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ON THE HISTORY OF THE DISCOVERY OF THE CARNOT THEOREM

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AN opinion is frequently advanced that logical deductions based on analogy are of great importance in the development of science. Polya, for example, even assumes that "analogy apparently plays a part in all discoveries, but in some of them it contributes the lion's share"[1].

It is usually assumed that deductions based on analogy are the more valuable the larger the number of similar attributes in the compared objects. Consequently, the more two objects have in common, the more probable are deductions based on analogy. In the history of science, however, one can encounter as examples where precisely an "incorrect" analogy, i.e., an analogy between fundamentally different phenomena, phenomena of different nature, has led to discoveries of prime significance. Such an example is the research by Sadi Carnot in the first half of the 19th century, reported by him in his work "Thoughts on the Moving Force of Fire and on Machines Capable of Developing This Force." In that paper Carnot, starting from a rather remote analogy and on the basis of an incorrect understanding of the work of a heat engine, obtained a correct result in the form of a theorem bearing his name.

Sadi Carnot undertook to investigate "whether the moving force of heat is bounded or infinite, whether there exists a limit in nature which cannot be exceeded by any method or, to the contrary, whether infinite improvements are possible"[2].

The question that Carnot posed to himself was at that time quite new, and it was necessary to find an approach for its solution. Naturally, Carnot's thought turned to work by his father, Lazare Carnot, devoted to mechanics. L. Carnot, in his "General Experiment on Machines"[3] (1783) and in the later "Fundamental Principles of Equilibrium and Motion"[4] (1803), introducing the concept of work, and investigated the question of how a machine should be constructed in order to minimize the loss of the energy transmitted by its parts. He established that the energy loss (we are using modern terminology) occurs during the work of the machine either as a result of friction or of impact. For the latter case, L. Carnot proved the theorem of the loss of momentum in the case of an inelastic impact, known as the Carnot theorem in mechanics.[5] It follows from this theorem, that in constructing a machine it is necessary to attain a situation wherein the transfer of motion from one part of the machine to the other occurs "smoothly", with equal velocities of the touching parts, i.e., that the transfer of motion occur without a jump in velocity.

S. Carnot, in the investigation of the operation of heat engines, in analogy with the investigation of a hydraulic engine, used the conclusions drawn by his father from the analysis of the work of a machine.

In a hydraulic engine, water falling from a higher level to a lower level performs work. In a heat engine, as reasoned by Carnot by analogy, the heat medium "falls" from the temperature of the heater to the temperature of the cooler, also performing work. Thus, Carnot drew an analogy between the mass of water and the amount of heat, on the one hand, and the head of water and the temperature difference on the other. Carnot wrote: "One can compare with sufficient justification the moving parts of heat with the force of falling water... The moving force of the falling water depends on the head and the amount of water, while the moving force of heat also depends on the amount of heat medium employed and depends on what can be called, and what we shall in fact call the height from which the heat falls, i.e., the temperature difference between the bodies between which the heat is exchanged ([2], pp. 26 and 30).

In order to solve the problem of the maximum work obtainable from a heat engine, Carnot devised an ideal heat engine, operating on the cycle bearing his name.

Here, again, S. Carnot used the analogy with ordinary machines, although he does not say so. For ordinary machines, as deduced by his father L. Carnot, it is necessary to strive to transfer the motion from one body to another at equal velocities of the bodies. In analogy, in a heat engine, the transfer of heat (or, according to the views of S. Carnot—the heat medium) should occur between bodies having the same temperature.

In an ordinary engine, to obtain maximum useful work, it is necessary to reduce friction to a minimum. In the heat engine, the analog of this condition is obviously that all processes be reversible.

Thus, S. Carnot arrived at the idea of investigating the "working ability" of an ideal engine operating in accordance with a reversible cycle consisting of two isotherms and two adiabats, where the working medium is an ideal gas.

Considering now any other heat engine with another working medium, but also working in accordance with a reversible cycle, Carnot could easily prove that its "moving force" should be equal to the "moving force" of the Carnot machine, operating at the same temperature of the heat source and heat sink. To this end it is necessary to connect together these two reversible engines and have one work in the forward direction, and the other in the reverse direction. As a result of one cycle we would find that the working matter has returned to the initial state, the heat medium taken from the heater and transferred to the cooler would return to the heater. And if one adheres to the opinion that a perpetual motion machine is impossible, then it must be assumed that no external work could be performed, i.e., that "the moving force of the heat does not depend on the agents use for its development; its amount is determined exclusively by the temperatures of the bodies, between which the heat transfer is carried out in final analysis[2], p. 30). This is indeed Carnot's theorem as he himself expressed it. If it is taken into account that the concept of "moving force of heat" must be taken to mean the efficiency of any Carnot machine, and if we exclude the mention of the heat medium, then we have here the modern formulation of the so-called first Carnot theorem. Thus, for example, "the efficiency of the Carnot cycle does not depend on the working medium, and is determined only by the temperature of the heater and the temperature of the cooler" ([6], p. 76)*.

Subsequently, while establishing the fundamentals of thermodynamics, R. Clausius and W. Thomson showed that although Carnot used incorrect concepts concerning the nature of heat, his conclusion was nevertheless correct. But in order to obtain this correct conclusion in a correct manner it is necessary to start not from the principle of the impossibility of an ordinary perpetual motion engine, but from a new premise, which was formulated by Clausius and Thomson in different versions, and which subsequently has been named the second law or principle of thermodynamics.

Together with the proof of the Carnot theorem, as given by Clausius and Thomson, it became clear that the analogy used by Carnot is quite limited and that the transfer of heat from a body with a higher temperature to a body with a lower temperature differs in principle from the process of dropping a mass from a higher altitude to a lower one. This analogy could no longer play any useful role and had to be discarded in thermodynamic research.

The situation here was similar to the position with mechanical models of the electromagnetic field, used by Maxwell in the derivation of the equations of this field. At the very beginning, they played a useful role, but then, when the equations were obtained, these analogies no longer were necessary and the role became even harmful. The analogy between the hydraulic engine and the heat engine was likewise very successfully used by Carnot, but subsequently it became unnecessary, since it could not yield anything positive. To the contrary, it could lead to errors, and to divert efforts to fruitless research. However, just as the searches for a mechanical explanation of electromagnetic phenomena continued long after Maxwell, Carnot's analogy, albeit in a somewhat modified form, continued as a subject of research on the part of many scientists.

We should note first that in one of the very first textbooks of thermodynamics, written by Zeuner[7] in 1886, he uses the analogy between the falling of a mass and the transfer of heat from a heated body to a cold body in the performance of the Carnot cycle.

If a mass falls from a height h_1 to a height h_2, then the work performed is

$$W=\frac{U}{h_1}(h_1-h_2),$$

where U is the potential energy of the mass at the height h_1.

On the other hand, the work performed by a Carnot engine, can be written in the form

$$W=\frac{Q}{T_1}(T_1-T_2),$$

where Q is the amount of heat taken from the heater at the temperature T_1, and T_2 is the temperature of the cooler.

These two expressions are similar in form. Consequently, Zeuner reaches the conclusion that there exists an analogy between the falling of a mass and the transfer of heat from a hot body to a cold one (in accordance with the reversible Carnot cycle). The weight U/h corresponds here to the quantity Q/T. Therefore Zeuner proposed to call this quantity the heat weight rather than the entropy.

A similar analogy was used even later. Great interest was attached to such analogies by the energy specialists[8], and also by Mach, who as late as in 1872, in a paper "Conservation of Work" reported this analogy. Mach treated this question in greater detail and more thoroughly in his "Principles of Heat"[9].

Mach's idea, in general outline, reduces to the following, It is possible to write for the Carnot cycle the relation

$$-\frac{Q'}{T_1}+Q\left(\frac{1}{T_2}-\frac{1}{T_1}\right)=0,$$

where Q' is the amount of heat converted into work, Q the amount of heat given up to the cooler, and T_1 and T_2 are the temperature of the heater and the cooler. It follows from this also that

$$-\frac{Q'+Q}{T_1}+\frac{Q}{T_2}=0,$$

or

$$\frac{Q'}{Q+Q'}=\frac{T_1-T_2}{T_1}.$$

But similar expressions can be written also for other cases of the conversion of any type of energy into work.

Indeed, for the case of a volume of water P flowing from a height h_1 to a height h_2 we can write a similar relation

$$-\frac{W'}{h_1}+W\left(\frac{1}{h_2}-\frac{1}{h_1}\right)=0,$$

where

$$W'=\frac{P}{2}(h_1-h_2)$$

is the performed work, and $W = Ph_2/2$ is the potential energy of the falling water. We can also write

$$-\frac{W_1}{h_1}+\frac{W_2}{h_2}=0,$$

where $W_1 = Ph_1/2$ and $W_2 = Ph_2/2$, and finally

$$\frac{W'}{W+W'}=\frac{h_1-h_2}{h_1}.$$

In exactly the same way we can consider that the work performed when an electric charge flows from a higher potential V_1 to a lower V_2. For example: the

*As mentioned above, this statement is frequently called now Carnot's first theorem. By Carnot's second theorem is meant the statement that the efficiency of an irreversible engine is lower than the efficiency of a Carnot engine operating at the same heater and cooler temperature. Carnot did not formulate this theorem specially, although it follows from his reasoning.

working body is a sphere, capable of expanding and contracting, and acquires a quantity of electricity $q' + g$ from a very large conductor with potential V_1, and then performs work (expanding) and changes its potential to a value V_2, after which it is connected to another large conductor of potential V_2. For this case we can also write

$$-\frac{W'}{V_1}+W\left(\frac{1}{V_2}-\frac{1}{V_1}\right)=0,$$

where W' is the energy converted into work, W is the energy transferred to the conductor with the smaller potential.

Generalizing the foregoing, Mach emphasized that for each form of energy we should distinguish between its amount (Energienwert) and its "potential" or "level" (Niveauwert), and that it is then possible to state the following general premise: "If a fraction W' of a total amount of some form of energy $W' + W$ at a potential V_1 is transformed into another form of energy, then the remainder should drop to a potential V_2, and the following equation holds[10]: $-W'V + W(V_2^{-1} - V_1^{-1}) = 0$" ([10], p. 331–332).

Of course, Mach understands the difference between the processes of performance of work as a result of equalization of the temperatures and performance of work as a result of equalization of the "potentials" (heights, electric potentials, etc.). He even emphasizes these differences, indicating, for example, that the temperature difference can vanish without performance of work, and also that the equalization of temperatures does not lead to its oscillations, as, for example, water starts to oscillate in connecting vessels when a difference between the levels in the vessels is produced. However, he attaches very great importance to the stated analogy.

Mach's opinion was shared by many physicists of the last quarter of the 19th century, and primarily by the energy specialists. Planck in his autobiography wrote: "Another dispute arose in connection with the question of the analogy between the transfer of heat from a body with a higher temperature to a body with a lower temperature and the dropping of a weight from a higher height to a lower one. Even earlier, I emphasized the need for distinguishing between these two processes, because they differ from each other in principle to the same degree as the first and second principles of thermodynamics are different. I met with the contradictions and objections that were widely held at that time, and I could not get my colleagues to agree with my point of view. Some physicists even believed that the reasoning of Clausius was too complicated and furthermore unclear. In particular, they did not agree that heat occupies an exclusive position among the various types of energy, owing to the interaction of the concept of irreversibility. To counteract the Clausius theory of heat, they produced the so-called energy theory, in which the first principle, just as in the case of Clausius, was the principle of energy conservation, but second principle, which should point to the direction of everything that takes place, consisted of a complete analogy between the transfer of heat from a body with a higher temperature to a body with lower temperature and the dropping of a weight from a larger height to a smaller one. It followed from this that the concept of irreversibility is not important for the proof of the second principle; the existence of an absolute zero of temperature was rejected by arguing that it is possible to measure only temperature differences, just as in the case of height differences[11].

How can one explain the fact that many prominent scientists could not understand this question, particularly Mach, Ostwald, and others? Without touching upon this entire question as a whole, we note here two circumstances. First, Mach and the energy school adhered to a formal approach to physical concepts and physical laws in general. Second, we wish to emphasize specially that Mach and other positivists were inclined to overestimate the role of analogy in physical theories. Indeed, the main methodological principle of science for Mach was the principle of the "economy of thought." But no one has to be convinced that analogy in science contributes to "economy of thought." In this connection it is known that Mach attached great significance to analogy in the cognition process. "The case of similarity is also... significant in the sense of economy of thinking, extending a known opinion to cover a larger region than before." so wrote Mach in[12]. And since, according to Mach, science does not reflect any objective reality, it follows that by establishing an analogy between different phenomena we move forward in science. Thus, establishment of an analogy between the work of a heat engine and any other engine is in this connection a forward step in science. Whether this analogy corresponds to reality or not, such a question could not occur to Mach or to other positivists.

The development of science has refuted the point of view of Mach, Ostwald, and other energy specialists and positivists. The analogy of Carnot, the analogy between the performance of work by a heat engine and the performance of work using any other type of energy no longer plays any role in physical science. And if we recall history and mention Carnot's discovery*, it is only to emphasize that even the most imperfect analogy is capable of playing an important heuristic role, but that subsequently any analogy sooner or later becomes unnecessary and even capable of retarding the subsequent development of science.

[1] G. Polya, Mathematics and Plausible Reasoning, Russ. transl. IL, 1957, p. 36. [Princeton, 1954].

[2] S. Carnot, Thoughts on the Moving Force of Fire in Machines Capable of Developing this Power, in: Vtoroe nachalo termodinamiki (The Second Law of Thermodynamics), ONTI, 1934, p. 19.

[3] L. Carnot, Essai sur les machines en général, Paris, 1783.

[4] L. Carnot, Principes fondamentaux de l'equilibre et du mouvement, Paris, 1803.

[5] Cf., e.g., N. E. Zhukovskiĭ, Teoreticheskaya mekhanika (Theoretical Mechanics), 2nd Ed., Gostekhizdat, 1952.

[6] I. P. Bazarov, Termodinamika (Thermodynamics), Fizmatgiz, 1961, p. 76.

*It is interesting to note that Carnot's analogy was recently recalled by Feynman and used in his well known lectures on physics [13].

[7] G. Zeuner, Grundlage der mechanischen Wärmetheorie, Leipzig, 1866.

[8] See: J. Popper, Electrische Kraftübertragung, Wien, 1884; G. Helm, Die Lehre von der Energie, Leipzig, 1887; R. Wronsky, Das Intensitätsgesetz, Frankfurt, 1888; W. Meyerhoffer, Der Energieinhalt, Zs. Phys., 544 (1891).

[9] E. Mach, Die Principien der Wärmelehre, Wien, 1896; 2. Auflage, Leipzig, 1900.

[10] E. Mach, Die Principien der Wärmelehre, Leipzig, 1900, p. 331.

[11] Maks Plank, Sb. k stoletiyu so dnya rozhdeniya Maksa Planka (Anthology in Honor of his 100th Birthday), AN SSSR, 1958, pp 19–20.

[12] E. Mach, Knowledge and Delusion (Russ. transl.) M. 1909, p. 232.

[13] R. Feynman, Feynman Lectures on Physics, No. 4, Ch. 44, Addison-Wesley, 1963.

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