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ELECTRONIC TRANSITIONS IN STRONG MAGNETIC FIELDS*

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I. INTRODUCTION

By the nature of their influence on the energy spectrum of a material, magnetic fields can be divided into three regions.

1. Weak magnetic fields (subquantum region) in which the Larmor radius of the current carriers exceeds appreciably the mean free path l, as a result of which the energy is not quantized. Weak fields are usually utilized for measuring the components of the galvanomagnetic tensor.

2. Strong or strong effective magnetic fields (the region of quasiclassical quantization): $l \gg R$, $kT \ll \hbar\omega_H \ll E_F$, where ω_H is the Larmor frequency and E_F is the Fermi energy. Strong or strong effective fields are widely utilized for determining the basic parameters of the energy spectrum, characterizing the behavior of the electrons on the Fermi surface, i.e., of electrons with an energy equal to the Fermi energy.

3. Ultrastrong magnetic fields (the ultraquantum region) in which the quantization energy $\hbar\omega H \gtrsim E_F$. The investigation of the properties of matter in the ultraquantum region is of interest from various points of view.

On the one hand, in the ultraquantum region the structure of the spectrum changes as a result of the shift of the zone boundaries, the changes in the Fermi energies in the bands, the densities of current carriers and their distribution over the individual isoenergy surfaces. The nature of this change is directly determined by the dispersion law of the current carriers, and for this reason the investigation of these changes furnishes valuable information on the nature of the dispersion law.

On the other hand, the shift of the Fermi level in a magnetic field makes it possible to investigate the structure of the energy spectrum in a sufficiently broad range of energies.

In the ultraquantum region isoenergy surfaces can disappear and new ones can appear at definite critical values of the magnetic field; open surfaces can go over into closed ones and vice versa, compensated metals turn into semiconductors and semiconductors into metals. Such changes in the spectrum should be accompanied by anomalies of the thermodynamic and kinetic quantities.[1]

It should also be noted that in the ultraquantum limit the electron gas in metals becomes one-dimensional, a fact which may in itself lead to a qualitative change in the commonly observed regularities. Effects connected with the transition to the ultraquantum region are most readily observed in metals having a low carrier density with relatively small Fermi energies, as well as in semiconductors with narrow energy gaps.

In this article we report the results of an investigation of the magnetoresistance in materials with a small controllable band overlap and a controllable energy gap—metallic or semiconducting bismuth-antimony alloys with an antimony concentration up to 16 at. percent—in pulsed magnetic fields up to 600 kOe in the 2–77°K temperature range. The investigation was carried out for the purpose of observing effects connected with qualitative changes of the energy spectrum of the material in the ultraquantum region of magnetic fields.

II. ENERGY QUANTIZATION IN THE ULTRAQUANTUM REGION OF MAGNETIC FIELDS

Let us consider in a purely qualitative manner the changes occurring in the energy spectrum of the current carriers in strong quantizing magnetic fields using as an example the quadratic dispersion law

$$\mathscr{E} = (p_x^2/2m_x) + (p_y^2/2m_y) + (p_z^2/2m_z); \qquad (1)$$

p_x, p_y, p_z, $1/m_x$, $1/m_y$, $1/m_z$ are the components of the quasimomentum and of the tensor of the reciprocal effective masses of the carriers respectively.

In a magnetic field $H = H_z \neq 0$ the energy associated with the motion of the carriers over closed trajectories in a plane perpendicular to the field is quantized:

$$\mathscr{E}(n) = \hbar\omega_H (n + 1/2), \quad \omega_H = eH/m^*c;$$

m^* is the cyclotron mass which is for a quadratic dispersion law $(m_x m_y)^{1/2}$; $n = 0, 1, 2, 3, \ldots$.

Each of the discrete levels (Landau levels) is degenerate in the spin. In a magnetic field this degeneracy is lifted and the Landau levels split into two levels separated from one another by an energy $2\mu_B^* H$ ($\mu_B^* = e\hbar/2m^s c$ is the effective Bohr magneton, and $m_s > 0$ is the effective spin mass of the current carriers).

The energy $p_z^2/2m_z$ of the motion of the carriers along the magnetic field is not quantized and constitutes a quasicontinuous spectrum.

Thus the expression for the energy in the magnetic field is of the form

$$\mathscr{E}(n, p_z) = (|e|\hbar/m^*c) H (n + 1/2) \pm (|e|\hbar/2m^s c) H + (p_z^2/2m_z). \qquad (2)$$

Near the energy minima (for electrons) $m^* = m_e^* > 0$ and $m_z = m_z^e > 0$. Near the energy maxima in the bands (for holes) $m^* = m_h^* < 0$ and $m_z = m_z^h < 0$.

The nature of the electron and hole spectrum in a magnetic field is illustrated in Fig. 1.

The minimum possible value of the energy of the

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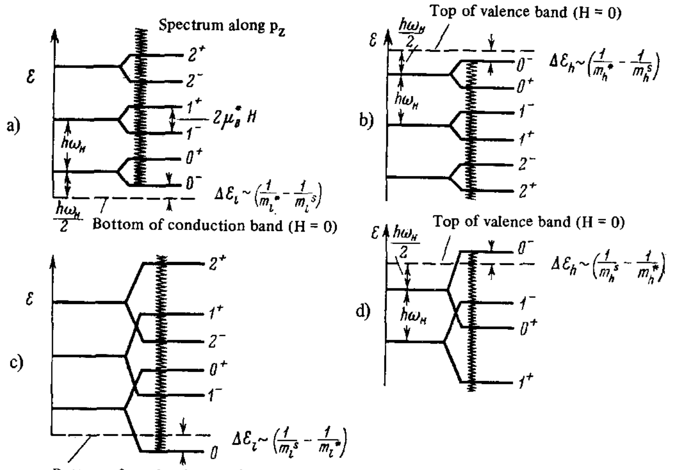

FIG. 1. Energy level diagram of electrons (a, b) and holes (c, d) in a magnetic field (quadratic dispersion law).

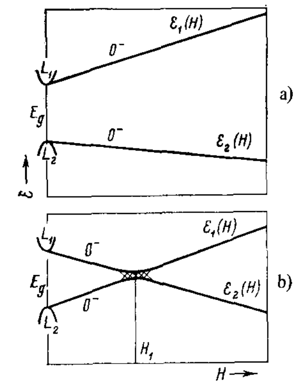

FIG. 2. Displacement of the extrema L_1 and L_2 in bismuth in strong magnetic fields. [2]

electrons (the bottom of the conduction band) and the maximum possible for holes (the top of the valence band) in a magnetic field correspond to n = 0 and $p_z = 0$. Depending on the ratio of the spin and orbital effective masses the bottom of the conduction band can either be raised or lowered in a magnetic field relative to its position for H = 0. The shift of the bottom of the band is

$$\Delta\mathscr{E}_e = (|e|\hbar/2c)[(m_e^*)^{-1} - (m_e^s)^{-1}]H. \quad (3)$$

If $B_e = (m_e^*)^{-1} - (m_e^S)^{-1} > 0$ then the bottom of the band will be raised in a magnetic field (Fig. 1a). For $B_e < 0$ the bottom of the band will be lowered (Fig. 1b). Analogously, for $B_h = (m_h^*)^{-1} - (m_h^S)^{-1} < 0$ the top of the band is raised and for $B_h > 0$ it is lowered.

For an arbitrary dispersion law, notwithstanding the ensuing considerable complication of the general pattern of energy quantization in a magnetic field, the shifting of the band boundaries can to a first approximation be described by the following phenomenological relations. For the shift of the bottom of the conduction band (energy minima) we have

$$|\Delta\mathscr{E}_e| = (e\hbar/c)H\,|[m_e^*(p_z,\mathscr{E})]^{-1} - [m_e^s(p_z',\mathscr{E})]^{-1}|_{max} = (e\hbar/c)H\,|B_e^*|_{max}, \quad (4)$$

if the bottom is lowered, and

$$|\Delta\mathscr{E}_e| = (e\hbar/c)H\,|B_e^*|_{min}, \quad (5)$$

if it is raised.

For the top of the band (energy maxima)

$$|\Delta\mathscr{E}_h| = (e\hbar/c)H\,|[m_h^*(p_z,\mathscr{E})]^{-1} - [m_h^s(p_z',\mathscr{E})]^{-1}|_{max} = (e\hbar/c)H\,|B_h^*|_{max}, \quad (6)$$

if the top is raised, and

$$|\Delta\mathscr{E}_h| = (eh/c)H\,|B_h^*|_{min}, \quad (7)$$

if it is lowered.

In these relations the effective masses $m^*(p_z, \mathscr{E})$ and $m_s(p_z', \mathscr{E})$ of the electrons at the bottom of the conduction band and for holes at its top refer to such values of p_z and p_z' for which at an energy $\mathscr{E}$ the modulus of B^* is an extremum. Extrema located at different points of phase space do not interact with one another and the nature of their shift is determined exclusively by the ratio of the spin and orbital masses of the current carriers.

The situation becomes considerably more complicated if the extrema are at the same point of phase space. Only one theoretical paper[2] which considers this problem is known at present.

This paper analyzes the various possible shifts of the extrema L_1 and L_2 in bismuth, separated by an energy gap E_g = 15 meV. The energy boundaries of these bands are levels of the type n = 0, S = $-\frac{1}{2}$ (0^- type levels). According to[2], in a magnetic field these levels can be displaced either linearly or hyperbolically (Fig. 2). In the latter case the energy gap between the bands decreases initially almost to zero (for $H = H_1$), and then the levels are "reflected" from one another and the gap increases again. The "reflection" of the levels occurs in such a way that after reflection the dependence of $\mathscr{E}_1$ on H at the lower level of the conduction band approaches asymptotically the dependence that represents the continuation of the motion of the valence band level $\mathscr{E}_2(H)$ in fields $H < H_1$, and vice versa. At the point of closest approach ($H \approx H_1$)[1] the levels become almost degenerate (almost the same energy corresponds to the different states ψ_1 and ψ_2). In the process of "reflection" the energy levels $\mathscr{E}_1(H)$ and $\mathscr{E}_2(H)$ "exchange" not only the directions of their shift in the magnetic field but also the wave functions ψ_1 and ψ_2.

III. VARIOUS TYPES OF ELECTRONIC TRANSITIONS IN A MAGNETIC FIELD

In the ultraquantum region the shift of the band boundaries reaches a magnitude comparable or exceeding the value of the Fermi energy in the bands in metals or of the energy gap in semiconductors. The following changes in the energy spectrum of substances can then be observed.

1. The Metal-semiconductor Transition

Transitions of this type can be expected in metals with equal numbers of electrons and holes,[1] whose conductivity is due to a small overlap of the bands. They are connected with a displacement of the extrema of the valence and conduction band, located at different points of phase space.

It follows from (3) that the band overlap existing at H = 0 will decrease in a magnetic field if

$$A = (m_e^*)^{-1} - (m_e^s)^{-1} + (m_h^*)^{-1} - (m_h^s)^{-1} > 0. \quad (8)$$

In this case for a certain magnetic field $H = H_c$ the

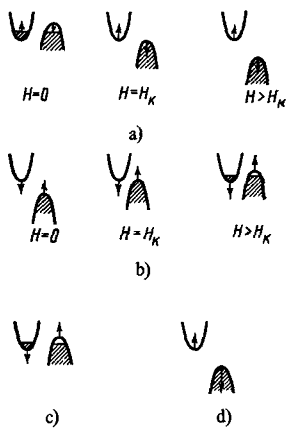

FIG. 3. Various types of changes in the energy spectrum of semimetals and intrinsic semiconductors in strong magnetic fields. a) Metal-semiconductor type transition; b) semiconductor-metal type transition; c) increasing band overlap; d) increasing energy gap.

overlap of the energy bands disappears and there appears an energy gap characteristic of the spectrum of semiconductors (Fig. 3a). The carrier density and hence the electrical conductivity decrease exponentially with further increase of the field.

It should be noted that the metal-semiconductor transition is impossible in metals with a single conduction band and a filled valence band. The effect of the magnetic field reduces in this case to a shift of the boundaries of the conduction band and to a change in its distribution of the density of states. This is not accompanied by any change in the number of sites or carriers. The conduction band can only become empty in the presence of free sites in the valence band.

The possibility of a transition of the semimetals to the semiconducting state in strong magnetic fields has been predicted by a number of authors[1,3,4] but this transition has so far not been observed experimentally.

2. The Semiconductor-metal Transition

If for $H = 0$ the material is a semiconductor with a small energy gap between the conduction and valence band, then when the condition

$$A = (m_e^*)^{-1} - (m_e^s)^{-1} + (m_h^*)^{-1} - (m_h^s)^{-1} < 0 \tag{9}$$

is fulfilled, an increase of the magnetic field will lead to a decrease of this gap and for $H > H_C$ it will lead to the appearance of an overlap of the band extrema (Fig. 3b). This phenomenon should be accompanied by a sharp increase in the number of carriers in the conduction band for $H > H_C$ and by the appearance of a "metallic" temperature dependence of the resistance. On further increasing the magnetic field the band overlap increases and the metallic properties of the material are enhanced.

3. Increasing Band Overlap in a Magnetic Field

In compensated metals the band overlap should increase in a magnetic field for $A < 0$ (Fig. 3c). An increase in the carrier density can at the same time lead to a strong change in the character of the dependences of the longitudinal and transverse resistance on the magnetic field, in particular—to a deviation of the transverse magnetoresistance from the classical $\rho \sim H^2$ law (see[5]).

4. Increasing Energy Gap in Intrinsic Semiconductors

For $A > 0$ in intrinsic semiconductors the gap in the energy spectrum increases in a magnetic field Fig. 3d). As a result of the decrease in the density of thermally excited current carriers one can observe, for example, an anomalously large increase of the resistance in a magnetic field.

5. The Semiconductor—"Quasimetal"—Semiconductor Transition

This type of transition is the result of the approach and subsequent separation of extrema located at the same point of phase space (Fig. 2b). If at $H = 0$ the material is a semiconductor, then as a result of the approach of the extrema the semiconductor should go into a peculiar state which can provisionally be referred to as "quasimetallic"; this state is characterized by an anomalously small energy gap between the bands. In this state the material is not a metal since the band overlap characteristic of the metallic state is absent, and the material is at the same time not a semiconductor within the generally accepted meaning of this term since the energy gap is anomalously small. In this state the material can have a number of unusual properties. On further increasing the magnetic field the extrema separate again and a gap appears in the spectrum which increases with increasing magnetic field. In the quasimetallic state there should be an increase of the electrical conductivity due to the increase in the density of current carriers.

6. Complex Transitions Connected with the Shifting of Several Extrema

When several extrema of various types are present in the spectrum of the semiconductors one can observe a transition from the semiconducting to the "quasimetallic" and from the "quasimetallic" to the metallic state. For such a transition extrema located at the same point of phase space approach one another initially; this is followed by the overlap of extrema located at different points of phase space and the material becomes a metal.

Depending on the initial disposition of the extrema (for $H = 0$) and on the nature of their shift in the magnetic field, transitions can occur under the same conditions from the semiconducting state to the "quasimetallic" state, from the "quasimetallic" again to the semiconducting state, and then from the semiconducting to the metallic state.

In the presence of a small overlap of the extrema (for $H = 0$) a transition is possible from the metallic to the "quasimetallic" and then to the semiconducting state.

IV. ANOMALIES OF THE ELECTRONIC CHARACTERISTICS IN ELECTRONIC TRANSITIONS

As was shown in[1], the metal-semiconductor and semiconductor-metal transitions are accompanied by a series of discontinuities of the electronic characteristics. The magnetic field dependences of the carrier density, of the thermodynamic potential, and of the magnetic moment of the electrons should have characteristic discontinuities (jumps of the derivative) for $H = H_c$ and $T = 0$. In the case of the metal-semiconductor transition when the magnetic field approaches the critical value, for $H < H_c$ and $H_c - H \ll H_c$, a small increase of the magnetic field leads to a sharp decrease of the Fermi energy and of the number of carriers, so that

$$E_F(H_c + H') - E_F(H_c) = \beta |H'|, \qquad H' = H - H_c < 0,$$
$$n_e,\ n_h \sim [E_F(H) - E_F(H_c)]^{1/2} \sim |H'|^{1/2}.$$

The thermodynamic potential associated with the conduction electrons is proportional to

$$n_{e,h}\{E_F(H) - E_F(H_c)\} \sim |H'|^{3/2}.$$

For $H > H_c$ ($H' > 0$) and $T = 0$ the current carriers disappear: $n_e = n_h = 0$, and the corresponding additions to the thermodynamic potentials also vanish. Consequently, for $H = H_c$ the derivatives of these quantities with respect to H change abruptly from ∞ to 0.

A detailed theoretical investigation of the dependence of the magnetic moment of semimetals on the field in the ultraquantum region[6] has shown that if the Fermi surface passes through two small bands and the number of electrons is equal to the number of holes then in the immediate vicinity of the metal-semiconductor transition ($0 < H_c - H \ll H_c$) the magnetic moment associated with the conduction electrons is proportional to $(|H'|)^{1/2}$. For $H > H_c$ the magnetic moment vanishes (it is assumed that $T = 0$). Consequently, for $H = H_c$ the field dependence of the magnetic moment has a discontinuity in the vertical derivative.

An increase in the magnetic field ($H \to H_c$) can also be accompanied by a change in the sign of the magnetic susceptibility. As the field approaches H_c a situation can be reached for which only a single lower energy level, nondegenerate in the spin quantum number, is occupied. The susceptibility of the metal may then become paramagnetic.

The metal-semiconductor transition should also be accompanied by anomalous behavior of the specific heat.[6] For $kT \ll E_F$ ($H = 0$) only electrons that are very close to the Fermi energy contribute to the electronic specific heat. The decrease of the Fermi energy for $H \to H_c$ lifts the degeneracy of the electrons at the bottom of the conduction band [when $E_F(H) \approx kT$]. Inasmuch as the density of states near the band boundary $\mathscr{E}_0$ increases in the ultraquantum limit in accordance with the law $\nu(\mathscr{E}) \sim (\mathscr{E} - \mathscr{E}_0)^{-1/2}$, the increase of the magnetic field is accompanied by an increase of the electronic specific heat near the metal-semiconductor transition point. Theoretical estimates indicate that at a sufficiently low temperature ($\sim 1°K$) the peak of the electronic specific heat can be clearly observed on the background of the lattice specific heat.

V. THE EFFECT OF A STRONG MAGNETIC FIELD ON THE SCATTERING OF CURRENT CARRIERS

The above metal-semiconductor and semiconductor-metal transitions, as well as the transition to the "quasimetallic" state, are connected with a change of the energy spectrum and the density of carriers in the bands in a magnetic field. At the same time, a strong magnetic field can affect the scattering mechanism of electrons which determines to a considerable extent the nature of the monotonic component of the dependence of the resistance on the magnetic field.

The magnetoresistance of bismuth in fields corresponding to the ultraquantum limit has been considered in[3]. The authors showed that when the scattering is predominantly by strongly localized impurities and when the numbers of electrons and holes are equal, the quadratic increase of the transverse magnetoresistance is retained.

A theoretical study of the magnetoresistance of semiconductors and semimetals in the ultraquantum region ($\hbar\omega_H > E_F$) has been carried out in[7,8] and in greatest detail in[9].

A quantum theory of electrical conductivity of semiconductors with a spherical Fermi surface was set up in[9]. It was shown that the quantum energies of the carriers as well as the distortion of the shape of the Fermi surface observed in the ultraquantum region of magnetic fields exert a considerable influence on the magnetoresistance. It was found that the dependence of the transverse and longitudinal resistance on the field for a degenerate and nondegenerate gas in the ultraquantum region are very sensitive to the scattering mechanism. However, the monotonic, nondecreasing nature of the dependence of ρ on H for an arbitrary scattering mechanism is characteristic.

VI. KNOWN EXPERIMENTAL DATA ON THE BEHAVIOR OF SEMIMETALS IN THE ULTRAQUANTUM REGION

An anomalous, close-to-linear increase of the resistance in strong magnetic fields was observed in Kapitza's classical experiments investigating the galvanomagnetic properties of bismuth in fields up to 300 kOe.[10] However, because of the insufficient information about the energy spectrum of the carriers in bismuth at that time, the regularities observed by Kapitza remained unexplained.

Recently, in connection with the considerable success of the theory of galvanomagnetic phenomena in metals and semiconductors there has been renewed interest in investigations of semimetals in strong magnetic fields. In[4] an attempt was made to observe the transition of antimony into the semiconducting state and to this end an investigation was carried out of the electrical conductivity of antimony in magnetic fields up to 350 kOe at a temperature above 77°K. However, the conclusion of the authors that they had observed this effect is not valid because of their incorrect interpretation of the obtained results. A subsequent investigation of the magnetoresistance and of the Shubnikov-de Haas effect in antimony in fields up to 420 kOe at liquid helium temperatures[35] [sic!] has shown that the maximum possible shift of the band boundaries for these fields does not exceed 10–15 percent, and one can therefore only expect a noticeable effect of the shift of the band boundaries on the electrical conductivity in magnetic fields of several million oersted.

Oscillations of the magnetoresistance of bismuth connected with the hole portion of the Fermi surface in fields up to 175 kOe at $T = 1.2°K$ were investigated

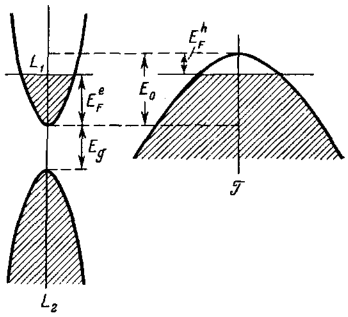

FIG. 4. Energy-band diagram in bismuth.

in[11]. The sharp increase in the magnitude of the spin splitting of the Landau levels for holes noted in[12] was observed when the magnetic field approached the trigonal axis of the crystal.

One must note in particular the detailed investigation of the magnetoresistance oscillations in bismuth carried out in[13] in magnetic fields up to 90 kOe at a temperature of 1.4°K; these made it possible to obtain new, complete data on the spin splitting of Landau levels. A calculation of the spin and orbital effective masses of the carriers was carried out, and the magnetic field dependence of the Fermi energy and carrier density in bismuth were evaluated on the basis of the experimental data. A considerable change was observed in the carrier density and in the Fermi energy in fields up to 90 kOe.

A change of the Fermi energy and a shift of the bottom of the conduction band in bismuth in a magnetic field up to 200 kOe directed parallel to the binary axis was observed in[14]. It was shown that the rate of this shift is determined to a large extent by the effect of other bands.

All these investigations were carried out at liquid helium temperatures in magnetic fields not exceeding 100—200 kOe which are insufficient for the observation of the qualitative changes in the energy spectrum of semimetals discussed above.

Of special interest as objects of investigation are the Bi-Sb alloys which form on the bismuth side a continuous series of solid solutions with monotonically varying properties from metallic to semiconducting; by changing the antimony concentration, one can vary the band overlap in the metallic alloys and the energy gaps in the semiconducting ones.

VII. THE ENERGY SPECTRUM IN Bi-Sb ALLOYS

The structure of the energy spectrum of bismuth is shown in Fig. 4.

Three electron triaxial "ellipsoids" are located at the three extrema L_1; the large axes of these are oriented perpendicular to the binary axes and are inclined at an angle of ~ 6° to the basal plane. The smallest axes of the ellipsoids are parallel to the binary axes. The dispersion law for the electrons is nonquadratic and is described sufficiently well by the ellipsoidal nonparabolic Lax-Cohen model.[15-17] At the extremum $\mathcal{T}$ there is a single hole ellipsoid which represents a figure of revolution elongated along the trigonal axis. The dispersion law for holes is to a first approximation parabolic.

H ‖ Trig. axis	Orbital m^*/m_0	Spin, m^s/m_0
Electrons	0.065	0.11
Holes	0.064	0.033
H ‖ Binary axis		
Electrons	0.128 0.0097	0.36 0.0091
Holes	0.21	1.5
H ‖ Bisec. axis		
Electrons	0.0084 0.0168	0.0079 0.158
Holes	0.21	1.5

Sample No.	C, at. percent Sb		$\rho_{4.2\,°K}/\rho_{300\,°K}$	ΔE, meV
	calculation	analysis		
1	6	5.5	1.2	—
2	5.9	6.45	3.5	—
3	8	8.8	24	6
4	9.0	8.9	45	13.0
5	7	9.1	215	15.6
6	10	10.5	300	13.5
7	10	10.5	155	13.5
8	12.0	12.0	236	15.7
9	15	15.8	26	19

According to the data of[18] the dispersion law at the three extrema L_2 is described by three triaxial ellipsoids located in analogy with the electron ellipsoids at L_1 and rotated by an angle of 8° relative to the basal plane. Six extrema of "heavy" holes $\mathcal{T}'$ [17] with an approximately parabolic dispersion law are apparently located near the extremum $\mathcal{T}$ below the Fermi level in bismuth. The values of the cyclotron and spin effective masses of the electrons in L_1 and of holes in $\mathcal{T}$[12] are cited in Table I.

The literature data on the nature of the change of the bismuth spectrum in the Bi-Sb alloys are contradictory. It is known that the overlap of the extrema L_1 and $\mathcal{T}$ in the energy spectrum of bismuth (E_0 in Fig. 4) decreases with increasing antimony concentration[19,20] and an energy gap appears in the spectrum at a certain antimony concentration. It has been assumed[19] that the change of the spectrum occurs at the expense of a downward shift of the extremum $\mathcal{T}$ relative to the extrema L_1 and L_2, the distance E_q between which does not change.

According to the data of[20] the magnitude of the band overlap vanishes at an antimony concentration of 5 at. percent. An analogous dependence of the energy gap on the antimony concentration was obtained in[21] in polycrystalline samples. On the basis of the results of measurements of the oscillations of the magnetic susceptibility[22,23] it was shown that the Fermi energy of the electrons decreases linearly with increasing concentration of the admixture of antimony; at the same time, while decreasing in their volume, the ellipsoids of the Fermi surface remain similar to a first approximation and the decrease of the effective masses is in satisfactory agreement with Cohen's theory (see[23]). However, quantitative data on the antimony concentration for which the overlap vanishes and on the size of the gap E_g obtained in[20] do not agree with the results of an investigation of the cyclotron resonance in $Bi_{95}Sb_5$ alloys and with the data of magneto-optic investigations.[24,25]

We have measured the dependence of the electrical resistance ρ on the temperature T in the 4.2–100°K range in samples of Bi-Sb alloys with an antimony concentration of more than 5.5 at. percent (Table II). In all investigated samples the electrical resistance increased with decreasing temperature from room temperature down to ~20°K.

For calculating the gap we have used the formula

$$\rho \sim \rho_0 \exp(\Delta E/2kT), \qquad (10)$$

which is only valid for semiconductors with intrinsic conductivity with the assumption that the mobility of the current carriers varies as $T^{-3/2}$.

It must, however, be borne in mind that an increase in the resistance on decreasing the temperature can also be a consequence of the small band overlap,[26,27] as a result of which the carrier density varies as $T^{-3/2}$. The T dependence of ρ should in this case be step-like

$$\rho = \rho_0 T^{-m} \quad (m > 0).$$

The gaps ΔE calculated with formula (10) for the Bi-Sb samples which we have investigated are also presented in Table II. It is interesting to note that for samples of the same composition but of differing quality (different ratios $\rho_{4.2°K}/\rho_{300°K}$) the gaps have the same value regardless of the fact that the T dependences of ρ of these samples below 15–20°K differ considerably.

The large values of $\rho_{4.2°K}/\rho_{300°K}$ for the majority of the investigated samples attest to their high degree of perfection. The gaps calculated from the $\rho(T)$ curves for the investigated samples are shown in Fig. 5. The width of the gap increases rapidly in the range of concentration of 8 to 9.5 at. percent antimony; after this it changes weakly. Such a nature of the variation of the gap means that at $C > 9.5$ at. percent antimony the extremum $\mathcal{T}$ becomes equal to the extremum of the light holes L_2. When $\mathcal{T}$ is shifted further down, the gap between the extrema L_1 and $\mathcal{T}$ becomes larger than the gap E_g. Therefore, the experimental data obtained for $C > 9.5$ at. percent antimony refer to the gap E_g and characterize its concentration dependence.

The proposed nature of the variation of the gap

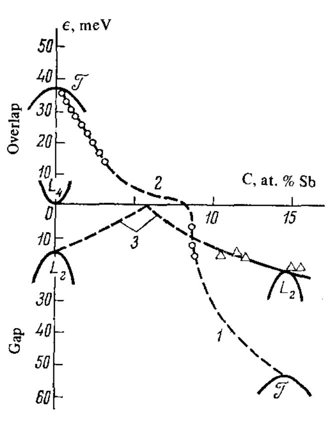

FIG. 5. Proposed dependence of the overlap (E_0) of the band extrema L_1 and $\mathcal{T}$, of the gap (ΔE) between them and of the gap (E_g) between L_1 and L_2 in Bi-Sb alloys on the antimony concentration.

width ΔE between the extrema L_1 and $\mathcal{T}$ for concentrations $C > 9.5$ at. percent based on the results of magnetoresistance measurements is shown in Fig. 5 by the dashed curve 1.

In order to reconcile the data on the variation of the band overlap in alloys with an antimony concentration up to 3.5 at. percent with the obtained dependence of the gap ΔE on the antimony concentration, one must assume that either the band overlap in Bi-Sb alloys changes in an irregular fashion in the range of antimony concentrations $3.5 < C < 8$ at. percent (Fig. 5, dashed curve 2), or that there is a systematic error in the concentrations from the data of the chemical analysis. It is, no doubt, of great interest to determine the exact nature of the transition from the metallic to the semiconducting state in Bi-Sb alloys. However, it should be noted that the present uncertainty regarding this problem in no way affects the results of this work.

Reliable data on the nature of the variation of the gap E_g in the range of concentrations 0 to 10 at. percent Sb are at present not available. If one interprets the decrease of the longitudinal magnetoresistance in alloys with an antimony concentration up to 8 at. percent for a field orientation parallel to the binary axis as the result of the closer approach of the extrema L_1 and L_2 (see below), then one can conclude that the gap E_g decreases with increasing antimony concentration, becomes very small for $C \approx 5$ at. percent, and then increases (dashed curve 3 in Fig. 5). It is interesting to note that such a dependence of E_g on C is in qualitative agreement with the results of recently published theoretical calculations,[28] and the concentration C = 8.6 at. percent Sb for which the overlap of the extrema $\mathcal{T}$ and L_1 vanishes is exactly equal to the value obtained in[18].

VIII. METHODS OF MEASUREMENT

The measurements were carried out in pulsed magnetic fields with intensities up to 600 kOe at temperatures of 2–77°K. The magnetic field was produced by the discharge of a capacitor bank with a total capacitance of 2000 μF through a solenoid in the form of a wholly turned winding with an inner diameter of 7 mm, submerged in a liquid nitrogen bath. The period of the magnetic field amounted to 320 μsec.

In low-temperature investigations of metallic samples in which the depth of the skin layer was less than the sample diameter, a constant field was first set up in which as a result of the increase of the resistance the depth of the skin layer increased to such an extent that the sample became practically transparent to a high-power short field pulse. The induction appearing in the circuit of the potential leads during the magnetic field pulse was compensated by a special coil in series with the sample. The sample was jacketed with a small thin-walled Plexiglas breaker filled with diffusion oil that froze on being cooled and fixed rigidly the size and position of the compensating coil and also that of the current and voltage electrodes. In order to exclude induction completely, oscillograms were taken for two identical field pulses with opposite directions of the measuring current. The magnitude of the potential difference across the sample, proportional to its resistance, was defined as half the sum of the amplitudes of these oscillograms.

The investigations were carried out on samples of various shapes (cylinders and parallelepipeds) and dimensions (from 1 × 1 × 3.5 mm to 0.2 × 0.4 × 2 mm). Potential electrodes of copper wire 20 microns in diameter were soldered to the samples by an electrospark method.[29] The maximum deviation of the soldering positions from the longitudinal axis of the face of the sample did not exceed 20 microns. The distance between the contacts was varied between 0.6 and 1.2 mm.

Following a careful investigation of the effect of the sample geometry, of the placing of the electrodes, of the measuring current that determines the superheating of the sample during the field pulse ("thermal" shock), and of the method of sample mounting, we chose such conditions under which one can practically neglect the influence of parasitic effects.

The obtained results were completely reversible and reproducible for field pulses of various magnitudes.

Forty seven samples were measured for all the principal field and current orientations relative to the crystallographic axes.[30-34]

When the orientation of the magnetic field relative to the crystallographic axes is changed the ratio of the spin and cyclotron masses in the extrema L_1, L_2 and $\mathcal{T}$ changes. This is accompanied by a change in the

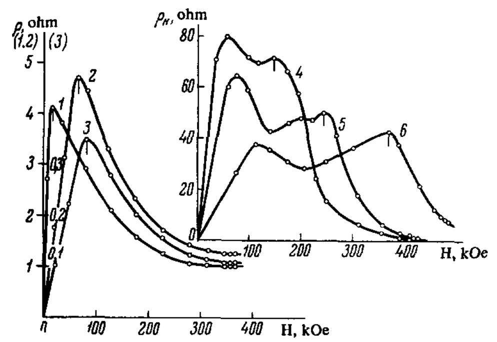

FIG. 6. Dependence of the transverse resistance on the magnetic field with H ∥ C_3 and i ∥ C_2. 1–8.8; 2–8.9; 3–9.1; 4–10.5; 5–12; 6–15.8; at. percent antimony. The scale for curves 1 and 2 is indicated on the left of the ordinate axis, for curve 3–on the right.

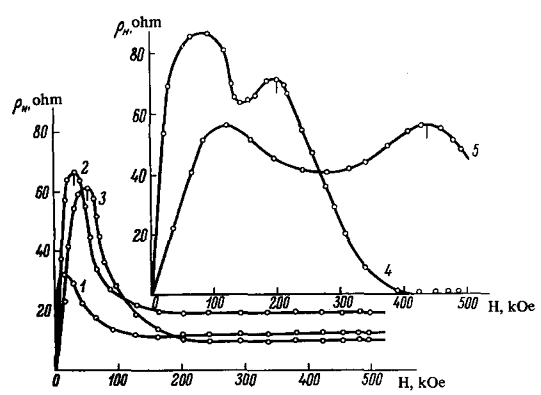

FIG. 7. Dependences of the transverse resistance on the magnetic field with H ∥ C_3 and i ∥ C_1. 1–8.8; 2–8.9; 3–9.1; 4–10.5; 5–15.8 at. percent antimony.

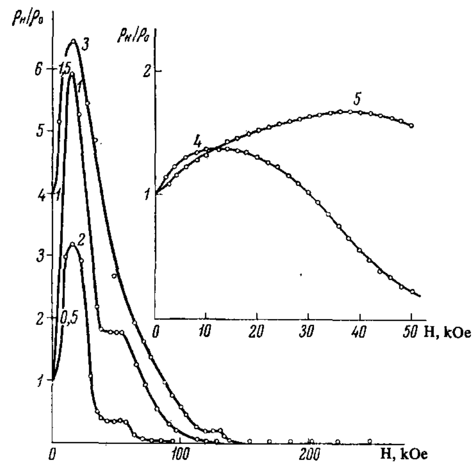

FIG. 8. Dependences of the relative change of the longitudinal resistance on the magnetic field with H ∥ i ∥ C_3. 1–10.5; 2–10.5; 3–11.5; 4–9.1; 5–12 at. percent antimony. The scale for curves 1 and 2 is indicated on the left of the coordinate axis, for curve 3–on the right.

nature of the shift of the extrema in a magnetic field, a fact which makes it possible to observe practically all the types of electronic transitions considered above.

IX. MAGNETORESISTANCE OF THE SEMICONDUCTING Bi-Sb ALLOYS IN A FIELD PARALLEL TO THE TRIGONAL AXIS

The field dependences of the transverse and longitudinal resistance of samples for three principal orientations of the measuring current at a temperature of 4.2°K are presented in Figs. 6–8.

The transverse magnetoresistance increases in a field, passes, depending on the composition of the alloy, one or two maxima, and then decreases sharply for $H = H_c$. Regardless of the fact that the decrease of the resistance for $H = H_c$ is the most characteristic feature of the behavior of alloys containing more than 8 at. percent antimony, a series of features observed in the ρ (and ρ_H) dependences on H in these alloys allow one to separate to subregions of antimony concentrations:

$$8 \leqslant C \leqslant 9.5 \text{ at.\% and } C > 9.5 \text{ at.\%}.$$

In the first subregion, on increasing the field oriented parallel to the trigonal axis, the resistance first increases, passes through a maximum for $H = H_c$, and then decreases, approaching some constant value which in the region of high fields depends little on the field strength. In the second subregion ($C > 9.5$ at. percent antimony) there appears in the region of the maximum on the curves of the dependence of ρ on H a local resistance minimum whose depth increases on increasing the antimony concentration; the position of the minimum shifts towards the region of strong fields.

From Figs. 6 and 7 it is seen that the nature of the dependences of the magnetoresistance is mainly determined by the orientation of the magnetic field and depends weakly on the orientation of the measuring current relative to the crystallographic axes. A certain shift of the maxima for $H = H_c$ on the $\rho(H)$ curves towards the left when the current is oriented parallel to the binary axis may be connected with the anisotropy

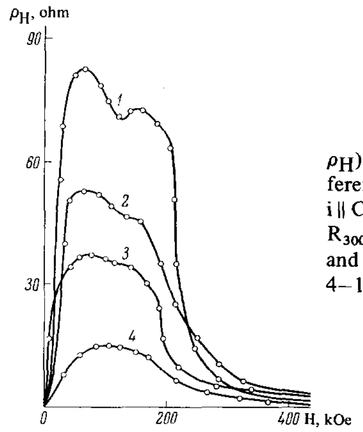

FIG. 9. The dependence of ρ (and ρ_H) on H for $Bi_{89.5}Sb_{10.5}$ with different $\rho_{4.2}/\rho_{300}$ ratios for H ∥ C_3, i ∥ C_2, and T = 4.2°K. 1–$R_{4.2°K}/R_{300°K}$ = 300, ρ_0 = 1.7 ohm; 2–175 and 1.3 respectively; 3–212 and 1.0; 4–135 and 0.4.

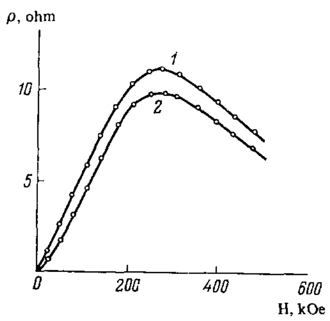

FIG. 10. Dependence of ρ on H for $Bi_{89.5}Sb_{10.5}$ samples with H ∥ C_3, ∥ C_2, and T = 78°K. 1–$\rho_{4.2°K}/\rho_{300°K}$ = 300, 2–212 respectively.

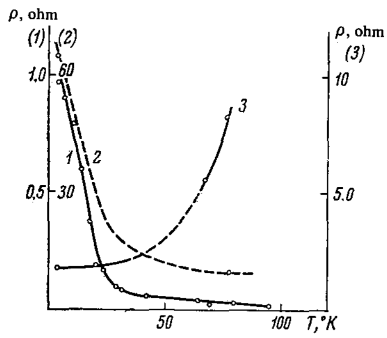

FIG. 11. Temperature dependence of ρ for a $Bi_{89.5}Sb_{10.5}$ sample for: 1–H = 0, 2–H < H_c, 3–H > H_c.

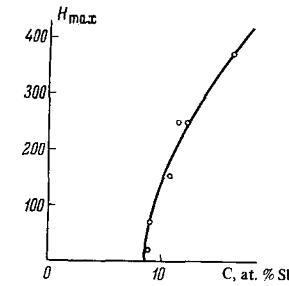

FIG. 12 Dependence of the field H_{max} on the antimony concentration.

of the isoenergy surfaces of the alloys and the resulting change in their contribution to the electrical conductivity when the current orientation is changed. It should also be borne in mind that the sharpness of the maximum and the magnitude of the subsequent resistance decrease is determined to a large extent by the quality of the samples; for this reason the position of maximum in different samples of the same composition may vary somewhat.

The depth of the local minimum depends also strongly on the quality of the sample. When a sample is destroyed the depth of the local minimum decreases gradually (Fig. 9). In the best samples the effect of the decrease of the resistance in a field is also observed at the liquid nitrogen temperature of 77°K (Fig. 10). However, the increase of the resistance with the field at this temperature is considerably smaller than at 4.2°K, the maximum is washed out, the local minimum disappears, the resistance begins to decrease at higher values of the magnetic field and decreases only by 30–40 percent. In less perfect samples the resistance at liquid nitrogen temperature increases monotonically with the field, exhibiting no tendency to saturation.

The longitudinal magnetoresistance was measured both in constant and pulsed magnetic fields. The field dependence of the longitudinal resistance when the field is oriented along the trigonal axis has the following features. The resistance increases in weak fields, passes through a maximum, decreases for C > 9.5 at. percent to some constant value (as a rule smaller than the value of ρ_0 for H = 0), and then again falls sharply when the field reaches a value close to H_c for the transverse magnetoresistance (see Fig. 8).

The nature of the temperature dependence of the electrical resistance of the samples changes qualitatively in fields larger than H_c. The dependence of ρ on T characteristic for semiconductors for H < H_c(C) becomes metallic for H > H_c (Fig. 11).

The appearance of the metallic dependence indicates that in a field H = H_c the energy gap in the spectrum of the alloys vanishes and for H > H_c there appears a band overlap that increases in the magnetic field. The values of the fields H_{max}(C) corresponding to a maximum on the $\rho_\perp$(H) curves are shown in Fig. 12. The dependence of H_{max} on C characterizes to a first approximation the variation of H_c in Bi-Sb alloys.

The minimum concentration of antimony in Bi-Sb alloys (C ~ 8.8 at. percent), starting with which the effect of the resistance decrease in a field appears, correlates well with the concentration for which there appears a gap in the energy spectrum of the alloys (see Fig. 5). Correlation is also observed between the dependences of H_c in the region of concentrations 8 < C < 9.5 at. percent and the magnitude of the gap ΔE in the corresponding range of concentrations.

The large negative value of the coefficient B_h [see formula (3) for holes in the extremum $\mathcal{T}$ in bismuth when the field is oriented parallel to the trigonal axis ($m_h^*/m_h^S \approx 2$)] as well as the correlation of the H_c(C) and ΔE(C) dependences for 8 < C < 9.5 at. percent noted above indicate that the decrease of the gap in semiconducting Bi-Sb alloys in a magnetic field parallel to the trigonal axis (leading to the appearance of overlap for H = H_c) is mainly connected with a rise of the extremum $\mathcal{T}$ in the magnetic field.

The most probable reason for the appearance of a local minimum on the transverse magnetoresistance curves and of a plateau-like section on the longitudinal magnetoresistance curves is the convergence of the extrema L_1 and L_2 which leads to the formation of a "quasimetallic" state that precedes the appearance of overlap of the extrema L_1 and $\mathcal{T}$.

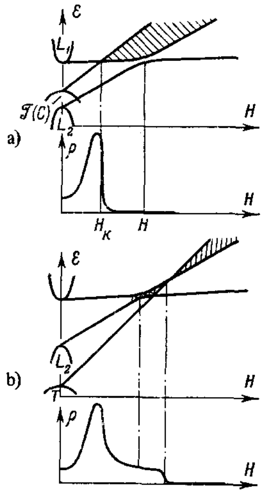

FIG. 13. Diagram of the shift of the band extrema of Bi-Sb alloy and the variation of ρ in a magnetic field with H ∥ C_3. a) C < 9.5 at. percent antimony; b) C > 9.5 at. percent antimony.

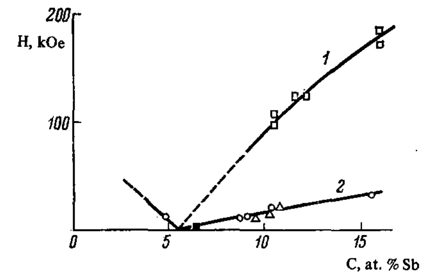

FIG. 14. Dependence of the magnetic field corresponding to the first resistance maximum due to the convergence of the extrema L_1 and L_2 on the antimony concentration. 1—H ⊥ i, 2—H ∥ i.

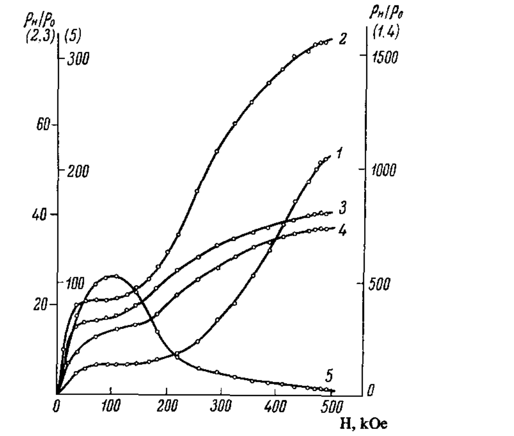

FIG. 15. Magnetic field dependence of the relative transverse resistance for H ∥ C_2 and i ∥ C_1. 1–8.8; 2–8.9; 3–10.5; 4–12; 5–15.8 at. percent antimony. On the left-hand ordinate axis the scale is indicated for curves 2 and 3 on the left, for curve 5—on the right. The right-hand ordinate axis indicates the scale for curves 1 and 4.

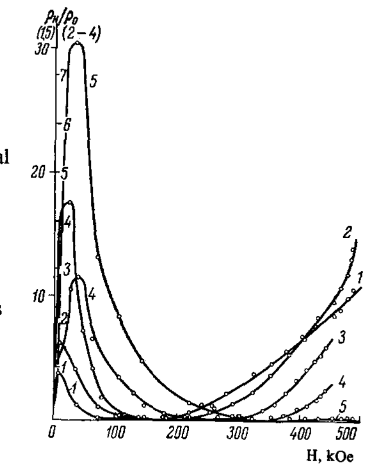

FIG. 16. Dependence of the relative change of the longitudinal resistance on the magnetic field for H ∥ i ∥ C_2. 1–8.8; 2–8.9; 3–10.5; 4–12; 5–15.8 at. percent antimony. The scale for curves 1 and 5 on the ordinate axis is indicated on the left and for curves 2–4 on the right.

All the above indicated features of the magnetoresistance of semiconducting Bi-Sb alloys in a magnetic field parallel to the trigonal axis are qualitatively explained by a shift of the extrema shown (in the linear approximation) in Fig. 13.

Alloys (8 < C < 9.5 at. percent) in which the extremum $\mathcal{T}$ is located between the extrema L_1 and L_2 (Fig. 13a) go over at once (for H = H_c) into the metallic state. The convergence of the extrema L_1 and L_2 in these alloys occurs for H > H_c and decreases the rate of increase of the overlap of the extrema L_1 and $\mathcal{T}$ in a magnetic field.

In alloys containing more than 9.5 at. percent antimony the extremum $\mathcal{T}$ is located (for H = 0) below the extremum L_2. Therefore in these alloys (Fig. 13b) on increasing the magnetic field a convergence of the extrema L_1 and L_2 takes place initially (the alloys go over to the "quasimetallic" state); this is followed by the overlap of the extrema L_1 and $\mathcal{T}$—a transition to the metallic state. The convergence of the extrema L_1 and L_2 leads to the first drop in the longitudinal magnetoresistance and to the appearance of a local minimum on the transverse magnetoresistance curves. The fact that the maximum of the longitudinal magnetoresistance $\rho_{\parallel}(H)$ is observed in weaker fields than the first maximum of the transverse resistance $\rho_{\perp}(H)$ (see Figs. 6–8) is not surprising. The maximum of $\rho_{\parallel}(H)$ corresponds to the beginning of a noticeable increase in the carrier density when the extrema L_1 and L_2 converge (since the mobilities change little in this case), and the maximum of $\rho_{\perp}(H)$ corresponds to an appreciable change in the transverse magnetoresistance, for the observation of which one requires a sufficiently strong increase of the carrier density (since the effect is masked by a strong decrease of the mobility in the field).

The fields $H_1'(C)$ and $H_1''(C)$ corresponding to the position of the first maximum on the transverse and longitudinal magnetoresistance curves are shown in Fig. 14 (curves 1 and 2).

If it is assumed that the rate of convergence of the extrema L_1 and L_2 does not depend on the antimony concentration, then the fields $H_2'(C)$ and $H_2''(C)$ should be proportional to the magnitudes of the gap $E_g(C)$ between the extrema L_1 and L_2 in the investigated alloys. When this assumption is justified, the extrapolation of the $H_1'(C)$ and $H_1''(C)$ dependences in the region of low concentrations should indicate the concentration for which the gap E_g becomes minimal (we recall that in bismuth E_g = 15 meV).

X. MAGNETORESISTANCE OF SEMICONDUCTING ALLOYS IN A FIELD PARALLEL TO THE BINARY AXIS

For this orientation of the magnetic field the transverse resistance increases in weak fields; this is fol-

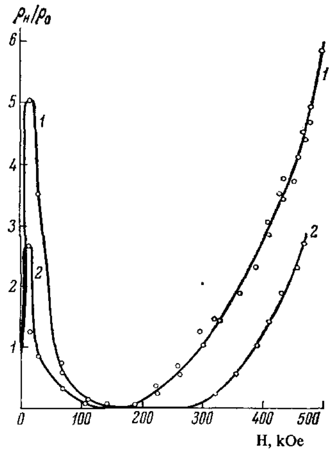

FIG. 17. Dependences of the relative change of the longitudinal resistance on the magnetic field for H ∥ i ∥ C_2. 1–T = 4.2°K; 2–T = 10°K.

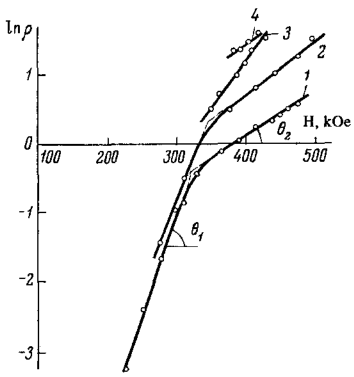

FIG. 18. Dependence of ln ρ on H for H ∥ i ∥ C_2 and $H > H_2$. 1–8.8; 2–8.9; 3–10.5; 4–15.8 at. percent antimony.

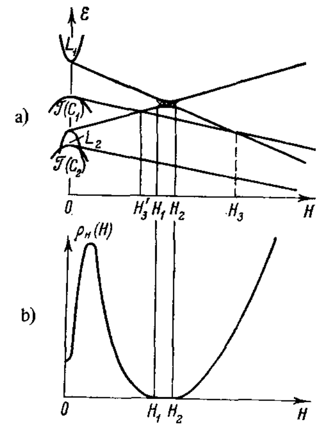

FIG. 19. a) Schematic diagram of the shift of the energy band extrema in Bi-Sb alloy in a magnetic field with H ∥ C_2. b) The variation of the longitudinal magnetoresistance for H ∥ C_2.

lowed by a slowing down of the increase, a plateau appears on the ρ(H) curves, after which the resistance again increases considerably (Fig. 15). In the strongest magnetic fields the dependences of ρ on H exhibit a tendency to saturation which manifests itself more clearly when the antimony concentration in the alloys is increased. In samples containing 15.8 at. percent antimony the ρ (H) curves pass through a maximum; however their resistance remains larger than ρ_0 (for H = 0), even in the strongest fields.

In a longitudinal field the resistance increases at first sharply and, having reached a maximum for $H = H_1(C)$, it drops to values lying within the accuracy of the measurements: ~0.1 ohm. With further increasing magnetic field the resistance remains very small up to a certain value of the field $H_2(C)$, above which it again increases with the field (Fig. 1b). On increasing the antimony concentration the maximum resistance and the second rising branch of the ρ(H) curve are shifted towards stronger fields. On increasing the temperature the maxima on the ρ(H) curves decrease and shift towards the region of weaker fields, whereas the second rising branch shifts towards stronger fields (Fig. 17).

A characteristic feature of the dependence of the longitudinal magnetoresistance after the minimum is the exponential nature of this dependence. The dependence of ln ρ on H is shown in Fig. 18. In samples with a concentration of $8 < C < 9.5$ at. percent antimony one observes on the ln ρ(H) curves a clear break which separates two linear sections. In samples with $C > 9.5$ at. percent antimony the rate of increase of the resistance in a magnetic field does not vary. The exponential character of the increase of the resistance indicates the appearance of a gap in the energy spectrum of the alloys whose magnitude increases with increasing magnetic field.

When the magnetic field is oriented along the binary axis the ratio ($m_h^*/m_h^s \approx 0.14$) in bismuth for holes in the extremum $\mathcal{T}$ is small, as a result of which the extremum $\mathcal{T}$ drops in a magnetic field. The spin splitting of the Landau levels in the extremum L_1 in bismuth exceeds the orbital splitting. Assuming that the ratio of the spin and orbital masses in Bi-Sb alloys up to 16 at. percent remains qualitatively the same, the extremum L_1 should drop in a magnetic field. Since the effective masses in the extremum L_1 should be close to the effective masses in the extremum L_2, one might expect that the extrema L_2 rise in a magnetic field. Such a motion of the extrema L_1 and L_2 should lead to the formation of a quasi-metallic state.

An explanation of the form of the dependences of the longitudinal and transverse magnetoresistance on the magnetic field in a field parallel to the binary axis can be illustrated by a scheme of the motion of band extrema shown in Fig. 19. The convergence of the extrema L_1 and L_2 for $H = H_1$ and their subsequent reflection for $H > H_2$ accompanied by an increase of the energy gap leads to the observed sequence of semiconductor-metal-semiconductor transformations. This is confirmed, first, by the nature of the change of the dependence of ρ on H when the temperature is raised (See Fig. 17). A temperature rise leads to an increase in the intensity of the thermal generation of carriers through the gap, as a result of which the region of decreased resistance becomes broader. Another confirmation is the exponential nature of the increase of ρ in fields $H > H_2$. The rate of increase of the resistance with the field for $H > H_2$ depends on the rate of change of the energy gap, which should in the case of reflection of the levels L_1 and L_2 be constant within a certain range of fields. This is accompanied by an exponential decrease in the number of carriers and the resistance increases exponentially:

$$\rho \sim \exp[\alpha_1 (H - H_2)/kT] \quad (H > H_2),$$

where α_1 is determined by the ratio of orbital and spin effective masses in the extrema L_1 and L_2. The presence of two linear sections with different slopes for alloys with concentrations of 8.8 and 8.9 at. percent antimony and the presence of a single linear section for alloys with a higher concentration of antimony is explained as follows. The slope $\theta_1 = \alpha_1/kT$ of the linear section of the dependence of $\ln \rho$ on H in weaker fields characterizes the rate of divergence of the extrema L_1 and L_2 for $H_2 < H < H_3$ (see Fig. 19). For the value of the magnetic field corresponding to the point of inflection of the logarithmic curve the energies of the extrema L and $\mathcal{T}$ coincide. The slope $\theta_2 = \alpha_2/kT$ of the second linear section of the dependence of $\ln \rho$ on H in fields above H_3 for samples with concentrations of 8.8 and 8.9 at. percent antimony characterizes the rate of separation of the extrema L_1 and $\mathcal{T}$.

On increasing the antimony concentration the extremum for H = 0 is shifted to a lower position, a fact which leads to an increase of the field H_3. For an antimony concentration above 8.9 at. percent the field H_3 becomes, apparently, higher than 500 kOe and the point of inflection on the logarithmic curve is not reached.

It is seen in Fig. 18 that the slope θ_1 decreases with increasing antimony concentration. This, apparently, indicates a change in the ratio of the cyclotron and spin effective masses with changing antimony concentration.

It follows from the model under consideration (see Fig. 19) that the slope of the logarithmic curve on the section H_3'—H_1 should have the same magnitude as on the section H_2—H_3, and should differ from the latter only in sign. However, the length of this section along the H axis is considerably smaller and it is therefore difficult to recognize this on the experimental curves.

The rate of change of the number of carriers in the range of fields from zero to H_3' is determined by the change in the gap between the extrema L and $\mathcal{T}$, which differs from the rate of change of the gap between the valence and conduction band in the remaining range of magnetic fields. It should be noted that this treatment does not take into account the possible change of the effective masses in the extrema L_1 and L_2 when they converge and the dependence of the carrier mobility on the magnetic field, and is therefore not exact.

In the case of the transverse magnetoresistance the strong dependence of the carrier mobilities on the magnetic field masks the effect of the increase of the number of carriers due to the convergence of the extrema L_1 and L_2. This effect is only revealed by the appearance of a plateau on the $\rho(H)$ curves (see Fig. 15).

The position H_1 of the first maximum on the curves of the longitudinal magnetoresistance determines the magnitude of the field preceding the transition to the "quasimetallic" state and should therefore characterize (on the basis of the above considerations) the change of the gap E_g in Bi-Sb alloys for H = 0. The dependence of the fields H_1 on the concentration (see Fig. 14) also indicates a minimum in the value of the gap E_g in Bi-Sb alloys for $C \sim 5$ at. percent antimony, which is in good agreement with the analogous data obtained for H parallel to the trigonal axis.

We note that the possible convergence of the extrema L_1 and L_2 in Bi-Sb alloys on increasing the antimony concentration was first considered in[28] and the values of the antimony concentrations calculated there that correspond the minimum gap E_g ($C \sim 6$ at. percent) are in good agreement with the results of our present work.

The exceptional character of the variation of the magnetoresistance in the $Bi_{84.2}Sb_{15.8}$ alloys (see Fig. 15) can be connected with the approach of extrema corresponding to the hole surfaces in antimony to the Fermi level.

XI. THE MAGNETORESISTANCE OF METALLIC Bi-Sb ALLOYS IN A FIELD PARALLEL TO THE BINARY AXIS

In investigating the semiconducting Bi-Sb alloys (in the range of concentrations C = 8–16 at. percent antimony when an energy gap appears between the extrema L_1 and $\mathcal{T}$) it was observed that when the field is oriented along the binary axis the longitudinal magnetoresistance passes through a maximum, decreases, and then again increases. Such a behavior of $\rho(H)$ was interpreted as a transition of the alloys from the semiconducting to the "quasimetallic" state, and from the "quasimetallic" state again to the semiconducting state, resulting from the convergence of the extrema L_1 and L_2.

It could be assumed that bismuth and the metallic Bi-Sb alloys (with an antimony concentration less than 8 at. percent) in which the extrema L_1 and $\mathcal{T}$ overlap should go over (when the magnetic field is oriented along the binary axis) in strong magnetic fields into the semiconducting (dielectric at T = 0°K) state (Fig. 20). In the range of fields $H < H_c'$ the extrema L_1 and $\mathcal{T}$ drop in a magnetic field. As a result of the fact that L_1 is shifted more rapidly than $\mathcal{T}$ the band overlap and the densities of the current carriers increase.[5,13] For $H > H_c'$ the nature of the displacement of the band boundaries changes qualitatively: the extremum $\mathcal{T}$ continues to drop (apparently, at the same rate) but the extremum L_1 begins to rise. As a result of this for $H = H_c''$ the overlap of the bands L_1 and $\mathcal{T}$ vanishes and for $H > H_c''$ an energy gap is formed and the metal is transformed into a semiconductor. Obviously, the field H_c'' is maximum in bismuth and should decrease in the alloys because of the decrease of the magnitude $E_0(C)$ of the overlap of the extrema L_1 and $\mathcal{T}$ for H = 0 (if the ratio of the spin and orbital masses does not change appreciably).

Figure 21 shows the curves of the longitudinal magnetoresistance of $Bi_{93.5}Sb_{6.5}$ (curve 1) and $Bi_{95.1}Sb_{4.9}$ (curve 2). In the upper part of the drawing we show the

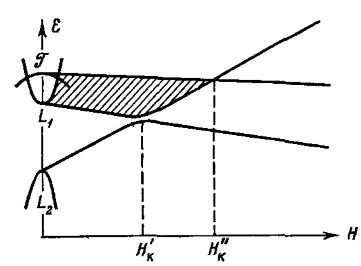

FIG. 20. Schematic diagram of the shift of the band extrema in metallic Bi-Sb alloys for H ∥ C_2.

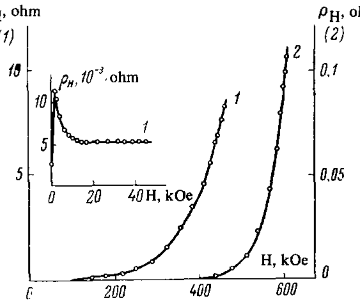

FIG. 21. Dependence of the variation of the longitudinal resistance in metallic $Bi_{93.5}Sb_{6.5}$ (curve 1) and $Bi_{95.1}Sb_{4.9}$ alloys (curve 2).

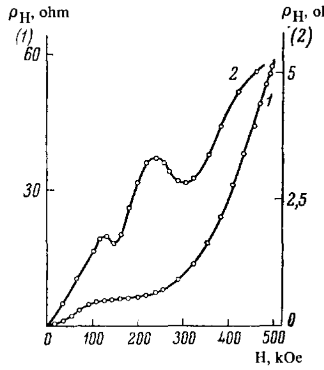

FIG. 22. Dependence of the variation of the transverse magnetoresistance for $Bi_{93.5}Sb_{6.5}$ (curve 1) and $Bi_{95.1}Sb_{4.9}$ samples (curve 2).

results of measurements on one of the samples in constant magnetic fields. The sharp increase of the resistance goes over into a decrease, after which the resistance retains an approximately constant value in a rather broad range of fields. In the $Bi_{93.5}Sb_{6.5}$ sample the resistance begins to increase sharply in a field of ~100 kOe. In a field of ~480 kOe the longitudinal resistance increases by a factor of 3000. An analogous increase of the resistance in the $Bi_{95.1}Sb_{4.9}$ sample takes place in fields exceeding 400 kOe. In a field of 600 kOe the longitudinal magnetoresistance increases by a factor of 100.

Figure 22 shows the dependences of the transverse magnetoresistance for samples of the same concentrations. In the $Bi_{93.5}Sb_{6.5}$ sample a characteristic feature of the curve is a region of retarded increase of the resistance in a magnetic field. In stronger fields the resistance increases sharply. On raising the temperature the increase of the magnetoresistance decreases and the increase begins in larger fields.

In the $Bi_{95.1}Sb_{4.9}$ sample in fields less than 400 kOe one observes on the $\rho_\perp(H)$ curve oscillations (the Shubnikov-de Haas effect) which distort strongly the monotonic component of the magnetoresistance.

We note that in all cases the resistance in strong magnetic fields increases exponentially:

$$\rho \sim \exp[\alpha(H - H''_c)/kT] \quad (\text{for } H > H''_c),$$

with similar values of the coefficient α. The metallic nature of the temperature dependence of the resistance for $H < H''_c$ goes over to the semiconductor dependence for $H > H''_c$.

The exponential increase of the magnetoresistance for $H > H''_c$, the appearance for $H > H''_c$ of the semiconductor temperature dependence of the electrical resistance, the dependence of the field H''_c on the

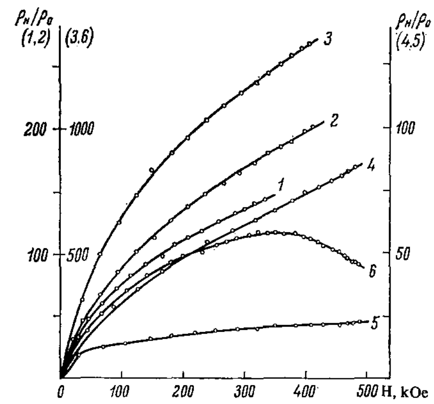

FIG. 23 Dependences of the relative change of the transverse resistance on the magnetic field for H ∥ C_1 and i ∥ C_2. 1–8.8; 2–8.9; 3–9.1; 4–10.5; 5–12; 6–15.8 at. percent antimony. The scale for curves 1 and 2 is indicated on the left-hand ordinate axis on the left, for curves 3 and 6—on the right; the scale for curves 4 and 5 is indicated on the right-hand axis.

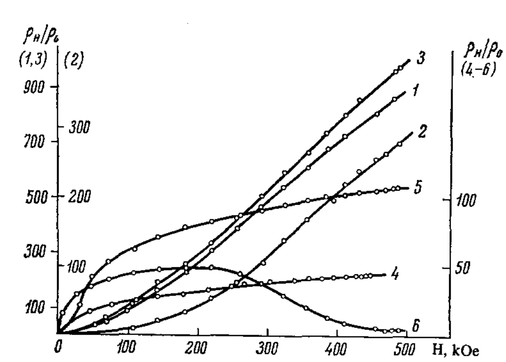

FIG. 24. Dependences of the relative change of the longitudinal resistance on the magnetic field with H ∥ i ∥ C_1. 1–8.8; 2–8.9; 3–9.1; 4–10.5; 5–12; 6–15.8 at. percent antimony. The scale for curves 1 and 3 is indicated on the left-hand ordinate axis on the left, for 2—on the right; the scale for curves 4–6 is indicated on the right-hand ordinate axis.

composition of the alloy [the magnitudes of the overlap $E_0(C)$ of the bands for $H = 0$] indicate that in a field H''_c the band overlap in the energy spectrum disappears and an energy gap appears for $H > H''_c$. At the same time the metallic Bi-Sb alloys go over to the semiconducting state. A preliminary estimate shows that in order to observe an analogous transition in bismuth for a magnetic field orientation along the binary axis one requires fields of about 1.5 MOe.

It is interesting to note that the position of the maximum connected with the convergence of the extrema L_1 and L_2 on the longitudinal magnetoresistance curve in $B_{93.5}Sb_{6.5}$ fits well on a curve obtained by extrapolating the dependence of H_1 on C for semiconducting alloys (see Fig. 14).

XII. THE MAGNETORESISTANCE OF SEMICONDUCTING Bi-Sb ALLOYS IN A FIELD PARALLEL TO THE BISECTRIX AXIS

For this orientation of the magnetic field one observes in all the investigated samples, with the exception of those of the alloy containing 15.8 at. percent antimony, a monotonic increase of both the transverse

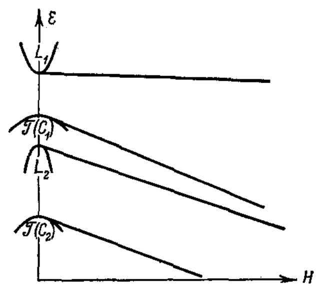

FIG. 25. Schematic diagram of the band shift in a magnetic field for Bi-Sb alloys with H || C_1.

and longitudinal magnetoresistance (Fig. 23 and Fig. 24). As the antimony concentration increases there appears an increasing tendency of the magnetoresistance towards saturation. For a concentration of 15.8 at. percent antimony the dependences of ρ on H have a maximum for H ~ 250 kOe in a longitudinal field (the current is parallel to the bisectrix axis) and for H ~ 350 kOe in a transverse magnetic field (the current is parallel to the binary axis).

For the field oriented parallel to the bisectrix axis one should note the unusually large increase of the resistance in a longitudinal magnetic field (the longitudinal effect is comparable in order of magnitude with the transverse effect).

The monotonic nature of the field dependence of the longitudinal and transverse resistance (with the exception of the alloy containing 15.8 at. percent antimony in which one can, as was indicated above, observe effects connected with the approach of extrema corresponding to the hole surfaces in antimony to the Fermi level) attests to the absence of electronic transitions attended by a fundamental change of the energy spectrum.

The possible nature of the shift of the extrema for this orientation is shown in Fig. 25. The increase of the gap between the bands and the related decrease in the carrier density are the reason for the anomalously large increase of the longitudinal resistance in a magnetic field. It should, however, be noted that the scheme for the shift of extrema shown in Fig. 25 does not agree with the known ratios of the spin and orbital masses in bismuth (see Table I).

XIII. CONCLUSION

The various types of electronic transitions in strong magnetic fields considered and observed are obviously not a specific and sole property of metallic and semiconducting Bi-Sb alloys. Transitions of this type should also be observed (in a readily attainable range of magnetic fields) in all materials with a small energy gap between the bands or with small band overlap, for example in the ternary alloy systems Hg-Cd-Te and Hg-Te-Bi. If the ratio of the spin and cyclotron effective masses in the metallic phases of these materials is such that the band overlap increases in a field, then in the corresponding semiconducting phases a transition should occur in a field into the metallic state, and, vice versa, if in the semiconducting phase the gap increases in a field, then the corresponding metallic phases should go over in a magnetic field to the semiconducting state. With increasing limits of attainable fields the class of materials in which it will be possible to observe electronic transitions will, of course, become more extensive.

Interesting changes of the magnetic, thermal, mechanical, and optical properties of materials should occur during electronic transitions in a magnetic field. Particularly interesting, from our point of view, is the investigation of materials in the "quasimetallic" state. In this state a material may have a whole series of specific properties which are not only of scientific but also of applied significance.

Note added in proof. Recently obtained more accurate data on the nature of the change of the energy spectrum of the Bi-Sb alloy are presented in the papers: 1) N. B. Brandt, H. Dittman, Ya. G. Ponomarev, and S. M. Chudinov, Semiconductor-"quasimetal"-semiconductor Transition in $Bi_{1-x}Sb_x$ Alloys Under the Influence of Pressure, ZhETF Pis. Red. 11, 250 (1970)[JETP Lett. 11, 160 (1970]. 2) N. B. Brandt et al., Electronic Transitions in Bismuth-antimony Alloys with a High Antimony Concentration in Strong Magnetic Fields, Zh. Eksp. Teor. Fiz. (1970).

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