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L. V. Keldysh. Electron-hole Drops in Semiconductors.

At sufficiently low temperatures, the non-equilibrium electrons and holes introduced into a pure semiconductor are bound together into excitons—systems similar to positronium, but differing from it in having macroscopically large Bohr radii ($a_0 \sim 10^{-6}$ cm) and very low binding energies ($\epsilon_0 \sim 10^{-2}$ eV).

Such a change in the length and energy scales in a system coupled by Coulomb forces is due to the decrease of the Coulomb interaction as a result of the large dielectric constants of the semiconductors, $\kappa \geq 10$, and the small effective masses of the electrons and holes, $m \sim 0.1\, m_0$ (m_0—mass of free electron). Substitution of these values into the known Bohr formulas for the binding energy and the radius of the hydrogenlike atom

$$\varepsilon_0 = e^4 m/2\varkappa^2\hbar^2, \qquad a_0 = \varkappa\hbar^2/me^2$$

leads to the estimates indicated above. An increase of the length scale by two orders of magnitude and a decrease of the energy scale by three orders of magnitude compared with the length and energy scales in ordinary substances is characteristic also of all the phenomena considered below which occur in a system of electrons and holes in a semiconductor. In particular, the criterion of high exciton density, wherein an important role is assumed by the interaction between them, corresponds obviously to concentrations $n_0 \sim a_0^{-3} \sim 10^{18}$ cm^{-3}, and the region of temperatures at which all these phenomena should be observed is $kT \lesssim 0.1\epsilon_0$, i.e., $T \lesssim 10°$ K.

If the electron concentration is large enough, the interaction between them can lead to a "liquefaction" of the exciton gas,[1] i.e., to the formation of a relatively dense electron-hole phase, in which all the particles are coupled by mutual attraction forces and the average distance between them is of the order of a_0, while their concentration is $n_0 \sim a_0^{-3} \sim 10^{17}$–$10^{18}$ cm^{-3}. This phase differs from the usual electron-hole plasma in semiconductors in the same manner as liquid metals (e.g., mercury) differ from an electron-ion plasma: it is contained by internal forces and has a perfectly well defined equilibrium density n_0. It does not diffuse over the entire sample, and occupies only that part of the sample volume which can be uniformly filled with a density n_0 at a specified total number of electrons and holes introduced into the sample. The transition from the gas of free excitons to the electron-hole "liquid" should have many characteristic features of a first-order phase transition. In particular, when the average exciton concentration in the sample reaches a certain value $n_c(T)$ that depends on the temperature T ($n_c(T) \ll n_0$ at sufficiently low temperatures), the system should become laminated into two phases: regions filled with the liquid phase—"drops"—with density n_0, and regions filled with an exciton gas having a much lower density. With further increase of the number of electrons and holes introduced into the sample, the volume of the liquid phase increases, but its density n_0 does not change so long as it does not fill the entire sample. A rigorous theoretical investigation of the properties of the liquid phase entails considerable difficulties, but its main properties can be predicted from general considerations. The absence of heavy ions from the sample makes it impossible to produce in such a phase any spatial ordering such as crystallization at arbitrary temperatures, since the amplitudes of the zero-point oscillations of the particles should be of the order of a_0, i.e., of the average distance between the particles. For the same reason, it is not very likely that such a liquid phase can consist of exciton molecules—biexcitons. The large zero-point oscillations and the low coupling energy of the biexciton should lead to an intense interaction of each particle with all the nearest neighbors, to a strong electron exchange, and as a consequence to a collectivization of all the electrons and holes. Therefore the phase under consideration is more likely to be similar to a liquid metal.

The electron-hole drops in pure semiconductors should have quite high mobility, since the scattering of the electrons and of the holes by the phonons, which is sufficiently small at low temperatures to start with, is suppressed even more by the presence of Fermi degeneracy in the drop, and the density of the effective mass in the drop is very small. Therefore such external actions as inhomogeneous deformations or inhomogeneous magnetic fields can relatively easily accelerate the drops to velocities of the order of the velocity of sound. It is not very likely that the drop can exceed this velocity, owing to the coherent emission of phonons. How-

ever, even these velocities suffice to move the drops over a distance on the order of several centimeters during the lifetime of the non-equilibrium electrons and holes.

An electron-hole liquid can acquire properties such as superfluidity or superconductivity. In the special case when the effective masses of the electrons and the holes are almost isotropic, it can be proved[3] that when the temperature is decreased there occurs a collective binding of the electrons with the holes, leading to the transition of the liquid into a dielectric superfluid state. In the presence of noticeable anisotropy, the question of the feasibility of such a transition remains open. In order for superconductivity to occur it is necessary that electrons be attracted to electrons (or holes to holes). Theoretically it is impossible to prove reliably or refute reliably the existence of such an attraction. Qualitatively it is possible as a result of the interaction of two electrons and one hole (or two holes) or with the vibrations of the liquid density ("phonon").

Finally, it should be noted that theoretically one cannot exclude one more possibility of the behavior of the system of excitons with decreasing concentration, an alternative to that described above. If the exchange repulsion of the biexcitons prevails over the Van der Waals attraction and if the coupling energy per particle in the biexciton is larger than in the liquid phase, then a Bose-condensed superfluid gas of biexcitons can exist at low temperatures,[4] and the coupling energy of the biexcitons tends to zero only gradually with increasing concentration.[5] Even if the coupling in the biexcitons is weaker than in a liquid, but at distances larger than a_0 repulsion between them prevails, there can exist a Bose condensate of biexcitons as a certain metastable phase. For "atomic" excitons, such a behavior is impossible, owing to the predominance of the attraction forces.

Until recently, the considerations advanced concerning the "liquefaction" of the excitons[1, 2] remained hypothetical to a considerable degree. A "metallization" of excitons was observed,[6] but the character of this transition remained unclear. During the last year and a half, however, there appeared a large number of experimental papers whose results are interpreted by their authors as the observation of drops of electron-hole liquid in such semiconductors as germanium and silicon.

In [7] they investigated the absorption of light in pure germanium in the exciton lines (in the region of the so-called direct exciton) at helium temperatures, as a function of the number of electron-hole pairs introduced into the sample. Analyzing their results, the authors of [7] reached the conclusion that in the concentration region $\bar{n} \sim 10^{15}-10^{16}$ cm^{-3} (here and below $\bar{n}$ denotes the concentration of the electrons and holes averaged over the volume without allowance for the possible lamination into phases), the sample breaks up into regions in which the exciton absorption remains practically constant, and regions in which the exciton line vanishes completely as a result of the screening of the Coulomb interaction between the electron and the hole, which appear in this region as free charges ("metallic" regions). An estimate of the equilibrium concentration n_0 in the drops of the liquid metallic phase, according to the data of [7], yields $n_0 \sim 2 \times 10^{16}$ cm^{-3}. It is shown in [8] that in the spectrum of the recombination radiation of the electron-hole pairs in germanium at low temperatures $T \lesssim 4.2°$ K, besides the usual emission line of the free excitons, after a certain critical electron and hole concentration is reached there appears a new line, corresponding to lower energies of the photon emitted upon recombination, i.e., to larger coupling energy between the electrons and the holes. With further growth of the concentration or with decreasing temperature, this line becomes rapidly dominating in the emission spectrum. The position and shape of this line, and also its practically complete independence of a great variety of impurities when their concentration is varied from values much smaller than n to values much larger than n, do not make it possible to ascribe this emission to any known impurities. At the same time, all its characteristic features, including the concentration and temperature dependences, as shown in [8], can be satisfactorily explained if this radiation is ascribed to recombination of the electrons and holes in the drops of the liquid phase of the type considered above with an equilibrium concentration $n_0 \approx 2 \times 10^{17}$ cm^{-3}. An analogous radiation was observed in silicon in [9, 10]. Further confirmation of the foregoing point of view was obtained in [11]. It turned out that under conditions of uniaxial compression of the germanium crystal, the above-described emission line behaves in a completely anomalous manner: whereas the lines of the free excitons and the different impurity levels, which fall in this energy interval, shift upon deformation in almost identical fashion (the binding energies do not change), the line ascribed to the emission of the drops is hardly displaced at small deformations along the ⟨111⟩ axis, so that its distance from the exciton line (the coupling energy) decreases by a factor of two, and only then, at still larger deformations, does it begin to shift together with all the other lines, i.e., no further decrease of the coupling energy takes place. Such a behavior of the coupling energy in the metallic liquid phase is attributed in [11] to the fact that in germanium, by virtue of the known singularities of the structure of its electron spectrum, deformation along the ⟨111⟩ axis decreases by a factor of four the density of the electronic states near the bottom of the conduction band. At a specified concentration of the electrons n_0, this would lead to a growth of the Fermi energy and of the electron-gas pressure, i.e., in order to maintain the equilibrium, the value of n_0, and with it also the binding energy per particle, should decrease (the drop "expands"). Whenever the deformations were not homogeneous through the sample, a number of new anomalies was observed, particularly a catastrophic drop (by two orders of magnitude) of the emission intensity in the described line. These anomalies can be interpreted as an indirect indication of the acceleration of the drops by the deformation gradients (the energy of the drop depends on the deformations). The drop of intensity of the radiation is in this case explained by the fact that the nuclei of the drops of the liquid phase, having time to grow to the equilibrium value determined by $\bar{n}$ and T, go off from the region where the non-equilibrium electrons and holes are produced and where consequently the drops can grow. Finally, it was shown in [12] that under the same conditions ($\bar{n}$ and T) at which

there appears the recombination-radiation line ascribed in [8, 11] to drops of an electron-hole "liquid," pure germanium begins to absorb in the far infrared region, where heretofore it was perfectly transparent. This absorption has, as a function of the wavelength λ, a distinct maximum in the region $\lambda \sim 100\,\mu$, which was interpreted as plasma resonance in the absorption (or scattering) by metallic drops whose linear dimensions are much larger than the wavelength λ. From the position of this resonance it is possible to estimate directly the concentration n_0 of the particles in the drop. It also turned out to be $\approx 2 \times 10^{17}$ cm^{-3}.

Thus, at the present time there is an entire series of facts that agree satisfactorily with the hypothesis that a condensed electron-hole phase exists in semiconductors. Some of these facts can be explained just as well as being due to the fact that at low temperatures the excitons become bound into "molecules" (biexcitons).[9] However, within the framework of the biexciton picture, there is still no satisfactory explanation of such facts as the absorption in the infrared region, the anomalous behavior of the radiation under uniaxial deformations, and the vanishing of the absorption line of the direct exciton. Therefore the existence of condensed-phase drops seems to be quite likely, but only further experiments can prove it (or refute it) conclusively. Such convincing experiments might be, for example, direct observation of the motion of the drops over a macroscopic distance, or scattering of light by these drops.

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V. N. Lugovoĭ and A. M. Prokhorov. Self-focusing of Intense Light Beams.

The phenomenon of self focusing[1] of intense light beams occurs in media whose refractive index depends on the intensity of the light. The dependence of the refractive index on the light intensity can be connected with different physical processes in a material medium. The largest attention was immediately attracted by the so-called Kerr self focusing,[2-15] in which the dependence of the refractive index on the amplitude $|\mathbf{E}|$ of the oscillations of the electric field is due to the Kerr effect: $n = n_0(1 + \frac{1}{2} n_2 |\mathbf{E}|^2)$, $n_2 > 0$. The self focusing phenomenon itself begins with the fact that the beam produces in the initial layer of the medium a distributed lens, which then focuses this beam if its initial power P exceeds a certain critical value P_{cr}.[2, 8] In 1964, Chiao, Garmire, and Townes[2] proposed that a stationary (in time) regime of waveguide propagation of the beam is produced behind the focusing point, in the form of thin filaments with large energy density (self-focusing filaments). Then in [3] and in later papers[4-7, 13-15] it was reported that self-focusing filaments were observed. After this, the concept of stationary waveguide propagation of a laser beam in a nonlinear medium became universally accepted in spite of its partly postulative character (from the theoretical point of view).

For a theoretical solution of this problem, without using any arbitrary assumptions concerning the behavior of the beam behind the focusing point, it is obviously necessary to investigate in sufficient detail the evolution of the light beam propagating in the interior of a nonlinear medium at a specified distribution in the initial plane ($z = 0$), corresponding to the real conditions. If the distribution in the initial plane is assumed to be stationary, then the field in the medium will also be stationary. Then the equation for a slowly varying (in space) complex amplitude $\mathbf{E}$ of the oscillations of the electric field in an axially-symmetrical beam can be written in the form[2, 8-12]

$$\frac{\partial^2 \mathbf{E}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{E}}{\partial r} + 2ik\frac{\partial \mathbf{E}}{\partial z} + k^2 n_2 |\mathbf{E}|^2 \mathbf{E} = 0 \qquad (1)$$

($k = \omega n_0 / c$). A correct analytic solution of this equation was obtained in [12]. However, it is valid only near the boundary of the medium in a narrow interval of values of the initial power of the light beam. In view of the complexity of the analytic solution, the authors of the present paper, in conjunction with A. L. Lyshko of the Computation Center of the U.S.S.R. Academy of Sciences, using in 1967 a computer for a numerical solution.[16] It should be noted that a numerical solution of Eq. (1) was obtained already by Kelley.[8] The calculations have shown that the intensity on the beam axis on approaching the focus increases strongly. However, the question of what takes place in the light beam behind the focus was not considered. Our calculations[16] obtained without a limitation on z, have led to the following picture of the phenomenon. When $P > P_{cr}$, in the process of propagation in the nonlinear medium, the beam breaks up, as it were, into annular zones and successive focusing of these zones takes place at different points on the beam axis. These points (focal points) constitute regions of small dimensions and large energy concentration. Their number is finite and is determined by the excess of the initial power over the critical value. At large excesses of the initial power above the critical value, the focal points lie close to each other.