NIKOLAĬ NIKOLAEVICH BOGOLYUBOV

(On his 60th Birthday)

D. N. ZUBAREV, B. V. MEDVEDEV, Yu. A. MITROPOL'SKIĬ, and M. K. POLIVANOV

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THE outstanding Soviet theoretical physicist, Academician Nikolaĭ Nikolaevich Bogolyubov, celebrates his 60th birthday on 21 August 1969.

When Bogolyubov began to be interested in theoretical physics, he was already a well known mathematician, who had blazed new trails and obtained outstanding results in a variety of fields of mathematics. His tremendous mathematical experience, which is unusual for a theoretical physicist, is reflected also in his physics research, and is the characteristic individual feature of his scientific creativity in physics.

His very earliest investigations, in which he developed direct methods for solving extremal problems, have won him great fame. In the theory of almost-periodic functions, he has demonstrated that the basic theorems of this theory follow from one general theorem concerning the behavior of an arbitrary bounded function, and by the same token he actually presented a new formulation of this theory.

Many of Bogolyubov's interesting papers are connected with the application of the difference method to variational calculus. These results were employed by him for the development of approximation methods in the theory of boundary-value problems for ordinary differential equations and for partial differential equations.

In 1932 Bogolyubov, together with his teacher Academician N. M. Krylov, started to develop an entirely new trend in mathematical physics—the theory of nonlinear oscillations (nonlinear mechanics). Bogolyubov's efforts were directed here primarily towards the development of methods for asymptotic integration of nonlinear equations describing vibrational processes. The asymptotic integration of differential equations with a small parameter were investigated, as is well known, in celestial mechanics, where, however, only conservative systems were considered. The extention of the methods developed there, and particularly the perturbation method, to the investigation of nonconservative vibrational systems, entailed great fundamental difficulties. Bogolyubov succeeded in developing perturbation-theory methods for the general case of nonconservative systems, and to devise many new methods of nonlinear mechanics.

In subsequent investigations, Bogolyubov presented a rigorous justification of the asymptotic methods of nonlinear mechanics. Basing himself on the Poincare-Lyapunov theory and on the Poincare-Denjoy theory of trajectories on a torus, he investigated the character of the exact stationary solution near the approximate solution for small values of the nonlinearity parameter, and proved a number of theorems concerning the existence and stability of the quasiperiodic solutions.

In addition, Bogolyubov has authored many papers on nonlinear mechanics, concerning the abstract theory of dynamic systems. In these papers he established the existence of an invariant measure, introduced the important concept of the ergodic set, and proved many theorems concerning the breakdown of an invariant measure into measures that are localized in ergodic sets.

The regular perturbation-theory methods developed by N. N. Bogolyubov in nonlinear mechanics turned out to be very effective in statistical mechanics. In his monograph "On Certain Statistical Methods in Mathematical Physics'' he considered the problem of the influence of a random force on a harmonic oscillator, and the establishment of statistical equilibrium in a system coupled to a thermostat (an aggregate of a large number of harmonic oscillators). Bogolyubov has shown that, depending on the choice of the time scale, the same random process can be regarded as dynamic, as a Markov process, or as a certain non-Markov process. By the same token, he first introduced the idea of the hierarchy of relaxation times in statistical mechanics, which determined all the further development of the statistical theory of irreversible processes.

In the investigations of statistical mechanics of classical systems, reported in the monograph ''Problems of Dynamic Theory in Statistical Physics'' (1946), Bogolyubov developed the chain method for the distribution functions of complexes of particles. This is the most effective method in equilibrium and non-equilibrium statistical mechanics, and has been named in the world literature the BBGKY method (Bogolyubov-Born-Green-Kirkwood-Yvon).

Bogolyubov proposed regular methods for solving chains of equations for the distribution functions of various systems, in both equilibrium and non-equilibrium cases. He noted that after the lapse of a certain time interval (on the order of the collision time) there occurs a ''synchronization'' of the distribution functions—all the higher distribution functions are determined completely by the single-particle function, and it is possible to construct kinetic equations for this stage. Instead of the Boltzmann hypothesis of molecular chaos, Bogolyubov used in the construction of kinetic equations the boundary conditions for the weakening of the correlation. He succeeded in obtaining, from a unified point of view, different types of kinetic equations for systems with short-range or long-range but weak forces. Whereas the Boltzmann method is based on complete neglect of the dynamic correlation between the colliding molecules, and is suitable only for rarefied gases, the Bogolyubov method makes it possible to take into account in the kinetic equation the higher order terms of the expansion in powers of the density. No less important results were obtained by Bogolyubov in quantum statistics. The method of constructing kinetic equations were generalized by him for non-ideal quantum gases.

Many of Bogolyubov's investigations were devoted to the spectrum of elementary excitations of quantum systems. He developed consistent methods in the microscopic theory of superfluidity and superconductivity.

Bogolyubov succeeded in calculating the spectrum of elementary excitations of a weakly-nonideal degenerate Bose gas and to show that such a gas should have superfluid properties. By the same token, a theoretical model was constructed for the superfluidity of helium-II, the spectrum of which turned out to be of the same form as predicted by Bogolyubov. Elementary excitations in a nonideal Bose gas, called ''Bogolyubovons'' in the world literature, are superpositions of the creations and annihilation operators for particles with oppositely directed momenta.

Similar elementary excitations in a nonideal Fermi gas, relating the creation and annihilation operators of particles with oppositely directed momenta and spins, offer a theoretical explanation of the superconductivity phenomenon. The physical cause for the appearance of such coupled pairs in a metal is the interaction between the electrons and the lattice vibrations. Bogolyubov has shown that superconductivity can be regarded as superfluidity of the electron gas in a metal. To study superconducting and superfluid systems, Bogolyubov developed the so-called generalized Hartree-Fock method (1958—1959), in which account was taken of the existence of correlated particle pairs. To investigate the spectra of the elementary excitations of ferromagnetic and antiferromagnetic crystals, Bogolyubov developed a method of approximate second quantization, which starts from a quasiclassical description of the ground state of the system by the Fock method. The elementary excitations correspond to small deviations from this state.

The development of statistical physics was greatly influenced by Bogolyubov's ideas concerning quasiaverages in the problems of statistical mechanics. In many statistical systems, the ground state turns out to be unstable against small perturbations, for example against the application of a small magnetic field in an isotropic ferromagnet, against an infinitesimally small source of pairs in a superconductor, etc. Bogolyubov has shown that a theoretical investigation of such systems is possible with the aid of the quasiaverage method, which makes it possible to ''lift'' the quasidegeneracy of the ground state. It is then possible to apply the regular methods of perturbation theory to the system. Connected with the quasiaverage method is Bogolyubov's $1/q^2$ theorem, according to which, in superfluid Bose and Fermi systems, the density of the continuous distribution of the particles with respect to the momenta q tends to infinity more slowly than $1/q^2$ as $q \to \infty$.

Bogolyubov's ideas concerning quasiaverages, which are deeply connected with the Goldstone broken-symmetry theorem, are the fundamental guiding ideas in modern theory of phase transitions. For example, Bogolyubov's inequality leads to strikingly simple but rigorous proofs of the absence of ferromagnetism or antiferromagnetism in one- and two-dimensional isotropic Heisenberg models and to the absence of superfluidity and superconductivity in the one- and two-dimensional cases.

The progressive development of the second-quantization formalism in statistical physics naturally attracted Bogolyubov's attention, since the early Fifties, to problems of quantum field theory. Actually, he always noted in his papers the close physical and methodological relation between the nonrelativistic many-body problem and quantum field theory. Working in both fields, he successfully used this close relation for a fruitful mutual exchange of ideas between field theory and statistical physics. As is well known, one of the main difficulties of quantum field theory is that the coefficients of the perturbation-theory expansion in terms of the coupling constant (in the case of quantum electrodynamics—in terms of the quantity $e^2 = 1/137$) are expressed in terms of integrals that diverge at large virtual momenta. The subtraction mechanism proposed for the elimination of these so-called ultraviolet divergences, based on the idea of renormalizing the mass and the charge, are for the most part formally prescriptive in nature. To justify this formalism, it was necessary to review critically all the main premises of quantum field theory and to analyze thoroughly the physical and mathematical nature of the difficulties arising therein.

In his investigations of the early Fifties, Bogolyubov performed this work and arrived at a new complete formulation of quantum field theory. First, he dispensed with the usual Hamiltonian formalism and took as the basis of his theory the scattering (S) matrix introduced by Heisenberg. In order to accurately introduce in such a formulation the hypothesis of adiabatic turning on of the interaction, which is necessary for the determination of the initial approximation—the free-field theory—he started to regard the S matrix as a functional on the interaction-switching functions g(x) with values $0 \leq g(x) \leq 1$, where $S[g] \to 1$ as $g(x) \to 0$ (free field), and $[g] \to S$ as $g(x) \to 1$ (physical S matrix). To determine the S matrix in all orders of perturbation theory, Bogolyubov used the main physical requirements that this matrix must satisfy—the requirements of relativistic invariance, unitarity, and causality.

The development of a new form of the latter condition played a special role in Bogolyubov's work on quantum theory. The point is that in the S-matrix theory there are in general no local operators initially, and therefore the formulation of the space-time relations, particularly of the causality requirement, is not trivial. To this end, he introduced local Heisenberg operators as variational derivatives of the S matrix with respect to the switching function g(x), namely, $H(x) = i(\delta S/\delta g(x))S^{+}$, and formulated the causality condition, now widely known as the Bogolyubov causality condition, in the form

$$\frac{\delta}{\delta g(x)}\left\{\frac{\delta S}{\delta g(y)} S^{+}\right\}=0$$

for $x \lesssim y$. The usual requirement of local commutativity of the Heisenberg operators $[H(x), H(y)] = 0$ when $x \sim y$ results from this condition as a particular case.

The theorem proved by Bogolyubov, that the scattering matrix is determined successively from the indicated requirements in all orders of perturbation theory, accurate to quasilocal operators, points to the source of the ultraviolet divergences—the singular nature of the coefficient functions of the S matrix, and gives a consistent recipe for its elimination, now called the R-operation. In this connection, it should be specially noted that the introduction of the function g(x) as the functional argument of the scattering matrix, and consequently of all the Heisenberg operators, led Bogolyubov naturally to the establishment of the now universally accepted point of view, namely that physical quantities of the quantum field theory must be considered from the mathematical point of view not as ordinary quantities but as (operator-valued) generalized functions, which are integrable in certain classes of "fundamental" functions g(x). (We use here S. L. Sobolev's definitions of the classes of fundamental functions, and are thus able to attain maximum "economy" in the choice of the powers of the arbitrary polynomials.)

Among the other results obtained by Bogolyubov in perturbation theory, let us mention the method of renormalization group. The gist of this method consists in the observation that multiplicative renormalizations in quantum field theory form a group. This makes it possible to improve the formulas of perturbation theory in the ultraviolet region, where the effective parameter of the expansion is not e^2 but $e^2 \ln(k^2/m^2)$, by reconstructing a form that is invariant with respect to the group.

In his investigations of that time, Bogolyubov was one of the first to develop the trend which later came to be called the axiomatic method of constructing quantum field theory. The advantages of this method became quite clearly evident in a cycle of his papers connected with the method of dispersion relations for amplitudes describing different processes of particle scattering and production.

The significance of this method lies in the fact that it requires no explicit assumptions concerning the form of the interaction, and is not based on perturbation theory. This is particularly important for strong interactions, for which perturbation theory is not applicable at all.

The basis for the derivation of the dispersion relations was the same system of physical requirements as in perturbation theory. Bogolyubov modified it only in part, adding also requirements not explicitly included in the system employed in perturbation theory, such as the condition of the stability of the vacuum and of the single-particle states. He also reformulated the causality principle. Inasmuch as the introduction of the special function g(x) no longer offered any advantages in the absence of an expansion in powers of the charge, he introduced as the functional argument of the S matrix the classical additions to the asymptotic fields, extending by the same token the scattering matrix beyond the energy shell, making it possible to postulate reduction formulas in a physically most lucid form. Finally, Bogolyubov was the first to point out the connection between the class of integrability of the elements of the scattering matrix and the causal properties of the theory, including among the conditions that separate the local theories also the requirement of integrability in any one of the classes C(q, r) with finite exponents. Later investigations have shown that it is precisely this criterion that can play the leading role in the classification of the degree of violations of locality of interaction of elementary particles.

The main difficulty in the proof of the dispersion relations was the need for considering physical quantities in the so-called unphysical region, where the momenta of the particles become complex. Bogolyubov succeeded in getting around this difficulty by using the method of double analytic continuation. At first the dispersion relations were derived and proved for imaginary values of the parameter—the particle mass, and in this case the indicated difficulty did not appear; the final expression was then continued analytically to the physical value of the parameter.

The proof constructed by Bogolyubov for the dispersion relations called for the development of a special mathematical apparatus of analytic continuation of generalized functions of many variables. The theorems proved by him in this field have found extensive use in the entire subsequent development of the theory of dispersion relations and in many other branches of quantum field theory.

It is impossible to overestimate the influence of this work on the entire mathematical style of theoretical physics of the last decade. Physicists had to learn new mathematical methods, they had to realize that they work with unbounded operators, to understand that the physical requirements can lead to important properties of holomorphy of certain usual physical objects, and that from this it is possible in turn to extract perfectly concrete physical information. As a result, a new standard of mathematical skill was established in the theory of quantized fields (and in statistical mechanics), calling for greater rigor of proof. Bogolyubov and his students have pointed out varied and extensive applications of the method of dispersion relations, such as the general construction of the axiomatic approach, asymptotic estimates at high energies, the description of low-energy regions by using the unitarity condition, etc.

The topics listed here far from cover the entire field of Bogolyubov's scientific activity. He originated many ideas and researches in other branches of relativistic dynamics of particles. Thus, he pointed out the important concept of "quasiaverages," which plays an exceedingly important role not only in statistical mechanics but also in quantum field theory and in all cases where there is degeneracy. Bogolyubov has paid much attention also to elementary-particle symmetry models. In his 1965—1966 lectures at the Moscow University, he presented a brilliant and simple exposition of the ideas of SU(3) and SU(6) symmetry. He proposed also a model of quarks with integer charges (the model of three triplets) and presented a simple explanation for the increase of the magnetic moment of the particle in the quark model.

Bogolyubov is the founder of three large scientific schools—in nonlinear mechanics and mathematical physics, in statistical mechanics, and in quantum field theory. He has trained an entire generation of soviet mathematicians and theoretical physicists. Many scientists call him their teacher with respect and with pride. A number of scientific staffs are being trained under his spiritual guidance in Kiev, Moscow, Dubna, Novosibirsk, Kishinev, Serpukhov, etc.

Bogolyubov pays much attention to the organization of science. At the present time he is the Academic Secretary of the Mathematics Division of the USSR Academy of Sciences and the director of the Joint Institute for Nuclear Research in Dubna.

Bogolyubov is a deputy of the 7th Superior Soviet of the USSR gathering, and is a participant of the Pugwash movement of scientists for peace.

His country has greatly valued his scientific and social activity: he was awarded the title of Hero of Socialist Labor, many orders and mdeals, including four Orders of Lenin. His work has been awarded many prizes, including the Lenin Prize and two state prizes. Many foreign scientific societies, universities, and academies have chosen Bogolyubov as a member in acknowledgement of his personal contribution to the development of science and of his great scientific and social authority.

Translated by J. G. Adashko