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Methodological Notes

RADIATION AND RADIATION FRICTION FORCE IN UNIFORMLY ACCELERATED MOTION OF A CHARGE

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I N physics there are many "perpetual problems," the discussion of which continues for decades in the scientific literature, let alone in text books. By way of examples from classical electrodynamics alone, we can indicate the choice of the energy-momentum tensor for a field in a medium (we refer here to the Abraham and Minkowski tensors), to the problem of the electromagnetic mass in the equation of motion with allowance for the radiation friction force, and to the problem of the radiation in the reaction of radiation accompanying uniformly accelerated motion of a charge.

1969 marks the sixteenth anniversary of M. Born's first analysis of the problem of the field of a uniformly accelerated charge[1]. It was concluded from Born's solution that a uniformly accelerated charge does not radiate—this same opinion was reflected in the well known book by W. Pauli[2]. At the same time G. Schott[3] and later many other authors reached to an opposite conclusion, namely that radiation exists for a uniformly accelerated charge, as in any other accelerated motion. At the same time, in the case of uniformly accelerated motion of the charge the radiation-friction force is equal to zero, which seems paradoxical in the presence of radiation. This and related problems connected with the study of uniformly accelerated charges have been the subject of many articles, among which we point out only the recent ones[4-7] (references to the earlier literature can be found in[4-7]; see also below). For example, in[6] the establishment of the energy balance in uniformly accelerated motion of a charge is characterized as the "most puzzling problem of classical physics."

The problem of radiation from a uniformly accelerated charge and most other "perpetual problems" are undoubtedly of no major significance, and this is precisely why they have remained insufficiently well explained for so long a time. On the other hand, however, neglect of such methodological types of problems sometimes incurs vengeance. For example, certain inaccuracies and misunderstandings, recently observed in the theory of synchrotron (magnetic bremsstrahlung) radiation are connected precisely with the frequent practice of equating of the radiated energy (total radiation flux) to the work done by the radiation friction force (see[8], and also[9], where many articles devoted to this problem in the theory of synchrotron radiation are mentioned). Actually, however, the radiated energy and the work of the radiation friction are not equal to each other in the nonstationary case. This indeed resolves the paradox concerning the radiation of a uniformly accelerated charge. It seems to us that this circumstance

was not taken into account to a sufficient degree in earlier discussions[4-7]. Nor was the behavior of a uniformly accelerated charge discussed in sufficient detail from the point of view of the equivalence principle. For these reasons and on the basis of the known experience, which indicates that the discussion of the corresponding problems is not sufficiently clear even to well-trained physicists, it seems appropriate to stop once more to discuss the radiation and radiation force in the case of uniformly accelerated motion of a charge.

I. RADIATION AND RADIATION FORCE IN THE CASE OF A UNIFORMLY ACCELERATED MOTION OF A CHARGE

When a charge e moves vacuum along a certain trajectory, the electromagnetic field is determined by the well known equations that follow from the Lienard-Wiechert potentials:

$$\mathbf{E} = \frac{e\,[1-(v^2/c^2)]}{[R-(\mathbf{vR}/c)]^3}[\mathbf{R}-(\mathbf{v}/c)R] + \frac{e}{c^2\,[R-(\mathbf{vR}/c)]^3}[\mathbf{R}\,[(\mathbf{R}-(\mathbf{v}R/c))\,\dot{\mathbf{v}}]], \quad (1)$$

$$\mathbf{H} = \frac{1}{R}[\mathbf{RE}]. \quad (2)*$$

The fields $\mathbf{E}$ and $\mathbf{H}$ are taken here at the point of observation at the instant t, and the quantities $\mathbf{R}$, $\mathbf{v}$, and $\dot{\mathbf{v}}$ in the right sides of the equations pertain to the "radiation time" $t' = t - [R(t')/c]$, the vector $\mathbf{R}$ being drawn from the point where the charge e is located to the observation point. Further, $\mathbf{v}(t') = d\mathbf{R}(t')\,dt'$ is the charge velocity and $\dot{\mathbf{v}} = d\mathbf{v}/dt'$. Obviously, expression for $R(t')$ determines the trajectory of charge motion, but it is more convenient to characterize the position of the charge by means of the vector $\mathbf{r}(t')$ and the observation point by the vector $\mathbf{r}(t) = \mathbf{r}(t') + \mathbf{R}(t')$, from which we get also $\dot{\mathbf{r}} \equiv d\mathbf{r}/dt' = -d\mathbf{R}/dt'$ (for a derivation of formulas (1) and (2) see, for example,[10-13]).

The first term in (1) corresponds to the field of a charge moving with velocity v; this term decreases with increasing R like $1/R^2$. The second term in (1) decreases like $1/R$ and is the principal term when $R \gg c^2[1 - (v^2/c^2)]/\dot{v}$; the field described by this term is transverse and represents the field of a certain electromagnetic wave. If the charge produces also a wave field, then such a charge is said to be radiating. Although this definition is not trivial, it does call for clarification. Indeed, one can consider the charge's wave field, which decreases like $1/R$, only in the wave

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*$[\mathbf{RE}] \equiv \mathbf{R} \times \mathbf{E}$.

zone, where only one such field exists in practice. It is possible, however, to verify the existence of a wave term (the second term in (1)) also at shorter distances from the charge. In such a case, however, the total field is by far not the radiation field that propagates with the speed of light. For reasons that will be made clear later, it seems to us that it is better to take the statement "the charge radiates" in a broader sense, i.e., in the presence of a wave field and independently of the presence or absence of other parts of the field. It must be emphasized also that measurements of the fields E and H at the instant t can lead to conclusions concerning the state (for example acceleration) of the electron only in the preceding instant $t' = t - [R(t')/c]$.

If we consider only the field of one given charge, then the energy flux through any closed surface surrounding the charge should differ from zero in the presence of radiation. Obviously, the energy passing a time $dt = [1 - (v \cdot n/c)]dt'$ through the area $d\sigma = R^2 d\Omega$ in the direction $n = R/R$ is equal to $(d\Omega$ is the element of the solid angle)

$$d\mathscr{E}_n = \frac{c}{4\pi} [EH] n R^2 \, d\Omega \, dt = \frac{e^2}{4\pi c} \frac{[n\,[(n-v/c)\dot{v}]]^2}{[1-(vn/c)]^6} \, d\Omega \, dt, \tag{3}$$

where the fields are represented by the wave field (the second term of (1) or (2)). For this reason, expression (3), generally speaking, is valid only in the wave zone.

Calculation of the total energy radiated in a unit time t' yields

$$P \equiv \frac{d\mathscr{E}}{dt'} = \frac{e^2}{4\pi c^3} \int \frac{[n\,[(n-(v/c))\,\dot{v}]]^2}{[1-(vn/c)]^5} \, d\Omega = \frac{2}{3} \frac{e^2}{c^3} \frac{\dot{v}^2 - [v/c\dot{v}]^2}{[1-(v^2/c^2)]^3} = -\frac{2e^2c}{3} w^i w_i; \tag{4}$$

here $w^i = (w^0, w) = du^i/ds$ is a four-dimensional vector of particle acceleration.* By virtue of the Lorentz invariance of expression (4), its calculation can be carried out in any inertial system. In the system in which $v = 0$, formula (3) is valid for any R, and consequently the calculation of the radiated energy and the establishment of the very presence of radiation can be carried out also near the charge, and not only in the wave zone[5,7]. This conclusion is understandable in part, of course, even from general considerations, since the field (particularly the wave field) is defined by formulas (1) and (2) at any distance from the charge.

The quantity $P = v\mathscr{E}/dt'$ characterizes the flux of energy through a sphere of radius R at the instant t, but it must be emphasized that the quantities in the right side pertain to the instant $t' = t - [R(t')/c]$, and the radiated energy also is referred to units of "radiation

time" t'. The difference between the intervals dt $= [1 - (v \cdot n/c)]dt'$ and dt' is a manifestation of the Doppler effect: a pulse of radiation emitted at the instant dt' will have a length c dt.

If the velocity of the charge at the instant of radiation t* is equal to zero (or in practice is sufficiently small), then the radiated energy is

$$P \equiv \frac{d\mathscr{E}}{dt'} = \frac{d\mathscr{E}}{dt} = \frac{2e^2}{3c^3} \dot{v}^2. \tag{5}$$

This expression is sometimes called the Larmor formula, and it is quite well known.

In the case of nonrelativistic uniformly-accelerated motion we have $\dot{v}$ = const. Relativistic uniformly-accelerated motion is defined as motion in which the acceleration is constant in the co-moving (proper) reference system, i.e., in the system in which the instantaneous velocity of the particle is zero. If we use the expression given in the foregoing footnote for $w^i w_i$, then we get $w^i/w_i = -w^2/c^4$ at $v = 0$ (we have introduced the symbol $\dot{v}$ (at $v = 0$) = w). This condition, obviously, defines relativistic uniformly-accelerated motion in an invariant manner.

We confine ourselves here to the particular case of linear motion (the vectors v and $\dot{v}$ are collinear); then the foregoing general expression for $w^i w_i$ and the condition $w^i w_i = -w^2/c^4$ = const yield immediately $\dot{v}/[1 - (v^2/c^2)]^{3/2} = w$, or $d[v/\sqrt{1 - (v^2/c^2)}]/dt = w$ = const. Choosing the velocity direction as the z axis and assuming, in order to obtain specially simple expressions, $v = dz/dt = 0$ and $z = c^2/w$ at $t = 0$, we get

$$z = c\sqrt{\frac{c^2}{w^2} + t^2}, \quad v = \frac{dz}{dt} = \frac{ct}{\sqrt{(c^2/w^2) + t^2}} = \frac{wt}{\sqrt{1 + (w^2t^2/c^2)}},$$
$$\dot{v} = \frac{dv}{dt} = \frac{c^3}{w^2\,[(c^2/w^2) + t^2]^{3/2}} = \frac{w}{[1 + (w^2t^2/c^2)]^{3/2}}. \tag{6}$$

Relativistic uniformly accelerated linear motion is also called hyperbolic, since the function $z(t)$ is a hyperbola*. In a homogeneous and constant electric field or in a gravitational field, either of which is parallel to the charge velocity, the motion is precisely hyperbolic, since the equation of motion is given by

$$\frac{d}{dt}\left[\frac{mv}{\sqrt{1-(v^2/c^2)}}\right] = F = \text{const},$$

which coincides with the expression given above for hyperbolic motion.

From formulas (4) and (5) and from the foregoing it is clear that in both nonrelativistic and relativistic uniformly accelerated motion the charge radiates, with $P = d\mathscr{E}/dt' = (2e^2/3c^3)w^2$. Moreover, from the point of view of the radiation, motion with constant acceleration does not differ qualitatively at all from radiation pro-

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*We use the notation of [11]. The four-dimensional velocity is

$$\frac{dx^i}{ds} = u^i \equiv (u^0, u) = \left(\frac{1}{\sqrt{1-(v^2/c^2)}}, \frac{v}{\sqrt{1-(v^2/c^2)}}\right),$$

$$u^i u_i = u_0^2 - u^2 = 1, \quad ds = c\,dt\,\sqrt{1-(v^2/c^2)}$$

and

$$w^i = \frac{du^i}{ds} = \left(\frac{v\dot{v}}{c^3[1-(v^2/c^2)]^2}, \frac{\dot{v}}{c^2[1-(v^2/c^2)]} + \frac{v\,(v\dot{v})}{c^4[1-(v^2/c^2)]^2}\right),$$

where $\dot{v} = dv/dt$. It is easy to see that

$$w^i w_i = -\frac{\dot{v}^2}{c^4[1-(v^2/c^2)]^2} - \frac{(v\dot{v})^2}{c^6[1-(v^2/c^2)]^3} = -\frac{\dot{v}^2 - [v/c\dot{v}]^2}{c^4[1-(v^2/c^2)]^3}.$$

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*In the general case of uniformly accelerated motion [1 − (v²/c²)] $\ddot{v}$ + (3/c²$\dot{v}$)($\dot{v}$ · v) = 0 (see [4] and the literature cited there). Hyperbolic motion is realized under the influence of a constant electric field only if the charge velocity v is collinear with the field E_{ext}. On the other hand, if the charge moves in the electric field at an angle to E_{ext}, i.e., if its velocity has a component transverse to the field, then such motion is not uniformly accelerated (in this case the charge is acted upon also by a magnetic field in the reference frame in which the charge is at rest) [7], let alone the obvious fact that it cannot be hyperbolic.

duced in arbitrarily accelerated motion. The last remark holds not only for the calculation of the total power $P = d\mathscr{E}/dt'$, but also for the spectral distribution of the radiation[7].

An accelerated charge, generally speaking, experiences a radiation-friction force f. In nonrelativistic motion $f = (2e^2/3c^3)\ddot{v}$, and the equation of motion is given by

$$m\ddot{v} = F + \frac{2e^2}{3c^3}\dddot{v};\qquad(7)$$

F is the external force. A relativistic generalization of this equation is

$$mc\frac{du^i}{ds} = \frac{e}{c}F_{ext}^{ik}u_k + \frac{2e^2}{3c}\left(\frac{d^2u^i}{ds^2} + u^i\frac{du^k}{ds}\frac{du_k}{ds}\right),\qquad(8)$$

where the external force is assumed to be a Lorentz force (F_{ext}^{ik} is the tensor of the external electromagnetic field). Sometimes Eq. (8) is written in a different form, taking into account the fact that $u^l(du_l/ds) = 0$, and consequently

$$u^k\frac{d^2u_k}{ds^2} = -\frac{du^k}{ds}\frac{du_k}{ds}.$$

In three-dimensional notation, Eq. (8) takes the form

$$\begin{aligned}&\frac{d}{dt}\left(\frac{mv}{\sqrt{1+(v^2/c^2)}}\right) = e\left\{E_{ext} + \left[\frac{v}{c}H_{ext}\right]\right\} + f,\\&f = \frac{2e^2}{3c^3[1-(v^2/c^2)]}\left\{\ddot{v} + \dot{v}\frac{3(v\dot{v})}{c^2[1-(v^2/c^2)]} + \right.\\&\left.+\frac{v}{c^2[1-(v^2/c^2)]}\left((v\ddot{v}) + \frac{3(v\dot{v})^2}{c^2[1-(v^2/c^2)]}\right)\right\}.\end{aligned}\qquad(9)$$

For nonrelativistic uniformly-accelerated motion it is immediately obvious that the radiation force $f = (2e^2/3c^3)\ddot{v}$ is equal to zero. In hyperbolic motion, the radiation force is also equal to zero, for in this case

$$\frac{\dot{v}}{[1-(v^2/c^2)]^{3/2}} = w = \text{const}$$

and consequently,

$$\ddot{v} + \frac{3v^2v}{c^2[1-(v^2/c^2)]} = 0.$$

The last equation leads to the vanishing of the radiation force in (9). This can be readily verified on the basis of Eq. (8), by substituting the solution $v = wt/\sqrt{1 + (w^2t^2/c^2)}$ or $u^l = (\sqrt{1 + (w^2t^2/c^2)}, 0, 0, wt/c)$. The radiation force is equal to zero also in the general case of uniformly accelerated motion.

A discussion of the character and of the conditions of applicability of the equations of motion (8) and (9) also constitutes one of the "perpetual problems" mentioned in the introduction. We refer here both to the electromagnetic mass and to the fact that Eqs. (8) and (9) have inadmissible self-accelerating solutions[11,12]. It seems to us, however, that this is of no significance in the analysis of the motion and radiation of a uniformly accelerated charge. It suffices to say that in the nonrelativistic region one could consider an extended charge and obtain expression (7) accurate to terms that are arbitrarily small when the radius of the charged sphere is made sufficiently small; at the same time, the electromagnetic mass remains finite (of course, we are considering the non-quantum case throughout). In such an approach it is also clearly seen that Eq. (7) is not valid at the initial instant of time (we have in mind the solution of the initial-condition problem; see, for example,

ple[14], and also[5]), by virtue of which the self-accelerating solutions cannot appear. Finally, and this may be particularly convincing in this case, Eqs. (7)—(9) have a fully defined meaning in the case of uniformly accelerated motion, and neither difficulties nor misunderstandings arise in this case (it will be shown below, in particular, that Eqs. (7) and (8) lead to a result that agrees with the equivalence principle).

II. WHAT IS UNCLEAR IN THE QUESTION OF RADIATION AND THE MOTION OF A UNIFORMLY ACCELERATED CHARGE?

In the discussion of the question of the radiation and motion of a uniformly accelerated charge, there are some unclear aspects or apparent paradoxes of several types.

First, we refer to the field obtained in[1] in the case of hyperbolic motion (see also[2,4,6]). The corresponding solution for the field turns out to be valid not for all values of z and t, this being connected with the consideration of uniformly accelerated motion from $t = -\infty$ to $t = +\infty$. Attempts to "correct" the solution of[1] were not successful. For example, one article[6] ends with the statement: "We thus arrive at the conclusion that Maxwell's equations are incompatible with the existence of a single charge that is uniformly accelerated all the time." Such a conclusion may turn out to be perfectly correct, since in hyperbolic motion that is not limited in time the total radiative energy is infinite, and when $t \to \pm\infty$ the kinetic energy of the charge is also infinite (the velocity of the charge is equal to c). But a solution for unbounded motion need not be sought for any real physical formulation of the problem, where the particle moves with uniform acceleration only for a finite time interval. For example, if we are dealing with motion in a homogeneous and constant electric field, and specifically in a capacitor, then the charge moves in the capacitor in the time $t_1' < t' < t_2'$, and when $t' < t_1'$ or $t' > t_2'$ its velocity, for example, may be constant (we recall that such a motion in a capacitor is uniformly accelerated, specifically hyperbolic, only if the velocity of the charge is parallel to the field vector). If this circumstance is taken into account, then the possibility of finding a solution for the field in the form of retarded potentials is subject to no doubt. A second question discussed in the literature[4,6,7] is connected with the interpretation of the solution[1] in Pauli's book[2], where it is concluded that "hyperbolic motion is thus also unique in that it is not connected with formation of a wave zone and of the corresponding radiation." Such a conclusion will subsequently become perfectly natural, since in hyperbolic motion the radiation force vanishes (see above) "as it should be, since in this case there is no radiation at all"[2].

It is shown in[4] that the solution used in[1,2] for the field of a uniformly accelerated electron, which is suitable for all z and t, can nevertheless be used when $t > -z/c$ and leads to the same result for the radiated energy

$$P = \frac{d\mathscr{E}}{dt'} = \frac{2e^2}{3c^3}w^2,$$

which follows from the more general proof presented above. As to Pauli's conclusion concerning the absence of a wave zone, it pertains to another case, namely,

when the distance $R = c(t - t')$ increases at a fixed observation time t. Consequently when $R \to \infty$ we have for the time $t' \to -\infty$. But when $t' \to -\infty$ we have for a particle executing hyperbolic motion (see formulas (6)),

$$1 - \frac{v^2}{c^2} = \frac{1}{1 + [w^2 (t')^2/c^2]} \approx \frac{c^2}{w^2 (t')^2} \quad \text{и} \quad \dot{v}(t') \approx \frac{c^3}{w^2 (t')^3},$$

from which we get

$$\frac{c^2}{\dot{v}} \left(1 - \frac{\dot{v}^2}{c^3} \right) \approx ct'.$$

At the same time, as follows from (1) and as was already indicated, in the wave zone we have $R \gg c^2[1 - (z^2/c^2)]/\dot{v}$, and for hyperbolic motion this condition cannot be satisfied at a fixed $t = t' + (R/c)$ and $R \to \infty$. By the same token, this explains the arbitrariness of the concept of energy radiated by the charge—it is necessary to stipulate whether we are dealing with the time t or with the time t'.

For a motion that is uniformly accelerated in a finite time interval, the situation is nevertheless perfectly well defined. Given the observation time t and given the law of charge motion, we obtain R(t') and the radiation time t'. If t' was in the interval (t_1', t_2') when the charge moved with uniform acceleration, then it can be stated that at this instant t' the charge was not acted upon by the radiation force, and that at the same time the charge radiated—the flux of energy through a sphere of radius R(t') at the instant $t = t' + (R/c)$ is different from zero.

The third and fundamental question connected with radiation and motion of the uniformly accelerated charge is raised precisely by the fact that the presence of radiation in the absence of a radiation-deceleration force is paradoxical.

The fourth question concerns the application of the equivalence principle, on which the general theory of relativity is based. According to this principle, all the physical phenomena occurring in an inertial reference frame K_g, in which there is a uniform gravitational force with acceleration due to gravity g, are perfectly equivalent to those in a uniformly accelerated system K_a moving with an acceleration g relative to an inertial reference frame without a gravitational field. In the presence of a uniform gravitational field, a charge which is not secured is uniformly accelerated relative to the inertial system, and will radiate in accordance with the foregoing. On the other hand, in an accelerated reference frame K_a, the charge apparently should not radiate, since it is not accelerated relative to the inertial system. The systems K_g and K_a are thus non-equivalent, i.e., the equivalence principle is violated. Actually, however, the charge in the system K_a radiates exactly in the same manner as the system K_g, i.e., the equivalence principle holds without qualifications.

In the last two sections we shall stop to explain the paradoxes we have just formulated.

III. CALCULATION OF THE RADIATED ENERGY BY DIFFERENT METHODS AND ON THE ENERGY CONSERVATION LAW IN ELECTRODYNAMICS

To determine the energy radiated by a charge, or the radiation intensity observed at a specified surface, the procedure is to calculate the Poynting vector $\mathbf{S} = c\mathbf{E} \times \mathbf{H}/4\pi$ far from the charge, and to find the energy lost by the charge one determines the flux of this vector through a closed surface. This is precisely how the standard formulas (4) and (5) are derived. However, the problem is not limited to the use of these formulas, particularly because they are valid only in vacuum. If the charge moves in a medium, then the results obtained in general are quite different. It suffices to state that even a uniformly moving charge can radiate in a medium—this is precisely what is observed in Cerenkov radiation or in transition radiation. The calculation of the Poynting vector and its flux through a surface remains, of course, the method of determining the radiated energy also when the charge moves in a medium (more accurately, we have in mind a medium without spatial dispersion, for in the presence of such a dispersion the energy flux density is not given by the Poynting vector). However, the energy lost by the charge or the radiated energy can be calculated also by two other methods; by determining the time derivative of the field energy

$$\frac{d}{dt} \int \frac{\mathbf{ED} + H^2}{8\pi}\, dv$$

or by determining the work $e\mathbf{V} \cdot \mathbf{E}' = \mathbf{v} \cdot \mathbf{f}$ performed by the charge against the field produced by itself (in other words, one calculates the work of the radiation friction force f, which in the presence of a medium is of course no longer determined by expressions (7)—(9)). For a frequently encountered case (which will be identified later), all three indicated methods lead to the same result; as one of the many examples, we point to the calculation of the Cerenkov-radiation energy*. Actually, however, the total energy flux is in general not equal to the change of the field energy by the work of the radiation force. Disregard of this circumstance has led, for example, to an inaccuracy in the theory of synchrotron radiation for helical (non-circular) particle motion[8] (see also[9], where references is made to certain articles published on the same topic in 1968).

The paradox arising in connection with the radiation from a uniformly accelerated charge is also connected with the incorrect identification of the energy flux with the work of the radiation force.

The electromagnetic field equations yield, by the well known method, the following relation (the Poynting theorem)

$$\frac{\partial}{\partial t} \left(\frac{E^2 + H^2}{8\pi} \right) = -\mathbf{j}\mathbf{E} - \operatorname{div} \mathbf{S}, \quad \mathbf{S} = \frac{c}{4\pi} [\mathbf{EH}]. \qquad (10)$$

Here and below we confine ourselves to the case of vacuum and consider the motion of one point charge, when $\mathbf{j} = e\mathbf{v}\delta(\mathbf{r} - \mathbf{r}_e(t))$. After integrating over a certain volume V bounded by the surface σ, we get

$$\frac{dW_{\text{e-m}}}{dt} = -e\mathbf{v}\mathbf{E} - \oint S_n\, d\sigma, \quad W_{\text{e-m}} = \int \frac{E^2 + H^2}{8\pi}\, dV. \qquad (11)$$

On the other hand, from the equation of motion (9) we get

$$\frac{dW_k}{dt} = e\mathbf{v}\mathbf{E}_{\text{ext}} + \mathbf{v}\mathbf{f}, \quad W_k = \frac{mc^2}{\sqrt{1 - (v^2/c^2)}}. \qquad (12)$$

In (11), we have by definition the total field $\mathbf{E} = \mathbf{E}_{\text{ext}} + \mathbf{E}'$, where $\mathbf{E}'$ is the field of the charge itself; at the point where this charge is located we have $e\mathbf{E}' = \mathbf{f}$, and

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*In their original paper, I. E. Tamm and I. M. Frank [15] calculated the energy flux, the change of the field energy per unit time was determined in [16] and the radiation-friction force corresponding to Cerenkov radiation was calculated, for example, in [17].

consequently in (11) $e\mathbf{v}\cdot\mathbf{E} = e\mathbf{v}\cdot\mathbf{E}_{\text{ext}} + \mathbf{v}\cdot\mathbf{f}$. Therefore, as expected, (11) and (12) lead to the conservation law

$$\frac{d(W_{\text{e-m}} + W_k)}{dt} = -\oint S_n \, d\sigma. \tag{13}$$

The field energy $W_{\text{e-m}}$ includes the energy of the external field $\mathbf{E}_{\text{ext}}$ and $\mathbf{H}_{\text{ext}}$, for example the energy of the field in the capacitor through which the charge under consideration moves and is accelerated. Therefore, only to simplify the problem, we assume that the charge is accelerated by some external field of non-electromagnetic nature (the influence of this field was not taken into account in (8) and (9); the same pertains to (12) and (13)).

Then the conservation law (11) assumes the form

$$\frac{dW_{\text{e-m}}}{dt} = -\mathbf{v}\mathbf{f} - \oint S_n \, d\sigma, \tag{14}$$

where $W_{\text{e-m}}$ is the energy of the field of the charge (all the remaining fields, as already mentioned, are assumed to be missing); we emphasize that in (10)—(14) the arguments which have not been written out involve only one time—the observation time t.

Equation (14), which has a perfectly clear meaning, shows that the work of the radiation force $\mathbf{v}\cdot\mathbf{f}$, the change of the field energy $dW_{\text{e-m}}/dt$, and the total energy flux of the field $\oint S_n d\sigma$ are all connected by a single relation, and in the general case are far from equal to each other in absolute magnitude. On the other hand, if we consider stationary motion, then $dW_{\text{e-m}}/dt = 0$ and $-\mathbf{v}\cdot\mathbf{f} = \oint S_n d\sigma$. We can further calculate the energy $W_{\text{e-m}}$ in all of space, by moving the surface σ to infinity, by virtue of which $\oint S_n d\sigma = 0$. Then $dW_{\text{e-m}}/dt = -\mathbf{v}\cdot\mathbf{f}$. The foregoing explains why, say, the energy lost by the particle $\mathbf{v}\cdot\mathbf{f}$ in the stationary regime can be determined by calculating $\oint S_n d\sigma$ or $dW_{\text{e-m}}/dt$.

Stationary radiation in the exact meaning of this word is not easy to realize (an example of a stationary process is Cerenkov radiation), and usually one deals with a periodic process, wherein $W_{\text{e-m}}(t_1)$ = $W_{\text{e-m}}(t_1 + T)$. This is precisely the situation in the case, for example, of an immobile oscillator or synchrotron radiation of a charge moving on a circular orbit (it is important here that the radiating particle returns to the same point after a period T). For a periodic process

$$\int_{t_1}^{t_1+T} \mathbf{v}(t)\,\mathbf{f}(t)\,dt = -\int_{t_1}^{t_1+T} \oint S_n(t)\,d\sigma\,dt. \tag{15}$$

Obviously, the fact that the observation time t does not coincide with the radiation time t' is immaterial here, since the choice of the instant t_1 does not play any role in a periodic process. On the other hand, if there is motion wherein the field energy $W_{\text{e-m}}(t < t_1)$ = $W_{\text{e-m}}(t > t_2) = W_{\text{e-m}}^{(0)}$, then relation (15) is again valid, except that $t_1 + T$ is replaced by any time $t > t_2$. This is precisely the situation, or almost so, in the case of radiation from a charge "reflected" from an electric field in a capacitor (it is assumed that when $t < t_1' \le t_1$ and $t > t_2' \ge t_2$ the charge velocity is constant). It must only be borne in mind that the energy $W_{\text{e-m}}(t)$ depends on the volume V bounded by the surface σ (thus, the time t_1 can be assumed to be the time

t_2' when the charge enters the capacitor, for the radiation field must have time to leave the volume V).

The radiation-friction force that enters in the non-relativistic equation (7) satisfies in obvious fashion the foregoing conclusions. Indeed,

$$\ddot{\mathbf{v}}\mathbf{v} = \frac{d}{dt}(\dot{\mathbf{v}}\mathbf{v}) - \dot{\mathbf{v}}^2,$$

and under conditions when $[\mathbf{v}\times\dot{\mathbf{v}}]_{t_1}^{t_1+T} = 0$ and relation (15) is valid we have

$$-\int \mathbf{v}\mathbf{f}\,dt = -\frac{2e^2}{3c^3}\int \ddot{\mathbf{v}}\mathbf{v}\,dt = \frac{2e^2}{3c^3}\int \dot{\mathbf{v}}^2\,dt = \int P\,dt = \int\oint S_n\,d\sigma\,dt. \tag{16}$$

We have taken into account here also the nonrelativistic formula (5), in which

$$P = \frac{d\mathcal{E}}{dt} = \oint S_n\,d\sigma,$$

and either $\mathbf{v} \to 0$ or the integral is calculated in the wave zone.

In the relativistic case, we can write for the time component of (8), after elementary substitutions,

$$\frac{d}{dt'}\left(\frac{mc^2}{\sqrt{1-(v^2/c^2)}}\right) = e\mathbf{v}\mathbf{E}_{\text{ext}} + \frac{2e^2}{3}\left(\frac{dw^0}{dt'} + cw^i w_i\right). \tag{17}$$

With allowance for (4), Eq. (17) takes the form

$$\frac{d}{dt'}\left(\frac{mc^2}{\sqrt{1-(v^2/c^2)}}\right) = e\mathbf{v}\mathbf{E}_{\text{ext}} + \mathbf{v}\mathbf{f} = e\mathbf{v}\mathbf{E}_{\text{ext}} + \frac{2e^2}{3}\frac{dw^0}{dt'} - P,$$
$$w^0 = \frac{\dot{\mathbf{v}}\mathbf{v}}{c^3[1-(v^2/c^2)]^2}, \quad P = \frac{d\mathcal{E}}{dt'} = -\frac{2e^2c}{3}w^i w_i. \tag{18}$$

In (17) and (18), the time is denoted by t'; this time characterizes the motion of the charge and represents the radiation time when radiation is considered. Yet in (14) and in the initial equations (2) and (11) the same time t is used for the charges and for the field. In this connection, even in the calculation of the fields in the wave zone, the radiated energy $P = d\mathcal{E}/dt'$ differs from

$$\frac{d\mathcal{E}}{dt} = \oint S_n(t)\,d\sigma.$$

A charge that enters a capacitor parallel to the decelerating field radiates electromagnetic waves during the entire time t' that it remains in the field (as indicated, $t_1' \le t' \le t_2'$), with

$$P = \frac{d\mathcal{E}}{dt'} = \frac{2e^2}{3c^3}\dot{\mathbf{v}}^2 = \frac{4e^4 E_{\text{ext}}^2}{3m^2c^3} = \text{const.}$$

This means that at a sufficiently large distance $R(t')$ from the charge there will be observed at the instant $t = [t' + R(t')]/c$ a radiation field with corresponding value of the energy flux. The radiation force does not act on the charge when $t' < t_1'$ and $t' > t_2'$. The charge then moves in accordance with the law

$$\frac{d}{dt'}\frac{m\mathbf{v}}{\sqrt{1-v^2/c^2}} = \mathbf{F}_{\text{ext}} = e\mathbf{E}_{\text{ext}}.$$

At the instants t_1' and t_2', the charge is acted upon by a friction force, and the work of this force during the entire time of accelerated motion is

$$\int_{t_1'}^{t_2'} \mathbf{v}\mathbf{f}\,dt' = -\int_{t_1'}^{t_2'} P\,dt' = -\frac{2e^2}{3c^3}w^2(t_2' - t_1'),$$

i.e., it is exactly equal to the radiated energy.

The vanishing of the radiation force during the course of accelerated motion of the charge is in no way

paradoxical, in spite of the presence of radiation. Indeed, a nonzero total energy flux through a surface surrounding a charge at a zero radiation force is exactly equal to the decrease of the field energy in the volume enclosed by this surface. In the general case, on the other hand, all three quantities, dW_{e-m}/dt, $v \cdot f$, and $\oint S_n d\sigma$ differ from zero (see relation (14)). There are all the more no grounds for expecting the work of the radiation force $v \cdot f$ to equal the energy flux

$$\frac{d\mathscr{E}}{dt} = \oint S_n \, d\sigma$$

or the flux $d\mathscr{E}/dt' = P$, since the force is applied to the charge, and the flux is calculated through a sphere of radius $R(t')$. In full accord with the spirit of field theory, the energy flux through a surface is determined directly by the field near this surface, and not by the field on the trajectory of a charge situated inside the surface*.

All these explanations may seem to be too detailed, as indeed they do seem to the author himself. But this was done because in a detailed article[4], devoted exclusively to the radiation of a uniformly accelerated charge, no use is made of the conservation law (14) at all. Instead, as in many preceding papers, the concept of the acceleration energy is introduced

$$Q = \frac{2e^2 w^0}{3} = \frac{2e^2}{3c^3} \frac{v\dot{v}}{[1-(v^2/c^2)]^2}.$$

As is clear from (18)

$$vf = \frac{dQ}{dt'} - P$$

and Eqs. (12), (17), and (18) are written in the form

$$\frac{dW_k}{dt'} - \left(\frac{dQ}{dt'} - P\right) = evE_{ext}, \quad W_k = \frac{mc^2}{\sqrt{1-(v^2/c^2)}}, \quad Q = \frac{2e^2 v\dot{v}}{3c^3[1-(v^2/c^2)]^2}. \quad (19)$$

In[4], the quantity Q is first interpreted as part of the "internal energy of the charge particle," and then, in the same article[4], and also in[18], Q is assumed to be a part of the energy of the field directly surrounding the particle, but making no contribution to its electromagnetic mass. From this point of view, at zero radiation force, it can be assumed that the radiated energy P is drawn from the "acceleration energy" Q or the "internal energy" ($W_k - Q$). On the other hand, if Q is assumed to be part of the field energy, then the radiation energy P is drawn from the field energy. Formally, the latter is perfectly true, since $P = d\mathscr{E}/dt'$ is the field-energy flux through a certain surface enclosing the charge, referred to a unit time t'.

It seems to us, however, that the introduction of some "acceleration energy" or "internal energy" of the charge not only fails to add anything to the under-

standing of the energy balance, but also confuses the question. The charge has only an energy $W_k = mc^2/\sqrt{1 - (v^2/c^2)}$; the subdivision of the radiation force f or of the work of this force $v \cdot f$ into two parts or into any other number of parts, of course, is not unique and therefore can no longer have a special meaning. More accurately speaking, if such a meaning is assigned, then this is possible only in connection with identification of part of work $v \cdot f$ with the expression obtained for P from independent considerations.

Thus, in discussing the question of the energy balance of the accelerated motion and of the radiation of a charge, we see neither reason nor necessity for going beyond the scope of the conservation laws (12) and (14). Of course, writing down the work $v \cdot f$ in the form of a sum of two terms (see (18)) is also convenient and natural, but there is no need to impart any new meaning to these terms.

IV. THE EQUIVALENCE PRINCIPLE AS APPLIED TO UNIFORMLY ACCELERATED MOTION

By virtue of the equality of the inertial and heavy masses, all the neutral bodies fall in a gravitational field with equal acceleration, and consequently, within the framework of classical mechanics, it is impossible to distinguish between motion in a uniform gravitational field with a gravitational acceleration g and motion relative to a reference frame K_a, which has an acceleration $-g$ relative to the inertial system K without a gravitational field*. A generalization of this statement to all physical processes is indeed the content of the equivalence principle (see, for example,[2]). If an inertial reference frame with a gravitational field (acceleration g) is designated as the system K_g, then, in accordance with the equivalence principle, the systems K_g and K_a are on par, i.e., at identical initial and boundary conditions, all the physical processes should occur in them in perfectly identical manner. As already stated, the validity of the equivalence principle in mechanics is guaranteed by the equality of the inertial and heavy masses.

We shall now assume that the moving particle is charged, confining ourselves for simplicity to the non-relativistic case, when Eq. (7) is valid and takes in the system K_g the form

$$\dot{v} = g + \frac{2e^2}{3m_3}\ddot{v}. \quad (20)$$

If, as assumed, g = const, then Eq. (20) has a solution $\dot{v}$ = g = const, and we see that the equivalence principle is actually valid for charged particles. This principle would already be incorrect, however, if attempts were made to extend it to homogeneous but time-varying

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*If one speaks not of the radiation force f itself, but of its work per unit time $v \cdot f$, then the difference between the aforementioned quantities can be said to appear in a rather trivial manner. Indeed, regardless of the value of f, its work per unit time $v \cdot f$ is equal to zero when $v = 0$, i.e., for a charge at rest at the given instant t'. On the other hand, the values of the flux and, say, the quantity P, are determined primarily by the acceleration of the charge at the same instant t', and do not vanish when $v = 0$.

*Our reasoning, obviously, is pursued on the classical (pre-Einstein) level and the gravitational field is understood in the Newtonian sense, by virtue of which the gravitational field is assumed to be independent of the choice of the reference frame. For example, one can choose as the system K_g (see below) a system connected to the earth (in this case, however, neither rotation of the earth nor the inhomogeneity of its gravitational field should have any effect; both these conditions can be satisfied in practice by considering a sufficiently small region of space near the earth during a sufficiently short time interval).

gravitational fields*. In fact, for uncharged particles the field g(t) can be "replaced" by choosing an accelerated frame K_a with acceleration $-g(t)$ relative to the system K. But the motion of the charged particle in the system K_g is described by Eq. (20), and, in particular, depends on the particle mass m. On the other hand, in the presence of a gravitational field the charged particle, like any other particle, moves with constant velocity relative to the inertial system K, and with an acceleration g(t) that does not depend on the mass m relative to the system K_a.

For the purpose of constructing a general relativity theory, and conversely as a consequence of the general relativity theory, it is necessary and sufficient to satisfy the equivalence principle "in the small," i.e., locally, in a sufficiently small space-time region, where the gravitational field can be regarded as homogeneous and constant. We conclude therefore that Eq. (20) is in full accord with the equivalence principle. It is curious, incidentally, that even on the basis of this principle it can be stated that the radiation force should vanish in the case of uniformly accelerated motion.

There remains, however, the question of radiation from a uniformly accelerated charge. It might seem that by detecting the presence of radiation one could distinguish between the systems K_g and K_a, since a charge placed in the first of these systems is accelerated and radiates, while a charge in the second system is not accelerated relative to the inertial reference frame K, and consequently it might seem that it should not radiate. This paradox was discussed in[4,5,20], where two arguments are presented. First, if the radiation is recorded in the usual manner in the wave zone, then the measurement does not have a local character and this can be referred to as meaning that in the wave zone we go beyond the region of homogeneity of the gravitational field. Such a conclusion[4,20] is based on rather deep reasoning (see below), although at first glance it does not seem to be sufficiently convincing and is refuted in[5]. In this connection, we must note immediately that, as already indicated, the presence of

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*In connection with the foregoing, we consider the following remark made in [11] (p. 291) to be incorrect: "A somewhat more general case is a uniformly accelerated reference frame—it obviously is equivalent to a homogeneous but variable gravitational field." Incidentally, this statement is a natural consequence of the fact that in [11], as in several other books, the equivalence principle is formulated essentially as purely mechanical (i.e., as tantamount to the equality of the inertial and heavy masses, or, which is the same, to the statement that acceleration in a gravitational field is independent of the mass of the body). It seems to us that the equivalence principle must be formulated (as was done by Einstein) in a more general manner—as applied to all physical phenomena. To show how large this difference can be, we recall the situation with the relativity principle. In classical mechanics this principle (the equivalence of all inertial reference frames) is valid if the Galilean transformations are used. But the same relativity principle, when extended to optics, leads already to the Lorentz transformations. Thus, from the logical point of view, the transition from the equality of heavy and inertial masses to the equivalence principle is similar to a generalization of the relativity principle of classical mechanics to include all of physics. Here, to be sure, there is a certain difference, which is already emphasized in [19], but it still does not change the gist of the matter and there is no need to dwell on it further.

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radiation in the sense that

$$P = d\mathcal{E}/dt' \neq 0,$$

can be established at any distance from the radiating charge, and therefore it is insufficient to refer to the fact that the wave zone is remote.

The second argument is as follows. The total radiated energy

$$P = -\frac{2e^2c}{3} w^i w_i$$

(see (4)) is Lorentz-covariant, i.e., it is the same in any inertial reference system K. However, on going over to non-inertial systems, particularly to a uniformly accelerated system K_a, the quantity P is no longer conserved. In the discussed case, P vanishes in the inertial system K, but should not vanish and does not vanish in the system K_a.*

By the same token, the paradox is immediately resolved qualitatively. To satisfy the equivalence principle it is necessary, further, that the energy radiated in the system K_a be equal to $P = (2e^2/3c^3)g^2$, for this is precisely its value in the system K_g (this system is inertial, and the charge in it has an acceleration g; consequently, the energy P can be calculated here by means of formula (5)). Sometimes one considers also a charged particle "lying on a table" in the presence of a gravitational force[20], i.e., a particle which is at rest in the system K_g because some force balances the force of gravity. Obviously, such a particle is always stationary in the system K_g and does not radiate.

We can finally consider the situation in one more reference frame, namely in the system K_{ga}, which falls freely in the K_g system (see also the table). In the system K_{ga}, obviously, the charge is immobile at all times and does not radiate, although in the inertial system K_g with a gravitational field an energy $P = (2e^2/3c^3)/g^2$ is radiated in a unit time. In this respect, and indeed in all others, the system K_{ga} is equivalent to the inertial system K and is called local-inertial.

The proof of the fact that there is no radiation in the system K_{ga} is given in[5]. This proof, to be sure, is not particularly lucid, but we do not consider it necessary to present the corresponding calculation in greater detail. The point is, first, that the absence of radiation in the K_{ga} system, which moves together with the charge (it moves not only with the same instantaneous velocity,

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*Let us assume that in the given system K a non-uniformly moving charge is at rest at the instant t = 0 and its magnetic field at the same instant of time is equal to zero everywhere (this is precisely the situation in the case of hyperbolic motion of a charge and when the solution of [1] is used for its field; see [2,4]). Then, obviously, at the instant t = 0 the Poynting vector $S = cE \times H/4\pi$ is also equal to zero everywhere (at all points) and there is no radiation. But at some other instant t it is always possible to find an inertial system in which the charge is at rest; from this it might seem that there is no radiation at any instant of time. As correctly noted in [21], such reasoning is perfectly analogous to the well known reasoning of Zenon: since a flying arrow is situated at any instant in only one place, it is immobile. In actual fact there is no such inertial (non-accelerated) reference frame in which the charge under consideration is always at rest and the energy flux is always equal to zero.

Reference frames

System	Acceleration of charge	Radiated energy	Character of the system.
K_g	g	$\dfrac{2e^2}{3c^3}g^2$	Inertial system in which there is a homogeneous and constant gravitational field with acceleration g.
K_a	g	$\dfrac{2e^2}{3c^3}g^2$	Noninertial reference system moving relative to the inertial system K with constant acceleration − g. There is no gravitational field in this system.
K_{ga}	0	0	The system K_{ga} falls freely in the system K_g, i.e., it has an acceleration g relative to the system K_g. The system K_{ga} is called local-inertial (from the point of view of general relativity theory, the system K_{ga} is equivalent to the system K i.e., to the inertial reference system without a gravitational field).
K	0	0	Inertial system without a gravitational field. The system K_a moves relative to this system K with acceleration − g.
K_g	0 (the charge is kept up by an external force)	0	We are dealing with a charge "lying on a table" in the system K_g, i.e., a charge that is immobile in this system as the result of compensation of the gravitation force by some other force.

In all the systems, the charge is immobile at the considered instant of time (it has a velocity of v = 0). We emphasize that the systems K_g and K_{ga} on the one hand, and the systems K_a and K, on the other, correspond to different physical situations. Thus, in the systems K_g and K_{ga} there is a gravitational field (in the classical sense), for example, the system K_g may be connected with the earth while the system K_{ga} is connected with a rocket that falls freely on the earth (the inhomogeneity of the gravitational field and the rotation of the earth are neglected here). In the systems K_a and K there is no gravitational field; for example, they may be located far from all stars, somewhere in interstellar or intergalactic space.

but also with the same acceleration), is very natural even from energy considerations (the charge is at rest at all times, and it has "nothing to radiate"). Second, the dependence of the radiated energy P on the acceleration of the reference frame is in full agreement with the fact that the quantity P and the radiation field itself (see (1)) are determined by the acceleration of the charge relative to the reference frame under consideration. Third, finally, the conclusion that there is no radiation in the system K_{ga} and that it exists in the system K_a follows directly from the equivalence principle. No calculation based on field theory can contradict this principle, for within the framework of the general theory of relativity one can solve, in principle, any electrodynamic problem; at the same time, the equivalence principle is contained automatically in the general relativity theory, where it reduces to the statement that in an infinitesimally small space-time region it is possible to replace the Riemannian space-time by a tangent pseudo-Euclidean space-time.

Following Einstein, the present author believes, like many others, that the equivalence principle is the true spirit of the general theory of relativity and must inevitably remain at present the foundation for the exposition of this theory to students, and in general for gaining familiarity with its principles (for details see[22]). But when an entirely different question is discussed (in this case, the question of the radiation of a uniformly accelerated electron), it is perfectly legitimate not to prove anew the validity of the equivalence principle, but to make use of this principle as a conse-

quence of the general relativity theory. Such an approach makes it possible to determine directly the energy radiated per unit time in the system K_a or K_{ga}. Incidentally, as already mentioned, this energy can be calculated also independently, and then we reach the conclusion that the equivalence principle is valid also for radiation of a uniformly accelerated charge.

The foregoing, however, does not settle the question and can give rise to new misunderstandings. After all, we have reached the conclusion that by choosing the reference frame it is possible to change the radiated energy P = d$\mathscr{E}$/dt′, and consequently to produce radiation in some manner. But such a conclusion would contradict the classical concept of electromagnetic waves or the quantum picture of radiation as an aggregate of photons. If, say, a photon exists in some reference frame; then it should exist also in any other reference frame. It is possible that this statement is even more strongly pronounced for particles with nonzero rest mass such as electrons, mesons, or protons. All these particles, from the quantum point of view, are "quanta" of corresponding wave fields—the electron-positron, meson, or nucleon field. It is clear that neither a uniform gravitational field* nor a changeover to an accelerated reference frame can generate new "free"

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*Furthermore, within the framework of the Newtonian theory of gravitation, which is the only one referred to in the discussion of the equivalence principle, the gravitational field should be weak (this means that the encountered gravitational-potential differences $|\varphi_2 - \varphi_1|$ should be small compared with the square of the speed of light c^2).

particles, particularly photons.

Such a conclusion is indeed valid, but it does not contradict the foregoing conclusions. The point is that the criterion employed for the presence of radiation, namely P = d$\mathscr{E}$/dt' $\neq$ 0 (see (4)), is by far not tantamount to the presence of free electromagnetic radiation or photons propagating with the velocity c. It is sufficient to state that for a charge at rest at a given instant t' the quantity P, can be determined by measurements performed as close as desired to this charge. In the general case P is only a suitably defined energy flux of the transverse field produced by the charge. On the other hand, a transverse electromagnetic field, generally speaking, does not reduce at all to a field of electromagnetic waves (a radiation field). The field of the charge is a radiation field only asymptotically, in the wave zone, and formally only when R(t') $\rightarrow \infty$.

Strange as it may seem, it has been almost standard procedure to forget this generally known circumstance in the exposition of the quantum theory of radiation*. We present therefore a trivial example to explain the foregoing, namely we consider the field of a uniformly moving charge. Such a field is described by the first term of (1) and obviously contains a transverse part (it suffices to state that the field H is always transverse, i.e., it satisfies the equation div H = 0). This transverse field moves with the velocity v of the charge and cannot be reduced in any way to a radiation field propagating with velocity c. The transverse part of the field

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*The radiation field, defined as an aggregate of electromagnetic waves, satisfies the free-field equations

$$\text{rot } H = \frac{1}{c}\frac{dE}{dt}, \quad \text{rot } E = -\frac{1}{c}\frac{dH}{dt}, \quad \text{div } H = 0, \quad \text{div } E = 0.$$

On the other hand, for an arbitrary transverse field, the first and last of these equations are

$$\text{rot } H = \frac{4\pi}{c}j + \frac{1}{c}\frac{dE}{dt}, \quad \text{div } E = 4\pi\rho$$

(the remaining equations are the same). Only a free electromagnetic field, or in practice only the field of charges in the wave zone, can be regarded quantum-theoretically as an aggregate of photons, namely particles with energy $\hbar\omega$ and momentum $\hbar k$, with ω = ck. On the other hand, the relation ω= ck no longer holds for an arbitrary transverse field expanded in terms of plane waves proportional to exp [− i(ωt − k · r)]; quantization of the field, of course, does not change anything, and for the corresponding "field quanta" we also have $\omega \neq$ ck. Yet in the text books on radiation theory known to us (including the latest of them [23]), an aggregate of photons is defined either explicitly or implicitly as a transverse field in the presence of sources. Such an approach and such terminology usually do not lead to any difficulties only because of the character of the problems and of the methods encountered in quantum theory of radiation (see, however, [24]). But even with the field of a uniformly moving electron as an example, it becomes quite clear [25] that it is necessary to distinguish between the radiation field (photon field) and the transverse field dragged by the charge (the latter, if convenient, can be regarded as an aggregate of virtual photons with $\omega \neq$ ck, and these photons are independent of one another). The case discussed in the present article, that of a field of a uniformly accelerated charge, can serve as one more illustration of the foregoing. At the same time, it must be emphasized, that the actual formulation of quantum electrodynamics is free of difficulties in this respect, and is not at all connected with identification of the quantized transverse field with the field of real photons. We find it only striking that this essentially trivial circumstance (see, for example, [25]) is usually not explained in the exposition of quantum electrodynamics.

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of a uniformly accelerated charge, at any finite distance from the charge, is likewise not the field of free radiation. In the case of unbounded hyperbolic motion, a manifestation of this fact is the already noted absence of a wave zone for any fixed observation time t.

Let us consider specifically a nonrelativistic uniformly accelerated motion in a gravitational field (with acceleration $\dot{v}$ = g). The wave zone is then at distances R $\gg$ c²/g (see Sec. I). If the gravitational field were to remain homogeneous also in the wave zone, then the potential difference in this field would be |$\varphi_1 - \varphi_1$| = gR $\gg$ c². Yet for a weak field, to which we should confine ourselves, we have |$\varphi_2 - \varphi_1$| $\ll$ c² (see also[26], page 305). Therefore, if we take the statement that radiation is present to mean the possibility of measuring the field in the wave zone, where this field is equivalent to some degree to the free radiation field, then a charge moving in a strictly homogeneous field cannot be regarded as radiating[20]. The same pertains to the uniformly accelerated reference frame K$_a$, since such a frame can be realized only within the limits in which the condition gR $\ll$ c² is satisfied (for details on the limitations imposed on realizable reference systems see[11], Sec. 84). By the same token, neither a homogeneous gravitational field nor a uniformly accelerated reference frame can actually "generate" free particles, especially photons.

On the other hand, when we spoke above of the presence of radiation in the systems K$_g$ and K$_a$, we had in mind, as already indicated, the appearance of a field described by the second (wave) term in (1), and by the ensuing existence of a flux P = (2e²/3c³)g² $\neq$ 0. The possibility of creating or annihilating the field of a uniformly accelerated charge (at limited distances from the charge) by choosing a corresponding accelerated reference frame is analogous to the possibility of creating or annihilating the transverse field of a uniformly moving charge by changing over to some inertial (Lorentzian) reference frame; in this case, in the reference frame co-moving with the charge, the transverse field is equal to zero, and in other systems it differs from zero.

It does not follow at all from the foregoing, of course, that a charge moving in a gravitation field does not produce a "true" radiation field, which can be observed in the wave zone. The point is only that to this end, as in any other real physical formulation of a problem, it is necessary to investigate the radiation for time-limited uniformly accelerated motion. Under such conditions, inasmuch as the acceleration of the charge is not always constant, the gravitational field should not be constant in time (or should be inhomogeneous in space). On the other hand, if the particle moves in a non-constant or inhomogeneous gravitational field, then, as we have seen, this field is no longer equivalent to an accelerated reference frame*.

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*An exhaustive analysis of the question of the motion in radiation of of a charged particle in a gravitation field it is possible only on the basis of general relativity theory. As applied to nonrelativistic motion of a charge in a weak static gravitational field, such an investigation was carried out in [27] and is quite instructive. We confine ourselves here only to the remarks that the result of [27] agrees fully with the statement that there is no radiation friction force when a charge moves in a homogeneous gravitational field.

Thus, in the question of the field (as well as radiation) of a charge, as in the question of its motion, the requirements connected with the equivalence principle are fully satisfied.

It is possible that the last remarks should be discussed in greater detail for methodological purposes, but this is hardly the place for it, for it would then be necessary to touch upon entirely different problems, too.

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Summarizing all the foregoing, we are able to conclude that on the sixtieth anniversary of Born's work[1], where radiation and the radiation force of a uniformly accelerated moving charge was first considered, this problem is already clear enough not to be regarded as a "perpetual problem" of classical physics.

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