tuations of the phase of the wave are valid even at even larger fluctuations of the level.

To describe the propagation of waves in media with large inhomogeneities one uses, besides the MSP and the method of geometrical optics also the method of parabolic equation (MPE).[8–10] This method was recently developed further as a result of the uses of the so-called Markov approximation.[11] This method makes it possible to go beyond the limits of the region of weak level fluctuations (although it is less convenient than the MSP in those cases when direct interest attaches to the statistical characteristics of the phase of the wave).

All the foregoing methods are constructed from the very outset as approximate ones. Much interest is evinced at present to the construction of an exact wave theory of multiple scattering. In this case equations are formulated for the average field and for its correlation function (the Dyson and the Bethe–Salpeter equations.[3, 7, 12–14] To be sure, they can be solved only in a number of particular cases, but the general formulation of the problem as given by them makes it possible to hope for further refinement of both the concrete results and of the comparative estimate of the approximate methods. Among the accomplishments of the general theory of multiple scattering is, for example, the consistent wave-theory derivation of the radiation-transfer equation,[15] which heretofore was derived on the basis of purely energy considerations. In other respects, the approximate methods mentioned above remain the most effective working apparatus for the investigation of concrete problems.

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Yu. A. Kravtsov. Geometrical-optics Method and Its Generalizations.

The method of geometrical optics occupies a definite place in the wave theory, and it might seem makes no claim at describing any diffraction phenomena.

However, during the last 10–15 years, the approach to the method has changed. Many attempts were made to revise the established concepts concerning the limits of applicability of the method. An idea is developing, or has already developed, that the geometrical-optics method contains much more than an intuitive representation of lines in space (rays), along which the field energy propagates, and that the formal solutions of the geometrical-optics equations contain definite information concerning diffraction processes. The present paper is devoted to a review of two extensions of this kind—the complex form of the method of geometrical optics and the asymptotic methods of finding the field in the vicinity of caustics. Both these extensions have been brought about by problems in which it is necessary to deal with an approximate description of the field in the presence of caustics. Such problems became timely recently in optics, radiophysics, plasma theory, acoustics, and in part in quantum mechanics.

As is well known, in the region of the caustic shadow, light rays do not penetrate and the usual ray approximation yields a zero value of the field there. Actually, the field in the shadow region differs from zero, although it is exponentially small. The complex form of the ray method is precisely intended for finding the field in this case. In addition, it is suitable also for the description of fields in strongly absorbing media.

The "complex" geometrical optics differs from the ordinary one in that one operates not with real but with complex rays, which are defined as complex solutions of the ray equations.[1–4] The amplitude and the phase are then determined in the form of quadratures on complex trajectories. The only difference from the ordinary ray method lies perhaps only in the fact that one introduces the so-called "selection rules" of complex rays. According to these rules, out of all the trajectories one selects only the "physical" branches corresponding to the damping of the field with increasing distance from the caustic (for details see [5]). By the same token it becomes possible to describe the diffractive penetration of the field into the shadow region. Neither the real nor the complex form of the ray method is suitable in the vicinity of the caustic, since they give infinite values of the field. Recently developed asymptotic methods lift this limitation of the geometrical-optics approximation.

In the method of standard functions one starts from the fact that the sought solution of the wave problem is expressed in terms of suitably chosen known functions with indeterminate arguments and amplitude factors.

In the simplest case of a smooth caustic without loops and kinks (the so-called simple caustic) it is natural to use as the stated function the Airy function v(t) (more accurately, the Airy function and its derivative in combination with an oscillating exponential function).[6-8] It turns out that the amplitude factors in front of the Airy function and its derivative, which are to be determined, as well as the arguments of the Airy function and the exponentials, are expressed algebraically in terms of the geometrical-optics solutions (i.e., in terms of the pair of solutions of the eikonal equation $(\nabla\psi)^2 = \epsilon$ and the transport equation div $(A^2 \nabla\psi) = 0$). At a distance away from the caustic, in the light region, such a standard solution goes over into a sum of incident and reflected waves of the ray approximation, while in the shadow region it goes over to the damped wave of the "complex" geometrical optics. However, unlike these geometrical expressions, the standard solution is finite on the caustic.

For caustics of more complicated form, the standard formulas can be suitably complicated, but the main "geometrical" statement remains in force: all the stated solution parameters to be determined are expressed in terms of solutions (both real and complex) of the geometrical-optics equations.[7, 8]

Similar results are obtained also from the asymptotic integral representations proposed by Maslov.[9] These representations are obtained from an analysis of the wave problems in a mixed coordinate-momentum space, but the obtained diffraction solutions also turn out to be "tied in" with the rays.

In addition, the paper describes briefly two other more known generalizations of the method of geometrical optics: the method of parabolic equation (the diffusion approximation)[10, 11] and the geometrical theory of diffraction developed by Keller and his followers.[1, 12] In these generalizations, the geometrical "skeleton" also plays an exceptionally important role.

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L. L. Goryshnik and Yu. A. Kravtsov. Correlation Theory of Radio Wave Scattering in the Polar Ionosphere.

The paper is an exposition of the correlation theory of scattering of radio waves in the polar ionosphere. A derivation is presented of a general expression for the space-time correlation function of the signals of auroral radio reflections, taking into account the main features of the polar ionosphere as a scattering medium and the main features of the emission and reception of pulsed sounding signals.

With the aid of the obtained expressions for the signals of radio reflections and their correlation functions, three questions are considered: 1) the study of the correlation properties of signals of radio reflections following the emission of either a single short pulse or a sequence of short pulses, 2) analysis of the operation of a spectrum analyzer measuring the frequency spectrum of the fluctuations of the electron concentration in the ionosphere, and 3) an estimate of the change of the polarization of the signal as it propagates and is scattered in the polar ionosphere.

Comparison of the results of the theoretical estimates of the change of the polarization with the experimental data shows good agreement.

Thus, the developed theory makes it possible to study, even without specifying concretely a model for the scattering medium, both some properties of radio reflections, and some principal possibilities of different methods of measuring the statistical properties of fluctuations of the parameters of the polar ionosphere. In addition, the general expression for the space-time correlation functions will undoubtedly be useful for the solution of the fundamental problem of the study of the auroral radio reflections, namely the clarification, on the basis of the properties and the statistics of the signal of the radio reflections, of the true structure and the time behavior of the inhomogeneities of the polar ionosphere.

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Z. I. Feĭzulin. Propagation of Bounded Wave Beams in Media with Random Inhomogeneities.

The paper considers two groups of problems: 1) the calculation of the fluctuations of the amplitude and of the phase in a spatially-bounded waveguide beam propagating in a medium with random inhomogeneities, and 2) investigation of the influence of the fluctuating medium on the shape of the bounded beam.

The mean value and the correlation characteristics of the complex phase in a bounded beam of electromagnetic radiation with a Gaussian amplitude distribution over the cross section, propagating in a medium with a fluctuating dielectric constant, are obtained in the approximation of the method of smooth perturbations. Values are obtained for the scales of the transverse correlation of the level of the amplitude for a model of a medium with a Gaussian correlation function of the fluctuations, and for a locally-isotropic turbulent model,