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RELATIVISTIC GRAVITATIONAL EXPERIMENTS

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I. INTRODUCTION

As is well known, it is very difficult to verify effects that follow from general relativity theory (GRT), in view of their smallness under terrestrial laboratory conditions. Therefore the experimental accomplishments in this field were limited for a long time (up to approximately 1960) to three classical experiments (the shift of the perihelion of Mercury, the deflection of a light ray in the sun's gravitational field, and the gravitational red shift; see the review[98]).

Since 1960, following the experiment of Pound and Rebka[85] where the red-blue shift of the photon frequency in the earth's gravitational field was checked, a "new wave" of experimental investigations of relativistic gravitational effects was initiated. The increased interest in such research is apparently connected with the advances in the experimental techniques and the mastery of new methods of measuring small quantities. In addition, by now there have been accumulated a number of variants of gravitation theory, competing with general relativity theory[97], for example: the Brans-Dicke scalar-tensor theory, the tensor theory of gravitation in flat space, the Jordan model with a variable gravitational constant, and others. The final choice depends to a considerable degree on the experiment.

This review includes the most interesting, in the authors' opinion, descriptions of experiments performed during the last five years, and also a number of projects and experimental schemes. Some of these were reported at the Fifth International Conference on the Theory of Relativity and Gravitation in Tbilisi, in September 1968.

II. GRAVITATIONAL RADIATION

One of the small experimental groups working on this problem of observing gravitational radiation is the group of Prof. Weber (University of Maryland). In 1959–1961, Weber[1,2] (see also the review[3]) analyzed in detail the possibility of realizing under terrestrial laboratory conditions a transmitter-receiver system for gravitational radiation, using mechanical oscillations of extended masses. The calculations performed have shown that the use of mechanical oscillations of extended masses leads to too large scales of the experiment (large transmitter and receiver masses, large powers necessary to excite oscillations in the transmitter, and very long time of separating the signal from the noise). At the present time this group is making vigorous attempts to observe gravitational radiation of extraterrestrial origin from certain possible hypothetical sources. These sources can in principle produce near the earth a much larger gravitational-radiation flux density than a laboratory radiator having reasonable dimensions. The principles on which the receivers of Weber's group are based consist of the following. By virtue of the known singularities of the gravitational fields, the gravitational wave cannot be observed with the aid of a single point mass m, as could be done in an electromagnetic field with the aid of an electric charge q. The field of the gravitational wave imparts to an observer located alongside the mass m the same acceleration as to the mass. This circumstance is based on the fundamental experimental fact called the principle of equivalence of inertial and gravitational masses.

Thus, in order to observe a gravitational wave, it is necessary to have two separated point masses or one extended body (a receiver of the quadrupole type). In the case of an extended body, in the geometrical language of special relativity theory (SRT) it can be stated that the time-varying bending of space (due to the gravitational wave) produces mechanical tensions in the extended body. Measurement of the resultant tensions (which lead to the occurrence of mechanical oscillations of the body) will make it possible to determine the characteristics of the bending, i.e., the components of the Riemann curvature tensor.

In the first variant of the detector[4,6] developed by Weber's group, the extended body used was an aluminum cylinder ~150 cm long, ~60 cm in diameter, and with mass ~1.5 ton. This cylinder (Fig. 1) was suspended on thin wires to a frame consisting of steel blocks interlined with rubber liners (anti-seismic filter). The cylinder and the frame were placed in a vacuum chamber, and the entire setup was placed outside the city limits, far from industrial noise. To observe the gravitational radiation, use is made of the very lowest quadrupole mode of oscillations of this cylinder. Its frequency is $\omega_0 \approx 10^4$ rad/sec, and the Q for this mode is $\approx 10^5$, so that only a relative narrow frequency band $\Delta\omega \approx 0.1$ rad/sec near $\omega_0 \approx 10^4$ rad/sec is "cut out" from the entire possible spectrum of the gravitational waves, if the time of separation is on the order of the relaxation time of this mode (approximately 30 sec).

Quartz piezoelectric pickups glued on the surface of the cylinder convert the mechanical vibrations of this

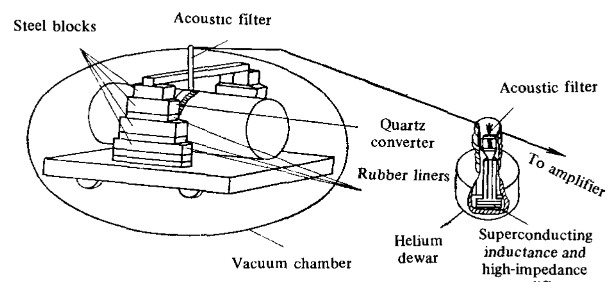

FIG. 1

mode into an electric signal. The problem of matching the electric signals from the pickups turned out to be quite complicated in this case, since the impedance of the quartz pickups glued on the cylinder was relatively high (about 10^9 Ohms). To solve this matching problem, it was necessary to use a superconducting inductance in the resonant preamplifier. As a result, the sensitivity of the gravitational detector was limited only by the Brownian oscillations of this mode of the aluminum cylinder[4]. This means, for example, that the minimum observable amplitude $\bar{\delta}$ of the oscillations of the ends of the cylinder (within a time on the order of the relaxation time) can be estimated from the condition

$$m\omega_0^2\bar{\delta}^2 \geqslant \varkappa T. \tag{1}$$

For room temperature $(\overline{\delta^2})^{1/2} \gtrsim 2 \times 10^{-14}$ cm, corresponding at a cylinder length ~ 150 cm to relative length changes (tensions) on the order of 10^{-16}. We note that such a device is in principle an instrument that measures mechanical stresses and not displacements.

This gravitational detector was calibrated both using a standard noise source and directly with the aid of a dynamic gravitational field. The latter variant of calibration, performed by Sinsky and Weber[5], is in fact a high-frequency variant of the Cavendish experiment. The dynamic gravitational field was produced by oscillations of a second aluminum cylinder, of somewhat smaller dimensions, located ~ 2 m from the main cylinder (distance between centers). The output power of the detector agreed approximately to the calculated value, but the accuracy of such a calibration was low. Both calibration methods have shown that the sensitivity corresponding to the minimal observed tension given by (1) was reached.

The equivalent "gravitational" sensitivity can be obtained with the aid of relations given by Wever[1], connecting the tension ϵ produced in the elastic body with the Riemann tensor component R_{i0j0}, producing the acceleration of different parts of the test body relative to one another. In the case when R_{i0j0} changes sinusoidally in time, with a frequency coinciding with the frequency of the lowest-frequency quadrupole mode of the cylinder oscillations, and the cylinder is oriented in the best fashion relative to R_{i0j0}, we have

$$\varepsilon \approx 2c^2 Q R_{i0j0}/\omega_0^2\pi, \tag{2}$$

where c is the velocity of light propagation and Q is the quality factor of the mode. Substituting in (2) $\epsilon \approx 10^{-16}$, $\omega_0 = 10^4$ rad/sec, and $Q = 10^5$, we obtain $R_{i0j0} \approx 2 \times 10^{-34}$ cm^{-2}. This corresponds to a gravitational-radiation flux density $t \approx 2 \times 10^4$ erg/sec-cm^2.

The second variant of a gravitational detector developed by Weber's group is based on the idea of using the earth as the extended body. On the one hand, this seems to be quite effective, since the cross section for the absorption of radiation is proportional to the mass of the detector. On the other hand, however, such a variant excludes under terrestrial conditions the possibility of using the coincidence scheme which will be considered later. The lowest-frequency quadrupole mode of the earth has a period of ~ 54 min and a $Q \sim 400$. A high sensitivity gravimeter was constructed for approximately these frequencies[6,7], and makes it possible to

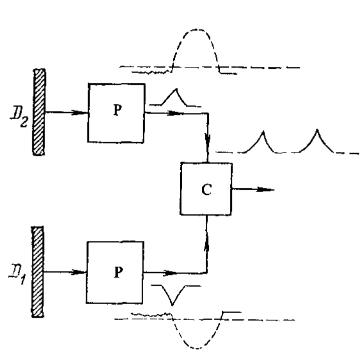

FIG. 2

register variations of the acceleration due to gravity g exceeding the level $\Delta g/g \approx 10^{-11}$. The results of an investigation of the earth's noise background during the quietest period (from the seismic point of view) yielded the following estimate for the spectral density of the accelerations near the frequency $\omega \approx 10^{-3}$ rad/sec: $\overline{[\Delta g(\omega)]^2} \approx 6.9 \times 10^{-14}$ gall^{2}sec/rad[6,8]. A comparison of this quantity with the calculated relation[8] between $\overline{[\Delta g(\omega)]^2}$ with the spectral density of the Riemann tensor has enabled Weber to lower somewhat the estimate for the upper limit of the cosmic background of gravitational radiation (in the frequency region $\omega \approx 10^{-3}$ rad/sec), namely $\overline{[R(\omega)]^2} < 6 \times 10^{-79}$ cm^{-4} rad^{-1}sec; the earlier estimate[1] was larger by 3–4 orders of magnitude. The value of this quantity is relative, since the corresponding energy flux density near the frequency under consideration should be ~ 10 erg/sec-cm^2, whereas binary stars produce in the same frequency range a gravitational flux density $\sim (10^{-9}-10^{-11})$ erg/sec-cm^2[3]. There is no doubt that the considered variant of the gravimetric-type detector will be more promising if it can be used in a coincidence system, by having, for example, one detector each on the earth and on the moon[6,8]. After the flight of Apollo 11, this variant appears to be quite realistic and apparently will be realized in the nearest years.

At the present time an experiment using a coincidence system is carried out by Weber with detectors of the first type. The idea of this system[1] consists of simultaneously using two detectors separated by a distance shorter than the length of the gravitational wave. Such a method makes it possible to detect "gravitational bursts" against the background of the internal fluctuations. Indeed, the gravitational radiation would lead to correlated readings on the outputs of both systems, whereas the internal thermal fluctuations cannot have such a correlation. Detectors of the first type were used, placed in concrete covered chambers initially at a distance ~ 2 km from each other[6,9]. One detector had the parameters described above, and the other was of smaller dimensions (the length was the same but the diameter was ~ 20 cm) and was equipped with a somewhat different electronic circuitry, with broader bandwidth and with adjustable central frequency. In addition, on the platforms of the detectors there were installed instruments to monitor the forces of non-gravitational nature, namely seismographs, magnetometers, acoustic pickups, and inclinometers. A block diagram of the experiment is shown in Fig. 2. The voltage from the piezoelectric pickups is fed to a threshold device, which is operated by signals exceeding a certain level established by the experimenter. The received pulses are fed to a coincidence circuit; the latter develops a signal if the pulses arriving from both channels coincide in time. The time resolution in the first experiment[8] was low (~30 sec), but was subsequently improved to ~0.2 sec[9], i.e., the pulses were received by the scheme as coincident if their leading fronts were shifted in time by not more than ~0.2 sec.

Measurements with a coincidence system were carried out for several months. Several coincidences of pulses exceeding the threshold level were registered (approximately once a month). The threshold level was so much higher than the average noise power, that the probability of random coincidences for some cases was negligibly small (less than 0.0001). It is very important that the coincidences, according to the testimonial by the author, were not accompanied by correlated bursts in the other monitoring instruments. On the basis of the exceedingly small probability of random coincidences, Weber discards purely statistical factors and assumes that he registered some rare synchronous action on the detector, which, generally speaking, can be due also to gravitational radiation (!). According to the latest communication[94], Weber repeated the experiments by lengthening the base between the two receivers to 1000 km (Chicago—Maryland); the coincidences were again registered.

In order to assess the results, let us turn once more to the sensitivity of Weber's gravitational receiver. We shall consider here a simplified model of the gravitational quadrupole in the form of two separated masses connected by a spring (Fig. 3). The corresponding relations between R_{i0j0}, F^M, t, etc. coincide in order of magnitude with the analogous ones obtained in the case of an extended mass, differing by insignificant factors.

The gravitational wave acting on the mass quadrupole leads to the occurrence of a force F that causes the masses of the detector to swing. For the component F^M we can obtain the following expression[1,3]:

$$F^M \approx -mc^2 L^j R^M_{0j0}; \tag{3}$$

here L^j is the component of the vector of length between masses, and m is the mass of the quadrupole. We can rewrite the preceding formula by using the flux density t of the gravitational-wave energy:

$$t \approx c^3 F^2 / 8\pi G m^2 L^2 \omega^2, \tag{4}$$

where G is the gravitational constant.

The sensitivity of Weber's receiver is limited by the thermal fluctuations, which are described by the Naiquist theorem: $\overline{F^2_{fl}} = 4\kappa T h \Delta f$; Here $\overline{F^2_{fl}}$ is the mean

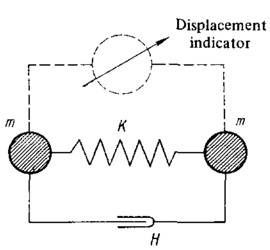

FIG. 3

square of the fluctuation force, Δf is the band width of the receiving system, h is the friction coefficient connected with the Q and with the masses that are contained in the detector, by $h = m\omega_0/Q$. Substituting in (4) the fluctuation force in lieu of F, we obtain an expression for the minimum observable flux t_{min}:

$$t_{min} \approx (c^3/2\pi G)(\kappa T/Q m \omega_0 L^2)\, \Delta f. \tag{5}$$

The parameters of the setup are $\omega_0 \approx 10^4$ rad/sec, $Q = 10^5$, and $T = 300°K$. The equivalent mass and the length are respectively $m_{eq} \approx 5 \times 10^5$ g and $L_{eq} \approx 10^2$ cm, we therefore obtain for t_{min}

$$t_{min} \approx 6 \cdot 10^5 \Delta f \text{ erg/sec-cm}^2. \tag{6}$$

The bandwidth of the receiving system in the described experiment was determined completely by the coincidence circuit: the system did not record difference pulses shorter than 0.2 sec, so that the equivalent bandwidth was $\Delta f \approx 5$ Hz, and hence the absolute value of the sensitivity was at the level $t_{min} \approx 3 \times 10^6$ erg/sec-cm^2. Recognizing that the correlated bursts were observed for the case when the threshold level was on the average several times lower than the noise level, it must be assumed that the bursts would correspond to a flux $t \approx 10^7$ erg/sec-cm^2 (Weber gives a figure $t \approx 2 \times 10^4$ erg/sec-cm^2 [4] corresponding to the threshold sensitivity of the detector at a band width $\Delta\omega \approx 0.1$ rad/sec, which is not decisive in the given experimental scheme).

In[96] are given energy estimates of the gravitational radiation corresponding to the noise level of Weber's receivers, under different assumptions concerning the structure of the radiation. In the case of resonant action for $\tau \sim 1$–2 sec, the response of the receiver at the noise level requires a gravitational energy flux $t \sim 10^6$ erg/sec-cm^2 (see above). For a single periodic pulse with $\tau \sim 1/\omega_0 \sim 10^{-4}$ sec the required flux is $t \sim 3 \times 10^{13}$ erg/sec-cm^2; finally, a quasiperiodic pulse with varying frequency ($d\omega/dt \sim 10^6$ rad/sec) corresponds to $t \sim 3 \times 10^{11}$ erg/sec-cm^2 (at the instant when $\omega = 10^4$ rad/sec). These three cases represent the following hypothetical sources of gravitational radiation: non-radial oscillations of a neutron star[23], frontal collision of two massive bodies[97], collapse of the components of the binary system in rotation on an orbit[97]. If, as was done in[96], we assume that the radiation comes from the center of the galaxy, we can estimate the absolute power of the source and compare it with the theoretically reasonable level for each of the three cases. It turns out[96] that only for oscillations of neutron stars are the two values close (~10^{52} erg/sec). In the remaining cases there is a discrepancy of 2—3 orders of magnitude.

Let us estimate now the possibilities of increasing the sensitivity of the receiving quadrupole mechanical detector[10]. First of all, by using a detector consisting of two separated masses, it is possible to obtain considerable gain by increasing the distance L between the masses (see relation (4)). The most powerful fluxes of gravitational radiation of extra-terrestrial origin must apparently be expected at low frequencies[3], $f \lesssim 1$ kHz. We note that although in this case t_{min} increases with decreasing frequency (4), the requirements imposed on the registered mechanical displacements ΔX_{min} are

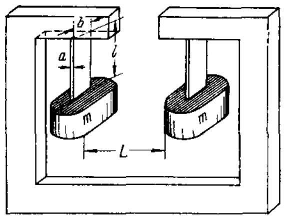

FIG. 4

less stringent. In this frequency region we can point to a large number of sources with exactly known frequency of mechanical or electromagnetic processes, for example binary systems or pulsars. Then, assuming that the gravitational radiation is correlated with these processes, it is possible to realize a scheme for synchronous separation of the signal from the noise and thereby increase the resolving power of the detector still more. Finally, the most important source for increasing the sensitivity is to decrease the friction coefficient h, and consequently also the fluctuation level $\overline{F_{\text{fl}}^2}$.

Let us consider a concrete variant of a detector designed for the frequency band near 1 Hz (the modulation frequency of pulsar radiation)*. A diagram of the receiver is shown in Fig. 4. The quadrupole detector is made up of two masses suspended in vacuum on thin quartz ribbons (l—length, a—thickness, b—width of the ribbon). The masses are located on a seismic platform having a period of oscillation of about 1 hr relative to the earth. The distance between masses is L. The friction introduced by the quartz suspension is $h \approx \eta ba^3/5l^3$, where η is the viscosity coefficient of quartz. It is thus convenient to reduce the ribbon thickness a, maintaining constant the product ab, so as to be able to maintain the value of the mass $m = ab\sigma_{\text{lim}}/g$, where σ_{lim} is the permissible tension stress in quartz. Formula (5) can be rewritten in this case in the form

$$t_{\min} \approx (c^3 \varkappa T g^2/10\pi)\,(\omega_0^2 l^3)^{-1}\,(\eta a/\sigma_{\lim} L^2 b)\,\Delta f. \tag{7}$$

In (7) the first factor is a constant that depends on the temperature, the second is determined by the frequency range, the third by the properties of the material, and the fourth by the scale of the experiment and by the technical requirements. For quartz $\eta \approx 10^6$ poise, $\sigma_{\text{lim}} \approx 2 \times 10^9$ dyne/cm^2. In addition, it is necessary to put $l = \omega_0^2 g = 50$ cm, since the frequency of the pendulums of the quadrupole is determined at small $a \lesssim 10^{-1}$ cm not by the elastic properties of the quartz but by the acceleration due to gravity. Putting further $a = 0.04$ cm, $b = 4$ cm ($a/b = 10^{-2}$, $m = 10^5$ g), and $L = 3 \times 10^3$ cm, $T = 300°$C, we get

$$t_{\min} \approx 30\Delta f \text{ erg/sec-cm}^2 \tag{8}$$

Comparison of (7) and (6) shows that the sensitivity of the receiver in this variant increases by approximately 10^4 times. For the band $\Delta f \approx 1$ Hz we have $t_{\min} \approx 30$ erg/sec-cm^2; for $\Delta f = 10^{-5}$ Hz (synchronous separation during the course of one day) $t_{\min} \approx 3 \times 10^{-4}$ erg/sec-cm^2. An analysis of the technical difficulties arising in the realization of such a system shows that it is realizable with contemporary laboratory techniques.

We emphasize once more that the main gain in the sensitivity should result from the large increase of the receiver Q: $Q \approx 10^9$ as against $Q \approx 10^5$ in Weber's experiments. The very high value of Q causes the observation time $\hat{\tau}$ to be much shorter than the relaxation time $\tau^* \sim Q/\omega_0$ of the receiving system; in this case it amounts to about 10^9 sec.

Under conditions when $\hat{\tau} \ll \tau^*$, the intrinsic thermal fluctuations of the receiver limit the maximum registered portion of energy obtained under the influence of the external action not to the level κT, but to a much lower level $\sim \kappa T \hat{\tau}/\tau^*$, of course so long as $\kappa T \hat{\tau}/\tau^* \gg \hbar\omega_0$. In this sense Weber's recording system is not optimal. Accordingly, under these conditions it becomes possible in principle to measure oscillation amplitudes $\Delta X \sim 10^{-15}$ cm, as is required, for example, in the pendulum variant under consideration, and smaller amplitudes (information on the technique of measurements at $\hat{\tau}/\tau^* \ll 1$ can be found in[11]).

An increase of τ^* causes the main factor limiting the resolving power to become not the intrinsic thermal fluctuations of the mechanical oscillator, but the fluctuations (due to the influence on the oscillator mass) of the instrument that registers its oscillations. Further increase of c* is meaningless in this case. By choosing the best (optimal) parameters of such an indicator it is possible to obtain maximum sensitivity. Referring the reader to more detailed explanations in the literature[12], we present here an expression for the minimum recordable sinusoidal force under "optimal strategy" conditions:

$$F_{\min} \approx (2/\hat{\tau})\,(\hbar\omega_0 m)^{1/2}; \tag{9}$$

here $\hbar$ is Planck's constant, ω_0 is the natural frequency of the oscillator oscillations, and $\hat{\tau}$ is the duration of the sinusoidal train. The formula was derived for the case of an optical registration system. It follows from (4) and (9) that

$$t_{\min} \approx (c^3/2\pi G)\,(\hbar/m\omega_0 L^2\hat{\tau}^2), \tag{10}$$

Substituting the proposed parameters of the "pendulum variant" of the gravitational receiver, namely $L = 3 \times 10^3$ cm, $m = 10^5$ g, and $\omega_0 = 6$ rad/sec, we obtain $t_{\min} \approx 1 \times 10^{-2}(\hat{\tau})^{-2}$ i.e., for $\hat{\tau} \approx 1$ sec we have $t_{\min} \approx 1 \times 10^{-2}$ erg/sec-cm^2, and for $\hat{\tau} \approx 10^5$ sec (one day) we have $t_{\min} \approx 10^{-12}$ erg/sec-cm^2.

Besides the installations indicated above, intended for the observation of the gravitational radiation of extraterrestrial origin, there have appeared preliminary communications concerning the development of one more gravitational detector, by the Forward-Miller-Berman group (see[18]). The setup is similar to the Weber receiver, but it is proposed that the electronic circuitry ensures reception in a wider frequency range, from 100 Hz to 2 kHz. The latter circumstance is important because the frequency of the gravitational radiation changes strongly in the case of collapsing binary systems.

The researches listed above apparently cover all the direct attempts to observe gravitational radiation. The number of publications devoted to proposed experiments,

*The authors of this article described this variant of the detector at the session of the Division of General Physics and Astronomy of the USSR Academy of Sciences on 24 October 1968.

calculations connected with different gravitational radiators and receivers, is much larger. We shall dwell briefly below on the most interesting, in our opinion, investigations performed in 1965—1968 (not included in the review[3]).

Let us return to question of the feasibility of producing a transmitter-receiver system for gravitational waves under laboratory conditions. As already noted above, the creation of such a system using mechanical oscillations of extended masses is exceedingly difficult. Kopvillem and Nagibarov proposed an experimental system with a receiver and transmitter in the optical frequency band[14]. The idea of their proposal is that effects analogous to spin and photon induction and "echo" should take place for gravitational radiation. Let us recall briefly the gist of these phenomena.

In a system of particles having spins or electric multipole moments, it is possible to produce a so-called "superradiant" state, in which the intensity of the coherent spontaneous emission is proportional to the square of the number of particles (the latter is due to the interaction between the individual particles via the electromagnetic field of the radiation). The "superradiant" state can occur under the influence of external resonant radiation of definite amplitude and duration (the so-called 90° pulse[15]). The effect itself of coherent spontaneous emission is called the effect of spin (or accordingly photon) induction. Since the frequency of the particle transitions can differ somewhat from one another as a result of inhomogeneity of the sample or of the internal and external magnetic fields, etc., a so-called "inhomogeneous" line broadening $\Delta\omega_{ib}$ in addition to the always existing "homogeneous" broadening $\Delta\omega_{hb}$, as a result of the effects of the spin-spin and spin-lattice relaxation. Owing to the "inhomogeneous" broadening, dephasing of waves from individual radiating centers, and loss of coherence, occur during the process of the spontaneous emission. The coherence can be restored by reversing the direction of the external magnetic field or, equivalently, by acting on the system with resonant radiation having an amplitude and duration corresponding to the so-called 180° pulse[15]. This effect is called spin (or, correspondingly, photon) "echo." The coherence can be restored several times (if $\Delta\omega_{ib} > \Delta\omega_{hb}$) until the irreversible relaxation processes destroy the excited state of the system. For a more detailed explanation we refer the reader to the review[16].

Following the authors of[14], it is possible to represent, by analogy, gravitational induction and "echo," which represent generation of a coherent gravitational wave as a result of collective oscillations not of electric (or magnetic) but of mass quadrupole systems of many particles—molecules, atoms, and nuclei. It is necessary to indicate here a method of transferring the gravitational quadrupoles into an excited "superradiant" state. It turns out, however, that this takes place "automatically" under *electromagnetic excitations, since the* particle charge distribution is closely related with the distribution of the mass that carries the charge. In order for coherent electromagnetic radiation not to interrupt the excitation too rapidly, it is necessary to increase the decay time, by enclosing and retaining the electromagnetic radiation in a resonator, and this increases the decay time by several orders of magnitude[17]; at the same time, the resonator does not obstruct the gravitational radiation.

For the reception of a coherent gravitational wave, the authors of[14] propose to use the generation scheme in reverse order—the detector can be a similar particle system, carefully shielded against electromagnetic radiation. The gravitational wave, interacting with the mass quadrupoles of the particles, produces an electromagnetic "superradiant" state (if the particles have electric moments). The emitted photons can be registered with a photomultiplier. The best numerical estimates were obtained in[14] for the quadrupole moments of the electron shells of ions. Making, as in[14], the most optimistic assumption that all the centers of a certain sample emit gravitons coherently, while going over to the ground state from the "superradiant" state, (i.e., the intensity is proportional to the square of the number of particles), we can obtain an exceedingly promising estimate $t \sim 10^2$ erg/sec-cm^2 = 10^{13} gravitons/sec-cm^2 for a sample of length $l = 10^3$ cm and a volume $V \approx 10^3$ cm^3 (for single-mode powerful lasers, the power of the gravitational radiation is $\sim 10^{-10}\, w_{em}$(W), where w_{em} is the electromagnetic power of the laser. Under the same conditions, the response of a receiver with $l = 10^3$ cm and $V = 10^5$ cm^3 to a gravitational flux $t \sim 10^2$ erg/sec-cm^2 would amount to $\sim 10^{-14}$ erg/sec-cm^2 $\approx 10^{-3}$ photons/sec of coherent electromagnetic radiation[14].

There exist, however, a number of fundamental difficulties, greatly reducing the capabilities of the given scheme (critical remarks in this respect are found in[18,19]). In the optical band, the dimensions of the crystal greatly exceed the wavelength. Therefore, in order for individual crystal centers excited by an electromagnetic wave to emit a gravitational wave coherently it is necessary that the phase velocities of both waves be equal (similar to the well known synchronism condition in nonlinear optics). In fact, these velocities are different: the phase velocity of the electromagnetic wave depends on the dielectric and magnetic properties of the medium, whereas the velocity of gravitational radiation, owing to the weakness of the interaction, remains practically equal to c.

Another inconvenience is that it is impossible to use the effective dipole excitation of the crystal by electromagnetic pumping, owing to the quadrupole character of the gravitational radiation: the selection rules forbid excitation of the atom via a dipole transition with subsequent quadrupole relaxation, since parity is not conserved in this case. Thus, it is necessary to choose media with a forbidden dipole and an open quadrupole electromagnetic transition, i.e., working with quadrupole-quadrupole interaction. The difference between the directivity patterns of electromagnetic and gravitational quadrupoles, noted in[20], will under certain conditions apparently not hinder the inversion process, but conversely, *contribute to its effectiveness.* In subsequent papers[21,22], Kopvillem and Nagibarov made more realistic quantum-mechanical estimates of the possible yield of gravitational radiation. To satisfy the synchronism requirement, they propose to use two opposing waves of electromagnetic pumping, detuned relative to a central frequency ω_0, with carrier frequencies

$\omega_1 = \omega_0[1 + (\epsilon\mu)^{-1/2}]/2$ and $\omega_2 = \omega_0[1 - (\epsilon\mu)^{-1/2}]/2$. Such a two-quantum electromagnetic excitation is equivalent to a certain effective field with frequency $\omega_1 + \omega_2 = \omega_0$ and a wave vector $\mathbf{k} = \omega_0\mathbf{n}/c$, where $\mathbf{n}$ is a unit vector parallel to the propagation direction of the electromagnetic wave (other methods of satisfying the synchronism condition are found in[19]).

In the "echo" regime, taking into account the limitations connected with the features of the directivity patterns of the electromagnetic and gravitational quadrupoles, the following estimates were obtained in[21]: for a rather large ruby crystal with length $l = 10^3$ cm and cross section $s = 10^5$ cm^2 the rate of emission of gravitons (during one pulse $\Delta f \approx 10^{-7}$ sec) $N_g \sim 10^{-8}$ gravitons/sec repetition of the pulses at a frequency 10^6 Hz for 10^5 sec yields a total of $\sim 10^3$ gravitons. Using a crystal with a larger concentration of selective centers, $N_0 \sim 10^{22}$ cm^{-3} (as against $\sim 10^{20}$ cm^{-3} for ruby) and with a larger quadrupole moment $Q_0/m \sim 10^{-15}$ cm^2 (as against $\sim 10^{-17}$ cm^2 for ruby), at a frequency $\omega_0 \approx 10^{16}$ rad/sec, it is possible to expect under the same conditions a yield $N_g = \sim 10^{11}$ gravitons. In addition, the authors of[21] hope that with a successful arrangement of the active centers the effect of focusing is capable of increasing the yield to $N_g = \sim 10^{21}$ gravitons; the estimates for molecular and nuclear levels are also somewhat improved[21,22].

In[19], and later also in[22], attention was called to the fact that it is possible to retain the regime of initial excitation via dipole transitions, if one bears in mind the two-quantum process of absorption, since the selection rules permit the existence of the process

$$\gamma + \gamma + x \rightleftarrows x + g, \tag{11}$$

where γ is the quantum of electromagnetic radiation of frequency ω_0 (spin S = 1), x is the atom, and g are quanta of gravitational radiation of frequency $2\omega_0$ (S = 2); at the same time, as already noted, two-quantum pumping solves the synchronism problem. In a receiver, the synchronism condition is even more important, but the experimenter does not have at his disposal two gravitational waves whose frequency and direction could be selected arbitrarily. The problem can be solved by applying to the receiver, simultaneously with the coherent gravitational wave of frequency $2\omega_0$, an electromagnetic coherent wave of frequency ω_0 (coherent phonon interaction is also proposed in[22]). In this case, the nonlinear interaction of the waves leads to coherent electromagnetic excitation of the receiver at frequency ω_0. In principle it is possible to separate the output radiation from the pump by using, for example, the different polarizations of these waves.

Quantum mechanical[22] and classical[19] estimates of the photon yield from the receiver under the optimal assumptions $N_0 \approx 10^{21}$ cm^{-3}, $\omega_0 \approx 10^{16}$ rad/sec, $V \approx 10^6$ cm^3, and a pump power $\sim 10^{10}$ W yields a quantity of approximately the same order: $\sim 10^5$ photon/sec within a pulse time $\Delta f \sim 10^{-7}$–10^{-8} sec.

Concluding the discussion of the proposals of Kopvillem and Nagibarov, we note the following. Such a method makes it possible to generate successfully gravitons of optical frequency, but the efficiency of the receiver is not clear. Apparently, a low receiver yield can be masked by spontaneous thermal emission, for example as a result of heating of the crystal by the pump wave, etc.

On the whole, in our opinion, these interesting proposals call for further theoretical analysis, which possibly will yield transmitter and receiver parameters that are feasible in modern laboratories.

Searches for possible sources and receivers of gravitational radiation are continuing. The most detailed analysis was performed by Thorne[23] for quadrupole oscillations in a number of models of neutron stars. In a typical case, the oscillation periods of neutron stars with masses $M \gtrsim 0.5\, M_\odot$, equal $\sim 10^{-3}$ sec, and a typical attenuation time is ~ 0.1–10 sec. Immediately after the formation of a neutron star by a supernova explosion, it should emit an energy $\sim 10^{52}$ erg, in a pulse of ~ 2 sec, in the form of gravitational waves of frequency ~ 1 kHz. We note that numerous discussions and doubts concerning the ability of gravitational waves to carry energy have apparently been eliminated to a considerable degree following the publication of the paper by Burke and Thorne[95], demonstrating the presence of radiative damping in the source as a result of energy lost to gravitational radiation.

Forward and Berman[24] again used the radiation of binary systems (binary neutron stars; mass falling on a collapsing star) from the point of view of the possibility of detection. They calculated the reaction of a gravitational antenna (two masses of ~ 1 ton at a distance ~ 1 meter) and reached the conclusion that it is possible to register radiation from systems located within a radius of ~ 3000 light years from the earth. In this region of our galaxy there are $\sim 10^5$ binary stars. (We note that this estimate was made for systems with $\hat{\tau} \sim \tau^*$ and therefore does not represent the limit.) Radiation from quasi-stellar systems was investigated by Cooperstock[25] for various types of proper oscillations.

A detailed review of the known processes that lead to generation of gravitational waves was presented by de Sabbata[26], who added some calculations (together with Boccaletti et al.[27]). Refining the older estimates[28,29] of mutual photon-graviton transformations, the authors of[26,27] noted that the effect increases in the presence of large masses such as quasars, and one can expect a graviton yield (due to the electromagnetic radiation of the quasars) of $\sim 10^{28}$ erg/sec. The quantum estimates of the photon-graviton transformations in an electrostatic field of nuclei have also shown that there is no appreciable effect. We note that a similar problem in the classical approximation, and for the receiver scheme, was considered by Lupanov[30], who investigated the generation of electromagnetic waves by a capacitor placed in the field of a gravitational wave. The results of the calculations predict a sensitivity not better than possessed by mechanical detectors.

An interesting discussion has centered around the problem of observing interstellar gravitational radiation by determining the oscillations of the intensity of the optical radiation of stars. Just as atmospheric fluctuations cause flickering of stars as seen by an observer on the earth's surface, fluctuations of the gravitational field (which can be described by equivalent fluctuations of the refractive index of the interstellar space) should cause flicker visible by an observer located outside the atmosphere. The quasistatic changes of the field, which are connected with the motion of masses in the universe, lead to exceedingly long-term oscillations, $\sim 10^4$ years, and are therefore unobservable. At the same time, the field variations produced, for example, by gravitational radiation of binary stars will have a period of ~ 3 hrs. The first calculations of the effect of gravitational stellar flickering were apparently carried out by Zipoy[31]. In a recent very optimistic paper by Winterberg[32] it is proposed, in addition, to use the flicker effect to determine the velocity of propagation of gravitational waves. Unfortunately, one must agree with the remarks made by Zipoy and Bertotti[33] that the effect of intensity fluctuations will be observed only in second order (in the weak-field approximation). The estimates in[32] are much too high, since the calculation made there, in the scalar wave approximation, ignores the features of the structure of the gravitational-wave field.

In connection with the discovery of pulsars, the first attempts were made to estimate their gravitational radiation[34,35]. According to calculations by Shklovskiĭ[35], the pulsar in the Crab nebula, which has an electromagnetic-radiation modulation frequency of ~ 30 Hz, should radiate a powerful flux of gravitational waves. The flux density of the gravitational radiation from this source near the earth should amount to about 10^{-6} erg/sec-cm^2 at ~ 30 Hz. This is larger by three orders of magnitude than the radiation flux density from the most "suitable" binary stars.

In conclusion we note a new rotational variant of a receiver for gravitational radiation, proposed in[96]. A plane gravitational wave incident normally on a plane in which a certain quadrupole ("dumbbell") rotates, can either accelerate or slow down its rotation, depending on the phase relations (the frequency of the wave should exceed by a factor of 2 the frequency of quadrupole rotation). One can visualize two quadrupoles so disposed that the wave accelerates one of them and decelerates the other. If the frequency of rotation of the quadrupoles is made to deviate slightly from half the frequency of the wave, beats are produced: the quadrupoles will alternately come together and move apart during the time of rotation. According to the estimates of[96], dumbbells with dimensions $l \sim 0.5$ meters and a detuning $\Delta f \approx 10^{-3}$ Hz from the radiation frequency will experience beats with amplitude $\sim 10^{-12}$ cm (a measurable quantity) under the influence of the flux expected from the pulsar in the Crab nebula.

III. GRAVITATIONAL EFFECTS IN THE PROPAGATION OF ELECTROMAGNETIC RADIATION IN THE SOLAR SYSTEM

I. Shapiro has proposed[36] (see also[37,99]) an experiment in which it was possible to observe the deceleration of a radar pulse of electromagnetic radiation in a gravitational field, expected from GRT. The experiment should consist of measuring the delay time of radio emission sent from the earth and reflected from Venus or Mercury. The magnitude of the expected relativistic delay should be ~ 160 μsec for a probing beam passing at the edge of the sun.

Preliminary results of the experiment (which can be regarded as the fourth verification of GRT) agree with the calculated value (the relative accuracy attained is $\sim 20\%$)*. This experiment is one of the most complicated and most laborious of recent years. Let us indicate the main parameters of the employed apparatus and discuss briefly the features of the measurement procedure.

The observations were carried out with two radio telescopes, on Venus and Mercury during those periods when these planets were close to the position of the far conjunction (a position in which the sun lies between the planet and the earth and all three are on one line). The transmitter power was about 300 kW, the frequency (7.84 GHz) was stabilized with a hydrogen maser. The power of the reflected signal was $\sim 10^{-21}$ W. An optimal separation technique was used, similar to the technique developed during the time of the first radar investigations of planets. The radiation of the transmitter was subjected to phase coding: every 60 μsec, the phase of the radiation was changed by 180°, and the total length of the code cycle was ~ 3.78 msec (i.e., the code had 63 elements). The transmitter radiated continuously for 30 min (the time necessary for the "echo signal" to return from the planet in the position of the far conjunction), and then the receiver was connected to the antenna. The "echo signal" reflected from different parts of the planet had different phase shifts, owing to the Doppler effect. The "echo signal" train corresponding to an individual code cell was directed to a multichannel receiver, in which each of the narrow-band channels corresponded to a definite range of possible Doppler frequency shifts. The accumulation of the signal was carried out in a two-dimensional matrix, the number of elements in which was equal to the product of the number of channels by the number of code cells.

The delay time of the "echo signal" reflected from the planet in the far conjunction was determined from the mutual correlation function between the "echo signal" and the standard signal, which was also obtained in the form of a two-dimensional matrix, but in the position of the near conjunction (the planet is located between the earth and the sun). During the time of passage of the planet through the position of the far conjunction, the sounding operation was carried out continuously, and thus it was possible to trace the successive changes of the delay as the sounding beam approached the edge of the sun. The accuracy with which this delay was measured was ± 10 μsec.

Simultaneously with the measurements of the delay, the parameters of the trajectories of the planets were determined with the aid of radar and optical methods. These parameters were used to calculate the "echo signal" without allowance for the GRT effect. A comparison of the calculated value of the delay with the observed ones has made it possible to separate the relativistic addition. In order to estimate better the volume of the work, let us indicate that it was necessary to know 23 parameters (the "initial" parameters of Mercury, Venus, the earth-moon mass center, etc.). Approximately 400 radar and 6000 optical measurements were used. The accuracy of the theoretical calculations was at level $\sim 1 \times 10^{-10}$, and in the most accurate measurements $\sim 5 \times 10^{-9}$ (the ratio of the resolution of the

*At the Conference on Relativistic Astrophysics in Dallas (1968), I. Shapiro reported that the accuracy of his measurements was raised to 5%.

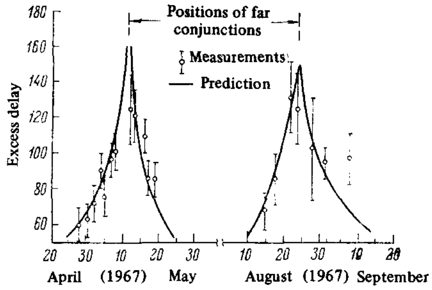

FIG. 5

delay ~10 μsec to the 30 min total delay). By way of illustration, Fig. 5 shows the results of measurements of the delay of the "echo signal" in the radar sounding of Mercury near two successive far conjunctions. The solid curves correspond to the theoretical value, and the experimental points constitute the relativistic addition obtained with the aid of the procedure described above.

In conclusion we note that the foregoing experiment aimed at verifying the GRT has a certain shortcoming: it is possible to separate the effect only by using calculated values. In this respect, this experiment is similar to the measurements of the secular shift of the perihelion of Mercury (which also follows from GRT).

According to[39], Shapiro and a group of co-workers at the Massachusetts Institute of Technology is planning also to measure the deflection of a radial beam passing near the edge of the sun. It is proposed to monitor by radio-interference methods the distance between quasars during the time that they approach to the sun. A similar suggestion was made also by Chikhachev[40]. There are several known quasars whose angle coordinates come close to the sun during the year[41]. The small angular dimensions of these objects make it possible to determine quite accurately their position on the celestial sphere. The measuring instrument can be a radio interferometer. It is sensible to choose for the observations a wavelength such that the effect of refraction in the solar corona be as small as possible (the effect is proportional to λ^2), and the spectral intensity of the quasar radiation is still sufficiently large. Such an optimum wavelength is close to $\lambda \approx 2$ cm. The total variation $\delta\varphi$ of the angular coordinate of the quasar relative to two positions—with the sun and without it—consists of the following main parts:

$$\delta\varphi = \psi + \alpha_{\odot} + \beta_{\delta}; \tag{12}$$

here ψ is the relativistic deflection of the electromagnetic beam in the sun's gravitational field, $\alpha_{\odot}$ is the deflection in the solar corona, and β_{δ} is the deflection due to refraction in the earth's atmosphere.

The angle $\alpha_{\odot}$ depends strongly on the frequency (the approximate formula is $\alpha_{\odot} \approx -af^{-2}$, where f is the frequency of the electromagnetic wave and a is a certain constant). The quantities $\alpha_{\odot}$ and ψ have opposite signs. Using the frequency dependence of $\alpha_{\odot}$ we can exclude the effect of refraction in the corona, carrying out the measurements at two different (but close) frequencies. Indeed, in this case we have

$$\psi + \beta_{\delta} = [f_1^2\delta\varphi(f_1) - f_2^2\delta\varphi(f_2)]/(f_1^2 - f_2^2). \tag{13}$$

(the $\beta_{\delta}(f)$ dependence is immaterial for close frequencies). It is possible to separate ψ and β_{δ}, for example, by observing simultaneously several quasars located at different angle distances from the edge of the solar disc—the values of β_{δ} are equal but the ψ are different.

The requirements imposed on the radio interferometer are determined by the angular dimensions of the quasars and by the intensity of their radiation*. The width $\delta\theta$ of the interferometer directivity pattern must not exceed the source dimension, ~0.1″. From the well known relation $\delta\theta'' = 206265\ \lambda/D$ it follows that the distance D between the antennas of the radio interferometer should be not less than 40 km; it is necessary in this case to stabilize the fluctuation phase drifts in the connecting circuits. At the present state of the art it is possible to realize stabilization accurate to $\sim 10^{-5}$ of the lobe width[42], which is perfectly sufficient for these purposes. The spectral density of the radiation flux of quasars in the $\lambda \approx 2$ cm band fluctuates in the interval $\tilde{Q} \sim 10^{-25}$–10^{-26} W/m²Hz from object to object. An antenna with an effective receiving area $s_0 \approx 500$ m² and a receiving device with a bandwidth $\Delta f \approx 10^6$ Hz ensures an output signal at a level $p_s = \tilde{Q}s_0\Delta f = 5 \times (10^{-17}$–$10^{-18})$ W. The noise level of the quantum receiver with temperature $T_n = 100$°K and a time constant $\tau = 10$ sec is of the order of

$$p_n \approx \varkappa T_n (\Delta f/\tau)^{1/2} \approx 5\cdot 10^{-19}\ \mathrm{W}. \tag{14}$$

Thus, in principle the reception is perfectly feasible. Additional difficulties are due to the solar noise received by the side lobes of the directivity pattern of the interferometer. However, an analysis in[40] shows that these and some other difficulties can be overcome. At the same time, such an experiment yields information concerning the physical properties of the solar corona.

A long-base interferometer can also be used to measure the deflection of a radar beam reflected from planets in the far conjunction[39].

In addition to these experiments, preparations are being made to repeat the experiment in which an electromagnetic ray in the optical band is deflected[39]. It is proposed to use a follow-up photoelectric telescope developed in the experiments of Dicke and Goldenberg on the observation of the non-sphericity of the sun[43]. This technique will be used to measure the distance between two stars during the time when the sun passes by[43]. The solar diameter can be used as the length scale. Apparently in this variant it is possible to overcome a number of systematic errors inherent in the older experiments carried out during the time of solar eclipses.

Interesting experimental data favoring GRT were obtained by Arifov and Kadyev[44] in studies of the catalogues of the annual stellar paralaxes. They call attention to the fact that the curving of the trajectory of a light ray in the sun's gravitational field changes the apparent position of the star on the celestial sphere and should necessitate a correction to the value of the stellar paralax. Calculations within the framework of the

*The estimates presented below are due to B. M. Chikhachev.

GRT lead to the following expression for the annual paralax of a star:

$$p_{\text{obs}} \approx p_{\text{real}} - \alpha r_0^{-1}\xi; \tag{15}$$

here α is the sun's gravitational radius, r_0 the radius of the earth's orbit, p_{obs} the value of the parallax measured by the trigonometric method, $p_{real} = r_0/R$ is the true value of the annual parallax of a star located at a distance R from the sun, and ξ is a certain parameter that depends on the heliocentric ecliptic lattitude of the star φ_0 and the mutual positions of the earth, sun, and the star; ξ varies in the range $1 \leq \xi \leq 1/\sin^2\varphi_0$.

It follows from (15) that, first, the relativistic corrections decrease the true value of the parallax; second, remote stars, for which $R > r_0^2/\alpha \approx 7.5 \times 10^{15}$ km, should have, in principle, negative parallaxes. To confirm these conclusions, the authors of[44] turned to the parallax catalogue of the Yale Observatory[45], which contains 2289 stars with annual parallaxes measured simultaneously by trigonometric and astrophysical methods. The astrophysical methods (e.g., based on the dependence of the intensity of the spectral lines on the absolute stellar magnitude, etc.) gives parallax values free of relativistic corrections due to the sun's gravitational field. These values can thus apparently be identified with p_{real}. It is further possible to compare the differences of the astrophysical and trigonometric values and to compare them with the expected value $\Delta = p_{real} - p_{obs}$. The theoretical prediction of Δ from formula (15) yields $\Delta \gtrsim 0.004''$ (the concrete value depends on the parameter ξ). The catalogue[45] lists values averaged over different positions of the earth on the solar orbit. By separating 135 stars for which the trigonometric and astrophysical parallaxes were measured with equal absolute errors, Arifov and Kadyev obtained the mean value $\overline{\Delta} \approx +0.004''$ (with allowance for the sign of the difference Δ), which agrees with the theory. We note that a systematic excess of the spectral parallaxes above the trigonometric ones was noted already by Adams and Joy in 1917[46], but no interpretation was given there.

The second theoretical conclusion, namely the existence of negative parallaxes for remote stars, could not be confirmed quantitatively. The authors of[44] satisfy themselves with a modest conclusion that there are no data contradicting the theory. At the same time, the very fact of the existence of negative parallaxes is subject to no doubt. In the catalog[45], out of 72 stars with $p_{real} \lesssim 0.004''$ there are about 11% of stars with essentially negative parallaxes (with small measurement errors). Up to now, negative values of the parallaxes were interpreted only as a result of measurement misunderstandings. Arifov and Kadyev, apparently, were the first to connect this fact with relativistic effects. The geometrical interpretation of the negative parallax is as follows: the sum of the angles of the parallactic triangle produced by the star and, for example, by two opposite points of the earth's orbit (observation points) turns out to be larger than π. Indeed, at the observation points one actually measures the angles α_1 and α_2 between the directions to the sun and to the star. The parallactic angle is defined as the difference $\pi - (\alpha_1 + \alpha_2)$. Thus, its negative value actually denotes that the geometric laws of space differ from the laws of Euclidean geometry.

At the present stage, it is advisable to perform new and more accurate parallactic measurements accompanied by an investigation of the dependence of the value of Δ on the lattitude φ_0 of the star for a fixed configuration of the earth, sun, and the star.

New possibilities of investigating the general-relativity laws of the propagation of electromagnetic waves in a gravitational field (and of other relativistic effects) arise in connection with the discovery of stable cosmic sources of pulsed radiation, namely pulsars. Counselman and Shapiro[47] and Richard[48] noted that the effect of the delay of the electromagnetic wave in the field of the sun can be registered by continuously following the frequency of pulsar radiation during the time when the pulsars pass through the position of far conjunction.

Hoffman uses a somewhat different approach[49], proposing to compare the pulsar radiation frequency with terrestrial atomic frequency standards during the time that the earth moves on its orbit around the sun. During the year, owing to the eccentricity of the earth's orbit, the earth is located at points with different gravitational potentials, and the frequency of the earth's standard should change. Hoffman estimates the frequency variation at a level of 3.3×10^{-10}; when account is taken of the special relativity theory effect connected with the change of the earth's velocity, the level becomes 4.9×10^{-10}. It is estimated that the stability of the pulsar radiation[50] at the present time is not worse than $\sim 10^{-9}$.

Finally, mention should be made of suggestions connected with the use of artificial satellites. According to Eshleman's variant[51], it would be possible to carry out an experiment (similar to that of[38]) in which one measures the delay of a radar pulse reflected from an artificial planet or a solar satellite. Such a satellite, equipped with a rebroadcasting unit, would constitute a target with much more accurately determined coordinates than the planets. NASA is planning to mount the required apparatus on board the next "Mariner," which will be launched on an orbit around Mars in 1971.

1. Relativistic Precession of a Gyroscope

With the aid of artificial satellites it is also possible to realize Schiff's program, on which it is worth while to dwell in greater detail. Back in 1960, Schiff[52,53] proposed an experimental observation of the laws of general relativity in the motion of a freely falling gyroscope. Without writing out the solution of this problem, which can be found in the book[54], we present only the end results.

A spherical top executing orbital motion in a gravitational field rotating about its axis, should experience relativistic precession with frequency $\mathbf{\Omega}_E$. In the absence of any forces of non-gravitational nature

$$\begin{gathered}\mathbf{\Omega}_E = \mathbf{\Omega}_1 + \mathbf{\Omega}_2,\\ \mathbf{\Omega}_1 = (3Gm/2c^2r^3)\,[\mathbf{r}\times\mathbf{v}],\qquad \mathbf{\Omega}_2 = (GI/c^2r^3)\,(3r^{-2}\boldsymbol{\omega}\mathbf{r} - \boldsymbol{\omega});\end{gathered} \tag{16}$$

here $\mathbf{r}$ and $\mathbf{v}$ are respectively the radius vector of the mass center of the top and its velocity, while m, $\boldsymbol{\omega}$, and I are the mass, angular velocity, and the moment of inertia of the central body.

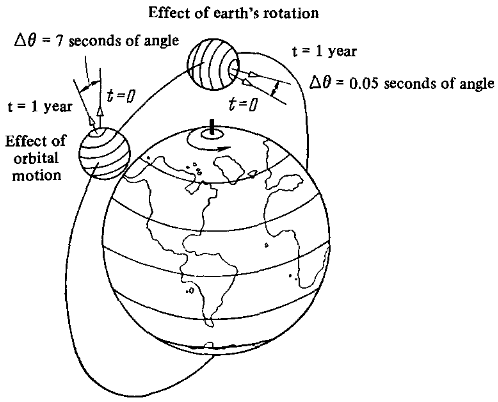

FIG. 6

The first term in (16) is connected with the non-Newtonian character of the centrally symmetrical field (see[55]). The second term in (16) is the result of the rotation of the central body (see[56]). Following Schiff, let us imagine two gyroscopes rotating on a polar orbit around the earth (the earth's axis lies in the plane of the orbit; Fig. 6). In this case the precession of the gyroscope with a spin parallel to the earth's angular momentum will be determined only by the value of Ω_1, and the precession of the gyroscope whose spin is perpendicular to the plane of the orbit only by Ω_2. According to Schiff's estimates, on an orbit with height ~ 800 km, the first effect should lead to a deflection of the gyroscope spin by ~ 7 seconds of an angle annually, while the second weaker one to a deflection by ~ 0.05 seconds of an angle annually. The proposed gyroscope parameters are as follows: it consists of a quartz sphere with diameter ~ 4 cm and rotation frequency ~ 300 rps.

The experimental technique necessary for the realization of this experiment was developed by Everitt and Fairbank[57]. In principle, the experiment should consist of comparing the orientation of the gyroscope spin with certain fixed directions, for example, a previously chosen star. However, the situation is greatly aggravated by the fact that the measurements should be carried out under conditions of an artificial satellite (under terrestrial conditions the effect is weaker by a factor of 15, and, in addition, there is a large number of disturbing factors). The requirements with respect to extraneous perturbations are very stringent: acceleration $< 10^{-4}$ cm/sec, magnetic fields $< 10^{-6}$ g, gas pressure $< 10^{-7}$ Torr, electric fields $< 10^2$ V/cm; the requirements with respect to the finish of the gyroscope are: nonsphericity $< 10^{-6}$, density inhomogeneity $< 10^{-5}$. Everitt and Fairbank have found that satisfaction of these conditions can be ensured with the aid of cryogenic techniques. According to their proposal, the quartz gyroscope is coated with a thin layer of a superconducting material and is placed together with a metallic screen in a helium dewar. The gyroscope body is electrostatically suspended in the center of the screen and is set to rotate by weak gas jets. The superconducting screen, owing to the effect of quantization of the magnetic flux in superconductors[58], ensures the required low level of extraneous magnetic disturbances. The telescope should also be located in a dewar: in this case the helium temperature ensures the absence of noticeable thermoelastic effects that disturb the rigid coupling (within 0.01″) of the telescope with the gyroscope.

The measurement of the deflection of the gyroscope axis is also a very complicated problem. The usual method of observing a marker on its surface is not suitable, owing to the stringent requirements with respect to the homogeneity of the body of the gyroscope (inhomogeneity would lead to the presence of disturbing torques as a result of the gradient of the earth's field). Everitt and Fairbank found an original solution—follow the orientation of the magnetic moment possessed by the superconducting rotating body, the so-called "London moment" (see[59]), the direction of which coincides with the direction of the gyroscope axis. A superconducting magnetometer suitable for this purpose was developed earlier[60]. Since it is necessary to measure only the change of the mutual orientation of the gyroscope relative to the telescope, the requirements with respect to stabilization of the position of the satellite itself in space are not too stringent. The permissible swings are determined by the value of the deflection at which the chosen star goes out of the field of view of the telescope. At a reasonable aperture of the telescopic objective (~ 11 cm), Everitt and Fairbank estimate the accuracy of satellite stabilization at ± 5 seconds of angle in angle and 0.1 seconds of an angle per second in velocity. Such a stabilization can be ensured by means of a servomechanism that controls the gas jets of helium evaporated from the same dewar in which the entire system is placed. Calculations show that in this case the jet reaction can exceed by 40–80 times all the remaining perturbations to which the satellite is subjected. The aberration of the star on which the telescope is aimed can be calculated very accurately, and it is thus possible to calibrate the system. A test of the described cryogenic apparatus on a satellite is planned for 1970–1971, and it is proposed to perform the experiment itself in 1973.

Recently Schiff[61] reached the conclusion that a gyroscopic experiment can be supplemented by testing one more general-relativistic effect. As was noted more than 30 years ago by Mathisson[62], the rotation of bodies leads to a deviation of their trajectory from the geodesic lines along which the same bodies would move in a gravitational field without rotation. According to Schiff's calculations, the nongeodesic singularities of the motion of a free gyroscope located inside a satellite moving on a circular orbit can become manifest in the presence of a periodic acceleration (at the satellite revolution frequency) of the mass center of the gyroscope relative to the hull of the satellite. The amplitude of this acceleration should reach $\sim 10^{-17}$ cm/sec^2, but the effect averaged over the orbit is equal to zero. However, if the orbit is elliptic, then there is a monotonic change of the velocity of the center of mass of the gyroscope and accordingly its position relative to the satellite. An estimate shows that this shift is of the order of $\sim 4 \times 10^{-4}\, y^2$ cm, where y is the number of years. The effect is large enough to be measured, but to this end it is necessary to ensure high stability of the parameters of the orbit itself.

Observation of relativistic precession of a gyroscope is also of interest from the point of view of comparing GRT with other theories.

One of the most frequently cited variants of hypothetical theories competing with GRT is the Brans-Dicke variant[63], in which a scalar field is considered besides the tensor (Einstein) field. The parameter ω peculiar to this theory characterizes the ratio of the interaction forces with the tensor and with the scalar fields. A preliminary estimate of the constant, $\omega \geq 6$, was made by Brans and Dicke, so that the theory explains the effect of the gravitational red shift, and the agreement between the shift of the Mercury perihelion with the known relativistic value is within 8%. According to Brans and Dicke, the precession of the perihelion of Mercury is 39.6 seconds of angle in a century, and is smaller by 3.43 seconds of angle than the Einstein value. The latest experiments of Dicke and Goldenberg[64], who observed the non-sphericity of the sun (the difference between the equatorial and polar radii is ~ 35 km), explain, as it were, the missing 3.43″ as being due to the presence of a quadrupole moment in the sun. However, as was correctly noted by O'Connell[65], the surface oblateness still does not say anything concerning the distribution of the internal masses, which are responsible for the contribution to the quadrupole moment (of course, if the oblate surface is not equipotential). Thus, the experiment of[64] is not critical enough with respect to the choice between the two theories. The accuracy of the experiment of Shapiro et al.[38] is likewise insufficient to draw unambiguous conclusions.

In the opinion of O'Connell[65] and Schiff[53], observation of effects of gyroscopic precession may turn out to be the decisive experiment favoring one of the theories. Let us explain this circumstance. As shown in[65], the precession of a free gyroscope within the framework of the Brans-Dicke theory, for a weak field, is described by the equation

$$\Omega_{BD}=\left(\frac{4-3\omega}{6-3\omega}\right)\Omega_1+\left(\frac{3+2\omega}{4+2\omega}\right)\Omega_2, \tag{17}$$

i.e., the deviation from the Einstein relation (16) lies in the small numerical coefficients. If we put, as follows from the experiments of Dicke and Goldenberg, $\omega = 6$, then

$$\Omega_{BD}=\frac{11}{12}\Omega_1+\frac{15}{16}\Omega_2.$$

and for the expected value $\Omega \approx 7$ seconds of angle per annum the accuracy of reading between 0.01″ and 0.001″, guaranteed in[57], is perfectly sufficient to observe the difference between Ω_E and Ω_{BD}. We note that according to[100], it is also possible to distinguish the GRT from the linear gravitation theory by observing the effect of the precession.

2. Equivalence Principle

As is well known, the equivalence principle is the basis of the consequences of GRT. In this connection, it is necessary to emphasize the need for a reliable experimental base to confirm this premise. Einstein[66] formulated the principle of the equivalence of the inertial system of measurement in a homogeneous gravitational field and in a certain uniformly accelerated reference frame (with acceleration g of the gravitational field). Recently, following Dicke[67], this statement is frequently divided into two:

a) Weak equivalence principle: in a gravitational field, in the region where the field gradient can be neglected ($(\partial g/\partial l)l^2 \ll c$, where l is the dimension of the region), all the bodies move along the same trajectories (of course, for identical initial conditions and in the absence of non-gravitational forces).

b) Strong equivalence principle: under the same limitations, all the physical laws are the same in all of the space-time continuum.

From the experimental point of view, these premises are verified, in particular, by testing the equality (or constancy of ratio) of the inertial mass m_i and the gravitational mass m_g of the body. Indeed, in this case the first statement is satisfied, namely, it is possible to cancel out the masses in the equation of motion $m_i a = m_g g$. The validity of the second statement can be seen by recognizing that the inertial mass contains contributions of the energies of all possible body particle interactions: if the ratio m_i/m_g does not change from point to point, then the ratio of the forces of the different interactions should also remain the same in all of space.

For a long time, the only classical experiment establishing the equality of the inertial and gravitational masses, were the Eotvos experiments[68], in which the acceleration of bodies having different internal structures was measured in the earth's gravitational field. The equality of m_i and m_g was verified to within the eighth decimal place (e.g., for the alloy 90% Al + 10% Mg and for platinum). In 1961–1963, the experiment was repeated by Dicke and co-workers[69,70], where the acceleration of the bodies was measured in the field of the sun and the accuracy was improved to $\sim 10^{-11}$ (for aluminum and gold).

In the evaluation of these experiments it was emphasized more than once[71,72] that the results confirm the strong equivalence principle only in a limited sense, namely, accurate to weak and gravitational interactions. If the interaction that changes the inertial mass of the body were not to introduce suitable changes in the gravitational mass, this could have been observed in the experiments of the Eotvos-Dicke type. The accuracy $\sim 10^{-11}$ makes it possible to extend, with a large margin, the strong equivalence principle to strong and electromagnetic interactions, but nothing can be said as yet with respect to the weak and gravitational interactions.

Besides experiments of the Eotvos-Dicke type, great interest attaches to direct measurements of the acceleration of the force of gravity of free neutral and charged particles and photons. It was in this field that considerable progress was reached recently.

The gravitational acceleration of free neutrons was measured in[73]. The idea of the experiment was to observe the effect of "sagging" of a neutron beam in the earth's gravitational field. The authors of[73] have employed very successfully velocity separation of the beam. In the experiment, the fast neutrons, which experience exceedingly small and practically non-existent deflection under the force of gravity, were used as a reference system to measure the "sag" of the slow neutrons.

A schematic diagram of the experiment is shown in

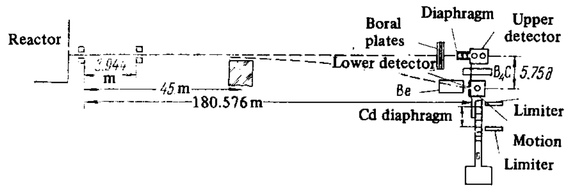

FIG. 7

Fig. 7. The flux of neutrons from the nuclear reactor passed through a collimating device with diaphragms of ~ 0.02 cm, and special measures were taken to eliminate diffraction effects. After traversing an evacuated beam ~ 180 m long, the beam reached the recording point. The expected magnitude of separation of the high-velocity particles from the slow ones was ~ 10—20 cm. Two separate diaphragms and two detectors were used to register both components. Boral filters (Al with 20% B) located along the path of the high-velocity beam, separated the sufficiently fast particles, which were not deflected noticeably in the gravitational field. Ahead of the slow-particle detector was placed a beryllium filter, the purpose of which will be explained below. The detectors were carefully insulated from each other by means of a bulky screen of B_4C. The entire construction (diaphragms, detectors, screens) could be moved in a vertical direction through a distance ~ 10 cm with velocity 1.9 cm/min. The neutron-counting results were recorded by a multichannel memory system, and each spatial position of the system corresponded to its own fixed set of channels (a total of ~ 500 channels distributed equally among the upper and lower detectors).

In order to assess the gravitational acceleration from the experimentally measured altitude distribution of the particles at the end of the flight base, it is necessary in addition to know exactly the particle velocities for at least one concrete altitutde. For this purpose, a diffraction filter of polycrystalline beryllium block, located in front of the slow-particle detector, was used. Owing to the Bragg scattering by the crystal structures ⟨100⟩ and ⟨002⟩, this filter ensured the possibility of accurately determining the angular position of the neutrons with velocities $V_1 = h/2m_n d_{100}$ and $V_2 = h/2m_n d_{002}$ (here d_{100} and d_{002} are constants of the corresponding crystal lattices, m_n is the neutron mass, and h is Planck's constant). Indeed, the filter passes only those neutrons which do not experience reflection at the Bragg angle $\sin\theta = \lambda/2d_{ikl}$, where $\lambda = h/m_n v$ is the de Broglie wavelength of the neutron. For neutrons with $\lambda \geq 2d$ $(v < h/2m_n d_{ikl})$, the scattering process terminates sharply and these neutrons arrive at the detector. At the registration point, the neutrons already subjected to terrestrial acceleration are distributed in altitude in such a way that the depth of the "sag" increases with increasing λ (with decreasing velocity). In this situation, the detector placed behind the beryllium filter, moving downward, should register a sharp growth of the beam intensity at two critical points corresponding to the coordinates of the neutrons with velocities v_1 and v_2. These interruptions should be observed against the background of a small general decrease, connected with the decrease in intensity of the main beam with increasing λ (the left slope of the Maxwellian distribution).

Figure 8a shows the corresponding experimental

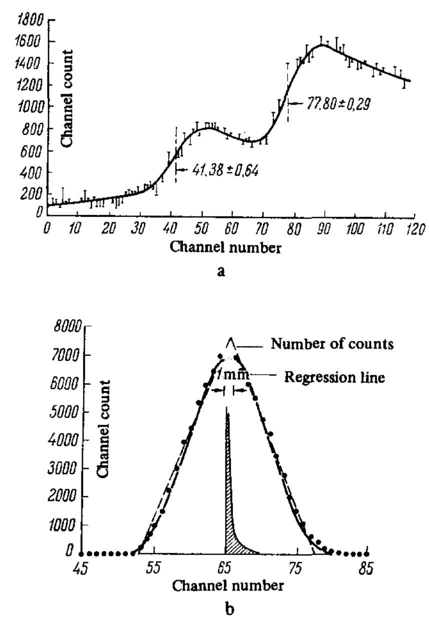

FIG. 8

curve. The finite slope of the rise in the region of the critical points is connected with the finite dimensions of the input diaphragms of the detector (the critical coordinate corresponds to the center of the rise). The results of the registration of the fast particles are illustrated in Fig. 8b. The number of the channel determines uniquely the vertical coordinate; the separation S of the fast and slow beams can thus be determined. From a comparison with the calculated value, such as $S = g_n(m_h \lambda h^{-1})^2 l^2/2$ (l is the length of the flight base), one can estimate the gravitational acceleration of the free neutrons:

$$g_{\text{н}}(002) = 973 \pm 7.4 \text{ cm/sec}^2,$$
$$g_{\text{н}}(100) = 975.4 \pm 3.1 \text{ cm/sec}^2. \tag{18}$$

The local free-fall acceleration is $g \approx 979.74$ cm/sec^2. Thus, the experiment shows, with accuracy not worse than 1%, the correspondence between the gravitational acceleration of the free neutrons and the normal acceleration of free fall.

The use of a two-structure diffraction filter in the slow-neutron beam has enabled the authors of the experiment to verify the effect of variation of g_n for particles with different spin orientations. In the earth's magnetic field, a beam of neutrons splits into two beams with spins ± 1/2. If g_n were to have spin anisotropy, the shape of the curve (see Fig. 8a) in the region of the maxima would be more complicated. However, the width of the experimental step was well described by the geometry of the diaphragms. It was therefore concluded in[73] that if the effect exists at all, it is exceedingly small, not exceeding several percent.

This result is of interest and is connected with the fact that recently, hypotheses were advanced in a number of papers[73,74] concerning the possible gravitational effects connected with nuclear polarization, and models of gravitational interaction were proposed[75], from which follow such effects. Morgan and Peres[74] have shown that the absence of the influence of orientation of the nuclear spin at the level of the mass defect in the Eotvos-Dicke experiment can serve as a confirmation of the strong equivalence principle. Experimentally, spin anisotropy of gravitation was investigated also in[76], where a sensitive balance was used to measure the variation of the weight of samples placed in a slowly varying resonant (with respect to the frequency of the balance) magnetic field. The measurement accuracy was limited by the degree of homogeneity of the magnetic field. No spin gravitation effects were registered at the level $\Delta p/p \approx 6 \times 10^{-10}$ for ${}_{13}Al^{27}$ and $\Delta p/p \approx 4 \times 10^{-9}$ for ${}_{1}H^{1}$.

In connection with the problem of separating matter and antimatter on a cosmological scale, there is a large number of known pronouncements[77-79], ascribing "negative" gravitational properties to antimatter. Since the inertial mass of the antiparticles is positive (as is evidenced with great accuracy by effects of annihilation and deflection of antiparticles in electromagnetic fields), the existence of a "negative" gravitational mass of antiparticles would denote violation of the equivalence principle. An estimate of the contribution of positrons produced as a result of polarization of the vacuum in the Coulomb field of a nucleus, to the inertial mass of the atom, was obtained by Schiff in[72]. His calculations have shown that this contribution (of the order of $m_{e^+}(Z/137)^2$, where Z is the charge of the nucleus) would have been observed in the Eotvos-Dicke experiments (in the case of "negative" gravitational properties). This fact indicates that the positron has a normal gravitational mass.

A direct experiment would consist of comparing the acceleration of particles and antiparticles in the earth's field. Witteborn and Fairbank[80] recently completed the first (easier) part of such an experiment, having measured the free-fall acceleration of electrons. The measurement method consisted of studying the flight times of electrons falling freely in a vertical metallic cylinder.

Preliminary calculations of Schiff and Barnhill[81] have shown that the distribution of the intrinsic electrons in a metallic cylinder under the influence of the earth's gravitation produces inside the cylinder an electric homogeneous field $E = m_e g/e$ (m_e and e are the mass and charge of the electron). For electrons located on the cylinder axis, this field cancels out exactly the acceleration due to gravity. Thus, in the absence of extraneous perturbations, the total acceleration of the electrons inside the cylinder is equal to zero, whereas positrons should move with an acceleration 2g.

The main problem is the performance of such an experiment is that of eliminating all the extraneous fields at the level $m_e g/e \approx 5.6 \times 10^{-13}$ V/cm. This pertains above all to the field of the electrostatic image in the walls of the cylinder in regions close to the cathode (injector) and the detector of the electrons. To decrease these effects, it would be necessary to maintain the inside diameter of a long copper tube (~ 1 m) surrounding the region of the falling electrons, with very high accuracy (± 0.0003 cm at a diameter d = 5 cm); the vertical component of the image field in the tube walls turned out in this case to be sufficiently small. The potentials in the region of the cathode and of the detector are approximately proportional to exp(− 24z/a) (z—distance to

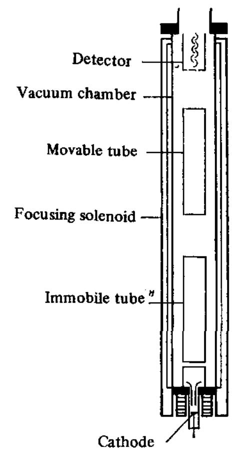

FIG. 9

the nearest end of the tube) and could be decreased to the required value by choosing z.

The chamber used in the experiment of Witteborn and Fairbank is illustrated in Fig. 9. The electrons were retained at the chamber axis by a magnetic field of a coaxial solenoid. The entire device was placed in a helium bath. The helium temperature ensured the immediate satisfaction of several necessary conditions. First, the temperature gradients of the surrounding walls could not exceed 10^{-3} deg/cm, meaning that the thermal potential drops were smaller than 10^{-13} V/cm (the Thomson coefficient for copper at 4.2°K amounts to ~ 10^{-6} V/deg). Second, the superconductivity of the solenoid ensured high stability of the controlling magnetic field, $\Delta H/H \sim 10^{-4}$. Third, it is necessary to freeze out the molecules of the residual gas in order to maintain the vacuum at a level ~ 10^{-11} Torr, at which it is possible to neglect the influence of the electron-molecule collisions on the electron motion. Fourth, finally, wall-surface-potential variations due to the random inhomogeneities of the crystal structure, which reach ~ 10^{-3} eV at room temperature, turned out to be less than ~ 10^{-13} V/cm, owing to the thin hydrogen-helium film covering the walls of the chamber when the residual gas is frozen out. Thus, all the extraneous electrostatic fields were smaller than the critical Schiff-Barnhill (SB) field.

At the same time, the incomplete magnetic screening has made possible the presence of magnetic gradients, owing to the external magnetic background of ~ 0.5 G/cm. In the general case, electrons having spin and orbital magnetic momenta should experience in such a field a vertical force at least of several times $m_e g$, with the exception of the electrons for which the orbital momentum exactly cancels out the spin momentum: it is precisely these electrons that can be used for the measurements. The selection of the electrons in the "ground" state (without the magnetic moment) was realized in the experiment[80] by superimposing a strong magnetic field (~ 7000 G) in the cathode region. By the same token, this resulted in spatial sorting of the electron bunches traveling towards the detector, in accordance with their initial velocities and with the field strength. The last to reach the detector were the electrons in the ground state. The detector was an electron multiplier. After passing through a number of auxiliary blocks, the signal reached a memory device, which fixed the temporal distribution of the electrons arriving at the cathode after each individual pulse. The pulse contained on the average $\sim 10^9$ electrons with an average energy lower than $\sim 10^{-9}$ eV.

Of practical interest was the time t_{max} of the flight of the slowest electrons (in the "ground" state), which corresponds experimentally to the time elapsed from the start of the pulse to the instant when the intensity of the input signal reaches the constant level of the background. During the course of the experiment it turned out to be convenient to produce a well known weak homogeneous electric field E_0 in the flight region of the electrons. This made it possible to estimate t_{max} with the aid of the formula

$$t_{\max} = [2hm_e/(m_e g_e - eE_{SB} - eE_0)]^{1/2}; \qquad (19)$$

here g_e is the acceleration of the electrons as a result of the force of gravity, E_{SB} is the Schiff-Barnhill field, and h is the flight path length. Formula (19) explains the need for including in the experiment the field E_0, namely, by performing measurements at two different values of E_0 it is possible to determine the mass m_e of the investigated particles and the value of the difference $(m_e g_e - eE_{SB})$.

Of course, the foregoing scheme describes only in principle .he course of the analysis of the experimental data, and the actual analysis performed by Witteborn and Fairbank was much more complicated. This is connected with allowance for the errors introduced by the statistical fluctuations of the counters and the background level, and also edge effects that disturb the homogeneity of the field near the cathode, near the ends of the transit tube, etc. Without stopping on the details of the actual analysis, we present only the final results. Figure 10 shows a plot of $T = m_e g_e - eE_{SB} - eE_0$ against the field E_0. The solid line corresponds to the theoretical value $F = eE_0$; the experimental points were obtained by least squares with the aid of a digital computer, using the results of an analysis of the temporal distributions of the electrons (from 10 000 individual measurements). As seen from Fig. 10, the difference $m_e g_e - eE_{SB}$ is quite small. The mean value is $\overline{m_e g_e - eE_{SB}} = 0.13 \times 10^{-10}$ eV/cm, and the mean square deviation (with allowance for the errors in the measurement of h, E_0, and t_{max}) is 0.51×10^{-10} eV/cm ≈ 0.09 mg. Thus, the vertical component of the force acting on an electron incident along the axis of the vertical metal tube is less than 0.09 mg (measurements of

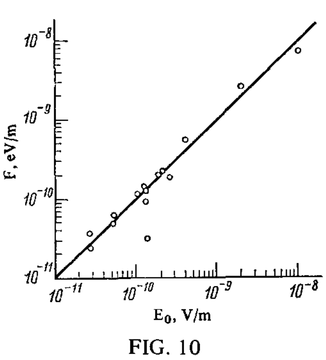

FIG. 10

m_e yielded the value of the electron mass), in accord with the calculations given in[81]. This also means that the gravitational acceleration of the electrons inside the metal is the same as in vacuum. (In the discussion of this experiment in the literature[82,83,61], the question why the deformation field of the ions (under the action of gravity), which is not completely cancelled out by the change of the local work function, remains unclear. The most probable cause is assumed to be the effect of screening the surface by a layer which apparently[84] experiences no deformation.)

Experiments with positrons are presently under preparation[61]. As proposed recently by Held[84], the influence of the wall should decrease the positron acceleration from 2g to g. Finally, Pound and Rebka's experiment[85] proves that photons experience normal acceleration by the earth's gravity. The photons have a normal g in the earth's gravitational field, with accuracy not worse than 0.1% as attained in recent experiments.[86]

Thus, at present there exists experimental proof that the inertial and gravitational masses are equal for ordinary bodies, for neutral and charged elementary particles, and for photons. It should be noted that these data concern the inertial and the so-called "passive" gravitational mass. Apparently, following Bondi[87], it is frequently customary in the physics literature to distinguish between two types of gravitational masses: "passive," proportional to the force acting on the body in the gravitational field ($F = -m \operatorname{grad} \varphi$, where φ is the gravitation potential), and "active," proportional to the force exerted by the gravitating body. The "active" gravitational mass characterizes the ability of the body to produce a gravitational field, and is the mass that enters, for example, in the Poisson equation. Actually such a distinction affects two fundamental laws, Newton's third law and the equivalence principle. In classical mechanics, where Newton's third law is postulated, the "passive" and "active" gravitational masses are identical, and their equality to the inertial mass is a separate empirical fact. Relativity theory is based on the equivalence principle, without specially postulating a relation similar to Newton's third law. In this connection, an experimental verification of the equality of the "active" and "passive" gravitational masses of bodies (or constancy of their ratio) is of interest. Until very recently, it was possible to estimate the ratio of the "active" and "passive" gravitational masses of bodies only on the basis of measurements of the gravitional constant G. Mainly these are experiments of the type of the Cavendish experiment, in which usually one measures the torque produced by the action of attraction forces between a certain massive body and small trial masses mounted on the ends of the beam of a torsion balance. The gravitational constant can be calculated if the trial masses, the geometry of the experiment, and the rigidity of the torsional suspension are known. The sizes of the trial masses are determined by exact weighing and consequently "passive" trial masses take part in the estimate of G. On the other hand, the twist angle of the beam is determined by the action of the "active" mass to the "passive" changes as a function of the internal structure of the bodies, then the results of the measurements should depend on the employed substances. An analysis of the existing data on the measurement of G with different substances[88] shows that G remains constant with accuracy $\sim 3 \times 10^{-3}$. With the same accuracy it can be stated that "passive" and "active" gravitational masses of the bodies are equal. This figure was recently improved in experiments by Kreuzer[88]. His estimate of the upper limit of the difference in the ratio of the "active" and "passive" masses is $\Delta m/m \lesssim 5 \times 10^{-5}$. In addition to the performed experiments, there are at present several suggestions involving testing the principle of equivalence at a level of still higher orders of magnitude.

The Eotvos-Dicke experiments establish the validity of the strong equivalence principle accurate to weak and gravitational directions. It is precisely at this level that further experiments may be to advantage. For example, these may be experiments with cosmic bodies, since the internal gravitational energy of laboratory bodies is too small: the ratio of the internal gravitational energy to the total energy of a body of mass m and radius a is $\Delta = Gm^2a^{-1}/mc^2 = Gm/c^2a \lesssim 10^{-25}$ for bodies with laboratory dimensions. The situation changes on going over to cosmic objects.

Let us recall Dicke's suggestion[67], made by him a few years ago and again considered in[89], concerning the observation of anomalies in the motion of Jupiter, which should occur if the internal gravitational energy of the planet makes no contribution to the gravitational mass. Accurate to the Eotvos-Dicke experiment, we can write

$$m_g/m_i = 1 + \eta\Delta, \tag{20}$$

where η is a coefficient of the order of unity, characterizing the degree of violation of the equivalence of the masses. In the case when $\eta = 1$ we have for the sun $\Delta_S \approx 10^{-5}$ and for Jupiter $\Delta_J \approx 10^{-18}$. If the gravitational energy makes no contribution to the heavy mass ($\eta = 1$), the corresponding corrections should be introduced, for example, into the third Kepler's law

$$4\pi^2R^3/T^2 \approx G(m_S + m_J + m_J\Delta_S + m_S\Delta_J). \tag{21}$$

The mass ratio is $m_J/m_S \sim 10^{-3}$, and it follows from (21) that the deviation from the Kepler motion should take place at the level $\sim 10^{-8}$. Measurement of T can be carried out at the present time with much better accuracy, but unfortunately the accuracy with which distances are measured in the solar system does not exceed $\sim 10^{-6}$. Thus, such an experiment is at present still impossible.*

Recently Nordtvedt[89] proposed to estimate the parameter Δ by determining the behavior of a small mass located at a libration point characteristic of the Lagrangian three-body problem. It is known from mechanics that an infinitesimally small mass will move along a stable orbit relative to two massive bodies that are in relative orbital motion, provided the small mass is located at a libration point (in the orbital plane of the fundamental bodies, at the vertex of an equilateral triangle). The characteristic configuration is shown in Fig. 11; the conditions superimposed on the angles and on the distances can be found in[89]. If the two large masses are the sun and a planet, then $\Delta_S \gg \Delta_p$ (i.e., at $\eta = 1$ the sun's gravitational field decreases much more strongly than the planetary one) and the libration point in the case of violation of the equivalence principle should shift in direction towards the planet by an amount $\delta r_2 \approx -\Delta_S(R_1 + R_2)/3$ (the notation is explained in Fig. 11). According to Nordtvedt's estimates, the shift of the asteroid Trojan, captured by the sun-Jupiter system, may amount to $\delta\theta \approx 1''$, which is a measurable quantity. On the other hand, it is realistic to consider an artificial satellite in the sun-earth system. In this case the delay time of a radio signal reflected from the satellite decreases by an amount on the order of $\delta t \approx 2\Delta_S(R_1 + R_2)/3c \sim 5 \times 10^{-3}$ sec. Unfortunately, the experiment cannot be performed in such a way as to measure δt directly (a situation analogous to the experiment of Shapiro et al.[38]). However, a semi-calculation experiment is possible in principle, since δt amounts to 10^{-5} of the total signal-echo time, and as noted above the distances are known with accuracy $\sim 10^{-6}$.

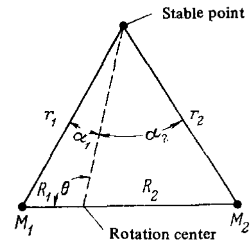

FIG. 11

To determine η we can consider also the earth-moon-sun system. The earth should lose much more in its force of attraction to the sun than the moon (if the equivalence principle is valid ($\eta \approx 1$)), since its gravitational energy greatly exceeds the lunar one. A detailed calculation[90] shows that for this reason there should be observed a periodic change of the radius of the lunar orbit (relative to the earth), $\delta r \approx -1200\eta \cos \Omega t$ (cm)—on the order of the angular frequency of the lunar revolution around the earth. This effect can apparently be observed with the aid of laser sounding of the moon[91].

The described proposals undoubtedly represent only the principles of the experiments. For a real estimate of their feasibility it is necessary to analyze the limitations imposed by numerous disturbing factors (other cosmic bodies, etc.). We note that when these experiments are performed, in Nordtvedt's opinion[92], it will be possible to separate the competing gravitational theories, particularly, $\eta \neq 0$ for the Brans-Dicke treatment, in contradiction to GRT, where $\eta \equiv 0$.

The improvement of the accuracy of the Eotvos-Dicke experiments is possible also in the Schiff gyroscopic experiment. Some suggestions in this respect were made by Schiff[61] in connection with an old remark by Lee and Yang, concerning the hypothetical vector field of "heavy-particle charges." We recall that about 15 years ago Lee and Yang[93], in an analysis of the analogy between the laws of conservation of the electric charge and of the number of heavy particles (baryons), pro-

*The installation of a laser reflector on the moon by the astronauts of "Apollo-II" apparently produces conditions for raising the accuracy with which the lunar trajectory is measured to the required value.

posed that the latter may be connected with the invariance against the gauge transformation $\psi \rightarrow e^{i\alpha}\psi$ for the wave function ψ of the heavy particles (similar to the gauge transformation in electrodynamics). In this case there should exist a vector field interacting with all heavy particles whose quanta possess a zero rest mass. The force of interaction between the two massive bodies would of necessity contain a contribution of the quasi-nucleon repulsion of the "heavy-particle charges." Such a force, exerted on the gyroscope and satellite by the earth, will cause periodic acceleration of the gyroscope relative to the hull of the satellite. This acceleration, directed towards the earth's center, should be approximately described by an expression in the form

$$a_N \approx kg \cos \omega_0 t; \tag{22}$$

here ω_0 is the orbital frequency, $k \approx (\epsilon^2/eM_N^2)\gamma$, ϵ is the "heavy-particle charge" of the nucleon, M_N is the mass of the nucleon, and γ is a dimensionless parameter proportional to the difference between the ratios of the mass number to the atomic number for the gyroscope and satellite materials.

The accuracy of the Eotvos-Dicke experiments limits the possible value of the Lee-Yang forces (or, generally speaking, any other nongeodesic forces) from above, $k < 10^{-11}$. Therefore a rough estimate of the upper limit of the amplitude of the periodic acceleration, in the case of non-equivalence of the inertial and gravitational masses, is $a_N < 10^{-8}$ cm/sec^2. This quantity greatly exceeds the effect of the non-geodesic shift connected with the rotation of the gyroscope (see Ch. III). It is apparently here that there is a great margin of accuracy for the improvement of the Eotvos-Dicke experiments.

The authors thank Ya. B. Zel'dovich for useful remarks during the compilation of the present review.

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