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PLASMA CONTAINMENT IN ADIABATIC TRAPS

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1. INTRODUCTION

MAGNETIC traps are magnetic-field configurations in which charged particles can be contained for a long time. Interest in magnetic traps has risen especially in connection with searches for the possibilities of prolonged containment of high-temperature plasma for the purpose of realizing a controlled thermonuclear reaction, and at the present time the greater part of research on magnetic traps is carried out as part of the program of controlled nuclear fusion.

The principles of creating magnetic configurations having trap properties follows naturally from the general laws of the motion of charged particles in strong magnetic fields. As is well known, in a strong magnetic field charged particles move along helical trajectories that wind around the force lines. Therefore the most natural method of producing such traps is to close a bundle of force lines into a torus. Indeed, a toroidal field was proposed in 1950 by Sakharov and Tamm[1] for containment of a high-temperature plasma. A more detailed consideration of the motion of the particles has shown that to eliminate the consequences of the so-called magnetic drift, the toroidal field must have magnetic surfaces that can be produced either by superimposing a longitudinal current[1] or else by twisting the magnetic axis or else by means of additional helical windings as proposed by Spitzer[2].

Besides toroidal traps, however, there exists one more rather extensive class of magnetic traps, called adiabatic. In adiabatic traps, prolonged containment of charged particles is based on the conservation of the transverse adiabatic invariant—the ratio of the transverse energy of the particle to the frequency of Larmor rotation. Stoermer has shown as early as in 1912 that in a dipole magnetic field there exists a definite group of particles that execute quasiperiodic motion and are contained by the field for a long time, and the idea for the use of this effect to contain a high-temperature plasma arose much later. It was advanced in 1952 by Budker[3] and independently by York and Post (see[4]). Budker also cautioned against the possible instability of containment of a diamagnetic plasma in a mirror-configuration field. A detailed theoretical analysis of this question[5,6] has shown that in the simplest axially-symmetrical adiabatic trap the plasma should indeed be subject to the so-called flute instability. However, in spite of the entire lucidity of the theoretical argumentation, no one succeeded in observing flute instability for a long time. It was found subsequently that this was caused by the insufficiently clean experimental conditions (the plasma made contact with the walls). By 1961, one of the authors of the present review and his co-workers succeeded not only in clearly demonstrating the presence of flute instability[7,8] but also in stabilizing it[9] with the aid of additional current-carrying rods, which made it possible to produce a magnetic field configuration with "minimum B."

The establishment of complete correspondence between theory and experiment with respect to the macroscopic behavior of the plasma has made it possible to proceed to the next stage of investigation of kinetic instabilities that are much subtler in nature. By now, certain clarity (albeit incomplete) has also been reached in this problem, and it has become possible to summarize the accumulated information to some degree. This is precisely the purpose of the present article. Since the question of plasma containment in adiabatic traps have been discussed in the pages of this journal only sporadically[10-12], we deemed it advantageous to cover as fully as possible, albeit briefly, the entire circle of problems connected with collective processes in a plasma contained in an adiabatic trap.

2. CONTAINMENT OF INDIVIDUAL PARTICLES, COULOMB COLLISIONS

In the simplest variant, the adiabatic trap is produced by two identical coaxial coils connected in the same direction (Fig. 1). In this case the magnetic field between the coils is somewhat weaker than in the plane of the coils, so that the central part of the field turns out to be contained between two magnetic "stoppers" or "mirrors," namely regions with enhanced field. The ratio of the field in the mirrors B_m to the field in the central part of the trap B_0 is customarily called the mirror ratio $\alpha = B_m/B_0$.

If there is no electric field, then the velocity $\mathbf{v}$ of a charged particle moving in the magnetic field remains constant (the Lorentz force, being perpendicular to $\mathbf{v}$, performs no work). In addition, in a strong magnetic field, when the Larmor radius $\rho = v_\perp/\omega_B$ ($v_\perp$—velocity component transverse to $\mathbf{B}$, $\omega_B = eB/mc$—Larmor frequency, e—particle charge, m—its mass, c—speed of light) is much smaller than the characteristic length of variation of the magnetic field, the quantity

$$\mu = mv_\perp^2/2B \quad (2.1)$$

is also conserved. This quantity, which has also the

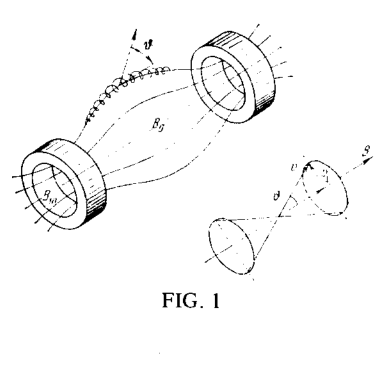

FIG. 1

meaning of the magnetic moment of the Larmor circle, is the adiabatic invariant of quasiperiodic motion[13-15].

Since μ = const, the transverse velocity component $v_\perp$ increases when the charge particle approaches the mirror, and since v = const, the longitudinal component of the velocity decreases and may vanish if α is sufficiently large. In this case the particle is reflected from the magnetic mirror.

Let us introduce the angle ϑ between the velocity vector and the direction of the magnetic field **B**. It is equal to $(\pi/2) - \psi$, where ψ is the so-called pitch angle. It is easy to see that the magnetic mirror reflects only those particles for which, in the central part of the trap,

$$\sin\vartheta > \alpha^{-1/2} = (B_0/B_m)^{1/2}. \qquad (2.2)$$

All the particles with angle $\vartheta < \vartheta_0 = \sin^{-1}[B_0/B_m)^{1/2}]$ fall in the "forbidden cone" of directions and escape from the trap. Thus, the adiabatic trap does not retain all the particles, but only those that are present inside the allowed direction cone.

The particles retained by the trap execute relatively fast oscillations between the reflection points and at the same time shifts only from one force line to the other, experiencing the so-called magnetic drift. The rate of this drift is of the order of $v_m \sim v\rho/R$, where ρ is the Larmor radius and R is the radius of curvature of the force line. In an axially-symmetrical trap (see Fig. 1) the particles drift in the azimuthal direction, and for particles that spend the greater part of their time in the region where the field decreases in the radial direction, the drift direction coincides with the direction of the Larmor rotation. The sign of the drift is opposite only for a small fraction of the particles, those experiencing reflection near the plane of the coils, where the field increases with increasing radius. In magnetic fields of more complicated configuration, the particles drift in such a way that their longitudinal adiabatic invariant is conserved:

$$I = \oint mv_\parallel dl = (2m)^{1/2} \oint \left(\frac{mv^2}{2} - \mu B\right)^{1/2} dl. \qquad (2.3)$$

The drift rate is then proportional to the gradient of I and is directed perpendicular to it[17-19]. Thus, the condition I = const determines the drift surface for each given particle. In the fields of complicated configuration, the drift surfaces for different particles may deviate quite strongly from one another. We note also that in the case when the magnetic configuration varies slowly compared with the drift rotation, there is conserved one more, third, adiabatic invariant, namely the magnetic flux Φ through the drift surface[18].

The quantity μ, like all adiabatic invariants, is conserved generally speaking only with exponential accuracy[13-16,20]. This raises first of all a purely practical question, whether the time of conservation of the adiabatic invariant attainable in laboratory conditions is sufficient to make the use of an adiabatic trap meaningful for a prolonged containment of particles. This question was carefully investigated in many experiments. Apparently the most convincing results were obtained in the experiments of Gibson[21] and Rodionov[22], who investigated the containment of β particles. It was shown that β particles are contained for more than 10^7 flights between mirrors, which undoubtedly is perfectly sufficient for a prolonged containment of the particles.

Later measurements[23-25] of the dependence of the electron lifetime in an adiabatic trap on the parameter $\epsilon = \max(\rho|\nabla B|/B)$ (max denotes the maximum value on the trajectory) revealed a number of fine features which are in qualitative agreement with the theoretical concepts developed to date[26,28].

The question of conservation of the adiabatic invariant of a charged particle as it moves in a trap with magnetic mirrors is a particular case of the more general problem of stability of motion as a whole in classical mechanics. In recent years, owing to the work of Kolmogorov, Arnold, and Moser, appreciable progress was reached in this field (see[26]). A development of these and analogous ideas for individual models, from the point of view of methods customarily used in physics and their application to different particular cases, was carried out by Chirikov, who analyzed, in particular, the question of motion of particles in an adiabatic trap[27]. The corresponding results can be briefly summarized as follows.

Let us consider a trap with axial symmetry of the type of Fig. 1, and assume a parameter $\epsilon = \max(\rho|\nabla B|/B) \ll 1$. Then, if we neglect the nonconservation of the adiabatic invariant, which is exponentially small in ϵ, and assume μ to be strictly constant, then the motion of the particle will be doubly periodic—it rotates in the azimuthal angle in velocity space with cyclotron frequency, and executes oscillations between the mirrors with frequency Ω. In order of magnitude, $\Omega \sim v/L$, where L is the length of the trap, so that when $\epsilon \ll 1$ we have $\Omega \ll \omega_B$. We introduce the average frequency ω_B per period of oscillation in length. If the ratio of ω_B to Ω is rational, i.e., $\omega_B = m\Omega/n$ (where m and n are integers), then the motion of the p particle is periodic (if we disregard its motion around the symmetry axis); otherwise the motion is conditional or quasiperiodic.

We now consider an exponentially small nonconservation of the adiabatic invariant during each lengthwise flight as a small perturbation (we note that this actually occurs on passing through minimum-B[20]. This raises the question: how will this perturbation "work" in the case of very many flights? It turns out that if the ratio of the frequencies ω_B and Ω is rational, then there is always a resonant harmonic of the perturbation at which the energy is transferred from the longitudinal component to the transverse one and vice versa. In other words, a slow "jiggling" of the adiabatic invariant takes place. At not too small ϵ, the amplitude of these oscillations may turn out to be sufficient to jump from one resonance to one of the neighboring, corresponding to another pair of integers m and n ($\omega_B = m\Omega/n$). With such an overlap of the resonances, the variation of μ assumes the character of random wandering—the process becomes stochastic, with the exception possibly of individual "islands" near those periodic motions (with a rational frequency ratio), where the average value of the perturbation randomly vanishes.

With decreasing ϵ, by virtue of the exponential smallness of the perturbation, no overlap of the resonances takes place, and at sufficiently small ϵ, as was proved rigorously by Arnold[26], the adiabatic invariant is conserved forever in a trap with axial symmetry. More accurately speaking, near values of μ corresponding to a rational ratio of the frequencies ω_B and Ω, the quantity μ can execute small oscillations, but the corresponding regions are separated by values of μ with a non-rational frequency ratio, where μ = const, so that in any case the adiabatic invariant cannot change by a large amount. If there is no axial symmetry, then the concluded permanent conservation of the invariant μ becomes meaningless, and "diffusion" of μ is possible even at very small ϵ[27,28]. In this case, however, we can expect much better conservation of the adiabatic invariant than predicted by the exponential dependence[27].

The results of an experimental investigation of the motion of electrons in a magnetic trap is in qualitative agreement with the theoretical concepts. Figure 2 shows the experimental dependence of the lifetime on the magnetic field, taken from[23]. We see that at moderate B the lifetime depends on the value of the magnetic field exponentially, and then the dependence becomes much steeper and is limited only by the pressure of the neutral gas (the scattering by the neutral-gas atoms transfers the electrons to the forbidden cone).

The critical value of the magnetic field at which a break occurs in the τ(B) plot corresponds to $\epsilon \approx 0.04$. This result might have been interpreted as an indication of perpetual conservation of the adiabatic invariant at small ϵ, but one should speak more accurately[29] of a transition to a steeper τ(B) plot, since an analogous phenomenon is observed in the absence of the axial symmetry (in addition, in any experiment there is always a small asymmetry). At low values of B, there is also observed a dependence of τ on the pressure[25], which can be interpreted as an indication of the existence of stability "islands." The direct proof of the influence of the resonances on the motion of the particles in the trap are the exponentially observed dips in the energy spectra of the particle in the trap, corresponding precisely to resonances between the Larmor rotation and the longitudinal oscillations.

Thus, both theory and experiment show that singly-charged particles are contained quite effectly by adiabatic traps. On going over from single particles to a rarefied plasma, the containment becomes less perfect, since interactions between particles come into play.

The simplest form of interaction is via paired Coulomb collisions. The particles scattered by the mutual collisions can sooner or later fall into the dan-

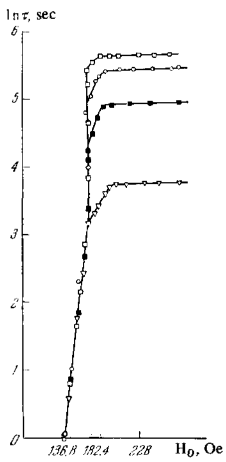

FIG. 2. Dependence of the lifetime of the electrons in the trap on the magnetic field at different gas pressures [23].

ger cone and escape through the mirrors. This imposes an upper limit for the time of plasma containment in adiabatic traps. Furthermore, if there exist in the plasma collective motions in the form of various instabilities, then the containment time may turn out to be much shorter, since the particle can be intensely scattered and displaced in the fields connected with the instabilities.

From this point of view, one should speak of two types of plasma losses from the trap: "classical" loss—owing to Coulomb scattering and anomalous loss under the influence of the instability.

As to Coulomb scattering, it is caused in the main by "remote" collisions, i.e., those accompanied by small-angle scattering. Owing to the long range of the Coulomb forces, the loss of the directional momentum by the particles occurs much more rapidly as a result of multiple "remote" collisions than as a result of the rarer "close" collisions, in which a strong deflection occurs in a single collision act. This feature of Coulomb interaction makes it possible to equate the problem of classical losses through mirrors to the problem of calculating the diffusion flux in velocity space through the surface of the "forbidden cone." Such a problem was solved by Budker[3] and by Judd et al.[29] under definite assumptions concerning the ion velocity distribution function. The final result can be represented in the form of the simple relation[30]

$$\tau_{ki(e)} = k\bar{\tau}_{i(e)}\lambda(\alpha); \quad (2.4)$$

here $\tau_{ki(e)}$ is the average time of retention of the ions (electrons) in the trap. k is a numerical factor on the order of unity, $\bar{\tau}_{i(e)}$ is the average time of loss of directional momentum by the ions (electrons), $\lambda(\alpha)$ is a function of the mirror ratio, the exact form of which depends on the velocity distribution of the ions injected into the trap; for realistically chosen distributions it can be approximated by the expression $\lambda(\alpha) = \log \alpha$. This shows, incidentally, that the change of the mirror ratio has relatively little influence on the duration of the containment.

The foregoing relation still does not characterize completely the time of plasma containment. The ions and electrons are scattered with different velocities: $\tau_i \sim M^{1/2}T_i^{3/2}$ and $\tau_e \sim m^{1/2}T_e^{3/2}$ (T_i, T_e—temperature of the ions and electrons, M and m—their masses). Therefore an ambipolar electric field should arise in the plasma in the vicinity of the mirrors, equalizing the fluxes of particles of both signs leaving the plasma, and ensuring quasineutrality of the plasma. This additional effect may cause the plasma containment time to be much shorter than the larger of the times τ_{ki} or τ_{ke}. Thus, according to calculations by Fowler and Rankin[31], for a plasma with thermonuclear parameters the containment time should decrease by a factor 2—3 compared with τ_{ki} (in such a plasma $\tau_{ki} \gg \tau_{ke}$). The influence of the ambipolar field on the containment of the plasma was considered also in[32-34].

Experimental data on classical plasma losses in adiabatic traps are quite limited and pertain mainly to a plasma with cold ions and electrons of high energy ($T_e \sim 10^3-10^5$ eV).

Under conditions when there are no plasma instabilities, containment of fast electrons in a plasma of density 10^{11}–10^{12} cm^{-3} was observed for hundredths and tenths of a second (see, for example,[35,36]). These times correspond, in order of magnitude, to the times of Coulomb scattering of fast electrons.

In numerous investigations with a hot-ion plasma, it has been impossible so far to separate the Coulomb losses against the background of the more intense losses due to plasma instabilities or ion charge exchange. Only recent experiments, with the British MTSE-II apparatus[37], which was used to study the containment of a dense plasma with hot ions ($n \sim 1 \times 10^{13}$ cm^{-3}, $T_i = 2$ keV), did the Coulomb collisions come to the foreground. In these experiments, to be sure, the containment time was short, only several hundred microseconds, but it was determined apparently by the cooling of the ions by the cold plasma flowing into the trap from the injector.

In the overwhelming majority of the other experiments, the main obstacle to prolonged containment of the plasma were the instabilities. This is precisely why the study of plasma instabilities and the physical phenomena associated with them is of greatest interest and occupies a leading place in the theoretical and experimental research.

3. FLUTE INSTABILITY OF A PLASMA

A plasma contained in an adiabatic trap is in a highly non-equilibrium thermodynamic state, and is therefore subject to a large number of instabilities. One of these instabilities—the roughest and most dangerous—was predicted theoretically by Rosenbluth and Longmire[5] and by one of the present authors[6], although in implicit form it was included already in the model problem of Kruskal and Schwarzschild[38]. This instability has been named convective or flute instability.

Flute instability is due to diamagnetism of the plasma; roughly speaking, the plasma should be repelled out of the magnetic field in which one attempts to contain it. In an adiabatic trap of the ordinary type (see Fig. 1), the magnetic field weakens in a radial direction, so that the plasma should be ejected to the side walls. Since the plasma is a fluid medium, the corresponding ejection might seemingly occur in a great variety of ways. This in fact is not so, for the presence of a strong magnetic field strongly limits the motion of the plasma. In order to visualize the plasma motion in a strong field, let us consider first an individual plasma element, namely the plasma tube 1 (Fig. 3). We consider just a tube because, if the plasma were to have the form of a small bunch 2, at the initial instant of time and its component particles were to have a rather wide spread of longitudinal velocities, then the plasma would expand very rapidly in the direction of the magnetic field. The particles in the loss cone would escape from the trap, and the remainder would make up the tube in question. With further motion of the plasma, rapid longitudinal oscillations of the electrons and ions cause the plasma bunch to remain elongated along the force lines all the time. Let us see now how such a tube will move.

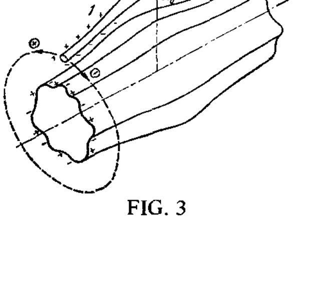
FIG. 3

In the main part of the plasma tube 1, the force lines of the magnetic field are convex outward and the magnetic field decreases towards the periphery. As a result, each charged particle is pushed out radially: the transverse motion along the Larmor circle produces a diamagnetic moment μ, which is acted upon by an expulsion force $-\mu\nabla B$, while the longitudinal motion along the curved line gives rise to an outwardly directed centrifugal force. As a result, the average force per particle becomes approximately equal to $|\mathbf{F}| = T/R$, where T is the temperature or the average energy of a given species of particles (electrons or ions), R is the average radius of curvature of the force lines of the given tube. Under the action of this force, a tube with a dense plasma should be expelled in the radial direction with an acceleration $g_0 = (T_i + T_e)/MR$, where T_i is the ion temperature, T_e the electron temperature, and M the ion mass.

However, at not too large a plasma density (which is not a rarity in the case of adiabatic traps), the process of plasma expulsion should slow down. In fact, individual charged particles shift very little radially under the influence of the radial force in a strong field, and begin instead to drift in azimuth at the rate of the so-called magnetic drift $v_m = cT/eBR$, so that the force F is balanced by the Lorentz force ev_m/cB. It can be stated that the charged particles in a magnetic field have the properties of a gyroscope—they move in a direction perpendicular to the applied force, with a velocity proportional to this force. The electrons and the ions are then analogous to gyroscopes rotating in opposite directions—they drift in azimuth in opposite directions. The latter circumstance causes the plasma tube to become "polarized": the charges inside the tube become separated and this gives rise to an azimuthal electric field.

With increasing density, the electrons and the ions become more and more "coupled" with each other by the electric field, and the plasma tube begins to behave like a diamagnet. The radial motion occurs in this case, in final analysis, as a result of the drift in the cross azimuthal electric field and the longitudinal magnetic field. From the equations of motion for the electrons and ions of the plasma tube, we can readily show[40] that the radial acceleration g due to the diamagnetic expulsion is equal to

$$g = g_0\Omega_0^2/(\Omega_0^2 + \Omega_B^2), \qquad (3.1)$$

where $\Omega_0^2 = 4\pi e^2 n_0/M$ is the square of the plasma (Langmuir) frequency for the ions, $\Omega_B = eB/Mc$ is the cyclotron frequency, n_0 is the ion density, and $g_0 = (T_i + T_e)/Mr$. We see therefore that when $\Omega_0 < \Omega_B$, the process of expulsion of the plasma tube from the trap slows down.

It is precisely the effect of expulsion of the diamagnetic plasma which serves as the basis of flute instability. This instability is manifest in the fact that under a small perturbation of the surface of azimuthally-symmetrical plasmoid, "tongues" elongated along the force lines with grooves between them (see Fig. 3) should appear and grow in time, and the plasma should be ejected towards the periphery. The characteristic growth increment γ of the flute perturbations is of the order of $\gamma \sim (g/a)^{1/2}$, where a is the transverse dimension of the plasmoid.

In spite of the extreme simplicity of the physical mechanism of flute instability and its rather convincing theoretical justification, for a long time it was impossible to observe it and investigate it experimentally. This created the feeling of disbelief in the correctness of the theoretical concepts. Since a plasma with hot electrons turned out to be especially stable, the opinion was even advanced that the deduced existence of the flute instability does not hold, for some unknown reason, for a plasma with hot electrons.

The first experiments in which the presence of flute instability was quite clearly and convincingly demonstrated, were performed with the "ion magnetron" setup[7,41] in 1959–1960.

This setup was used to study the containment of a hot-ion plasma produced by accelerating ions with a radial electric field. The field was applied between a cold-plasma column passing along the axis of the trap and the metallic walls of the vacuum chamber, in the form of a single pulse of 30 μsec duration. After the end of the pulse and simultaneous stoppage of the "supply" of the cold plasma, the trap turned out to be filled with a high-temperature plasma of density $\sim 10^9$ cm^{-3} average ion energy $\sim$ 1 keV, and electron temperature $\sim$ 20 eV. The free decay of such a plasma was then investigated.

In the case when there were no instabilities whatever, the decay should have been determined by the charge exchange between the fast ions and the residual gas. Under the conditions of these experiments, the charge exchange time amounted to several milliseconds.

However, measurements of the time dependence of the plasma density based on the flux of the charge-exchange neutral particles have shown that the decay has a much shorter characteristic time, $\sim 10^{-4}$ sec, i.e., anomalously large plasma losses occur, greatly exceeding the charge-exchange losses. Direct measurements of the currents flowing from the plasma to the chamber walls have demonstrated that the plasma vanishes on the side walls of the chamber. It was also established that the flow of plasma to individual wall elements has the nature of spikes that are irregular in time, and are well correlated along the magnetic force lines. In other words, the plasma is ejected to the wall in the form of individual "tongues" extending over the entire length of the trap.

All these facts offered direct and unambiguous evidence of the flute nature of the instability causing the plasma loss. However, the very mechanism of plasma transport was not completely cleared up. The point was that the experimentally measured containment time exceeded by many times the theoretically predicted instability-development time, and even its dependence on the field intensity and ion energy was entirely different. These problems were resolved within the framework of the concepts of turbulent plasma convection[40].

In order to visualize the mechanism of plasma transport to the side walls, it must be taken into account that the plasma tube coming in contact with the metallic wall cannot vanish immediately. This is connected with the fact that in the thin layer of plasma next to the wall, with thickness on the order of the average Larmor radius ρ of the ions, the transverse electric fields should be very small: the excess ions can leave this layer relatively freely to the side walls, while the electrons can leave along the force lines, so that the charge fluctuations die out. This means in turn that near the wall the radial plasma motion should be slowed down, since this motion is connected with the azimuthal component of the electric field, which should vanish near the wall (and all the more on the wall itself). Consequently the transport of plasma to the wall assumes the character of convection—the tubes arriving at the wall can lose only part of their plasma, and after this they are forced into the interior of the trap by other denser tubes. If we denote by ξ the fraction of the plasma lost upon coming in contact with the wall, then one should expect near the wall density fluctuations $n' \sim \xi n_0$, where n_0 is the average density. Near the wall itself, the plasma transport in turbulent convection is effected by pulsations of minimum scale $\lambda_\perp \sim \rho$. The velocity v' of such pulsations is the result of the Archimedean force of buoyancy of the tubes depleted of plasma, and is determined by the balance of the kinetic and potential energies: $v'^2 \sim n'\lambda_\perp g/n_0 \sim \xi \rho g$. Consequently, the plasma flux to the wall is $q = \langle n'v' \rangle = A\xi^{3/2} n_0 (\rho g)^{1/2}$, where A is a numerical coefficient of the order of unity. With increasing distance x from the wall, pulsations of increasing scale come into play, up to $\lambda_\perp \sim x$, so that the effective diffusion coefficient increases rapidly and the density in the interior of the trap is close to the density near the wall. Therefore n_0 in the expression for q can be taken to mean the average density in the trap, by suitably redefining the constant A. Hence, knowing the radius of the chamber a, we obtain, taking into account (3.1), the lifetime $\tau = \pi a^2 n_0 / 2\pi a q$:

$$\tau = Ca\,(MR/T\rho)^{1/2}\,[1 + (\Omega_B^2/\Omega_0^2)]^{1/2}, \qquad (3.2)$$

where T is the average ion energy and C is a numerical factor of the order of unity at ξ = const $\sim 1/2$.

Expression (3.2) is in satisfactory agreement with the experimental data on the measurement of the plasma lifetime. By way of an example, we can show a comparison of the theoretical and experimental density dependences of the lifetime (Fig. 4). Figure 5 shows the radial distribution of the density, from which it is seen

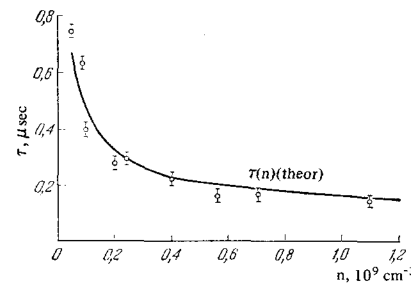

FIG. 4. Experimental and theoretical dependences of the lifetime of the plasma (after subtracting the charge-exchange losses) on the density [8].

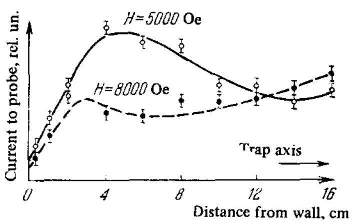

FIG. 5. Radial distribution of the plasma density [8].

that the plasma actually is "poured out" on the periphery of the trap and is decelerated near the external wall.

Further experimental confirmation of the concepts of turbulent conduction of a rarefied plasma was obtained following a more detailed study of the spatial and temporal characteristics of the plasma pulsations[8]. Such information was obtained with the aid of a system of miniature Langmuir probes, which introduced no noticeable perturbations in the plasma. The local oscillations of the plasma density could be assessed from the oscillations of the saturation ion current to the corresponding probe, and the dimensions of the plasma inhomogeneities could be estimated from the correlations of the current to several probes located at known distances from one another. It turned out that in the plasma next to the wall there exists a broad spectrum of deeply modulated irregular oscillations of duration from several microseconds to several dozen microseconds. The high-frequency part of the spectrum corresponds to the smallest-scale pulsations with dimensions (across the field) 3—4 cm, which are close to the Larmor diameter of the ions.

On going over to the deeper layers of the plasma, the amplitude of the small-scale pulsations, as expected, decreases sharply and all that remains is the low-frequency component, which is connected with pulsations of a scale of the order of the transverse dimensions of the trap. In the central region of the trap there is observed a very weak low-frequency modulation of the probe signals, indicating the absence of initial density perturbations from this region. Such a picture is in qualitatively good agreement with the model of turbulent plasma transport across a magnetic field. With decreasing density, when Ω_0 becomes smaller than Ω_B, one should expect a slowing down of the convection process, as is indeed observed experimentally (Fig. 6).

Flute instability was observed also in a rarefied plasma in apparatus with stationary injection of high-energy beams of charged and neutral particles, such as Ogra-I[42], Phoenix[43], and Alice[44]. In these installations it is possible to vary the plasma density smoothly, and therefore they make it possible to obtain additional information on the variation of the character of manifestation of the flute instability with changing density.

It is clear that at very low densities, when the electric fields produced upon separation of the charges are too weak to bind the electrons and ions, there should be no flute instability. Accordingly, two branches of oscillations should be observed, with the frequency of the magnetic drift of the ions Ω_m, corresponding to perturbation of the spatial distribution of the ions, and with zero frequency, corresponding to perturbation of the electron cloud (it is assumed that the magnetic drift of the cold electron is negligibly small)[45]. With increas-

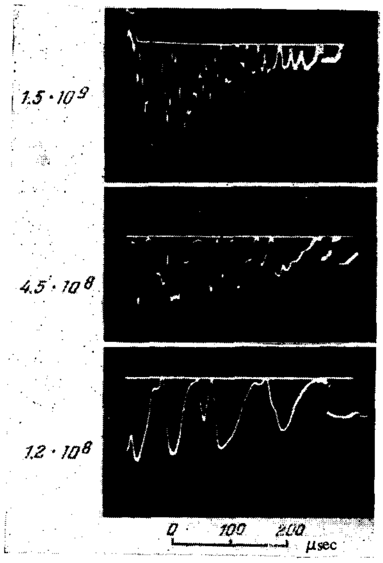

FIG. 6. Change in the frequency of plasma pulsations with decreasing density [8].

ing density, the oscillations of the electron and ion clouds become coupled with each other, and accordingly the frequencies of the two branches come closer and at a certain critical density they become equal and assume the value $\Omega_m/2$[40,45,46] (Ω_m—frequency of the ion magnetic drift). Starting with this density, flute instability of minimal mode m = 1 should develop in the plasma. The critical density corresponds approximately to the equality[45]

$$r_d \approx (aR)^{1/2}, \quad (3.3)$$

where $r_d = v/\Omega_0$ is the Debye radius, a is the radius of the plasmoid, and R is the average radius of curvature of the force lines.

The critical density depends, in addition, on the decompensation of the electron and ion charges[45,47] and on the radial distribution of the electric field, which can lead to rotation[48,49].

When the plasma density is not much higher than critical, it is natural to expect[50,51] steady-state oscillations with finite amplitude—the analog of the laminar convection of an ordinary liquid. All these theoretical considerations agree qualitatively (but unfortunately not always quantitatively) with the experimental data.

Intense low-frequency density oscillations, localized mainly in the surface layer and in-phase along the magnetic force lines, were registered in "Ogra-I" in a plasma of density ~ 10^7 cm^{-3}, produced by injecting 1 160-keV H_2^+ ions. From phase measurements at different azimuths it follows that the oscillations correspond to rotation of the deformed plasma cylinder with principal deformation mode m = 1, as expected at low densities. The oscillation frequency lies in an interval from several kHz to several dozen kHz. In most injection regimes, it is determined not only by the inhomogeneity of the magnetic field, but also by the radial electric field, which exists in the plasma as a result of the incomplete compensation of the positive charge of the ions by the electrons. (Strong violation of the quasineutrality is caused apparently by the accelerated departure of electrons as a result of the buildup of ion-cyclotron waves in the anisotropic plasma.) As a result there is added to the frequency of the magnetic drift of the ions also the frequency of rotation of the entire plasma in the crossed E and B fields; this determines the observed linear dependence of the frequency on the plasma potential.

At quite small injected-beam currents, when the plasma density does not exceed 10^6 cm^{-3} and the electric fields are weak, the frequency of the flute oscillations is close to $\Omega_m/2$. After turning off the beam, during the course of the plasma decay, the oscillations are damped and vanish completely at a density $\sim 10^5$ cm^{-3}, corresponding to the critical density for flute instability (3.3).

In "Ogra-I" there was observed also the effect of improvement of the plasma stability when the electron and ion charges become decompensated, in accordance with the theoretical conclusion[48].

Qualitatively similar results were obtained also in the "Phoenix" and "Alice" apparatus, in which the plasma was produced by Lorentz ionization of excited neutral H^0 atoms. In "Phoenix" at densities above $\sim 10^8$ cm^{-3} there were observed low-frequency oscillations of the flute type with m = 1. As a rule, their frequency was close to $\Omega_m/2$, but sometimes it decreased to smaller values, and in this case it was proportional to the density. The experimental data on the dependence of the threshold density at which flute oscillations are excited on the magnetic field still differs noticeably from that calculated[52,53] with allowance for the finite Larmor radius, the geometry, and the degree of decompensation of the charges (Fig. 7). This is possibly connected with the insufficiently accurate allowance for the differential rotation of the plasma[49,48] in the calculation of the theoretical field dependence at the critical density.

The results obtained with "Alice" are analogous in many respects with the data obtained with "Phoenix." Here, too, two modes of oscillations were observed: with frequency proportional to the density and independent of the density (Fig. 8). Sometimes the first mode jumps over unexpectedly to the second, and the latter continues to exist down to very low densities. As seen from Fig. 8, the mode with a frequency increasing with the density can be interpreted as the electronic branch of the oscillations, while the oscillations that are independent of the density apparently correspond to the third branch (ionic), which, as shown by Varma[54], can take place in a plasma with a monochromatic velocity distribution of the ions (see also[55], where a detailed analysis of the third branch is given, and[56]).

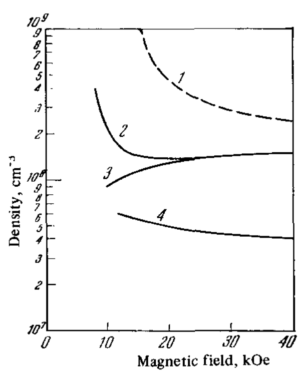

FIG. 7. Threshold densities at which flute instability sets in vs. the magnetic field [52]. 1–Experimental data of the "Phoenix" apparatus [43]; 2–results of numerical calculations with account taken of the stabilizing effect of the finite Larmor radius; 3–the same but without allowance for the effect of the finite Larmor radius; 4–results of calculations of Kuo et al. [43] for cylindrical geometry.

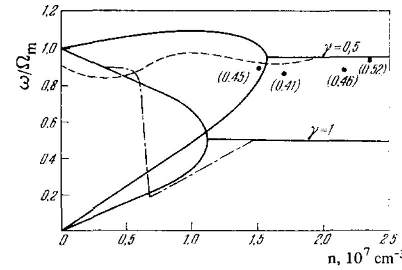

FIG. 8. Dependence of the frequency of the drift oscillations on the ion density. The theoretical curves (solid lines) are drawn for two values of the degree of decompensation γ: the experimental curves (dashed and dash-dot) were obtained during the time of plasma decay [44].

4. METHODS OF STABILIZATION OF FLUTE INSTABILITY

At the present time we can point out three different methods of stabilizing flute instability in open adiabatic traps.

The first and most direct consists of modifying the configuration of the magnetic field in such a way that the field intensity increases in all directions from the regions occupied by the plasma, i.e., not only in a longitudinal direction, as in the case in an ordinary trap, but also radially. Such configurations, called "minimum-B" configurations, exclude in principle the possibility of occurrence of flute instability, for when the outward distance from the plasma boundary increases, the magnetic field increases everywhere within the limits of the trap, and consequently, the diamagnetic plasma is in a potential well.

The two other stabilization methods are based on suppressing the polarization electric fields arising in the plasma as a result of the magnetic drift of the ions and electrons in the inhomogeneous field of the ordinary trap. In this case this is attained by producing sufficiently good conductivity between the contained plasma and equipotential metallic electrodes located outside the plasma—on end faces beyond the mirrors. In the latter case, the fields are produced with the aid of a special system of electrodes placed around the plasma and not in direct electric contact with it; at each given instant of time there are produced between the electrodes artifically (automatically) electric fields opposite in direction to the fields in the plasma and preventing by the same token the growth of the initial flute perturbations.

Let us consider in greater detail each of these methods and the experimental results obtained with their aid.

a) Minimum B

The use of fields of the "minimum-B" type is the most radical method of stabilization, for in this case the very cause of the instability is eliminated. In producing such configurations, it is only necessary to bear in mind that the adiabatic properties of the trap must be conserved. This means, first of all, that one excludes from consideration the class of fields with B = 0 at the

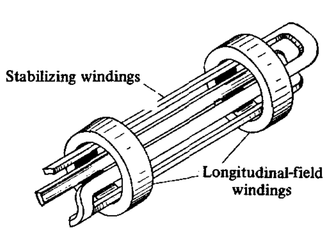

FIG. 9

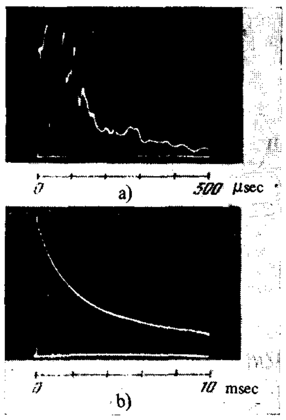

FIG. 10. Oscillogram of flux of charge-exchange neutrals: a) $\alpha_\perp = 1.0$, b) $\alpha_\perp = 1.22$ ($p = 1 \times 10^{-7}$ Torr) [58].

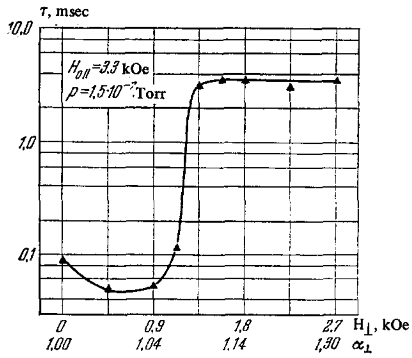

FIG. 11. Dependence of plasma lifetime on the stabilizing field ($p = 1.5 \times 10^{-7}$ Torr) [57].

minimum point, at which the constancy of the magnetic moment of the charged particles is certainly violated.

Among the various magnetic systems satisfying the indicated requirement, we shall point out here one of the simplest variants, which is widely used in many experiments. To the usual coils producing the basic longitudinal field with the mirrors ($B_\parallel$) there is added a so-called stabilizing winding, which constitutes a system of linear conductors placed along the force lines of the main field, as shown in Fig. 9. The currents in the neighboring conductors flow in mutually opposite directions. The magnetic field of the stabilizing winding ($B_\perp$) is zero on the axis of the trap and increases monotonically along the radius. Therefore, by passing a sufficiently strong current through the winding, it is possible to compensate for the radial decrease of the main field and to make the summary field increase from the central region of the trap towards the periphery.

The force lines of the summary field have a rather complicated structure. Only a narrow beam of near-axis force lines passes along the entire trap without reaching the side walls. The lines farther from the axis in the central section of the trap cross the wall at locations of gaps between the conductors of the stabilizing winding.

The theory of plasma stability in "minimum-B" traps was developed in [59-61]. The effectiveness of the combined field with respect to suppression of the instabilities was demonstrated in experiments with the "ion magnetron"[9] and PR[57] apparatus. It was shown that an increase of the stabilizing field $B_\perp$ (at a fixed field $B_\parallel$) leads to the vanishing of the low-frequency oscillations characteristic of the flute instability, and simultaneously to an increase of the plasma containment time τ. These results are illustrated in Figs. 10 and 11, taken from[57,58] (the parameter $\alpha_\perp$ in these figures characterizes the ratio of the total field at the wall in the central cross section of the trap to the field on the

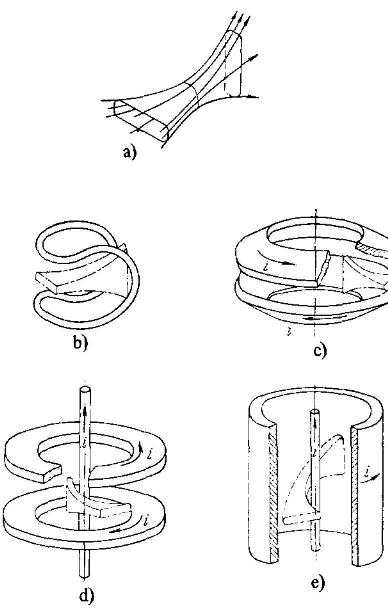

FIG. 12. Different types of magnetic configurations with minimum B. a) "Quadrupole element," b) "Baseball" [109], c) Andreoletti configuration [67], d) acute-angle geometry with axial current [107], e) helicoidal configuration [40].

axis). Attention is called to the almost threshold-like dependence of τ on $B_\perp$. The transition from small containment times to large ones occurs precisely at such values of $B_\perp$ at which the radial gradient of the summary field near the wall reverses sign from negative to positive, i.e., when the "minimum-B" configuration is established. At fields $B_\perp$ larger than the threshold value, the plasma is completely stable. The lifetime of the plasma is determined in this case only by the charge-exchange loss of the fast ions. In subsequent experiments with PR-5, τ reached 50—60 msec at plasma densities 10^9—10^{10} cm^{-3} and at an average ion energy 5 keV[58].

The results on stabilization of flute instability obtained with the PR-5 apparatus was subsequently repeated with many other installations[52,62-66], and at the present time fields with "minimum-B" of the type used with PR-5 or similar to them are used in most adiabatic traps.

A great variety of different minimum-B configurations have been proposed (Fig. 12). It turned out that all contain as the "constructive element" or "building block" a mirror trap with a quadrupole transverse field (Fig. 12a). It is easy to understand why this is so. To this end, let us consider the conditions under which it is possible to attain a minimum magnetic field (in absolute magnitude) at a certain point. Let this point coincide with the origin and let us assume that the magnetic field there is $B = B_0$ and is directed along the z axis. Since the magnetic field varies linearly in the transverse direction near a curved force line, in order for the point r = 0 to be a minimum-B point it is necessary that the axial force line be a straight line (more accurately, its curvature at the point r = 0 must be equal to zero). In this case the expansion of the scalar potential ψ for the magnetic field ($B = \nabla\psi$) should contain no terms linear in x or y, so that

$$\psi = B_0 z + az\left(\frac{z^2}{3} - \frac{x^2+y^2}{2}\right) + \frac{b}{2}(x^2 - y^2) + \ldots \quad (4.1)$$

In this expansion we have taken into account the fact that $\nabla^2\psi = \operatorname{div} B = 0$, and assumed that the axes x and y are turned in such a way that the last quadrupole term has a diagonal form. All the remaining terms in (4.1) are of higher order in x and y, and can therefore be discarded. It follows from (4.1) that near r = 0 the magnetic field is of the form

$$B = B_0 + az^2 + \left(\frac{b^2}{B_0} - a\right)\frac{x^2+y^2}{2} + \ldots, \quad (4.2)$$

where the dots stand for terms of higher order of smallness. We see therefore that the superposition of magnetic mirrors, i.e., the intensification of the field along z ($a > 0$) automatically leads to its weakening in the radial direction, but this can be compensated for by means of a sufficiently strong quadrupole field ($b^2 > aB_0$).

Figure 12 shows a number of minimum-B configurations, in each of which the structural element can be seen to be a field of the form (4.2). This circumstance was noted by Andreoletti[67].

We note that the cycle of investigations aimed at suppressing flute instability in adiabatic traps by minimum-B fields (such configurations are frequently called "magnetic wells") served as a definite stage in the research on high-temperature plasma physics. This was the first convincing demonstration of the possibility of stabilizing the magnetohydrodynamic instability of a plasma. The minimum-B principle was extended to toroidal systems[68]. Various authors have proposed many toroidal systems having a minimum magnetic field on the average (more accurately, a maximum of the integral along the force lines $\int dl/B$, which determines the properties of the hydrodynamic stability of the toroidal plasma).

b) Stabilization by Conductivity on the Ends

In many experiments performed on simple adiabatic traps, particularly those in which containment of a plasma with hot electrons (and cold ions) was investigated, no clear symptoms of flute instability were observed[35,36,69-75]. The containment times in these experiments exceeded by many orders of magnitude the times of development of the flute instability, and reached tenths of a second in certain cases.

Such experiments were performed usually in insufficiently perfect vacuum conditions. As a result, upon ionization of the residual gas there could be formed a sufficiently dense cold plasma, which, flowing out along the force lines, ensured good electric contact between the hot plasma and the conducting surfaces beyond the mirrors, and by the same token "short circuited" the polarization electric fields in the contained plasma.

The influence of the vacuum conditions on the stability of the plasma was systematically investigated in[73], where it was possible to vary in a controllable manner the pressure of the neutral gas during the course of the experiment. It was shown in that investigation that stable containment of a plasma is attained only starting with pressures 10^{-5} mm Hg, i.e., at neutral-gas densities not much smaller than the density of the contained plasma. At lower pressures, distinct symptoms of flute instability are observed and the containment of the plasma is greatly disturbed.

The results of most other investigations have a sporadic and sometimes random character, and are governed by various specific peculiarities of the experiment, such as the value of the initial pressure of the residual gas, the degree of increase of pressure during the time of plasma injection, the outflow of the "tail" of cold plasma from the injector, etc.

Unfortunately, the aggregate of the available experimental data does not include reliable quantitative information on the density that the cold plasma must have beyond the mirrors in order to stabilize the flute instability. The existing results can be regarded only as a qualitative configuration of the fact that such a stabilization does take place.

c) Stabilization of Flute Instability by the Feedback Method

A recently employed method of stabilizing plasma instabilities with feedback was suggested by Morozov and Solov'ev[36] and developed in detail by Arsenin and Chuyanov (see[77-78]). The gist of this method is applied to flute instability consisting of the following:

The polarization charges of flute perturbations produce on the plasma surface local azimuthal electric fields, which cause a growth of the initial perturbations and a drift of the plasma to the wall. If the plasma is surrounded by a system of insulated electrodes and the potentials of these electrodes are varied in such a way that at each instant of time there are produced everywhere along the azimuth electric fields of direction opposite to that of the polarization fields in the plasma, then it is possible to weaken the growth of the flute perturbations by this method, and ultimately suppress it completely. The potentials of the individual electrodes should, of course, be controlled automatically. This is done with a system of small capacitive pickups placed near the electrodes; these pickups follow the variations of the potential on the surface of the plasma. The signals from the pickups are amplified and are fed back to the corresponding electrodes at the required phase.

Such a stabilization system was used in the "Ogra-II" installation[77]. The experiment was performed in a simple mirror field with a rarefied plasma of $\sim 10^7$ cm^{-3} density, produced by Lorentz ionization of

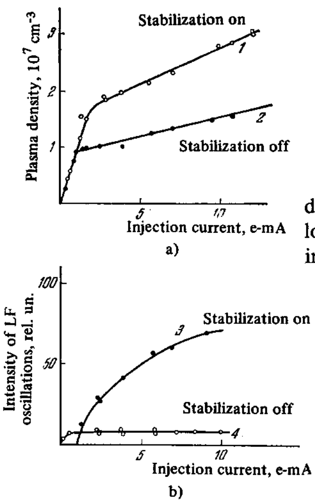

FIG. 13. Dependence of the plasma density (a) and of the intensity of the low-frequency oscillations (b) on the injection current [37].

a beam of neutral atoms of hydrogen with energy 70 keV. In the absence of stabilization, at a density (7–8) × 10^6 cm^{-3}, flute instability of the first azimuthal mode sets in and limits the density of the accumulated plasma. This is illustrated in Fig. 13, taken from[77]; the plot of the plasma density against the injection current reveals a sharp kink at the indicated density, corresponding to the appearance of high-frequency oscillations characteristic of flute instability (see Fig. 13b). When the stabilization system is turned on, the amplitude of the low-frequency oscillations decreases sharply and does not depend on the injection current. The maximum density of the plasma approximately doubles, and the observed kink on the linear dependence of the plasma density due to the injection current is now already connected with the buildup of high frequency oscillations corresponding to ion-cyclotron instability, but the stabilization system was not designed for the suppression of the latter instability.

This experiment confirmed convincingly the possibility of suppressing large-scale flute oscillations in a rarefied plasma by means of feedback. It turns out to be effective for the suppression of the first mode also in a denser plasma[79]. However, with increasing density there can appear perturbations of higher and higher modes, with respect to which the feedback system becomes less effective. The authors of[77] propose that the higher modes can be stabilized by the effect of the finite Larmor radius, as previously predicted by the theory[112]. If nonetheless this effect turns out to be insufficient (as was the case, for example, in the aforementioned experiments with the "ion magnetron"), then it is apparently impossible to suppress the instability completely with the aid of feedback.

5. KINETIC INSTABILITIES OF A PLASMA

In flute instability of plasma, an important role is played only by the gross characteristics of the plasma—the presence of a transverse energy that leads to magnetism, and the location of the plasma with respect to the minimum magnetic field. In this sense, flute instability can be called hydrodynamic; just as in hydrodynamics, the concepts of a continuously flowing medium suffice in this case. At the same time, however,

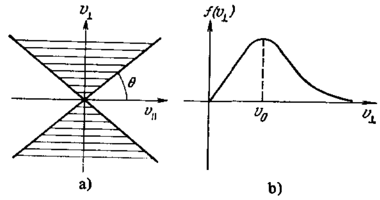

FIG. 14

there can develop in the plasma more subtle kinetic instabilities, which are sensitive to the details of the particle velocity distribution function. Once the flute instability is suppressed, it is precisely these kinetic instabilities that come to the forefront.

An adiabatic trap has a built-in defect—it retains only particles that do not fall into the forbidden cone (loss cone). This leads automatically to population inversion of the energy levels. In fact, let us consider velocity space (Fig. 14a). The particle velocity distribution function $f(v_z, v_\perp)$ differs from zero only in the shaded region. Even if $f(v_z, v_\perp)$ is approximately constant in a certain region of not too large $v_\perp$, the transverse-velocity distribution function $f(v_\perp) = \int f(v_z, v_\perp) dv_z$ will increase simply because $\Delta v_z \sim v_\perp$. At sufficiently large $v_\perp$, the function $f(v_z, v_\perp)$, together with $f(v_\perp)$, should decrease, so that the complete plot of $f(v_\perp)$ is as shown in Fig. 14b. In the region $v_\perp < v_0$, the distribution function increases with increasing $v_\perp$, and this means that there is an inverted population with respect to the transverse energy—there are more particles with large energy than with small energy.

In analogy with lasers, it can be concluded here that if there is in the plasma a wave that interacts in resonant fashion with the inverted population of the particles, then induced generation of the waves can take place, i.e., an exponential growth of small oscillations with time. Thus, the question of the buildup of kinetic instabilities consists of determining which waves in the plasma can enter in resonance and how this resonance is realized. These questions are formulated in somewhat different manners for hot-ion and hot-electron plasma.

Let us consider first a hot-ion plasma. It is clear that only low-frequency oscillations, with frequencies on the order of the characteristic ion frequencies Ω_B or Ω_0, can interact with the ions effectively. In a laboratory plasma, the quantity $\beta = 8\pi nT/B^2$ is exceptionally small, and therefore the corresponding waves are longitudinal (electrostatic), and the magnetic field in them is not disturbed. These include oblique Langmuir waves propagating at an angle to the magnetic field, ion-acoustic and cyclotron waves, and also their various combinations. In an inhomogeneous plasma, one can add to them also drift waves.

As to resonance between the waves and the ions, in the presence of a strong magnetic field it is realized at harmonics of the cyclotron frequency Ω_B. Although the corresponding effect, is, of course, purely classical, it is simpler to consider it from the quantum point of view, as a limiting case of induced transitions in which a large number of quanta take part—the quantum concepts have so deeply penetrated in modern physical thought, that in the purely classical concept of resonance it is simpler to formulate in the quantum language.

From the quantum point of view instability is induced emission of a quantum $\hbar\omega$ with momentum $\hbar k_z$ along the magnetic field. Inasmuch as in a strong magnetic field the transverse motion of the charge particles is quantized (Landau levels with intervals $h\Omega_B$ between levels), the conservation laws for the energy and the longitudinal momentum in the transition take the form

$$\hbar\omega + Mv_z\,\Delta v_z - l\hbar\Omega_B = 0, \quad (5.1)$$

$$\hbar k_z + M\,\Delta v_z = 0, \quad (5.2)$$

where Δv_z is a small change of the longitudinal velocity in the transition and l is an integer corresponding to the transition of the particle to a lower level located at a distance $l\hbar\Omega_B$ from the initial one, so that $Mv_\perp\Delta v_\perp = -l\hbar\Omega_B$. From (5.1) and (5.2) we obtain the relation

$$\omega - k_z v_z - l\Omega_B = 0, \quad (5.3)$$

which is the condition of the resonance of the particle with the wave. It has a simple meaning—in the coordinate system moving z together with the particle, the oscillation frequency is exactly equal to the l-th harmonic of the cyclotron frequency. At $l > 0$, when the particle goes over to the lower transverse level upon emission of the wave, one speaks of a normal Doppler effect, and when $l < 0$ one uses the term "anomalous Doppler effect." The instability takes place only in the case when the population at the upper (initial) energy level is larger than at the lower one, i.e., if

$$\int \{f(v_z,\ \varepsilon_\perp) - f(v_z + \Delta v_z,\ \varepsilon_\perp - l\hbar\Omega_B)\}\,\delta(\omega - k_z v_z - l\Omega_B)\,dv_z\,d\varepsilon_\perp > 0, \quad (5.4)$$

where $\epsilon_\perp = Mv_\perp^2/2$ is the transverse energy, and the δ function selects only resonant particles.

Letting $\hbar \to 0$, we obtain from (5.4), with allowance for (5.1) and (5.2),

$$\int \left(k_z \frac{\partial f}{\partial v_z} + \frac{l\Omega_B}{v_\perp}\frac{\partial f}{\partial v_\perp}\right)\delta(\omega - k_z v_z - l\Omega_B)\,dv_z\,d\varepsilon_\perp > 0. \quad (5.5)$$

As follows from (5.2) and (5.3), in the transition

$$v_\perp\,\Delta v_\perp + [v_z - (\omega/k_z)]\,\Delta v_z = 0, \quad (5.6)$$

i.e., the particles move in velocity space along trajectories

$$v_\perp^2 + [v_z - (\omega/k_z)]^2 = \text{const}, \quad (5.7)$$

in the form of concentric circles with center at the point $v_z = v_{ph}$, where $v_{ph} = \omega/k_z$ is the phase velocity. Accordingly, $f(v_\perp^2 + (v_z k_z^{-1}\omega)_\perp^2)$ causes (5.5) to vanish.

It is easy to see that on moving along the trajectory (5.7) the particle loses an energy ($v_\perp\Delta v_\perp + v_z\Delta v_z < 0$) if the change of Δv_z has a sign opposite to that of v_{ph}, i.e., the particle is decelerated by the wave along B. We choose for concreteness $v_{ph} > 0$, i.e., $k_z > 0$. Then when $v_z < v_{ph}$ resonance is realized at the normal Doppler effect ($l > 0$), and the transverse energy of the particle decreases upon emission of the wave. In this case the instability is most strongly connected with the population inversion with respect to the transverse energies ($\partial f/\partial v_\perp > 0$ in the region making the main contribution to (5.5)), although even simple anisotropy ($T_\perp > T_\parallel$) may suffice to make (5.5) positive (at $v_z < 0$). When $v_z > v_{ph}$, the resonance is realized only on the anomalous Doppler effect, when the particle longitudinal motion slows down, but their transverse energy increases. This case is important only in the presence of beams.

At very low densities, when even the plasma electron frequency $\omega_0 = (4\pi e^2 n_0/m^{1/2}$ is lower than Ω_B, there can be no resonance between the waves and the ions, and there should be no kinetic instabilities in the hot-ion plasma in general. With increasing density, ω_0 reaches Ω_B and resonance sets in between the Langmuir oscillations of the electrons and the cyclotron rotation of the ions—the Harris cyclotron instability develops (see[80,81]). If we continue to increase the density, then the resonance will be realized most effectively on oblique Langmuir waves with $\omega = k_z\omega_0/k$, where k_z is the longitudinal component of the wave number. Under such oscillations, the strongly "magnetized" electrons move along the force lines of the magnetic field, i.e., at an angle to the electric field, and accordingly they become so to speak "heavier." With increasing density, the resonance goes over to higher and higher harmonics of Ω_B, and finally when $\Omega_0 > \Omega_B$ we arrive at the so-called cone instability[82]*. In this case the influence of the magnetic field on the motion of the ions can be completely neglected, the oscillations become quasineutral and assume the form of electronic sound—the electron execute inertial oscillations along the magnetic field, and the ions, being drawn across the magnetic field, produce pressure perturbations and, as a result of the inverted population, build up waves corresponding to the inverse Landau damping. The cone instability develops on perturbations that are strongly elongated along the magnetic field, and is therefore very sensitive to the longitudinal dimensions of the plasmoid. A decrease of the longitudinal length to approximately 100 Larmor radii can probably stabilize the cone instability[86].

In a dense inhomogeneous plasma it is possible to have, in addition, a drift-cone instability[85-88] connected with the build up of drift oscillations of the electron bunch by the ions. To stabilize the drift-cone instability, in accordance with the theory[86], it suffices to decrease the transverse density gradient by increasing the transverse dimension of the plasmoid to 100 or 200 ion Larmor radii. Thus, from the point of view of the rough theory, without allowance for any details of the interaction between the waves and the inhomogeneous plasma, the stabilization of the dense-plasma cone and drift-cone instabilities having the largest growth increment can be attained by a suitable choice of the dimensions and the shape of the plasma—it is necessary to produce an almost spherical plasmoid with dimensions on the order of $10^2\,\rho$. An additional means of stabilization may be the addition of "warm" plasma, partly filling the region with the small $v_\perp$[89].

Besides the cone instabilities, which can be regarded as non-resonant, in a plasma contained in adiabatic traps it is possible to build up resonant cyclotron oscillations with much smaller increments and with frequencies closely adjacent to the harmonics of the cyclotron frequency[90]. The resonant instabilities should be

*The cone instability was discussed in a somewhat abstract form by Dnestrovskiĭ [83], Krasovitskiĭ and Stepanov [84], and Mikhailovskiĭ [85]; it was separated most distinctly and analyzed by Rosenbluth and Post [82,86].

very sensitive to the inhomogeneities of the magnetic field, and therefore it is precisely the inhomogeneity of the field which may possibly permit their stabilization. However, this can give rise to new instabilities of the negative-mass type.

The instabilities listed above can be called fundamental in the sense that they are less sensitive to the details of the distribution function of the ions with respect to the transverse velocities—with very existence of the loss cone suffices for these instabilities. In addition, certain additional instabilities can appear in adiabatic traps. If, for example, the trap contains besides hot plasma also cold plasma, then an instability on the "double-humped" distribution function may develop—in this case the hot ions build up cyclotron oscillations or oscillations at the upper hybrid frequency $(\Omega_0^2 + \Omega_B^2)^{1/2}$ in the cold plasma[91-92]. In addition, at a sufficiently large monochromaticity of the distribution function $f(v_\perp)$, the so-called ion instability is possible, in which, as it were, two neighboring cyclotron harmonics, analogous to a certain degree to two beams in a plasma, build each other up mutually[93]. The corresponding instability can appear also at a frequency lower than the cyclotron frequency. In this case it turns out that the wave energy can be negative[94,97], i.e., the energy of the plasma and of the perturbation field turned out to be smaller than in the unperturbed plasma. Besides the electrostatic instabilities, a plasma with sufficiently large $\beta = 8\pi nT/B^2$ is subject to a possible instability at the magnetosonic (the so-called lower instability[98]) and at the Alfven branches[99] of the oscillations. We shall not enter in the details of the theoretical analysis of all these instabilities, since there are at present reviews specially devoted to the theory of instability of an anisotropic plasma[92,100]. In the succeeding sections we shall consider only the main kinetic instabilities of the plasma as they bare on the existing experiments.

In a hot-electron plasma there can also develop electrostatic instabilities similar to those listed above, but on the electronic branch of the oscillations. In addition, instability on transverse waves of the "whistler" (helicon) type is possible[99,100]. The corresponding physical mechanism of oscillation buildup and the experimental data will be considered briefly in Ch. 9.

6. CYCLOTRON ELECTROSTATIC INSTABILITY

Cyclotron instability of a rarefied plasma has by now been investigated in considerable detail in a number of experimental installations. It was first registered in "Ogra-I"[102,103]. It was shown that in accordance with the theory, the instability leads to the development of oblique Langmuir waves, interacting with the transverse cyclotron oscillations of the ions. Subsequently, the cyclotron instability, connected with resonance on the Langmuir waves, was observed in the "Alice"[52], "Phoenix"[83,104], DCX[105], "Ogra-II"[64], AC[106,107], DECA-II[66], and others. Before we proceed to describe and summarize the experimental results, let us consider briefly what is to be expected from the theory.

The Harris cyclotron instability is developed as a result of a resonant coupling between the cyclotron oscillations of the ions on a certain harmonic of the cyclotron frequency, i.e., $\omega = l\Omega_B$ (l = 1, 2, ...), with longitudinal Langmuir oscillations of the electrons. The frequency of the Langmuir oscillations in a strong magnetic field is equal to $\omega = k_z\omega_0/k$, where $\omega_0 = (4\pi e^2 n_0/m)^{1/2}$ is the ordinary Langmuir frequency, and k_z is the projection of the wave number on the direction of the magnetic field. The instability is possible only in the case when the longitudinal phase velocity of the oscillations ω/k_z is larger by several times (say by a factor of 3) than the thermal velocity of the electrons $v_e = (T_e/m)^{1/2}$, for in the opposite case the Landau damping on the electrons comes into play. The buildup of the oscillations has an optimum at a transverse wave number determined by the relation $k_\perp v_\perp \sim l\Omega_B$, where $v_\perp$ is the transverse thermal velocity of the ions. At smaller $k_\perp$, the wave cannot "probe" the distribution function and to "feel" that the distribution with respect to the velocity is inverted, and at large $k_\perp$ the transverse wavelength is too short—the shallow oscillation in space deteriorates the resonance between the ions and the wave. Thus, at the optimum we have

$$\omega \approx l\Omega_B \approx (k_z/k)\,\omega_0 \approx k_\perp v_\perp > 3k_z v_e. \tag{6.1}$$

Hence, taking into account the fact that $k = (k_\perp^2 + k_z^2)^{1/2}$, we obtain the density threshold for the cyclotron instability

$$\omega_0/l\Omega_B \gtrsim [1 + (9MT_e/mT_i)]^{1/2}. \tag{6.2}$$

We see that with increasing density there should first appear instability at the first harmonic, then at the second, third, etc. with the corresponding threshold values of the density proportional to l^2.

Further, from the condition $(k_z/k)\omega_0 \approx l\Omega_B$ it follows that with increasing density the wave with a given l should stretch more and more along the magnetic field. But since the length of the plasmoid is limited, k_z cannot decrease without limit; consequently, starting with a certain density, the transverse component $k_\perp$ should increase. Accordingly the growth increment of the small oscillations should decrease.

All these considerations are in splendid correlation with the experimental data obtained in different setups[102-111]. The most systematic investigations were performed on "Phoenix." We present here the main results obtained with this setup. Figure 15 shows a com-

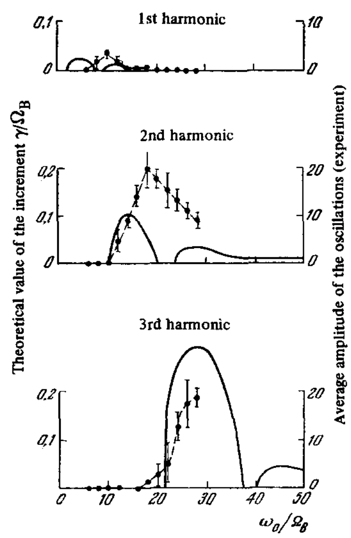

FIG. 15. Comparison of the average amplitude of ion-cyclotron oscillations with the theoretically calculated increment for the model of finite geometry [108].

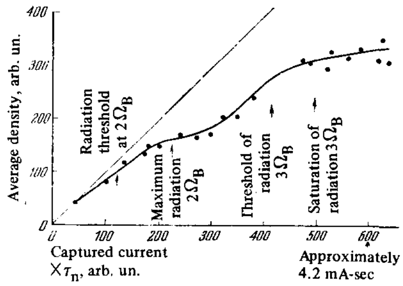

FIG. 16. Correlation of average ion density with appearance of harmonics of ion-cyclotron radiation [106].

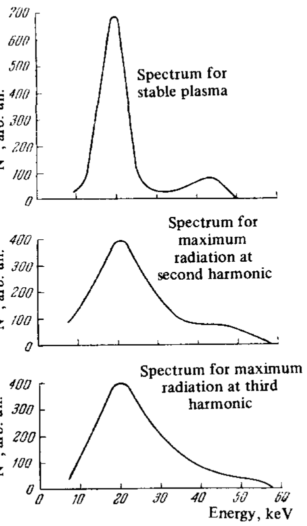

FIG. 17. Variation of the energy spectrum of the protons under the influence of ion-cyclotron instability [104].

parison of the experimentally measured oscillation amplitude[108] at the cyclotron-frequency harmonics with the theoretically calculated increment (its non-monotonicity is connected with allowance for the oscillations of the electrons along the trap as a result of reflection from the mirrors) as a function of ω_0/Ω_B. Special measurements of the phase shift of the oscillations along the trap have shown that these oscillations actually represent the first mode of the Langmuir oscillations of the density with a node in the middle and with two antinodes (in antiphase) on the ends.

The oscillations lead to ejection of some of the ions from the trap and to a limitation of the plasma density[108], as shown in Fig. 16, which represents the dependence of the density on the fast-neutral flux injected into the trap (it is the ionization of the neutrals that produces the hot plasma). As we see, the deviation from the dashed straight line, characterizing the particle intensity loss, correlates with the successive development of oscillations at increasing harmonics of the cyclotron frequency with increasing density.

At the same time, the oscillations lead to an increased exchange of energy between the particles and to a "growth of a tail" of hot particles[104], as illustrated in Fig. 17.

It was noted above that if the phase velocity of the wave ω/k_z does not greatly exceed the thermal velocity of the electrons, the buildup of the oscillations should be hindered by the Landau damping. This conclusion was also confirmed experimentally. In experiments with "Phoenix"[108,109,79] and "Alice" the electrons were purposely heated additionally with the aid of electron-cyclotron resonance, and it was established that at increased plasma density, when the oscillations at the second and third harmonics of Ω_B predominates, the heating leads to disappearance of these oscillations. To be sure, the heating leads to disappearance of these oscillations. To be sure, this gives rise to intense oscillations of the fundamental frequency Ω_B, but of a different nature. The corresponding waves change $k_z = 0$ and correspond possibly to either drift-cone or to modified negative-mass instability[79].

The instability on the ion-cyclotron frequency and its harmonics, and also with $k_z = 0$, and consequently not associated with longitudinal oscillations of the electrons, was observed at high densities in the DECA-II installation[140]. A number of experimentally-investigated characteristics of this instability lead the authors of[140] to the conclusion that in this case there develop in the plasma oscillations due to the double-humped nature of the ion distribution function.

7. CONE AND DRIFT-CONE INSTABILITIES

With increasing plasma density, the oscillations in the cyclotron instability should go over to higher and higher cyclotron harmonics and accordingly to larger values of the transverse wave number $k_\perp$. The growth increment of the oscillations in a plasma of unlimited length also increases in this case, and if it exceeds Ω_B, then the influence of the magnetic field on the motion of the ions in the oscillations can be neglected completely, for in this case the amplitude of the oscillation changes significantly during the time of revolution on the Larmor circle. Accordingly, the configuration of the magnetic field determines only the equilibrium distribution function of the ions and the form of the wave along B.

Since the ions are not magnetized, the distribution of the ions over the velocity components are transverse with respect to the wave vector is insignificant, so that for example for a wave propagating along the y axis there enters into the dispersion equation only $f(v_y) = \int f(v)dv_x dv_z$. It turns out that the distribution function $f(v_y)$ has a positive derivative $df(v_y)/dv_y$ for a distribution function $f(v_\perp)$ with a "cut-out" cone, i.e., at small v_y there is an inverted population also with respect to v_y. Accordingly, for a wave with a small transverse phase velocity $\omega/k_\perp < (T_i/M)^{1/2}$ there can take place a buildup of oscillations. Waves with such a small transverse velocity have the character of electron sound—ions in such waves take upon themselves the role of elasticity, and the electrons execute inertial oscillations along the magnetic field. Then $\omega \sim (T_i/m)^{1/2}k_z$, $k_z \sim k_\perp(m/M)^{1/2}$, the increment is $\gamma \sim \Omega_0$ at $\Omega_0 > \Omega_B$, and the transverse wavelength is somewhat smaller than the Debye radius of the ions. Since the frequency ω is proportional to k_z, the group velocity of the wave packet along B is of the order of the phase velocity, so that the instability has a convective (drift) character—an individual perturbation building up in time, drifts away quite rapidly along B. Therefore the instability requires for its development a sufficiently large intensification of waves over the length of the trap or a certain reflection from the ends. By shortening the trap and obtaining as complete an absorption of the waves as possible on the ends, one can hope to stabilize the cone instability[86,89]. The critical length of the trap L, starting with which the instability should develop, is determined from the condition[86]

$$\frac{L}{\rho} > 20K\left(\frac{M}{m}\right)^{1/2}\left(\frac{m}{M}+\frac{\Omega_B^2}{\Omega_0^2}\right)^{1/2}, \quad (7.1)$$

where ρ is the average Larmor radius of the ions, and K is a constant that depends on the form of the distribution function of the ions with respect to velocities and on the degree of reflection of the waves from the ends. For the so-called collision-equilibrium distribution (maximum smeared-out type of Maxwellian distribution with a cut-out cone) and in the absence of reflection, typical values of K range from 4 to 8, depending on the mirror ratio. We see therefore that for a dense plasma ($\Omega_0 > \Omega_B$) the length of the trap should be not larger than 100—200 Larmor radii. At narrower distribution and at stronger reflection of the wave from the ends, the critical length is even smaller.

The cone instability is very strong and can upset the plasma containment completely: according to the theory, the containment time should not exceed the travel time of the ions along the trap[113].

At the present time there are no experimental data on cone instability. In all the existing installations (with the possible exception of the 2X installation[114]), the condition (7.1) has not yet been attained. Therefore the question of the existence of this instability and its threatening circumstances remains so far only a subject of theoretical predictions.

Close to the cone instability is the drift-cone instability[85,86]. The difference between them is that in the second case the electrons move mainly not along the magnetic field but across it, executing the so-called drift oscillations. Since these oscillations are less known than the Langmuir oscillations, let us explain here briefly their origin. Let us assume that a small perturbation of the electron density, constant in the z direction, has been superimposed on a layer of plasma which is inhomogeneous along the x axis and has a density $n_0(x)$ (the z axis is directed along the magnetic field): $n' \exp(-i\omega t + ik_x x + ik_y y)$. If the ions are immobile, then this perturbation leads to the appearance of an electric field with potential φ, which can be determined with the aid of the Poisson equation:

$$k^2\varphi = -4\pi e n'. \quad (7.2)$$

In the electric field of the wave, the electrons execute electric drift with velocity $\mathbf{v} = c\mathbf{B} \times \nabla\varphi/B^2$, so that in the linear approximation the continuity equation for the electrons takes the form

$$-i\omega n' - cB^{-1} ik_y\varphi\,(dn_0/dx) = 0. \quad (7.3)$$

From (7.2) and (7.3) we obtain the frequency of the drift oscillations:

$$\omega = (4\pi eck_y/Bk^2)\,dn_0/dx. \quad (7.4)$$

We see that these oscillations take place only in an inhomogeneous plasma, but under laboratory conditions the plasma is always inhomogeneous, and the density gradient is no longer so small.

The drift oscillations (7.4), like the Langmuir ions, can be built up by the ions as a result of their inverted population.

The corresponding instability is called drift-cone instability. According to[86] it should develop if the following condition is satisfied:

$$\frac{\rho}{a} > 0.4\left(\frac{m}{M}+\frac{\Omega_B^2}{\Omega_0^2}\right)^{2/3}, \quad (7.5)$$

where a is the characteristic dimension of the density gradient,

$$a^{-1} = n_0^{-1}\,|\,dn_0/dr\,|.$$

Unlike the convective cone instability, the conditions under which one can expect the appearance of the drift-cone instability have definitely been attained recently in two installations, namely 2X[114] and PR-6[115]. In the former of these, the threshold density determined from (7.5) was exceeded by at least several dozen times (during the last stage of decay, when relatively long periods of stable plasma containment are observed), while in the latter they were exceeded by 5—10 times. Nonetheless, no instability symptoms were observed in either experiment. The containment time of the freely decaying plasma amounted to milliseconds and were determined mainly by the charge-exchange losses, whereas the drift instability should lead to a loss of plasma within a time on the order of 10^{-6} sec[116].

The noted contradiction between experiment and theory has not yet been unambiguously explained. It is not excluded that it is connected with the insufficiently "clean" conditions of the experiments: it is possible that besides the main component of the "hot" plasma there is a rather small admixture of "warm" plasma, which partly fills the forbidden cone and stabilizes the instability in accordance with[89].

8. INSTABILITY OF THE NEGATIVE-MASS TYPE

Whereas no one has succeeded as yet in observing the cone or drift-cone instability experimentally, it can be stated that the negative-mass instability in adiabatic traps turned out to be an uninvited guest—it appeared just where it was not expected at all. A few years ago, an interesting effect was observed in the PR-5 installation, namely, a powerful ejection of plasma from the

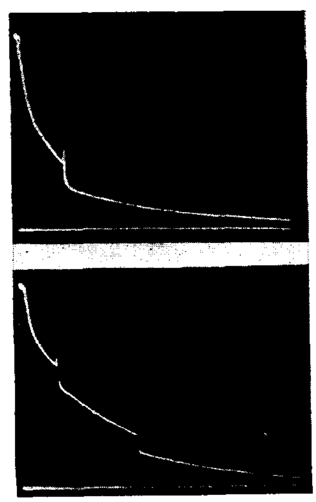

FIG. 18. Oscillograms of the flux of charge-exchange neutrals, illustrating the drops of the density following bursts of instability during the time of free decay of plasma (sweep duration 3 msec) [117].

trap, accompanied by cyclotron radiation[117,118]. Sometimes, as shown in Fig. 18, the plasma density during the time of collapse decreased by a factor 2—3. A detailed investigation of this phenomenon[118] has shown that during the time of the plasma ejection there develop in it intense oscillations with a frequency close to the frequency oscillation, corresponding to the second or third (more rarely fourth) azimuthal mode (the number of the mode corresponds to the number of wavelengths subtended by the perimeter of the plasma in azimuth). The oscillations have the character of a traveling wave directed in the direction of rotation of the ions. Along the magnetic field, their phase is constant, i.e., they have a flute character. The amplitude of the oscillations is very large—sometimes it corresponds to oscillations of potential on the order of the ion energy (1—1.5 kV). What is furthermore most important is that the transverse wavelength of such oscillations is not small, corresponding to $k_{\perp}\rho \ll 1$. In the arsenal of instabilities of a uniform plasma, there was nothing that could correspond to these facts to any degree.

They were explained on the basis of the negative-mass instability, or more accurately, a modified instability that differs greatly (and therefore turns out to be unexpected) from the ordinary negative-mass instability that originated in the theory of cyclic accelerators[119,120]), and is manifest also in certain plasma experiments natural for this instability[121,122].

Since the physics of all the instabilities of the negative-mass type is the same, we begin with the initial variant, observed independently by Lebedev and Kolomenskiĭ[119] and by Nielsen et al.[120] Let us consider a certain group of positively charged particles distributed uniformly over the cyclotron circle (Fig. 19) and rotating in a clockwise direction. This can be either a homogeneous bunch of accelerated particles in a cyclotron, or simply a certain isolated group of ions in a plasma. Let us assume now that a small perturbation A has been produced against the uniform background. In the electric field of this perturbation, the ions of the right-hand semicircle will lose transverse energy $\epsilon_{\perp}$ (they move opposite to E), while in the left-hand semicircle they gain energy. If the particle rotation frequency ω is a decreasing function of the energy, then the ions of the right semicircle will lead the particle with the initial energy, while those of the left semicircle will lag; this will result in motion of the ions in phase towards the point A. In other words, when $d\omega/d\epsilon_{\perp} < 0$ the initial perturbation will increase. Since in this case the particles move, as it were, against the applied force, this instability is called negative-mass instability. Its density threshold depends on the initial spread with respect to ω, tending to zero for a monochromatic beam.

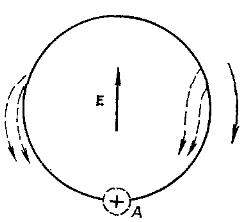

FIG. 19

In accelerators, the condition $d\omega/d\epsilon_{\perp} < 0$ is the result either of relativism or of the decrease of the magnetic field in the radial direction. A similar effect takes place also in traps with magnetic mirrors, when the Larmor radius of the ions is sufficiently large. The corresponding instability was observed experimentally[121]. On the other hand, in the PR-5 installation the magnetic field does not decrease, and rather increases in the radial direction (because of the stabilizing windings), and in addition the Larmor radius of the ions is very small compared with the curvature radius of the force lines.

However, an effect leading to modified negative-mass instability does take place[123-125]. The point is that the ion losing the transverse energy penetrates deeper into the magnetic mirrors, and therefore its average cyclotron-rotation frequency increases. In other words, the effect $d\omega/d\epsilon_{\perp} < 0$ is reached as a result of the longitudinal inhomogeneity of the magnetic field, which is not eliminated in adiabatic traps.

The modified negative-mass instability has made it possible to determine the qualitative theoretical picture of the density drops in PR-5 as being the result of the buildup of surface oscillations in the peripheral plasma on the upper hybrid resonance $\omega = (\Omega_0^2 + \Omega_B^2)^{1/2}$. In such oscillations, the ions inside the plasma are gathered in phase on their Larmor circles, and this leads to incompressible oscillations of the macroscopic velocity. As a result, charges appear on the side surface of the plasma, and travel in the form of a wave in an azimuthal direction towards the ion side.

In the experiments with the PR-5, the oscillation amplitude of the electric field of the wave turns out to be quite large. One can therefore assume that in such a field the resonant ions experience phase oscillations and exchange energy with the wave. In that phase, when the transverse ion energy is maximal, their points of reflection from the magnetic mirrors shift towards the center of the trap. Conversely, in the phase when the ions give up part of their transverse energy to the wave, they penetrate deeper into the mirrors and can leave the trap. This effect, which leads to the ejection of only the "spent" particles that gave up part of their energy to the wave, makes build up of oscillations possible also during the nonlinear stage, and the growth time can be sufficiently large—on the order of period of the phase oscillations of the particles. These considerations agree to some extent with the experimentally observed picture of the slow development and equally slow decrease of the oscillations with sufficiently large amplitude, and the oscillations are accompanied by a loss of a noticeable fraction of the hot ions. The fact that after the loss the plasmoid becomes shorter in the direction of the magnetic field also indicates that particles that have penetrated sufficiently far into the mirrors are ejected.

A modified negative-mass instability was independently observed also in experiments with the DCX-II installation[125] where it turned out that it led to quite unexpected and curious consequences.

In the DCX-II, the plasma is produced by injection and subsequent dissociation of the molecular ions of the hydrogen with energy 270 keV per proton. Under the optimal injection conditions, the density of such a

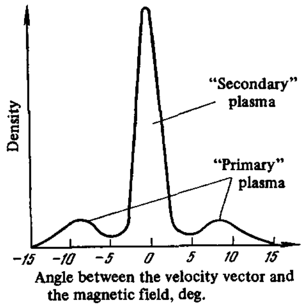

FIG. 20. Angular distribution of ions in the central cross section of DCX-11 [125].

plasma does not exceed 5×10^8 cm^{-3}. Besides this "primary" plasma, there is produced in the trap a "secondary" plasma, which is much denser and hotter: $n_{sec} \approx 5 \times 10^9$ cm^{-3}, $T_{i.sec} \approx 800$ keV; the ions have then an unusually anisotropic angular distribution: $T_\perp/T_\| \approx 10^3$ (Fig. 20).

It has been ascertained that the source of the secondary plasma are the ions produced from the residual gas by ionization and charge exchange; the initially cold ions are accelerated by the ion-cyclotron waves, in which the electric field reaches 100 V/cm.

The spatial structure of these waves, the dependence of the increment on the density, and a number of other characteristics that have been experimentally investigated, agree well with the theory of the negative-mass-type instability due to longitudinal inhomogeneity of the magnetic field.

9. INSTABILITY OF A PLASMA WITH HOT ELECTRONS

We have described above experiments with a hot-ion plasma. By now, many experiments have been performed also with hot-electron plasma. To produce such a plasma one uses a great variety of methods: high frequency heating at electron-cyclotron resonance[35,75,126], heating with electron beams[73,127,128], heating by pulses of longitudinal magnetic[129] or electric fields[130] (turbulent heating[131,132]), and adiabatic compression of the plasma[133]. Different methods lead to somewhat different plasma parameters, but the density of the hot electrons lies in the approximate interval from 10^{10} to 10^{11} cm^{-3}, and the energy ranges from several keV to several hundred keV. As a rule, besides the hot plasma there exists in the traps a cold plasma with a density not lower than that of the hot electrons, and sometimes even larger by one order of magnitude. The presence of the cold plasma usually leads to stabilization of the field instability, and therefore one rarely resorts to "minimum B" in experiments with hot-electron plasma.

As a rule, bursts of instability are observed in experiments with hot-electron plasma of sufficiently high density. They are observed in the form of very rare interruptions of the diamagnetic signal (i.e., the pressure of the plasma nT), intense bursts of microwave radiation at frequencies on the order of the electron-cyclotron frequency ω_B, and ejection of electrons along the magnetic field and across it.

A thorough frequency analysis of the bursts was carried out in[75]. In that investigation, the plasma was produced and heated in a resonator with the aid of electron-cyclotron resonance ($\omega_{gen} \approx 2\omega_B$) at an initial neutral-gas pressure on the order of 10^{-4} Torr. It consisted of two components—hot with $n_e \approx 10^{10}$ cm^{-3} and $T_e \approx 15-30$ keV, and cold with density larger than 10^{10} cm^{-3}. It turned out that during the time of an individual burst the radiation is monochromatic, but the frequencies vary slightly from burst to burst. The frequencies cluster with greatest probability about values $\omega \approx 2\omega_B$, $\omega \approx 0.75\omega_B$, and $\omega \approx 0.6\,\omega_B$. The maximum radiation intensity occurs at $\omega = 0.75\,\omega_B$, when the maximum plasma energy loss is also observed (a drop in the diamagnetic signal to 80%).

Attention is called to the fact that the frequency of the oscillations (if we disregard the second harmonic) lies lower than the cyclotron frequency. It is observed also in other installations. For example, in[134], the authors of which produce with the aid of microwave heating a two-component plasma with $n_{hot} \approx 10^{10}$ cm^{-3}, $T_{e.hot} \approx 150$ keV, $n_{cold} \approx (1-10)\,n_{hot}$ and $T_{e.cold} \approx 20$ eV, the bursts observed were either spontaneous or excited artificially by an external microwave pulse at a frequency $\omega = 0.66\,\omega_B$. Figure 20 shows by way of another example the results of an experimental investigation of the dependence of the oscillation frequency during the time of the burst on the density of the cold plasma[135]. In that investigation, the plasma was heated by a turbulent method in which pulses of longitudinal magnetic field were applied, and the plasma had the following parameters: $n_{hot} = 10^{10}-10^{11}$ cm^{-3}, $T_{e.hot} = 10-20$ keV, and $n_{cold}/n_{hot} \sim 10$.

Figure 22 shows how the density of the plasma and its anisotropy vary during the time of the flash[129]. We see that after the flash the anisotropy of the plasma decreases as expected.

A noticeable change in the anisotropy and in the distribution function of the hot plasma, and also an intense energy exchange between the hot and the cold plasma, were observed in[133]. A "tail grows," i.e., a group of high-energy electrons appears, in the hot plasma. At the same time, the density of the cold plasma increases with temperature at $T_e \approx 200$ eV, this being interpreted by the authors as a result of heating in the high frequency fields of the burst of the rather cold electrons with $T_e \approx 20$ eV.

There is no doubt that the burst of microwave radiation and the plasma-pressure drops are connected with the buildup of oscillations due to the nonequilibrium character of the distribution function of the hot electrons. It is however not yet fully clear just what kinds of oscillations are excited in the plasma. It was shown theoretically that in a hot-electron plasma the presence of only the anisotropy of the distribution function (transverse temperature $T_\perp$ larger than the longitudinal temperature $T_\|$) should lead not only to electrostatic[136] but also to electromagnetic instability, i.e., to the buildup of transverse oscillations with circular polarization—the so-called "whistlers" (this instability was predicted by Sagdeev and Shafranov[99] and subsequently considered by Sharev and Trivelpiece[101,137]). Both types of instabilities should develop at frequencies lower than the cyclotron frequency: by substituting the Maxwellian two-temperature distribution function ($T_\perp \neq T_\|$) into the condition (5.5) for the buildup of the oscillations, we can readily obtain the condition

$$\omega/\omega_B < (\theta - 1)/\theta, \qquad (9.1)$$

where $\theta = T_\perp/T_\parallel$ is the anisotropy parameter.

The buildup conditions (9.1) pertain to any instability, but the maxima of the corresponding increments are reached nevertheless at several different frequencies. The solid line in Fig. 21 represents a plot of the frequency corresponding to the maximum increment against the cold-plasma density, for electrostatic and electromagnetic instabilities. We see that the experimental data on the $\omega(n_{cold})$ curve do not make it possible to decide with sufficient assurance on either of the two instabilities. Numerical calculations of the increments for this region of parameters also leads to close values for the two types of waves[135]. Nevertheless, the authors[135] were inclined to conclude settle on the buildup of "whistlers" i.e., transverse electromagnetic waves, since they lend themselves better to the explanation of the experiments on the amplification of artificially excited waves in a plasma. The conclusion that "whistlers" can build up is reached also by the authors of[75,134,139]. In particular, it was shown in[134,138] that the radiation emitted from the plasma actually has circular polarization in the direction of rotation of the electrons, as should be the case for waves of the "whistler" type.

On the other hand, however, the experimental data[133] are in good agreement with calculation[92] on the excitation of cyclotron electrostatic instability for a double-humped distribution function, namely, the theory predicts correctly the threshold density at which the instability is excited ($\omega_0 \sim 0.1\,\omega_B$), the magnitude of the relative density of the cold plasma ($n_{cold}/n_{hot} \approx 0.5$), the increment $\sim 2 \times 10^7$ sec^{-1}, and the frequency of the oscillations. Incidentally, one cannot exclude the possibility that these results agree also with the hypothesis of excitation of "whistlers" (helicons).

Summarizing, it can be stated that at the present time it is more probable that in a hot-electron plasma contained in an adiabatic trap there develops an electromagnetic instability of the "whistler" buildup type, although the possibility of electrostatic instabilities is not excluded.

10. CYCLOTRON-RESONANCE MASERS

Instabilities and the microwave bursts accompanied by plasma loss are, of course, undesirable phenomena in adiabatic traps. But a phenomenon of the same phys-

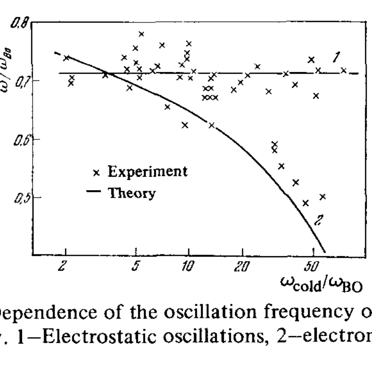

FIG. 21. Dependence of the oscillation frequency on the cold-plasma density. 1—Electrostatic oscillations, 2—electromagnetic oscillations [135].

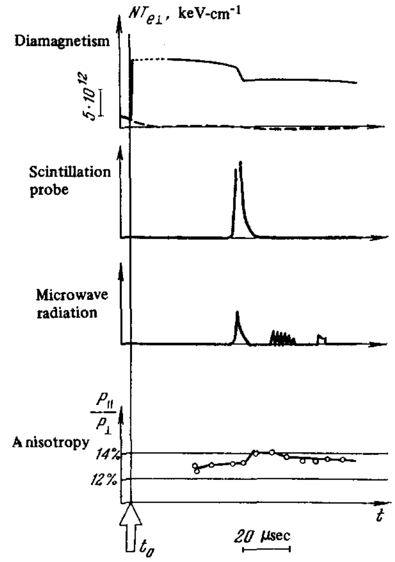

FIG. 22. Signals from different pickups and measurement of anisotropy in an instability burst.

ical nature is used successfully in radio electronics. We have in mind cyclotron-resonance masers[141-143]. Like masers, one likewise deals in this case with excitation of a microwave resonator, but the excited molecules are replaced by helical electron current in which the electrons execute, besides a certain longitudinal motion, also rotation on the Larmor circles. The oscillation buildup is due precisely to the interaction with the transverse motion. Thus, the term "maser" is used in this case not in the sense of a quantum generator; in essence, the interaction of the electron beam with the resonator is purely classical and can be described in customary radioelectronic terms. However, the analogy with masers turns out to be quite fruitful, making it possible to analyze theoretically from a unified point of view, on the basis of the mechanism of stimulated emission of classically excited oscillators, a large number of electronic devices using beams of oscillating electrons and to indicate ways of their development[143].

In cyclotron-resonance masers (CRM) the role of the excited oscillators is played by electrons revolving on Larmor circles in a helical beam. The corresponding excitation or amplification of the oscillations with frequency close to cyclotron frequency in the CRM does not differ in fact in its physical nature from cyclotron instabilities in adiabatic traps. The only difference is that in the CRM one deals with excitation of a vacuum resonator, and in adiabatic traps the oscillating medium is a plasma.

However, from the point of view of a more detailed picture of the interaction between the resonant electrons and the medium, there is nevertheless a definite difference between the CRM and the adiabatic trap. The point is that in adiabatic traps the particle velocity distribution function is always smooth, so that only a small fraction of the particles is in resonance with the wave. In the CRM, to the contrary, it is desirable that all the electrons experience resonant interaction with the wave, and it is therefore advantageous to use beams with small velocity spreads. Accordingly, instead of the method used in Ch. V to describe the interaction of the particles with the waves, in which the analogy with the Landau damping we considered the contribution made to the wave buildup of only the resonant particles, it is convenient to change over to a concept more familiar to radio electronics, that of bunching of beam particles as they interact with the wave. This also makes it possible to consider the oscillation-buildup mechanism from a somewhat different point of view.

It turns out that there exist two mechanisms of oscillator bunching in the wave field, as a result of which the coherent stimulated emission is produced[143,158]. One of them is connected with the longitudinal displacement of the oscillators, which is analogous to the "recoil" (5.2) of the oscillator following emission of a photon. This displacement leads to a spatial bunching of the oscillators. In addition, if the oscillators are not isochronous, i.e., if their frequency depends on the energy (for electrons this is the result of the relativistic dependence of the mass on the velocity), a possibility appears of phase grouping of oscillators, analogous to that considered by us earlier in the discussion of the negative-mass instability. It turns out, that in the CRM the predominance of any particular type of bunching is determined by the value of the phase velocity v_{ph} of the electromagnetic wave: spatial bunching predominates when $v_{ph} < c$ and phase bunching when $v_{ph} > c$[143,159]. Thus, even at relatively low electron velocities compared with the velocity of light, if $v_{ph} > c$ it is important to take into account the relativistic effect that leads to phase bunching[159]. This conclusion is not so unexpected if it is recognized that at low densities of the electrons in the beam the frequency of the oscillations should be very close to the cyclotron frequency.

The phase bunching due to the relativistic effect can become manifest also in adiabatic traps, as was demonstrated recently in[160], where buildup of oscillations of a cold plasma by hot electrons at double the cyclotron frequency was observed.

11. EARTH'S RADIATION BELTS

The earth's magnetic field is a natural magnetic trap, the efficacy of which with respect to containment of a rarefied plasma was clearly demonstrated by the discovery of the radiation belts[144,145]. This discovery immediately raised a number of problems, basically of purely plasma character: where do the high-energy particles come from and how are the radiation belts maintained, what are the physical processes that occur in the belts, what causes their apparent stability, etc. A prolonged and detailed investigation of these problems revealed an abundant and interesting number of physical phenomena[146,147], quite close in their nature to those occurring in laboratory adiabatic traps. An important role in the understanding of these phenomena were played by those concepts that were developed during the process of investigations in the physics of high-temperature plasma and controlled thermonuclear reactions.

The picture of the dynamics of radiation belts can be described briefly, and of necessity in a very simplified manner, as follows: The radiation belts are regions with increased corpuscular radiation, i.e., zones in which the earth's magnetic field traps high-energy charged particles. The belts lie in the earth's magnetosphere, i.e., in a cavity containing the earth's magnetic field compressed by the solar wind—a continuous flux of plasma from the sun. On the side facing the sun, the boundary of the magnetosphere lies approximately at $L \approx 10$ (L—distance from the center of the earth to the point of the intersection of the force line with the plane of the equator, expressed in earth's radii), while on the right side the force lines are elongated in the tail of the magnetosphere, which extends over a distance on the order of a million kilometers. If we confine ourselves only to the internal region of the magnetosphere, then we can say that the earth has one proton belt* and two electron belts. The proton belt with particle energies from 100 keV to several dozen MeV has a maximum intensity at $L \approx 3–4$. In approximately the same place, at $L \approx 3$, is located an electron-radiation minimum, separating the two electron belts (electrons at energy larger than 100 keV)—the outer belt, with maximum at $L \approx 1.5$. The particle flux density in the belts is of the order of $10^7–10^8$ cm^{-2} sec^{-1}.

Besides the high-energy particles there exists in the magnetosphere a cold plasma with particle energy from 0.1 to several eV and density of the order of 10^3 cm^{-3} at $L \lesssim 4$. At a certain L, whose value varies with the level of the magnetic activity in the interval $L \approx 3–6$, a sharp decrease of the cold-plasma density takes place, by one or two orders of magnitude. This decrease region is called "knee" or the plasmopause.

The radiation belts are formed and shaped via collisionless collective processes that are so characteristic of plasma physics. As to the proton belt, it is due to the transfer of particles in the direction towards the earth across their drift surfaces. This proceeds as follows. During the time of the disturbances of the magnetosphere, under the influence of the fluctuations of the solar-wind parameters, strong perturbations of the magnetic-field configurations occur and are connected, in particular, with the realignment of the peripheral force lines, which can go off to the tail and then return.

These perturbations lead to injection of fast particles from the tail to the interior of the magnetosphere (in addition, particles of solar wind may become directly captured). This is followed by a unique diffusion mechanism[147,148] connected with the fact that geomagnetic perturbations such as sudden pulses distort the drift surfaces for the charged particles, making it possible for the particles to become displaced along L.† The diffusion is such that the transverse (and also the longitudinal) invariant is conserved, so that when the relativistic particle moves towards the earth its energy increases like L^{-3}. It turns out that this mechanism, in conjunction with ionization losses, is responsible for the formation of the proton belt with particles from 100 keV to 30 MeV. Only in the most interior regions ($L \lesssim 2$) is the diffusion slowed down sufficiently to permit the mechanism of proton capture from the decay of the neutrons of the cosmic-ray albedo to assume a predominant role at energies above 40 MeV.

*An internal proton belt is sometimes considered separately, but if one bears in mind the bulk of the particles with energy less than 30 MeV, there is no need for this.

†Under such assumptions, the third adiabatic invariant Φ is not conserved.

The proton belt, generally speaking, is stable. As to the flute instability, it does not play a noticeable role in the radiation belts. This is connected with the rapid decrease of the magnetic field along the radius, $B \sim L^{-3}$. Just as convective instability develops in the atmosphere only in the case of a superadiabatic temperature gradient, flute instability in a rapidly decreasing field also requires a sufficiently large pressure gradient for its development[6,39]. The instability will develop if, in the case of displacement along the radius of a certain tube with plasma, the pressure in the tube is larger than the pressure of the surrounding plasma. But when the tube displacement is δL, its volume V increases by $\delta V = 4(\delta L/L)V$ as a result of the elongation and the increase of the transverse cross section, so that in the case of adiabatic expansion with adiabatic exponent $\gamma = 5/3$ the pressure decreases by $\delta p = -4\gamma p \delta L/L \approx -7p\delta L/L$. It follows therefore that no flute instability should develop if the pressure decreases with the radius more slowly* than L^{-7}. (Allowance for the real distribution of the particles with respect to the pitch angles change this exponent somewhat, but not strongly[146].) With the possible exception of the region of the "knee," this condition is always satisfied. The proton belt is also stable against drift instability.

As a rule, the kinetic instabilities connected with the presence of the forbidden cone in velocity space of the particles retained by the trap do not develop in the proton belt—the proton density is usually lower than critical. However, at the maximum the intensity of the fast protons is close to critical, and the stability condition is violated in the case of disturbances of the geomagnetic field or of the ionosphere. In this case cyclotron instability develops, corresponding to a buildup of Alfven waves in a cold plasma at cyclotron resonance with fast protons (unlike in laboratory experiments, what develops is not electrostatic but electromagnetic instability, this being connected with the relatively large density of the plasma). This instability becomes manifest in the form of the beautiful geomagnetic pulsations of the "pearl" type[150].

A much more important role is played by cyclotron instability in the formation of the electron belts. It turns out that both the diffusion of the electrons from the outer regions and the neutron mechanism (decay of neutrons of the albedo of cosmic radiation) are perfectly sufficient to overfill the belts in excess of the critical density in the region of $2 < L \lesssim 5$. Therefore the form of the belts in this region is determined by the cyclotron instability[147], i.e., by the buildup of oscillations of the type of "whistlers" on cyclotron resonance, as is indeed the case in laboratory experiments with an electron-hot plasma. Calculation shows[147], in particular, that the gap between the internal and external electron belts ($2.8 < L < 3.2$) corresponds to the minimum of absorption, in the ionosphere, of waves with frequency close to the electron-cyclotron frequency at the vertex of the force line†. Thus, besides the transport of particles in geomagnetic perturbations, cyclotron instability is one of the main collisionless mechanisms of formation of electronic radiation belts.

*In rapid displacement of the plasma, stabilization by means of the end surfaces, namely the conducting ionosphere, assumes a greater role, so that the exponent may be even larger [149].

†The frequency of the resonant wave is in this case much smaller than the electron cyclotron frequency but several times larger than the ion-cyclotron frequency in the ionosphere.

12. CONCLUSION

Experiments on plasma containment in adiabatic traps, initiated and developed as part of the research program on controlled thermonuclear reactions, rapidly revealed a rather extensive and interesting group of physical phenomena. Experiments with individual particles have permitted a deeper understanding of the picture of quasiperiodic motions in classical mechanics; on the other hand, investigations with plasma have shown that the dynamics of an aggregate of charged particles is determined not so much by pair collisions as by the remote interactions connected with the excitation of superthermal electric and magnetic fields, i.e., with the instability of the plasma. In experiments with adiabatic traps, a detailed investigation was made of what was seemingly one of the most dangerous instabilities, the so-called flute or convective instability, and this instability was successfully stabilized. An investigation was made also of the peculiar turbulent plasma motion due to this instability. Further investigations of the properties of an adiabatically contained plasma revealed a large number of collective effects connected with the so-called kinetic instabilities; their study continues to this date. The launching of artificial satellites has led to the discovery of the earth's radiation belts and by the same token demonstrated that the effects of plasma containment in magnetic traps play a role not only in artificially produced laboratory devices, but under natural conditions, and the experimental data obtained under laboratory conditions are in good agreement with the observational data pertaining to the earth's magnetosphere. The physics of these phenomena, i.e., the induced excitation of waves, turned out to be close to physics of masers; in particular, cyclotron-resonance masers are closely linked with adiabatic traps with hot-electron plasma.

By now, our knowledge of the physical properties of plasma in adiabatic traps has reached a sufficiently high level, making it possible to consider, with full justification, the question of whether adiabatic traps can actually lead to the creation of a thermonuclear reactor, as was predicted initially by Budker and Post.

A similar question was discussed many times during the early stages of the research on controlled fusion[30,31,151-154]. It should be noted that the conclusions arrived at by various authors carry a certain imprint—in some cases of distinct optimism, and in others of cautious skepticism. This is easily understood if consideration is taken of the fact that open traps do not have with respect to their energy balance the reliable "safety" margin possessed, at least in principle, by closed systems*.

The point is that even without any plasma instabilities and the associated energy losses, open traps have an unavoidable and quite appreciable source of losses in

*By energy balance is meant here the ratio of the nuclear energy released in the fusion reaction (with allowance for the efficiency of conversion of the nuclear energy into electric energy) to all the energy losses consumed in maintenance of the reaction (also in the form of electric energy).

the form of the flux of particles into the "forbidden" cone, owing to the Coulomb scattering. At plasma parameters that are optimal from the point of view of the energy balance, the time τ_c of containment of ions in the trap turns out to be much smaller than the lifetime of the ions τ_n up to the nuclear reaction, and only a negligible fraction of the injected particles, on the order of 10^{-2}, has time to react during the time τ_c. As a result, the condition for the attainment of a positive energy yield turns out to be very sensitive to the true value of τ_c.

An exact calculation of τ_c is a complicated kinetic problem. The analytic[3,29,155] and numerical[31,156,157] calculations performed to date are based on various simplifying assumptions; accordingly, the results obtained by various authors differ by several times, and they must be regarded more readily as approximate estimates. In view of the complexity and the large number of factors that influence on the Coulomb losses, it is hardly possible to hope to be able to determine τ_c by calculation with much higher accuracy. Therefore the last word will obviously belong to experiment, which must be carried out at plasma parameters that approach as close as possible the thermonuclear parameters.

As to plasma instabilities, many years of experience in their study have shown that in spite of their a priori threat, a detailed study of the physics of the instabilities and a clarification of the conditions of their development makes it possible to find also means of stabilization.

It is precisely these circumstances which make the study of the plasma behavior in adiabatic traps not only advantageous from the purely physical point of view, but also fully justified from the point of view of the possible prospects of developing a controlled thermonuclear reactor.

Note added in proof. At the conference on thermonuclear reactors, held in September 1969 in Culham England, R. Post advanced a scheme of direct conversion of kinetic energy of the ions passing through the mirrors into electricity. Such a scheme admist, in principle, of a high efficiency of conversion, about 90%, and in this case the energy balance of the open traps becomes less critical to the leakage of plasma through the mirrors (R. F. Post, Mirror Systems: Fuel Cycles, Loss Reduction and Energy Recover, UCRL-71743 abstract (1969)).

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