In other words, the synchronous waves are connected with the displacements of the center of the orbital motion relative to the system axis, and, conversely, the existence of cyclotron waves is due to the presence of cyclotron rotation of the mass centers of the individual discs.

In perfect analogy with the foregoing, using the method of variations of the constants in (2.27) and (2.26), subject to the supplementary conditions

$$\frac{d\mathbf{R}_{1\pm}}{dz}\, e^{-j(\pm\beta_e-\beta_c)z} + \frac{d\mathbf{R}_{2\pm}}{dz}\, e^{\pm j\beta_e z} = 0, \tag{2.30}$$

we can obtain from (2.25) a system of first-order differential equations describing the behavior of the normal waves of the beam in the presence of external perturbing fields:

$$d\mathbf{R}_{1\pm}(z)/dz = -j(\beta_c v_0^2)^{-1}\,\mathbf{F}_\pm(z)\, e^{j(\pm\beta_e-\beta_c)z}. \tag{2.31}$$

$$d\mathbf{R}_{2\pm}(z)/dz = j(\beta_c v_0^2)^{-1}\,\mathbf{F}_\pm(z)\, e^{\pm j\beta_e z}. \tag{2.32}$$

The ideas of averaging, of expansion in terms of a small parameter, etc. can be applied to these fundamental equations of the theory of coupled waves if it is necessary to simplify them and to separate the most intense couplings between the waves.

Both methods of describing the transverse oscillations of the beams start from the same equation of motion of the mass center of the individual disc (2.2). The separation of the variables z and t in the second case has made it possible to change over to a description of a sequence of discs (to take into account the longitudinal configuration of the transverse oscillations in the beam).

It should be noted, however, that if the system (2.9) and (2.10) is exact and the approximations are introduced only in the subsequent solution of the system, then the very derivation of the analogous equations for

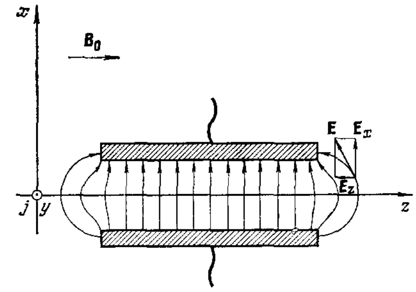

FIG. 2

the normal waves (2.31) and (2.32) is subject to limitations due to the validity of using relations (2.22)–(2.24).

Finally, both methods can be extended also to the more general case, when, for example, the proper parameters of the system ω_c and v_0 vary in space and in time.

2.3. Kinetic Powers of the Waves

The power of the energy exchange of the beam with the external transverse electric field $\langle W^\perp\rangle$, averaged over the period, was calculated by Siegman[20] and Dubravec[21]. For cyclotron waves

$$\langle W^\perp_{1\pm}\rangle = I_0 U_0 \beta_c^2\, |\mathbf{R}_{1\pm}|^2, \tag{2.33}$$

where I_0 and U_0 are the current and potential of the beam, and for synchronous waves there is no energy exchange,

$$\langle W^\perp_{2\pm}\rangle \equiv 0, \tag{2.34}$$

since the transverse displacements have only longitudinal velocities and do not interact with the transverse field. Frequently, however, the transverse fields are accompanied by longitudinal ones. Thus, for example, owing to the fringing of the fields on the ends of the simplest capacitive coupling devices (Fig. 2), the longitudinal component of the field **E** differs from zero away from the axis. The results of the interaction with the longitudinal fields is given approximately by the expressions

$$\langle W^=_{1\pm}\rangle = -I_0 U_0 \beta_c(\beta_c \pm \beta_e)\, |\mathbf{R}_{1\pm}|^2, \tag{2.35}$$

$$\langle W^=_{2\pm}\rangle = \mp I_0 U_0 \beta_e \beta_c\, |\mathbf{R}_{2\pm}|^2, \tag{2.36}$$

and the total energy-exchange power is given by

$$\langle W_{1\pm}\rangle = \langle W^\perp_{1\pm}\rangle + \langle W^=_{1\pm}\rangle = \pm I_0 U_0 \beta_e \beta_c\, |\mathbf{R}_{1\pm}|^2, \tag{2.37}$$

$$\langle W_{2\pm}\rangle = \langle W^=_{2\pm}\rangle = \mp I_0 U_0 \beta_e \beta_c\, |\mathbf{R}_{2\pm}|^2. \tag{2.38}$$

When solving self-consistent problems, the approximate allowance for the influence of the longitudinal fields reduces formally to replacing the transverse electron velocity $\mathbf{v}_{e\perp} = d\mathbf{R}/dt$ in the expression for the induced current by the transverse beam velocity $\mathbf{v}_{b\perp} = \partial\mathbf{R}/\partial t$[20,21].

In conclusion we note that excitation of a slow cyclotron wave in the beam, unlike the case of excitation of longitudinal waves, still does not mean that the electrons will be slowed down, since the reversal of the

*In the literature, however, the wave $\mathbf{R}_{2+}$ is frequently called the slow synchronous wave and $\mathbf{R}_{2-}$ the fast synchronous wave. The reason for this is the difference in the signs of the kinetic powers of these waves (see Sec. 2.3).

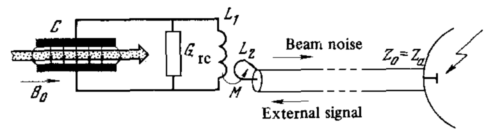

FIG. 3. Capacitive input coupling device with homogeneous electric field. C – resonant-circuit capacitance, G_c – equivalent active loss conductance in the circuit, L_1, L_2 – inductances of the circuit and of the coupling loop, M – mutual induction coefficient, Z_0, Z_a – wave impedances of the feeder and of the antenna.

sign of the kinetic power for this wave is due to longitudinal fields, the presence of which in the case of transverse interaction is not essential. The beam can exchange energy with a TEM wave when $E_z = 0$.

3. PRINCIPLES OF FORMATION OF LOW NOISE LEVEL OF CYCLOTRON WAVES

In the main, it is possible to indicate two methods of attaining a low noise level in amplifiers using transverse modulation of the electron beam. The simplest of them is based on the "cooling" of the fast cyclotron wave in a passive coupling device, and the second is connected with obtaining a low noise level of any particular wave directly during the formation of the beam, i.e., in the electron gun.

3.1. Cooling of a Fast Cyclotron Wave of an Electron Beam in a Passive Coupling Device

The most widely used coupling device, in the form of a parallel-plate capacitor, is illustrated in Fig. 3. The uniform field of the parallel-plate capacitor can be regarded as a section of the field of a wave with infinite phase velocity. In the cyclotron-resonance region ($\omega = \omega_c$), the phase velocity of the fast cyclotron wave of the electron beam also tends to infinity and consequently, effective energy exchange between this wave and a coupling device of this type is possible. Conversely, the finite and rather low phase velocity of the remaining waves hinders interaction with them.

When (2.31) and (2.32) are used, it is possible to write down a system of self-consistent equations describing the interaction of transverse waves with the field of a parallel-plate capacitor. The result of its solution for a fast cyclotron wave of the electron beam is of the form

$$\mathbf{R}_{1+}(l) = \mathbf{R}_{01\pm}\left(1 - \frac{2G_{e1+}}{Y}\right) - \frac{2G_{e1+}}{Y}\left[\mathbf{R}^*_{01-}\frac{k(\beta_-\theta_l)}{k^*(\beta_+\theta_l)} + (\mathbf{R}_{02} + \mathbf{R}^*_{02-})\frac{k(\theta_l)}{k^*(\beta_+\theta_l)}\right] + \mathbf{S}, \quad (3.1)$$

where $\mathbf{R}_{01\pm}$ and $\mathbf{R}_{02\pm}$ are the initial noise amplitudes of the waves at the input into the interaction region, $k(x) = (e^{jx} - 1)/jx$, $\beta_\pm = \pm 1 - \beta_c\bar{\beta_e}$, $\theta_l = \beta_e l$, l—length of the interaction region, $\mathbf{S}$—term due to the external signal entering into the beam from the resonant circuit, $Y = Y_e + Y_L + Y_{rc}$, Y_{rc}—equivalent admittance of the resonant circuit proper, Y_e, Y_L—admittances introduced into the resonant circuit by the electron beam and by the external load, respectively, with

$$\begin{aligned} Y_{e1\pm} &= G_{e1\pm} + jB_{e1\pm} = \pm\frac{\omega}{\omega_c}G_0\left\{\left[\frac{\sin(\beta_\pm\theta_l/2)}{\beta_\pm\theta_l/2}\right]^2 \mp 2j\,\frac{\beta_\pm\theta_l - \sin\beta_\pm\theta_l}{(\beta_\pm\theta_l)^2}\right\}, \\ Y_{e2\pm} &= G_{e2\pm} + jB_{e2\pm} = \mp\frac{\omega}{\omega_c}G_0\left\{\left[\frac{\sin(\theta_l/2)}{\theta_l/2}\right]^2 - 2j\,\frac{\theta_l - \sin\theta_l}{\theta_l^2}\right\}, \end{aligned} \quad (3.2)$$

$$Y_e = Y_{e1+} + Y_{e1-} + Y_{e2+} + Y_{e2-}, \quad G_0 = I_0 l^2/8U_0 d^2, \quad (3.3)$$

d—gap between plates.

In the general case, the interaction is with all the waves. However, if the length of the plate is such that $\theta_l \gg 1$, then in the region of cyclotron resonance ($\omega \approx \omega_c$), at

$$2G_{e1+} = Y \quad (3.4)$$

the amplitude of the fast cyclotron wave of the beam is close to zero (at $\mathbf{S} = 0$), and practically complete transfer of the oscillation energy (particularly the noise oscillations) of the fast cyclotron wave of the electron beam to the external circuit is possible.

Since $|Y_{e1+}| \gg |Y_{e1-}|$ and $|Y_{e2\pm}| \ll |Y_{e1+}|$, the condition (3.4) corresponds to complex-conjugate matching of the conductivities of the beam and of the circuit:

$$G_e = G_L + G_{rc}, \quad (3.5)$$

$$B_L + B_L + B_e = 0. \quad (3.6)$$

The condition for the matching of the signal source (antenna) to the input of the amplifier, owing to the finite loss conductivity in the circuit ($G_c \neq 0$), has a somewhat different form:

$$G_L = G_e + G_{rc}, \quad (3.5')$$

$$B_L + B_L + B_e = 0. \quad (3.6')$$

This deteriorates the noise characteristic of the instrument as a whole although not very appreciably, since in practice it is possible to ensure $G_{rc}/G_e \sim 0.1$ and less.

Lee-Wilson[22] has established that the power drawn from the beam and dissipated in the external circuit is smaller by several orders of magnitude than the power of the transverse noise oscillations in the beam at the entrance to the system. Nonetheless, the amplitude of the fast cyclotron waves at the output of the system can be equal to zero. This indicates that physically the mechanism of "cooling" the beam is a rather complicated process, in which the main energy is lost as a result of balancing (or phase adjustment) of the individual electron orbits in such a way that the vector sum of the transverse velocities of the electrons inside each disc that leaves the capacitive gap vanishes*.

The experimental data indicate that such a method is effective and that very deep "cooling" of the beam is possible. The residual noise temperature of the fast cyclotron wave at the output of the coupling element (without allowance for the thermal noise that enter from the circuit into the beam) can be of the order of only 20–30°K[23].

The band in which the noise is drawn off depends on the bandwidth within which the matching conditions (3.5) and (3.6) are satisfied. If the circuit is such that its natural frequency (ω_{rc}) coincides with the cyclotron frequency (ω_c), and the "cold" loaded Q (without the electron beam) is chosen equal to

*We are referring, of course, to that part of the sum which is responsible for the excitation of only the fast cyclotron wave in the beam "cooling" band.

$$Q_{rc,\,L} = \theta_l G_0 / 6\,(G_L + G_{rc}), \qquad (3.7)$$

then it is possible to cancel out the reactances of the beam and of the circuit in a definite frequency band (Fig. 4). The situation is more complicated with the active conductances. If $G_0 = G_{rc} + G_L$ (curves 1 of Fig. 4), then the minimum value of the noise is attained at the point of cyclotron resonance. It is more advantageous to choose G_0 somewhat larger than $G_{rc} + G_L$. In this case the matching of the active conductances is observed already at two points that are symmetrical relative to ω_c, and the working frequency band becomes broader (curves 2 on Fig. 4).

It is possible to use two identically tuned resonators in series, separating by the same token the functions of the modulation and demodulation of the beam. This broadens the working bandwidth by a factor 1.5–2, and the amplifier becomes less critical to a mismatch of input circuits[24]. A broader band can be obtained at the expense of further complicating the construction of the instruments using for beam cooling a chain of coupled[24] or uncoupled[26] resonators at the input.

In those cases when the interaction must be ensured away from cyclotron resonance ($\omega \neq \omega_c$), planar coupling devices are no longer effective. Adler[27] proposed and used (see[28]) a lumped coupling device in the form of two twisted plates (Fig. 5). It can, naturally, ensure coupling with any of the four transverse waves, depending on the longitudinal velocity of the beam and on the direction of the magnetic field.

Finally, various distributed coupling devices were proposed to expand the bandwidth (see, for example[29-31]). The difficulty lies in this case apparently in the fact that none of them have the great homogeneity of the field as in the case considered above. In each of them, a major role is played by longitudinal fields. These give rise to an additional longitudinal modulation of the beam, which is homogeneous over the cross section of the disc. And even if the "cooling" (balancing) is attained, it turns out to be unstable and is rapidly lost. In approximately the same manner, a natural scatter of longitudinal velocities of the beam electrons appears away from the cyclotron resonance[32].

3.2. Electron Guns, Noise Transformers

The development of electron guns with low noise levels of the electron-beam transverse waves is a most complicated problem and is therefore least discussed in the literature.

During the earlier stages, principal attention was paid to the static problems of formation of homogeneous beams of small cross section with high space-charge density[33]. These problems are most important for low-frequency parametric amplifiers (up to 1 kMHz), where the resonant magnetic fields are small in magnitude (up to 300 Oe) and are not always sufficient for hard focusing of the beam in the near-cathode region. The high space-charge density in the beam is useful also in the amplification zone, making it possible to avoid expansion of the beam and its premature interception[15,34].

With increasing interest in the electrostatic nonparametric amplification principle, in which an important role is played by one of the slow waves of the beam,

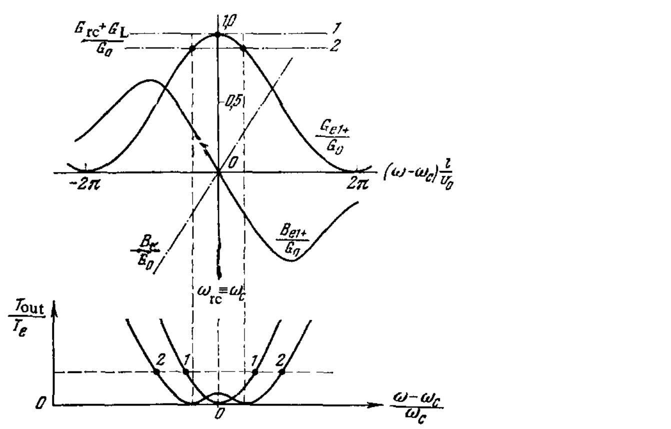

FIG. 4. Types of matching of conductances and beam "cooling" band. The reactances are cancelled out by choosing the slope of the curves B_{el+}/G_0 and B_{rc}/C_0 in such a way that the sum $(B_{el+} + B_{rc})$ vanish in the region of small detunings. With respect to the active conductances, two variants are possible: either $G_0 = G_L + G_{rc}(1)$, or $G_0 > G_L = G_{rc}(2)$. In the former case the beam cooling curve T_{out}/T_e (T_e is the equivalent noise temperature of the fast cyclotron wave of the electron beam at the input to the interaction region, and T_{out} is its residual part at the output) has a single minimum, and in the latter case there are two minima.

the question arose of the magnitude and ways of decreasing the noise level of the transverse waves of the beam produced by the electron gun.

Blotekjaer[35], starting from the model of a filamentary beam, and assuming a Maxwellian distribution of the transverse velocities on the cathode and an equally-probable emission over its surface, calculated the elements of the noise matrix and showed that the equivalent temperature of the transverse waves on the surface of the cathode is

$$T_{1\pm}(\omega) = \omega\omega_{cc}^{-1}T_c, \qquad T_{2\pm}(\omega) = \omega_{cc}^{-1}T_c(1+\varkappa^2), \qquad (3.8)$$

where T_c is the temperature of the cathode, ω_{cc} is the value of the cyclotron frequency on its surface,

$$\varkappa^2 = \frac{(\overline{|x|^2} + \overline{|y|^2})\,\omega_c^2}{(\overline{|v_x|^2} + \overline{|v_y|^2})} = \Gamma^2 \frac{m\omega_{cc}^2 d^2}{16kT_c}$$

characterizes the ratio of the powers of the fluctuations of the transverse displacements and the transverse velocities, Γ^2 is the depression factor at the minimum of the potential, k is Boltzmann's constant, and d is the diameter of the cathode.

If the relative level of the fluctuations of the transverse coordinates is negligible, then a linear lossless noise transformer can be used to redistribute the energy of the noise in the beam so as to lower the noise level of at least one of the cyclotron waves. This idea was advanced in the literature, essentially, many times, although from several different points of view[36,37].

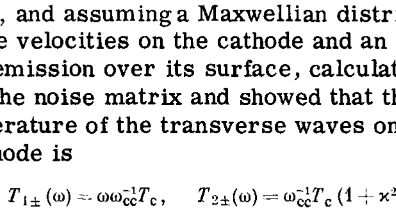

FIG. 5. Twisted coupling device of the lumped type.

Wessel-Berg and Blotekjaer[37] calculated the minimum value of the equivalent noise temperature of cyclotron waves

$$(T_{\pm})_{\min} = \omega\omega_{cc}^{-1} T_c \{[\varkappa^2 + (\varkappa^4/4)]^{1/2} - (\varkappa^2/2)\}. \tag{3.9}$$

When $\omega \approx \omega_{cc}$ it is possible to lower the temperature of the cyclotron waves only at small values of κ, and this requires very small cathode diameters (0.05 mm and less), especially for high-frequency tubes*.

In this connection, a number of ideas have been advanced to overcome the theoretical minimum calculated by Wessel-Berg and Blotekjaer, by using aid of current interception[38] or the scatter of longitudinal velocities[39] (i.e., with the aid of converters with losses). Several results of general type were obtained in this direction by Hart[40]. So far, however, it is impossible to indicate a sufficiently reliable mechanism for such a process, and this question remains in fact open. Nor is the role of the Coulomb interaction in the most interesting near-cathode multivelocity zone sufficiently clear. It would be useful to have information on the influence of the type of the cathode and of the character and type of activating coating on the noise level of the transverse waves.

4. ELECTRON-BEAM PARAMETRIC AMPLIFIERS (EPA)

Parametric amplification of cyclotron waves of an electron beam is possible in any inhomogeneous high-frequency field. In the first amplifiers, proposed by Adler[41], the pump fields were produced by a dipole capacitor, which, unlike the input capacitor, had plates of curved form. Later Wade (see[42]) proposed to use for these purposes the field of a quadrupole capacitor, which is strictly linear in the radial direction (this ensures linearity of the amplification processes) and is periodic in the azimuthal direction†.

The complex field of a lumped twisted quadrupole structure, in the notation employed here, can be written in the form

$$\mathbf{E}(\mathbf{R}, t) = E_x + jE_y = -2V_0 a^{-2}\mathbf{R}^* e^{2j\beta_q z}\cos[\omega_p(t-t_0)], \tag{4.1}$$

where V_0 is the potential of the quadrupole plates, a is the minimum distance from the axis of the system to the plate, β_q is the constant of the spatial rotation of the quadrupole structure, and ω_p is the pump frequency.

Expression (4.1) describes the fundamental types of the quadrupole fields used in supernoiseless amplifiers of cyclotron waves. In EPA of the degenerate there is no spatial rotation of the field ($\beta_q \equiv 0$), and the pump frequency is approximately equal to double the single frequency ($\omega_p \approx 2\omega \approx 2\omega_c$). In the nondegenerate variant $\beta_q \neq 0$ and $\omega_p \approx n\omega$, $n > 2$. In electrostatic structures $\omega_p \equiv 0$ and $\beta_q \neq 0$.

The decisive process, from the point of view of the intrinsic noise, in parametric amplifiers is the removal of the noise by the input coupling device. Therefore, in the amplification zone (more accurately, in all the succeeding elements of the device), the attained balancing of the orbits must not be disturbed; the result of the action of external fields on the individual cyclotron orbit should not depend on its position inside the disc. Mathematically this requirement can be satisfied by making the external fields adiabatic, and making the equation for $\dot{\boldsymbol{\beta}}$ in the first approximation linear and independent of $\boldsymbol{\alpha}$ or $\boldsymbol{\alpha}^*$, i.e., it is necessary to have in (2.12)

$$\mathbf{B}_1 = C\boldsymbol{\beta} + D\boldsymbol{\beta}^* \neq \mathbf{f}(\boldsymbol{\alpha}, \boldsymbol{\alpha}^*) \quad (\varepsilon \ll 1), \quad \dot{\boldsymbol{\alpha}} \neq \dot{\boldsymbol{\alpha}}(\boldsymbol{\beta}, \boldsymbol{\beta}^*) \tag{4.2}$$

where C and D are constant coefficients.

With respect to an electron moving with velocity v_0 ($z_{rc} = v_0(t - t_0)$), a field of the type (4.1) splits into two components with frequencies $\omega_{pe_1} = \omega_p + 2\beta_q v_0$ and $\omega_{pe_2} = -\omega_p + 2\beta_q v_0$. With each of them, depending on the magnitude and direction of the magnetic field, resonant interaction can take place. Let us assume that this is ω_{pe_1}. Then $\omega_{pe_1} = 2\omega_c$,* $\omega_{pe_1} \equiv 2\omega_{0e}$, and the function $\mathbf{f}(\mathbf{R}, \mathbf{R}^*, \theta)$ from (2.9) takes the form

$$\mathbf{f}(\mathbf{R}^*, \theta) = \mathbf{R}^*(e^{2j\theta} + e^{-2j\gamma\theta}), \quad \varepsilon = e'V_0/a^2\omega_{0e}^2, \tag{4.3}$$

where $\gamma = \omega_{pe_2}/\omega_{pe_1}$.

If γ is an integer, the requirement (4.2) is satisfied, and the solution of the problem, in accordance with (2.1)–(2.21), takes the form

$$\mathbf{R}(\theta) = \boldsymbol{\alpha}(\theta) + \boldsymbol{\beta}(\theta)e^{j\theta} - \varepsilon[(\boldsymbol{\alpha}^* e^{j2\theta}/2) + (\boldsymbol{\alpha}^* e^{-j2\theta}/6) + (\boldsymbol{\beta}^* e^{-j3\theta}/12)], \tag{4.4}$$

with

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 e^{-2j\varepsilon^2/3}, \tag{4.5}$$

$$|\boldsymbol{\beta}|^2|\boldsymbol{\beta}_0|^{-2} = \mathrm{ch}^2\tau + (s^2\sin^2\varphi_0 + s^{-2}\cos^2\varphi_0)\,\mathrm{sh}^2\tau - [(s + s^{-1})\sin 2\varphi_0\,\mathrm{sh}\,2\tau/2] \tag{4.6}$$

$$e^{j\varphi} = (\mathrm{ch}\,\tau e^{j\varphi_0} - \{\mathrm{sh}\,\tau[(s^{-1} - s)e^{j[\varphi_0 + (\pi/2)]} + (s^{-1} + s)e^{-j[\varphi_0 - (\pi/2)]}]/2\})|\boldsymbol{\beta}|^{-1}|\boldsymbol{\beta}_0|, \tag{4.7}$$

where

$$\tau = \{(1 - \varepsilon\Delta)^2 - [\Delta - (13\varepsilon/12)]^2\}^{1/2}\varepsilon\theta, \quad s = \{(1 - \varepsilon\Delta) - [\Delta - (13\varepsilon/12)]\}^{1/2} \times \{(1 - \varepsilon\Delta) + [\Delta - (13\varepsilon/12)]\}^{-1/2},$$

$\boldsymbol{\alpha}_0$ and $\boldsymbol{\beta}_0$ are the initial values of the cyclotron and synchronous radii at the input to the quadrupole zone, and $\varphi = \arg\boldsymbol{\beta}$.

The synchronous waves (synchronous radii) are not amplified in the quadrupole field. There is only a weak drift of the phase of the synchronous motion of second order of smallness in ϵ.

The amplification of the cyclotron motion is linear but not phase-selective. The gain depends on the initial phase φ_0, reckoned from half the pump phase (at $\omega_p \approx 2\omega_c$). The maximum of the gain in the first approximation is reached exactly at $\omega_c = \omega_{pe}/2$, and in the second approximation at a slight detuning ($\sim \epsilon^2$).

For an incoherent input signal, φ_0 is a linear function of the time. The presence of deep beats, in accordance with (4.6), indicates in the wave representation the

*It is apparently physically feasible and technically much simpler to effect minimization for a ribbon beam (when only one of the cathode dimensions in negligibly small).

†It can be shown that the latter requirement is essential if the difference wave is to be a cyclotron wave.

*It can be shown that growing solutions are possible only when $\omega_{pe} \approx 2\omega_c/n$. The case n = 1 will be considered below; n = 2 corresponds to periodic exchange of energy between the cyclotron and synchronous motions ((4.2) is not satisfied), and when n > 2 (amplifiers with "low-frequency" pumping) it is likewise impossible to satisfy the condition (4.2), although the amplification of the cyclotron waves is possible.

creation of an additional (cyclotron) wave the amplitude of which is close to the signal wave, and the frequency is equal to the difference $\omega_p - \omega$. The amplitudes of all the remaining combination frequencies are smaller than the fundamental amplitude by a factor not less than ϵ (see (4.4)). From (4.4)—(4.7) and (2.37) we get for the power gains of the signal and difference waves, respectively,

$$G\,(\text{дб}) = 10\lg\{\mathrm{ch}^2\tau + [(s-s^{-1})^2(\mathrm{sh}^2\tau/4)]\}, \tag{4.8}$$

$$G_i\,(\text{дб}) = 10\lg[(s+s^{-1})^2(\mathrm{sh}^2\tau/4)]\,(|\omega_p-\omega|/\omega)^{-1}. \tag{4.9}$$

The noise existing in the beam at the difference wave $\omega_i = \omega_p - \omega$, becoming amplified in the quadrupole pump zone, leads to the appearance of difference components $\omega_p - \omega_i = \omega_p - (\omega_p - \omega) = \omega$, which fall into the signal channel and greatly deteriorate the noise figure of the instrument. Analyzing (4.4), we can show that the contribution of the noise from the waves of the remaining combination frequencies can always be made insignificant by choosing a suitable value of small parameter ϵ.

4.1. Degenerate EPA

The degenerate variant of the instrument was historically the first and has been most widely used. The pump frequency used in such an amplifier is $\omega_p \approx 2\omega_c \approx 2\omega$, and consequently the difference frequency is close to the signal frequency $\omega_i \approx \omega$. There are known amplifiers of this type operating at frequencies from 200[43] to 4000 MHz[23].

The low-frequency variants employ as coupling devices lumped LC circuits located usually directly in the envelope of the tube (Fig. 6). The quadrupole field is excited by hyperbolic plates or by four round rods (Fig. 7).

The high-frequency devices usually employ cavity resonators with plates forming interaction gaps of the capacitive type (Fig. 8).

The main shortcoming of the degenerate systems is connected with the transfer of the noise from the difference channel, as a result of which the minimum value of the equivalent single-channel temperature of the intrinsic noise is finite and is determined by the noise temperature of the signal source (antenna) at the difference frequency

$$T_{\mathrm{EPA\,min}} \approx \approx [(\omega_p-\omega)/\omega]^{-1}\,\mathrm{th}\,\tau T_a \sim T_a, \tag{4.10}$$

where T_a is the antenna noise temperature. In addition, if the frequency of the received signal ω turns out to be close in magnitude to $\omega_p/2$, then the difference component together with the signal component enter into the intermediate frequency band and distorts the waveform of the received signal. Therefore the central part of the pass band of degenerate EPA turns out to be usually unsuitable for use.

4.2. EPA of the Nondegenerate Type

The first attempts to construct EPA of the nondegenerate type were based on the desire to separate the signal and difference channels (symmetrically with respect to $\omega_c \approx \omega_p/2$, so as to decouple the input and output circuits of the amplifier at the difference frequency. A diagram of such an amplifier is shown in Fig. 9b. The phase velocities of the signal and difference waves are

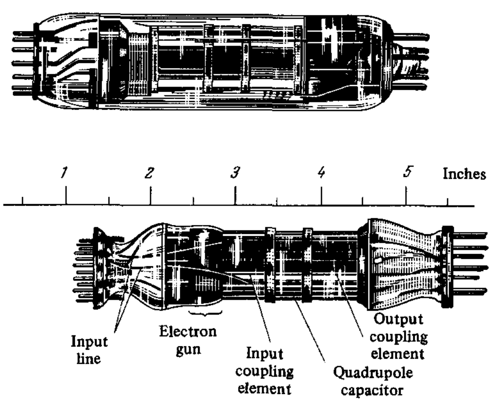

FIG. 6. Overall view and construction of adverse variant of the low-frequency amplifier. The working frequency of the instrument is 560 MHz, the gain is 20 – 30 dB, the equivalent intrinsic noise temperature is ~ 100°K.

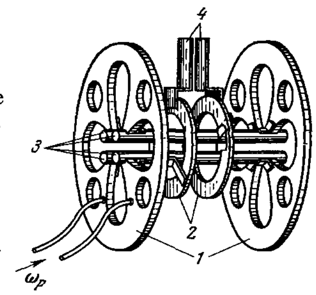

FIG. 7. Low-frequency structure for the excitation of the quadrupole pump field. 1 – Inductive elements, 2 – couplings to fix the quadrupole type of oscillation, 3 – round cylindrical rods exciting the quadrupole field, 4 – capacitive-type tuning element.

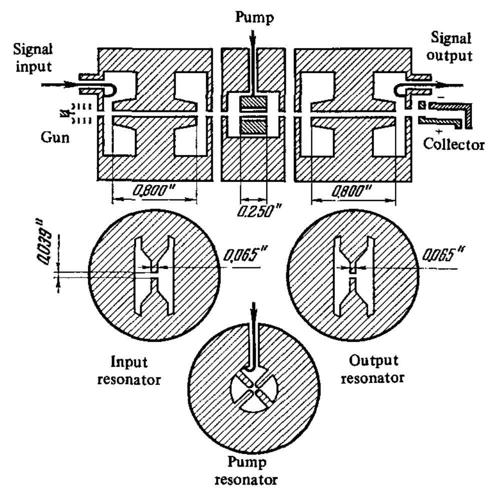

FIG. 8. Construction of Ashkin's high-frequency amplifier (the dimensions are in inches). Working frequency 4137 MHz, equivalent intrinsic noise temperature 58°K, gain 20 dB, dynamic range 100 dB.

in this case finite, of opposite sign, and equal in absolute magnitude. The minimum value of the noise coefficient of such an amplifier is determined in practice by the level of the thermal noise of the tuned circuit operating at the resonant frequency. A combined coupling device is also possible, interacting with the beam simultaneously at the signal and at the difference frequencies (Fig. 9a). This, however, results in a lower efficiency of interaction compared with the twisted system[28].

A more advanced design of a nondegenerate EPA was

$$\omega_{pe_1} = 2\beta_q v_0 \approx 2\omega_c, \quad \omega_p \equiv 0. \tag{5.1}$$

Relations (4.1)–(4.9) remain valid also in this case. The essential difference, from the wave point of view, is that the role of the fast cyclotron wave (when $\omega_p > \omega$) at the difference frequency is now assumed by the slow cyclotron wave at the signal frequency ω. Its gain is determined by the same expression (4.9) with $\omega_p \equiv 0$.* This wave is practically not excited in the input coupling device and is not extracted from the beam in the output device. By the same token, it cannot lead to a distortion of the wave form of the signal at the output of the ESA, thus eliminating the resultant bandwidth limitation characteristic of degenerate parametric systems. To the contrary, the slow cyclotron wave, interacting with the helical field of the electrostatic quadrupole and becoming amplified, gives rise to a fast cyclotron wave of the same frequency. Consequently, during the course of the electrostatic amplification, the noise of the slow cyclotron wave of the electron beam can fall into the signal channel, i.e., into the channel of the fast cyclotron wave. The intrinsic noise level of the electrostatic amplifier will therefore be determined by the noise level of the slow cyclotron wave of the electron beam at the input to the amplifier.

5.2. Formation of Low Noise Level in an ESU

The temperature of the cyclotron waves of the electron beam directly on the surface of the cathode can be made arbitrarily low by placing the cathode in a magnetic field whose intensity is such that $\omega_{cc} \gg \omega$ (see (3.8)). Conversely, in the working region of the ESA it is necessary to have $\omega_c \approx \omega$, if use is made of coupling devices of a capacitor type.

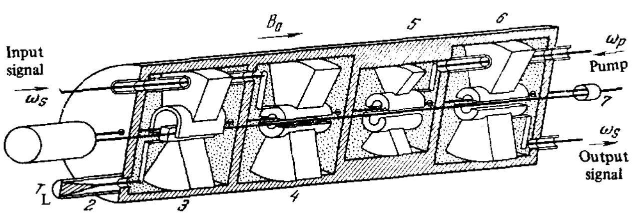

FIG. 11. Construction of two-beam amplifier. 1 – Electron gun, 2 – matched load of correlating resonator, 3 – antiphase resonator correlating the beam noises, 4 – input resonator with in-phase fields, 5 – pump resonator with antiphase quadrupole fields (using Ashkin's structure [52], which excites one quadrant of the quadrupole field), 6 – output resonator with in-phase fields, 7 – collector.

*Generally speaking, the mechanism of amplification in the ESA differs somewhat from that in the EPA. The increase of the energy of the transverse (cyclotron)motion of the beam is due to the decrease of the energy of its longitudinal motion. On the other hand, the deceleration of the electron orbits causes a loss of synchronism between the beam and the external field ($\omega_{pe}(z) \neq 2\omega_c$, $\Delta(z) \neq 0$), and this can greatly reduce the resultant gain. This effect is important also at very low input signals, since the transverse energy of the noise oscillations in the beam, in the case of a narrow parametric-resonance band ($\epsilon \ll 1$), may turn out to be sufficient to lead to a rather appreciable detuning Δ. This effect can be neutralized, for example, by choosing the profile of the magnetic field such that the condition $\Delta = 0$ is maintained along the entire length of the quadrupole system.

The problem of obtaining a low level of intrinsic noise in ESU will thus be solved if it is possible to shape (focus, accelerate) the beam and reduce the intensity of the magnetic field, while leaving the low cyclotron-wave noise level obtained on the surface of the cathode unchanged.

Under certain conditions this turns out to be possible. The transverse motion of the mass center of the disc in static azimuthally-symmetrical electric (gun lenses) and magnetic fields is described in the paraxial approximation by the equation[46]

$$\frac{d^2\mathbf{R}}{dt^2} - j\omega_c(z)\frac{d\mathbf{R}}{dt} = \frac{1}{2}\left[j\frac{d\omega_c(z)}{dt} - e'\frac{d^2\Phi_0(z)}{dz^2}\right]\mathbf{R}, \tag{5.2}$$

where $\Phi_0(z)$ is the electron potential on the system axis. Using the concept of the form of the coupled waves and assuming that in (2.22)–(2.32) ω_c and v_0 are functions of the longitudinal coordinate z, we obtain

$$\frac{d\mathbf{R}_{1\pm}}{d\theta} = -[\mu(\theta) - j\varepsilon(\theta)]\mathbf{R}_{1\pm} - [\mu(\theta) + j\varepsilon(\theta)]\mathbf{R}_{2\pm}e^{-j\theta}, \tag{5.3}$$

$$\frac{d\mathbf{R}_{2\pm}}{d\theta} = -[\mu(\theta) - j\varepsilon(\theta)]\mathbf{R}_{1\pm}e^{+j(\theta)} - [\mu(\theta) + j\varepsilon(\theta)]\mathbf{R}_{2\pm}, \tag{5.4}$$

where $\theta = \int_{z_0}^{z} \beta_c dz$ is the transit angle, $2\mu(\theta) = \omega_c^{-1} d\omega_c/d\theta$ is the relative change of the cyclotron frequency per radian of the transit angle, $\epsilon(\theta) = (e'/2\omega_c^2)d^2\Phi_0/dz^2$ is a parameter characterizing the relative intensity of the radial electric forces.

It follows immediately from (5.3), (5.4) and (2.37), (2.38) that the summary kinetic power of waves having identical polarization in azimutually-symmetrical fields remains unchanged:

$$\langle W_{1\pm}\rangle + \langle W_{2\pm}\rangle = \mathrm{const}_\pm, \tag{5.5}$$

and waves with opposite signs of the polarization do not interact with one another, regardless of the intensity of the external fields.

If the parameters $\mu(\theta)$ and $\epsilon(\theta)$, which enter in the system (5.3)–(5.4), are small in absolute value and do not depend on the transit angle θ (or at least are sufficiently slow functions of θ), then in first approximation we obtain from (5.3) and (5.4) (after averaging over θ) the required result*:

$$\langle W_{1\pm}\rangle = \mathrm{const}_\pm, \quad \langle W_{2\pm}\rangle = \mathrm{const}_\pm. \tag{5.6}$$

Thus, in adiabatic azimuthally-symmetrical fields the kinetic powers of the waves (and their equivalent noise temperatures) remain unchanged, and the degree of accuracy with which this is satisfied improves when the inequalities

*In the kinematic analysis, the corresponding invariants take the form

$$\omega_c(z)(|\beta|^2 - |\alpha|^2) = \mathrm{const}, \tag{5.5'}$$

$$\omega_c(z)|\beta|^2 = \mathrm{const}, \quad \omega_c(z)|\alpha|^2 = \mathrm{const}. \tag{5.6'}$$

The first condition (5.5′) is similar to the Busch theorem, written out for azimuthally-symmetrical fields in terms of α and β, and was formulated several times by Gordon [46,47]. Finally, the most important invariant $|\beta|^2\omega_c = \mathrm{const}$ reflects the known premise of mechanics, namely that the ratio of the oscillation energy of a one-dimensional harmonic oscillator ($W^\perp \sim \omega_c^2|\beta|^2$) to the natural frequency ω_c of its oscillations is adiabatically invariant [48].

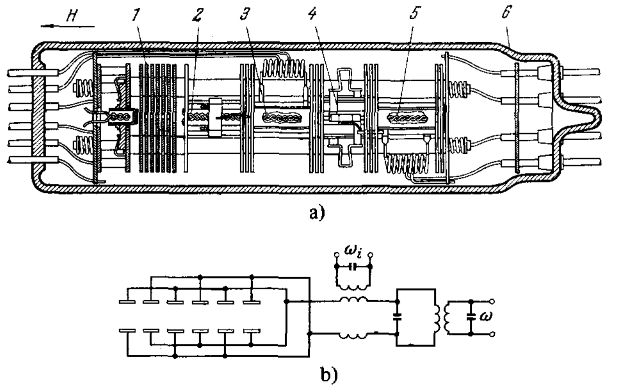

FIG. 9. Diagram of amplifier of nondegenerate type with separated signal and difference channels (coupling devices of the twisted type are used) (b), and two-frequency coupling device for interacting with the electron beam simultaneously at the signal and the difference frequencies (a). 1 – Electron gun, 2 – device for cooling the fast cyclotron wave of the electron beam at the difference frequency, 3 – input coupling device, 4 – quadrupole amplification zone, 5 – output coupling device (similar to the input device), 6 – glass housing of the amplifier.

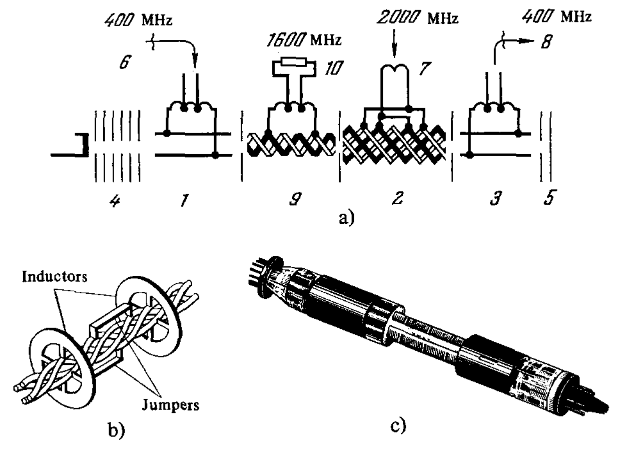

FIG. 10. a) Schematic diagram. b) Twisted quadrupole structure. c) Exterior view of nondegenerate EPA of Adler and Hrbek. 1 – Input coupling device of flat type, 2 – twisted quadrupole system, 3 – output coupling device, 4 – electron gun, 5 – collector, 6 – signal-source load, 7 – inductance of pump circuit, 8 – output load of the amplifier, 9 – twisted coupling device for cooling the beam at the difference frequency, 10 – matched load of difference circuit. The noise figure of the amplifier was 1.9 dB (1.3 dB when the load 10 was cooled to liquid-hydrogen temperature).

realized by Adler and Hrbek[28]. The pump frequency was chosen much higher than the signal frequency ($\omega_p \approx 5\omega$). One of the channels (signal) operates here as before in the cyclotron resonance zone ($\omega_c \approx \omega$, and the longitudinal scatter of the velocities does not play any significant role). Cooling of the beam at the difference frequency is by a twisted coupling device (since $\omega_i \approx 4\omega_c$), and the amplification takes place in a twisted quadrupole system (in order to retain $\omega_{pe} \approx 2\omega_c$, i.e., $\Delta = 0$, and to ensure by the same token the parametric resonance of the fundamental type).

The additional noise temperature due to the difference channel $\Delta T = [\omega/\omega_p - \omega)]T_L \approx T_L/4$ (T_L is the load temperature of the difference circuit) determines the minimum value of the noise figure of the device. A diagram and an overall view of such an amplifier are shown in Fig. 10.

The twisting of the system greatly complicates the technical construction of nondegenerate devices, especially in the short-wave part of the decimeter band and in the centimeter band. The way out of this situation may be a two-beam amplifier, constructed in accordance with the compensation principle[44,45]. Its construction is shown in Fig. 11[45]. Two electron beams are used in the amplifier. The useful signal is introduced into both beams in phase, and is also extracted at the output in phase. The amplification occurs in antiphase quadrupole fields of equal intensity, as a result of which the waves of the difference frequency at the output are in antiphase with each other (their phase depends on the phase of the pump field) and does not excite any currents in the output coupling device. The beam noise cooling is produced by a system of two resonators. The first has antiphase plane fields, and the second (output) has in-phase fields. The noise oscillations in the beam at the output of the first resonator are correlated (are equal to each other[45]), and are therefore subsequently completely eliminated in the input coupling device. All this pertains to the intrinsic noise of the beams. The thermal noise of the matched load of the correlating resonator is introduced into the beam in antiphase and does not interact with the input coupling device, so that the noise figure is limited to the value

$$F_{\min} = 1 + T_H T_0^{-1}, \qquad (4.11)$$

where T_L is the temperature of the matched load of the correlating resonator and $T_0 = 293°$ K.

This is a shortcoming of the two-beam amplifier, although the noise source is localized outside the amplifier and the possibility of its cooling is not excluded. The identity of the conditions under which the beams operate and the associated reliability of suppression of the difference channel, the simplicity of construction (there are no twisted systems), the low pump frequency ($\omega_p \approx 2\omega_c$ as before), and the absence of channels working at a finite phase velocity (the longitudinal velocity scatter does not play any special role) are undisputed advantages of such a device.

5. ELECTROSTATIC AMPLIFIERS (ESA)

5.1. Mechanism of Electrostatic Amplifier

The main idea of the electrostatic amplification of cyclotron waves of the beam is based on the use of the fact that in order for the cyclotron radius β to increase it is sufficient that the effective Doppler-shifted frequency of the external (quadrupole) field acting on the individual moving electron be close to double the cyclotron frequency (see (4.3)–(4.7)). For this purpose, high-frequency pumping is not obligatory ($\omega_p \neq 0$). Resonant interaction (amplification) will be observed also in a twisted electrostatic quadrupole structure, if the pitch of its spatial rotation and the longitudinal velocity of the beam are chosen such that

$$\max|\varepsilon(\theta)| \ll 1, \quad \max|\mu(\theta)| \ll 1, \quad |\varepsilon^{-1}\, d\varepsilon/d\theta| \ll 1, \quad |\mu^{-1}\, d\mu/d\theta| \ll 1, \tag{5.7}$$

become stronger; these inequalities determine the requirements imposed on the rates of change of the profiles of the electric and magnetic fields in the section between the cathode and the input coupling device of the electrostatic amplifier.

5.3. Construction of ESU. Experimental Results

The construction of a low-noise electrostatic amplifier (diftron) proposed by Adler[12] is shown in Fig. 12.

To produce high values of the magnetic field on the surface of the cathode, a ferromagnetic pole piece is used with a magnetization coil separate from the main solenoid. The law governing the variation of the longitudinal component of the magnetic field in the section between the cathode and the working region is chosen close to hyperbolic. In the beam drift region (v_0 = const) this ensures a constant relative change of the cyclotron frequency (μ = const).

The twisted electrostatic quadrupole field is shaped by means of the quadrupole helix. Its diameter can be increased somewhat at the beginning to decrease the role of the lens effects on the boundary of the quadrupole region. The input and output coupling devices are of the usual capacitor type.

A breadboard model of the diftron[13] was constructed for $\omega_c/2\pi \approx 400$ MHz and had on the surface of the cathode a magnetic field intensity corresponding to $\omega_{cc}/2\pi \approx 4000$ MHz. The drift region of the beam, where the field decreases to the resonant value, at a length ~ 50 mm. The beam diameter increased from 0.13 (near the cathode to 0.4 mm (in the working region). The adiabaticity parameter μ was close to 0.015. The beam current was 15 μA at the potential of 6 V. The proper temperature of the slow wave of the beam was obtained to be about 120° K, and the noise figure of the amplifier was 2 dB at a gain of 20 dB.

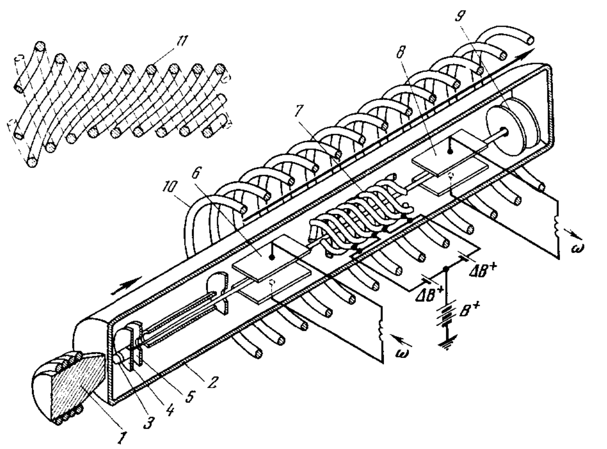

FIG. 12. Construction of ESU (diftron). 1 – Ferromagnetic pole piece with magnetizing coil, 2 – metallic housing, 3 – wafer cathode, 4 – anode passing a beam of 0.13 mm diameter, 5 – focusing electrode, 6 – plates of input device, 7 – quadruple helix producing a twisted electrostatic quadrupole field, 8 – plates of output device, 9 – collector, 10 – turns of solenoid, 11 – quadruple helix with an increased initial diameter.

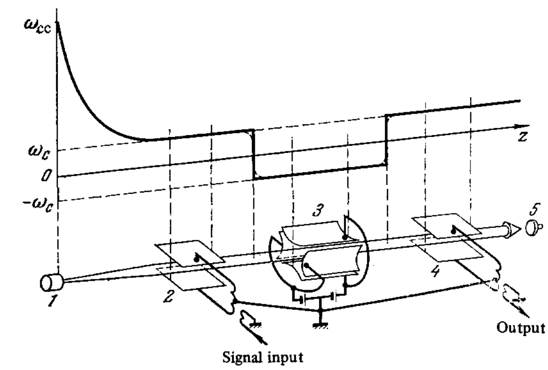

FIG. 13. Diagram of ESU with reversing magnetic field. 1 – Electron gun, 2 – input coupling device of the planar type, 3 – straight electrostatic quadrupole, 4 – output coupling device, 5 – collector.

A somewhat larger noise figure, 3.5 dB, was obtained as expected for a higher-frequency model ($\omega/2\pi$ ~ 1300 MHz) with a smaller field differential (ω_{cc}/ω = 4). These figures, of course, are not limiting and can be greatly improved.

Besides the quadrupole helix, there are other known geometrical structures which make it possible to realize amplification as a result of the active coupling between the fast and slow cyclotron waves (the system of turned quadrupoles, planar systems of the ridge type, etc.). The "smoothest" field can be obtained, of course, in systems with smooth spatial rotation.

In any case, however, spatial rotation of the structure and replacement of the hyperbolic plates by round ones (helix) leads to a screening of the field, causing a decrease of the effective value of V_0 in expression (4.1). For this reason, additional field components of higher order than the quadrupole are excited. All these are undesirable phenomena, and the problem of optimizing the structures still awaits its solution on the engineering and physics levels.

The need for spatial rotation of the quadrupole field can be eliminated in a diftron with a reversible magnetic field[50]. Its diagram is shown in Fig. 13.

If the magnetic field intensity changes jumpwise relative to the moving electron and the transit angle θ does not have time to change appreciably during that time ($|\delta\theta| \ll 1$), then, regardless of the concrete profile of the variation of the field, the amplitudes of the waves are determined only by the initial (ω_{c0}) and final (ω_c) values of the cyclotron frequency*:

$$\mathbf{R}_{1\pm} = [(\omega_c + \omega_{c0})/2\omega_c]\,\mathbf{R}_{01\pm} + [(\omega_c - \omega_{c0})/2\omega_c]\,\mathbf{R}_{02\pm}, \tag{5.8}$$

$$\mathbf{R}_{2\pm} = [(\omega_c - \omega_{c0})/2\omega_c]\,\mathbf{R}_{01\pm} + [(\omega_{c0} + \omega_c)/2\omega_c]\,\mathbf{R}_{02\pm}, \tag{5.9}$$

where $\mathbf{R}_{01\pm}$ and $\mathbf{R}_{02\pm}$ are the amplitudes of the waves before the jumplike change of the magnetic field, and $\mathbf{R}_{1\pm}$ and $\mathbf{R}_{2\pm}$ after the change.

This makes it possible, using a symmetrically-

*This result can be obtained by integrating the system (5.3) and (5.4) at $\epsilon = 0$, $\exp(\pm j\theta) \approx 1$. For the particular case of cosinusoidal variation of the profile of the field in a short magnetic lens, similar expressions were derived in a somewhat different manner in [51].

reversible jump of the magnetic field ($\omega_c = -\omega_{c0}$) to transfer the information from the cyclotron waves to the synchronous waves, to amplify them in an electrostatic (not twisted) quadrupole field, and then extract the amplified signal with usual planar-type coupling devices, using once more the reversing field.

The practical realization of such an amplifier is naturally facilitated by the fact that the concrete profile of the magnetic field in the region of the jump does not play any role. In addition, it is possible to attain complete energy exchange always if the correcting field of an electrostatic lens is introduced in the region of the reversal of the magnetic field[46].

6. CONCLUSION

The low intrinsic noise level (~ 100–200°K and less), the high electric strength (an EPA is known to have operated in a radar without a discharge gap), the operating stability connected with the absence of internal feedback via a slow-wave structure or the beam, the exceptional linearity in a wide dynamic range (100 dB and more), the high phase stability, and the independence of the phase shift in the amplifier of the level of the transmitted power, all these qualities are typical of cyclotron-wave amplifiers and justify the attention paid to them.

The most promising among the parametric devices are amplifiers of the nondegenerate type, which make it possible to eliminate in part or fully the shortcomings connected with the presence of a difference wave. In many cases, however, when the level of the noise coming from the idling channel is negligible or is simultaneously accompanied by useful signals (cosmic communication, radioastronomy, etc.), the advantages of degenerate EPA are most fully manifest.

The electrostatic amplifier (diftron) does not require a high-frequency pump source at all, and has no difference channel, retaining all the basic advantages of supernoiseless amplifiers for cyclotron waves, and in this sense it is the most promising. In addition, the low noise level of the cyclotron waves in the beam makes the bandwidth of the electrostatic amplifier larger and its operation less critical to a mismatch of its input circuits. Most promising for the diftron are apparently distributed coupling devices, since the mechanism of "cooling" the electron beam in this device is valid independently for each individual electron orbit. Therefore, unlike the EPA, the presence of small longitudinal fields in the input and output coupling devices of this type should not be reflected to a noticeable degree in the equivalent temperature of the intrinsic noise of the amplifier. Very promising is the use of distributed systems for the construction, on the basis of the diftron, of supernoiseless amplifiers of the narrow-band type, for the construction of an easily-tuned filter with increased interference immunity[53], etc.

Although definite progress has been attained in the design of amplifiers, and with respect to their technical parameters these devices are already among the most highly perfected input amplifiers for radio receiving installations, many problems have not yet been sufficiently investigated and solved.

The existing theory is highly primitive and makes it possible to describe essentially only the most pronounced effects. In engineering practice it is still necessary to resort frequently to qualitative considerations and to intuition, in view of the lack of reliable theoretical results concerning various questions. In practice, the most effective ways of broadening the bandwidth of the cyclotron-wave amplifiers are still not clear. Although individual theoretical investigations have already been carried out, in the main, only resonant coupling elements are presently used, and these provide a bandwidth on the order of several percent relative to the central frequency.

The question of obtaining a low noise level of cyclotron waves directly in the electron gun has not been sufficiently well investigated. Many possibilities in this respect have apparently not yet been clarified or used. The solution of the problem would greatly facilitate also the preceding problem, for in this case the main function of the input coupling element would be modulation of the electron beam by the external signal, and the demodulation of its noise, which is the most critical to detuning, would no longer play a decisive role. It is possible that individual types of distributed coupling devices, i.e., essentially structures with greater bandwidth, will be developed for low-noise beams.

There are still no published reliable experimental data on the noise level of synchronous waves in electron beams, although knowledge of this level is very important for the development of devices, and the accuracy and validity limits of the theoretical formula (3.8) have long been subject to doubt[35].

Of definite promise for amplifiers is the use of a ribbon beam in lieu of a round one. This makes it possible to reduce the role of the edge effect (the coupling with the synchronous and slow cyclotron waves), but would make it possible to expand the dynamic range, etc. It is possible that such a beam configuration will turn out to be most convenient when distributed coupling devices of the planar types are used.

Finally, due attention has not been paid as yet to problems of micro-miniaturization of the devices. Task-oriented research aimed at decreasing the dimensions and weights, to the use (in part or exclusively) of permanent magnets, etc., can undoubtedly result in great improvements in the technical and operational characteristics of supernoiseless amplifiers of cyclotron waves. It is difficult to predict all the ways and all the results of research aimed at further improvement of the devices. There is no doubt, however, that they offer great promises and that there is much room for improvement.

[1] L. P. Smith and C. I. Shulman, Proc. IRE 35, 644 (1947).

[2] C. L. Cuccia, RCA Rev. 10, 270 (1949).

[3] A. Ashkin, USA Patent, Class 330–4.7, No. 3, 101, 449 (20 August 1963).

[4] A. Ashkin, USA Patent, Class 330–4.7, No, 3, 009, 078 (14 October 1961).

[5] R. Adler and G. Wade, USA Patent, Class 330–4.7, No, 3, 090, 925 (21 May 1963).

[6] R. Kompfner, USA Patent, Class 330–4.7, No. 2, 972, 702 (13 October 1958).

[7] C. F. Quate, USA Patent, Class 330–4.7, No. 2, 974, 252 (7 March 1961).
[8] P. J. Bridges, USA Patent, Class 330–4.7, No. 2, 972, 081 (24 February 1962).
[9] W. H. Louisell, Coupled-mode and Parametric Electronics, Wiley.
[10] V. M. Lopukhin, V. B. Magalinskiĭ, V. P. Martynov, and A. S. Roshal', Shumy i parametricheskie yavleniya v elektronnykh priborakh (Noise and Parametric Phenomena in Electronic Microwave Devices), Nauka, 1966.
[11] V. M. Lopukhin and A. S. Roshal', Electronno-luchevye parametricheskie usiliteli (Electron-Beam Parametric Amplifiers), Soviet Radio, 1968.
[12] R. Adler, USA Patent, Class 330–4.7, No. 3, 315, 173 (18 April 1967).
[13] R. Adler and G. Hrbek, Proc. of 5th Intern. Congress on Microwave Tubes (Paris, 1964), N.Y.–London, Acad. Press, 1965, p. 17.
[14] V. M. Lopukhin and A. S. Roshal', Usp. Fiz. Nauk 85, 297 (1965) [Sov. Phys.-Uspekhi 8, 117 (1965)].
[15] R. Adler, A. Ashkin and E. I. Gordon, J. Appl. Phys. 32, 672 (1961).
[16] J. E. Carrol, J. Electr. and Control., Ser. 1, 16, 633 (1964).
[17] P. L. Kapitza, Elektronika bol'shikh moshchnosteĭ (High-Power Electronics), AN SSNR, 1962, Vol. 1, Ch. I.
[18] N. N. Bogolyubov and Yu. A. Mitropol'skiĭ, Asimptoticheskie metody v teorii nelineĭnykh kolebaniĭ (Asymptotic Methods in the Theory of Nonlinear Oscillations), Fizmatgiz, 1963.
[19] V. Dubravec, Arch. elektr. Übertragung 18, 585 (1964).
[20] A. E. Siegman, J. Appl. Phys. 31, 17 (1960).
[21] V. Dubravec, Electron. Commun. 39, 558 (1964).
[22] C. P. Lea-Wilson, Proc. IRE 48, 255 (1960).
[23] A. Ashkin, Proc. IRE 49, 1016 (1961).
[24] B. M. Lopukhin and V. K. Mitsenko, Vestnik, Moscow State University, Physics and Astronomy Series, No. 1, 80 (1968).
[25] V. I. Kanavets, V. G. Rassadin, and A. S. Roshal', Izv. Vuzov (Radiotekhinika) 9, 622 (1966).
[26] M. M. Kovalevskiĭ and A. S. Roshal', ibid. 6, 1195 (1963).
[27] R. Adler, USA Patent Class 330–4.7 No. 3, 296, 542 (3 January 1967).
[28] R. Adler and G. Hrbek, IEEE Trans. ED-10, 1 (1963).
[29] N. B. Chakraborty, J. Electr. and Control. 10, 147 (1961).
[30] C. C. Wang, USA Patent, Class 330–4.7, No. 3, 251, 001 (10 May 1966).
[31] Yu. Ya. Yurov, Izv. Vuzov (Radiotekhnika) 9, 15 (1966).
[32] E. I. Gordon, Proc. IRE 49, 1208 (1961).
[33] R. A. H. Hart and C. Weber, Philips Res. Rept. 16, 376 (1961).
[34] V. A. Vanke and V. B. Magalinskii, Izv. Vuzov (Radiophysica) 9, 832 (1966).
[35] K. Blötekjaer, J. Appl. Phys. 33, 2409 (1962).
[36] C. P. Lea-Wilson, T. J. Bridges, and J. Vokes, J. Electr. and Control. 10, 261 (1961).
[37] T. Wessel-Berg and K. Blötekjaer, IRE Trans. ED-9, 1388 (1962).
[38] R. A. H. Hart, Philips Res. Rept., Suppl. 4, 1 (1964).
[39] R. A. H. Hart, Proc. IRE 50, 227 (1962).
[40] R. A. H. Hart, Proc. of 5th Intern. Congress on Microwave Tubes (Paris, 1964), N.Y.–London, Acad. Press, 1965, p. 29.
[41] R. Adler, USA Patent, Class 330–4.7, No. 3, 233, 182 (1 February 1966).
[42] R. Adler, G. Hrbek and G. Wade, Proc. IRE 47, 1713 (1959).
[43] G. O. Chalk, Proc. IEE B 108, 125 (1961).
[44] R. Adler, USA Patent, Class 330–4.7, No. 3, 275, 943 (published 27 September 1966).
[45] V. A. Vanke, S. P. Kantyuk, V. M. Lopukhin, and V. L. Savvin, Radiotekhnika i élektronika 14, 478 (1969).
[46] E. I. Gordon and R. C. Miller, J. Appl. Phys. 38, 4911 (1967).
[47] E. I. Gordon, J. Appl. Phys. 31, 1187 (1960).
[48] L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics), Nauka, 1965, Sec. 49 [Addison-Wesley].
[49] A. S. Bondarev and A. I. Tolpeko, Izv. Vuzov (Radiotekhnika) 9, 683 (1966).
[50] R. C. Miller, USA Patent, Class 330–4.7, No. 3, 122, 710 (25 December 1964).
[51] Yu. Ya. Yurov, Izv. Vuzov (Radiotekhnika) 8, 1 (1965).
[52] A. Ashkin, USA Patent, Class 330–4.7, No. 3, 178, 646 (13 April 1965).
[53] S. Mao, Proc. of 5th Intern. Congress on Microwave Tubes (Paris, 1964) N. Y.–London, Acad. Press., 1965, p. 109.

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