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THE COUNTS OF RADIO SOURCES

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INTRODUCTION

RADIO Astronomy's association with cosmology began in 1950 when it was discovered that one of the most intense of the "radio stars" which had been discovered within the previous few years was associated with a distant galaxy. This "radio star," the well-known radio source Cygnus A, was intrinsically so powerful at radio wavelengths that similar objects could be detected at very great distances—in fact, at distances well beyond the limit of optical telescopes. These distances are so great that the radio waves were emitted by the sources when the Universe was significantly younger than its present age and hence the hope was that by studying very faint radio sources it might be possible to determine the large-scale structure and evolution of the Universe.

It was a simple matter to modify all the "classical" cosmological tests which had been investigated for optical observations to radio frequencies. Unfortunately, most of these tests involve a knowledge of the distance of the objects being considered and from the radio data alone there is no way of determining distances. Radio spectra are generally featureless so that it is impossible to determine redshifts as can be done from the lines in optical spectra. Even at the present day when thousands of radio sources are known, the exact distances of only a small fraction of these are known since the only way to determine the distance of a radio source is to associate it with an optical object for which a redshift may be determined. This can only be done when very accurate radio positions are known and in general these are only available for relatively intense sources. Even when such accurate positions are known there is no guarantee that it will be possible to identify the radio source with an optical object since it may be too distant to be detected by the largest modern telescopes.

Besides these difficulties which will presumably become less with time as more effort is expended on the study of radio sources, there are other more fundamental ones in using radio sources as cosmological probes. The aim of "classical" cosmology was the determination of the large scale dynamics of the Universe from observations of distant galaxies. The solutions of the Einstein field equations derived by Friedmann show how the propagation of light signals is related to the dynamics and matter distribution in the Universe. Most classical cosmological tests involve selecting a suitable "standard candle" or "rigid rod" and noting how the observed properties change when the standard is observed at different distances. The variation of the observed properties of the standard with distance reflects the geometry and hence the dynamics of the Universe.

It is now known that it is very difficult to select such standard sources from the radio source population because of the very great dispersion in the intrinsic properties of extragalactic radio sources. Their intrinsic radio luminosities cover a range of at least $10^6:1$ whilst their physical dimensions range from hundreds of kiloparsecs to tens of parsecs. It has proved impossible to find criteria by which to select radio sources of similar intrinsic properties although the search continues to discover new correlations which have not been noticed before. Under these circumstances, the only way of performing these tests of world models is on a statistical basis in which the average properties of a local and distant sample are compared. There are many difficulties in these types of test but it is now becoming possible to perform such tests and an example of this using radio observations will be discussed in Sec. 4.

There is, however, another fundamental difficulty about all such cosmological tests. In the classical cosmological tests it is always essential to make observations of very distant objects so that the large-scale dynamics of the Universe when the light or radio waves were emitted are different from those at the present day, that is, at epochs significantly earlier than the present. There is therefore no way of ensuring that the "standard candles" or "rigid rods" have not changed with time. There is no way in which such an effect could be distinguished from changes due to the difference between the geometries of different world models.

Because of the large scatter in the properties of radio sources it has proved impossible to determine their evolution directly from the observations. At present there does not exist a well defined correlation diagram corresponding to the Hertzprung-Russell diagram for stars from which it is possible to infer the evolution of stars of different masses. In the future it may be possible to determine this evolution directly from the observations but at present there does not seem to be any order in the observed properties of sources. Of course, if the astrophysics of these objects were understood in sufficient detail, it would be possible to make allowance for such evolutionary changes but the physics of extragalactic radio sources and galaxies are very poorly understood at present and there is no prospect of making satisfactory estimates of evolution from present-day theory.

It therefore appears that it is difficult to obtain any useful cosmological information from a study of radio sources. However, this is not so. It turns out that one of the simplest cosmological tests gives a result which is in complete contradiction with the predictions of all simple world models and which demands a cosmological explanation. This test is the counts of radio sources.

Counting sources is, in principle, a particularly simple and attractive test since it merely involves determining the total number of sources brighter than different limiting values of observed intensity. At optical wavelengths, this test has proved very difficult because of first and second order clustering of galaxies and, in addition, the only objects for which statistics are available extend to redshifts, Z, less than 0.2 where the differences between different world models are very small. As will be discussed later, however, there are no such problems with the distributions of radio sources.

The counts of radio sources indicate that there exist many more faint radio sources than would be expected in all simple world models. The simplest explanation of this effect is that, in the past, there were many more powerful radio sources than there are at the present day—that is, the properties of the radio source population have changed or evolved with cosmological epoch.

These observations and their interpretation have caused much controversy between the proponents of the steady-state theory and the observers. The steady-state theory in its simplest form results in a unique world model in which the large scale structure and dynamics of the Universe are completely determined and therefore this theory is particularly susceptible to observational verification. The reason for this is that the "cosmological principle" of relativistic cosmological models according to which the Universe looks isotropic to any observer is replaced by the "perfect cosmological principle" according to which the Universe not only looks isotropic to any observer but presents exactly the same aspect to all observers at all times. Since the Universe is expanding it is necessary to replace the matter which is dispersing and for this it is necessary to introduce the concept of the continuous creation of matter. The observation that at large distances there exists a greater density of radio sources than there is locally is in direct contradiction with the perfect cosmological principle. There have been many attempts to reconcile the counts of radio sources with steady-state cosmology but none of them is consistent with all the present observations.

It might be thought that a way of circumventing these difficulties would be to postulate that the redshifts of extragalactic radio sources are not entirely of cosmological origin. For example, they might be partly due to the presence of strong gravitational fields in radio sources. However, whatever the origin of the redshift, the counts of radio sources indicate that the Earth must be situated in a minimum of the distribution of radio sources. It can be shown that the likelihood of the Earth being located at the centre of such a region by chance is very small (Ryle and Pooley, 1968). For these reasons it seems likely that the sources belong to a distant cosmological population.

Recently there has been much interest in world models which include the cosmological constant, Λ. It has been introduced in an attempt to explain some anomolies in the distribution of emission and absorption redshifts in the spectra of quasi-stellar objects. In principle, it is possible in certain models (Pertrosian, 1969) to obtain a source count having slope greater than 1.5 but if we take into account the dispersion in luminosity of radio sources, the slope of the counts cannot exceed 1.5.

Besides this direct evidence of evolution, the counts tell us nothing about the geometry of the Universe. We may, however, learn a great deal about the distributions of extragalactic radio sources throughout the Universe and how they have evolved with time.

An analysis of the source counts shows that there are rather powerful limits which may be set to possible modes of evolution of the radio source population. Firstly, the effects of evolution are very strong so that at a redshift of 2 the relative space density of sources was roughly 1000 times greater than it is at the present day. Secondly, this powerful evolution refers only to the most powerful radio sources, the evolution of intrinsically weaker sources being much less pronounced or non-existent. These conclusions which were originally derived from the counts of radio sources have been confirmed by recent data on the redshifts of quasi-stellar objects and by the most recent surveys of radio sources.

The physical nature of cosmological evolution of this type at radio wavelengths is not at all clear. As has already been stated above, our understanding of the physics of radio sources is in a very primitive state. The simplest interpretations of this behaviour might be that in the past radio sources were born at a much greater rate or that the properties of a radio source depend upon the physical conditions at large redshifts. However, problems such as why an extragalactic object becomes a radio source and the detailed physics of the evolution of individual radio sources are still unsolved.

In the present paper, we will not discuss this aspect of cosmological evolution which is undoubtedly the most important of the physical problems arising from the counts of radio sources. We will concentrate on showing in how much detail we can determine this evolution from the observations.

Besides its obvious importance for the physics of radio sources where, by good fortune, we may study their behavior in different physical conditions automatically by studying them at large redshifts, the evolution of radio sources is related to a wide range of other problems ranging from the origin of cosmic rays to the observed X- and γ-ray background radiation.

In view of the importance of a knowledge of the cosmological evolution of radio sources, it is important to clarify.

a) What is the evidence for the necessity of cosmological evolution?

b) In what detail can we determine the evolution of the radio source population?

In Sec. 2, I will discuss the observations and in Sec. 3 the interpretation of these results. In Sec. 4, I will discuss the most recent observations and their interpretation and finally in Sec. 5 some of the applications of these results to other branches of astronomy.

2. THE OBSERVATIONS

Throughout the 1950's, there was great rivalry between the Cambridge and Sydney groups, each observatory producing alternately catalogues which superceded and disagreed with the previous catalogues. It is now clear that these discrepancies arose from an incomplete understanding of the various selection effects, both observational and instrumental which are important in the reduction of complete surveys and counts of radio sources. The catalogues of sources produced since 1960 are now in complete agreement.

The important catalogues of radio sources produced since 1960 which are now the standard reference catalogues and on which the counts of radio sources are based are.

a) The Revised 3C Catalogue (Bennett, 1962) contains the 328 most intense radio sources observed at 178 MHz in the Northern sky having declinations greater than $\delta \geq -05^\circ$ and flux densities, $S_{178} \geq 9.0 \times 10^{-26}$ w m^{-2}Hz^{-1}.

b) The 4C Catalogue (Pilkington and Scott, 1965; Gower, Scott and Willis, 1967) contains almost 5000 sources having $S_{178} \geq 2 \times 10^{-26}$ w m^{-2}Hz^{-1} in the range. $-07^\circ \leq \delta \leq 80^\circ$. This survey was made with an interferometer and therefore includes only sources of small angular size, in general less than 2.5. This is a great advantage from the point of view of extragalactic astronomy since it automatically excludes nearly all galactic sources which have much larger angular sizes.

c) The Parkes Catalogue (Bolton, Gardner and Mackay, 1964; Day et al, 1966; Shimmins et al., 1966). This survey includes the area of sky $+20^\circ > \delta > -90^\circ$ to a lower limit of flux density of about 2×10^{-26} wm^{-2} Hz^{-1} at 408 MHz. In this survey, flux densities are also measured, at 750 MHz and 1400 MHz so that it is possible to obtain estimates of the counts at different frequencies. It must, however, be remembered that the catalogue is only complete at the finding frequency of 408 MHz.

Another survey of importance for the extension of the counts to small flux densities is the North Polar Survey of Ryle and Neville (1962). The survey, the first supersynthesis survey, covered only a small area of sky around the North Celestial Pole but included sources of flux density as small as 0.25×10^{-26} wm^{-2} Hz^{-1}.

The source counts at 178 MHz from the 4C survey and the North Polar Survey were derived by Gower (1966) who performed a careful analysis of the various selection effects which could bias the counts. Examples of particular effects are a) the linearity of the flux density scale b) the effect of resolution of extragalactic sources. c) the effects of clustering of sources and d) the effect of weak sources lying within the beam of the telescope when a radio source is observed (confusion). This last effect which was responsible for much of the earlier troubles was discussed and analyzed in detail.

After these effects were taken into account, Gower derived the counts shown in Fig. 1. In this figure N is the total number of sources per steradian having flux densities greater than different limiting values of flux density, S. This relationship between N and S is referred to as the counts of radio sources. Within the range $20 > S_{178} > 2 \times 10^{-26}$ wm^{-2} Hz^{-1} the slope of the counts is -1.8. Below 2×10^{-26} wm^{-2} Hz^{-1}, the slope decreases suggesting that the counts are beginning to converge.

Such a convergence had already been predicted by Hewish (1961) from a statistical analysis of some of the original records of the first 4C survey strips. Hewish, using the method developed by Scheuer (1957) determined the distribution h(D) which is the probability distribution of deflections D on the original survey records. The deflections D are measured from the zero of the interferometer records. This function contains information about the distribution of sources at flux densities far below those at which sources can be resolved individually because of confusion. Hewish showed that the counts must begin to converge below 2×10^{-26} Wm^{-2} Hz^{-1} and this result was confirmed by the survey of Ryle and Neville.

The counts from the surveys of the Southern Hemisphere confirmed the slope of -1.8 at high flux densities at 408, 750 and 1400 MHz. The slope of -1.8 was also found by the group working at Illinois (Macleod et al., 1965) and by the group at Bologna (Bracessi et al., 1966) but since in these cases the statistics and the range of flux densities are smaller than the Cambridge results, I will only discuss the latter.

Before turning to the interpretation of these results, it is important to discuss the isotropy of the radio source population. On large scales, $\theta \sim 90-180^\circ$ an anisotropic distribution of sources might be related to an anisotropic expansion of the Universe. On small scales, $\theta \lesssim 10^\circ$, any anisotropies might be connected with clusters or superclusters of radio sources. The distribution of sources in the 4C catalogue was analyzed statistically by Holden (1966) and no departures from isotropy and homogeneity detected on any scale. It is interesting that in this analysis the region of the Galactic Plane was included but no excess of sources found in these directions. This confirms the supposition that nearly all of the sources in the 4C catalogue are extragalactic.

By applying Scheur's statistical method to all the original records of the 4C survey, Hughes and Longair (1967) were able to show that there is no evidence for any anisotropy in the distribution of sources having $S_{178} > 0.2 \times 10^{-26}$ Wm^{-2} Hz^{-1}.

Nowadays, observations of the relict radiation provide much more powerful tests of the isotropy and homogeneity of the Universe at much larger redshifts than those accessible by observations of radio sources (e.g. Partridge and Wilkinson, 1967; Conklin and Bracewell, 1967). However, from the point of view of the relation between radio sources and clusters of galaxies and the population of galaxies in general, it is very important to know whether, for example, they tend to form clusters of radio sources or not. There is no evidence yet for any clustering of radio sources.

A further very important piece of observational data is the observed background radiation attributable to extragalactic radio sources. Bridle (1967) has derived a figure of $30^\circ \pm 7^\circ$ at 178 MHz. This piece of data places a powerful restriction upon possible models of the distribution of radio sources once evolution is included.

3. INTERPRETATION

The counts of radio sources have been discussed by many authors. The principle discussion of the counts and Steady State cosmology is that of Ryle and Clarke (1961). Detailed interpretations of the counts in evolutionary cosmological models have been given by Davidson and Davies (1964), Longair (1966) and McVittie and Schusterman (1966). Below I will follow my own presentation of the problem (Longair, 1966).

In a static infinite Universe which is filled uniformly with radio sources of intrinsic radio luminosity P having space density $\rho(P)$, the counts of radio sources can easily be shown to be given by

$$N(S) = \frac{1}{3} S^{-\frac{3}{2}} \int \rho(P) P^{\frac{3}{2}} dP, \qquad (1)$$

The integration sums the counts over all radio luminosities. Of course, this formula assumes that there is no

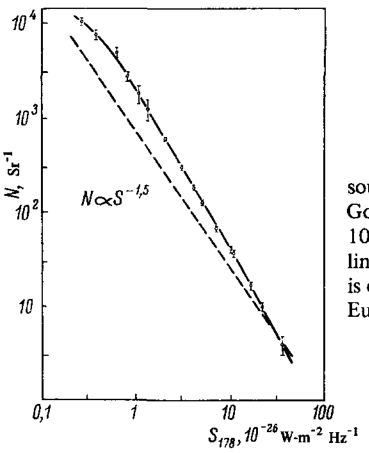

FIG. 1. The counts of radio sources at 178 MHz derived by Gower (1966) to $S_{178} = 0.25 \times 10^{-26}$ wm^{-2} Hz^{-1}. The dashed line represents $N \propto S^{-1.5}$ which is expected in a homogeneous Euclidean Universe.

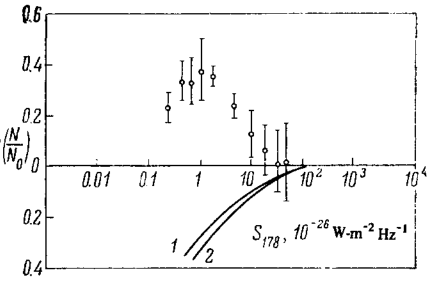

FIG. 2. The counts of radio sources replotted as N/N_0 where N_0 is the number of sources expected in a Euclidean Universe, $N_0 \propto S^{-1.5}$. The thin lines show the predictions for (1) the Einstein-de Sitter model for which $q_0 = ½$, $\Omega = 1$, $\lambda = 0$ and (2) steady state cosmology.

cosmological evolution. This is referred to as the Euclidean counts of radio sources and always has $N \propto S^{-3/2}$. This relationship is compared with the observations in Fig. 1.

In the world models of general relativity Eq. (1) is replaced by a more complicated expression which takes into account the expansion of the Universe, according to the different world models and the effects of time dilation. All the simple expanding world models predict less sources at low flux densities than the Euclidean model (Scheuer, 1969). This is also true of steady-state cosmology. To illustrate more clearly the discrepancies between the counts and the predictions, it is convenient to plot the counts as N/N_0 where N_0 is the Euclidean value of the counts. This is illustrated in Fig. 2.

It might be thought that it is possible to normalize the counts of radio sources at 1×10^{-26} Wm^{-2} Hz^{-1} so that rather than an excess of distant sources, we have a local deficit of sources. In practice, however, we require an irregularity, or local deficit, on the scale of the Universe itself which amounts to the same thing as requiring cosmological evolution on a large-scale.

From Figs. 1 and 2 it is clear that there is a great discrepancy between the observations and theory. The simplest way of explaining this difference is to postulate that in the past there were many more powerful sources than there are at the present day. Note that this statement refers to the mean properties of sources since generally their lifetimes are much shorter than cosmological time-scales. Such a postulate is inadmissible in steady-state cosmology since by definition the Universe always presents the same aspect to any observer.

Table I. The luminosity function of extragalactic radio sources at the present epoch
(P is radio luminosity at 178 MHz; the space densities in column 2, $\rho'(P)$ are defined by

$$\rho'(P) = \int_{P/10^{1/5}}^{10^{1/5}P} \rho(P)dP,$$

i.e., $\rho'(P)$ is the density of sources in equal logarithmic intervals of luminosity P.

P, $w\text{-}Hz^{-1}$ sr^{-1}	$\rho'(P)$, pc^{-3}	P, $w\text{-}Hz^{-1}$ sr^{-1}	$\rho'(P)$, pc^{-3}
$2.5 \cdot 10^{27}$	$3.5 \cdot 10^{-29}$	$6.3 \cdot 10^{23}$	$9.0 \cdot 10^{-24}$
$1.0 \cdot 10^{27}$	$1.7 \cdot 10^{-28}$	$2.5 \cdot 10^{23}$	$9.4 \cdot 10^{-24}$
$4.0 \cdot 10^{26}$	$9.0 \cdot 10^{-28}$	$1.0 \cdot 10^{23}$	$1.8 \cdot 10^{-23}$
$1.6 \cdot 10^{26}$	$4.4 \cdot 10^{-27}$	$4.0 \cdot 10^{22}$	$4.7 \cdot 10^{-23}$
$6.3 \cdot 10^{25}$	$2.4 \cdot 10^{-26}$	$1.6 \cdot 10^{22}$	$2.0 \cdot 10^{-22}$
$2.5 \cdot 10^{25}$	$9.0 \cdot 10^{-26}$	$6.3 \cdot 10^{21}$	$8.3 \cdot 10^{-22}$
$1.0 \cdot 10^{25}$	$3.2 \cdot 10^{-25}$	$2.5 \cdot 10^{21}$	$2.1 \cdot 10^{-21}$
$4.0 \cdot 10^{24}$	$9.6 \cdot 10^{-25}$	$1.0 \cdot 10^{21}$	$6.7 \cdot 10^{-21}$
$1.6 \cdot 10^{24}$	$3.3 \cdot 10^{-24}$	$4.0 \cdot 10^{20}$	$4.5 \cdot 10^{-21}$

Once we attempt to express this evolution in some quantitative form, however, there are many difficulties. We must know the following.

i) The local luminosity function of extragalactic radio sources—$\rho(P)$. This is defined to be the space density of sources of different luminosities at the present epoch. (Longair and Scott, 1966).

ii) The choice of world model,

iii) The choice of the form of evolution.

The local luminosity function can only be determined for relatively local sources which have been optically identified. For high luminosity sources which have, typically, redshifts greater than 0.2, it becomes very difficult to determine the local luminosity function for the following reason—we know that evolution is important in that there are more powerful sources present in our samples than we expect and hence it is impossible to distinguish how much of the observed distribution of powerful sources is due to evolution and how much corresponds to the local luminosity function without evolution. This problem is fundamental to the determination of the function for high luminosities and can only be solved by a process of model fitting. We make a reasonable guess at the local function and then incorporate sufficient evolution to satisfy the counts—if the guess was a good one, the predicted distribution of sources will correspond to the observed number of powerful sources. This procedure and the resulting luminosity function are described by Longair (1966). Modifications to the luminosity function on the basis of more recent data related to the optical identifications of bright galaxies have been derived by Caswell and Willis (1967). In Table I, we give the best present estimates of the luminosity function of extragalactic radio sources over a wide range of radio luminosities at the

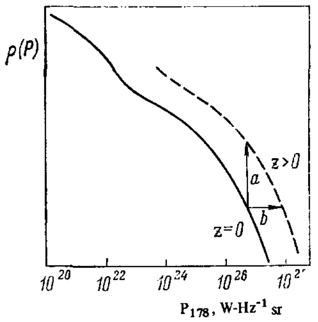

FIG. 3. Changes with epoch of the luminosity function of extragalactic radio sources illustrating the equivalence of the density and luminosity evolution hypotheses.

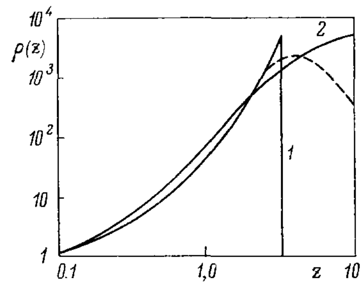

FIG. 4. Possible variations of the evolution of radio sources at large redshifts which are compatible with the source counts. (a) was considered by Longair (1966), (b) by Doroshkevitch, Longair and Zeldovich (1969).

present epoch (i.e., allowance has been made at high radio luminosities for the effects of evolution).

The choice of cosmological model is relatively unimportant since the differences between the geometries of different world models are small compared with the differences between the observations and these predictions. Therefore, the simplest world model has been used, the Einstein—de Sitter model for which $\Omega = 1$, $q_0 = \frac{1}{2}$, $\Lambda = 0$. This model is consistent with optical determinations of the decelaration parameter $q_0 = 1 \pm 1$ (Sandage, 1968).

The evolutionary changes to the luminosity function are shown schematically in Fig. 3. (Note that this diagram refers to comoving coordinates—that is, coordinates which expand with the Universe). The important change is that there were relatively more sources in the past and how we care to describe this mathematically is merely a matter of computational convenience. Note that these changes are necessary, over and above the increase in density of sources in the past as a result of the expansion of the Universe. Extreme ways of describing the changes are to postulate that either

a) the average density of sources was greater in the past, corresponding to (a) in Fig. 3 or

b) the mean luminosity of sources was greater in the past, corresponding to (b) in Fig. 3.

Note, however, that these are introduced for mathematical convenience—there is not necessarily any physical difference between the two types of evolution.

Evolution of the form

a) $\bar{\rho}(Z) \propto (1 + Z)^{\beta}$

or b) $\bar{P}(z) \propto (1 + z)^{\gamma}$

were considered. The bars symbolize the fact that the evolution refers to average values rather than to individual sources.

It was found that it was not easy to obtain satisfactory agreement with all the observational data and that the observations placed rather severe restrictions upon the evolution of radio sources.

a) Strong evolutionary effects are necessary. To obtain the observed slope of the counts -1.8, it is necessary to include strong evolution and in the forms considered above $\beta \sim 5.4$ and $\gamma \sim 3.3$. This means, for example, that in the density evolution hypothesis, the co-moving space density of sources (or the probability of occurrence of a radio source) was 40 times greater at a redshift of 1 than at the present epoch.

b) The strong evolution must terminate at a redshift in the range $2.3 < z < 4$. In the forms chosen, as z tends to infinity, the space density of sources and hence the source counts and integrated radio background emission diverge. To prevent this happening, the strong evolutionary effects must be terminated at a redshift in the range $2.3 < z < 4$. It is impossible to determine the behavior of the radio source population beyond this cut-off redshift except to say that the evolution must be much less rapid. The forms of evolution illustrated in Fig. 4 are permissible. (for discussion, see Doroshkevitch, Longair and Zel'dovich (1969)). This topic will be considered briefly in Sec. 5.

c) Not all luminosity classes of radio source can evolve in this way. There are two related reasons for this. Firstly, if all luminosity classes of radio source evolve in this way, the integrated background radio emission will exceed the observed value of 30°K at 178 MHz by a large factor. Secondly, the observed convergence of the counts below 1×10^{-26} Wm^{-2} Hz^{-1} cannot be reproduced. This can be understood qualitatively from the fact that the distribution of radio luminosity at a given flux density has a range of about $10^5:1$ whereas the excess of sources occurs over a range of only $10^2:1$ in flux density. Only by restricting the powerful evolutionary effects to high luminosity sources having $P_{178} > 10^{26}$ W-Hz^{-1} sr^{-1} is it possible to satisfy all the observational data. It is interesting that this class of powerful radio sources coincides with the quasi-stellar radio sources (QSO's) which must form a substantial fraction of the evolving component of the radio source population (see Sec. 4 (b) below).

These are the conclusions which could be drawn from the radio source counts up to 1966. In the next section, I will consider the most recent developments.

4. RECENT DEVELOPMENTS

a) The 5C surveys of Radio Sources.*

The Cambridge one-mile radio telescope came into operation in 1964 and one of its main programs has been the extension of the counts of radio sources to very small flux densities. These surveys which are referred to as the 5C surveys cover only a small area of sky (approximately 3° in diameter at 408 MHz) but within this area sources can be detected to $S_{408} = 0.01 \times 10^{-26}$ Wm^{-2} Hz^{-1}. The survey frequency of the 5C surveys is 408 MHz and it was therefore necessary to redetermine the complete counts at this new frequency.

*For an excellent survey of the counts of radio sources and a detailed discussion of the 5C surveys, reference should be made to the article by Ryle (1968).

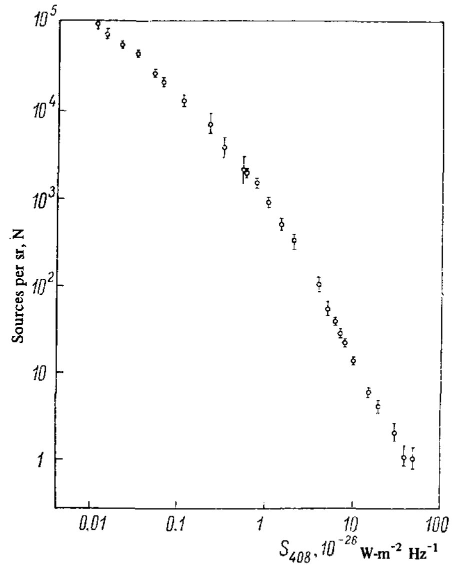

FIG. 5. The counts of radio sources at 408 MHz derived by Pooley and Ryle (1968) to $S_{408} = 0.01 \times 10^{-26}$ wm^{-2} Hz^{-1}.

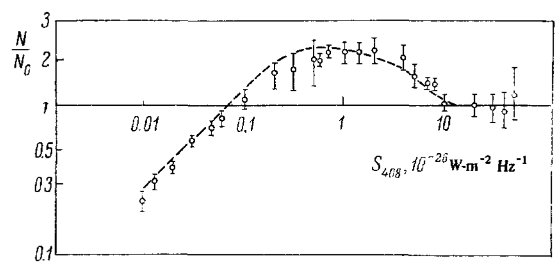

FIG. 6. The counts of radio sources at 408 MHz replotted as N/N_0 (as in figure 2). The dashed line shows the predictions of one of the models discussed in Section 3.

It is not a safe procedure to extrapolate counts between different frequencies since there is no guarantee that the evolutionary behavior of radio sources is the same at different frequencies.

The analysis of the counts at 408 MHz has been described by Pooley and Ryle (1968) who used the 5C1 and 5C2 surveys (Kenderdine, Ryle and Pooley, 1966; Pooley and Kenderdine, 1968) in conjunction with an additional survey (Bailey and Pooley, 1968) which used the one-mile telescope as a transmit instrument and which enabled the counts at higher flux densities to be determined. The resulting counts are shown in figure 5. There is complete agreement with the counts at 178 MHz and it can be seen that the convergence implied by the work of Hewish (1961) and the North Polar Survey is dramatically confirmed. It is interesting that the slope of the counts in the range $0.1 > S_{408} > 0.01 \times 10^{-26}$ Wm^{-2} Hz^{-1} is -0.8 which is very different from the value of -1.5 to which all expanding world models approximate locally. Therefore, in any cosmological model, the radio sources in the 5C surveys are a very distant sample and it is not surprising that it has been impossible to identify more than a few percent of the sources, ($\sim 5\%$) despite the very high accuracy with which the positions of the sources are known.

In Fig. 6, the counts are replotted as N/N_0 and compared with the predictions of the evolutionary world models discussed in Sec. 3. It is clear that the old models are in very good agreement with the new observations and all the conclusions drawn in Sec. 3 may be applied to the new observations. It is interesting that the total integrated background radiation from discrete sources detected in the 5C surveys account for roughly one half of the observed background and extrapolation of the counts a further factor of 10 in flux density with the same slope of -0.8 could account for all the observed background emission.

The most recent surveys at other frequencies are completely in agreement with the new Cambridge observations. Particularly interesting is the recent survey of Shimmins, Bolton and Wall (1968) at 2700 MHz. They found the slope of the counts from their deep survey was -1.4 and there was a suggestion that perhaps there was some disagreement with the usual figure of -1.8. When the appropriate ranges of flux density are compared, however, it is found that the slopes of the Cambridge counts are also -1.4 (Pooley, 1968). The slope of -1.8 refers to the high flux density region and there is general agreement from other recent surveys that this is also true at high frequencies, namely, from the work of Galt and Kennedy (1968) at 1420 MHz and Kellermann et al at 6 cms.

b) The Distribution of Quasi-Stellar Objects

Since the discovery of quasi-stellar objects (QSO's) in 1963, there has been a great effort to elucidate their properties and understand their physical nature (for review, see Burbidge and Burbidge, 1968). It was found that many QSO's are among the most powerful radio sources and by 1967 there was sufficient data available to test whether or not the distribution of QSO's agreed with the types of evolutionary cosmological models described in Sec. 3. There have been two detailed discussions of this problem which will be discussed here—those of Longair and Scheuer (1967) and Schmidt (1968).

The analysis of Scheuer and the author was stimulated by the work of Burbidge and Hoyle (1967) who claimed that a redshift—magnitude relation for QSO's at radio and optical wavelengths did not exist. On the cosmological hypothesis of the origin of the redshift, it is expected that the most distant QSO's with the largest redshifts have the faintest apparent magnitudes or flux densities but in fact the correlation diagrams of apparent magnitude and flux density against redshift had the appearance of "scatter" diagrams. We made a detailed statistical analysis of these relationships using 75 QSO's for which data was then available.

Indeed, it was found that there was no evidence for any relationship between small radio fluxes and large redshifts for all the sources in the sample, although there appeared to be a significant relationship for compact sources. At optical wavelengths, however, a significant redshift-magnitude relation was found, contrary to the claim of Burbidge and Hoyle.

This relationship, however, does not necessarily imply that redshift is related to distance for the following reason. The relationship between flux density (or apparent magnitude) and luminosity may be written

$$S = \frac{P}{[\sin(Ar)/A]^2 (1+z)^{1+\alpha}},$$

where r is a measure of the geodesic distance of the QSO, A^2 is the curvature of the cosmological model at the present day and α is the spectral index of the source (defined by $S \propto \nu^{-\alpha}$). The first factor of the denominator describes the fact that the radiation is spread out over a surface of $(\sin Ar/A)^2$ and contains the dependence of apparent intensity upon distance. The second factor describes the effects of time dilation and is present no matter what the source of the redshift which may be an ordinary Doppler shift (e.g. Terrell, 1964), a gravitational redshift (Hoyle and Fowler, 1967) or a cosmological redshift. Note that in this discussion, the luminosity of the source in the case of a gravitational redshift refers to that measured by an observer situated at the source rather than that measured by a distant observer. The latter is the usual definition of the luminosity of a source in a strong gravitational field. Therefore before we can test whether there is a genuine relationship between apparent magnitude and distance, we must first remove this effect of time dilation. Fortunately, this can be done simply since the spectra of sources at optical and radio wavelengths are now well-known.

At radio wavelengths, it is found that what was originally a "scatter" diagram becomes a redshift-magnitude relation, but one in which the sources which are observed to be brightest have the largest redshifts—that is, the exact opposite of our usual expectations! The redshift-magnitude relation for compact sources disappears. These effects are readily explained, however, in the evolutionary cosmologies described in Sec. 3 since at large redshifts there are many more intrinsically bright sources and this can entirely account for the reversal of the redshift-magnitude relation. The observed diagrams are found to be in excellent quantitative agreement with the prediction of the models of Sec. 3.

At optical wavelengths, the relation between redshift and apparent magnitude disappears, the distribution of points being statistically indistinguishable from a random distribution. We performed a detailed analysis of possible causes of this effect, in particular, whether it may be due merely to the fact that there is a very broad spread in the absolute magnitudes of QSO's which would blur out any real relation. We concluded that this was unlikely and suggested two possible explanations of this effect. Either redshift is not related to distance, as has been suggested by Burbidge and Burbidge (1967) or the effects of evolution are also present at optical as well as at radio wavelengths. If the latter interpretation is correct, then it implies that it is impossible to use QSO's in cosmological tests involving the redshift-magnitude relation since as was stressed in Sec. 3, when evolution is present it is impossible to distinguish changes in the spatial density of sources from changes in their mean luminosity.

The analysis of Schmidt (1968) is based upon a complete sample of QSO's from the 3C catalogue. The test used is the luminosity-volume test in which the volume of space enclosed by a QSO, V, is compared with the volume, V′, within which it could lie and still appear in the complete sample of sources. For a uniform distribution of sources (in comoving coordinates) the mean value of V/V′ should be 0.5. A detailed analysis of this test for a wide range of different cosmological models has been performed by Rowan-Robinson (1967). In fact, for Schmidt's complete sample, the mean value of V/V′ is 0.7 implying that sources are concentrated towards the limit of their observable volume. Schmidt explained this as an evolutionary effect of the density of sources. The amount of evolution required to explain the excess of sources at large redshifts agreed very well with the amount of evolution derived from the total source counts—in fact surprisingly well! It appeared as if the new redshift data provide an independent confirmation of the evolution derived from the counts but this is not so.

The important point is that for most of the QSO's in Schmidt's complete sample, the limiting volume V′ is determined by the lower flux density limit of the Revised 3C catalogue and not by the limiting optical magnitude. The effect of this can be illustrated by considering a simple Euclidean Universe in which the counts of sources are described by $N \propto S^{-\beta}$. Then the mean value of V/V′ is $\beta/(1.5+\beta)$—that is, $\overline{V/V'}$ is related to the slope of the source counts. For the actual distribution of sources where redshifts of 1 and greater are encountered, this relation of V/V′ and β is not correct since proper world models must be used but this formula serves to illustrate the interdependence of $\overline{V/V'}$ and β. Therefore the two tests are not independent and, essentially, Schmidt's method is a rather complicated way of determining the counts of radio sources. Incidently, it proves beyond doubt that QSO's have a source count with slope steeper than −1.5, which has been disputed by some authors. (e.g. Hoyle and Burbidge (1966)).

It is therefore reassuring that the two methods give similar results for the evolution of the radio source population. Schmidt's analysis also enables the local luminosity function of QSO's to be determined more accurately than has been possible in the past.

"The redshift-magnitude relation expected in world models with positive cosmological constant, which corresponds to the negative gravitational effect of a vacuum has been discussed by Petrosian, Salpeter, and Szekeres (1967). When the Λ term just exceeds the critical value for which the Universe would have attained a stationary state (an Einstein Universe) there is some redshift at which the expansion almost stops. During this period of slow expansion, light can travel one or more times round the Universe and this leads to the focusing of the light rays from sources situated near the antipodes of the observer. In this case we might expect a concentration of bright sources at some particu-

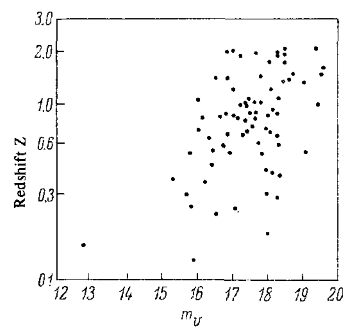

FIG. 7. The redshift – magnitude relation for QSO's at optical wavelengths illustrating the apparent limit to the redshifts of QSO's about z = 2.3.

lar redshift. The redshift-magnitude relation (Fig. 7) shows that in fact there is no concentration of sources at any particular redshift. The absence of this effect is unlikely to be due to the effects of selection which operate against faint rather than bright sources.

Similar models with positive cosmological constant have been discussed by Kardashev (1968) in order to explain the concentration of absorption redshifts around $z = 1.95$ discovered by Burbidge and Burbidge (1967). In this case the absorption is associated with the period of very slow expansion which for an appropriate choice of parameters can take place at $z = 1.95$. Arguments against models with positive cosmological constant have been given by Sunyaev (1969).

In all cases, the counts of radio sources can not be explained by the effects of geometry alone.

A further interesting feature of the distribution of QSO's is the apparent cut-off in their distribution of redshifts about $z \sim 2.3$. This is particularly clearly seen in figure 7 which is taken from Longair and Scheuer (1967). Since this diagram was drawn, redshifts as large as 2.36 have been measured but the deficit of sources at larger redshifts is still striking. It is clear from the diagram that QSO's exist which could be detected at greater redshifts since intrinsically some of them are very bright optically.

It may be that it is difficult to distinguish QSO's with such large redshifts from ordinary stars by their UBV colors (Lari and Setti, 1967). It is, however, interesting that the cut-off required by the counts of radio sources also occurs about this redshift and the two cut-offs may be related.

c) An Angular Diameter—Redshift Test Using Faint Radio Sources.

As was discussed in the introduction, because of the broad dispersion in the physical properties of radio sources, it is only possible to perform the classical cosmological tests in a statistical manner and the data must fulfill the following requirements.

a) The availability of data over a very wide range of flux densities since, in the Euclidean model, for example, a factor of 100 in flux density corresponds to a factor of only 10 in distance.

b) Over this range of flux densities the samples of sources must be complete within well-defined limits and must be sufficiently large that statistical errors are small.

c) At the higher flux density, it is necessary to know the physical properties of the sources in the sample.

It has become possible to perform such a test using the angular diameters of radio sources from new surveys recently completed at Cambridge. These are the 5C2 and 5C3 surveys (Pooley and Kenderdine, 1968; Pooley, 1969) and a new survey of the structures of a complete sample of sources in the Revised 3C Catalogue with a resolution of 23″. (Macdonald, Kenderine and Neville, 1968; Mackay and Elsmore, 1969).

The procedure used is a hybrid of the classical angular diameter—redshift test and the counts of radio sources (for details, see Longair and Pooley, 1969). The angular diameters and redshifts of the sources in the complete sample from the Revised 3C Catalogue are used to predict the angular diameter distribution which would be expected at the much lower flux densities corresponding to the 5C surveys, a factor of 200 in flux density. In practice, because the half-power beam width of the one-mile telescope is 80″ at 408 MHz, it is only necessary to predict the numbers of sources having angular diameters greater than 70″. This greatly simplifies the analysis since at the higher flux density, it is only necessary, in general, to consider sources having angular sizes greater than 70″. A fortunate consequence of using such large angular sources is that they are generally associated with radio sources having luminosities in the range $10^{24} < P_{178} < 10^{26}$ W-Hz^{-1} sr^{-1}. As a result, the identifications for the 3C sample are more than 75% complete and we can make predictions for different world models without cosmological evolution which are accurate within statistical errors.

Table II. The total number of radio expected in the 5C2 and 5C3 surveys with angular diameters $\theta \geq 70''$ and $S_{408} \geq 0.03 \times 10^{-26}$ W-m^{-2} Hz^{-1} for different cosmological models. ($\Lambda = 0$)

q_0	Number of sources
0.1	2.0
0.5	2.3
1.0	2.2
3.0	2.5
10.0	2.6

Some of the results of these calculations are shown in Table II as the total number of sources which would be expected in the 5C2 and 5C3 surveys having angular diameters greater than 70″ at a flux density of 0.03 $\times 10^{-26}$ W-m^{-2} Hz^{-1}.

It is clear that the test with the present statistics is not a very sensitive one since there is very little difference between the predictions for different world models. Allowing for statistical errors, 2.2 ± 1 sources are expected. In fact, 10 sources are found in the 5C2 and 5C3 surveys having $\theta \geq 70''$ and $S_{408} \geq 0.03 \times 10^{-26}$ W-m^{-2} Hz^{-1} and this is significantly greater than the predictions. This has been interpreted as evidence for the evolution of sources of large angular diameter in the range $10^{24} < P_{178} < 10^{26}$ W-Hz^{-1} sr^{-1}. However, the evolutionary effects associated with these sources are much weaker than those associated with sources of luminosity $\rho_{178} \geq 10^{26}$ W-Hz^{-1} sr^{-1}, evolution of the form $\rho(z) \propto (1 + z)^{\beta}$ where $\beta \sim 3$ being sufficient to explain the discrepancy. This result is entirely consistent with the previous analysis which showed that the powerful evolutionary effects could only be associated with powerful extragalactic sources.

5. OTHER ASPECTS OF THE COSMOLOGICAL EVOLUTION OF RADIO SOURCES

Besides its importance for the physics of radio sources, the fact that there were many more powerful radio sources in the past has implications for other branches of astronomy. As is well known, the radio emission observed from powerful radio sources is directly related to the number of cosmic rays in the source and total fluxes of the order of 10^{62} ergs may be released by any individual powerful radio source (Ryle and Longair, 1967). Therefore the fact that there were many more such sources in the past must be taken into account in any calculations related to the origin of cosmic radiation.

This has found particular application with regard to the x-ray background radiation at energies greater than 1 keV. The relativistic electrons within radio sources produce not only radio waves as a result of the synchrotron mechanism but also x-rays as a result of inverse Compton scattering (Ginzburg and Syrovatskiĭ, 1964; Felten and Morrison, 1965). In extragalactic space, the scattering is due to interactions between the relativistic electrons and the relict black-body radiation. Since the energy density of the relict radiation increases very rapidly with redshift as $(1 + z)^4$, this is a very powerful source of x-rays at large redshifts. However, it can be shown that without the inclusion of cosmological evolution, the observed x-ray background cannot be obtained (Brecher and Morrison, 1967; Bergamini, Londrillo and Setti, 1967). Once evolution of the type required to satisfy the radio source counts is included, the observed intensities can be obtained. There are still difficulties with this model, however, because of the decrease in slope of the x-ray background spectrum below about 40 keV. (see, for example, Gould, 1967, Vainstein et al, 1968). It then appears that it is only possible to obtain the correct total spectrum of the x-ray background for energies greater than 1 keV by considering the detailed physical processes involved in the evolution of individual radio sources. (Longair, 1969).

A similar application of evolution to explain the x-ray background has been made by Silk (1968) but this model requires the evolution of the properties of normal galaxies having x-ray emissivities similar to our own Galaxy. It is, indeed, possible that such evolution may take place at x-ray wavelengths but such evolution of normal galaxies which are intrinsically very weak radio sources is totally inconsistent with the observed radio background. As has been emphasized, only the most powerful classes of radio source can exhibit strong evolutionary effects.

Low and Tucker (1968) have also suggested that if evolution is important for QSO's and systems such as N-galaxies and Seyfert galaxies at infra-red wavelengths, the background radiation at wavelengths less than 0.3 mm may be dominated by their integrated emission rather than by the relict black body radiation. This conclusion must await confirmation from the counts of sources at infrared wavelengths.

Evolution has also been invoked in the metagalactic theory of the origin of cosmic rays by Burbidge (1967) and Hillas (1968). The local distribution of radio sources can provide only about $10^{-(3-4)}$ of the observed energy density of cosmic rays and again evolution has been called upon to eliminate this discrepancy. In Hillas's treatment of the problem, the cosmic rays observed at the present day mainly originate at $z \sim 10$. This figure is adopted since losses due to pair production by interactions between the cosmic rays and the relict radiation can provide the observed break in the cosmic ray spectrum at about 10^{15} eV. It can then be shown that the amount of evolution to $z \sim 10$ must be similar to that derived from the counts of radio sources—i.e., $\rho(z) \propto (1 + z)^\beta$ where $\beta \sim 5-6$. However the counts of radio sources require such evolution to terminate at $z \sim 3-4$. This difficulty is overcome by postulating that inverse Compton losses become important at $z \sim 3-4$ so that the observed density of radio sources beyond this redshift is greatly depressed. This, however, is only obtained at the expense of producing x-rays and γ-rays and it can be shown that there is a simple relation between the observed flux of x-rays and the local energy density of cosmic rays, independent of the exact details of the evolution of radio sources (Longair, 1969). These calculations indicate that the amount of x-rays produced in Hillas's and indeed a very wide range of models of the metagalactic origin of cosmic rays exceed the observations by a factor of about 500. It appears that it may be possible to explain the high-energy tail of the cosmic ray distribution, $E > 10^{17}$ eV, as of extragalactic origin.

These calculations have bearing upon another aspect of the distribution of radio sources, the nature of the cut-off at $z \sim 3-4$. The discussion above and considerations of the radio background show that it cannot be wholly due to inverse Compton scattering but that there must be some physical change in the evolution of the radio source population. It may be that this results from a poor choice of the laws of the evolutionary behavior and as has been shown (Doroshkevitch, Longair and Zel'dovich, 1969), it is possible to obtain models of the evolution of radio sources in which their distribution extends to redshifts beyond $z = 4$. These models however, still imply rapid evolution at small redshifts and weak evolution at large redshifts.

When we consider the evolution from the point of view of proper time rather than redshift, this apparent change in the evolution disappears. It may be that there is a genuine cut-off to the distribution of radio sources at this redshift which may be associated with the epoch of galaxy formation (see, e.g., Zel'dovich and Novikov, 1967). Only further observations of faint radio sources will help resolve this problem.

6. CONCLUSIONS

It now appears that the general features of the evolution of the radio source population can be understood within the framework of evolutionary world models as discussed in Sec. 3. Namely.

i) Powerful evolutionary effects are essential.

ii) This evolution refers only to small redshifts, less than $z \sim 3-4$ beyond which either the evolution is much weaker or there is a termination to the radio source population.

iii) The strong evolutionary effects refer only to the most powerful classes of radio source, although there may be weaker evolution associated with intrinsically weaker radio sources.

It appears that these effects are all intrinsic to the radio source population rather than being influenced by effects such as inverse Compton scattering.

There is much work to be done to elucidate this evolution in detail and to understand it in terms of the physics of radio sources. There is also a wide field of application of these results to other branches of astronomy of which the discussion in Sec. 5 is only a beginning.

There is no lack of important programmes of research to be carried out such as

a) the determination of source counts at different radio frequencies to very low flux densities,

b) the counts of radio-quiet QSO's,

c) the counts of infra-red objects,

d) the detection of x-rays from distant radio galaxies and their counts at x-ray wavelengths,

e) the nature of radio sources in the 4C catalogue.

This last program is particularly important for the radio source problem. In general, when the excess of sources is greatest over the predictions of the simple world models, there is the greatest probability of detecting very distant sources where the effects of evolution are most important. Fortunately, sources at the bottom of the 4C catalogue are ideal for this purpose. Particularly important are optical identifications, redshifts, radio structures and spectral data.

All of these problems are now being tackled although they are all taxing the ingenuity and skill of observers to their technological limits and many involve much difficult and painstaking work. However, the importance of this work for our understanding of the Universe, its evolution and the nature of the physical processes which have influenced this evolution more than justifies the large amount of effort expended on these problems.

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