PROPAGATION OF A LIGHT PULSE IN A RESONANTLY AMPLIFYING (ABSORBING) MEDIUM

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1. INTRODUCTION

THE investigation of the propagation of a light pulse in an amplifying medium is closely connected with one of the most significant accomplishments of quantum radiophysics—the generation of short and ultrashort high-power pulses of light. The first ruby lasers[1,2] emitted light pulses with energy on the order of 1 Joule in a time of approximately 10^{-3} sec. After 20 years, the laser power has been increased by approximately 10^4 times as a result of shortening of the pulse duration by suddenly increasing the Q (Q-switching)[3-5]. This method is based on accumulating energy in the active medium at low Q, and subsequently emitting of the stored energy, after suddenly raising the Q, in the form of a short-duration powerful pulse. A typical Q-switched laser emits a pulse with energy on the order of a Joule in a time 10—30 nsec. Such lasers are widely used in physical and applied research. Thus, for example, the rapid progress in nonlinear optics research (see, for example[6,7]) is connected to a considerable degree with the use of Q-switched lasers.

The next step towards the increase of the power, by one or two more orders of magnitude, was made with the aid of a method for amplifying light pulses[8-11]. It is possible to attain a reduction of the pulse duration in the case of a nonlinear interaction between a powerful pulse and a medium, when the pulse emits an appreciable fraction of the energy stored in the amplifying medium. For example, in a chain of optical amplifiers the power can be increased to 10^{10} W by increasing the energy to 20—100 J and decreasing the pulse duration to 2—5 nsec[9-11]. In addition, the effect of superluminal motion of the pulse maximum in the medium was observed in investigations of the propagation of a powerful light pulse in a nonlinearly amplifying medium[8,12].

The power and the energy of the pulse are limited ultimately by the destruction of the active medium under the action of the intense light field of the pulse. The destruction threshold of ruby and neodymium glass by nanosecond-duration pulses is determined by the pulse energy[13,9,11]. Therefore, to obtain maximum power it is necessary to reduce the pulse duration[10]. This holds true until the effects of saturation of the power as a result of multiphoton losses begin to play a role[29,34,37].

Indeed, the increase of the light-pulse power to a level of approximately 10^3 GW[16] has become possible after the development of a laser for ultrashort light pulses with self-locking of the modes by means of a linear absorber[14]. Unlike the ordinary multimode lasers, the resonator of such a laser contains a nonlinearly absorbing medium whose transparency is proportional to the instantaneous power of the light. The initial light pulse produced upon addition of fields with random phases in a large number of axial modes, moving between the mirrors of the laser, passes many times through the nonlinear absorber. As a result, the pulses are gradually compressed, and owing to the simultaneous increase of the Q, their power increases appreciably. The process of formation of light pulses inside such a laser can be regarded as a result of establishment of definite phase relations between the fields in different modes as a result of their interaction with a nonlinear absorber, i.e., as a self-locking of the modes. In the regime of simultaneous Q-switching and self-locking of the modes, a neodymium-glass[144] or ruby[144] solid-state laser emits a train of several dozen of very short light pulses spaced $T = 2L/c$ apart (L—optical length of the resonator, c—velocity of light) and with duration $\tau_p \approx T/m$ (m—number of locked axial modes). The energy of one pulse amounts to 10^{-3}—10^{-2} J, and its duration is $\tau_p \approx 10^{-11}$—10^{-12} sec[145,146,121], i.e., the power of these pulses is of the order of 10^8—10^9 W. The pulse power can be greatly increased by separating it from the train and passing it through an optical quantum amplifier. This is a rather difficult task (the time interval between the pulses in the train is only several nsec), but is perfectly feasible by modern experimental techniques[15]. Thus, a power on the order of 10^{12} W was obtained[16] by amplifying an ultrashort light pulse*.

To give an idea of the attained pulse power level, we note that when the pulse radiation is focused by a real optical system on an area measuring 100 λ^2 ($\lambda = 1.06\ \mu$ is the radiation wavelength), a power of 10^{18} W/cm^2 is attained. The electric field intensity of the light wave at the focus amounts to 1.5×10^{10} V/cm, and the magnetic field intensity is 5×10^7 G. In other words, the energy density at the focus is of the order of 3×10^7 J/cm^3, and the photon density is of the order of 2×10^{26} photons/cm^2. If the diffraction limit of the divergence of the radiation of such powerful pulses is ever attained, then these figures will increase to 3×10^9 J/cm^3 and 2×10^{28} photons/cm^3.

Strong efforts on the part of many researchers in many countries were needed to obtain powerful ultrashort light pulses. It was therefore quite unexpected to observe spontaneous picosecond light pulses in a solid-state laser of practically any type[147-149]. This turned out to be possible as a result of the development of a new method of registering picosecond pulses, based on the collision between the pulses in a medium with

*On the average, in 1960 – 1968 the light-pulse power increased exponentially at a rate of approximately one order of magnitude annually.

two-photon luminescence excitation[121]. The spontaneous picosecond pulses occur apparently in any multimode laser. The mechanism producing such pulses is not fully clear, and investigations of the dynamics of the generation of a multimode laser have begun only recently. The most likely explanation for the appearance of such pulses is the fluctuation mechanism, according to which the pulses represent the fluctuation peaks produced by the interference of a large number of modes with random phases[112]. A detailed investigation of the dynamics of the generation of a multimode laser will undoubtedly exert an influence on the development of methods of obtaining powerful ultrashort pulses of light.

Three chapters of the present review (Ch. 2–4) are devoted to the evolution of a light pulse propagating in an amplifying (Ch. 2), absorbing (Ch. 3), and two-component (amplification plus absorption) medium (Ch. 4). Principal attention is paid here to the physical aspect of the problem.

The main feature of the interaction of a light pulse with an amplifying (absorbing) medium is the nonlinearity of the interaction. The coefficient κ of resonant amplification (absorption) of the medium per unit length, is determined by two parameters of the radiating particles (atoms, ions, molecules, etc.), namely the cross section σ of the radiative transition between two considered levels having N_1 and N_2 particles, and the density $N = N_2 - N_1$—of the inverted population of these levels. These two parameters, generally speaking, depend on the radiation intensity:

$$\varkappa(I) = \sigma(I) N(I). \tag{1.1}$$

The nonlinearity of the amplification or absorption usually results from the change in the population difference of the levels under the influence of the intense radiation (the saturation effect[26]). It is precisely this effect which serves as the basis for nonlinear amplifiers on absorbers and plays the principal role in the propagation of a light pulse in a resonantly amplifying (absorbing) medium. The influence of the saturation-type nonlinearity has been investigated most thoroughly, so that in the present review greatest attention will be paid to processes connected with the saturation effect.

The nonlinearity can also be due to the dependence of the cross section of the radiative transition on the intensity. In the general case, this dependence can be represented in the form

$$\sigma(I) = \sigma_0 + \sigma_1 I + \sigma_2 I^2 + \ldots \tag{1.2}$$

For single-quantum resonant transitions, only the first term of the expansion (1.2) differs from zero, and the remaining terms can be neglected. For two-quantum transitions $\sigma_0 = 0$ and the first nonvanishing term is the second, etc. For multiquantum transitions, the nonlinearity of the amplification or of the absorption arises even when the level population remains unchanged, but becomes appreciable at sufficiently high power levels. Although multiquantum nonlinearity was investigated in many papers (see, for example, the review[27]), its influence on the propagation of the light pulse has hardly been studied. Investigations in this direction have only just begun, and interesting results can be expected here. Thus, for example, the possibility of formation of powerful ultrashort light pulses with duration on the order of 10^{-13} sec by propagation in a medium with two-quantum amplification was demonstrated in[28].

Another interesting feature of the interaction between a short pulse of coherent light and a medium is the coherent character of the interaction. By coherent interaction of a radiating particle with a field is meant an interaction such that the high-frequency dipole moment (polarization) induced in the particle by the field does not attenuate spontaneously during the time of the interaction. In practice this means that the duration of the pulse should be much shorter than the damping time T_2 of the polarization of the medium due to different relaxation mechanisms. In this case the polarization of the medium does not "follow" the field in a quasistationary manner, but is determined by the field in the preceding instants of time. The presence of a "phase memory" changes qualitatively the picture of the evolution of the pulse. For example, in the absence of inverted population, the particle in the coherent state can continue to radiate or to absorb, etc. Whereas in the radio band coherent interaction is well known[87,90,93], in the optical band it has been observed only in several experiments ("optical echo"[91,22] and "self-transmission" of a pulse through an absorbing medium[21,73]). In ordinary experimental conditions, the interaction is incoherent. However, the development of methods of generation of ultrashort pulses of light and the use of gas amplifying and absorbing media, apparently makes possible a detailed study of coherent interaction of a light pulse with a medium.

In the last two chapters we consider the problem of obtaining coherent-light pulses with maximum brightness. By brightness is meant, as usual, the power of the radiation flux per unit solid angle ($W/cm^2 sr$). It is precisely this characteristic which determines the radiation power when it is focused or when it is observed in the far zone. In this review we consider two of the presently most effective methods of increasing brightness: the method of increasing the pulse power (Ch. 5) and the method of cascade transformation of the radiation (Ch. 6). In the amplification method, the main increase of the brightness is obtained by increasing the energy density and the power density of the radiation. In amplification of ultrashort pulses, the brightness amplification coefficient reaches 10^3.[16] In the cascade-transformation method the main brightness amplification is reached as a result of decreasing the divergence and the cross section of the transformed radiation with a slight decrease of the total energy. In the case of transformation with the aid of stimulated Raman scattering, the brightness gain reaches 10^2 even now.[135,136] At the present time, the amplification method has yielded pulses with brightness 10^{17}–10^{18} $W/cm^2 sr$[16]. This is approximately 10^4 times brighter than the surface of the sun, and corresponds to a blackbody brightness with temperature $T = 2 \times 10^7$°K.* Even now, ways of increasing the brightness to 10^{19}–10^{20} $W/cm^2 sr$ are feasible in principle. The high rad-

*We have in mind the effective temperature, corresponding to the total radiation flux, and not to the radiation flux in a unit spectral interval.

iation brightness of powerful pulses makes it possible in principle to heat matter to thermonuclear temperatures[167]. The prospects of such an application of lasers have given rise to the development of an independent trend in methods of obtaining high-temperature plasma. A temperature at which neutrons are produced as a result of a thermonuclear reaction has already been attained[139].

We hope that the present review will contribute to the development of research on the production of superpowerful light pulses and their application in physical research.

2. AMPLIFYING MEDIUM

At the present time, amplification has been obtained in various substances (luminescent crystals and glasses, semiconductors, liquids, gases). The main properties of these amplifying media, such as the gain, the width of the amplification band, the character of the amplification line broadening, the mechanisms of level population relaxation, etc. differ strongly in different media. For example, the gain per unit length ranges from 0.01–0.1 cm^{-1} (gases, luminescent crystals, glasses, liquids) to 10^2–10^3 cm^{-1} (semiconductors).

The amplification line broadening differs just as strongly. In condensed media, the main mechanism of broadening is the interaction of the radiating particles with phonons. This interaction leads to a uniform broadening of the line, and the amount of broadening is $\Delta\omega_\alpha \approx (10^{-4}-10^{-2})\omega_0$ at 300°K, and can decrease by one or two orders of magnitude when cooled to nitrogen or helium temperatures. In many substances (for example gases, glasses, and cooled crystals), the resonant frequencies of the particles do not coincide and are distributed in a certain spectral region with a width exceeding the homogeneous width. In this case, the spectral line is inhomogeneously broadened. In particular, this takes place in luminescent crystals at low temperature, owing to the inhomogeneity of the stresses, in glasses as a result of the indefinite position of the ion in the irregular matrix, and in gases as a result of the Doppler effect when the particles move. The reciprocal homogeneous width determines the time during which the high frequency dipole moment induced by the light field attenuates as a result of the relaxations. In the case of a particle with two levels, this relaxation time is called the transverse relaxation time T_2[64].

Relaxation of the inverted population takes place, as a rule, as a result of radiative transitions. In luminescent crystals operating in accordance with a three-level scheme, as well as in semiconductors, the principal role is played by radiative decay of the upper level. In gases, besides radiative decay of the upper level, a role can be played by the population of the lower level as a result of transitions from other levels. The relaxation time of the inverted population, corresponding to the time of longitudinal relaxation T_1 in the two-level scheme[64], ranges from 10^{-3} sec for metastable levels of ions in crystals and gases to 10^{-9} sec for allowed transitions in gases, semiconductors, etc. Usually the relaxation of the level population occurs much more slowly than the relaxation of the polarization ($T_1 \gg T_2$). For example, for condensed media $T_2/T_1 = 10^{-5}$–10^{9}. Physically this is explained by the fact that for relaxation of the polarization it is sufficient to "collapse" the phase of the wave function of the particle, something which does not entail an appreciable change of the particle energy, and the relaxation of the population is connected with an appreciable change of the particle energy. However, in low-pressure gases, when the only mechanism of all the relaxations is spontaneous decay or a final interaction time with the field, the case $T_1 = T_2$ is possible.

The variety of the properties of amplifying medium makes it very difficult to present a unified analysis. A reasonable way out is to consider a certain unified model that takes qualitatively into account the most important properties that are common to all media. Such a model of the active medium is an aggregate of uniformly spatially distributed non-interacting two-level quantum systems with two phenomenological relaxation times T_1 and T_2[64]. The inhomogeneously broadened line can be regarded in this model as an aggregate of inhomogeneously broadened lines with a definite spectral distribution of the resonant frequencies.

The propagation of a light pulse in a resonantly amplifying medium was considered in a number of papers. In the theoretical papers[30-41,183] they considered the variation of the pulse waveform during propagation*. The main conclusion of these investigations was that in the nonlinear amplification regime one can obtain an appreciable reduction of the duration of the light pulse and an increase in its power. Qualitatively this effect was considered even earlier in[42]. Attempts to obtain a reduction of the pulse duration from a Q-switched laser were undertaken in[43-46], but did not lead to significant results. It was shown theoretically and experimentally in[8,47] that a light pulse from a Q-switched laser, owing to the smooth leading front, is not reduced by nonlinear amplification. Instead of a reduction in the pulse duration, one obtains a shift of its maximum over its leading front. The conditions for reducing the pulse duration were determined in[48]. After the shift of the maximum was eliminated, pulse compression and an appreciable power increase were attained[8-11]. The effect of shifting the maximum of the pulse over the leading front can lead to a propagation of the maximum pulse in the medium with superluminal velocity[8-10,47]. The conditions for the occurrence of this effect were discussed in[12,49].

Coherent interaction of a light pulse with an amplifying medium was first considered in[36-38], and later in[41,54,55]. In[36-38,41] it was shown that a stationary pulse that reverses (but does not equalize) the inverted population is produced during the propagation (– pulse). Its duration can be much shorter than the time T_2. It is shown in[52,53] that stationary pulses that leave the level population unchanged after the passage of the pulse (2π-pulses) are produced in an ideal amplifying medium without linear losses of radiation. Such pulses are similar to stationary pulses produced

*Certain problems involved in the interaction of a radiator with an amplifying medium were considered in [59].

in an absorbing medium[21]. The general picture of the evolution of an ultrashort pulse in an amplifying medium was presented in[53].

2.1. Fundamental Equations

We consider the interaction between a field and a medium made up of identical particles with two energy levels having an inverted population. We describe the medium by the Boltzmann equation for the density matrix $\hat{\rho}$ with longitudinal and transverse relaxation, and the field E by Maxwell's equation in a medium having also a linear nonresonant radiation losses:

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} + c^2 \operatorname{rot}\operatorname{rot}\mathbf{E} + \gamma c \frac{\partial \mathbf{E}}{\partial t} = -4\pi N_i \frac{\partial^2}{\partial t^2} \operatorname{Sp}(\mu\hat{\rho}), \tag{2.1}$$

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = [\mathscr{H}_0 - \mu\mathbf{E}, \hat{\rho}] - i\hbar\Gamma\hat{\rho}, \tag{2.2}$$

where c is the velocity of light in the medium without active particles, γ is the coefficient of nonresonant linear radiation losses per unit length in the medium, N_i is the particle density, $\mathscr{H}_0$ is the unperturbed Hamiltonian of the particle, and μ is the operator of the electric dipole moment of the particle. The term $\Gamma\hat{\rho}$ describes phenomenologically the relaxation of the elements of the density matrix: the diagonal elements $(\Gamma\rho)_{nn} = (\rho_{nn} - \rho^0_{nn})/T_1$, describing the level population, attenuate within the longitudinal relaxation time T_1, and the non-diagonal elements $(\Gamma\rho)_{mn} = \rho_{mn}/T_2$ (m, n = 1, 2), which describe the high-frequency dipole moment, attenuate within the transverse relaxation time T_2.

In the representation in which $\mathscr{H}_0$ is diagonal, the equation for the density matrix can lead to equations for the polarization $\mathbf{P} = N_i \operatorname{Tr}(\mu\hat{\rho})$ and for the inverted-population density $N = N_i(\rho_{22} - \rho_{11})$:

$$\frac{\partial^2 \mathbf{P}}{\partial t^2} + \frac{2}{T_2}\frac{\partial \mathbf{P}}{\partial t} + \omega_0^2 \mathbf{P} = -2N\mathbf{E}\frac{\omega_0}{\hbar}|\mu|^2, \tag{2.3}$$

$$\frac{\partial N}{\partial t} + \frac{1}{T_1}(N - N_0) = \frac{2}{\hbar\omega_0}\mathbf{E}\frac{\partial \mathbf{P}}{\partial t}, \tag{2.4}$$

where $N_0 = N_i(\rho^0_{22} - \rho^0_{11})$ is the density of the inverted population in the absence of a field, $\omega_0 = \omega_{21}$, and small terms have been omitted under the assumption that $\omega_0 \gg 1/T_2$. For the case $T_1 = T_2$, these equations were obtained in[56,57].

The field equations (2.1) and the material equations (2.3) and (2.4), together with the initial conditions, describe completely the propagation of a light pulse in a resonantly amplifying medium. In luminescent crystals and practically in all other amplifying media, the variation of the light-wave parameters over distances on the order of the light wavelength and within times on the order of the light period is very small. We can therefore go over to "slow" variables $\mathscr{E}$, $\mathscr{P}$, φ, and ψ:

$$\begin{aligned} E &= \tfrac{1}{2}\mathscr{E}(t, x)\exp\{i[\varphi(t, x) + \omega t - kx]\} + \text{c.c.} \\ P &= \tfrac{1}{2}\mathscr{P}(t, x)\exp\{i[\psi(t, x) + \omega t - kx]\} + \text{c.c.} \end{aligned} \tag{2.5}$$

where we confine ourselves to the case of a light pulse in the form of a linearly polarized plane wave, moving in the positive direction of the x axis. Then the equation for the field and the material equation reduced to the following system of five equations:

$$\left.\begin{aligned} &\frac{\partial \mathscr{E}}{\partial t} + c\frac{\partial \mathscr{E}}{\partial x} + \frac{\gamma}{2}c\mathscr{E} = 2\pi\omega\mathscr{P}\sin(\psi - \varphi), \\ &\mathscr{E}\left(\frac{\partial \varphi}{\partial t} + c\frac{\partial \varphi}{\partial x}\right) = -2\pi\omega\mathscr{P}\cos(\psi - \varphi), \\ &\frac{\partial \mathscr{P}}{\partial t} + \frac{1}{T_2}\mathscr{P} = \frac{\mu^2}{\hbar}N\mathscr{E}\sin(\psi - \varphi), \\ &\left[\frac{\partial \psi}{\partial t} + (\omega - \omega_0)\right]\mathscr{P} = \frac{\mu^2}{\hbar}N\mathscr{E}\cos(\psi - \varphi), \\ &\frac{\partial N}{\partial t} + \frac{1}{T_1}(N - N_0) = -\frac{1}{\hbar}\mathscr{P}\mathscr{E}\sin(\psi - \varphi). \end{aligned}\right\} \tag{2.6}$$

In the envelope approximation, Eqs. (2.6) are exact, and take into account the effects of coherent interaction of the pulse with the medium. They become much simpler, however, in the case of incoherent interaction.

Incoherence of the interaction can arise either as a result of the incoherent state of the medium during the pulse time, or as a result of an incoherent state of the field. The condition for incoherent interaction between a coherent pulse and a medium is of the form

$$\tau_p \gg T_2. \tag{2.7}$$

However, even if $\tau_p \sim T_2$, the interaction can be incoherent if the coherence time of the field τ_{coh} is much shorter than T_2:

$$T_2 \gg \tau_{coh} \tag{2.8}$$

If the interaction is incoherent as a result of the incoherent state of the medium ($\tau_p \gg T_2$) then, as follows from the third equation of (2.6), the polarization "follows" the field amplitude to quasistatically:

$$\mathscr{P} = \frac{\mu^2}{\hbar}T_2 N\mathscr{E}\sin(\psi - \varphi).$$

Taking this into account and changing over to the radiation flux density

$$I = \frac{1}{\hbar\omega_0}\frac{c}{8\pi}\mathscr{E}^2 \text{ photons/cm}^2\text{sec}$$

we can reduce (2.6) to the following three equations:

$$\left.\begin{aligned} &\frac{\partial I}{\partial t} + c\frac{\partial I}{\partial x} = c(\sigma N - \gamma)I, \\ &\frac{\partial N}{\partial t} + \frac{1}{T_1}(N - N_0) = -2\sigma I N, \end{aligned}\right\} \tag{2.9}$$

$$\frac{\partial \varphi}{\partial t} + c\frac{\partial \varphi}{\partial x} = (\omega_0 - \omega)\frac{T_2}{2}c\sigma N, \tag{2.10}$$

where $\sigma = \sigma(\omega)$ is the cross section of the radiative transition at the frequency ω, defined by the expression

$$\sigma(\omega) = \frac{4\pi T_2 \omega_0 \mu^2}{\hbar c}\cdot\frac{T_2^{-2}}{(\omega - \omega_0)^2 + T_2^{-2}}. \tag{2.11}$$

The first two equations are the usual transport equations that follow from the energy conservation law. The third phase equation describes the effect of variation of the phase velocity as a result of the anomalous dispersion within the limits of the negative-absorption line.

The propagation of a pulse of incoherent light is described only by two equations of the type (2.9), since the phase of the field is an indeterminate quantity. However, the equations differ somewhat from (2.9), owing to the fact that the radiation is not monochromatic.

2.2. Incoherent Interaction

a) Theory. Let us consider the propagation of a

pulse of coherent light with duration $\tau_p \gg T_2$, when the interaction of the pulse with the medium is incoherent, i.e., it is described by Eqs. (2.9) and (2.10). It is easy to see that these equations are nonlinear. If the duration of the pulse $\tau_p \gg T_1$, then the inverted population is determined by the instantaneous value of the intensity:

$$N = \frac{N_0}{1 + 2\sigma T_1 I}. \qquad (2.12)$$

The nonlinearity parameter in this case is the pulse power. The radiation power that decreases the inversion, say, by a factor of two, can be regarded as the saturation power I_S. The value of I_S is obviously determined by the expression

$$I_s = (2\sigma T_1)^{-1}. \qquad (2.12a)$$

In the other limiting case, when the pulse duration is $\tau_p \ll T_1$, the change of the inverted population is determined by the pulse energy:

$$N = N_0 \exp\left[-\frac{1}{2\sigma}\int_{-\infty}^{\infty} I(t', x)\,dt'\right]. \qquad (2.13)$$

In this case the nonlinearity parameter is the pulse energy. The radiation energy decreasing the inversion, say, by a factor e, can be taken as the saturation energy E_S. Its value is determined by the relation

$$E_s = (2\sigma)^{-1}. \qquad (2.13a)$$

If the pulse energy $E \ll E_S$, then the pulse is amplified linearly, without changing its shape or spectrum. If the pulse energy is comparable with E_S, the amplification becomes nonlinear. In nonlinear amplification, obviously, the front part of the pulse, with energy E_S, is much more strongly amplified than the remaining part. This should lead to an appreciable change in the waveform of the pulse, and in the presence of detuning ($\omega \neq \omega_0$), the spectrum may also be distorted.

The most interesting is the propagation of a short pulse ($\tau_p \ll T_1$), which we shall consider in detail. Equations (2.9) reduce in this case to

$$\frac{\partial I}{\partial t} + c\frac{\partial I}{\partial x} = cI\left\{\sigma N_0 \exp\left[-2\sigma\int_{-\infty}^{t} I(t', x)\,dt'\right] - \gamma\right\}. \qquad (2.14)$$

This equation can be solved analytically only if there are no linear losses ($\gamma = 0$). The solution takes the form[30-32]

$$I(x, t) = \frac{I_0\left(t - \frac{x}{c}\right)}{1 - \left\{1 - \exp\left[-\sigma\int_0^x N_0(x')\,dx'\right]\right\}\exp\left[-2\sigma\int_{-\infty}^{t-\frac{x}{c}} I_0(t')\,dt'\right]}, \qquad (2.15)$$

where $I_0(t)$ is the shape of the pulse at the boundary of the medium ($x = 0$), $N_0(x)$ is the initial distribution of the inverted population. A feature of the ideal case of absence of losses is the unlimited linear growth of the energy at $E \gtrsim E_S$. This corresponds to total emission of the energy stored in the medium by the propagating pulse. The change of the waveform can be readily traced in the simplest example of an initial pulse in the form of a step function. In this case the shape of the pulse is given by the expression

$$I(x, t) = I_0\left\{1 - [1 - \exp(-\sigma N_0 x)]\exp\left[-\frac{E_0}{E_s}\frac{t - (x/c)}{\tau_0}\right]\right\}^{-1}, \quad 0 \leqslant t - \frac{x}{c} \leqslant \tau_0, \qquad (2.16)$$

where it is assumed that the pulse begins at the instant $t = 0$ and has a duration τ_0, an initial power I_0, and an energy $E_0 = I_0\tau_0$, while the initial inverted population is homogeneous. It is easy to see that when $E \gtrsim E_S$ a strong sharpening of the leading front begins. The change of the pulse waveform during propagation is shown in Fig. 1. When $\sigma_0 N_X \gg 1$, the compression of the pulse is quite appreciable:

$$\frac{\tau_p}{\tau_0} = \frac{E_s}{E_0}\exp(-\sigma N_0 x),\ \exp(-\sigma N_0 x) \ll 1. \qquad (2.17)$$

This gives grounds for hoping that in such a way it is possible to attain an appreciable shortening of the duration and an increase of the power of the pulse. However, such a simple picture occurs only for pulses with a steplike leading front. For pulses with a gradual leading front (and real high-power pulses of light, such as from a Q-switched laser, have smooth extended leading fronts), the picture of the evolution of the pulse is quite different. The effect of preferred amplification of the frontal part of the pulse leads to a gradual "shift" of the pulse maximum over the leading front, and the magnitude of the shift is determined essentially by the character of the leading front of the initial pulse. If the magnitude of the shift is characterized by the relation $W = d\tau/dx$, where $d\tau$ is the shift of the pulse over the leading front on passing through a layer of medium with thickness dx, then the following expression holds[48]:

$$W = \frac{d\tau}{dx} = -(\sigma N_0 - \gamma)\frac{\int_{-\infty}^{\tau_s} I_0(\tau)\,d\tau}{I_0(\tau_s)}, \qquad (2.18)$$

where $\tau = t - (x/c)$, and τ_S is the point on the leading front of the pulse corresponding to a definite (say, 20%) level of gain saturation.

The shift of the maximum of the pulse prevents its compression. Therefore, in nonlinear amplification, only the pulses for which $W = 0$ (pulses with a stepwise leading front) or $W \to 0$ are shortened as the pulse is shifted over the leading front. The condition

$$\lim_{\tau\to-\infty}\left[\frac{1}{I_0(\tau)}\int_{-\infty}^{\tau} I_0(\tau')\,d\tau'\right] = 0 \qquad (2.19)$$

is satisfied, for example, by the leading front of a pulse of Gaussian shape $I_0(\tau) \sim \exp(-\tau^2/\tau_0^2)$. In spite of the infinite length of the leading front, a Gaussian pulse is shortened when propagating in a nonlinearly amplifying medium. Figure 2 shows the results of an electronic-computer solution of Eq. (2.14) for a Gaussian initial pulse.

Pulses for which $W = \text{const}$ or $W \to \text{const}$ as the pulse propagates, i.e.,

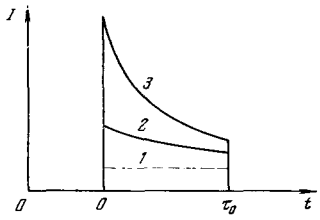

FIG. 1. Change of waveform of a step pulse of light in nonlinear amplification. 1 – $x = 0$; 2 – $x\sigma N_0 = 1$; 3 – $x\sigma N_0 = 2$ for the case $E_0 = 0.5E_S$. E_0 – initial pulse energy, E_S – saturation energy, x – distance covered by the pulse.

$$\lim_{\tau\to-\infty}\left[\frac{1}{I_0(\tau)}\int_{-\infty}^{\tau} I_0(\tau')\,d\tau'\right]=\text{const}, \qquad (2.20)$$

tend to a stationary shape I[t − (x/u)] without a decrease in duration. In particular, this condition is satisfied by pulses with an exponential growth of the leading front, $I_0(\tau)\sim\exp(\tau/\tau_0)$. Numerical solutions

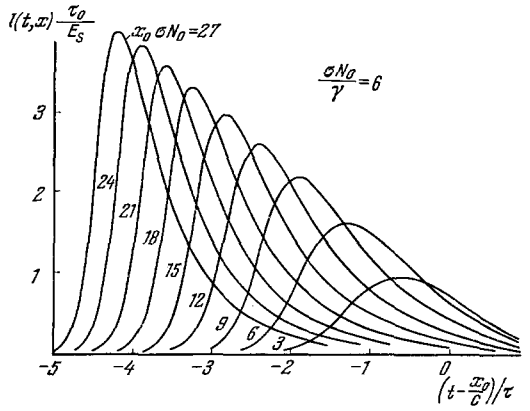

FIG. 2. Change of form of a Gaussian pulse of light in nonlinear amplification [48]. x_0 – distance traversed by the pulse in the medium from the boundary.

of the nonstationary equation (2.14) for a pulse with an exponential leading front are shown in Fig. 3, where the approach of the stationary state can be seen.

Finally, pulses satisfying the condition $|W|\to\infty$, i.e.,

$$\lim_{\tau\to-\infty}\left[\frac{1}{I_0(\tau)}\int_{-\infty}^{\tau} I_0(\tau')\,d\tau'\right]=\infty,$$

experience an infinite lengthening of duration. Since the total energy of the pulse, regardless of its form, is limited, the intensity of such pulses tends to zero. Including among the "broadening" pulses are initial pulses with power-law growth of the leading front, $I_0(\tau)\sim|\tau_0/\tau|^n$, $n>1$. Figure 4 shows the results of a numerical integration of Eq. (2.14) for an initial pulse with a leading front of the type $|\tau_0/\tau|^8$.

The change of the pulse energy

$$E=\int_{-\infty}^{\infty} I(t,\ x)\,dt'$$

does not depend on its form and is described by an equation that follows directly from (2.14):

$$\frac{dE}{dx}=\frac{N_0}{2}[1-\exp(-E/E_s)]-\gamma E. \qquad (2.21)$$

It is easy to see that when $E\ll E_S$, the energy in-

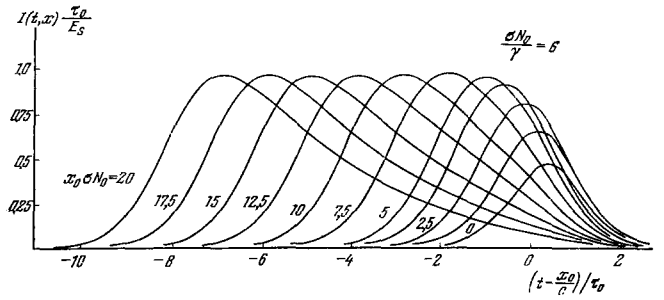

FIG. 3. Change of shape of an exponential light pulse in the case of nonlinear amplification [8,48].

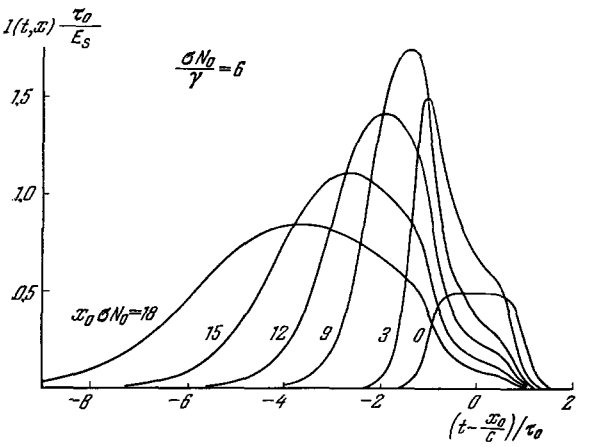

FIG. 4. Change of shape of a light pulse with a leading front having a power-law growth in the case of nonlinear amplification [48].

creases exponentially, when $E\lesssim E_S$, the growth of the energy is practically linear, but it stops gradually because of the loss factor γ. The limiting energy E_m is determined by Eq. (2.14) with dE/dx = 0. If $\gamma\ll\sigma N_0$, then the limiting energy is determined by the simpler expression[23,34,35]

$$E_m=\frac{N_0}{2\gamma}. \qquad (2.22)$$

The linear losses limit the maximum length of the medium that gives up energy to the pulse. This fact was already pointed out in[23].

A pulse from a Q-switched laser has the form shown in Fig. 5. The shape of the pulse is determined by the operating principle of this laser[3,4]. The development of generation begins with the level of spontaneous noise in the oscillation modes I_{sp} at the instant of Q switching (t = 0). The exponential growth of the power $I_{sp}\exp(t/\tau_0)$ continues for a relatively long time τ_d (50–500 nsec), called the delay time until the power reaches the level I_{sat} ($I_{sat}\approx 10^{15}I_{sp}$), sufficient to start saturation of the gain of the active medium. After this period, the energy stored in the laser is emitted, and this emission lasts for a short time, $\tau_p\approx$ 5–50 nsec, which is smaller by approximately one order of magnitude than the delay time. During that time, the giant radiation pulse proper is emitted. It is clear that the leading front of such a pulse satisfies the condition (2.20). Therefore the pulse of the Q-switched laser should not be shortened but tend to a certain stationary form. However, a similar displacement of the pulse over the leading front continues until the maximum of the pulse reaches the start of the pulse (t = 0). After this, the compression of the pulse can begin. In order for the compression of the duration to occur from the very start, it is necessary to cut off the smooth leading front of the pulse.

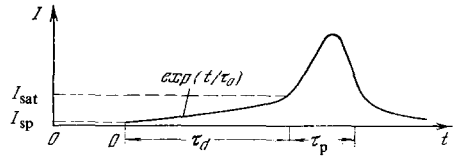

FIG. 5. Shape of radiation pulse of a Q-switched laser. The ordinate scale is strongly compressed ($I_{sat}/I_{sp}=10^{12}$–10^{15}).

The shift of the maximum of the pulse over the leading front can lead to a motion of the pulse in the medium with a velocity exceeding the velocity of light[47,8,10]. Indeed, the velocity of a point on the leading front $t_S(x)$ with definite saturation level is determined by the relation

$$u=\left[\frac{dt_s(x)}{dx}\right]^{-1}, \tag{2.23}$$

and the connection between the velocity u and the displacement of the pulse over the leading front W is determined by differentiating the relation $\tau_S(x) = t_S(x) - (x/c)$. As a result, the velocity of the pulse u satisfies the relation

$$\frac{1}{u}-\frac{1}{c}=W=-(\sigma N_0-\gamma)\frac{\int_{-\infty}^{\tau_s} I_0(\tau)\,d\tau}{I_0(\tau_s)}. \tag{2.24}$$

For pulses with an exponential leading front $\exp(\tau/\tau_0)$, expression (2.24) takes the form

$$\frac{1}{u}-\frac{1}{c}=-\varkappa_0\tau_0, \tag{2.25}$$

where $\kappa_0 = (\sigma N_0 - \gamma)$ is the initial gain of the medium per unit length. It is easy to see that when $c\kappa_0\tau_0 < 1$ the velocity of propagation of the maximum pulse $u > c$, and when $c\kappa_0\tau_0 > 1$, the velocity $u < 0$[12,49]. To understand all these effects, let us consider in greater detail the propagation of pulses of stationary form*.

Equation (2.4) has a stationary solution $I[t - (x/u)]$. If this equation is rewritten in terms of the variables $\tau = t - (x/t)$ and $R = \int_{-\infty}^{+\infty} I(\tau')d\tau'$ with allowance for the initial conditions $R = I = 0$ as $\tau \to -\infty$, then it takes the form

$$\left(\frac{1}{c}-\frac{1}{u}\right)\frac{dR}{d\tau}=\frac{N_0}{2}[1-\exp(-2\sigma R)]-\gamma R. \tag{2.26}$$

In this equation, the propagation velocity u is an indeterminate quantity and must be obtained from additional conditions. From the condition that the pulse energy is limited $E = R(+\infty)$ when $\sigma N_0 > \gamma > 0$, we get a condition for the propagation velocity[8,12]

$$u>c. \tag{2.27}$$

Consequently, the stationary pulses in the medium can propagate only with superluminal velocity.

An analysis of the propagation of the leading front of the pulse shows that the velocity u is determined by the expression (2.25). The condition for the existence of the stationary state then takes the form

$$\varkappa_0\tau_0 c<1. \tag{2.28}$$

Physically condition (2.28) is due to the exponential spatial growth of the intensity of the leading front in the amplifying medium. This is illustrated in Fig. 6, which shows the instantaneous propagation of the intensity of the pulse inside and outside a layer of an amplifying medium, for three values of the parameter $\kappa_0\tau_0 c$. From Figs. 6b and 6c it follows that intensity at

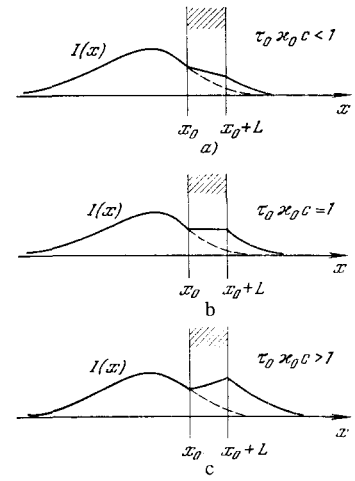

FIG. 6. Spatial distribution of the intensity of a light pulse passing through a nonlinear amplifier, for different values of the parameter $\kappa_0\tau_0 c$ [12].

infinity (as $\tau \to -\infty$) is not satisfied when $\kappa_0\tau_0 c \geq 1$ in an unbounded medium. It follows from Fig. 6c that when $\kappa_0\tau_0 c > 1$ the saturation of the gain begins at the output boundary of the layer, and then moves towards the input boundary in a direction opposite to the propagation of the light ($u < 0$)[49,12]. In this case there is no stationary state of the pulse in an unbounded medium, but if we consider a medium of finite length, then the expression (2.25) for the velocity u is meaningful in this case, too.

The form of the stationary pulses, i.e., pulses that do not change their form as they propagate through a nonlinear amplifying medium, satisfies Eq. (2.26). It was investigated in[8,12]. A typical form of such pulses is shown in Fig. 7. The pulse stationary form I(t), after passing through a layer of amplifying medium of thickness L, is given by

$$I\left(t-\frac{L}{c}+L\varkappa_0\tau_0\right),$$

where τ_0 is the slope of the exponential leading front of the given stationary pulse.

If the condition (2.28) is satisfied, then the stationary pulse moves in the medium with superluminal velocity. This, of course, does not contradict the causality principle. After all, strictly speaking, a pulse of stationary form has an infinitely long leading front and therefore is not a signal. If somewhere far

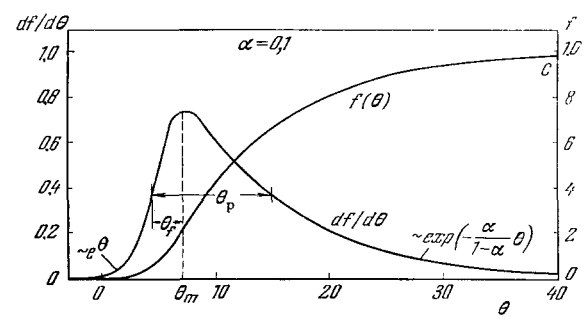

FIG. 7. Form of stationary light pulse. The following dimensional symbols are used: $f = 2\sigma R$, $\theta = t/\tau_0$, $I = (1/2\sigma\tau_0)df/d\theta$, $\alpha = \gamma/\sigma N_0$ [12].

*It is shown in [125] that a pulse of stationary form is also produced in the case of nonlinear amplification of a light pulse moving inside a laser with periodic loss modulation.

on the leading front, at the point $\tau = \tau'$, the front is cut off in such a way that it takes the form shown in Fig. 5, then the motion of the front with velocity $u > c$ will continue until the maximum of the pulse reaches the point $\tau = \tau'$. After this, compression of the pulse begins, and the zero-intensity point $\tau = \tau'$ always moves with the velocity of light in the medium c. This question can also be approached in a different manner. Assume that some "notch" carrying unity information, has been made on the initial pulse. On passing through a linearly amplifying medium, the maximum of the pulse shifts forward, and the "notch" remains in place (Fig. 8). Consequently, the message cannot be transmitted with velocity $u > c$.

However, the gradient of the refractive index, the gradient of the inverted population, a condensation of the average volume polarization of the medium[94], and other changes of the medium parameters, which depend on the intensity of the radiation[47], can propagate with velocity $u > c$ in the medium. This can lead in principle to a number of new effects similar to Cerenkov radiation. This phenomenon is therefore of definite physical interest.

Certain active media have an inhomogeneously broadened amplification line (for example, neodymium glass[81] and gas media). The change of the shape of a pulse whose spectrum is narrower than the homogeneous line width occurs in the same manner as in homogeneous broadening. However, the character of the growth of the energy changes somewhat. The difference is due to the fact that in nonlinear amplification a "hole" is burned in the amplification line, and the width of the "hole" increases with increasing energy of the pulse. The equation for the pulse energy (2.21) takes the following form:

$$\frac{dE}{dx} = \frac{N_0}{2}\int g(\omega - \omega_0 - \Omega)\{1 - \exp[-2\sigma(\Omega)E]\}\,d\Omega - \gamma E, \quad (2.29)$$

where $g(\Omega)$ is the form factor of the inhomogeneous line ($\int g(\Omega)d\Omega = 1$), ω is the frequency of the pulse, and ω_0 is the frequency of the center of the amplification line. In the region of nonlinear amplification, the pulse energy increases almost quadratically with length, until the width of the hole $\Delta\omega$, determined by the relation $\sigma(\Delta\omega/2) = 1/2E$, becomes close to the total width of the amplification line. After this, the growth of the energy will be linear, just as in the case of homogeneous broadening, until saturation of the energy sets in as a result of the nonlinear radiation loss.

Although we have considered a simple ideal case of propagation of the plane wave with homogeneous transverse distribution of the intensity in the medium and with homogeneous distribution of the gain, the results are applicable to and help explain the more general real case when the beam and the medium have transverse inhomogeneity. Owing to the inhomogeneity of the transverse distribution of the gain, narrowing of the width of the beam at half the maximum intensity takes place in the region of the linear amplification if the amplification is maximal at the center of the distribution, and broadening of the beam takes place in the opposite case. Owing to the inhomogeneous transverse distribution of the beam intensity in the region of nonlinear amplification, a broadening of the beam takes place, and an appropriate decrease of the divergence. This phenomenon was observed in[186]. If the transverse distribution of the intensity of the beam has a Gaussian shape, then the beam broadens by a factor $\beta \approx \sqrt{\kappa_0 L_{nonlin}}$, where l_{nonlin} is the length of the nonlinear gain in the medium, and if the beam has exponential wings, then $\beta \approx \kappa_0 L_{nonlin}$. At a gain $K = \exp(\kappa_0 L) = 10^2$, we can expect a beam broadening by a factor 2–4. In addition, the off-axis parts of a pulse having a smooth leading front begin to lag the central part of the pulse, which enters the nonlinear amplification regime sooner. This recalls the occurrence of a transverse structure in a Q-switched laser[58]. All these effects can be seen in Fig. 9, which shows the qualitative picture of the spatial evolution of the pulse in the case of propagation in an amplifying medium.

The change of the spectrum of the pulse is described by the phase equation (2.10) and by Eq. (2.9). From the condition $\omega t - kx + \varphi(t, x) = \text{const}$ and from Eq. (2.10) there follows an expression for the phase velocity of the pulse

$$w = c\left[1 + \frac{\omega_0 - \omega}{k}\frac{T_2}{2}\sigma(\omega)N\right]. \quad (2.30)$$

A powerful light pulse changes the inverted population of the medium, and when $\omega \neq \omega_0$ this changes the phase velocity. It is clear that when the pulse moves towards an immobile observer, who receives light at the output of the amplifying medium, its phase changes, and this leads to a distortion of the spectrum. However, the shift of the spectrum of the pulse is small and lies within the limits of its width. Indeed, the instantaneous shift of the spectrum $\Omega(t, x) = \partial\varphi(t, x)/\partial t$ at the output of the amplifying medium is determined by the expression[50]

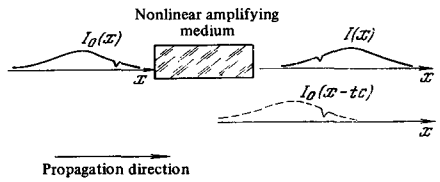

FIG. 8. Illustration of the possibility of using the effect of motion of the maximum of the pulse in a nonlinearly amplifying medium with velocity $u > c$ to transmit a message with superluminal velocity. The dashed line shows the shape of the pulse in the absence of nonlinear amplification.

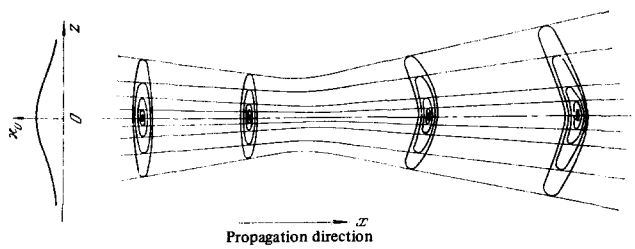

FIG. 9. Evolution of the shape of the pulse in space. The lines join points of equal relative intensity (they do not coincide with the propagation direction).

$$\Omega(\tau, x) - \Omega_0(\tau) = -c\sigma N_0(\omega - \omega_0)\frac{T_2}{2} \times \times \int_0^x \frac{I(x', \tau)}{E_s} \exp\left[-\int_{-\infty}^{\tau} \frac{I(x', \tau')}{E_s} d\tau'\right] dx', \quad (2.31)$$

where $\tau = t - (x/c)$ and $\Omega_0(\tau) = \Omega(\tau, x = 0)$. In the region of linear amplification, a small shift of the spectrum from the center of the line occurs. In the nonlinear regime, the shift increases and in the limiting case, when, for example, the maximum has shifted over the leading front by an amount of the order of the pulse duration, the shift of the spectrum reaches $\delta\Omega \approx T_2/2\tau_0$, where τ_0 is the slope of the pulse front. Under optimal conditions, when the detuning is $|\omega - \omega_0| \approx 1/T_2$, the shift of the spectrum does not exceed the width of the pulse spectrum $\sim 1/\tau_0$.

b) Experiment. Experiments on amplification of a powerful pulse from a Q-switched laser are described in[8-11,43-47]. The main purpose of these investigations was to increase the power of the pulse (the first optical amplifiers were realized in[105-107,42,181]). In these investigations, the amplifying media were ruby crystals or neodymium-glass rods. The choice of these substances was governed by the fact that when they are optically pumped by electric flash lamps it is possible to obtain a gain on the order of 0.1 cm^{-1} at a stored-energy density of the order of 0.2–0.5 J/cm^3. The table lists the main characteristics of ruby and neodymium glass at 300°K as active media. In addition, by the Q-switching method it is easy to obtain from ruby and neodymium-glass lasers pulses with energy on the order of 1 J. Such pulses have an energy close to the saturation energy E_S (see the table), so that the amplification regime should be nonlinear.

The change of the waveform of a Q-switched laser pulse was investigated in[47] for nonlinear amplification. It was shown there for the first time that the pulse duration of such a laser is not decreased by nonlinear amplification. Instead, the maximum of the pulse shifts over the leading front without a noticeable reduction of the duration. The reason is the extended exponential leading front inherent in the Q-switched laser pulse.

Main parameters of amplifying ruby and neodymium glass media at 300°K

Medium	Wavelength, frequency of transition, quantum energy	Line width, cm^{-1}	Lifetime of upper level, T_1, sec	Lifetime of lower level, τ_s, sec	Cross section of transition σ, 10^{-20} cm^2	Saturation energy E_S, J/cm^1
Ruby[1] (Al_2O_3: Cr^{3+}), $^2E \rightarrow {}^4A_2$	6943 Å, $4.33 \cdot 10^{14}$ Hz, $2.86 \cdot 10^{-19}$ J	11 [1,2]	$2.5 \cdot 10^{-3}$ [1,2]	∞	2.5 [1]	5.8
Neodymium glass[3] (glass: Nd^{3+}) $^4F_{3/2} \rightarrow {}^4I_{11/2}$	10 600 Å, $2.83 \cdot 10^{14}$ Hz, $1.87 \cdot 10^{-19}$ J	250 [3]	$0.6 \cdot 10^{-3}$ [3]	~ 60 nsec[4]	0.8–1.3[5]	~ 13 at $\tau_p \gg \tau_s$[6]; 7 at $\tau_p \ll \tau_s$[7]

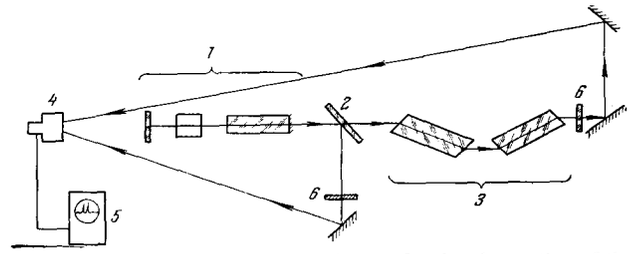

FIG. 10. Diagram of experimental setup for the observation of the change of the waveform of a light pulse in nonlinear amplification [8]. 1 – Master ruby laser Q-switched with a Kerr cell; 2 – glass plate for splitting the light beam; 3 – ruby crystals of traveling-wave amplifier; 4 – coaxial photocell; 5 – high-speed oscilloscope; 6 – neutral filters to attenuate the light fluxes.

Indeed, no noticeable reduction in pulse duration was observed in all the investigations[43-46] of nonlinear amplification of such pulses.

The experimental setup used in[47,8], is shown in Fig. 10. The ruby laser was Q-switched by a Kerr cell (shutter) and emitted a light pulse that passed through two stages of optical traveling-wave quantum amplification. The end faces of the ruby crystal in the amplifier were cut at the Brewster angle relative to the polarized light of the pulse. This prevented reflection of the radiation and self-excitation of the amplifiers by parasitic feedback. In such an amplifier, a gain on the order of 50–60 is reached for a weak signal. Part of the flux of the input pulse was diverted to the cathode of a coaxial photocell. The amplified light pulse was applied to the same photocell after delay and attenuation. As a result, it was possible to register simultaneously on the screen of a high-speed oscilloscope the input and output signals. Figure 11a shows the oscillograms of the input and output pulses following linear amplification, when the input pulse was attenuated by a factor 3×10^3 (the energy of the input pulse before attenuation was 1.3–1.5 J, its duration at half-height was 16 nsec, the duration of the leading edge from the 0.5 level to maximum was 8 nsec, the start of the pulse determined by the instant of switching of the electrooptical shutter was 45–47 nsec ahead of the peak of the pulse). Figure 11b shows a similar oscillogram for the case of the input pulse without attenuation. From a comparison of the oscillograms of Figs. 11a

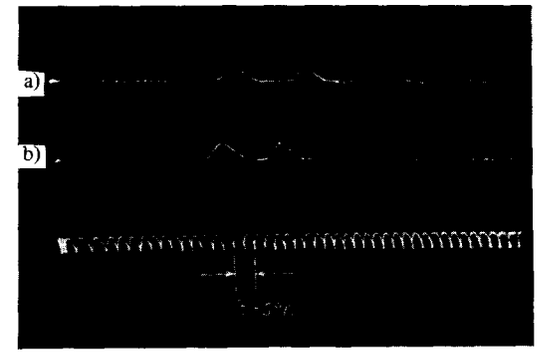

FIG. 11. Oscillograms of input (left) and amplified (right) light pulses [8]. a) Weak signal – linear amplification; b) strong signal – nonlinear amplification.

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and b we see that there is no noticeable reduction in the pulse duration, but the output pulse in Fig. 11b is closer to the input pulse than in Fig. 11a. This shift is equal to $\Delta\tau = 9 \pm 0.5$ nsec for both the maxima and half-heights of the leading fronts. Consequently, in nonlinear amplification of the pulse with exponential leading front, a shift of the pulse maximum takes place over the leading front, without a reduction in its duration. The magnitude of the shift, according to (2.18), is given by the expression

$$\Delta\tau = \varkappa_0 L_{\text{nonlin}} \tau_0,$$

where κ_0 is the gain of the medium per unit length, L_{nonlin} is the distance traversed by the pulse with energy $E > E_S$, i.e., the length of the nonlinear gain, and τ_0 is the time constant of the input pulse. In the experiment[4,7,8], $\kappa_0 \approx 0.12$ cm^{-1}, $L_{\text{nonlin}} \approx 10-15$ cm, and $\tau_0 = 4$ nsec. Consequently, $\Delta\tau_{\text{theor}} \approx 5-7$ nsec, which agrees with the experimental value $\Delta\tau_{\text{exp}} \approx 9$ nsec. The main error lies in the determination of the length of L_{nonlin} the nonlinear amplification of the pulse in the medium.

A more detailed comparison of theory and experiment can be obtained by observing the dependence of the shift $\Delta\tau$ on L_{nonlin}. Such an experiment was carried out in[11]. The amplifying medium in this investigation was neodymium glass. Figure 12 shows a plot of the shift against the pulse energy at the entrance to the medium[11]. In the region of linear amplification, when the energy density of the amplified pulse is $E = E_0 K\gamma$ (E_0—energy density of input pulse, K—gain of the medium, γ—coefficient of energy-density attenuation due to divergence) is much smaller than the saturation energy of the amplification E_S, there is no shift towards the start of the pulse, With increasing energy of the input pulse, a certain shift appears, and in the region $E > E_S$ the shift $\Delta\tau$ depends linearly on $\ln E_0$, i.e., on the nonlinear amplification length L_{nonlin}. To be sure, on the experimental curve the linear section occupies a small interval of values of E_0. The observed deviation at large values of E_0 is due to the occurrence of a coupling between the amplifier and a generator, leading to a decrease in the gain, owing to the intense superionization prior to the arrival of the pulse. In[11], the gain reached 10^4 at an active-medium length of 240 cm.

To reduce the pulse duration in nonlinear amplification, it is necessary to cut off in some way the gently sloping leading front of the input pulse. In[8], the leading front was cut off by means of an additional electro-optical shutter, with transmissions 3% and 50% in the closed and opened states, respectively. The total gain of the ruby medium was 10^2. The duration of the pulse propagating in the amplifying medium decreased from 9 to 5 nsec. An ideal supplementary shutter, which effected complete cutoff of the leading front, should have zero initial transmission. This cannot be attained in practice, and at sufficiently high gain the transmitted leading front causes the active medium to emit. As a result, broadening of the pulse takes place. Figure 13a shows the calculated change in the form of a pulse with an incompletely cut-off leading front. It is clearly seen that with further propagation the compression of the pulse gives way to expansion due to the amplification of the leading front. Figure 13b shows an oscillogram of the output pulse when the gain is increased to 500. It shows the initial stage of the broadening of the pulse[10].

To decrease the initial transmission, a cell with a solution of phthalocyanine of vanadium, with initial transmission 10^{-3}, was added to the Kerr shutter in[9,10]. Figure 14 shows oscillograms illustrating the effect of reduction of the pulse after passing through the amplification stages. The duration of the output pulse decreases to 2 nsec. The pulse energy reached 15 J in this experiment.

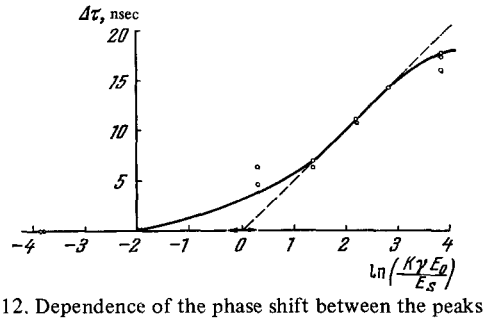

FIG. 12. Dependence of the phase shift between the peaks of the input and output pulses on the energy density at the input of the amplifying medium E_0 [11]. K – gain of the medium, γ – coefficient of attenuation of the energy density as a result of the divergence.

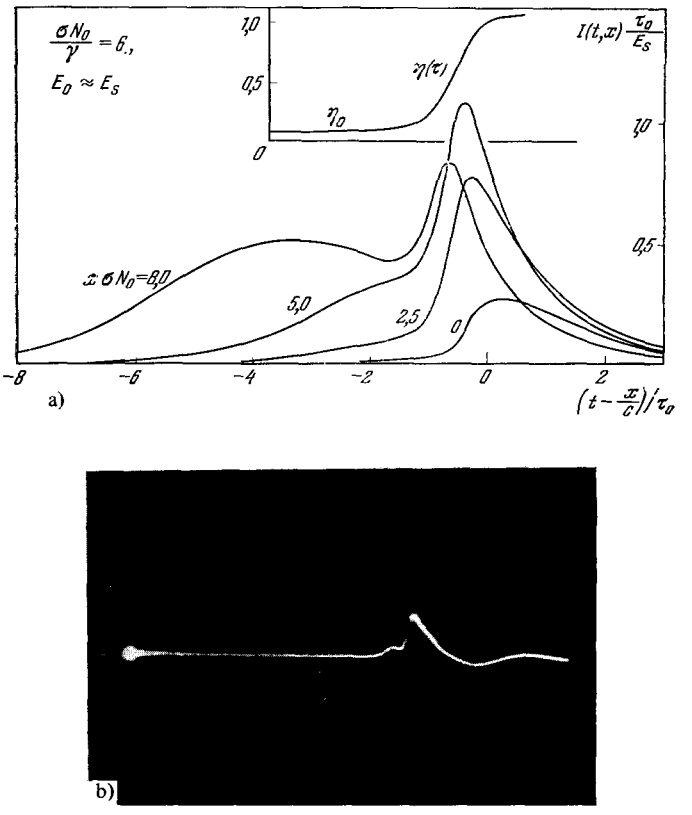

FIG. 13. Change of waveform of a light pulse with incompletely cut off leading front. a) Calculation change of pulse following amplification, if the shutter is opened in accordance with the $\eta(\tau)$ law; b) experimentally observed rise of the leading front at an initial shutter transmission $\eta_0 = 3\%$ [10].

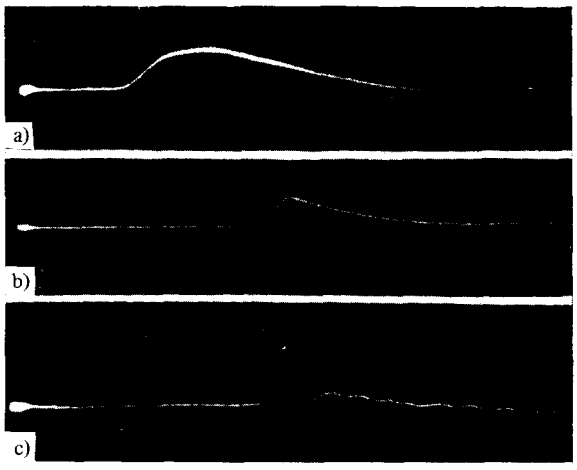

FIG. 14. Oscillograms of pulses after passage through stages of a ruby amplifier when the leading front is cut off by a combined shutter: a) after passage through the shutter; b) after the second amplification stage; c) output pulse [9].

The approximate experimental values of the parameter $\kappa_0 T_0 c$ were 8 in[47] and 7 in[11]. In accordance with the condition (2.28), one could not speak in these cases of the rate of propagation of the pulse maximum in the medium. In the experiment of[9], the pulse in the last stage of the amplifier had a rise time $\tau_0 \approx 0.4$ nsec. The pulse energy density, amounting to 7 J/cm^4, was sufficient to saturate the ruby gain. For such a pulse, $\kappa_0 T_0 c \approx 0.7$ ($\kappa_0 \approx 0.1$ cm^{-1}), and consequently one can speak of superluminal velocity of propagation of the pulse maximum in the medium. In this experiment, the slope of the pulse front was not constant, owing to the decrease in the duration, so that the speed of the maximum changed as the pulse propagated in the medium. At the input end of the last stage, where $\tau_0 \approx 0.4$ nsec, the speed of the maximum was $u \approx 3c$. To observe effects resulting from the propagation of the maximum of pulse with superluminal velocity, it is necessary to have shorter pulses with an energy exceeding the gain saturation energy of the medium. So far there are no experiments in which such effects could be observed.

2.3. Coherent Interaction

In the propagation of ultrashort light pulses with duration of the order of or smaller than T_2, it is necessary to take into account the coherent interaction. In this case the propagation of the pulse is described by Eqs. (2.6), which contain a number of effects (finite width of the amplification line and its in the field of a powerful pulse, "phase memory" of the medium during the time T_2, and an oscillatory response to a strong field) not taken into account by the rate equations (2.9).

In the case of exact resonance ($\omega = \omega_0$), when the phase velocity of the pulse in the medium is equal to c, the system (2.6) simplifies greatly:

$$\begin{cases} \frac{\partial \mathcal{E}}{\partial t} + c\frac{\partial \mathcal{E}}{\partial x} = -\frac{\gamma}{2} c\mathcal{E} + 2\pi\omega\mathcal{P}, & (2.32) \\ \frac{\partial \mathcal{P}}{\partial t} + \frac{1}{T_2}\mathcal{P} = \frac{\mu^2}{\hbar} N\mathcal{E}, & (2.33) \\ \frac{\partial N}{\partial t} = -\frac{1}{\hbar}\mathcal{E}\mathcal{P}. & (2.33a) \end{cases}$$

The reaction of the medium to the light pulse depends strongly on the field intensity. If the field is weak, so that $\mu\mathcal{E}/\hbar \leq \tau_p^{-1}$, then the population of the levels does not change noticeably under the influence of the field, and the amplification is linear. Nonetheless, owing to the fact that $\tau_p \lesssim T_2$, i.e., the width of the spectrum of the pulse is comparable with or even larger than the width of the amplification line $2/T_2$, the shape of the pulse changes during the propagation. This can be readily understood by considering the reaction of the medium to a weak δ-pulse $\mathcal{E}_0\delta(t - t_0)$, where $\delta(t)$ is the delta function. The polarization of the medium in the field of such a pulse is given by $\mathcal{P} = (\mu_2/h)N_0\mathcal{E}_0\exp[(t_0 - t)/T_2]$, where $t > t_0$. Since the polarization is conserved for a time $\sim T_2$, the medium radiates during that time independently of the pulse duration. It is clear that a pulse of duration $\tau_p \ll T_2$ will broaden upon propagation to a value on the order of T_2, and its propagation velocity will be somewhat smaller than c. This question is discussed in[60].

An entirely different picture is presented by the evolution of a powerful pulse, for which $\mu\mathcal{E}/\hbar \gtrsim \tau_p^{-1}$. In this case the response of the medium is oscillatory. Indeed, when $\tau_p \ll T_2$ we can neglect the relaxation of the polarization in (2.33), and we obtain the following solution for the material equations:

$$N = N_0\cos\Phi, \quad \mathcal{P} = \mu N_0 \sin\Phi, \qquad (2.34)$$

where

$$\Phi = \frac{\mu}{\hbar}\int_{-\infty}^{\infty}\mathcal{E}(t, x)\,dt. \qquad (2.35)$$

The change of N and $\mathcal{P}$ can be represented as the rotation of a unit vector in the (X, Y) plane in such a way that the X component of the vector corresponds to N/N_0, and the Y component to $\mathcal{P}/\mu N_0$. Then the function Φ is the angle of rotation of this vector: $\Phi = \pi$ corresponds to a complete transition of the particle to the lower level, and $\Phi = 2\pi$ corresponds to a complete return to the upper level.

The change of the pulse energy is described by the equation

$$\frac{dE}{dx} = N_0(1 - \cos\Phi) - \gamma E, \qquad (2.36)$$

which differs significantly from the analogous equation (2.21) in the case of the incoherent interaction. In the absence of linear losses ($\gamma = 0$), the growth of the pulse energy is determined completely by the change of the angle of rotation Φ under the influence of this pulse. Figure 15 shows the dependence of the gain dE/dx on the angle Φ (curve 1). If the pulse causes the vector to rotate through an angle $\Phi = 2\pi m$ (m is an integer), then such a pulse propagates without an energy gain. If the angle of rotation under the influence of the pulse is $\Phi = 2\pi m + \delta$, where $\delta > 0$ is a momentarily small quantity, then such a pulse, obviously, will become amplified until the angle of rotation becomes equal to $2\pi(m + 1)$. If $\delta < 0$, then the amplification continues until the rotation angle reaches $2\pi m$. In this sense, the "$2\pi m$-pulses" are unstable. From the curve of Fig. 15 we can guess that the transmission of the pulse occurs in "batches" each of which changes

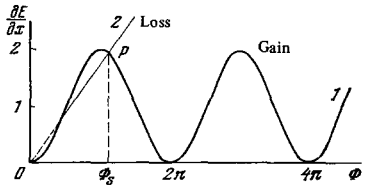

FIG. 15. Dependence of the gain and loss of the pulse energy on the angle of rotation Φ [53].

the rotation angle Φ by 2π. This corresponds to a gradual breakdown of the pulse into "2π-pulses."

If linear radiation losses take place ($\gamma > 0$), then the evolution of the pulse is radically changed. This can be understood by replacing the pulse energy in (2.36) by $\beta\Phi$, where β is a certain coefficient with the dimension of energy flux (photons/cm^2). The loss $\gamma E \approx \gamma\beta\Phi$ is plotted in Fig. 15 (curve 2). Owing to the nonlinear loss, a stationary value of the phase Φ_S sets in, and to determine this value it is necessary to know the value of β. When the pulse propagates, the coefficient varies until the pulse assumes the stationary form. For a pulse of stationary form we have $\beta = 2N_0/\pi\gamma$, and consequently $\Phi_S = \pi$. Thus, in the presence of linear losses there should form one "π-pulse" independently of the initial value of the phase $\Phi_0 = \Phi(x = 0)$.

These conclusions are confirmed by the results of a numerical integration of the system (2.32) and (2.33), presented in[53]. The propagating pulses considered there had on the boundary of the medium the form $\mathcal{E}_0(t) = \mathcal{E}_0\cosh^{-2}(t/\tau_0)$ with duration $\tau_0 \ll T_2$ and amplitude $\mathcal{E}_0$. Such a pulse corresponds to a rotation angle $\Phi_0 = 2\mathcal{E}_0\tau_0(\mu/\hbar)$. In Fig. 16 ($t = \tau_0$) are shown the results of the solution for the case $\alpha = \gamma/\sigma N_0 = 0$, $\tau_0/T_2 = 0.33$, and $\mathcal{E}_0\mu T_2/\hbar = 3$, when $\Phi_0 < 2\pi$. As the pulse propagates in the medium without linear losses,

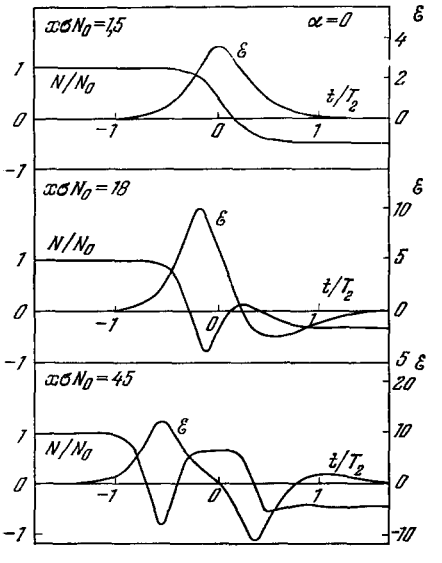

FIG. 16. Evolution of a pulse of coherent light propagating in an amplifying medium without linear losses (x – distance covered by the pulse in the medium). The initial angle of rotation under the influence of the pulse is $\Phi_0 = 2$ [53].

it becomes transformed into two pulses, a stationary "2π-pulse" and a stationary "π-pulse." An important factor is, first, that the "π-pulse" is produced even in the presence of polarization relaxation (the formation of a "2π-pulse" in an amplifying medium without relaxation ($T_2 = \infty$) is demonstrated in[52]). Second, the "2π-pulse" is followed by a "π-pulse," which effects the relaxation of the populations to the absorbing state. Figure 17 shows the results of the solution for $\gamma = 0$ in the case $\tau_0/T_2 = 0.33$ and $\mathcal{E}_0\mu T_2/\hbar = 15$, when $2\pi < \Phi_0 < 4\pi$. Such a pulse is gradually transformed into two "2π-pulses" and one "π-pulse." This tendency remains also with further increase of the pulse power. Thus, if the angle of rotation under the influence of the initial ultrashort pulse is $\Phi_0 = 2\pi(m + \delta)$, where m is an integer, and $0 < \delta < 1$, then in the case of propagation in an amplifying medium with nonlinear losses, such a pulse becomes transformed into m + 1 "2π-pulses" and one "π pulse."

In a medium with linear radiation losses, one stationary "π pulse is always produced. For the case of initial pulses with low intensity ($\Phi_0 \ll 2\pi$), this was demonstrated already in[36] by numerical integration of Eq. (2.32) and (2.33). This result remains in force also when $\Phi_0 > 2\pi$. It is shown in[53], likewise by numerical integration, that the pulse ultimately is transformed into a stationary "π pulse," regardless of the initial power. The initially produced "2π-pulses" gradually attenuate, and the first "2π-pulse" is transformed into a "π-pulse." This is clearly seen in Fig. 18, which shows the results of the solution for the case when $\Phi_0 = 3\pi$ and $\gamma/\sigma N_0 = 0.2$.

The form of the stationary "2π-pulse" $\mathcal{E}[t - (x/u)]$ is obtained by solving Eqs. (2.32) and (2.33) with $\tau_p \ll T_2$. The solution takes the form[52,53]

$$\mathcal{E}\left(t-\frac{x}{u}\right) = \frac{2}{\tau_0}\frac{\hbar}{\mu}\operatorname{sch}\left[\frac{t-(x/u)}{\tau_0}\right], \qquad (2.37)$$

where the parameter τ_0 is determined by the slope of the initial pulse, and the propagation speed is determined by the expression[53]

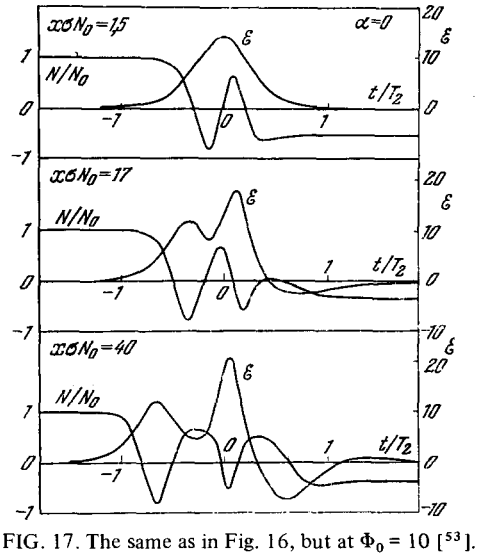

FIG. 17. The same as in Fig. 16, but at $\Phi_0 = 10$ [53].

$$\frac{1}{u}=\frac{1}{c}-\frac{1}{2}\frac{\tau_1^2}{T_2}\sigma N_0. \qquad (2.38)$$

It follows from (2.38) that the speed of propagation of the stationary "2π-pulse" is $u > c$. This phenomenon is analogous to the previously considered effect of superluminal propagation of a pulse having an exponential leading front and a duration $\tau_p \gg T_2$. Moreover, the expression for the speed (2.38) coincides with expression (2.25). Since the width of the spectrum of the ultrashort pulse is larger by T_2/τ_0 times than the width of the amplification line, the amplification of such a pulse per unit length $\kappa_0 = \sigma N_0(\tau_0/T_2)$. Substituting this value of κ_0 in (2.25) and recognizing that the rate of growth of the front intensity is twice the rate of growth of the amplitude, we obtain expression (2.38). In spite of the fact that the velocities coincide, the forms of the stationary pulses in these two cases are entirely different. This is due to the different loss mechanisms that maintain the stationary character of the pulse. In the case of incoherent interaction, the establishment of the stationary form is due to the additional linear losses of the radiation γ[8,12], whereas in the case of the "2π-pulse" the absorption occurs in the amplifying medium itself after half the pulse passes through it, i.e., after the population inversion, and this absorption is nonlinear.

The form of the stationary "π-pulse" $\mathcal{E}[t - (x/u)]$ in the medium with $\gamma > 0$ was obtained analytically in[37,38,41]. It was assumed there that the velocity of the pulse is $u = c$. As shown by numerical calculations performed in[36], u is somewhat smaller, but when $\gamma \ll \sigma N_0$ this difference is so small that this effect can be neglected. The form of the stationary "π-pulse" is determined by the expression

$$\mathcal{E}\left(t-\frac{x}{c}\right)=\frac{1}{\tau_0}\frac{\hbar}{\mu}\operatorname{sch}\left[\frac{t-(x/c)}{\tau_0}\right], \qquad (2.39)$$

where

$$\tau_0=\frac{T_2}{(\sigma N_0/\gamma)-1}. \qquad (2.40)$$

In the region $\gamma \approx \sigma N_0$ it is necessary to take into account the difference between u and c, and this leads to a certain asymmetry of the "π-pulse"[36]. During its propagation, the "π-pulse" acquires practically the entire energy of the excited particles. This is a feature of the coherent interaction, when the particle is transformed by the field into a coherent superposition of the ground and excited states, in which it continues to emit until it goes over completely to the lower state.

It is seen from (2.40) that the duration of the "π-pulse" can be much shorter than T_2. This raises the question: how is such a pulse produced in the medium if its spectrum is much broader than the spectral amplification line? In a resonant electromagnetic field, broadening of the spectral line takes place as a result of the effect of saturation, by an amount[26]

$$\omega_{\mathcal{E}}=\frac{\mu\mathcal{E}}{\hbar}. \qquad (2.41)$$

Then the condition of the population inversion by the "π-pulse" $(\mu\mathcal{E}/\hbar)\tau_p = \pi$ can be written in the form

$$\Delta\omega_{\mathcal{E}} \approx \Delta\omega_p, \qquad (2.42)$$

where $\Delta\omega_p \approx \pi/\tau_p$ is the width of the spectrum of the pulse. Thus, the field intensity of the "π-pulse" is maintained such as to broaden the amplification line to a value equal to the width of the spectrum of the pulse. In other words, when a powerful ultrashort pulse propagates, self-broadening of the spectral lines takes place. From this point of view, the "π-pulse" should be formed in a gaseous amplifying medium with inhomogeneous broadening of the levels, if the line broadening in the field exceeds the Doppler width. In this case the line becomes uniformly broadened, owing to the broadening by the field.

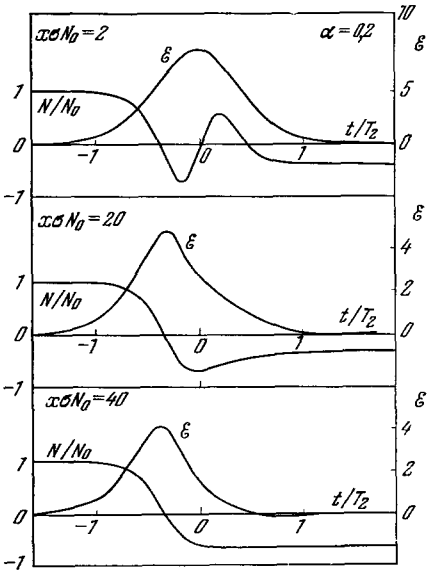

FIG. 18. Evolution of a pulse of coherent light propagating in a resonantly amplifying medium with linear losses ($\alpha = \gamma/\sigma N_0 = 0.2$). The initial angle of rotation under the influence of the pulse is $\Phi_0 = 3\pi$ [53].

3. ABSORBING MEDIUM

Any resonantly absorbing medium in an intense field should possess nonlinear absorption. The power necessary to saturate the absorption of solid and liquid media exceeds 1 MW/cm^2 as a rule, but if the excited particle falls into a metastable state, the saturation power may drop to a value on the order of several dozen kW/cm^2. Therefore certain nonlinear absorbers were revealed with the aid of pulsed sources of incoherent light[17,80]. The nonlinear absorbers employed are glasses doped with absorbing ions (uranium glass[17], neodymium glass[95], etc.), solutions of complex organic molecules (phthalocyanines[80,19], cryptocyanines[76,77], polymethine dyes[82-84], etc.), and gases (SF_6[108,85], BCl_3[86], etc.). The most widely used are organic bleachable solutions, which are used for laser Q-switching[76,77,19], to sharpen the leading front of a pulse[9,10], for mode self-locking[20,14], etc.

The relaxation of the bleached state of many absorbers, for example saturable solutions of organic molecules, differs from the simple two-level scheme. This is due to the fact that the excited particles can go over to a certain intermediate level, and only then do they relax to the ground state[74,25,88]. However, it suf-

fices completely to examine in detail the propagation of the pulse in an absorbing medium made of two-level particles. It is then possible to understand the picture of the evolution of the pulse in the case of more complicated character of the relaxation of the level population.

Just as in the case of an amplifying medium, the evolution of the pulse depends essentially on the relation between the pulse duration and the polarization relaxation time T_2. When $\tau_p \gg T_2$, the interaction of the field with the medium has an incoherent character. When $\tau_p \ll T_2$, the interaction is coherent and effects of the "2π-pulse" type appear[21].

3.1. Incoherent Interaction

The incoherent interaction of a light pulse with a two-level absorbing medium is described by Eqs. (2.9), in which it is necessary only to replace the sign of the inverted population N. The picture of the evolution of the pulse having a duration much longer than the relaxation time of the saturated state T_1 (more accurately, $1/I\,\partial I/\partial t \ll T_1^{-1}$), is particularly simple. In this case the absorption coefficient of the medium per unit length, κ, is determined by the instantaneous intensity:

$$\varkappa = \frac{\varkappa_0}{1+2\sigma T_1 I(x,t)}, \tag{3.1}$$

and the propagation of the pulse is described by the equation

$$\frac{\partial I}{\partial t} + c\frac{\partial I}{\partial x} = -cI\left[\gamma + \frac{\varkappa_0}{1+(I/I_s)}\right], \tag{3.2}$$

where $I_s = (2\sigma T_1)^{-1}$ is the saturation power (photons/cm^{2}sec), under the influence of which the absorption decreases by a factor of two. The quantity I_s is the nonlinearity parameter.

Inasmuch as the absorption decreases with increasing intensity, the maximum of the pulse is absorbed more weakly than the fronts. As a result, obviously, an increase of the slope of the fronts and a reduction of the pulse duration takes place during the propagation. It is easy to deduce from (3.2) that the rate of reduction of the pulse duration τ during the propagation is determined by the expression[22,25]

$$W = \frac{d\tau}{dx} = \varkappa_0 \frac{\tau P}{(1+P)(2+P)}, \tag{3.3}$$

where $p = I/I_s$ is the dimensionless intensity of the pulse at the maximum. The rate of compression of the pulse is maximal when $I = \sqrt{2}I_s$. When $I \gg I_s$, the compression is small because the bleaching of the medium occurs far on the leading and trailing edges, and when $I \ll I_s$ the compression is small because of the weak nonlinearity of the absorption. In an absorbing medium, a pulse of any initial intensity must attenuate. Since the maximum intensity will be of the order of I_s for some finite time during the course of attenuation, the compression of the pulse is finite. From the general expression obtained in[25] it is possible to obtain the following connection between the maximum input intensity I_0 and the intensity I at the output of the medium, and the pulse durations τ_0 and τ at the start and at the end of the propagation:

$$\frac{\tau}{\tau_0} = \left[\frac{1+(I/I_s)}{1+(I_0/I_s)}\right]^{1/2}. \tag{3.4}$$

The pulse compression is obtained at the expense of decreasing the intensity. For example, when I changes by a factor of 10, from $3I_s$ to $0.3I_s$, the pulse is compressed by a factor of approximately 2. Therefore this method is effective for the formation of short light pulses if the attenuation of the pulse is compensated for by propagation in an amplifying medium. This can be done in a two-component medium having nonlinear absorption and linear gain[22]. This is considered in detail in Ch. 4 below.

The effect of compression of the pulse by a nonlinear absorber is essentially the basis of the ultrashort-pulse laser with self-locking of the modes[20,14]. The formation of an ultrashort pulse in this laser is regarded as an increase in the number of the locked modes with the aid of a nonlinear absorber. However, this process can be regarded as multiple passage of a pulse through a nonlinear absorber. In this way it is possible to investigate in great detail the dynamics of an ultrashort-pulse laser with a nonlinear absorber[25].

The compression of the pulse propagating in a nonlinear absorbing medium, in the region of the incoherent interaction, was observed experimentally in a number of investigations[72,73,74]. The most convincing experiment is the one reported in[73], where a reduction of duration by a factor of 4 was observed for propagation in a gaseous absorbing medium. Since the main purpose of[73] was an investigation of the coherent interaction of a pulse with a medium, this experiment will be discussed in greater detail in Sec. 3.2. We note that the observation of pulse compression is a convenient method of estimating the relaxation time T_1[74], since a reduction of the duration resulting from simultaneous sharpening of the leading and trailing edges can be attained only when $\tau_p \gg T_1$.

If the pulse duration is comparable with or shorter than T_1, then the response of the medium is determined by Eqs. (2.9). The evolution of the pulse in this case was considered theoretically in[69,70,75]. The main results were obtained in these investigations by numerically integrating the equations. However, the qualitative picture of the evolution of the pulse is quite clear. The bleached state of the medium, produced by the leading front of the pulse, relaxes either incompletely or not at all on the trailing edge. Therefore, only the leading front becomes sharpened during the propagation, and the trailing edge is deformed much less, since it moves in a bleached medium. As a result, the shape of the pulse becomes asymmetrical with a steeper leading front, and the peak of the pulse lags gradually, becoming shifted along the trailing edge. The sharpening of the leading front on passing through a nonlinearly absorbing medium was observed experimentally in[9,10]. This effect was used to increase the slope of the leading front of a Q-switched laser pulse so as to reduce the pulse duration in a nonlinear amplifier.

The picture of propagation of a powerful pulse in a medium with $T_1 = \infty$ is quite clear. In this case the propagation causes gradual bleaching of the medium as

a result of the continuous absorption of the leading front. If the pulse is sufficiently long and has a constant intensity, then the propagation of the leading front and of the bleaching boundaries become quasistationary and have a certain constant speed u[62,67]. The effect of motion of the bleaching boundary was first considered in[62,67], and was then investigated in[67,68,71]. The speed of the bleached region, for a pulse of constant intensity, can be obtained from the following considerations. In order for the bleached region to penetrate to a depth Δx it is necessary to excite $N_0\Delta x/2$ particles per unit cross section. The excitation consumes $N_0\Delta x/2$ photons of the leading front of the pulse, i.e., a leading-front section with duration $\Delta\tau = N_0\Delta x/2I_0$ is absorbed, where I_0 is the intensity of the pulse. Taking into account the motion of the photons with velocity c ($\tau = t - (x/c)$), we obtain for the rate of motion of the bleached region $u = \Delta x/\Delta t$ the following expression[62,67]:

$$u \approx \frac{c}{1+(N_0/2I_0)c}. \qquad (3.5)$$

In the limiting case $u \ll c$, this expression reduces to the following:

$$u \approx \frac{2I_0}{N_0}. \qquad (3.6)$$

The form of the stationary bleaching wave can be obtained from the propagation equation

$$\frac{\partial I}{\partial t} + c\frac{\partial I}{\partial x} = -c\sigma N_0 \exp\left[-2\sigma\int_{-\infty}^{t} I(t', x)\,dt'\right] I. \qquad (3.7)$$

To this end it is necessary to go over to the variable $t - (x/u)$ and to carry out the appropriate calculations. The solution is given by[71]

$$I(t, x) = I_0\left\{1 + [\exp(\sigma N_0 x) - 1]\exp\left[-\left(t - \frac{x}{c}\right)\cdot 2\sigma I_0\right]\right\}^{-1}, \qquad (3.8)$$

$$N(t, x) = N_0\left\{1 + \exp(-\sigma N_0 x)\left(\exp\left[\left(t - \frac{x}{c}\right)\cdot 2\sigma I_0\right] - 1\right)\right\}^{-1}. \qquad (3.9)$$

The distributions of the level population difference N and of the intensity on the front of the bleaching wave are shown in Fig. 19. The degree of smearing of the front is of the order of $1/\sigma N_0$ and does not depend on the radiation intensity.

In certain absorbers, the excited particles fall on an intermediate metastable level with a lifetime $T^* \gg \tau_p$, and only then do they relax to the ground state. The obtained relations are valid also in this case, if the intensity I_0 is decreased by one half. This corresponds to a doubling of the saturation energy E_s.

Bleaching waves can apparently be investigated experimentally by observing secondary phenomena accompanying their propagation. This is possible in those cases in which the absorption leads to a sharp change in the state of the absorbing medium, for example in the case of photo-dissociation of molecules. It is clear that the bleaching wave will be accompanied by a pressure jump, etc. Certain aspects of this question are discussed in[89]. It is possible that the bleaching waves play an important role in natural optical pumping of the molecules in the interstellar medium in outbursts of stars and contributes to the occurrence of amplification in large values of outer space[71].

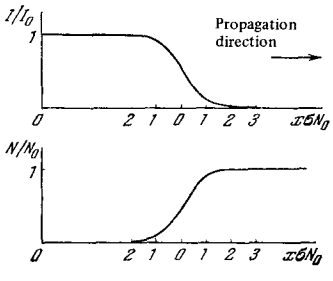

FIG. 19. Form of stationary bleaching wave [71].

3.2. Coherent Interaction

In the case of propagation of a pulse of duration $\tau_p \gg T_2$, the bleaching of the medium requires an irreversible expenditure of pulse energy, and a pulse of finite energy must of necessity attenuate. The nonlinearity of the interaction between the pulse and the medium only causes the rate of attenuation of a high-power pulse to be smaller by several orders of magnitude than that of a weak pulse. When $\tau_p \ll T_2$, the polarization does not attenuate during the pulse, and the interaction is not only nonlinear but also coherent. Naturally, to this end it is necessary that the field of the pulse be coherent, i.e., that the width of the spectrum of the pulse be due only to its finite duration. In this case the excited particle is in a coherent state, in which it emits coherently and goes over to the lower state ("2π-pulse"). As a result, the energy absorbed on the leading front of the pulse is returned on the trailing edge. Therefore a pulse with energy sufficient to raise the particles to an excited state propagates in a resonantly absorbing medium without attenuation. This effect, called "self-transparency" of the pulse, was first considered in[21]. Its physical cause is the same as that of a stationary "π-pulse" in an amplifying medium[36-38] or of radiation echo in the radio and optical bands[90-93].

a) Theory. In the case of homogeneous broadening of the levels, the propagation of the pulse is described by Eqs. (2.32) and (2.33), in which it is necessary to reverse the sign of the inverted population N. However, in the case of homogeneous broadening, the condition $\tau_p \ll T_2$ means that the spectrum of the pulse is much broader than the absorption line, i.e., even the linear absorption is small and depends on the pulse duration. It is therefore advantageous to consider from the very beginning an inhomogeneously broadened absorption line, such that the inhomogeneous width is much larger than the width of the spectrum of the pulse, and the attenuation of the weak pulse is independent of the pulse duration. We shall thus assume that the duration of the pulse satisfies the condition[21]

$$T_2^* \ll \tau_p \ll T_2, \qquad (3.10)$$

where $2/T_2$ is the homogeneous width of the line, and $2/T_2^*$ is the inhomogeneous width of the absorption line. The inhomogeneous broadening is characterized by a distribution function $g(\Omega)$ of the resonant frequencies of the particles relative to the central frequency ω_0. Then the initial population difference of the lower and upper levels at the frequency $\omega_0 + \Omega$ is

$$n_{0\Omega} = N_0 g(\Omega), \quad \int_{-\infty}^{\infty} g(\Omega)\, d\Omega = 1, \tag{3.11}$$

where N_0 is the total initial density of the population difference.

In the case of inhomogeneous broadening, the equations for the field are the same as for homogeneous broadening (the first two equations of the system (2.6)), in which $\mathcal{P}$ and ψ are the amplitude and phase of the summary polarization of all the particles. The summary polarization of all the particles is given by

$$\mathcal{P}\cos(\omega t + \psi) = \int_{-\infty}^{\infty} \mathcal{P}_\Omega \cos(\omega t + \psi_\Omega)\, d\Omega, \tag{3.12}$$

where $\mathcal{P}_\Omega$ and ψ_Ω are the amplitude and phase of the polarization of the particles with the resonant frequency $\omega_0 + \Omega$. The polarization and population of the particles at the frequency $\omega_0 + \Omega$ is determined by the material equations

$$\left.\begin{aligned} \frac{\partial \mathcal{P}_\Omega}{\partial t} + \frac{1}{T_2}\mathcal{P}_\Omega &= -\frac{\mu^2}{\hbar} n_\Omega \mathcal{E} \sin(\psi_\Omega - \varphi), \\ \left[\frac{\partial \psi_\Omega}{\partial t} + (\omega - \omega_0 - \Omega)\right]\mathcal{P}_\Omega &= -\frac{\mu^2}{\hbar} n_\Omega \mathcal{E} \cos(\psi_\Omega - \varphi), \\ \frac{\partial n_\Omega}{\partial t} &= \frac{1}{\hbar}\mathcal{P}_\Omega \mathcal{E} \sin(\psi_\Omega - \varphi), \end{aligned}\right\} \tag{3.13}$$

where all the symbols are the same as in (2.6).

In the case of exact resonance ($\omega = \omega_0$) in the approximation (3.10) it is possible to obtain from the propagation equation and from the material equations (3.13) an equation for the angle of rotation under the influence of the pulse $\Phi = (\mu/\hbar)\int_{-\infty}^{\infty} \mathcal{E}(t', x)\, dt'$ [21]:

$$\frac{d\Phi}{dx} = -\frac{\gamma}{2}\Phi - \frac{\alpha_0}{2}\sin\Phi, \tag{3.14}$$

where $\alpha_0 = 8\pi^2\mu^2\omega_0 g(0) N_0/\hbar c$ is the absorption coefficient of a weak signal per unit length. In a medium without linear losses ($\gamma = 0$), the solution of (3.14) is[21]

$$\operatorname{tg}\frac{\Phi(x)}{2} = \operatorname{tg}\frac{\Phi_0}{2} e^{-\alpha_0 x/2}, \tag{3.15}$$

where Φ_0 is the angle of rotation for the particles under the influence of the initial pulse. The branches of the solution (3.15) are shown in Fig. 20a. We see that when $\Phi_0 < \pi$ the pulse gradually attenuates. If $\Phi_0 > \pi$, then the angle of rotation tends to a stationary value 2π. Examples of a numerical solution of $\mathcal{E}(x, t)$ are shown in Fig. 20b for $\Phi_0 = 0.9\pi$ and $\Phi_0 = 1.1\pi$[21]. The attenuation of the pulse at $\Phi_0 < \pi$ and its transformation into a stationary "2π-pulse" when $\Phi_0 > \pi$ are clearly seen. The formation of the "2π-pulse," just as of the "2π-pulses" in an amplifying medium, is explained by the fact that the field causes the particle to go over into a coherent superposition of the ground and excited states, in which it emits until it goes over completely to the ground state. If $2\pi(m - 1/2) < \Phi_0 < 2\pi(m + 1/2)$, where m is an integer, then, as shown by computer calculations, the input pulse in the case of propagation in a medium is divided into m individual stationary "2π-pulses"[21,96]. The picture of such a breakdown is apparently analogous to the breakdown of a pulse in the case, considered in Sec. 2.2, of propagation in an amplifying medium.

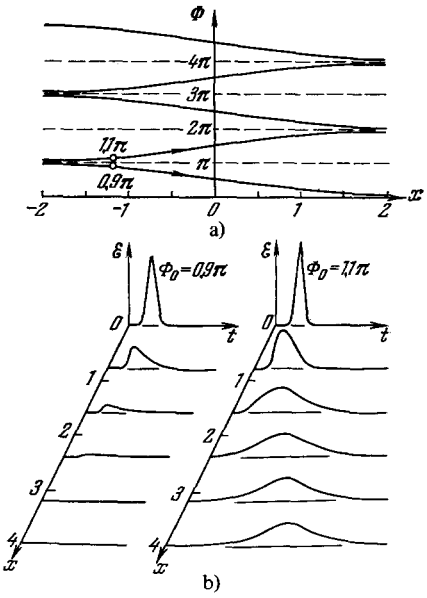

FIG. 20. Effect of self-transparency of a pulse in an absorbing medium. a) Branches of the solution of Eq. (3.14) at zero linear radiation loss. b) Evolution of the pulse in a medium at $\Phi_0 = 0.9\pi$ and $\Phi_0 = 1.1\pi$. The initial pulse has a Gaussian shape [21].

The form of the stationary "2π-pulse" is determined by the expression

$$\mathcal{E}(x, t) = \frac{2\hbar}{\mu\tau}\operatorname{sch}\left[\frac{1}{\tau}\left(t - \frac{x}{u}\right)\right], \tag{3.16}$$

where τ is the parameter of the pulse duration (the slope of the exponential), u is the velocity of pulse propagation. The form of this pulse coincides with the forms of the "2π-pulse" and "π-pulse" in an amplifying medium.

The velocity of pulse propagation is given by the relation[21]

$$\frac{1}{u} = \frac{1}{c} + \frac{\alpha_0\tau}{2}. \tag{3.17}$$

The velocity of "2π-pulse" in an absorbing medium is $u < c$ ($u > c$ in an amplifying medium[53]). The deceleration of the pulse is due to the fact that the energy of the pulse is absorbed by the medium on its leading front, and is returned with a certain delay on the trailing edge. It is interesting that the velocity of the pulse u can be smaller by several orders of magnitude than the phase velocity of the light c. For example, in a gaseous absorbing medium with an absorption coefficient $\alpha_0 \approx 10^{-2}$ cm^{-1}, a pulse with $\tau \approx 10^{-5}$ sec has a velocity $u \approx 2 \times 10^7$ cm/sec, i.e., smaller than c by a factor 10^3.

The decrease of the pulse velocity in the medium is obviously accompanied by its spatial compression in the medium. Indeed, the characteristic dimension, say of the leading front of the pulse, $1/\alpha_0$ is in the medium and $2/\tau c$ outside the medium (the rate of increase of the intensity is double that of the amplitude). When $u \ll c$, the velocity ratio $u/c \approx 2/\alpha_0\tau$ coincides exactly with the ratio of the dimensions of the pulse inside and outside the absorbing medium.

The decrease of the pulse velocity in the medium

is connected also with the large energy density u_{med} stored in the resonant particles, compared with the field energy density u_f. This connection can be obtained from the following considerations[109]. If we integrate the average energy density of the pulse and of the medium over the length of the pulse $u\tau$, then we obtain $u\tau(u_{med} + u_f)$. On the other hand, we can integrate the energy flux through a unit cross section with respect to time. The energy flux through the cross section is connected only with the electromagnetic wave and has a velocity equal to the phase velocity c of the wave in the medium. Consequently, the energy density is equal to $c\tau u_f$. Equating these expressions, we get[109]

$$\frac{1}{u} = \frac{1}{c} + \frac{1}{c}\frac{u_{med}}{u_f}. \tag{3.18}$$

This expression coincides exactly with expression (3.17) if one substitutes the values of u_{med} and u_f, which can be easily calculated from the expressions for the form of the pulse (3.16). It is clear that $u \ll c$ if the energy density in the medium u_{med} is much larger than the energy density of the field u_f.

It is shown in[109], that an anomalously large Faraday rotation is produced under the conditions of self-transparency of a stationary "2π-pulse" with velocity $u \ll c$. The expression obtained in[109] for the Verdet constant can be represented in the form

$$\rho = \gamma\left(\frac{1}{u} - \frac{1}{c}\right), \tag{3.19}$$

where $\gamma = g\beta/\hbar$ is the gyromagnetic ratio. When $u \approx 10^{-3}c$, the Verdet constant is of the order of $\rho \approx 0.3$ rad/cm-Oe, i.e., it is quite large. The anomalous Faraday rotation can take place also in the case of propagation of "2π-pulses" in an amplifying medium. The Verdet constant is determined in this case, as before, by expression (3.19), but the velocity u is determined by relation (2.38). It is clear that ρ reverses sign in this case.

Analogously, in the case of the propagation of circularly polarized "2π-pulse" in a medium, an axial magnetic field is produced, with the same time dependence as the intensity of the pulse[109] (the inverse Faraday effect).

b) Experiment. The first experimental observation of the self-transparency of a pulse was reported in[21], where the absorbing medium was a ruby crystal cooled with liquid helium. The inhomogeneous broadening of the R_1 line was due to stress inhomogeneities[110] and amounted to $T_2^* \approx 10^{-10}$ sec. The source of the input pulse was a ruby laser with a crystal cooled to liquid-nitrogen temperature. By thermally tuning the laser it was possible to make the generation line $\overline{E}(2E) \longleftrightarrow {}^4A_2(\pm 3/2)$ coincide with the absorption line ${}^4A_2(\pm 1/2) \longleftrightarrow \overline{E}(2E)$ of the sample. The radiation pulse had an energy on the order of 3 mJ and a duration of 10 nsec. The transmission of the weak signal was 10^{-5}. When the pulse energy was increase six times, the transmission increased 10^4 times. The main argument confirming the observation in[21] of the self-transparency effect is the presence of a pulse delay, the magnitude of which agrees with the expression for the velocity of the pulse and is too large for the case of incoherent interaction.

The self transparency effect was observed in[73] in the propagation of a pulse in a gaseous absorbing medium. Gases have considerable advantages over solids in experiments of this type, since the relaxation time of the absorbing levels, the inhomogeneous Doppler width, and the absorption coefficient can be readily controlled by varying the pressure or temperature and by adding a buffer gas. In the experiment of of[73], the source of the coherent-light pulse was a CO_2 laser[111], operating in a periodic pulse regime at $\lambda = 10.5915\ \mu$. The absorbing medium was gaseous SF_6, the transition of the rotational-vibrational band ν_3 of which coincides with the $\lambda = 10.5915\ \mu$ emission line of the CO_2 laser[85,108]. CO_2-laser pulses with power ~ 2kW and duration 200—300 nsec were linearly attenuated and then passed through a two-pass absorbing element 4.7 m long, filled with SF_6 at low pressure (10^{-2}—4×10^{-2} Torr. The energies and the waveforms of the pulses were measured at the input and at the output. At small input energies, the absorption was linear and proportional to the pressure ($\alpha_0 \approx 0.344$ cm^{-1}Torr^{-1}). When the power was increased to ~10 W/cm^2, bleaching of the medium began. An increase of the power by ten times led to an increase of the transmission by 10^4 times. The power threshold was independent of the pressure below 4×10^{-2} Torr, thus indicating that the condition $\tau_p < T_2$ was satisfied. The waveforms of the input and output pulses obtained in that investigation are shown in Figs. 21a and b. The output pulse is more symmetrical than the input pulse, and is delayed by $\tau_d \approx 0.5\ \mu$sec. Since the power of the output pulse exceeds for the greater part the power of the input pulse, this cannot be the ordinary saturation of the absorption and incoherent interaction discussed in Sec. 3.1.

An experiment was also performed[73] aimed at observing incoherent interaction of a pulse with an absorbing medium. To this end, He with pressure 2 Torr was added to the cell, and this greatly decreased the relaxation time T_2, to a value smaller than τ_p. In this case a narrowing of the pulse during the course of propagation was observed (Figs. 21a and c). The slopes of both the leading and trailing edges of the pulse increased in the propagation. This is evidence that the condition $\tau_p \gg T_1$ is also satisfied. Of course, there was no delay of the pulse in this case.

4. TWO-COMPONENT MEDIUM

The so-called two-component medium, consisting of amplifying and absorbing particles of different types with identical resonant lines, has a number of interesting properties. Besides the effects peculiar to amplifying and absorbing media (see Ch. 2 and 3), such a medium exhibits the effect of threshold transmission of pulses. The properties of two-component media were first considered in[62,24]. In[62], the two-component medium was analyzed from the point of view of developing a quantum neuristor for an optical computer. Two-component media are of considerable interest for the production of powerful short and ultrashort light pulses. In fact, lasers that are Q-switched by a nonlinear absorber[18,19,76,77] and lasers with mode self-locking by means of a saturable solution[14,20] can

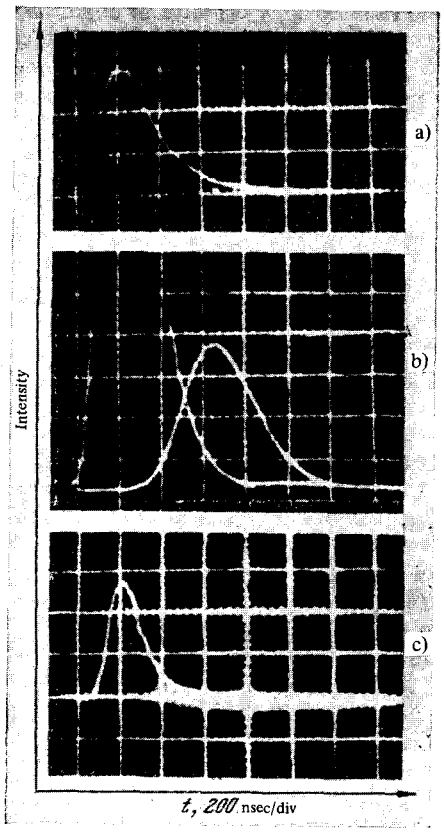

FIG. 21. Shapes of input and output pulses at a power sufficient to saturate the absorption. a) Typical shape of output pulse after transversing 9.4 m in an element without SF_6; b) double exposure with input and output pulses at an SF_6 pressure of 0.04 Torr (the vertical scale is increased four times); c) shape of output pulse at an SF_6 pressure of 0.04 Torr and He pressure 2 Torr [73].

be regarded as two-component media placed in an optical resonator. It should be noted immediately that the presence of the absorbing component does not necessarily lead to appreciable loss of the pulse energy. If the cross section of the radiative transition for the absorbing particles σ_b is much higher than the cross section for the amplifying particles σ_a ($\sigma_b \gg \sigma_a$), then a small fraction of the pulse energy is consumed in the bleaching of the absorber if the absorption and amplification coefficients are equal. This is precisely what makes the combination of the two media effective.

In practice, a two-component medium can be realized in several ways. It is possible to introduce into a matrix (glass, crystal) impurity ions of two types, such that optical pumping causes the ions of one type to amplify the radiation, and the ions of the other type to absorb it. In[101] is described glass doped with Nd^{3+} (amplification) and NO_2^{2+} (absorption), and glass with Nd^{3+} (amplification) and Yb^{3+} (absorption) is described in[95]. When the amplifying and absorbing ions are introduced in the same medium, effective medium resonant transfer of the excitation from the ions of the amplifying component to the ions of the absorbing component takes place (energy transfer from the "hot" system, which is made up of the inversely populated ions, to the "cold" system, represented by the ions of the absorbing component). A two-component medium in the form of an optical waveguide, the envelope of which contains absorbing impurity ions and the core contains amplifying ions, was considered in[98]. Under certain conditions, the light waves penetrate appreciably into the absorbing envelope (this takes place for the fundamental surface wave[103,104]). In such a two-component medium, the amplifying and the absorbing particles are spatially separated and there is practically no resonant transfer of the energy. Finally, it is possible to alternate amplifying and absorbing layers in the propagation direction, as was done, for example, in[9].

To create a two-component medium it is necessary to have different probabilities of induced transition in the components. This is attained, for example, by using particles of two kinds. The same effect can be obtained with particles of the same kind, by making the radiation intensity in the components different. In particular, this can be realized by decreasing the diameter of the light beam in the absorber with the aid of an optical system[102]. Another possibility exists for semiconducting media. The probability of interband radiative transition in a semiconductor depends on the position of the Fermi quasi levels, and their position in the bands depends in turn on the excitation level. Consequently, in an inhomogeneously excited semiconductor (for example, in a slotted laser diode) it is possible to produce a two-component medium. From this point of view, an injection laser with non-uniform excitation was considered in[113-115].

A laser with a two-component medium in the resonator has a great variety of dynamic regimes (a hard excitation regime[16], self-Q-switching[18,19,76,77,88], periodic pulsations[117], mode self-locking[20,14], etc.). We shall not consider these questions in the present review, which is devoted to the propagation of light pulses.

The propagation of light pulses in a two-component medium were considered in[62,24,8,9,97-99,22,102]. In the present review we cannot consider all the possible combinations of the properties of these components. We therefore confine ourselves only to several cases of practical interest.

4.1. Incoherent Interaction

In the region of incoherent interaction between the pulse and the amplifying and absorbing components, the following two limiting cases are of interest:

$$\tau_p \ll T_1^a, T_1^b, \tag{4.1}$$

$$T_1^b \ll \tau_p \ll T_1^a, \tag{4.2}$$

where the index a pertains to the amplifying component and b to the absorbing component. In the case (4.1), saturation of the absorption or the amplification is determined by the energy of the pulse, and in the case (4.2) the absorption saturation is determined by the instantaneous power of the pulse. Case (4.1) was investigated in[62,24,8,9,97-99], and case (4.2) in[22,102]. Let us consider them separately.

a) Case of absorption saturation. The propagation of the pulse is described in this case by equations of the type (2.14), but with allowance for the presence of

two components:

$$\frac{\partial I}{\partial t}+c\frac{\partial I}{\partial x}=cI\left\{\sigma_a N_{a0}\exp\left[-2\sigma_a\int_{-\infty}^{t}I(t',\,x)\,dt'\right]-\right.$$
$$\left.-\sigma_b N_{b0}\exp\left[-2\sigma_b\int_{-\infty}^{t}I(t',\,x)\,dt'\right]-\gamma\right\}, \quad (4.3)$$

where $N_{a0} = N_2^a - N_1^a$ is the initial density of the inverted population of the amplifying particles. $N_{b0} = N_1^b - N_2^b$ is the initial density of the population difference between the levels of the absorbing particles, and the remaining symbols are the same as before. From (4.3) we can obtain an equation for the pulse energy E

$$\frac{dE}{dx}=\frac{N_{a0}}{2}[1-\exp(-2\sigma_a E)]-\frac{N_{b0}}{2}[1-\exp(-2\sigma_b E)]-\gamma E. \quad (4.4)$$

It is seen from (4.4) that when

$$\sigma_a N_{a0}-\sigma_b N_{b0}-\gamma<0 \quad (4.5)$$

a weak pulse ($E \ll 1/2\sigma_b, 1/2\sigma_a$) is attenuated, since the loss in the absorbing component and the linear loss exceed the gain.

However, if the saturation of the absorption sets in earlier than the saturation of the gain ($\sigma_b \gg \sigma_a$), and the gain $\sigma_a N_{a0}$ exceeds the linear loss γ, the medium becomes amplifying under the influence of a pulse with energy exceeding a certain threshold value E_{thr}. Figure 22 shows a plot of the right-hand side of (4.4), i.e., of the gain of the medium dE/dx, against the pulse energy in such a case. We see that pulses with energy $E < E_1$ and $E > E_2$ will attenuate, and a pulse with energy $E_1 < E < E_2$ will be amplified, until its energy reaches the stationary value E_2. Therefore the energy E_1 can be called the threshold energy, and E_2 can be called the limiting energy of the pulse. In calculating the threshold energy E_{thr}, it is possible to neglect the saturation of the gain, and when calculating the stationary energy, the absorbing component can be regarded as completely transparent. The expressions for E_{thr} and E_{max} are[98]

$$E_{thr}=\frac{1}{2\sigma_b}f\left(\frac{\sigma_b N_{b0}}{\sigma_a N_{a0}-\gamma}\right), \quad (4.6)$$

$$E_{max}=\frac{1}{2\sigma_a}f\left(\frac{\sigma_a N_{a0}}{\gamma}\right), \quad (4.7)$$

where $f(x) = x[1 - \exp(-x)]$.

The limiting energy, just as in a single-component amplifying medium, can be attributed to the linear radiation loss. The threshold property is a result of a combination of two media under the condition $\sigma_b \gg \sigma_a$. Such a medium can be called stable, since, unlike an amplifying medium, it is stable against external signals with energy $E < E_{thr}$[24,97,99]. In particular, a two-component medium can be stable against spontaneous noise of the amplifying component. This property is of practical importance for optical amplifiers with large gain, in which the amplification of the spontaneous radiation leads to intense superradiance and to saturation of the gain.

When a pulse propagates in a two-component medium with $\sigma_b \gg a$, an appreciable contraction of the duration takes place regardless of the wave form of the input pulse[8]. The nonlinearly absorbing component increases continuously the slope of the leading front in this case, and in the case of nonlinear amplification of a pulse with a shaped stepwise front, a contraction of the duration takes place in the amplifying component. These two stages of pulse deformation are clearly in Fig. 23, which shows the change of the form of an exponential pulse. In a single-component amplifying medium, such a pulse tends to a stationary form with constant duration, while in a two-component medium it shortens continuously. This phenomenon was observed experimentally in[9] and used to obtain powerful short light pulses. This experiment is described in Sec. 2.2b and in Ch. 5.

b) The case of saturation of absorption by power is realized under the condition (4.2). Under this condition, the absorption depends on the instantaneous power of the pulse I(t, x) and is determined by expression (3.1). As before, greatest interest attaches to the case when the saturation of the absorption begins much earlier than the saturation of the gain, for in that case it is possible to obtain the effect of threshold amplification. When $\sigma_b \gg \sigma_a$, the main contribution to the change in the form of the pulse is made by the nonlinearity of the absorption. The role of the amplifying component is reduced only to compensation for the losses, and

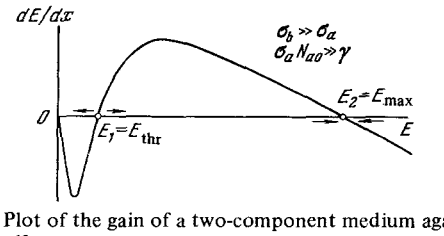

FIG. 22. Plot of the gain of a two-component medium against the pulse energy [62].

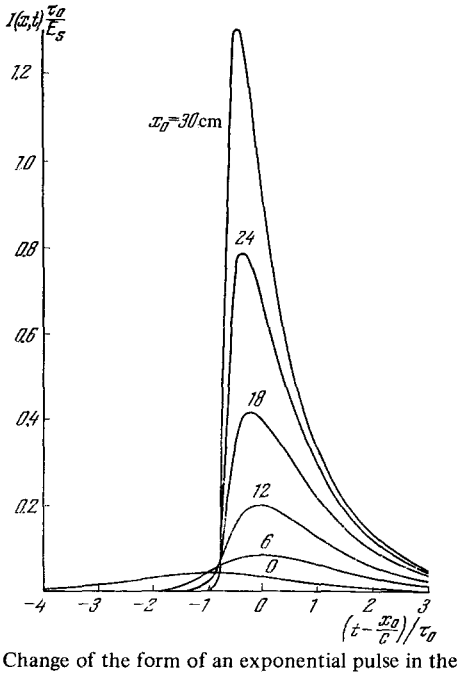

FIG. 23. Change of the form of an exponential pulse in the case of propagation in a two-component medium, when the initial energy of the pulse exceeds the threshold value. The parameters of the medium are: $\sigma_b/\sigma_a = 50$; $\sigma_a N_{a0}/\sigma_b N_{b0} = 5$; $\sigma_a N_{a0} = 0.2$ cm^{-1}; $\gamma = 0.03$ cm^{-1}.

therefore, when considering the evolution of the form of the pulse, the gain can be regarded as linear. The propagation of a light pulse is then described by an equation of the type (3.2)

$$\frac{\partial I}{\partial t}+c\frac{\partial I}{\partial x}=cI\left[(\sigma_a N_{a0}-\gamma)-\frac{\sigma_b N_{b0}}{1+(I/I_s)}\right], \qquad (4.8)$$

where $I_s = (2\sigma_b T_b^1)^{-1}$ is the absorption saturation power (photons/cm^{2}sec).

Figure 24 shows the dependence of the gain on the intensity I. It is seen that whereas for a weak signal the medium is absorbing ($\sigma_a N_{a0} < \sigma_b N_{b0} + \gamma$), at a certain threshold intensity I_{thr} it becomes amplifying. If the initial pulse has an intensity $I_0(t) < I_{thr}$, then such a pulse attenuates in the course of propagation. If $I_0(t) > I_{thr}$, then the parts of the pulse satisfying this condition become amplified, and the remaining ones are attenuated. As a result, an effective compression of the pulse takes place.

The rate of compression of the pulse, just as in the case of a single-component nonlinearly absorbing medium, is determined by (3.3). It is maximal when $I \approx I_s$ and tends to zero in the limiting cases $I \ll I_s$ and $I \gg I_s$. However, the presence of the threshold-amplification effect makes it possible to maintain $I \approx I_{thr} \approx I_s$ for a sufficiently long time (but not for an infinitely long time, since the point $I = I_{thr}$, as is obvious from Fig. 24, is unstable). If the initial light pulse has smooth fronts, an intensity I_0 at the maximum, and a duration τ_0, then during the propagation its duration τ is connected with the maximum intensity I by the relation[22,25]

$$\tau=\tau_0\left[\frac{I_{thr}-I_0}{1+(I_0/I_s)}\,\frac{1+(I/I_s)}{I_{thr}-I}\right]^{1/2}. \qquad (4.9)$$

It is easy to see that $\tau = 0$ if $I_0 = I_{thr}$. If $I_0 I_{thr}$, then the pulse either attenuates ($I_0 < I_{thr}$) or becomes amplified ($I_0 > I_{thr}$) or becomes amplified ($I_0 > I_{thr}$), and the pulse duration tends to stationary values that decrease with increasing proximity of the maximum intensity of the initial pulse I_0 to the threshold intensity. The shape of the pulse tends to the Gaussian $\exp\{-([t - (x/c)]/\tau)^2\}$. Of course, the process of pulse compression by a nonlinear absorber continues only up to a time on the order of T_1^b. When the pulse duration is $\sim T_1^b$, the absorber does not have time to relax on the trailing edge, and the saturation is determined by the pulse energy. Further contraction is possible as a result of the nonlinearity of the amplifying component, if the pulse energy is sufficient to saturate the gain.

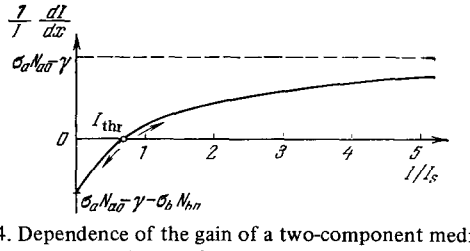

FIG. 24. Dependence of the gain of a two-component medium with absorption power saturation on the intensity

$$I_{thr}=I_s\frac{\sigma_b N_{b0}+\gamma-\sigma_a N_{a0}}{\sigma_a N_{a0}-\gamma}.$$

In practice, the effect of pulse compression in a two-component medium is used in a laser with mode self-locking by a linear absorber[14,20]. Such a laser constitutes amplifying and absorbing media placed in an optical resonator (Fig. 25a). The light pulse produced inside the laser as a result of the beats of the axial modes passes many times through both components. Every time the pulse passes through the nonlinear absorber, it contracts somewhat. In the case of multiple passage through the resonator, a sufficiently appreciable contraction is accumulated[184]. The dynamics of this process was investigated theoretically in[25]. The process of gradual contraction of pulses in a laser with mode self-locking was observed in[100]. Figure 25b shows an oscillogram of a train of pulses, demonstrating this process. The pulse repetition period is equal to the time of total passage through the resonator. We see that the pulse becomes somewhat shorter in each successive passage through the two-component medium.

The evolution of a pulse propagating through a two-component medium with absorption power saturation was investigated in[182]. In that experiment, a pulse from a Q-switched ruby laser was passed many times through an amplifying medium (ruby crystal) and an absorbing medium (two cells with cryptocyanine). A diagram of this experiment is shown in Fig. 26. The initial transmission of each cell with cryptocyanine was $\eta_0 \approx 0.25$, and the gain of the ruby per pass was $K \approx 10$, and the mirror reflection coefficient was $r = 0.65$. In the absence of external radiation, such a system is in a stable "locked" state, since the absorption per pass exceeds the gain by a factor of four. A laser pulse of power 1–10 mW/cm^2 and duration at half-height $\tau_0 = 10$ nsec was applied to the input. The threshold power I_{thr}, i.e., the power at which the total loss becomes comparable with the gain, was several mW. Whenever the maximum intensity of the input pulse I_{max} coincided with I_{thr} within several percent, effective compression of the pulse by an approximate factor of two per pass took place. This is clearly seen in Fig. 27a, which shows oscillograms of the initial pulse and of the pulses after the first, third, and fifth

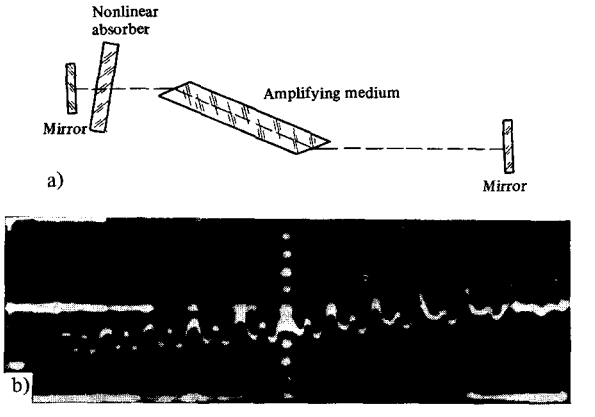

FIG. 25. Evolution of a pulse in a laser with self-locking of the modes by a nonlinear absorber, as in a two-component medium. a) Laser scheme; b) oscillogram of a train of pulses from the laser, obtained in [100].

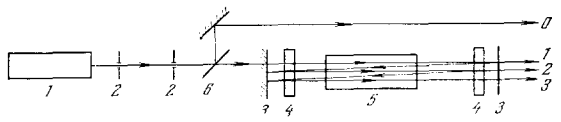

FIG. 26. Diagram of experiment aimed at observing the evolution of a pulse propagating in a two-component medium with rapid relaxation in the absorber [182]. 1 – ruby laser with saturable shutter of phthalocyanine of vanadium; 2 – diaphragms of ~1 mm diameter; 3 – mirrors with reflection coefficient ~65%; 4 – cells with solution of cryptocyanine, with transmission 25%; 5 – amplifying ruby medium with gain ~10 per pass.

passes. These results are in good agreement with relations (3.4) and (4.8). After the second pass, the pulse compresses to 2–3 nsec, i.e., to a value comparable with the time resolution of the receiver. The expected duration after the fifth pass is approximately 0.3 nsec. If I_{max} exceeds I_{thr} by 5–10%, then a sharp increase of the pulse power takes place, at which the compression is small (Fig. 27b), and conversely, if I_{max} is smaller than I_{thr}, then the pulse attenuates (Fig. 27c).

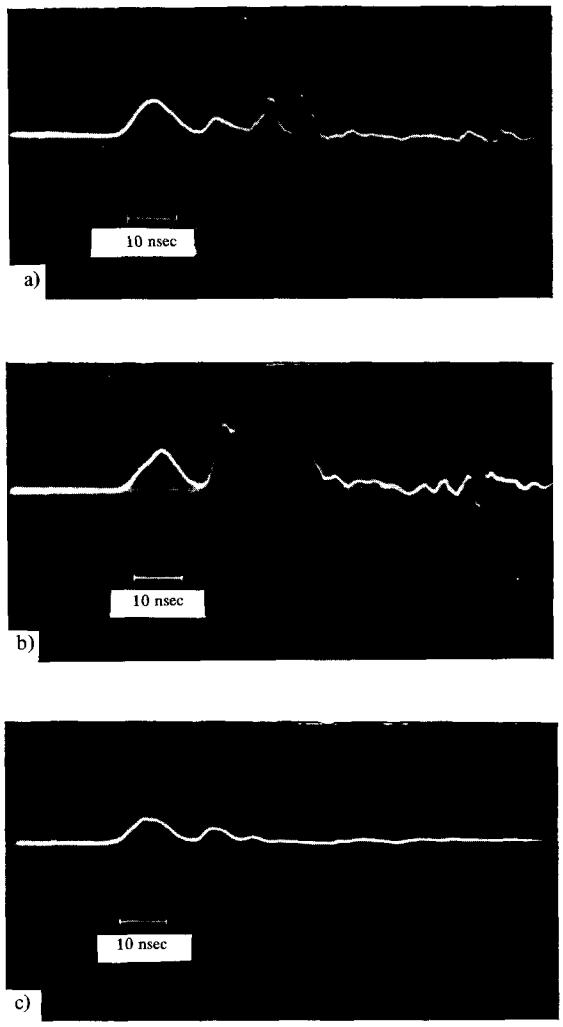

FIG. 27. Oscillograms of initial pulse and of the pulses after the first, third, and fifth passes through a two-component medium, as obtained in [182]. a) Maximum intensity of initial pulse I_{max} is close to the threshold intensity I_{thr}; b) I_{max} exceeds I_{thr}; c) I_{max} is lower than I_{thr}.

A two-component medium of this type is essentially a threshold element that transmits pulses with intensity larger than a certain threshold value. In particular, if the input pulse contains several fluctuation peaks with intensity larger than I_{thr}, then several ultrashort pulses are produced during the propagation. This tendency can be noted on the oscillogram of Fig. 27a. It can be attributed to the presence of fluctuation peaks in the initial laser pulse. These fluctuation peaks in the initial pulse are insufficiently deep and are strongly smooth by the inertia of the receiver. The property of the two-component medium, that of emphasizing the fluctuation peaks, can be utilized for the investigation of the fluctuation properties of radiation and for the formation of ultrashort light pulses. For example, superradiant or multimode laser radiation contains ultrashort fluctuation peaks with an amplitude exceeding by several time the average radiation intensity[112]. If such radiation is made to pass through a two-component medium under the condition that the average intensity is smaller than at threshold, then it is possible to separate only the most intense peaks of ultrashort duration.

4.2. Coherent Interaction

The effects of coherent interaction of a light pulse propagating in a two-component medium were hardly touched upon in the literature. There are only a few remarks in[62,9,25]. This is due to the mathematical difficulties of solving such problems. Thus, even in the case of exact resonance and homogeneous broadening, the propagation is described by five equations, namely, Eq. (2.32) and two pairs of equations of the type (2.33) for each of the components. However, it is clear from qualitative considerations that one can expect here a superposition of effects, arising upon interaction with each of the components, which were considered in Secs. 2.3 and 3.2. Let us illustrate this with numerical solutions of the equations as examples.

Let, for example, $T_2^b \ll \tau_p \sim T_2^a$, so that the pulse interacts coherently with the amplifying component and incoherently with the absorbing component. Owing to the amplifying component, a stationary "π-pulse" should be formed in the medium with the linear loss. If $\sigma_b \gg \sigma_a$, then the absorber becomes bleached on the leading front of the pulse, and by the same token the slope of the pulse is increased. In addition, owing to the nonlinear absorption, the pulse velocity is $u < c$. Figure 28a shows the results of the numerical solution of the corresponding equations for such a case, and confirm this qualitative picture. We see that the stationary "π-pulse" has an asymmetrical form and its propagation velocity is $u < c$.

If the interaction is coherent with both components, then a stationary pulse is likewise formed ($\gamma \neq 0$). If $T_2^a > T_2^b$ and $\sigma_b \gg \sigma_a$, then this pulse is an "$n\pi$-pulse" for the amplifying component, and a "π-pulse" for the absorbing component. The pulse has the shortest duration in the case when it is a "π-pulse" for the amplifying medium and a "2π-pulse" for the absorbing medium (when $T_2^a = 2T_2^b$). Figure 2b shows the results

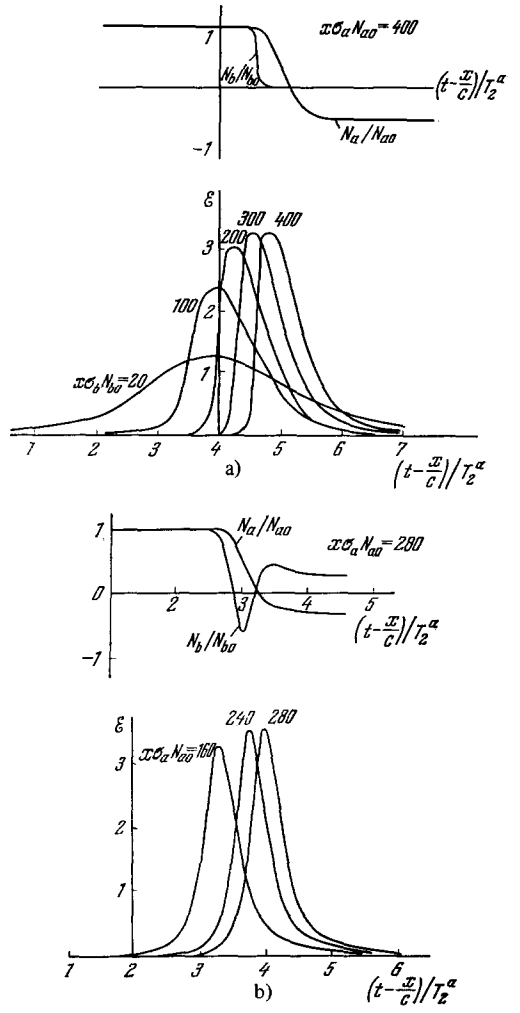

FIG. 28. Evolution of a pulse of coherent light propagating in a two-component medium with linear loss. a) Coherent interaction with the amplifying component and incoherent interaction with the absorbing component; b) coherent interaction with both components. The parameters of the medium are: $\sigma_b/\sigma_a = 5$, $\nu/\sigma_a N_{a0} = 0.2$, $\sigma_b/N_{b0}/\sigma_a N_{a0} = 1$, $T_2^a/T_2^b = \infty$ (a) or 2 (b). The field intensity is expressed in units of $(cT_2^a\sigma_a/4\pi\hbar\omega)^{1/2}$.

of a numerical solution of the corresponding equations for such a case. When $T_2^a \gg T_2^b$, the response of the absorber consists of several oscillations and the form of the stationary pulse becomes more complicated.

5. OBTAINING POWERFUL LIGHT PULSES BY THE AMPLIFICATION METHOD

At the present time, to obtain powerful light pulses, the active media employed are ruby and neodymium glass (see also the table on p. 649). Optical pumping with the aid of flash lamps makes it possible to store usually an energy not higher than 1–2 J/cm^3. Of course, only a fraction of this energy (10–50%) goes over into coherent radiation. Therefore, in order to obtain an energy of say 100 J from a generator only, it would be necessary to use an appreciable volume of active medium (on the order of 10^3 cm^3). This is practically impossible for the following reasons. The increase of the length leads to an increase of the duration and according to a decrease of the pulse power. In addition, an increase of the length leads to the occurrence of strong superradiance as a result of the amplification of the spontaneous radiation and the s emission of the stored energy. If the transverse dimension is appreciably increased, effective pumping of the internal regions of the medium becomes impossible. In addition, this leads to a sharp deterioration of the parameters of the ouput pulse, since the finite rate of generation development in the transverse dimension causes of the increase of the pulse duration[58,150], and the excitation of non-axial modes of higher order decreases strongly the directivity of the radiation. Besides these factors, the increase of the output power is limited by damage produced in the laser by its own radiation. This damage can occur both in the active medium and in the other elements of the laser (shutter, mirrors), and these elements frequently have the lowest strength.

Therefore, to obtain pulses of high energy and high power, the method of amplification is presently used. A generator pulse of relatively low energy and low power is passed through an amplifying medium of sufficiently large dimensions. Unlike the case of a generator, an increase of the cross section of the active medium does not increase the duration of the output pulse, since it is determined by the driving generator. In the amplification scheme, it is possible to decrease the influence of superradiance by separating the amplifying medium into individual stages and decreasing the coupling between them. Damage of the active med medium by the powerful radiation limits the growth of the energy and of the power from a unit surface section, but the limiting values of the power are higher in this case than in the generator, owing to the absence of mirrors etc. In the amplifiers it is possible to increase greatly the cross section. The difficulties connected with pumping of a rod of large diameter are alleviated here by lowering the concentration of the impurity ions, and the corresponding decrease of the gain and of the stored energy is compensated for by an additional increase of the length. As a result, by amplifying a pulse from a Q-switched laser it is possible to obtain pulses with energy up to several hundred Joules at a duration of 5–50 nsec[11 118]. Using as the driving generator a laser producing ultrashort pulses with mode self-phasing, it is possible to obtain pulses with energies up to 20 J and durations 10^{-11}–10^{-12} sec[68].

Let us consider the experimental installations in which pulses with maximum power and energy were obtained.

5.1. Short Pulses

A laser installation with ruby as the active medium is described in[9]. It consists of a driving generator with a Kerr cell as a Q-switch, and three amplification stages. The ruby crystal used in the generator is 120 mm long and 10 mm in diameter. The leading front of the pulse was cut off by a shutter consisting of a Kerr cell and saturable solution of phthalocyanine of vanadium in toluene with initial transmission 4×10^{-2}. The pulse then passed through three amplification

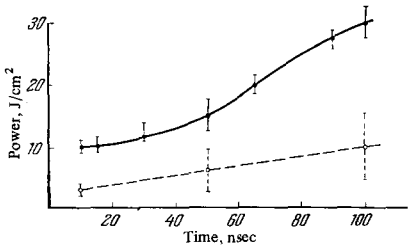

FIG. 29. Dependence of the threshold energy of self-damage of ruby and neodymium glass on the pulse duration [13]. ● – ruby; ○ – neodymium glass.

stages, in which ruby crystals 240 mm long and 60 mm in diameter were used. The end faces of the rods were cut at the Brewster angle to prevent self excitation. The total gain reached 10^4. A cell with phthalocyanine of vanadium, having an initial transmission 10%, was placed between the first and second stages. This nonlinear absorber prevented self excitation of the amplifier and contributed to sharpening of the leading front of the pulse. As a result of the change in the shape of the pulse in the nonlinear amplification regime, the duration of the output pulse was decreased to 2 nsec at an energy of 15 J.

The maximum energy density reached in this apparatus amounts to 7–8 J/cm^2 and was determined by the damage produced in the ruby crystals by the strong light field. The dependence of the threshold energy of self-damage of ruby and neodymium glass on the pulse duration was investigated earlier in[13]. Figure 29 shows the data obtained in that paper. It is seen that for ruby, in the pulse-duration range from 10 to 30 nsec, the damage is determined only by the pulse energy. The data obtained in[9] for τ_p = 2 nsec confirm this tendency down to 10^{-9} sec. If this tendency remains in force for shorter pulses, then it becomes necessary to decrease the pulse duration in order to increase the limiting output power[10].

High pulse energies can be obtained by using neodymium glass, since it is possible to prepare from it rods of practically any length and any cross section. Glass has a higher optical quality than ruby, thus ensuring a much lower divergence of the laser radiation and making longer installations possible.

An installation with neodymium glass as the active medium, producing a light pulse with energy 100 J at 5 nsec duration, is described in[11]. The overall view of the installation (viewed from the side of the output stages) is shown in Fig. 30. This installation uses a driving generator with a rod of neodymium glass of 6 mm diameter and 130 mm length, with reflection-free end phases. The rod of the driving generator is excited in an illuminator with a helical lamp (pump energy 15 kJ). The resonator is made up of two mirrors with reflection coefficients 98 and 37%. The Q-switching is with an electrooptical Kerr cell. The generator radiates a light pulse with energy 0.3 J and duration 10–15 nsec. The generator radiation passes through a second Kerr cell, a cell with a saturable filter, and strikes the input of an optical quantum amplifier with a weak-signal gain of approximately 500. Such a gain was attained by using a rod of 10 mm

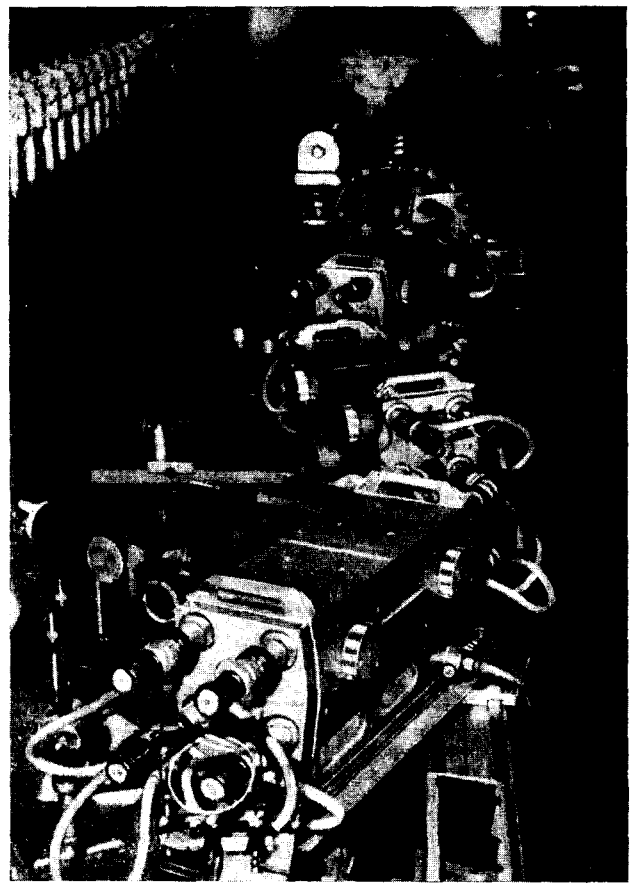

FIG. 30. Photograph of the high-power neodymium-glass laser setup described in [11].

diameter and 600 mm length, the ends of which were cut off at the Brewster angle. The pulse energy of this amplifier reached 3–5 J. Further increase of the pulse was in the rods of the main amplifier, of 30 mm diameter, since at powers 0.5–1 GW/cm^2 the rod becomes strongly damaged. To fill the cross section of the rods of the main amplifier, a telescope was used, which broadened the light beam by a factor 2.5. The surfaces of the telescope lens were coated for transmission at a wavelength 1.06 μ. The main amplifier consisted of four rods of 3 mm diameter and 60 mm length. The end faces of these rods were also cut at the Brewster angle. Each of the rods of the amplifier was pumped by four straight pump lamps in an elliptical illuminator (total pump energy 60 kJ). The gain of the main amplifier reached 10^4. At such a high value of the gain, self-excitation can arise as a result of feedback during scattering from the side surface of the rods, leading to a depletion of the inverted population. A similar depletion of the inverted population is produced by amplification of the spontaneous emission. In both these cases, the gain of the medium decreases sharply. In addition, when an electrooptical shutter is turned on in a system with high gain, a photon cascade may develop in the form of a rather short giant superradiance pulse[61]. To prevent all these phenomena, which deplete the inverted population in the amplifier, the rods of the amplifier were offset relative to each

other by 0.7 m, and cells with saturable filters were placed between the amplification stages. The duration of the output pulse at half height was 5 nsec, and the energy reached 100 J. The divergence of the radiation did not exceed 5×10^{-3} rad. In the case when a second driving generator was used, radiating a pulse with 7 nsec duration, the output was a pulse of 2.8 nsec duration and energy 55 J. At the obtained values of the pulse energy, damage in the form of bubbles was observed after the very first flash inside the rod of the last stage. Optical breakdown was produced on the output surface, causing the latter to become dull. After two or three flashes, the output energy decreases by one half, and 10 flashes damaged the output end of the last rod completely.

The French firm Companie Generale d'Electricite reported the development of a powerful laser installation using neodymium glass, which radiates pulses with energy 500 J at a duration of 30 nsec or with energy 250 J at 5 nsec[118]. The installation consists of a driving generator and a multistage amplifier. The amplifier employs neodymium glass rods with right-angle end faces and with successively increasing diameters 16, 23, 32, 45, and 60 mm. The beams of any two diameters are matched by means of telescopic systems. The self-excitation of the amplifier is prevented by depositing anti-reflection coatings on the end phases of the rods and by tilting the rods relative to each other. The driving generator is either Q-switched with a rotating total internal reflection (pulse duration 30 nsec), or else uses an electrooptical Q-switch (pulse duration 5 nsec)[119]. The large cross section area of the last stage makes it possible to obtain a high power, 50 GW. At a beam divergence of 10^{-3} rad this corresponds to a radiation brightness 2×10^{15} W/cm^{2}sr.

As noted earlier, neodymium glass had good optical properties. This makes it possible to obtain exceedingly high directivity of the ouput radiation, which is limited only by diffraction. To this end, it is necessary to employ a driving generator with a diffraction divergence and to take measures against the occurrence of distortion in the amplifier rods during pumping. In[120], an output power of 4 GW was obtained at a brightness 2×10^{17} W/cm^2 sr. When such radiation is focused by a real optical system on an area with dimensions 10λ, a power of 10^{15}–10^{16} W/cm^2 is reached, corresponding to an electric field intensity in the light wave of 5×10^8–10^9 V/cm and a magnetic field intensity 2×10^6–6×10^6 G.

5.2. Ultrashort Pulses

Exceedingly high powers (on the order of 10^3 GW) can be obtained also in an unfocused beam[16]. This can be done by amplifying ultrashort light pulses from a laser with mode locking by a nonlinear absorber[15]. Laser with mode locking emits a train of ultrashort pulses, the repetition periods of which are equal to twice the time of passage of the light between the mirrors (several n/sec). It is therefore necessary to use a special procedure to separate an individual pulse from the train. Generation and amplification of ultrashort light pulses using this method was reported in[15]. The experimental setup is shown in Fig. 31. The laser uses a neodymium-glass rod of 53 cm length and 1.3 cm diameter, with ends cut at the Brewster angle. The resonator was made up of two mirrors (reflection coefficient higher than 99%) spaced 70 cm apart. The cell with the saturable dye, the polarization Glan prism, and the Kerr cell were placed inside the resonator. The generator operates in the following manner. In the absence of voltage on the Kerr cell, the shutter is opened, thus ensuring development of generation. Part of the radiation from the polarizer was focused in the discharge gap of a pulse high-voltage generator. The discharge gap was regulated in such a way that breakdown takes place at a definite laser-radiation power, and a high voltage is then applied to the Kerr cell. The plane of polarization of the light is rotated by the double passage through the Kerr cell through 90°, and the radiation of the generator is deflected by the polarizer to the side--to the input of the amplifier. Of course, after rotation of the plane of polarization, the light does not reach the second mirror and the generation is stopped. Thus, the pulse preceding the instant of the shutter switching reaches the input of the amplifier. The energy of this pulse is of the order of 0.05 J, its duration is of the order of 1 nsec, and the peak power is 50 MW. The amplifier is a neodymium rod 76 cm long and 1.9 cm in diameter. After amplification, the energy increased to 1.8 J and the power to 1.8 GW. The relatively large pulse duration, in the authors' opinion, is connected with the use of a long rod in the generator and with the placement of the shutter elements inside the resonator. By replacing the 53-cm rod in the generator with a rod 16.5 cm long, they succeeded in obtaining an output power of 40 GW and a duration $\sim 2 \times 10^{-11}$ sec.

A more powerful installation is described in[16]. In it, the electrooptical shutter for separating the single pulse was placed outside the resonator, making it possible to obtain shorter pulses from the driving generator, and the amplification of the separated pulse was effected by five stages with neodymium-glass rods, each 60 cm long. The diameter of the rods of the two last stages was 40 mm. The overall gain reached 10^4 (for a weak signal of nanosecond duration). Cells with solutions of saturable dye were placed between the Kerr shutter in the first stage of the amplifier, and also between the first and second stages of the amplifier, in order to cut off that part of the radiation which penetrates through the closed Kerr shutter. The pulse energy at the output reached 20 J. Measurements of

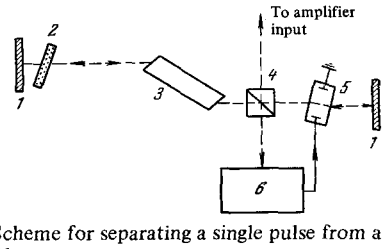

FIG. 31. Scheme for separating a single pulse from a train of ultrashort pulses [15]. 1 – Mirrors, 2 – cell with saturable solution, 3 – amplifying medium of neodymium glass, 4 – Glan polarization prism, 5 – Kerr cell, 6 – discharge gap of high-voltage pulse generator.

the duration, performed by the procedure of[121], have shown that the generator radiation consists of seven pulses of 2–3 psec duration, following each other at intervals of 100 psec. The pulse output power was $\sim 10^3$ GW, corresponding to a power density on the order of 10^{11} W/cm^2.

The energy density reached in[16] without appreciably damaging the neodymium glass was 2 J/cm^2. On the basis of these data and the data of[13,9,11] it can be assumed that damage in luminescent crystals and glasses by short and ultrashort pulses of light of duration 10^{-8}–10^{-11} sec is governed by the pulse energy. The self-damage energy threshold lies in the interval 2–10 J/cm^2. Consequently, the self-damage power threshold increases with increasing pulse duration. From these considerations, it was proposed back in[10] that the pulse duration be decreased in order to attain maximum power.

It has been established by now that the most probable mechanism of damage to crystals and glasses by powerful pulses is the electron cascade produced in the strong optical field[124,171,172]. In a strong optical field, free electrons are produced and are heated to an energy sufficient for impact ionization of the lattice. As a result of electron cascades, the concentration of the electrons increases to such an extent, that practically total absorption of the radiation takes place, together with heat and destruction of small sections of the medium. Let us obtain some estimates, following[124]. The rate of accumulation of energy by the electron in the conduction band is

$$\frac{dE_e}{dt} = \hbar\omega\sigma_{el} I, \tag{5.1}$$

where $\sigma_{e_1} = 4\pi e^2\nu_{eff}/m\omega^2 c$ is the effective cross section for the absorption of light of frequency ω by an electron, ν_{eff} is the effective frequency of electron collisions, e and m are the charge and effective mass of the electron, and I is the radiation intensity (photon/cm^{2}sec). The electron acquires an energy E sufficient for ionization of the lattice and for the occurrence of two electrons within the time $\tau_0 = E_i\sigma_{e_1}\hbar\omega I$. The electron multiplication follows the law $n^0\exp(t/\tau_0)$, where n_0 is the initial electron concentration. In order for the radiation to be completely absorbed, the electron concentration must reach the critical value $n_{cr} = m\omega^2/4\pi e^2$, which for $\lambda \approx 1\ \mu$ amounts to about 10^{19} cm^{-3}. Such a concentration is reached within a time

$$t = \frac{E_i}{\sigma_{e_1}\hbar\omega I}\ln\left(\frac{n}{n_0}\right),$$

i.e., after the passage of a pulse with energy

$$E_{cr} = \frac{E_i}{\sigma_{el}}\ln\left(\frac{n_{cr}}{n_0}\right). \tag{5.2}$$

At $\omega = 2\times 10^{15}$ sec^{-1} and $\nu_{eff} = 10^{14}$ sec^{-1} the cross section for the absorption by the electron is $\sigma_{e_1} \approx 3\times 10^{-18}$ cm^2. At $E_i = 8$–10 eV and $n_{cr}/n_0 \approx 10^{10}$–$10^{20}$ the critical energy lies in the interval 10–20 J/cm^2. This value agrees in order of magnitude with the experimental data[13,9,11,16]. There is also a better agreement with experiment in that the damage is determined by the pulse energy. The establishment of the existence of a mechanism of self-damage of active media justifies the proposal made in[10] to increase the limiting power of the pulse by changing over to shorter durations. In this way it is possible to obtain a power $I_{lim} \approx E_{cr}/\tau_p$. When $\tau_p \approx 10^{-10}$ sec and $E_{cr} \approx 10$ J/cm^2 it is possible to reach a power $I_{lim} \approx 10^{11}$ W/cm^2. Of course, this is valid until an appreciable role is assumed by the power limitation due to the nonlinear absorption of the light in processes of the type of multiquantum excitation and ionization of ions and other impurities in the active medium[34,29,37].

Multiquantum absorption processes limit primarily the pulse power at the level I_{lim}, determined by the expression

$$\alpha_0 = \gamma(I_{lim}), \tag{5.3}$$

where α_0 is the gain of the active medium per unit length, and $\gamma(I)$ is the coefficient of nonlinear absorption per unit length. Multiquantum absorption can greatly distort the form of the amplified pulse[143]. Thus, for example, if the power at the top of the pulse is stabilized, preferred amplification of the leading and trailing edges takes place, and this leads to a corresponding increase of the pulse duration. The rate of increase of the duration depends strongly on the shapes of the edges. For example, for a pulse with exponential edges of the type $\exp(t/\tau_0)$, the increase of the pulse duration in m-photon absorption follows the law[143]

$$\tau_p \approx 2(1-2^{1-m})\tau_0\alpha_0 x_{nonlin}$$

where x_{nonlin} is the distance traversed by the pulse in the amplifying medium in the power-limitation regime. In practice one can expect an increase of the duration of the ultrashort pulse by a factor of several times. In the case when the multiphoton absorption leads to the appearance of free electrons in the conduction band, and this takes place for both ruby[122,123] and neodymium glass[173], the wave form of the pulse may become distorted as a result of absorption by free electrons[143]. It is qualitatively clear that the electron concentration is maximal at the end of the pulse, and therefore predominant absorption of the trailing edge will occur. As a result, the maximum of the pulse shifts forward along the leading front. In the case of propagation of a pulse with an exponential front, such a shift assumes a stationary character, and the maximum of the pulse moves with superluminal velocity. The picture of the pulse deformation is very similar to the case of propagation of an exponential pulse in a nonlinearly amplifying medium with linear loss, considered in Sec. 2.2. To observe pulse-wave form distortion effects of this type it is necessary that the pulse energy density E_p satisfy the condition $E_p \gtrsim m\hbar\omega/\sigma_{e_1}$.

Multiphoton processes in a strong field in neodymium glass have not yet been investigated, so that no estimate can be given for I_{lim}. It is not excluded, however, that these effects play a role at the attained power density $\sim 10^{11}$ W/cm^2[16]. Thus, it was noted that the total gain of a powerful ultrashort pulse in the apparatus of[16] was smaller by a factor 2.5 than the gain for a weak signal. For ruby, these processes were investigated in detail in[122,123,172]. The coefficient of two-photon absorption of the λ = 6943 Å radiation in

ruby is $\gamma = 2 \times 10^{-2}$ cm^{-1} at a power $I \approx 1$ GW/cm^2[123]. At an initial ruby gain per unit $\alpha_0 = 0.1$ cm^{-1} the limiting pulse power in a ruby amplifying medium is $I_{lim} = 5$ GW/cm^2. It is clear therefore that neodymium glass is preferred for the production of powerful pulses.

The radiation brightness attained in the amplification of ultrashort pulses of light in[16] was 10^{17} W/cm^{2}sr. This brightness corresponds to the power of 10^{11} W/cm^2 and a divergence of 10^{-3} rad. It is possible in principle to increase the brightness to 10^{20} W/cm^{2}sr by decreasing the divergence to a value close to the diffraction limit. One of the possibilities of increasing the brightness of laser radiation is based on the method of cascade conversion of the radiation, which is considered in the next chapter.

6. PRODUCTION OF POWERFUL COHERENT-LIGHT PULSES BY THE CONVERSION METHOD

6.1. Idea of Conversion

The radiation brightness of a pulsed laser optically pumped by thermal radiation of electric lamps is smaller by several orders of magnitude than the theoretical limiting value. This is connected with the fact that, owing to the low efficiency of conversion of the pump radiation with the continuous spectrum into coherent light (several per cent at best), the active medium becomes appreciably heated during the course of pumping. As a result of the heating, noticeable distortion of the optical homogeneity of the resonator takes place[126-128], as well as excitation of transverse modes of high order. Therefore the radiation divergence of powerful lasers is worse by one or two orders of magnitude than the limiting radiation divergence determined by the divergence of the radiation of the fundamental mode TEM_{00}. At low pumping levels, and consequently at relatively low output power, it is possible to effect a certain degree of control of the transverse modes of the laser and to decrease the radiation divergence to the diffraction limit. For example, in[129] is described a Q-switched ruby laser with transverse-mode selection, emitting a light pulse of power 5×10^6 W and brightness 10^{15} W/cm^{2}sr. Further increase of the brightness can be obtained by the amplification method considered in the preceding chapter. It must be borne in mind, however, that thermal inhomogeneities that distort the wave front of the beam and increase its divergence arise also in amplifiers with optical pumping by electric lamps, owing to the low pumping efficiency. Although special schemes for compensating for the inhomogeneities are possible here, there is a definite brightness limit for laser installations with optical pumping by means of thermal sources.

High optical-pumping efficiencies, in principle close to 100%, can be obtained by using a laser as the pump source. In this case it is possible to obtain exact matching of the pump radiation spectrum to the absorption band of the active medium. The high power of laser radiation makes it possible to produce inverted populations in media with very short excited-level relaxation times, thus greatly increasing the number of active media. This has made it possible to develop lasers based on solutions of organic dyes[130-132], with high efficiency and large radiation brightness[133]. Powerful laser radiation makes it possible to produce negative absorption at combination frequencies in a nonlinear medium, for example in a Raman-scattering medium. The efficiency of conversion in Raman-scattering lasers is quite high and makes it possible to increase greatly the radiation brightness[134-136]. Other schemes of converting laser radiation and increasing the brightness are also possible.

The general scheme of increasing the brightness of laser radiation by a two-stage conversion method is shown in Fig. 32. The first stage of the laser setup is a laser that is optically pumped by a thermal source. The powerful radiation of this laser has a brightness many orders of magnitude higher than the brightness of the pump source, but much lower than the limiting brightness, for example as a result of the large number of transverse modes. The second stage is a laser optically pumped by the radiation of the first stage; the efficiency of the second stage is quite high, and the parameters of the output beam are close to the limiting values. Naturally, the first stage can contain several lasers. Then the second stage makes it possible to sum their radiation simultaneously into a highly-coherent beam.

The presently-realized two-stage converters for increasing the brightness employ stimulated Raman scattering in gas[134,136] and in liquids[135], as well as stimulated emission in solutions of organic molecules[130-132]. At the same time, they effect a conversion of the generation frequency, which is also of great interest. We shall consider these converters only from the point of view of the increase of the radiation brightness.

6.2. Converters Based on Stimulated Raman Scattering

When powerful optical radiation passes through a substance (crystal, liquid, gas), stimulated Raman scattering (SRS) of the incidence radiation by the oscillations or rotations of the molecules takes place. Negative absorption occurs in the medium at the combination frequency $\omega_1 = \omega_0 - \Omega$ (ω_0—frequency of incident radiation, Ω—vibrational or rotational frequency of the molecules). At a pump power on the order of 10^8 W/cm^2 in a condensed medium, one can realistically obtain at the first Stokes frequency ω_1 a gain of $\kappa_0 \approx 1$ cm^{-1}.[7] Owing to the negative absorption of the

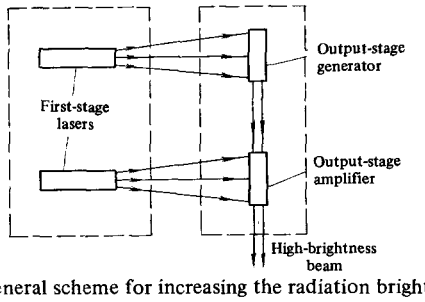

FIG. 32. General scheme for increasing the radiation brightness by the method of cascade conversion.

frequency ω_1, amplification of the spontaneous radiation develops, and generation takes place when the medium with SRS is placed in the resonator. The occurrence of a strong field at the frequency ω_1 leads to the development of the next Stokes and also anti-Stokes waves. It is meaningless to stop to discuss this in any detail here, since the SRS phenomenon has been considered in a number of books[7,137] and in a review[138].

The efficiency with which the pump radiation is converted into Stokes radiation can be quite high, since the only losses are the Stokes radiation losses. From the point of view of high optical homogeneity, gases and liquids are more suitable for conversion. A comparison of gases and liquids for conversion by the SRS method is given in[140]. An advantage of liquids is the higher value of the gain. To obtain the same gain in gases, it is necessary to increase their pressure to several hundred atmospheres. However, an essential shortcoming of liquids is that they have a lower threshold for self-focusing than for SRS[141]. This pertains to organic liquids, but there are possible exceptions (liquid N_2[135]). The situation is reversed in the case of gases[143], and they were used to obtain an appreciable increase of brightness both in the generation regime[134,136] and in the nonlinear amplification regime[66].

a) The generation regime is realized when the SRS medium is placed in a resonator. If the pump radiation is directed along the resonator axis, one speaks of "longitudinal pumping." and if it is perpendicular to the axis one speaks of "transverse pumping." In a generator with longitudinal pumping, the power density (W/cm^2) of the generated radiation, of course, does not exceed the pump power density[78]. An increase of brightness is possible here as a result of an improvement in the directivity of the radiation. The efficiency of conversion into the first Stokes component, under the condition that the generation at the second Stokes component is suppressed, can be close to the limiting value $1 - (\Omega/\omega_0)$[78]. In the case of transverse pumping, further increase of radiation can be obtained by decreasing the area of the cross section of the generation beam compared with the area of the cross section of the pump beam. In particular, this occurs when the medium is pumped with the aid of several lasers. A Raman laser with transverse pumping has many specific features (instability of the stationary regime or generation of one Stokes component in the case of homogeneous losses in the resonator), which were theoretically considered in[78]. It was shown there that a high conversion efficiency can be obtained in the regime in which two Stokes components are generated, and also by introducing transversely-inhomogeneous losses into the resonator.

A Raman laser with compressed N_2 and longitudinal pumping by a pulse of a Q-switched ruby laser is described in[134]. The radiation at the first Stokes component (9755 Å) has a divergence $(1.5-2)\times 10^{-3}$ rad, which is 4–5 times better than the divergence of the ruby-laser beam. There was no self-focusing of the beam in the active medium. This reference, as well as[142], shows that it is realistic to expect development of good laser-radiation converters based on SRS.

A generator and amplifier operating at the first SRS Stokes component in an active medium of compressed hydrogen with longitudinal pumping by a pulse of a Q-switched ruby laser are described in[136]. The ruby laser emitted a pulse of 40 nsec duration, 5 J energy, 5×10^{-3} rad divergence, and 1 cm beam diameter, corresponding to a brightness 10^{13} W/cm^2sr. The efficiency of the conversion in the generator was 10%, and the divergence was three times larger than the limiting diffraction value (for the TEM_{00} mode). The SRS amplifier was 300 cm long, 3 cm in diameter, and had a weak-signal gain of 30 dB. The limiting brightness of the radiation should have amounted to 4×10^{16} W/cm^2sr. The attained brightness was somewhat lower than this value (approximately 10^{16} W/cm^2sr), but much higher than the pump radiation brightness. The limitation of the brightness is due to transverse inhomogeneity of the pumping beam and to errors in the employed optical system.

In[135] is described an SRS laser with active medium of liquid nitrogen and non-axial pumping by a pulse from a Q-switched ruby laser. The efficiency of energy conversion of the pump radiation into radiation at the first and second Stokes components (0.828 μ and 1.026 μ) was 3%, but a 60-fold increase of brightness was obtained by decreasing of the beam divergence by almost one order of magnitude (to 5×10^{-4} rad) and reducing the duration of the pulse by a factor of 2. The laser parameters were far from their limiting values, owing to the fact that the pump duration (75 nsec) was insufficient to establish a quasistationary regime with maximum efficiency and minimum divergence. The results of[135] show that it is possible to find a liquid substance with a self-focusing threshold higher than the SRS threshold.

b) The regime of nonlinear amplification of radiation at the combination frequencies makes it possible to increase the brightness by greatly reducing the pulse duration compared with the pump-pulse duration. The mechanism of compressing the pulse is similar in many respects to the mechanism of compressing the duration of the pulse in a nonlinear amplifying medium, as considered in Ch. 2. What is amplified in this case, however, is not the external light pulse, but the pulse of the intrinsic spontaneous Raman scattering. Owing to the very large gain in SRS, the spontaneous radiation is increased to such a degree that it causes saturation of the pumping. This recalls the amplification of superradiance pulses described in[61,63].

Nonlinear amplification of intrinsic radiation of a Raman-scattering medium at the first Stokes component and the formation of very short pulses of light were observed experimentally for the first time in[51,65]. This method was subsequently developed in[66]. The experimental setup is shown in Fig. 33. A pulse of a Q-switched ruby laser was used for pumping. As a result, negative absorption was produced at the first Stokes frequency, and spontaneous emission at this frequency was propagated and amplified in two opposite directions. The radiation in the direction opposite to the pump radiation has more favorable amplification conditions, since it always encounters "fresh" amplification (in the case of Raman amplification, the role of the active particles is played by the pump photons,

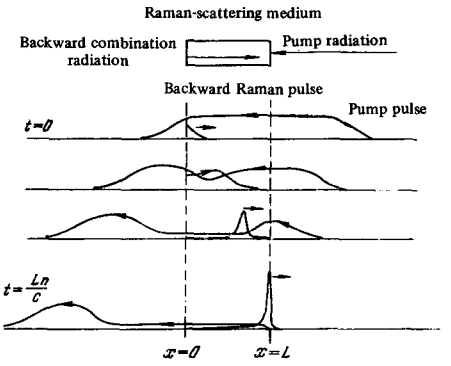

FIG. 33. Diagram showing the formation of a pulse of backward stimulated Raman radiation.

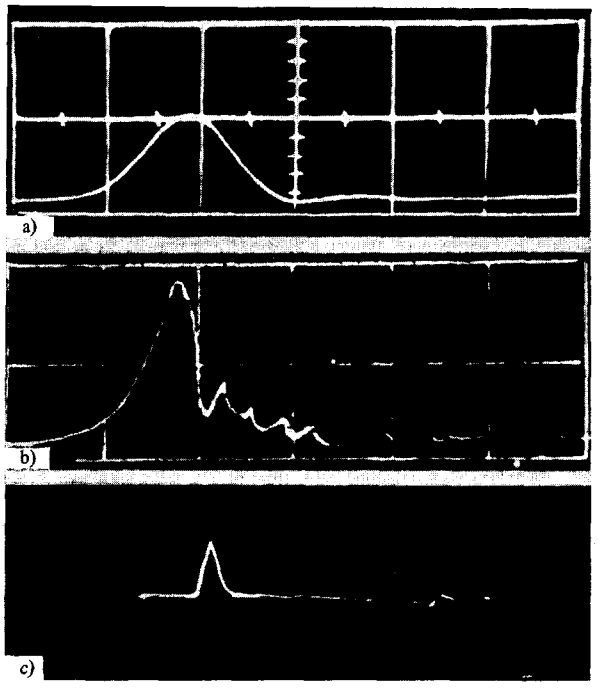

FIG. 34. Oscillograms illustrating the formation of a pulse of backward Raman scattering [66]. a) Pump pulse; b) pulse passing through the Raman-scattering medium; c) pulse of backward Raman scattering. Sweep – 10 nsec/div.

and therefore the "Raman amplification" propagates together with the pump radiation[51]). The backward Stokes radiation is amplified to such an extent, that it causes saturation of the gain, i.e., a depletion of the pump radiation. In the nonlinear-amplification regime, a short compression of the pulse takes place (Fig. 33). It can be shown that the mechanism of pulse compression is essentially connected with the presence of a steep leading front in the backward Stokes wave. This is called for by the conditions derived in[48] for the reduction of the duration in the case of nonlinear amplification. A steep leading front of a backward Stokes wave can occur either when the pump radiation is focused inside a cell with a SRS medium, or else by reflection of the forward Stokes wave from the output end of the cell.

The limiting duration of such pulses is apparently limited by the width Γ of the molecule transition line. In[61], the scattering medium was a liquid (CS_2) with a relatively broad line (Γ amounted to several cm^{-1}), and pulses with duration of the order of 30 psec, comparable with $1/\Gamma$, were obtained. In[66], the scattering medium was compressed hydrogen and pumping by a pulse of a Q-switched laser yielded pulses of backward Stokes radiation (λ = 9755 A) with duration 3×10^{-10} sec. Figure 34 shows the shapes of the pump pulses at the input of the cell (a), at the output of the cell (b), and of the pulse of the backward SRS (c) obtained in that reference.

6.3. Conversion With the Aid of Optical Pumping

By now, many lasers pumped by laser radiation have been constructed. The large family of lasers of this type include semiconductor lasers pumped with giant pulses (single-photon excitation of GaAs by a pulse from a ruby laser[152], two-photon excitation of GaAs by a pulse of a neodymium laser[153], etc.), with pumping by radiation from another semiconductor laser (InSb and CdSe were pumped by an injection GaAs laser[154,158]), lasers using luminescent crystals and pumped by radiation from a solid-state laser ($CaF_2:Sm^{2+}$ and $CaF_2:Dy^{2+}$ pumped by a ruby laser[155,156], $CaF_2:U^{3+}$ and YAG:Nd^{3+} pumped by an injection GaAs laser[157,159]), lasers using solutions of dye molecules pumped by giant pulses of ruby and neodymium lasers[130-132,79], etc. In some cases the laser pumping was chosen in order to increase the power to reach the threshold[152,153,130-132], and in some cases the laser radiation was purposefully converted to improve the characteristics of the coherent radiation[154,157-159].

The most attractive is the possibility of obtaining a very high brightness (up to 10^{20} W/cm^{2}sr), by the pulse conversion method. From this point of view, interest attaches to lasers using luminescent crystals and glasses, as well as liquid lasers.

Promising results were obtained so far with lasers using dye solutions. This, however, is more readily due not to the special properties of the liquids but to the initial stage of the research in this direction. Conversion coefficients up to 50% were attained in lasers using dye solutions[133]. The radiation-divergence angle of some of them amounts to 5×10^{-4} rad, i.e., it is much smaller than the divergence of the pumping ruby-laser radiation. As a result, the brightness of the generated radiation exceeds by several times the brightness of the pump-laser radiation.

When complex organic-dye molecules with a rich spectrum are used to obtain high powers on the order of 10^{10} W/cm^2 it is necessary to avoid processes of two-quantum absorption. From the point of view of the transmission of high-power light fluxes, up to 10^{11} W/cm^2, the best active medium at the present time is neodymium glass[16]. To obtain a radiation brightness on the order of 10^{19} W/cm^{2}sr it is necessary to attain a power of 10^{11} W/cm^2 at the diffraction divergence of a beam of 1 cm diameter. The use of neodymium glass in the converter is difficult, since there is no suitable powerful pump laser. To be sure, neodymium glass has a

weak absorption band at the second-harmonic frequency of the neodymium laser ($\kappa \approx 0.3$ cm^{-1} at a 2% weight content of Nd in the glass[151]). The radiation conversion efficiency at the second harmonic in nonlinear crystals reaches 50%[100]. This gives reason for hoping that it is possible to use neodymium glass in high-power converters.

The two-stage conversion method is a promising means of obtaining ultrapowerful light pulses of high brightness. Research in this direction has started only very recently, and interesting results can be expected.

7. CONCLUSION

In conclusion, we note that the present review is devoted mainly to the propagation of a light pulse in an amplifying and (or) absorbing medium with nonlinearity as a result of the saturation effect. We have therefore not considered a number of problems connected with the propagation of pulses in media with nonlinearities of other types (nonlinearity of multiquantum processes[28,143], nonlinearity of the refractive index[174,175], nonlinearity of a parametrically amplifying medium[176,177,185], etc.). The influence of nonlinearities of this type on the pulse propagation began to be considered only recently. The results have already suggested the use of new methods of obtained pulses of subpicosecond duration[28,176,178]. Relatively little attention has been paid also to the change of the pulse spectrum upon propagation. This question has been little investigated either theoretically or experimentally. However, as shown in[178], the spectrum of an ultrashort pulse can change appreciably during propagation, and this effect can also be used to shape powerful pulses of subpicosecond duration.

Short powerful pulses of coherent light serve as an effective and precise tool for the investigation of the interaction between an optical field and matter. The great possibilities of using powerful short pulses of light are connected both with the short duration and with the high power of the pulse. Pulses of picosecond duration make it possible to measure directly the relaxation times of atomic or molecular processes (for example, direct measurement of the lifetime of the excited state in the picosecond region) was effected in[179], to register rapidly flowing processes[180], to measure directly the response of optical media and instruments, to investigate the coherent interaction of a field with an optical medium, etc. The large power of the light pulses makes it possible to act on deep electrons in the shells of atoms and ions. Light pulses of power 10^4–10^6 W (electric fields at the focus 10^4–10^5 W/cm) have made it possible to observe nonresonant multiquantum processes in which external optical electrons of the atoms take part[6,7]. It is possible to use pulses of power 10^8–10^9 W (field at the focus of the order of 10^7 V/cm) to heat a dense plasma produced during optical breakdown of a gas.[162,163] or by action on a solid target[164,165], up to temperatures 10^{-5}–10^{-6}°K[164-166]. At such fields and temperatures, a large number of electrons are detached from the atoms and multiply charged ions are produced (for example, Ca XVI)[161]. The dense plasma obtained when matter is heated by a powerful light is a source of intensive x-radiation[168,169,170]. The use of 10^{12} W pulses has made it possible to heat a deuterium plasma to a temperature above 10^7°K and to observe neutrons produced as a result of thermonuclear reactions[139]. When brightnesses of the order of 10^{19}–10^{20} W/cm^{2}sr are attained, it will apparently be possible to obtain intense fluxes of x-rays, neutrons, and multiply-charged ions in a dense plasma. In particular, one can expect the development of coherent-radiation sources in the x-ray band on this basis.

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