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PHYSICAL PRINCIPLES OF QUANTUM GYROSCOPY

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I. INTRODUCTION

QUANTUM gyroscopy is a collective term designating a new branch of quantum electronics and engaged in the study of the principles and the possibilities of developing devices whose action is based on the gyroscopic properties of the particles making up the working parts of the instrument pickup. These properties may be due to the spin and orbital momenta of atomic nuclei, atoms, electrons, or photons. The gyroscope is in its working state when the mechanical moments of the particles are oriented beforehand in a certain direction. Devices of this kind are called collectively quantum gyroscopes, to distinguish them from classical mechanical gyroscopes, the necessary element of which is a rotating or vibrating body.

The measured quantities in instruments of this type are, for example, the frequency of the stimulated coherent quantum transitions between energy sublevels corresponding to different possible values of the projections of the mechanical momentum of the particles on a physically selected direction. The rotation of the instruments produces a displacement or a splitting of the energy sublevels of the system, and measurement of the frequencies of the transitions between them makes it possible to detect this rotation, and also to determine the direction and magnitude of its angular velocity.

In spite of the fact that the already existing quantum gyroscopes have reached a certain degree of perfection, the accuracy and sensitivity obtained in practice are far from their limiting values and can be greatly improved. Quantum gyroscopy as a branch of science is barely in its growth state, and its capabilities and limits have not yet been fully assessed. However, the instruments it developed by now have already found a variety of applications, some of them unexpected, in laboratory practice and in technology. The development of methods of quantum electronics uncovers ways

of using both new phenomena and phenomena heretofore well known but not yet employed in practice.

In this review we consider certain physical phenomena that are sensitive to changes in the position and orientation of physical objects in space, and underline the operating principles of quantum gyroscopes.

The existing quantum-gyroscopy instruments can be divided into two groups: those orienting themselves along the force lines of the geomagnetic or interplanetary magnetic field, and those orienting themselves relative to an inertial coordinate frame.

Devices of the first type can be called magnetic theodolites or direction finders. Together with quantum magnetometers, they can be extensively used to solve many problems of theoretical magnetometry, in preparing magnetic maps, in revealing magnetic anomalies, in prospecting for magnetic minerals, etc. Instruments of this type can be quite reliable, sensitive, accurate, and compact. For navigation purposes, however, they are of secondary interest at the present time^[1,2], primarily because not enough is known as yet about the topography of the magnetic field of not only the space around the sun but also of the geomagnetic field. These fields are furthermore subject to random time variations.

The most promising for navigation are instruments of the second type called quantum inertial gyroscopes. Much attention has been paid to the development of such instruments. It was found that the use, for example, of ring lasers in the optical band opens up prospects for the development of devices that are sensitive to rotation and can easily detect a relativistic change of the course of time in a gravitational field of attracting masses and in an acceleration field of inertial origin. An exceedingly high sensitivity to rotation should be possessed by the so-called superconducting interferometers, in which interference beats of the wave functions of electrons in a superconducting state can be observed.

It should be noted that in spite of the unusual sensitivity, quantum gyroscopes of the types presently known do not have a practical sensitivity higher than the better mechanical gyroscopes. Further increase of their sensitivity entails serious difficulties^[3]. They do possess, however, many practical advantages over classical gyroscopes.

II. OPERATING PRINCIPLES OF NUCLEAR MAGNETIC THEODOLITES

Soon after the discovery of nuclear magnetic resonance it was noted^[4] that the equations of motion of the resultant vector of homogeneous nuclear magnetization \mathbf{M} in a magnetic field \mathbf{H} are analogous to the equations of motion of a symmetrical top. The motion of a mechanical gyroscope under the influence of a time-dependent torque was considered in^[5] and its analogy with the motion of the nuclear magnetic moment was demonstrated. This gave grounds for proposing that nuclear magnetic moments can be used to produce gyroscopes of a new type^[6].

The system of nuclear magnetic moments in a substance located in an external magnetic field \mathbf{H} , which we assume to be uniform within the confines of the sample, acquires the direction of this field under the influence of relaxation processes. The resultant equilibrium magnetization \mathbf{M}_0 is equal to $\kappa_0 \mathbf{H}$, where κ_0 is the static nuclear susceptibility.

The changes in the components of the nuclear-magnetization vector \mathbf{M} in a weak magnetic field are described by equations of the Bloch type^[7]

$$\left. \begin{aligned} \frac{dM_x}{dt} + \frac{M_x}{T_2} &= \gamma [\mathbf{M}, \mathbf{H}]_x, \\ \frac{dM_y}{dt} + \frac{M_y}{T_2} &= \gamma [\mathbf{M}, \mathbf{H}]_y, \\ \frac{dM_z}{dt} + \frac{M_z}{T_1} &= \frac{M_0}{T_1} + \gamma [\mathbf{M}, \mathbf{H}]_z, \end{aligned} \right\} \quad (1)^*$$

where γ is the gyromagnetic ratio, which is positive for most nuclei of practical interest. For electrons it is negative. We shall henceforth assume throughout that γ is positive.

The physical constants T_2 and T_1 in (1) have the meaning of transverse and longitudinal relaxation times. They depend on the composition and on the state of the substance, and are very sensitive to paramagnetic impurities and to the temperature. For pure liquids and gases, $T_2 = T_1$.

Equations (1) can be used also to describe optical pumping in gases and vapors of atoms. In this case, however, the expressions for T_2 and T_1 have a more complicated meaning and depend on the intensity of the pumping light^[23].

Let us imagine that a sample containing nuclear magnetic moments is magnetized in a magnetic field $\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_0$, where \mathbf{H}_1 is a strong field and \mathbf{H}_0 is a weak field perpendicular to the strong one, but quite stable and homogeneous. Then, if the field \mathbf{H}_1 is turned off sufficiently rapidly, then a non-equilibrium but coherent state of the magnetic-moments sets in, and the magnetization vector \mathbf{M} , which initially is practically parallel to \mathbf{H}_1 , begins to precess about the vector \mathbf{H}_0 ,

decreasing in absolute magnitude as a result of the relaxation, and rotating in the direction of \mathbf{H}_0 .

For time intervals that are small compared with the relaxation times, i.e., for time intervals during which the nuclear gyroscope operates, the relaxation processes can be neglected, and the equation of motion of the system of magnetic moments can be written in the form

$$d\mathbf{M}/dt = \gamma [\mathbf{M}, \mathbf{H}]. \quad (1')$$

By taking in succession the scalar products of (1') with \mathbf{M} and \mathbf{H} , it is easy to verify that in constant fields, within the limits of validity of Eq. (1'), the length of the vector \mathbf{M} remains unchanged, i.e., the magnetization and the angle α between \mathbf{M} and the direction of the field remain unchanged ($\mathbf{M} \cdot \mathbf{H} = MH \cos \alpha$). Thus, in a homogeneous constant magnetic field the system of isolated magnetic moments precesses about the direction of the external magnetic field.

The frequency and direction of the precession can be readily determined by changing over to a coordinate system in which the vector \mathbf{M} is immobile, i.e., to a system of coordinates rotating together with the vector \mathbf{M} about the direction of the vector \mathbf{H} . This procedure will be discussed in greater detail later on. By using this procedure, it is easy to find that the precession can be described by the angular-velocity vector $\boldsymbol{\omega} = \gamma \mathbf{H}$, the length of which is $\omega = \gamma H$ (Fig. 1). The magnetic energy of the system in the absence of relaxation is an integral of the motion and remains equal to $-\mathbf{M} \cdot \mathbf{H}$ during the entire time. The precession frequency is independent of the angle α and of the value of the vector \mathbf{M} .

Let us imagine that the vector \mathbf{H} rotates with a constant angular velocity $\boldsymbol{\Omega}$ about an axis that is arbitrarily oriented in inertial space. As in the case with gyroscopes, the entire device with the aid of which the precession signals are registered, rotates together with the magnetic field^[8]. We change over to a coordinate frame rotating relative to the inertial frame with angular velocity $\boldsymbol{\Omega}$. To this end we use a theorem concerning the local derivative, expressed by the relation^[8]

$$d\mathbf{M}/dt = (d\mathbf{M}'/dt) + [\boldsymbol{\Omega}, \mathbf{M}'], \quad (2)$$

where the primes denote that the components of the vector \mathbf{M} are defined in terms of projections on the axes of the new coordinate system. In this system we have

$$d\mathbf{M}'/dt = \gamma [\mathbf{M}, \mathbf{H} + (\boldsymbol{\Omega}/\gamma)]. \quad (3)$$

From (3), in analogy with the procedure used above, it

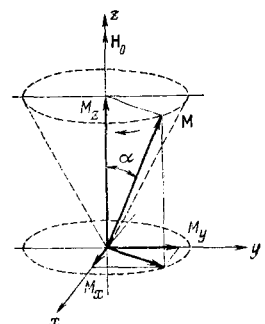


FIG. 1. Free precession of the magnetization vector about the magnetic-field vector.

* $[\mathbf{M}, \mathbf{H}] \equiv \mathbf{M} \times \mathbf{H}$.

is easy to obtain two integrals of motion showing that the vector \mathbf{M} precesses about the vector $\mathbf{H} + \Omega\gamma^{-1}$ of the effective field.

The motion of the vector \mathbf{M} relative to the initial inertial frame is usually called nutation. In order to determine the nutation frequency, it is necessary to change over to a new coordinate frame, in which the vector \mathbf{M} is immobile. After making this change, we obtain the nutation frequency

$$\omega_1 = \omega [1 + 2(\omega\Omega/\omega^2) + (\Omega/\omega)^2]^{1/2}, \quad (4)$$

which is equal to the length of the nutation vector $\omega_1 = -(\gamma\mathbf{H} + \Omega)$.

If the rotation of the device is slow, so that

$$\Omega \ll \gamma H,$$

then, expanding (4) in a series, we obtain

$$\omega_1 = \omega + \Omega \cos \theta + (\Omega^2/2\omega) \sin^2 \theta, \quad (5)$$

where θ is the angle between the vectors \mathbf{H} and Ω .

Thus, slow rotation of the device in which the precession of the magnetic moments is observed leads to a change of the precession frequency; this change is equal, in first approximation, to the projection of the vector of the angular velocity Ω on the field direction. This property of the system of magnetic moments, namely that the precession frequency changes with rotation of the coordinate system in which they are located, serves as the basis of the operation of nuclear gyroscopes of this type.

At low rotation velocity, the character of the motion of the vector \mathbf{M} , as seen from the foregoing, changes little. On the basis of (4) and (5) we can assume that this vector precesses about the vector \mathbf{H} approximately as if the latter were not to rotate at all. In other words, it can be assumed that as the vector \mathbf{H} moves it drags with it the vector \mathbf{M} completely. This conclusion is well confirmed by exact calculations performed with allowance for the relaxation^[10]. Such a motion of the magnetization vector is called adiabatic. The energy of the system of magnetic moments in the rotating coordinate frame remains practically unchanged in such a motion.

The character of the precession is significantly altered if the field rotation is rapid. To analyze this case, it is convenient to resolve the vector \mathbf{H} into two components, one of which, \mathbf{H}_0 , is parallel to the vector

Ω , and the other, \mathbf{H}_1 , is perpendicular to it. Then expression (4) for the precession frequency takes the form

$$\omega_1 = [(\gamma H_0 \pm \Omega)^2 + (\gamma H_1)^2]^{1/2}. \quad (6)$$

The sign in the first parentheses depends on the relative position of the vectors \mathbf{H} and Ω . If the angle between them is smaller than $\pi/2$, it is necessary to use the upper sign. The motion of the vector \mathbf{M} in the rotating coordinate frame is shown in Fig. 2.

It is easily seen from (6) that if the conditions $\Omega = \gamma H_0$ and $\theta > \pi/2$, called the resonance conditions, are satisfied, then the nutation frequency is minimal and equal to γH_1 . The vector \mathbf{M} will precess in the rotating coordinate frame about the vector \mathbf{H} . There is no nutation if the vector \mathbf{M} is parallel to the vector $\gamma\mathbf{H} + \Omega$. The effective-field vector makes a maximum angle with the vector Ω at resonance.

The relaxation processes, and also the reaction of the tank circuit whose coil delivers the signal, lead to damping of the free precession and by the same token limit the time during which a nuclear gyroscope of the described type can operate. To increase the operating time of the instrument it is necessary to have materials and operating conditions such that the relaxation times are large enough. A gyroscope based on the phenomenon of coherent free precession of the magnetic moments acts only during the relaxation time of the system or somewhat longer, depending on the noise level. In many cases this time can reach several minutes or even hours.

It is possible, however, to maintain the precession by replenishing the relaxation losses from an external source and using feedback. In this case the spin gyroscope can operate for an unlimited time. The frequency pulling due to the presence of feedback in the magnetic-moment system, however, makes its readings not absolute.

It is possible to excite undamped nutation simultaneously with forced precession by modulating the magnetic field. The modulation frequency necessary to excite the nutation is given by (6). The signals obtained in this case can be resolved into a spectrum consisting of a number of sideband frequencies separated by distances equal to the nutation frequency. The frequency of the first component of this spectrum is equal to the nutation frequency.

Nutation can also be excited by modulating the magnetic field with nuclear-resonance signals. In this case we obtain a spin gyroscope that operates as a generator at one of the sideband frequencies of the aforementioned spectrum. Non-steady-state nutation and free precession of the magnetic moments were observed first by Torrey^[11] and Hahn^[12].

In conclusion we note that the system of magnetic moments responds to rotation in the same manner as to an additional magnetic field. Indeed, in an inertial coordinate frame the angular-velocity vector of the precession is equal to $-\gamma\mathbf{H}$, while in a rotating frame it is equal to $-\gamma(\mathbf{H} + \Omega\gamma^{-1})$. Thus, rotation of the reference frame is equivalent to action of a magnetic field Ω/γ on the spin system. In this sense, any magnetometer based on spin precession can be used as a velocity gyroscope. To this end, however, it is necessary to

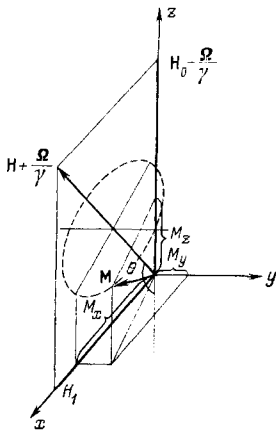


FIG. 2. Precession of the magnetization vector in a rotating coordinate frame.

propose and realize a method of separately measuring the true field H and the field Ω/γ .

This is precisely what is done in the gyroscopes described below, and distinguishes them from magnetometers.

III. NUCLEAR GYROSCOPES

The coherent properties of the system of weakly-interacting magnetic moments, considered above, uncover a possibility of creating quantum gyroscopes. We shall consider below gyroscopes in which the magnetic properties of atomic nuclei are used.

The possibility of creating nuclear gyroscopes with static and dynamic methods of recording their rotation is discussed in^[3].

In a magnetically-isotropic diamagnetic substance - a nuclear paramagnet - characterized by large nuclear-relaxation times, nuclear magnetization can be produced by any one of several known methods. If the nuclei have a gyromagnetic ratio γ , then the magnetization M corresponds to a mechanical moment M/γ per unit volume. Let us imagine that the sample is completely screened against the action of external magnetic fields after the nuclear magnetization has been produced. Then the direction of the mechanical and magnetic moments will remain constant in inertial space. By measuring the angle of inclination of the magnetic moment to the initial direction in a rotating coordinate system it is possible to assess the change of its orientation.

The technical realization of a device based on the foregoing idea entails considerable technical and fundamental difficulties. Exceedingly stringent requirements are imposed in this method on the magnetic screening of the pickup. Even a very weak magnetic field remaining in the screen can rotate the nuclear magnetization vector during the operating time of the instrument, and by the same token distort the measurement results appreciably. For protons $\gamma/2\pi = 4257$ Hz/Oe, and therefore in a field of $H = 10^{-5}$ Oe the precession frequency amounts to ~ 0.04 Hz. In such a field, the magnetization vector makes one revolution in 25 sec. As shown in^[3], to measure the velocity of the earth's daily rotation, 15 deg/hr, it is necessary that the residual field inside the screen be less than 3×10^{-9} Oe.

Relaxation of magnetization, as already noted, greatly limits the operating time and the accuracy of the instrument even at large relaxation times^[13].

The most promising is liquid He^3 , for which the relaxation time is ~ 2 hrs. For a number of organic liquids, it can reach several times ten minutes.

One of the possible ways of solving this problem is consecutive reading and recording the measurements of the orientation of the vector M , with periodic restoration of its magnitude by polarization^[14,15]. One must not forget that the extraction of information concerning the orientation of the magnetic moment entails measurement of the magnetic energy of the magnetized sample. It is clear from the foregoing that nuclear gyroscopes of this type can find application in measurements of relatively large angular velocities, the occurrence of which can be predicted beforehand.

Unlike the considered static measurement method,

in the dynamic method the vector of the nuclear magnetization precesses about the vector of the constant magnetic field, which is fixed in the apparatus. The rotation of the sample together with the field, as we have seen, leads to a change in the precession frequency.

To obtain high accuracy and sensitivity it is also necessary to have an exceptionally high stability and homogeneity of the magnetic field. To observe the earth's daily rotation it is necessary that the relative stability of the magnetic field be not worse than 10^{-9} . It is desirable to have the intensity of the field used to produce the precession as small as possible.

A method was proposed^[3] in which the requirements and the stability of the magnetic field in the dynamic method could be relaxed. It consists of using two samples containing different atomic nuclei and located in the same magnetic field. Let the precession frequencies of the nuclei in the first and second substances be respectively

$$\omega_1 = \gamma_1 H + \Delta\omega, \quad \omega_2 = \gamma_2 H + \Delta\omega, \quad (7)$$

where γ_1 and γ_2 are the gyromagnetic ratios of these nuclei, and $\Delta\omega$ is the change of the precession frequency under the influence of the rotation of the device. Measurement of two precession frequencies in the same magnetic field constitutes two independent physical observations, from which it is possible to determine the two quantities H and $\Delta\omega$. In many cases it is more convenient to use one sample containing both types of nuclei, for example a mixture of two isotopes.

The measurement of $\Delta\omega$ makes it possible to determine the projection of the vector of angular velocity on an axis parallel to the field. By using three pickups in which the field directions are mutually perpendicular it is possible to measure the magnitude of the pickup total velocity vector.

It is difficult to choose substances that are convenient for the described method in all respects. The well investigated isotopes of mercury Hg^{199} and Hg^{200} , and also the isotope Rb^{87} , are quite convenient. A spin generator based on Hg^{199} and Hg^{201} , developed specially for gyroscopic purposes, is described in^[16]. The energy spectrum of the ground state of the Rb^{87} atom consists of two closely-lying hyperfine levels. In a magnetic field, each hyperfine level splits into several Zeeman sublevels, two of which have close resonant frequencies and gyromagnetic ratios of opposite signs^[3]. This makes it possible to use the foregoing method of measuring the speed of rotation.

Together with dynamic gyroscopes, in which nuclear precession is used, a gyroscope based on the method of nuclear induction was also proposed^[17]. The authors propose to place the same in a constant magnetic field H_0 and a parallel alternating magnetic field with amplitude H_1 larger than H_0 and with frequency ω_1 close to the precession frequency γH_0 . Such a field, in the absence of rotation, produces no precession of the nuclear moments. Rotation of the device about an axis perpendicular to the magnetic field is equivalent to the action of a weak transverse magnetic field. As a result, the summary constant field $H_0 + \Omega\gamma^{-1}$ is inclined to the direction of the alternating field whose transverse component produces the precession. The rotation of

the device about an axis perpendicular to the magnetic field produces a nuclear-induction emf in a receiver coil placed along this field.

If we orient the z axis in the direction of the field H and assume that $\gamma(H_0 - H_1) \leq \omega_1 \leq \gamma(H_0 + H_1)$, then the Bloch equations^[7] describing the time behavior of the nuclear magnetization assume, with allowance for the relaxation times, the form

$$\left. \begin{aligned} \dot{\tilde{M}} + [T_2^{-1} + i\Omega_0 f(t)] \tilde{M} &= 0, \\ \dot{M}_z + (M_z - M_0) T_1^{-1} &= 0, \\ f(t) &= 1 - \kappa \cos \omega_1 t, \end{aligned} \right\} \quad (8)$$

where

$$\Omega_0 = \gamma H_0, \quad \Omega_1 = \gamma H_1, \quad \kappa = \Omega_1 / \Omega_0 \gg 1, \quad \tilde{M} = M_x + iM_y,$$

and T_1 and T_2 are the times of longitudinal and transverse relaxation.

Assume that at the instant $t = 0$ we have

$$t=0 \quad \tilde{M}(0) = M_0^0 = [M_z^0(0) + iM_y^0(0)]^{1/2},$$

then, as can be readily seen

$$\left. \begin{aligned} M_0(t) &= M_0^0 e^{-t/T_2}, \\ M_x(t) &= M_0^0 e^{-t/T_2} \cos(\Omega_0 t + \epsilon \sin \omega_1 t), \\ M_y(t) &= M_0^0 e^{-t/T_2} \sin(\Omega_0 t + \epsilon \sin \omega_1 t), \end{aligned} \right\} \quad (9)$$

where $\epsilon = \Omega_1 / \omega_1$.

The oscillations of the transverse components are modulated in frequency and attenuate with time. If $M_0^0 \neq 0$ at $t = 0$, then the transverse field components vanish after the lapse of a time larger than T_2 .

The electromotive force induced in the coil located along the y axis is proportional to dM_y/dt .

The oscillation spectrum can be easily obtained by using the known expansions^[18]

$$\cos(\epsilon \sin \omega_1 t) = J_0(\epsilon) + \sum_{n=1}^{\infty} J_{2n}(\epsilon) \cos(2n\omega_1 t), \quad (10)$$

$$\sin(\epsilon \sin \omega_1 t) = 2 \sum_{n=0}^{\infty} J_{2n+1}(\epsilon) \sin[(2n+1)\omega_1 t],$$

where $J_n(\epsilon)$ is a Bessel function. Calculating the derivative dM_y/dt , using (10), and expanding the resultant expression in a Fourier integral, we obtain the frequency spectrum $E(\omega)$ of the signal emf in the receiving coil.

The power spectrum picked off the coil with an active resistor and proportional to $|E(\omega)|^2$, is represented in the form of a sum of Lorentz lines in the form

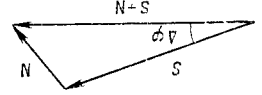
$$\varphi(\omega) = [1 + (\Omega_0 + n\omega_1 - \omega)^2 T_2^2]^{-1}, \quad (11)$$

whose amplitudes are proportional to the squares of the Bessel functions.

The spectrum of the signal emf contains resonant frequencies $\omega = \Omega_0 + n\omega_1$. If there is no constant field then, as can be readily seen from (9), the signal contains only odd harmonics. Even a weak field H_0 leads to the appearance of even harmonics. The amplitudes of the harmonics are determined by the order of the Bessel functions and by the ratio $\epsilon = \Omega_1 / \omega_1$, which can always be made large.

The rotation of the entire device about the x axis with angular velocity Ω is equivalent to the action of a magnetic field of intensity Ω/γ . It leads to the oc-

FIG. 3. Vector diagram showing the change of the phase of the magnetic-moment precession under the influence of noise.



currence of precession of the magnetic moments around the field and to the appearance of even harmonics of the signal emf. The rotation velocity is determined from the amplitude of the second harmonic of the emf induced in the coil. The instrument requires good screening and careful adjustment of the field-producing coils.

On the basis of the data of^[3], let us consider the question of the accuracy of a nuclear gyroscope. The limiting accuracy of such a device depends mainly on two factors: the radiofrequency noise and the magnetic noise. Both factors cause fluctuations of the phase of the nuclear precession. The fluctuations due to noise at the precession frequency is estimated in^[3] at

$$\Delta\varphi \leq NS^{-1}(t/T)^{1/2}, \quad (12)$$

where NS^{-1} is the signal to noise ratio, t is the observation time, and T is the relaxation time. Figure 3 shows a vector diagram illustrating the change of the precession phase under the influence of the noise. At relaxation times on the order of 1 sec, an observation time of 1 hr, and a signal/noise ratio on the order of 10^4 , the phase fluctuation due to the radiofrequency noise turns out to be 6×10^{-3} rad.

The magnetic noise is due to spontaneous magnetization occurring in the absence of a magnetic field. The phase fluctuations due to the magnetization fluctuations, as indicated^[3], can be estimated from the formula

$$\Delta\varphi \leq 1/n^{1/2}\sigma, \quad (13)$$

where n is the number of nuclei present in the sample and σ is the degree of orientation of the nuclear moments. If we assume $n = 10^{23}$ and $\sigma \approx 10^{-9}$ for protons of water, and $n \approx 10^{12}$ and $\sigma = 0.1$ in the case of optical pumping, then we find that the instability of the gyroscope in the latter case amounts to 10^{-3} deg/sec, and for a gyroscope using water protons it amounts to 1 deg/hr.

From 1955 through 1965, eight patents were issued in the USA for nuclear gyroscopes. Let us consider some of them. The first patent^[19] was issued for a gyroscope containing two bridge pickups for the NMR signals, amplifiers, high- and low-frequency generators (the latter to modulate the magnetic field), and a servomotor (Fig. 4). Both pickups are in magnetic fields of equal magnitude but opposite direction. The modulation amplitude is chosen to be smaller than the line width. Therefore, when the pickup is not rotating and is tuned to resonance, the modulation is symmetrical with respect to the center of the line, and there are no signals at the outputs of the two amplifiers. If the change of the magnetic field is the same in both pickups, signals of identical phase appear at the outputs of the amplifiers, and these do not cause the servomotor to rotate.

If the device rotates, then the resonant frequency increases in one pickup and decreases in the other. The resonance conditions are violated and the modula-

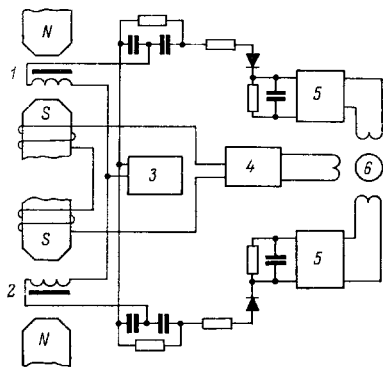


FIG. 4. Diagram of nuclear gyroscope: 1 and 2—nuclear-resonance-signal pickup coils containing the samples; 3—high-frequency generator; 4—low-frequency amplifier; 5—servomotor amplifier; 6—servomotor.

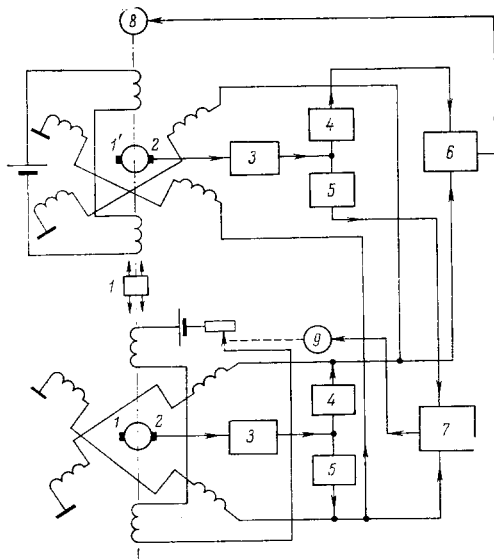


FIG. 5. Diagram of nuclear gyroscope in which optical pumping is used: 1—light source for nuclear polarization; 2—photodetectors; 3—amplifiers; 4, 5—band filters; 6, 7—frequency-comparison circuits; 8, 9—servomotors.

tion becomes asymmetrical. This causes the output signals of the amplifiers to differ in phase. The servomotor will consequently rotate. Rotation of the servomotor will be continuous until the device itself rotates. There is no indication in the literature that such a gyroscope was actually realized. This is apparently due to the fact that it is very difficult to produce two fields of exactly the same intensity and to maintain this equality of the fields for a sufficiently long time.

Let us examine another patent^[20], which is known to have been realized^[16]. The gyroscope has two pickups containing mercury vapor enriched with the isotopes Hg^{199} and Hg^{201} (Fig. 5). In each pickup there are three coils. One serves to produce a magnetic field and the other two are used to excite precession, one for Hg^{199} and the other for Hg^{201} . Optical pumping is used in the gyroscope^[23]. The pickups are illuminated by a light beam parallel to the constant field. This light produces nuclear polarization. In addition, each pickup is illuminated by an additional light source, 1 and 1', the beam of which is directed perpendicular to the magnetic

field. The latter light serves to register the precession. After passing through the substance, it strikes the photodetectors 2. The photodetectors generate voltages that vary periodically with the nuclear precession frequency. The lower pickup in the diagram operates on the spin-generator principle. Its signals, after amplification and filtering, are fed to the precession-excitation coils. The upper pickup in the diagram is fed with signals from the lower pickup.

The signals of both pickups are fed to two frequency-comparison circuits 6 and 7, the operation of which is based on measurement of a small phase difference. If the magnetic fields of both pickups do not coincide, then the precession frequencies likewise do not coincide. The comparison circuit 7 then produces an error signal and motor 9, with the aid of a rheostat, changes the current in the coil of the lower pickup in such a way that the fields become equalized. This control system has a small time constant and operates quickly.

If the device rotates, then the system for regularizing the magnetic field cannot compare the precession frequencies of the nuclei of both isotopes, since these frequencies change by different amounts. The magnetic field changes in this case in such a way, that if, for example, the precession frequencies of the Hg^{199} isotope becomes the same in both signals, then the precession frequencies of Hg^{201} in both pickups are different. This causes the frequency comparison circuit 6 to operate (it has a time constant larger than circuit 7) and to produce an error signal that causes motor 8 to operate. This motor can rotate the platform on which the device is mounted. Rotation of the device relative to the base causes the former to become immobile in the inertial space, and the precession frequencies of the Hg^{201} isotope become equal in both pickups. The total device consists of three identical gyroscopes that orient the system relative to three mutually perpendicular axes. There is a published description of a laboratory model of a gyroscope operating on the aforementioned principle. There are indications that the model will serve as a basis for the development of an instrument proposed for use for navigation purposes^[21].

Other devices of similar type are also described in foreign patent literature, but there are no indications that models of these devices have successfully passed laboratory tests. We shall therefore not consider them here.

In the nuclear gyroscopes described above, the constant magnetic field H_0 , which determines the precession frequency, was produced in the device and was connected with it. This led to the need for thoroughly screening the instrument against the action of extraneous magnetic fields, or for greatly reducing their action on the instrument, using two types of nuclei in the pickup.

If we imagine that the picture of the distribution of the external magnetic (geomagnetic) field in space is known, and disregard random changes of its magnitude and direction in time, then the devices described above without screening and without parts that produce this field, can be used for orientation in space relative to the external field.

The angular dependence of the signal amplitude on the angle β between the field and the magnetization

vector is proportional to $\cos^2 \vartheta$. Magnetic orienting devices of this type are of interest not so much for navigation as for the study of the topography of magnetic fields and for the observation of their anomalies. It is possible to separate with their aid, and to measure individually, changes of the magnetic field both in magnitude and in direction. We shall describe below one such magnetic direction finder.

IV. ELECTRONIC GYROSCOPES AND ORIENTERS

Applications of electron precession for the purpose of quantum magnetometry and gyroscopy are at first glance made difficult by the short transverse relaxation times T_2 of the electron spins, or the large width $\Delta\nu_e = 1/T_2$ of the magnetic-resonance lines. This circumstance, however, is not decisive. The gyromagnetic ratio for the electron $\gamma_e/2\pi = 2.8 \times 10^6$ Hz/Oe is 658 times larger than for the proton. The electron-spin precession frequency in the same field is accordingly higher. This circumstance is very important for applications. There are known substances, for example, certain stable free radicals, the resonance-absorption line widths of which are of the order of 1 Oe or less^[8]. The lines of hyperfine structure resolved in a weak field are even narrower.

An essential factor on which the measurement accuracy depends is also the signal to noise ratio. The measurement accuracy can be greatly increased by increasing this ratio. It is therefore of interest to compare magnetic-field measurement methods based on the use of electron precession.

A line width of 1 Oe corresponds, for proton resonance, to a relaxation time 0.2×10^{-3} sec, whereas for electron resonance, at the same line width, the relaxation time is smaller by almost three orders of magnitude and amounts to 0.4×10^{-6} sec. Furthermore, the signal to noise ratio in the case of registration of electron resonance is much larger than in the case of nuclear resonance. We can therefore conclude that electron resonance makes it possible to register faster variations of the field than nuclear resonance. It goes without saying that the use of electron resonance in weak fields excludes the application of the free-precession method, since the number of periods of free

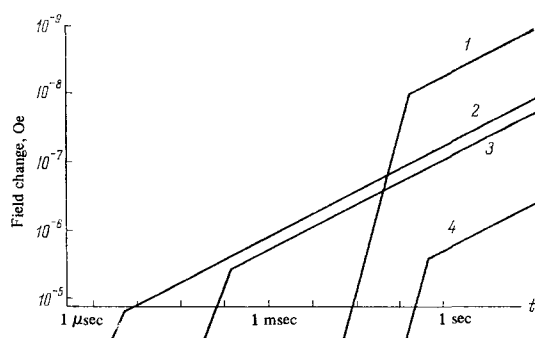


FIG. 6. Dependence of the minimum measurable field changes on the measurement time at a signal to noise ratio equal to unity, for different types of magnetometers. 1—Magnetometer using free nuclear precession; 2—magnetometer using electron precession in DPPH; 3—magnetometer using rubidium vapor; 4—precession magnetometer of the Varian Company.

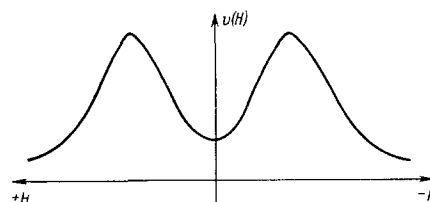


FIG. 7. Dependence of the absorption-signal curve on the magnetic field intensity in the case of a fine-alternating field.

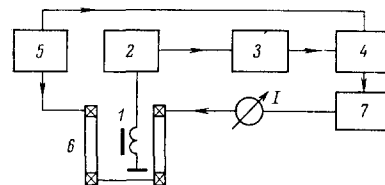


FIG. 8. Diagram of magnetometer in which electron precession is used. 1—Coil with sample; 2—weak-oscillation generator; 3—amplifier; 4—synchronous detector; 5—audio-frequency generator; 6—modulation coils; 7—dc source.

precession within the relaxation time is small. However, there is a method of synchronous detection under stimulated precession, which makes it possible to record the absorption and the dispersion curves in a rather large scale. This makes it possible to measure the magnetic field by electron precession with high accuracy. Contributing to the high accuracy is also the large signal to noise ratio possessed by electron precession.

To register rapidly varying weak magnetic fields, an original method was proposed in^[22] and permits rapid measurement of a weak magnetic field at an accuracy on the order of 10^{-5} of the electron-resonance line width. Its advantage is illustrated in the cited paper by the diagram of Fig. 6, which shows a plot of the changes of the magnetic field, registered by different methods, against the registration time at unity signal to noise ratio. It follows from this diagram that the method of free nuclear precession is the worst from the point of view of speed of measurement, since it does not make it possible to register magnetic field changes larger than 10^{-7} Oe within a time shorter than 1 sec. The use of electron precession with the stable organic free radical diphenylpicrylhydrazyl (DPPH) as the specimen has made it possible to register magnetic-field changes reaching 10^{-5} Oe within a time on the order of $2 \mu\text{sec}$.

Attention is called in^[22] to the fact that the magnetic-resonance absorption line is an even function of the field, i.e., the sign of the absorption curve does not change when the sign of the magnetic field changes. The dependence of the absorption in electron resonance on a sign-alternating magnetic field is shown in Fig. 7. If the sample producing the EPR signal is placed not in a constant magnetic field but in a purely alternating (sinusoidal) magnetic field, then in the case of total absence of a dc component the EPR signals obtained at the output of the pickup will contain only even harmonics of the fundamental frequency Ω of the alternating field, and the component with frequency Ω will be absent from the signal. In the presence of a weak ex-

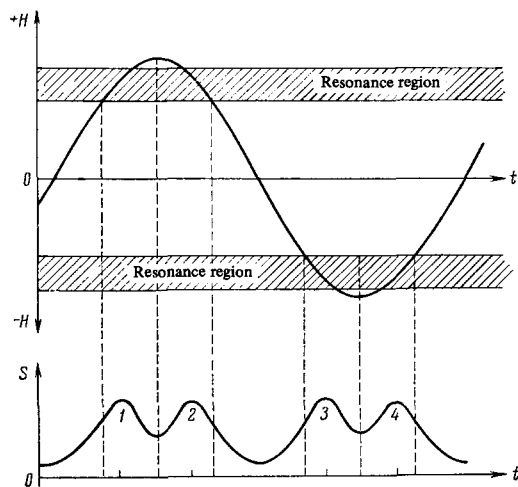


FIG. 9. Occurrence of EPR signals in an alternating magnetic field in the absence of a dc component.

ternal field (i.e., the earth's magnetic field) the symmetry of the absorption curve, shown in Fig. 7, is disturbed and the signal will contain a component of frequency Ω . The magnitude of this component will be approximately proportional to the intensity of the constant field. If the external field (relative to the pickup field) changes in time without reversing sign, then it can be registered by recording the amplitude of the EPR-signal harmonic of frequency Ω . For measurements with higher accuracy a compensation method, described below, is used.

A diagram of an experimental setup based on the use of the above-described property of the absorption curve are shown in Fig. 8. The pickup coil 1 is connected in the circuit of a weak-oscillation generator (autodyne) 2. The generation frequency amounts to 10 MHz. The amplifier 3, tuned to a frequency 4 kHz, amplifies the periodic changes of the generation amplitude. These changes, which result from resonant absorption in the sample, are obtained by applying to the sample an alternating field of amplitude 2 Oe and frequency 4 kHz. The absorption line width for DPPH is 0.5 Oe. In the absence of an external magnetic field, the amplitude of the autodyne generator changes with a frequency 8 kHz and is not registered by the amplifier tuned to the 4 kHz frequency. The 8 kHz frequency is obtained as the result of the fact that the resonance occurs four times during the oscillation period. This is shown in Fig. 9. It follows from the figure that the distance between signals 1-3 and 2-4 does not change when the generator frequency changes. In the presence of an external field, the resonance curve (see Fig. 7) becomes asymmetrical, the distances between signals 1-3 and 2-4 (see Fig. 9) change, odd harmonics appear in the signal spectrum, and the amplitude of the autodyne generator contains a component having a frequency 4 kHz. After amplification, the signal with frequency 4 kHz is applied to synchronous detector 4, to which there is applied simultaneously a signal from generator 5 which produces an alternating field of frequency 4 kHz in the coils of the instrument 6.

A voltage proportional to the intensity of the external magnetic field appears at the output of the syn-

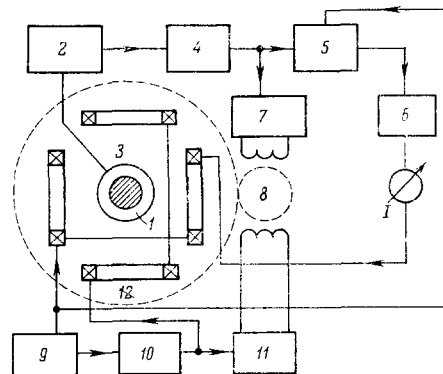


FIG. 10. Diagram of a direction finder (magnetic theodolite) that orients itself along the horizontal component of the magnetic field of the earth: 1—sample; 2—weak-oscillation generator; 3—generator coil; 4—amplifier; 5—synchronous detector; 6—dc source; 7—servomotor amplifier; 8—servomotor; 9—audiofrequency generator; 10—phase shifter; 11—power amplifier; 12—y-coils; 13—x-coils.

chronous detector. The intensity of the external field is measured by a null method. In this method, the dc voltage obtained at the output of the synchronous detector is amplified (7) and is fed to the same coil that produces the alternating magnetic field at the location of the sample. The sign of the field produced by the voltage of the output of the synchronous detector is opposite to the sign of the external terrestrial magnetic field. With the aid of such feedback, the dc component of the magnetic field in the region of the sample is reduced to zero. The current I that must flow through the coil to produce the compensation is proportional, to a high degree of accuracy, to the external magnetic field intensity.

It should be noted that changes in the frequency of the autodyne generator have practically no effect on the measurement accuracy, for in this case the absorption curve still remains even with respect to the magnetic field and no component of the 4-kHz signal appears. This variant of the apparatus makes it possible to measure one component of the weak magnetic field, namely the one perpendicular to the plane of the coils producing the alternating magnetic field.

The described measurement method can be used also to produce a magnetic theodolite or direction finder. To this, two pairs of coils are used (Fig. 10) producing an alternating magnetic field and make it possible to orient the instrument in the direction of the horizontal magnetic-field component. The voltage applied to one pair of coils (x-coils) 13, is shifted in phase by $\pi/2$ relative to the voltage applied to the other pair of coils (y-coils). Thus, the DPPH sample is in a rotating magnetic field. The EPR signal picked off the pickup, in the presence of an external magnetic field, will have a frequency 4 kHz and will be shifted in phase relative to the reference signal.

If the external magnetic field is directed along the x axis, then the phase of the EPR signal coincides with the phase of the oscillations in the x-coils. On the other hand, if the external magnetic field is directed along the y axis, then the phase of the EPR signal will coincide with the phase of the oscillations in the y-coils. In the intermediate case, there will be a certain phase

shift, determined by the ratio of the components of the external field along the x and y axes. This is equivalent to the presence in the EPR signal of two components shifted relative to each other in phase by $\pi/2$.

The EPR signal is fed after amplification to synchronous detector 5, and also to servomotor 8. The reference voltage for the synchronous detector is the signal for the x-coils. The output of the synchronous detector is a voltage proportional to the magnetic-field component directed along the x axis. The presence of a signal due to the y-component of the external magnetic field does not affect the synchronous detector output voltage, since this signal is shifted in phase by $\pi/2$ relative to the reference signal, and in the case of synchronous detection it produces a voltage equal to zero. The signal obtained at the output of the synchronous detector controls the compensating field in a manner similar to that used in the previously described device. This part of the instrument constitutes a magnetometer.

The EPR signal from the amplifier output is fed, in addition, to power amplifier 7, which supplies the servomotor. The reference voltage for the servomotor is the voltage feeding the y-coils. Therefore the servomotor responds only to that EPR component which is due to the external-field component directed along the y axis. The signal component due to the magnetic field directed along the x axis does not act on the servomotor, since it is shifted in phase by $\pi/2$ relative to the reference voltage applied to the latter. The servomotor will rotate the device until the signal component determined by the external field directed along the y axis becomes equal to zero.

Such a device, rotating about the vertical axis, becomes oriented along the magnetic field just like a magnetic compass, and measures the horizontal component of the earth's magnetic field. The instrument described above makes it possible to measure magnetic inclination with accuracy $\pm 30''$ of arc and the magnetic field intensity with accuracy up to 5γ .

In conclusion it should be noted that the foregoing measurement method has an important feature, namely the sensitivity of the apparatus is not decreased when the measured field tends to zero. This advantage of the method is not restricted to devices based on electron precession.

V. GYROSCOPES AND DIRECTION FINDERS WITH OPTICAL PUMPING

The advantages of nuclear precession and electronic gyroscopes and direction finder may be combined by using as the working medium a gas or a metal vapor in which the magnetic moments of the atoms or atomic nuclei are polarized with light^[23]. The width of the electronic-transition lines between magnetic sublevels of atoms can be made very small if certain conditions are satisfied. The frequencies of the magnetic transitions are then determined by the gyromagnetic ratios of the orbital electrons, which, as already indicated, are larger by hundreds of times than the gyromagnetic ratios for the nuclei.

Optical pumping of atoms can be used for orientation in space along the lines of an external magnetic

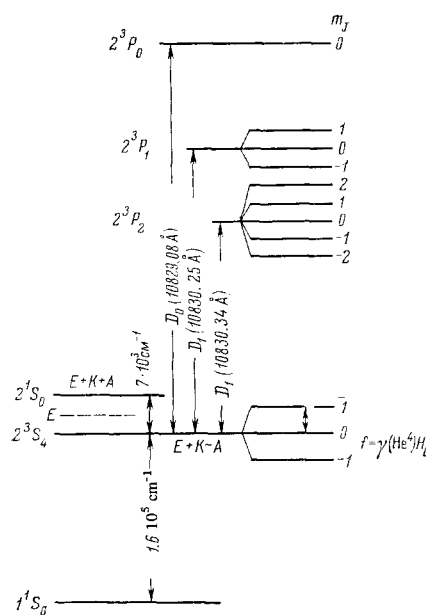


FIG. 11. Energy level scheme of the helium atom.

field. In this case the optical-pumping method has great advantages over other methods. A gyroscope using the polarization of nuclear moments with the aid of light was already described in Chap. III.

The dependence of the intensity of a beam of light passing through a gas-filled cell (pickup) on the angle between its axis and the external field can be made in many cases much more sensitive to the angle than in the case of magnetic resonance. One of such devices is the helium direction finder described in^[24].

The energy spectrum of ortho-helium is shown in Fig. 11. Its ground state is metastable and is excited by gas discharge. When illuminated with unpolarized light, the helium atoms go over to the excited state and stay in this state on the average for 10^{-8} sec. After this time, they return to the ground state. The ground level is split in a magnetic field into three sublevels, and the probability of absorption of light by atoms at the sub-

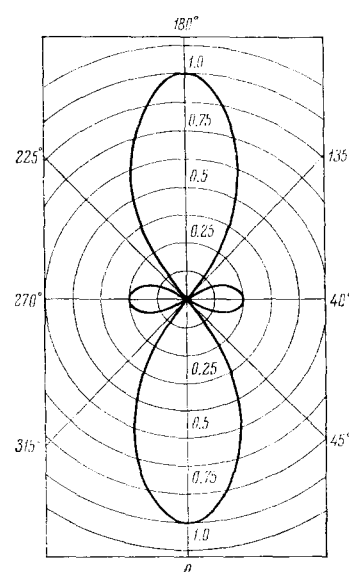


FIG. 12. Dependence of the relative intensity of the light signal on the angle between the direction of the beam producing the pumping and the direction of the constant magnetic field.

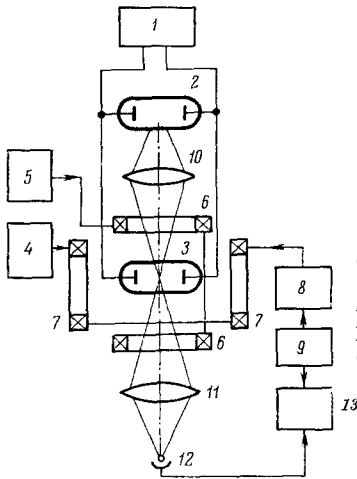


FIG. 13. Model of a device for verifying the possibility of constructing an orienter based on optical pumping. 1—Generator; 2—helium lamp; 3—absorbing chamber; 4, 5—dc sources; 6, 7—coils producing magnetic field; 8—square wave generator; 9—audio-frequency generator; 10, 11—lenses; 12—photodetector; 13—synchronous detector.

level with $m = 0$ is larger than the probabilities of absorption of light by the atoms at the sublevels with $m = 1$ and $m = -1$. Therefore illumination of the sample with light having $\lambda = 10829 \text{ \AA}$ leads to depletion of the $m = 0$ level, since the probability of returning from the excited levels to all three sublevels of the ground state is the same. The intensity of the light passing through the sample then increases.

Such a situation, however, takes place only if the axis of the light beam producing the pumping is parallel to the magnetic field intensity vector. If it makes a certain non-zero angle φ with the field, then the probability of transitions from the sublevels with $m = \pm 1$ to the excited level approaches the probability of the transition from the sublevel with $m = 0$, and at an angle $\varphi_0 = 54^\circ 43'$ the probabilities of the transitions from all the sublevels become equal. The dependence of the light intensity I on the angle φ is determined by the formula

$$I = I_0 (3 \cos^2 \varphi - 1)/2. \quad (14)$$

This dependence is shown in Fig. 12. The diagram of a setup for verifying the possibility of orientation on the basis of the described principle is shown in Fig. 13.

The entire installation is placed in a magnetic screen (not shown in the figure). The generator 1 (frequency 30 MHz) feeds a helium lamp 2 and an absorbing chamber 3, in which ortho-helium in the metastable state is produced. The dc sources 4 and 5 feed the coils 6 and 7, which imitate the earth's field. With the aid of a square-wave generator 8, fed from audio-generator 9, the magnetic field is modulated at the frequency $\Omega/2\pi$, and the field vector rotates through equal angles in both directions relative to the direction of the light beam.

We denote the component of the external constant magnetic field H_0 in the direction of light propagation by H_z , and the field perpendicular to it by $H_x + \epsilon H_1$, where H_1 is the field produced by the square-wave generator. Then the angle φ between the direction of the magnetic field

$$\{H_z^2 + [H_x + \epsilon(t) H_1]^2\}^{1/2}$$

and the z axis is determined by the expression

$$\cos^2 \varphi = H_z^2 / \{H_z^2 + [H_x + \epsilon(t) H_1]^2\}, \quad (15)$$

where $\epsilon(t)$ is the unit sign-reversing step function

with period $2\pi/\Omega$. This angle changes jumpwise from a value corresponding to $\epsilon(t) = -1$ to a value corresponding to $\epsilon(t) = +1$.

The signal $S(t) = I/I_0$ is equal to

$$S(t) = \{[2 - (h_x + \epsilon h_1)^2] / [1 + (h_x + \epsilon h_1)^2]\}^2 / 4, \quad (16)$$

where $h_x = H_x/H_z$ and $h_1 = H_1/H_z$. The spectrum of this signal is

$$S(t) = \sum_{n=-\infty}^{+\infty} S_n e^{in\Omega t}, \quad S_n = T^{-1} \int_0^T S(t) e^{-in\Omega t} dt. \quad (17)$$

Calculating the integrals

$$S_n = S_+ T^{-1} \int_0^{T/2} e^{-in\Omega t} dt + S_- T^{-1} \int_0^{T/2} e^{-in\Omega t} dt,$$

where

$$S_{\pm} = \{[2 - (h_x \pm h_1)^2] / [1 + (h_x \pm h_1)^2]\}^2 / 4, \quad (18)$$

we obtain

$$S_0 = (S_+ + S_-)/2, \quad S_n = i(S_- - S_+)/\pi n. \quad (19)$$

We note that $S_0^* = S_0$ and $S_n^* = -S_n$.

If $h_x = 0$, then the inclination of the vector of the total magnetic field will be symmetrical relative to the direction of light propagation, $S_+ = S_-$, and $S_n = 0$. If the detector is connected to the input of a narrow-band amplifier tuned to the frequency $n\Omega$, the latter will register a signal with amplitude S_n that decreases with increasing n .

The largest amplitude is possessed by the first harmonic $n = \pm 1$, for which

$$S_1 = 2(S_+ - S_-)(\sin \Omega t)/\pi, \quad (20)$$

where

$$S_+ - S_- = -6h_x h_1 (1 + h_x h_1) [1 + 2(h_x^2 + h_1^2) + (h_x^2 - h_1^2)^2]^{-1}. \quad (21)$$

In the absence of modulation ($h_1 = 0$), and also when $h_x = 0$, the signal $S_1(t) = 0$. The optimum of the signal takes place at $h_1 = h_x$. Using (19), we can easily calculate signals from the second and higher harmonics.

Thus, if the vector of the constant magnetic field is parallel to the light beam passing through the sample 3 from lamp 2 (this light is focused by lenses 10 and 11 on the detector 12), then the detector 12 does not register the change of the light (the curve in Fig. 12 is symmetrical relative to the light propagation direction). If the vector of the constant magnetic field is inclined to the pump-beam direction by not more than $54^\circ 43'$, then the oscillations of the summary vector become asymmetrical and the detector 12 separates the ac components of the signal, the frequency of which is equal to the modulation frequency of which is equal to the modulation frequency (66–70 Hz). The phase of the signal from detector 12 can be used to determine the direction of the field-rotation vector, by comparing it, with the aid of the synchronous detector 13, with the phase of the reference voltage from generator 9. The signals can also be observed with the aid of an oscilloscope or can be used to rotate the system with the aid of servomotors (not shown in the figure) until the direction of the vector of the constant field coincides with the direction of the light beam. In this case the signal from detector 12 serves as an error signal that controls the mechanism that orients the apparatus.

The use of helium as a working medium makes it unnecessary to control the pickup thermostatically. The sensitivity of the instrument to the field is very

high. The described system makes it possible to register a field component smaller than 10^{-7} Oe in the direction of the instrument axis; this makes it possible to orient the instrument along the earth's field and to measure the field at distances up to 10^5 km from the earth. The modulation method employed in this system is of great interest and can be used successfully in devices for other purposes.

In^[44] there was proposed a quantum gyroscope system operating in the cyclic regime. In this scheme, the atoms are first optically oriented by turning on a polarizing field and illuminating the sample by the pump lamp. Then the magnetic field and the pump lamp are turned off. The magnetization vector remains stationary relative to the inertial space. If the device is rotated through a certain angle, then the directions of the field and of the apparatus no longer coincide when the magnetic field is turned on. Thus, the operation of the instrument consists of three cycles. During the time of the third cycle, at the start of which the magnetic field is rapidly turned off, the precession signal is detected by illuminating the sample with light from another lamp, the light of which, after passing through the sample, falls on the photodetector. The detecting device action registers the signal produced by precession of the magnetization around the field vector.

The initial amplitude of the precession signal is proportional to the angle of rotation of the magnetization vector, since it is proportional to the magnetization-vector component perpendicular to the magnetic-field vector.

Owing to relaxation, the free precession attenuates and the magnetization vector, decreasing in magnitude, becomes parallel to the magnetic-field intensity vector. It is therefore necessary to illuminate the sample again by light from the polarizing lamp, i.e., to repeat the first cycle. Periodic repetition of all three cycles ensures continuous operation of the device.

A shortcoming of the described gyroscope is that during the time of the first and third cycles the information concerning the rotation of the device is lost. This shortcoming can be eliminated by using a second gyroscope, which operates when the first produces no information. In addition, the device calls for very careful screening, since during the time of the second cycle even the least field to which the apparatus may be subjected produces additional rotation of the vector and results in a false signal.

The literature on gyroscopes and direction finders in which optical pumping is used is quite scanty. Several communications appeared reporting instruments of this type^[25,44], but in most cases there are no indications that they were successfully tested in the laboratory.

VI. PHOTON GYROSCOPES

In the quantum gyroscopes considered above, the elementary carriers of the angular momentum were atomic nuclei, electrons, or atoms. Because the mechanical moments of these carriers are connected with their magnetic moments, it has become possible to produce with the aid of a magnetic field coherent states of the spins of macroscopic assemblies of these

carriers, and to observe their motion with the aid of radio devices. Rotation of the laboratory coordinate frame in which the entire device as a unit is located influences the variation of the frequency of the emf induced in the receiving coil by the precessing magnetic moments. A sufficiently accurate measurement of the precession signal frequency makes it possible to measure the angular velocity of rotation of the system.

The main shortcoming of instruments of this kind is that they are subject to the influence of different parasitic magnetic fields, which makes it necessary to screen the pickups very carefully to ensure high sensitivity. This imposes very stringent requirements on the relative stability and homogeneity of the controlling magnetic fields, and satisfaction of these requirements calls for great engineering sophistication. This naturally suggests the idea that the working medium of the pickup be an aggregate of such carriers of mechanical moment which have no magnetic moments. Such carriers can be, for example, coherent beams of photons, phonons, or all particles that have no magnetic moments. The macroscopic angular momentum produced by them can be made up either of the spins of these particles or of their orbital angular momenta. Great interest attaches to the use of photons. The appearance of lasers has made it possible to produce optical gyroscopes and by the same token uncovered new ways of development of quantum gyroscopy.

The simplest optical quantum gyroscope is a ring laser partly or completely filled with an active medium.

In the closed loop of a ring resonator made up of mirrors, there are generated two waves propagating through the common optical channel in the form of narrow light beams directed oppositely to each other (Fig. 14 and 15).

A small fraction of the energy of these light beams passes through one of the mirrors and is combined

FIG. 14. Initial construction of ring laser. 1—Flat mirrors; 2—mirror with curvature; 3—semitransparent mirror; 4—photodetector; 5—gas-discharge tubes.

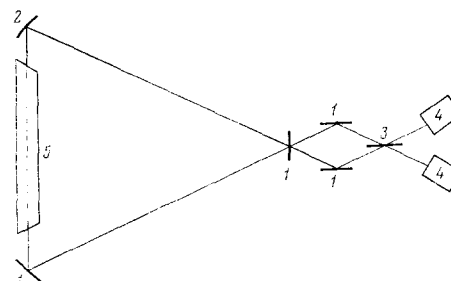
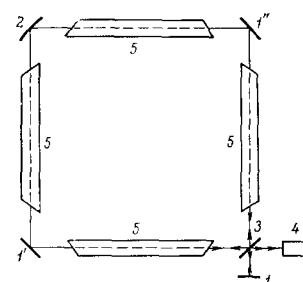


FIG. 15. Improved ring laser which makes it possible to determine the rotation direction. 1—Flat mirrors; 2—mirror with curvature; 3—semitransparent mirror; 4—photodetector; 5—gas-discharge tube.

("downed") with the aid of additional mirrors or a prism into a common beam, which falls on a quadratic detector 4. If the optical frequencies of the opposing waves $\omega_+ = \omega + \Delta\omega$ and $\omega_- = \omega - \Delta\omega$ differ somewhat, with $\Delta\omega \ll \omega$, then the current at the output of the photodetector is

$$I = I_0 \langle (\sin \omega_+ t + \sin \omega_- t)^2 \rangle \quad (22)$$

and varies with a frequency $\Delta\omega = 2\pi f$.

When the ring gyroscope rotates with angular velocity Ω , the energy E_0 of the opposing beams changes:

$$E = E_0 - L\Omega, \quad (23)$$

where L is the orbital angular momentum of the beam, and accordingly their frequency will be

$$\omega_{\pm} = \omega \mp (L_Z \Omega / \hbar). \quad (24)$$

If the direction perpendicular to the plane of the instrument is taken to be the z axis and if we put $L_Z / \hbar = m$, then, according to (24),

$$f = 2mF, \quad (25)$$

where $F = \Omega / 2\pi$. The quantum number m depends on the geometry of the instrument and on the length of the generated wave.

For a helium-neon ring laser with an average contour radius 50 cm we have $m \sim 10^7$. Therefore photon gyroscopes have a tremendous sensitivity to rotation.

The unique possibilities of such an instrument were theoretically predicted in^[26] on the basis of general relativity theory. The first successful experiments were performed by Macek and Davis^[27,28]. The theory and operating principles of a ring laser is the subject of an extensive literature^[29-32].

The laser or photon gyroscope is one of the few quantum-electronic instruments whose operation requires a high degree of coherence of the generated light.

After the first experiments have revealed the amazing properties of gyroscopes of this type and confirmed with high accuracy the relation (25), a large number of articles were published, mostly short communications, containing information concerning various improvements of the instrument^[33-41]. By now, instruments have been developed, making it possible to measure not only the magnitude of the component Ω_z of the angular velocity vector Ω , but also its sign^[42-43] (Figs. 15 and 16).

It is not our purpose to present here the theory of the ring laser, for which we refer the reader to the already cited reviews. We shall consider only certain physical features of the instrument and its operating principle. The operating principle of the gyroscope can be understood both from the classical and from the quantum points of view.

The time required by the light to cover a closed loop depends, according to general relativity theory, not only on the perimeter P of the loop, but also on whether the loop is immobile or whether it rotates with a certain angular velocity Ω . This time τ is equal to

$$\tau = \tau_0 [1 + (2\Phi/cP)], \quad (26)$$

where τ_0 is the time required to cover the immobile

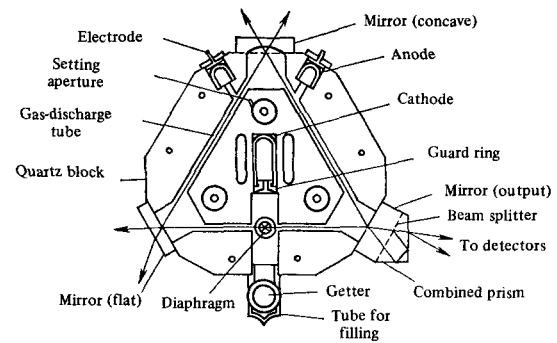


FIG. 16. Vibration-resistant ring laser.

loop, and $\Phi = \int \Omega dF$ is the flux of the angular-velocity vector through the surface F of the loop.

This conclusion does not depend on whether the path is covered by particle displacement or by wave displacement. The path time depends on the direction of motion in the loop. If two coherent light beams are made to travel in the same loop in opposite directions, then a path difference is produced between them, amounting to

$$c\tau = 4\tau_0\Phi/P. \quad (27)$$

This phenomenon, connected with the change of the readings of a clock in a rotating coordinate system, is well known. It was observed experimentally by Sagnac^[45,46], who detected the displacement of the interference fringes of a rotating ring interferometer. It is described in detail in^[47,48]. Formula (26) given above turns out to be a first approximation in the expansion of the exact expression in powers of $2\Phi/cP_0$. As a first-order effect, it can be interpreted within the framework of nonrelativistic kinematics*.

Devices based on the Sagnac effect and making it possible to register rotation have been known for long time^[46,49], but their sensitivity was low. The rotation of the instrument was revealed by the shift of interference fringes. To obtain measurable shifts it was necessary to construct interferometers with a very large contour area.

The appearance of lasers has uncovered the possibility of placing the active element inside the ring resonator, and by the same token alter its properties considerably. The main difference between a ring resonator and the previously employed ring interferometers is that when the ring resonator rotates its natural frequency changes, and consequently also the frequency of the oscillations generated by it. The measured quantity, as we have already seen, is the frequency beats between highly monochromatic beams, and not the displacement of interference fringes. This leads to an exceptional increase in the sensitivity of the instrument and makes it possible to decrease its dimensions appreciably.

A description of a ring resonator can be found in^[26]. In^[50] there is developed a theory of a generalized ring resonator containing focusing elements; a triangular resonator is considered in^[51], and the possibility of

*We note that $2\Phi/P = \Omega \langle R \rangle$ ($\langle R \rangle$ is the average radius of the loop, i.e., this ratio is equal to the average linear velocity of rotation of the loop).

developing a ring laser operating in the microwave band is discussed in^[29], where a bibliography of papers on this question is given.

Let us consider the elementary electrodynamic theory of a ring-laser gyroscope. A narrow monochromatic light beam with cross section area Δf , traveling around a closed contour, has an orbital angular momentum

$$\mathbf{L} = c^{-2} \int [\mathbf{RS}] dv = Nc^{-2} \oint [\mathbf{R} d\mathbf{l}], \quad (28)$$

where \mathbf{S} is the Poynting vector and $N = S_a \Delta f$ is the beam power.

The integral in the right-hand side of (28) is double the loop-area vector \mathbf{F} , therefore

$$\mathbf{L} = 2N\mathbf{F}/c^2. \quad (29)$$

If the resonator rotates with angular velocity Ω about a certain direction Ω/Ω , then the energy U of the field of the radiation dragged in the resonator changes in accordance with

$$U = U_0 - \mathbf{L} \cdot \Omega. \quad (30)$$

If the beam is circularly polarized, then, in accordance with the well known theory of electrodynamics, it carries a spin momentum^[52]

$$L_s = U/\omega = (N/\omega_0) P/c, \quad (31)$$

where ω is the frequency of the optical oscillations and P/c is the time that light takes to cover the loop. Now, according to (29)–(31), we obtain for the frequency of the light generated in the rotating ring laser

$$\omega = U/L_s = \omega_0 [1 - (2\mathbf{F}\Omega/cP)]. \quad (32)$$

Therefore the frequency difference $\Delta\omega = 2\pi f$ between the beams is, in accord with (25), given by

$$f = 2mF = 2(4\pi/\lambda_0)(\Omega F/P) \cos \theta, \quad (33)$$

where λ_0 is the wavelength of the radiation of the generator at rest, and θ is the angle between \mathbf{F} and Ω . If the beam is linearly polarized, then the same result can be obtained by expanding it into a sum of two waves polarized in opposite directions.

Formula (33) can be derived by means of an elementary quantum-mechanical calculation that illustrates clearly its mechanical origin and admits of simple generalizations.

The spectral line width of the laser radiation $1/\tau_c$ is of the order of 10^3 Hz, i.e., the coherence length of the radiation $l_c = c\tau_c$ is of the order of 3×10^6 cm. Therefore, at a perimeter $P = 3 \times 10^2$ cm the light quantum executes on the average $\sim 10^4$ circuits along the loop prior to becoming absorbed or prior to leaving the resonator. This means that it is quite reasonable to introduce the concept of the orbital angular momentum of each individual light quantum.

For further calculations, it is convenient to introduce the effective radius $\langle R \rangle$ of the contour:

$$\langle R \rangle = 2F/P, \quad (34)$$

where F is its vector area. It is easy to show that for a plane contour the quantity $\pi \langle R \rangle^2$ is equal to the area of the circle inscribed in the contour.

The projection of the orbital angular momentum of the light quantum of frequency ω_0 , traveling along the

contour of the resonator, on the direction normal to this contour is

$$L_z = \pm \hbar \omega_0 \langle R \rangle / c \pm \hbar P / \lambda_0, \quad (35)$$

where the \pm signs correspond to opposite directions of motion.

According to the rules of spatial quantization of the projection of the momentum on a preferred direction, we have

$$L_z = m\hbar, \quad (36)$$

where $m = P/\lambda_0$ is the quantum number corresponding to the macroscopic orbit of the light quantum. It is equal to the number of radiation wavelengths subtended by the resonator length. The ratio c/P is equal to the distance between the longitudinal modes in frequency units, therefore $m = \nu_0/\Delta\nu$. For an He-Ne laser with perimeter $P = 3 \times 10^2$ cm we have $m \approx 10^7$.

If the entire instrument as a whole rotates with angular velocity Ω about the z axis, then, in accordance with (30), we can write

$$\hbar \omega \pm \hbar \omega \mp L_z \Omega, \quad (37)$$

whence, taking (36) into account, we get

$$\omega = \omega_0 \mp m\Omega \cos \theta, \quad (38)$$

or

$$\omega = \omega_0 [1 \mp (\langle R \rangle / c) \Omega \cos \theta], \quad (39)$$

which coincides with the expression (32) above, if it is recognized that $\langle R \rangle = 2F/P$.

Thus, a ring laser is an instrument whose rotation causes a change of the orbital angular momenta of macroscopic photon orbits and of the energy of the photons generated in the resonator. The mode frequencies c/P are shifted, depending on the direction of the light travel along the contour by $\mp \Omega_z$, and the radiation frequency mc/P shifts accordingly by $m\Omega_z$, making the beat frequency not $2\Omega_z$ but $2m\Omega_z$.

The beams of light traveling along the contour of the gyroscope in opposite directions correspond to tremendous orbital angular momenta ($\sim 10^7 \hbar$) of photons whose projections on the rotation direction have opposite signs. The rotation of the coordinate system lifts the degeneracy with respect to the orientation of the orbital angular momentum. The linear proportionality of the difference of the beat frequency to the rotation frequency remains in force up to quite appreciable angular velocities and is violated primarily because of the deformation of the contour by centrifugal forces.

The phenomenon considered above is analogous to the splitting of energy levels of atoms in a magnetic field, and is called the Coriolis-Zeeman effect. Its existence in a gravitational field was noted by Zel'dovich^[59]. It is well known in the mechanics and electrodynamics of general relativity theory^[60,61,29].

The wave addition realized at the output of the ring laser is a modification of optical heterodyning. The ring laser, when regarded as an optical heterodyne, is an instrument sensitive to circulation of the vector of gyration of the gravitational field along its contour. It is also exceedingly sensitive to changes of the optical characteristics of the medium filling the resonator.

When light propagates along a rotating contour, the plane of polarization is rotated^[61,75] whenever opposing modes are captured. To the contrary, when the modes are independent, the deviation of the circulation of the gyration vector

$$\oint \mathbf{g} d\mathbf{l} \approx c^{-1} \oint [\Omega \mathbf{R}] d\mathbf{l}$$

along the laser loop from zero leads to the already mentioned change in the frequency of the opposing modes. The angle of rotation φ of the polarization plane can be readily found by using the fact that propagation of light in a non-inertial system of coordinates is equivalent to its propagation in a gyrotropic medium without dispersion^[75].

In the case of interest to us, the refractive indices for the opposing waves are equal to

$$n_{\pm} = 1 \pm g \cos \theta,$$

where θ is the angle between \mathbf{g} and $d\mathbf{l}$. The plane of polarization is therefore rotated after one circuit of the light through the loop by an angle equal to

$$\varphi = 2\pi (n_+ - n_-) P \lambda_0^{-1} = 4\pi \langle R \rangle \Omega_z P / c \lambda_0.$$

Besides the ring-laser regime in which the difference frequency is measured, it is possible to have a regime in which the change of the phase difference of the opposing waves is measured. A particular case of such a measurement of the phase difference is the determination of the number of oscillations of the difference frequency. The ring laser then operates like an integrating gyroscope. Its readings are proportional in this case to the angle of rotation^[62]. The use of this method is of interest to navigation^[63].

The theoretical limit of the sensitivity of a laser gyroscope is determined by the spontaneous emission. The phase fluctuations $\Delta\Phi$ due to the spontaneous emission are of the order of 10^{-18} rad and have an average frequency $\sim 10^{18}$ Hz. This leads to frequency fluctuations of $\sim 1/2\pi$ Hz. The accuracy with which the frequency is measured is determined by the minimum value of the phase which can still be measured. At a signal/noise ratio $\sim 10^4$, the minimum observable phase deviation is $\sim 10^{-4}$ rad. At a measurement time ~ 1 sec, the apparent frequency fluctuation is $\sim 10^{-5}$ Hz, and at a measurement time of several milliseconds it amounts to about 10^{-4} Hz.

If it is assumed that a difference frequency of 1 Hz corresponds to a rotation velocity ~ 1 deg/hr, the theoretical limit of the accuracy of the ring-laser gyroscope turns out to be 10^{-3} deg/hr. The accuracy of the instrument can be increased in this case only by increasing the measurement time.

We note for comparison that an angle 10^{-8} rad subtends a circular-arc segment 10 cm long at a radius equal to the distance from the earth to the moon (386 000 km).

The development of laser gyroscopes entails a number of difficulties inherent in devices of this type, which greatly reduce their accuracy and sensitivity^[64].

One such difficulty is the dependence of the oscillation frequency of the ring laser on its geometrical dimensions (perimeter). A laser gyroscope is very sensitive to vibrations, which change the frequency of

the circuit, and to low-frequency parasitic oscillations, which distort or even destroy the useful signal. These difficulties can be overcome only by increasing the rigidity of the instrument structure. In some models this is done by constructing the laser of single block of quartz or pyroceram, or some other suitable material^[65] (see fig. 16). The mirrors are placed in optical contact by using molecular adhesion. The construction shown in Fig. 16 turned out to be quite rigid and made it possible to ensure stable operation of the instrument in navigation systems.

If for some reason the perimeters are different for the circuits in the opposite directions, $P_{\pm} = P \pm \Delta P$, the frequency difference of the opposing modes is

$$\delta\nu = \nu_- - \nu_+ = mc (P_-^{-1} - P_+^{-1}) \approx 2\nu \Delta P / P. \quad (40)$$

Even in the case of a rigid construction of the instrument, the misalignment of the channels for the optical beams can lead to parasitic modulation of the beat frequency.

To maintain the perimeter constant, piezoquartz or piezoceramic elements are used, which make it possible to modify the perimeter somewhat. Modulation with the aid of piezoelements makes it possible to generate an error signal and to effect automatic frequency control. The device described in^[54], where such a possibility is realized, can measure an angular velocity of ~ 0.1 deg/hr, which is 150 times slower than the earth's rotation (~ 15 deg/hr). The velocity-measurement time amounts to 3 min.

When the discharge tubes are fed with dc, the ionized particles drift and this also leads to errors in the velocity measurement. The direction of the drift and one of the possible means of its elimination are shown in Fig. 17.

The accuracy of a ring laser is greatly influenced by the independence of the refractive indices of the medium, due to unequal saturation of the medium by the oppositely propagating waves when their intensities are not strictly equal^[65], and also by gyrotropy due to the action of external magnetic fields^[76,77]. When the difference between the per unit powers of the waves in the resonator is $0.1-0.15$ W/cm², the change of the refractive index is in the tenth decimal place. However, even such an insignificant change of the refractive index leads to noticeable errors in measurement of small angular velocities.

Another major difficulty encountered in the develop-

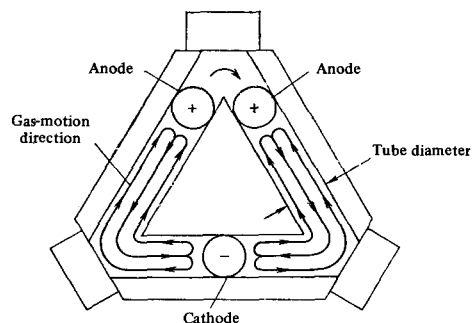


FIG. 17. Motion of ionized particles in the gas-discharge gap of a ring laser. The illustrated resonator construction ensures freedom from frequency shifts due to particle drift.

ment of gyroscopes is the mode-locking effect. This effect appears at low rotational velocity and is the analog of the frequency pulling of two coupled generators in radio engineering. In a ring laser, this effect is due to the interaction of the opposing waves that propagate in a non-linear medium (plasma). Its occurrence is aided by the diffuse scattering of the light by mirrors, prisms, and other elements of the optical system. The locking phenomenon is greatly influenced by the operating regime of the ring laser, and also by the position of the generating mode on the amplification curve. The most convenient is the line position near the maximum. On the other hand, to obtain a single-mode regime it is necessary to feed minimum power to the generator.

The locking region can be decreased somewhat by imposing stringent requirements on the purity and quality of the reflecting surfaces^[31,68] or by using total internal reflection prisms in place of mirrors^[54,30]. It is impossible to get rid completely of the locking of opposing modes, owing to the inevitable scattering by surface molecular waves^[56], to the finite dimensions of the mirrors and the beam cross section, and to non-linear phenomena in the discharge plasma and in the reflection region. The lower limit of locking is determined by the quantum locking phenomenon, due to the interference interaction between the beams^[57,58]. It can be shown that near the capture region there should be observed jump-like changes of the beat frequency.

In the ring lasers employed for gyroscopy purposes, the capture region reaches several dozen Hz. A number of methods have been proposed to help get rid of this undesirable phenomenon. It should be noted, however, that this problem has not been finally solved to this day.

The only real measure that helps in practice to get rid of this undesirable phenomenon and makes it possible to measure very small angular rotation velocities is to produce an artificial initial frequency difference exceeding the threshold locking frequency. It is possible to produce a frequency difference in many ways. The simplest method of producing a constant frequency difference is to rotate the entire apparatus with constant angular velocity. The practical value of such a procedure is low, owing to the low stability of the frequency difference obtained in this manner. It is easy to show that the initial frequency shift must be maintained with a relative accuracy $\sim 10^{-6}$ in order to observe and measure the earth's rotation with accuracy up to 1%.

One of the realized methods of producing an initial phase difference^[43,65] was to impart to the platform on which the gyroscope is mounted small rotary vibrations with frequency on the order of 10–40 Hz. During the greater part of the period of such oscillations, the angular velocity of motion is sufficient to cause the difference frequency to exceed the threshold locking frequency. This method was used in ring lasers operating in the integrating-gyroscope regime. Symmetrical frequency deviations due to the vibrations of the apparatus do not affect the readings of the gyroscope, owing to the integration effect. At the same time, it operated for the greater part of the period in the frequency region where there is no locking, and its readings are proportional to the angle of rotation.

If a part of the ring resonator is filled with a medium that moves at a certain angle to the direction of the light beam (solid, liquid, or gaseous), then as a result of the Fizeau effect, a certain frequency difference is produced between the opposing waves. This is due to the fact that the motion of the medium changes its relative refractive index, and consequently also the optical length of the resonator. The theory of the Fizeau effect in a ring resonator was considered in^[26,55,71-73]. Experiments on its study in solid, liquid, and gaseous media are described in^[53,67,73,74].

The refractive index of a moving medium is determined on the basis of the relativistic law of velocity addition

$$u = cn_0^{-1} \pm v [1 - n_0^{-2} + \omega n_0^{-2} (dn_0/d\omega)] \quad (41)$$

in accordance with the usual formula $n = c/u$, where v is the velocity of the medium relative to the laser body and n_0 is the refractive index of the immobile medium. The \pm signs in (41) are chosen in accordance with whether the directions of the light propagation and of the motion of the medium are the same or opposite. It is easy to find from (41), accurate to quantities of the order v/c , an expression for the refractive index of the moving medium

$$n_{\mp} = n_0 \mp v^{-1} [n_0^2 - 1 + n_0 \omega (dn_0/d\omega)]. \quad (42)$$

If the part of the resonator filled with the medium has a length l , then the frequency difference f between the opposing waves is

$$f = mc \{ [P + (n_- - 1)l]^{-1} - [P + (n_+ - 1)l]^{-1} \} \approx 2v l^{-1} \Delta n [P + (n_0 - 1)l]^{-1},$$

or

$$f = 2l n_0^{-1} \{ n_0^2 - 1 + n_0 \omega (dn_0/d\omega) \} v [P + (n_0 - 1)l]^{-1}. \quad (43)$$

By choosing the velocity of the medium it is possible to make the initial frequency shift large enough to get rid of mode locking. Knowing the parameters of the resonator and the refractive index of the medium moving in it, it is possible to determine its velocity. This makes it possible to determine very small flow velocities of liquids and gases (Fig. 18)^[65], and also to determine velocity fluctuations. Mention must also be made of the fact that the ring laser is the most sensitive instrument for the measurement of small natural and artificial optical activity. Its use as a polarimeter

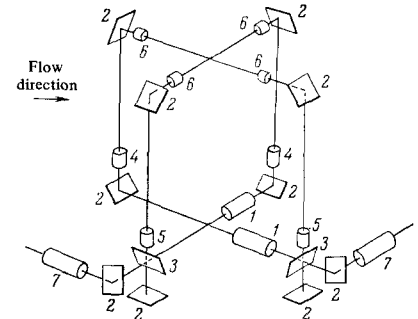


FIG. 18. Precision anemometer of the Sperry Corporation, consisting of two orthogonally mounted ring lasers, part of the optical paths of which pass through moving air. 1—Gas-discharge tubes; 2, 3—flat mirrors; 4—Faraday cells; 5, 6—windows inclined at the Brewster angle and used for emergence of the beams; 7—photodetectors.

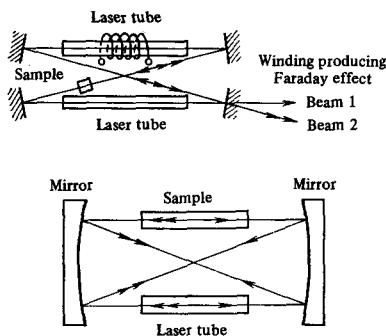


FIG. 19. Ring lasers insensitive to rotation.

makes it possible to observe rotations of the plane of polarization on the order of 0.0002° ^[74].

The influence of external magnetic fields on the laser elements and the active medium^[76] makes it possible, on the one hand, to use the phenomenon of artificial optical activity to separate the frequencies of the opposing modes, and on the other hand to produce ring lasers insensitive to effects of this type.

A ring laser insensitive to mechanical rotation is shown in Fig. 19.

The use of the Faraday effect for frequency separation has been the subject of a large number of papers^[31,67,69,70]. Mode separation is readily ensured. However, introduction of a Faraday cell into the resonator greatly increases the back scattering and increases the critical locking frequency.

In^[54] it is proposed that the intrinsic anisotropy of the active medium of the laser fed with direct current be used. This method makes it possible to obtain easily beat frequencies up to several kHz by varying the pump current. The method does not call for the introduction of any extraneous elements into the resonator. However, the change of the supply current leads to excitation of undesirable modes.

A Faraday cell usually consists of a plate made of an optically-active medium and two quarter-wave plates perpendicular to each other (Fig. 20). The linearly polarized light generated in the ring laser acquires circular polarization after passing through the suitably oriented plate. Therefore two waves circularly polarized in opposite directions propagate in the space between these plates in opposite directions. The refractive indices of the cell material for the waves with different circular polarizations differ little, and this leads, in accordance with formula (44), to a

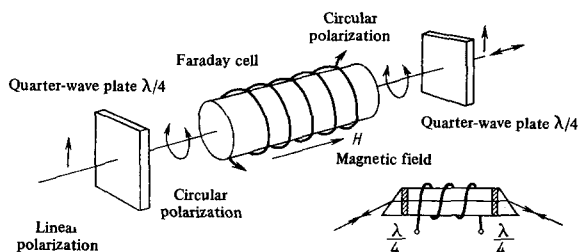


FIG. 20. Non-reciprocal element producing a frequency shift in a ring laser. The lower right figure shows a construction ensuring incidence of the rays at the Brewster angle, and consequently a minimum loss to reflection.

frequency shift. Besides naturally active substances (quartz), it is possible to use also artificially active substances (glass). In the latter case, however, the active element (glass cylinder) must be placed in a magnetic field.

The initial phase shift can be obtained, for example, with the aid of the device shown in Fig. 21^[67]. Inside the ring laser, on the path of the light rays, are mounted two Faraday cells that rotate the plane of polarization by 45° each. The same figure shows the change of the orientation of the plane of polarization upon passage of the right- and left-polarized beams through the system. As seen from the figure, the plane of polarization of the right-hand beam is turned 90° relative to the plane of polarization of the left-hand beam. There is practically no interaction between the beams polarized in mutually perpendicular directions. Such a device calls for the use of special mirrors, which do not change the character of polarization upon reflection. For optical glass, it is easy to make the difference between the refractive indices of the order of 10^{-7} . This is sufficient to cause the frequency difference between the orthogonally polarized waves to reach 100–200 kHz on a path of 2–3 cm.

If the axis of the laser gyroscope is aimed at the sun, then, by measuring the beat frequency f and knowing the earth's angular velocity, it is possible to determine, accurate to fractions of a degree, the latitude angle θ :

$$f = 4F \cos \theta / \lambda P.$$

In this case the instrument serves as a sextant. The capabilities of such an application can hardly be overestimated^[55].

Integrating the beat frequency, we transform the gyroscope into a goniometer

$$\Phi(t) = 2\pi \int f(t) dt = 2m \int \Omega dt = 2m\Phi(t).$$

The rotation angle $\Phi(t)$ is transformed into a phase advance with an amplification coefficient $2m$.

If the instrument is oriented in such a way that its axis is perpendicular to the earth's axis, then it turns out to be insensitive to its rotation; but if the axis of the instrument is parallel to the earth's axis, the beat frequency is maximal. Thus, a ring gyroscope can be used as a compass.

Photon gyroscopes are still in the development stage. Their parameters are continuously improved and the designs are being perfected. The technological

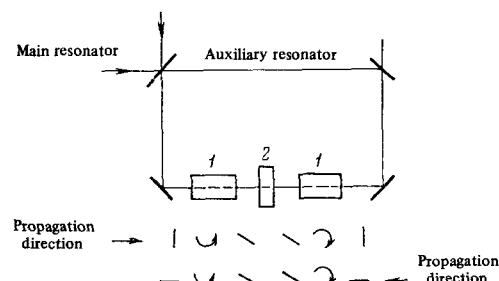


FIG. 21. Device for reducing wave locking, based on the Faraday effect. 1—Faraday cell; 2—Nicol prism. The semicircular arrows show the direction of rotation of the polarization.

process of the development of individual elements of the instrument is still far from perfected. The theoretical possibilities of the instrument have not yet been exhausted. In spite of this, the laser gyroscope already competes successfully with the better models of mechanical gyroscopes.

Photon gyroscopes and certain types of nuclear gyroscopes, described above, have a number of undisputed advantages over mechanical gyroscopes. The readings of quantum gyroscopes are absolute. The measured quantity is the frequency of an alternating voltage. The signal can be transmitted by wires or by radio. The signal frequency is strictly proportional to the angular velocity of rotation. The instruments do not contain moving parts, can be made sufficiently rigid, and withstand large accelerations. The devices do not call for locking devices. One of the most important advantages of quantum gyroscopes of all types is the absence of inertia.

Laser gyroscopes have already found use not only as highly sensitive rotation indicators, but also as gyrocompasses, gyro direction finders, sextants, liquid-flow and wind-flow velocity meters (see Fig. 18), flow meters, refractometers, polarimeters, frequency standards, etc.

VII. CONCLUSION

We have considered certain physical phenomena that can serve as a basis for the development of instruments sensitive to rotation. In all the cases under consideration, coherent ensembles of mechanical or magnetic properties of atomic nuclei, atoms, electrons, or photons are used. The devices in which the gyromagnetic properties are used were called by us quantum gyroscopes.

Problems in which it is necessary to use gyroscopic devices are so varied, with respect to purpose as well as with respect to required accuracy, that it is hardly possible to point to a single universal device. Almost each of the instruments described above, particularly the original methods used in their development, will perhaps find some application. The assembly of technical conditions determining the concrete requirements imposed on the instrument can make a gyroscope based on any of the foregoing principles a competitive instrument.

One of the most interesting quantum gyroscopes is the ring photon gyroscope. The beat frequency between the opposing modes of the ring laser is proportional to the average working frequency and to angular rotation speed. By increasing the working frequency it is possible to increase the sensitivity of the instrument. An appreciable increase of the sensitivity can be obtained by changing over from electromagnetic waves to coherent electron currents corresponding to de Broglie waves.

Unfortunately, to produce coherent closed electron currents in vacuum it is necessary to have unattainably high stabilization of the accelerating voltage and of the controlling magnetic field. The latter determines the phase shifts of the electron waves, and by the same token their coherent properties. On the other hand, electromagnetic interactions between uncompensated

electron charges lead to fluctuations of the density of their distribution over the orbits, and by the same token to a pulsed character of the operation of such a device.

A favorable situation is produced in metals that are in the superconducting state. The electrons joined in Cooper pairs form a condensate of Bose particles that are in one energy state. The high degree of coherence of such an electron ensemble makes it possible to control its phase and by the same token to observe interference between macroscopic electron currents.

In this sense, it turns out to be possible to construct an analog of the Sagnac interferometer, in the form of a superconducting interferometer containing two or more Josephson junctions^[78,79], and an analog of the ring laser, namely a Josephson ring generator. Instruments of this type, owing to the quantization of the magnetic flux, linking the superconducting loop, exhibit an unusually high sensitivity to external magnetic fields. A simple double Josephson interferometer, containing two Josephson junctions, at a loop area of 2 cm^2 , makes it possible to determine changes of the magnetic field much smaller than 10^{-7} Oe , corresponding to a quantum of magnetic flux. This field sensitivity can be increased by several orders of magnitude by using a number of parallel-connected interferometers. The sensitivity of such an instrument to rotation is determined from the Larmor formula $\Omega = 10^7 \text{ H}$. Thus, a change of the field by 10^{-7} Oe is equivalent to rotation of the instrument with angular velocity of 1 Hz . Gyroscopes of this type require an exceptionally strong magnetic screening or complete magnetic vacuum. The use of a ring superconducting interferometer-resonator is expected to increase the sensitivity of the instrument by a factor of 10^3 compared with the optical gyroscope. One might think that superconducting gyroscopes will be the subject of serious investigations in the nearest future.

Finally, it should be noted that the realistic possibilities of producing a complete magnetic vacuum make it realistic to develop nuclear gyroscopes in which the working medium can be, for example, a solution of He^3 nuclei in He^4 . (The relaxation time of He^3 in He^4 at a concentration $\sim 10^{-3}\%$, reaches one year.) It is proposed to use an instrument of this type to observe the electric dipole moment of the neutron, and also to verify the conclusions of general relativity theory^[80].

¹R. Robertson, in: *Upravlenie poletom kosmicheskikh apparatov* (Control of Flight of Space Ships) [Russian Transl.] IL, 1963, p. 9.

²A. Schneider, *ibid.*, p. 112.

³I. H. Simpson, *Astronom. and Aeronaut.* 2 (10), 42 (1964).

⁴N. Bloembergen, *Nuclear Magnetic Relaxation*, Leiden, 1948.

⁵W. Braunbek, *Zs. angew. Math., Mech.* 33, (5/6), 174 (1953).

⁶I. Holahan, *Space Aeronaut.* 31, 130 (1959).

⁷A. Abragam, *Principles of Nuclear Magnetism*, Oxford, 1961.

⁸N. M. Pomerantsev, V. M. Ryzhkov, and G. V. Skrotskiĭ, in: *Geofizicheskaya apparatura* (Geophysics

- Apparatus), Nedra, No. 33, 1967, p. 13; No. 34, 1968, p. 8.
- ⁹H. Goldstein, *Classical Mechanics*, Addison-Wesley, 1950.
- ¹⁰V. M. Ryzhkov, G. V. Skrotskiĭ, and Yu. I. Alimov, *Izv. Vuzov (Radiofizika)* 6, 884 (1959).
- ¹¹H. C. Torrey, *Phys. Rev.* 76, 1059 (1949).
- ¹²E. L. Hahn, *Phys. Rev.* 77, 297 (1950).
- ¹³W. Calver, *Raketnaya tekhnika*, (Rocket Engineering) [Russian transl.], No. 6, 110 (1962).
- ¹⁴M. F. Wolf, *Electronics*, No. 7, 6 (1962).
- ¹⁵M. F. Wolf, *Electronics*, No. 52, 36 (1962).
- ¹⁶P. I. Klass, *Aviation Week* 79, (7), 88 (1963).
- ¹⁷S. M. Formans and M. I. Minneman, *IEEE Trans. Military Electronics Mil-7*, 40 (1963).
- ¹⁸G. N. Watson, *A Treatise on the Theory of Bessel Functions*, Cambridge, 1944.
- ¹⁹A. D. Leete, U. S. Patent No. 2720625.
- ²⁰I. T. Fraser, U. S. Patent No. 3103621.
- ²¹*Electronics*, No. 3, 8 (1963).
- ²²P. Jung and I. Van Cakenberge, *Arch. Sci., Fasc. spec.* 14, 132 (1961).
- ²³G. V. Skrotskiĭ and T. G. Izyumova, *Usp. Fiz. Nauk* 73, 423 (1961) [*Sov. Phys.-Usp.* 4, 177 (1961)].
- ²⁴H. E. Debolt, *Electronics* 15, 49 (1960).
- ²⁵V. A. Belyĭ and A. M. Skripkin, *Opt. Spektrosk.* 21, 130 (1969).
- ²⁶A. H. Rosenthal, *J. Opt. Soc. Amer.* 52, (10), 1143 (1962).
- ²⁷W. M. Macek and D. T. M. Davis, *Appl. Phys. Lett.* 2, (3), 67 (1963).
- ²⁸W. M. Macek, D. T. M. Davis, R. W. Olthus, J. R. Schneider, and G. R. White, *Proc. Sympos. Optical Masers*, Brooklyn, 1963.
- ²⁹C. V. Heer, *Quantum Electronics*, Proc. of the 3rd Intern. Congress, Paris, 1964, p. 1305.
- ³⁰P. Laures, *Mem. de l'artillerie* 427, fasc. 2, 427 (1965).
- ³¹W. M. Macek and E. J. McLartney, *The Ring Laser*, Sperry Engineering Review, 1966, p. 8.
- ³²V. E. Privalov and S. A. Fridrikhov, *Usp. Fiz. Nauk* 97, 377 (1969) [*Sov. Phys.-Usp.* 12, 153 (1969)].
- ³³*Electronics News*, No. 359, pt. 1, 27 (1963).
- ³⁴*Electronics* 36 (7), 82 (1963).
- ³⁵*Aviation Week*, No. 6, 98, 103 (1963).
- ³⁶*Missiles and Rockets*, No. 7, 24 (1963).
- ³⁷*Electronic Design*, No. 5, 14 (1963).
- ³⁸*Machine Design* 35 (6), 12 (1963).
- ³⁹*Electronic Products* 5 (10), 38 (1963).
- ⁴⁰*Aviation Week* 78, 92 (1963).
- ⁴¹*Control* 6, 116 (1963).
- ⁴²*Electronics News*, No. 381, 16 (1963).
- ⁴³*Electronics* 36 (26), 82 (1963).
- ⁴⁴J. N. Anders, U.S. Patent No. 3158802.
- ⁴⁵G. Sagnac, *Compt. rend.* 57, 708 (1913).
- ⁴⁶G. Sagnac, *Ann. de Phys.* 4, 177 (1914).
- ⁴⁷S. I. Vavilov, *Éksperimental'nye osnovaniya teorii otноситel'-nosti* (Experimental Foundations of Relativity Theory), GIZ, 1928.
- ⁴⁸L. D. Landau and E. M. Lifshitz, *Teoriya polya*, Nauka, 1966 [Field Theory, Addison-Wesley, 1969].
- ⁴⁹A. A. Michelson and H. C. Gale, *Astrophys. J.* 61, 1401 (1921).
- ⁵⁰S. A. Collins, *Appl. Optics* 3 (11), 1263 (1964).
- ⁵¹S. A. Collins and D. T. M. Davis, *Appl. Optics* 3 (11), 1314 (1964).
- ⁵²A. Yu. Ardashev, V. A. Kashin, and G. V. Skrotskiĭ, *Izv. Vuzov (Fizika)* 3, 133 (1968).
- ⁵³B. V. Rybakov, Yu. V. Demidenkov, A. M. Khromykh, and S. G. Skrotskiĭ, *Zh. Eksp. Teor. Fiz.* 57, 1184 (1969) [*Sov. Phys.-JETP* 30, 646 (1970)].
- ⁵⁴B. Pessing, J. M. Catherin, and J. Migue, *Compt. rend.* 262, 1961 (1964).
- ⁵⁵J. M. Catherin and B. Dessus, *IEEE J. Quantum Electron.* VQE-3 (11), 449 (1967).
- ⁵⁶G. S. Landsberg, *Usp. Fiz. Nauk* 36, 284 (1948).
- ⁵⁷A. S. Davydov, *Kvantovaya mekhanika* (Quantum Mechanics), Fizmatgiz, 1953.
- ⁵⁸W. F. Chel'tzov, *Phys. Lett.* 29A (1), 34 (1969).
- ⁵⁹Ya. B. Zel'dovich, *ZhETF Pis. Red.* 1, No. 3, 40 (1965) [*JETP Lett.* 1, 95 (1965)].
- ⁶⁰V. I. Pustovoit, *Priroda*, No. 7, 7 (1960).
- ⁶¹G. V. Skrotskiĭ, *Dokl. Akad. Nauk SSSR* 114, 73 (1957) [*Sov. Phys.-Dokl.* 2, 226 (1958)].
- ⁶²P. I. Klass, *Aviation Week*, No. 12, 88 (1965); No. 11, 103 (1966).
- ⁶³J. Institute of Navigation 17, 289 (1964).
- ⁶⁴E. P. Markin, *Candidate's Dissertation*, (FIAN, 1966).
- ⁶⁵P. I. Klass, *Aviation Week*, No. 11, 103 (1966).
- ⁶⁶F. Arnowitz and R. I. Collins, *Appl. Phys. Lett.* 9, 55, 1966.
- ⁶⁷R. C. Smith and L. S. Watkins, *Proc. IEEE* 53 (2), 161 (1965).
- ⁶⁸E. M. Belenov, E. P. Markin, V. N. Morozov, and A. N. Oraevskii, *ZhETF Pis. Red.* 3, 54 (1966) [*JETP Lett.* 3, 32 (1966)].
- ⁶⁹W. M. Macek, J. R. Schneider, and R. M. Salamon, *J. Appl. Phys.* 35, 2556 (1964).
- ⁷⁰T. J. Huterings, J. Winocuz, R. H. Durrett, E. D. Jacobs, and W. L. Zingery, *Phys. Rev.* 152 (1), 467 (1966).
- ⁷¹I. L. Mazan'ko, *Zh. prikl. spektroskopii* 1, 153 (1964).
- ⁷²C. V. Heer, *Phys. Rev.* 134, A799 (1964).
- ⁷³A. M. Khromykh, *Zh. Eksp. Teor. Fiz.* 50, 281 (1966) [*Sov. Phys.-JETP* 23, 185 (1966)].
- ⁷⁴H. J. Raterink, H. V. Stadt, C. H. F. Velzel, and Dijkstra, *Appl. Optics* 6 (5), 813 (1967).
- ⁷⁵G. V. Skrotskiĭ and A. A. Izmet'ev, *Dokl. Akad. Nauk SSSR* 178, 77 (1968) [*Sov. Phys.-Dokl.* 13, 30 (1968)].
- ⁷⁶M. I. D'yakonov and S. A. Fridrikhov, *Usp. Fiz. Nauk* 90, 565 (1966) [*Sov. Phys.-Usp.* 9, 837 (1967)].
- ⁷⁷W. Culshaw and Kannerand, *Phys. Rev.* 133, 691 (1964).
- ⁷⁸I. M. Dmitrienko, *Kvantovye efekty v sverkh-provodnikakh* (Quantum Effects in Superconductors), Znanie, 1968.
- ⁷⁹R. Feynman, R. Leighton, and M. Sands, *Feynman Lectures in Physics*, Vol. 3, Addison-Wesley, 1963.
- ⁸⁰*Phys. Today*, No. 6, 59 (1968).

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