

Meetings and Conferences*SCIENTIFIC SESSION OF THE DIVISION OF GENERAL PHYSICS AND ASTRONOMY, USSR  
ACADEMY OF SCIENCES (24-26 December, 1969)*

Usp. Fiz. Nauk 101, 71-83 (May, 1970)

A session of the Division of General Physics and Astronomy was held on December 24, 25, and 26 at the conference hall of the P. N. Lebedev Physics Institute. The following papers were delivered:

1. V. A. Belinskiĭ, E. M. Lifshitz, and I. M. Khalatnikov, Relativistic Cosmology with a Singular Point Oscillating in Time.
2. L. M. Ozernoi, Present Status of the Problem of Origin of Galaxies.
3. A. I. Larkin, Fluctuations in Superconductors.
4. V. V. Shmidt, Critical Currents in Superconductors.
5. V. L. Tal'roze, Chemical Lasers.
6. V. V. Fadeev, Ultraviolet Lasers Using Organic Scintillators.
7. A. P. Sukhorukov, Thermal Self-action of Intense Light Waves.

We publish below brief contents of some of the papers.

L. M. Ozernoi. Present States of the Problem of the Origin of Galaxies.

1. Gist of the problem. The observed universe is statistically homogeneous and isotropic only with scales  $\gtrsim$  Mps ( $3 \times 10^{26}$  cm). On a smaller scale, the presence of inhomogeneities makes it essentially different from the idealized Friedmann model. The degree of inhomogeneity  $\delta\rho/\rho = (\rho - \bar{\rho})/\bar{\rho}$  increases with decreasing scale. In galaxies, the average density differs by a factor  $10^6 - 3 \times 10^6$  from the density of their matter "smeared out" in space ( $\sim 3 \times 10^{-31}$  g/cm<sup>3</sup>). This means that when the characteristic dimensions of the expanding universe were smaller than the present-day ones by a factor  $\sim 100$ , the galaxies did not have their individuality, being "dissolved" in a homogeneous background (with the possible exceptions of their cores). In the more remote past, the degree of inhomogeneity should be even smaller. However, it cannot become vanishingly small. Some initial "nuclei" of inhomogeneities are essential if we are to obtain the presently observed picture of the galaxies and their clusters.

Therefore, the theory of galaxy production should clarify the following:

- 1) the physical nature of the initial perturbations;
- 2) the mechanism of their changeover into galaxies, with indication of (a) the main parameters of the galaxies (mass, angular momentum, dispersion of random velocities, large-scale magnetic field) and (b) the main parameters of galactic clusters;
- 3) the origin of the initial perturbations.

The question of the occurrence of galaxies has grown into an important physical problem, especially in connection with item (3). Following Lifshitz<sup>[1]</sup>, it

was proved in a number of papers that the aforementioned "nuclei" of the inhomogeneity should be accompanied with finite perturbations of the metric (which do not vanish as  $t \rightarrow 0$ ). It is not very likely that these perturbations are the result of elementary "thermal noise" (thermodynamic fluctuations). On the other hand, if the perturbations constitute a definite initial structure with a nonthermal spectrum, then its nature and origin are inseparable from the physical properties of the universe, concerning which, in essence, almost nothing is known. By the same token a solution of the problem of the origin of galaxies, albeit in general outline, will contain also fundamental cosmological information.

In connection with the latter, it is of interest to investigate as broad a class as possible of spatial perturbations capable of leading to the modern structure of the universe. Apparently, all reduce to the following types of perturbations: (1) of the total density (or pressure) and corresponding potential velocities; (2) of the composition, including (a) the entropy, (b) the baryon charge, and (c) the electric charge, currents, and corresponding magnetic field; (3) of the vortical velocity.

Although these perturbations could exist for a long time (both "initially" and as a result of generation of some of them by the others), they could hardly play an equal role in the formation of the observed structure. The difference in the existing concepts concerning the formation of the galaxies are based on a priori preferences for one of the initial perturbation or another. Such a situation is possible, of course, only to the extent that one ignores various theoretical requirements that follow from the extensive factual material. The purpose of the present paper is therefore to outline schematically various approaches to the theory of galaxy production, indicate the difficulties they encounter in attempts to interpret the observed phenomena.

2. Development of adiabatic and entropy perturbations. The evolution of adiabatic perturbation (the quantum density  $\rho_{\mathbf{r}}$  and the matter density  $\rho_{\mathbf{m}}$  fluctuate simultaneously at a constant specific entropy), and of entropy or isothermal perturbations (only  $\rho_{\mathbf{m}}$  fluctuates and  $\delta T_{\mathbf{r}} = 0$ , i.e., the specific entropy fluctuates) has been considered in many papers and summarized in a number of reviews<sup>[2-4]</sup>. It is determined by the competition between growth under the influence of gravitational instability and damping as a result of dissipative processes. The rate of one or the other dependence on the scale of the perturbations and, in addition, it is different on intervals separated by two critical moments preceding the separation of the inhomogeneities from the expanding background. The first crisis occurs at the instant  $t_{\text{eq}} \approx 3 \times 10^3 \Omega^{-2}$  years (red shift  $z_{\text{eq}} \approx 2 \times 10^4 \Omega$ ), when the equality  $\rho_{\mathbf{m}} = \rho_{\mathbf{r}}$

$\approx 10^{-16} \Omega^4 \text{ g/cm}^3$  took place (the temperature then was  $T_{\text{eq}} \approx 6 \times 10^4 \Omega^\circ \text{K}$ , where  $\Omega$  is the factor by which the total density of matter differs from the critical density in the Friedmann model; according to observations of galaxies,  $1/30 < \Omega < 2$ ). The second crisis is connected with the start of recombination of the hydrogen plasma ( $t_{\text{rec}} \approx 2 \times 10^5 \Omega^{-1/2}$  years,  $z_{\text{rec}} \approx 1.5 \times 10^3$ ,  $\rho_{\text{rec}} \approx 3 \times 10^{-20} \Omega \text{ g/cm}^3$ ,  $T_{\text{rec}} \approx 4 \times 10^3 \text{K}$ ).

The character of the growth of a perturbation as a function of its scale is determined by the relation between the mass of the perturbation and the so-called Jeans mass  $M_J$ , in which the gradient of the pressure is balanced by the gravitational force. When  $\rho_r \gg \rho_m$ , the pressure is determined by the radiation and  $M_J \approx 3 \times 10^{15} \Omega^{-2} (t/t_{\text{eq}})^{3/2} M_\odot$ , coinciding in order of magnitude with the mass contained inside the cosmological horizon. When  $t_{\text{eq}} < t < t_{\text{rec}}$  we have  $M_J = \text{const} \approx 3 \times 10^{15} \Omega^{-2} M_\odot$ . Starting with  $t = t_{\text{rec}}$ , the medium becomes transparent to the radiation and  $M_J (t_{\text{rec}} + 0)$  drops rapidly to  $10^5 \Omega^{-1/2} M_\odot$ , being now determined by the gas pressure, which is much smaller than the radiation pressure.

When  $t < t_{\text{eq}}$ , the perturbations of all the masses with  $M \gg M_J$  increase monotonically ( $\delta\rho/\rho \propto t$ ) as a result of gravitational instability. This pertains also to the typical galactic mass ( $M \sim 10^{10} M_\odot$ ), so long as its dimensions are larger than the horizon. But at  $t > 0.6 \Omega^{-2/3}$  years this mass, "burrowing" under the horizon, turns out to be smaller than the Jeans mass and the adiabatic perturbations  $\delta\rho/\rho$  can oscillate only like sound waves. In the adiabatic approximation, their amplitude is constant when  $t < t_{\text{eq}}$  and decreases like  $t^{-1/6}$  when  $t > t_{\text{eq}}$ . Only after recombination, when  $M_J$  drops sharply, is the growth of such perturbations again possible. Now  $\delta\rho/\rho$  increases like  $t^{2/3}$ , and in order for it to have time to grow to unity, say to  $z \sim 10$ , it is necessary to set "manually" at the instant  $t_{\text{rec}}$  an amplitude  $\delta\rho/\rho \sim 1\%$ . Much greater density inhomogeneities (or, accordingly, potential velocities larger than  $\sim 10 \text{ km/sec}^{[5]}$ ) lead to separation of objects with average density exceeding that observed in normal galaxies.

This picture is greatly altered quantitatively by dissipative damping of the perturbations, considered in<sup>[6-9]</sup> and in greater detail in<sup>[10]</sup>. For adiabatic perturbations, the main contribution to the damping is made by diffusion of the quanta from the regions of increased pressure. By the instant  $t_{\text{rec}}$ , the perturbations attenuate exponentially up to  $M \approx 10^{12} \Omega^{-5/4} M_\odot$ . If account is taken of the delayed course of the recombination, when the perturbations become gradually transparent, the damping extends over even larger masses, up to  $M \approx 10^{14} \Omega^{-1/2} M_\odot$ , in accordance with preliminary estimates<sup>[10]</sup> which will be verified for different conditions by numerical calculation.

As a result of the dissipative processes, the amplitude of the adiabatic perturbations becomes smaller in the galactic-mass scale than in the scale of galaxy clusters, provided only the initial (undistorted) spectrum of the perturbation is not too steep. This imposes important limitations on the still non-existent theory of transformation of adiabatic perturbations into galaxies. In principle, the first to be separated gravitationally from the background can be the inhomogeneities of the

very largest masses with  $M > M_J (t_{\text{rec}} - 0)$ , the growth of which was never suppressed by damping and was not altered by oscillations. In this case the theory must explain why the average density of galactic clusters, to the contrary decreases rapidly with the dimension of the cluster. The existence of superclusters (clusters of second order) and the absence of accumulations of higher order raise additional difficulties for the hypothesis with the earlier separation of the largest masses.

Independently of the course of the spectrum of the perturbations in the region of the largest masses, the formation of galaxies must be regarded in view of the dissipative damping of the perturbations of the galactic scales as a result of fractionalization of much larger masses than possessed by the galaxies. This concept is difficult to relate with the observed large difference (by a factor ( $\sim 10^3$ )) in the average densities of clusters, which is accompanied by a much smaller difference (a factor of  $\sim 30$ ) in the average densities of the galaxies themselves.

Finally, the greatest difficulty in the theory of adiabatic perturbations lies in the explanation of the rotation of galaxies and their clusters. It is assumed<sup>[11,12]</sup> that the angular momentum of the galaxies was acquired in the pre-galactic stage as a result of tidal gravitational interaction with the surrounding protogalaxies. However, the closeness of the estimate<sup>[12]</sup> of the angular momentum of the galaxy to the observed value is illusory, since the quadrupole moment was not calculated in a self-consistent manner (as a result of the non-sphericity due to tidal forces), but was stipulated, and furthermore strongly exaggerated. No account was taken here of the dissipative damping of the perturbations of the galactic masses and of the need of their subsequent reproduction, which should change the result of the calculations. The observational data likewise do not support the tidal origin of the angular momentum of galaxies. Thus, dense galaxy clusters, where the action of the tidal forces might appear to be particularly effective for the generation of rotation, consist mainly of elliptic (E) galaxies, i.e., objects with much larger specific angular momentum than in spiral (S) galaxies. To the contrary, in more rarefied clusters, consisting mainly of spiral galaxies, one observes symptoms of general rotation. Further evidence against tidal forces as the mechanism of acquisition of angular momentum by the galaxies is provided also by physical pairs of galaxies forming isolated systems (the contribution to the potential energy from neighboring galaxies does not exceed 30%). A list of 87 such pairs is given, for example, in<sup>[12]</sup>, among which there are 30 EE (34.5%), 23 ES (26.4%), and 34 SS (39.1%) pairs, i.e., the number of ES differs little from that of EE. This indicates that the angular momentum of S galaxies is more readily native than acquired. Confirming this is the fact that the number of SS galaxy pairs with oppositely directed rotation does not exceed the number of pairs having the same rotation direction<sup>[14]</sup>.

The same difficulty with the explanation of the rotation of galaxies is present also in the theory of entropy perturbations. Unlike the adiabatic perturbations, these perturbations are not smoothed out so catastro-

phically by radiation as the adiabatic ones. By the instant of recombination, there attenuate only perturbations with  $M < M_J \sim 10^6 M_\odot$ <sup>[3]</sup>. However, radiation friction very rapidly damps out any motions of entropy perturbations against the background of the radiation, so that by the instant of recombination they are practically static inhomogeneities. Their ultimate fate is not clear. In<sup>[15]</sup> these perturbations serve as the basis for a multistep scheme of galaxy formation. It is assumed that the first condensations with  $M \sim 10^6 M_\odot$  form unstable "protostars," the explosions of which heat the surrounding neutral gas and increase  $M_J$  to  $10^9 M_\odot$ . The inhomogeneities of these masses are identified by the authors of<sup>[15]</sup> with quasars. Their explosions heat the gas still more and in turn produce inhomogeneities of the type of protocusters. The galaxies are assumed to be the last generation produced as a result of fragmentation of such protocusters, i.e., in analogy with the previously outlined scheme with adiabatic perturbations (although as a result of different causes).

In another variant<sup>[16]</sup> of the theory of entropy perturbations it is also assumed that the inhomogeneities "surviving" the dissipation had a decreasing spectrum, so that at first masses  $\sim 10^6 M$  were separated. However, unlike in<sup>[15]</sup>, it is assumed here that they do not form supermassive stars, but are fragmented as a result of cooling by the molecular hydrogen into a large number of stars, producing spherical clusters. The succeeding evolution is visualized, in main outline, as a unification of the spherical clusters into galaxies, and these in turn into clusters. However, that part of the theory is unsatisfactory even at the level of qualitative arguments.

In both variants of the theory of entropy perturbations it was impossible to obtain a characteristic parameter of the galaxy mass, and all the more to develop some detailed quantitative scheme.

We shall not stop to discuss the evolution of other types of composition perturbations<sup>[17,18]</sup> listed in Sec. 1, for which the possibility of their transformation into galaxies has not yet been demonstrated.

3. Evolution of vortical perturbations, Weizsäcker<sup>[19]</sup> advanced the hypothesis that the primary structure, from which the galaxies were formed, had the character of a turbulence. However, the notion<sup>[20]</sup> of a high radiation density in the past ( $\rho_r \gg \rho_m$ ) is incompatible, as shown in<sup>[21]</sup>, with the turbulence of plasma against the unperturbed background of radiation: the motions should attenuate within a time much shorter than cosmological. In order to retain an idea of the primary turbulence, it is necessary to consider an entirely different type of compatible motions of radiation and the plasma associated with it, having a solenoidal character ("photon vortices"). The main phases of the evolution of such vortices, superimposed on an isotropically expanding cosmological background, were considered in<sup>[22,23]</sup>.

In the linear hydrodynamic approximation, solenoidal (s) motions are not accompanied by density inhomogeneities and by the corresponding potential (p) motions. However, in second order  $v_p/v_s \sim (v_s/u)^2$ ,  $\delta\rho/\rho \sim (v_s/u)^2$ . During the  $\rho_r \gg \rho_m$  stage, the sound velocity  $u = c/\sqrt{3}$ , and  $v_s = \text{const}$ , and if  $v_s < u$ , then

subsonic solenoidal motion generates only quadratically small inhomogeneities.

It is possible to indicate uniquely a spectrum for a subsonic cosmological turbulence. The reason is that owing to the large Reynolds number in the past ( $Re = vl/\nu_r \propto z$ ), the initial motions with arbitrary spectrum acquire the character of turbulent pulsations. As a result, a Kolmogorov velocity spectrum  $v \propto l^{1/3}$  should be established within scales of  $l$  bounded from below by dissipative damping (principally as a result of radiative viscosity) and bounded from above by the equality of the characteristic hydrodynamic and cosmologic times.

The subsonic character of the turbulence is possible only up to the instant of recombination  $t_{rec}$ , when the situation changes qualitatively. After completion of the recombination, the sound velocity will be determined by the elasticity of the gas and not of the radiation, as a result of which it decreases by a factor equal to  $(\text{quantum density/nucleon density})^{1/2} \approx 10^4 \Omega^{-1/2}$ . Therefore in scales where  $v_s > u$ , the vortices generate potential motions and corresponding inhomogeneities in the distribution of matter.

The amplitude of the inhomogeneities produced in a certain scale  $l$  depends on the ratio of the characteristic hydrodynamic time  $t_h = l/v$  to the characteristic time of cosmological expansion  $t_{exp} = (d \ln \rho/dt)^{-1}$ . We consider first the scales where  $t_h < t_{exp}$  at the instant  $t_{rec}$ . For these, as a result of nonlinear effects in the steady-state turbulence, we can expect  $v_p \sim v_s$  and accordingly relatively large inhomogeneities  $\delta\rho/\rho \sim 1$ . Such an equipartition between the solenoidal and potential motion does not allow plane shock waves to collapse and produce large density discontinuities. The dissipating motions are reproduced as a result of energy pumping from the largest scales, where  $t_h > t_{exp}$ . The excess kinetic energy at the instant  $t_{rec}$  also prevents an immediate separation of these inhomogeneities. Separation from the expanding background becomes possible when the kinetic energy drops to a value admitted by the virial theorem. In<sup>[24]</sup>, in the approximation of the adiabatic decrease of the energy of the solenoidal and potential motions with expansion of the universe (without allowance for their partial attenuation), we calculated the principal cosmogonic parameters of the produced galaxies: the instant of gravitational separation, the radius-mass ratio, the specific angular momentum, and the velocity dispersion. The appearance of spiral galaxies is expected in those sections of the medium having predominately solenoidal motions, whereas elliptic galaxies are expected in sections where potential velocities predominate. The fraction of the mass contained in the regions of the null lines of the field of the solenoidal velocities makes it possible to estimate the relative number of the S and E galaxies and the fraction of the mass falling in the spherical clusters. Numerical estimates, in which use is made of the natural assumption that the characteristic scale and velocity in it at the instant  $t_{eq}$  were close respectively to  $ct_{eq}$  and  $u_{eq} \approx c/\sqrt{3}$ , lead to values close to those observed. It is interesting that the theory explains the existence of an upper limit of the galaxy mass (as the mass in a scale where  $t_h = t_{exp}$  at the instant  $t_{rec}$ ); its numerical value, close

to that observed, depends only on the fundamental physical constants ( $c$ ,  $G$ ) and on the specific entropy of matter in the universe.

Let us turn now to the fate of scales where  $t_h > t_{\text{exp}}$  at the instant  $t_{\text{rec}}$ . Potential motions and the corresponding density inhomogeneities are relatively small here, and, unlike the smaller scales, their evolution is determined not by the hydrodynamic but by the gravitational instability. If in a given scale the potential velocity is able to grow within a time  $t_{\text{exp}}$  to a value sufficient for the extinction of the velocity of the differential cosmological expansion, then the corresponding inhomogeneity is "switched off" from the background and a gravitationally bound system is produced. Obviously, the perturbations of these scales correspond to agglomerates (clusters) of galaxies with different numbers. The spatial correlation of the velocities in the metagalactic turbulence indicates that the inhomogeneities in which potential velocities prevail over solenoidal ones (elliptical galaxies) are predominantly grouped into spherical and furthermore denser clusters than clusters of irregular form (which reflects the presence of a common angular momentum), which will contain predominantly spiral galaxies and are separated later (at a lower background density) owing to the relatively smaller potential component of the velocity<sup>[25]</sup>. A principal fact is that the galaxies are separated from the background independently of the completion of the formation of their clusters, which continues up to  $z \sim \Omega^{-1}$  and is not completed at all in sufficiently large scales. The maximum possible mass of the "gas" of the galaxies, within which  $\delta\rho/\bar{\rho}$  can grow to unity towards  $z = 0$ , is, as can be shown, about  $10^{14} M_{\odot} - 10^{15} M_{\odot}$ . This theoretical estimate is close to the mass of the largest galactic clusters observed at present.

Unlike clusters, the formation of the main part of the galaxies was completed in the relatively ancient cosmological epoch ( $z \sim 100$ ). Direct observation of such galaxies is very difficult. On the other hand, observation of a young galaxy at small values of  $z$  (which in itself would be of considerable interest) will not have a direct bearing on the verification of the "solenoidal" theory, since such galaxies should be more readily an object of second or even later generation. The mechanism of their formation can be radically different than that of objects of the fundamental generation (e.g., thermal instability<sup>[4,7]</sup>). To verify the theory, the most promising are searches for young clusters, i.e., a decreasing clustering of the galaxies into the most massive clusters with increasing red shift.

The solenoidal theory of galaxy formation, which is now in the stage of intense further development, is faced with a number of important problems. It is necessary to go outside the framework of the adiabatic approximation in<sup>[24]</sup>, i.e., to perform a detailed analysis of the behavior of turbulence in the post-recombination stage. We note that its dissipation can become manifest in the distortion of the Wien part of the blackbody spectrum of the relict radiation<sup>[26]</sup>. Unfortunately, important details of the dissipation can be obtained only on the basis of the still non-existent physical theory of supersonic turbulence. It is necessary to analyze the

physical processes during the stage of transformation of the protogalaxy into a stationary system. Interesting prospects arise when one considers the transformation of the cause of galaxies and their possible cosmological role in the formation of galaxies<sup>[27]</sup>. Rotating cores are also regarded as responsible for the occurrence of the large-scale magnetic field of galaxies, a field shown to be explained within the framework of the solenoidal model in<sup>[28]</sup>.

Problems of the solenoidal theory inevitably turn into cosmological problems when attempts are made to explain the origin of the vortices, and particularly their aforementioned rather natural parameters at the instant  $\rho_r = \rho_m$  (limitations on these parameters, connected with small-scale angular inhomogeneities of vortical motion in the relict radiation background, were calculated in<sup>[29]</sup>). Of course, these vortices can be regarded just as an inevitable property of the universe, as its expansion, but this "explanation" casts little light on the cosmological problem. Extrapolation of the anisotropic properties of vortices into the past has led to the conclusion<sup>[22]</sup> that earlier stages of the cosmological expansion were not of the Friedmann type. Further investigations of the possible character of the cosmological expansion near the singularity<sup>[30,31]</sup>, and especially of the rotational fluctuations in this "boiling kettle," will possibly be provided by a solution of the intriguing problem of the origin of the vortices.

4. Conclusion. At the present time there are three competing hypotheses concerning the nature of the primary perturbations as potential, solenoidal, and entropy-type. It is possible that they form a "linearly-independent basis," to which it is possible to reduce any new hypothesis concerning galaxy formation. It is clear even now, however, that not all the foregoing types of perturbations are responsible with equal probability for the basic features of the future theory. The most abundant material of astrophysical observations narrows down greatly the conceivable possibilities. Thus, static (entropy) perturbations, although they are inevitable in the damping of other types of perturbations, have apparently played a secondary role in the formation of galaxies and their groups; nonetheless, one cannot exclude the possible genetic connection between entropy perturbations and the subgalactic structure such as spherical clusters, a connection that calls for further analysis. On the other hand, the formation of the galaxies themselves, and particularly of their clusters, can be justifiably connected with perturbations of dynamic type, i.e., potential or turbulent motions. The final choice between the latter will become possible when the theories become more detailed and their fundamental consequences are reliably interpreted.

<sup>1</sup>E. M. Lifshitz, Zh. Eksp. Teor. Fiz. 16, 587 (1946); E. M. Lifshitz and I. M. Khalatnikov, Usp. Fiz. Nauk 80, 391 (1963) [Sov. Phys.-Usp. 6, 495 (1964)].

<sup>2</sup>Ya. B. Zel'dovich and I. D. Novikov, Relyativistskaya astrofizika (Relativistic Astrophysics), Nauka, 1967.

<sup>3</sup>E. R. Harrison, Mon. Not. Roy. Astron. Soc. 141, 397 (1968).

<sup>4</sup>G. B. Field, Stars and Stellar Systems, vol. 9:

Galaxies and Cosmology (ed. by A. and M. Sandage), 1969.

<sup>5</sup> P. J. E. Peebles, *J. Roy. Astron. Soc. Canada* **63**, 4 (1969).

<sup>6</sup> P. J. E. Peebles, *Astrophys. J.* **142**, 1317 (1965).

<sup>7</sup> L. M. Ozernoĭ, in: "Problems of Star Evolution and Variable Stars" (Proceedings of Symposium, Moscow 24-27 November 1964), Nauka, 1968.

<sup>8</sup> A. G. Doroshkevich, Candidate's Dissertation (Moscow, 1968).

<sup>9</sup> J. Silk, *Astrophys. J.* **151**, 459 (1968).

<sup>10</sup> G. V. Chibisov, Diploma Paper (Moscow Physico-technical Institute-Physics Institute, USSR Academy of Sciences, 1969).

<sup>11</sup> F. Hoyle, *Problems of Cosmical Aerodynamics*, Dayton, Ohio, 1951, p. 195.

<sup>12</sup> P. J. E. Peebles, *Astrophys. J.* **155**, 393 (1969).

<sup>13</sup> I. D. Karachentsev, *Astrofizika* **2**, 81 (1966).

<sup>14</sup> B. A. Vorontsov-Vel'yaminov, *Astron. Circular, AN SSSR* No. 457, 1 (1968).

<sup>15</sup> A. G. Doroshkevich, Ya. B. Zel'dovich, and I. D. Novikov, *Astron. Zh.* **44**, 295 (1967) [*Sov. Astron.-AJ* **11**, 233 (1967)].

<sup>16</sup> P. J. E. Peebles and R. H. Dicke, *Astrophys. J.* **154**, 891 (1968).

<sup>17</sup> E. R. Harrison, *Phys. Rev. Lett.* **18**, 1011 (1967).

<sup>18</sup> Ya. B. Zel'dovich, *Astron. Zh.* **46**, 775 (1969) [*Sov. Astron.-AJ* **13**, 608 (1970)].

<sup>19</sup> C. F. von Weizsäcker, *Astrophys. J.* **114**, 165 (1951).

<sup>20</sup> G. Gamow, *Phys. Rev.* **86**, 251 (1952).

<sup>21</sup> L. M. Ozernoĭ and A. D. Chernin, *Astron. Zh.* **44**, 1131 (1967) [*Sov. Astron.-AJ* **11**, 907 (1968)].

<sup>22</sup> L. M. Ozernoĭ and A. D. Chernin, *ZhETF Pis. Red.* **7**, 436 (1968) [*JETP Lett.* **7**, 342 (1968)]; *Astron. Zh.* **45**, 1137 (1968) [sic]

<sup>23</sup> L. M. Ozernoĭ, *ZhETF Pis. Red.* **10**, 394 (1969) [*JETP Lett.* **10**, (1969)].

<sup>24</sup> L. M. Ozernoĭ and G. V. Chibisov, *Astron. Zh.* **47**, (1970) [*Sov. Astron.-AJ* **14** (1970)].

<sup>25</sup> L. M. Ozernoĭ, *Astron. Circular AN SSSR* No. 536, 1 (1969).

<sup>26</sup> R. Weymann, *Astrophys. J.* **145**, 560 (1966).

<sup>27</sup> L. M. Ozernoĭ, *Astron. Zh.* **47** (1970) [*Sov. Astron.-AJ* **14**, (1970)].

<sup>28</sup> E. R. Harrison, *Galaxy Formation in the Early Universe Preprint* (1969).

<sup>29</sup> G. V. Chibisov and L. M. Ozernoy, *Astrophys. Lett.* **3**, 189 (1969).

<sup>30</sup> V. A. Belinskiĭ, E. M. Lifshitz, and I. M. Khalatnikov, Paper at the present session.

<sup>31</sup> C. Misner, *Phys. Rev. Lett.* **22**, 1071 (1969).

#### A. I. Larkin, Fluctuations in Superconductors.

Near the temperature of transition into the superconducting state, just as near other second-order transition points, the role of fluctuations increases. However, the region of temperatures where the fluctuations exert a noticeable influence on the properties of superconductors is very small. According to Ginzburg's estimate<sup>[1]</sup>, for pure bulky superconductors it equals  $10^{-15}$  degrees. The fluctuations play a noticeable role in thin films with small electron mean free paths,

and they were observed experimentally in precisely such films of bismuth<sup>[2]</sup>. It turned out that on approaching the transition temperature the film resistance  $R$  decreases smoothly and at temperatures not too close to the transition temperature  $T_C$  it is given by

$$1/R = R_\infty^{-1} [1 + \tau_0 T_c (T - T_c)^{-1}], \quad (1)$$

where the parameters  $R_\infty$  and  $\tau_0$  do not depend on the temperature.

Simultaneously, the fluctuations of the resistance of the superconductors was investigated theoretically<sup>[3]</sup>. At temperatures higher than  $T_C$ , the superconducting pairs do not form a Bose condensate, but can be produced by fluctuation in noticeable amounts. Their density  $n_p$  obeys a kinetic equation that can be derived from the Ginzburg-Landau temporal equation

$$\frac{\pi \hbar}{16} \left( \frac{\partial}{\partial t} - 2eE \frac{\partial}{\partial p} \right) n_p + \left( T - T_c + \frac{p^2}{2M} \right) n_p = T, \quad (2)$$

where  $E$  is the electric field intensity,  $p$  the pair momentum, and  $M$  the parameter of the Ginzburg-Landau theory.

The contribution of the fluctuation pairs to the current density is called paraconductivity, and for a thin film of thickness  $d$  it is equal to

$$j = \frac{2e}{Md} \int \frac{d^2 p}{(2\pi)^2} n_p. \quad (3)$$

Substituting the solution of Eq. (2) in formula (3), we obtain in a weak low-frequency field the second term of expression (1). The ratio  $\tau_0/R_\infty$  for the resistance of a film square is equal to the universal value

$$\tau_0/R_\infty = e^2/16\hbar = 3 \cdot 10^{10}/16 \cdot 137 \text{ cm/sec} = 1.52 \cdot 10^{-5} \text{ cm}^{-1}. \quad (4)$$

The universality of this ratio was later verified for films of different thicknesses.

Many investigations were made of the dependence of paraconductivity on the electric field and its frequency. As seen from (2), the pair density, and consequently their contribution to the conductivity, decrease with increasing field and with increase of its frequency. Equation (2) takes into account the deviation of the pair density from their equilibrium distribution under the assumption that the unpaired electrons come into the equilibrium state more rapidly than the pairs. This is correct if there are magnetic impurities in the film and the film is placed in a magnetic field, or if the energy relaxation of the electrons, which is connected with the electron-photon and electron-electron interactions, is sufficiently large. Otherwise it is necessary to take into account the fact that the fluctuation pairs influence also the conductivity of the unpaired electrons. The current density then acquires the so-called "anomalous" term, which leads to an effective increase of the parameter  $\tau_0$ <sup>[4]</sup>. In some experiments with lead films<sup>[5]</sup> the measured value  $\tau_0$  turned out to be half as large as the theoretical one. There is still no satisfactory explanation of this fact.

In addition to the contribution of the fluctuations to the conductivity of thin films, there was observed<sup>[6]</sup> an influence of fluctuations on the tunnel current. A noticeable contribution is made by fluctuations to the magnetic susceptibility of superconductors at a temperature higher than the transition point<sup>[7]</sup>. In a wide