

FIG. 1

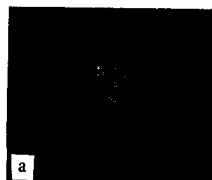


FIG. 2

and the convection on the self focusing, we used a cw argon laser of 0.3 W power. In the case of horizontal passage of a tubular beam through a cell with a liquid, deformation took place of the outer boundary of the beam, owing to convection, but this does not prevent formation of the bright spot of self-focusing (Fig. 2a). The time of establishment of the stationary picture was a fraction of a second. Figure 2b shows the trace of a beam passing vertically through the cell with the liquid; to the left of it is a control beam passing outside the cell.

The liquids investigated were methyl iodide, solution of iodine in alcohol, etc. Color film was used to investigate the dynamics of the process.

No self-focusing was produced when the cell with the liquid moved across the beam to one side or else in reciprocating motion at a velocity on the order of several centimeters per second.

The described experiments open up a possibility of controlling nonlinear optical effects by choosing the profile of the beam-intensity distribution. These effects can appear spontaneously in the presence of regions of decreased intensity in the laser beam, owing to the inhomogeneities of the generation power pumping, in the region of the shadow behind the absorbing or scattering centers in the medium, they can increase the intensity of the light on the beam axis when incident on a semi-infinite medium, etc.

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Yu. N. Barabanenkov, Yu. A. Kravtsov, S. M. Rytov, and V. I. Tatarskiĭ. Status of the Theory of Propagation of Waves in Randomly-inhomogeneous Media.

The investigation of wave propagation in randomly-inhomogeneous media has recently attracted more and more attention. The increased interest in this problem

is due primarily to the large number of timely applied problems that have arisen in radiophysics, acoustics, optics, plasma theory, etc. The variety of these problems has stimulated the development of various methods for calculating the statistical parameters characterizing the wave field propagating in a randomly-inhomogeneous medium or passing through such a medium. The results obtained by these methods were partly summed in a number of monographs^[1-5] and reviews,^[6,7] but since the time of publication of these works, the statistical theory of scattering and propagation of waves has experienced definite changes.

First, new methods were developed for calculating the fluctuations of the wave field, for example the Markov approximation in the method of parabolic equation,^[8-11] and procedures borrowed from other branches of physics, particularly methods of summing series of perturbation theory, analogous to those used in quantum electrodynamics,^[3,7,12-14] have been developed. Second, "older" methods have been improved, making it possible to use them for the description of a larger group of phenomena than heretofore. Third, in many cases it became possible to refine the limits of applicability of different calculation methods, which, of course, contributed to the clarification of the entire picture.

In this review, the authors have attempted to give an idea of the existing methods of the theory of wave propagation in randomly-inhomogeneous media, the limits of their applicability, and the role of the recently developed new methods. The most clearly outlined are the limits of applicability of the theory of single scattering of waves (the Born approximation). It is suitable under conditions when the total intensity of the scattered field is small compared with the intensity of the incident wave. In many problems this condition is satisfied, making it possible to use all the advantages of the Born approximation, namely, to take into account the set of effects accompanying the scattering under complicated conditions (the presence of regular refraction, anisotropy of the medium, the pulse character of the scattered signal, etc.). It is important that the Born approximation is applicable to scattering both by small inhomogeneities (compared with the wavelength) and by large ones, but when the dimensions of the inhomogeneities are increased the region of applicability of this approximation decreases.

In contrast, the method of smooth perturbations (MSP for short) and the related method of geometrical optics are suited for the description of fluctuations of the wave field precisely in media with large-scale inhomogeneities. The question of the limits of applicability of the MSP has been discussed many times in the literature. At the present time it can be regarded as established that for media whose parameter fluctuations are characterized by a single scale, the first approximation of the MSP (just as the Born approximation) is suitable only at sufficiently small fluctuations of the phase of the wave and its level (the logarithm of the amplitude). On the other hand, if there is an entire spectrum of scales, and the large inhomogeneities are most strongly represented (as is the case, e.g., in the turbulent atmosphere), then the first MSP approximation is suitable up to mean-square fluctuations of the level of the order of unity, and the results concerning the fluct-

tuations of the phase of the wave are valid even at even larger fluctuations of the level.

To describe the propagation of waves in media with large inhomogeneities one uses, besides the MSP and the method of geometrical optics also the method of parabolic equation (MPE).^[8-10] This method was recently developed further as a result of the uses of the so-called Markov approximation.^[11] This method makes it possible to go beyond the limits of the region of weak level fluctuations (although it is less convenient than the MSP in those cases when direct interest attaches to the statistical characteristics of the phase of the wave).

All the foregoing methods are constructed from the very outset as approximate ones. Much interest is evinced at present to the construction of an exact wave theory of multiple scattering. In this case equations are formulated for the average field and for its correlation function (the Dyson and the Bethe-Salpeter equations.^[3, 7, 12-14] To be sure, they can be solved only in a number of particular cases, but the general formulation of the problem as given by them makes it possible to hope for further refinement of both the concrete results and of the comparative estimate of the approximate methods. Among the accomplishments of the general theory of multiple scattering is, for example, the consistent wave-theory derivation of the radiation-transfer equation,^[15] which heretofore was derived on the basis of purely energy considerations. In other respects, the approximate methods mentioned above remain the most effective working apparatus for the investigation of concrete problems.

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Yu. A. Kravtsov. Geometrical-optics Method and Its Generalizations.

The method of geometrical optics occupies a definite place in the wave theory, and it might seem makes no claim at describing any diffraction phenomena.

However, during the last 10-15 years, the approach to the method has changed. Many attempts were made to revise the established concepts concerning the limits of applicability of the method. An idea is developing, or has already developed, that the geometrical-optics method contains much more than an intuitive representation of lines in space (rays), along which the field energy propagates, and that the formal solutions of the geometrical-optics equations contain definite information concerning diffraction processes. The present paper is devoted to a review of two extensions of this kind—the complex form of the method of geometrical optics and the asymptotic methods of finding the field in the vicinity of caustics. Both these extensions have been brought about by problems in which it is necessary to deal with an approximate description of the field in the presence of caustics. Such problems became timely recently in optics, radiophysics, plasma theory, acoustics, and in part in quantum mechanics.

As is well known, in the region of the caustic shadow, light rays do not penetrate and the usual ray approximation yields a zero value of the field there. Actually, the field in the shadow region differs from zero, although it is exponentially small. The complex form of the ray method is precisely intended for finding the field in this case. In addition, it is suitable also for the description of fields in strongly absorbing media.

The "complex" geometrical optics differs from the ordinary one in that one operates not with real but with complex rays, which are defined as complex solutions of the ray equations.^[1-4] The amplitude and the phase are then determined in the form of quadratures on complex trajectories. The only difference from the ordinary ray method lies perhaps only in the fact that one introduces the so-called "selection rules" of complex rays. According to these rules, out of all the trajectories one selects only the "physical" branches corresponding to the damping of the field with increasing distance from the caustic (for details see^[5]). By the same token it becomes possible to describe the diffraction penetration of the field into the shadow region. Neither the real nor the complex form of the ray method is suitable in the vicinity of the caustic, since they give infinite values of the field. Recently developed asymptotic methods lift this limitation of the geometrical-optics approximation.

In the method of standard functions one starts from the fact that the sought solution of the wave problem is expressed in terms of suitably chosen known functions with indeterminate arguments and amplitude factors.

In the simplest case of a smooth caustic without loops and kinks (the so-called simple caustic) it is natu-