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PHYSICAL PHENOMENA IN SEMICONDUCTORS WITH NEGATIVE DIFFERENTIAL CONDUCTIVITY

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INTRODUCTION

In solids, as in a gas plasma, the carrier energy distribution function deviates under the influence of a strong electric field from the equilibrium distribution function. "Heating" of the electron (or hole) takes place. This phenomenon is the basis of a large number of interesting physical effects. The most important of them is the nonlinearity of the current-voltage characteristic (CVC) of a solid in a strong field. The strongest manifestation of this effect is the appearance of a decreasing section on the CVC, in which the differential conductivity $\sigma_d = dj/dE$ is negative. A characteristic is defined as S-shaped if the current density is a multiple-valued function of the field (Fig. 1), and as N-shaped if the current is a single-valued but nonmonotonic function of the field (Fig. 2).

The decreasing branch of the CVC results in this case from definite singularities of either the energy spectrum of the carriers (electrons and/or holes), or their interaction with the lattice vibrations, with the impurities, and with one another. A study of the concrete mechanisms leading to a decreasing branch on the characteristic is in itself an important problem of solid-state physics. Interest in this group of phenomena has become particularly intense since the discovery of microwave generation in certain semiconductors—the Gunn effect. It turned out that there is a direct connection between this effect and the N-shape of the CVC of the material in which it is observed.

In this review we deal only with homogeneous conductors. We do not consider, for example, the tunnel diode, which is an interesting system with an N-shaped characteristic. The point is that in homogeneous semiconductors the presence of a decreasing branch on the CVC leads to phenomena that cannot arise in lumped elements. The principal feature of homogeneous conductors is that their stationary states with homogeneous field and current distributions, corresponding to the decreasing branch of the characteristic ($\sigma_d < 0$), are unstable against inhomogeneous fluctuations [1,2]. The development of this instability results in an inhomogeneous distribution of the current over the cross section ("pinching" of the current) in the case of S-shaped characteristic, and an inhomogeneous distribution of the field in the form of moving regions (domains) of strong or, conversely, weak field is produced in the case of N-shaped characteristics [3,4].

The instability leads to different effects in semiconductors with S-shaped or N-shaped characteristics. We shall see, however, that these effects have many common features, so that we are actually dealing with a unified group of phenomena.

The plan of this review is as follows: We consider first the main mechanisms of S-shaped and N-shaped characteristics, assuming in the discussion that the distributions of the current and the field in the sample are homogeneous. Indeed, as shown in Ch. 2, the homogeneous states corresponding to the decreasing CVC branch are unstable against inhomogeneous perturbations. It is important to determine precisely which perturbations possess the largest growth increment (Ch. 2). This facilitates the determination of those states with inhomogeneous current or field distribution, into which the semiconductor goes over as a result of the instability (Ch. 3 and 5). Among the possible inhomogeneous distributions, it is necessary to find the most stable ones (Ch. 4 and 5) and use them for the interpretation of the experimental data. In Ch. 4 we consider certain applications of the described phenomena.

1. MECHANISMS OF OCCURRENCE OF VOLUME S- AND N-SHAPED CHARACTERISTICS

1.1. Superheating Mechanisms

This name can be given to a number of mechanisms based on the change of the characteristics scattering times of the momentum (τ_p) and energy (τ_e) of the carriers with increase of their effective temperature (at a fixed concentration n). Some of these mechanisms pertain to semiconductors in which the impurities are fully ionized and the momentum scattering occurs

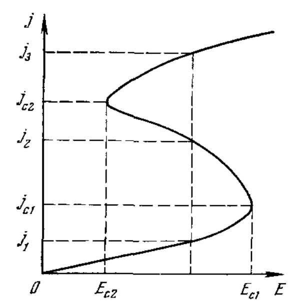

FIG. 1. S-shaped current-voltage characteristic.,

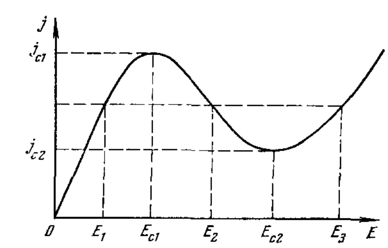

FIG. 2. N-shaped current-voltage characteristic.

principally on the impurity ions (or on carriers with relatively low mobility[7]; see below). Use is made of the rapid increase of τ_p of the electrons with increasing temperature of the electron gas.

The stationary values of the electron temperature T are determined by equating the specific Joule power jE acquired by the electron from the field (j—current density, E—field) to the specific power P transferred to the electrons to the phonons. When $\tau_p \ll \tau_e$ (this condition will be discussed later), the drift velocity v is small compared with thermal electron velocity v_T. The specific electric conductivity $\sigma = j/E$ and the power P are then functions of only the electron temperature T (and also the concentration n). The equation

$$\sigma(T)E^2 = P(T) \tag{1.1}$$

determines the function T(E), meaning also the form of the CVC $j = \sigma(T(E))E$. We represent $\sigma(T)$ and $P(T)$ in the form

$$\sigma(T) = \left(\frac{ne^2}{m}\right)\tau_p(T),$$
$$P(T) = \frac{nc_e(T-T_0)}{\tau_e(T)}; \tag{1.2}$$

here m is the effective electron mass, nc_e is the specific heat of the electron gas, and T_0 is the temperature of the lattice (of the phonons), which frequently can be regarded as constant.

For a given E, Eq. (1.1) may have more than one solution. If the temperature is a triple-valued function of the field in a certain field interval from E_{c2} to E_{c1}, then the characteristic has an S-shaped form (see Fig. 1).

Let us find the condition under which $\sigma_d < 0$. The expression for $\sigma_d \equiv dj/dE$ follows from (1.1) and (1.2):

$$\sigma_d = \sigma\frac{(dP/dT)+E^2(d\sigma/dT)}{(dP/dT)-E^2(d\sigma/dT)} = \sigma\frac{1-[1-(T_0/T)]\,d\ln(\tau_e\tau_p)/d\ln T}{1-[1-(T_0/T)]\,d\ln(\tau_e\tau_p)/d\ln T}. \tag{1.3}$$

If the characteristic is S-shaped, then σ_d becomes infinite at each reversal of the sign (in field E_{c1} and E_{c2}; Fig. 1), and consequently the denominator (1.3) vanishes. The latter occurs if $d\sigma/T > 0$, i.e., the electric conductivity increases with the heating. The differential conductivity is negative in the electron temperature interval in which

$$\frac{d\ln(\tau_e\tau_p)}{d\ln T} > \frac{T}{T-T_0}. \tag{1.4}$$

This inequality can be satisfied if

$$\frac{d\ln(\tau_e\tau_p)}{d\ln T} > 1. \tag{1.5}$$

The possible occurrence of an S-shaped characteristic as a result of variation of τ_p and τ_e with T was indicated by A. V. Gurevich[6]. He considered an electron gas interacting with free ions and neutral molecules. In collisions with ions, as is well known (see, for example,[5]),

$$\frac{d\ln\tau_p}{d\ln T} = \frac{d\ln\tau_e}{d\ln T} = \frac{3}{2}, \tag{1.6}$$

and in collisions with neutral molecules

$$\frac{d\ln\tau_p}{d\ln T} = \frac{d\ln\tau_e}{d\ln T} = -\frac{1}{2}. \tag{1.7}$$

We see that condition (1.5) is satisfied if τ_p and τ_e are determined by scattering from ions.

In semiconductors, when the electrons are scattered by charged impurity centers, as in the scattering by ions in a gas plasma, we have $\tau_p \propto T^{3/2}$. The energy scattering by impurities is usually neglected (at a small atomic concentration of the impurities), since the impurity atoms are not free. Bok[7] considered a case when carriers with small effective mass and large mobility, which make the main contribution of the current, scatter energy on carriers with large effective mass (for example, electrons on holes), and the latter rapidly transfer the energy to the phonons. Then $\tau_e \propto T^{3/2}$ (cf. (1.6)).

In scattering from the deformation potential of the acoustic lattice vibrations, τ_p and τ_e satisfy Eq. (1.7)[8]. Thus, a semiconductor in which the scattering of energy or momentum is from "heavy" carriers and from the deformation potential, is from this point of view a perfect analog of the systems investigated by A. V. Gurevich[6]. The S-shaped CVC observed by Bok in p-NSb at 20°K[7] is related by the author to the mechanism proposed by him.

In piezoelectric crystals, the acoustic lattice vibrations produce not only a deformation potential but also a piezoelectric potential. In scattering from that potential we have $\tau_e \propto T^{1/2}$[9,10], so that the role of the piezoelectric scattering of the energy increases with decreasing temperature, compared with the role of scattering by the deformation potential (see (1.7)). In piezoelectric semiconductors at low temperatures, when the electron momentum is scattered by the charged impurities and the energy by the piezoelectric potential of the phonons, the CVC can have a decreasing section, since $d\ln(\tau_e\tau_p)/d\ln T = 2$ (see (1.5))[10].

It follows from (1.3) that σ_d reverses sign in this case at $T_{c1} = 2T_0$. The corresponding field (Fig. 1) is

$$E_{c1} \sim \left(\frac{mT}{\tau_p\tau_e e^2}\right)^{1/2}.$$

In a plasma with Coulomb scattering of the momentum, the electrons with sufficiently high energy are on the average continuously accelerated by the field. This phenomenon is called electron runaway (see the reviews[5,11]). The characteristic runaway field, in which this process affects the bulk of the (thermal) electrons, is of order of

$$E_{ra} \sim \frac{(mT)^{1/2}}{e\tau_p}.$$

The ratio $E_{c1}/E_{ra} \sim (\tau_p/\tau_e)^{1/2} \ll 1$, and in fields $\sim E_{c1}$ the fraction of runaway electrons with energies much higher than thermal is exponentially small. We can therefore expect that runaway will not influence the occurrence of a decreasing branch of the CVC*. At sufficiently high average electron energies, higher than T_{c1}, the scattering by optical phonons and by the deformation potential becomes appreciable, and this leads to a new reversal of the sign of σ_d (in a field E_{c2}; see Fig. 1).

The CVC of a semiconductor placed in a quantizing magnetic field ($\hbar\omega_H \gg T$, $\omega_H = eH/mc$—cyclotron

*Effects such as electron runaway in the absence of collisions between electrons were discussed by Bass [18] and by Levinson [12].

frequency) has a decreasing section if the carrier momentum is scattered by acoustic phonons or (and) by impurities, and give up the energy to the acoustic phonons[13].

In a magnetic field, the motion of an electron in a plane perpendicular to the field, as is well known, is quantized[14,15]. If $T \ll \hbar\omega_H$ and the electron gas is not degenerate, practically all the electrons are in the oscillator ground state (below the Landau band). The scattering of electrons by the lattice vibrations differs greatly in this case from the scattering in the absence of a magnetic field or in a "classical magnetic field" ($T \gg \hbar\omega_H$). Since the wave function of the electron in the Landau ground band is concentrated (in the (x, y) plane normal to the field) in a region of the order of the "quantum length" $\lambda_H = (\hbar/m\omega_H)^{1/2}$, it emits and absorbs phonons with wave-vector components $f_\perp$ in the (x, y) plane on the order of λ_H^{-1}. The energy of such phonons is $\sim\hbar s\,|f_\perp| \sim (ms^2\hbar\omega_H)^{1/2}$, where s is the speed of sound, and does not depend on the energy of the electrons (or on their temperature T). On the other hand, in the absence of a magnetic field the characteristic wavelength of the emitted phonons is of the order of the De Broglie wavelength of the electron, and the energy is $\sim (ms^2T)^{1/2}$, i.e., smaller by a factor $(T/\hbar\omega_H)^{1/2}$ than when $h\omega_H \gg T$. For this reason, the time τ_p of energy scattering by the electrons in a quantizing magnetic field is smaller than in the absence of the field, but increases rapidly, like $T^{3/2}$, with the electron temperature[10].

In the case of scattering by the deformation and by the piezoelectric potentials of the lattice vibrations, the probability of scattering depends differently on the phonon energy. This is precisely why the temperature dependence $\tau_e(T)$ at $H = 0$ is different for these two scattering mechanisms. On the other hand, in a quantizing magnetic field the characteristic phonon energies do not depend on T and the corresponding times τ_e depend on the same manner on $T(\tau_e \propto T^{3/2})$.

In a quantizing longitudinal magnetic field ($j \parallel H$), the time of momentum scattering by a charged impurity is $\tau_p \propto T^{3/2}$[16], and the time of scattering by the deformation potential is $\propto T^{1/2}$[16], so that the condition (1.5) is satisfied (S-shaped CVC).

In a transverse magnetic field, the form of the CVC depends on the measurement regime. As is well known, two measurements regimes are possible: with zero h hole current ($j_y = 0$) and with short-circuited hole current ($E_y = 0$). The differential conductivity σ_d is given by formula (1.3), in which σ should be replaced by $\rho_{xx}^{-1} \propto \tau_p$ in the former case and by $\sigma_{xx} \propto \tau_p^{-1}$ in the latter (we assume that $\omega_H\tau_p \gg 1$). Since τ_p increases with T in a quantizing transverse field (like $T^{3/2}$ in scattering by ions and by the deformation potential[16]), then the characteristic in the absence of a hole current has an S-shape[13,10,17], and an N-shape in the case of a short-circuited hole current in the same material (σ_{xx} decreases rapidly with the field)[13,17-19].

At very high electron temperatures, the condition $T \ll \hbar\omega_H$ is ultimately violated. In addition, strong energy scattering by optical phonons sets in. Therefore σ_d becomes positive.

Let us make a remark concerning the possibility of using the concept of effective temperature. The method of effective temperature can be correctly utilized in the cases when the frequency of the interelectron collisions τ_{ee}^{-1} is larger than the reciprocal energy relaxation time τ_e^{-1}[5]. We have seen that the superheating mechanism of the occurrence of the S-shaped CVC is realized most frequently when the scattering of the momentum is by a charged impurity, and the energy is given up to the acoustic phonons. Since the momentum scattering by the impurity is predominant, the time of momentum relaxation by phonons is larger than τ_p, and all the more $\tau_e > \tau_p$ (the inelasticity of the scattering by acoustic phonons is small). The time of redistribution of energy among the electrons as a result of collisions between them is $\tau_{ee} \sim (N_i/n)\tau_p$, where N_i is the concentration of the scattering centers[5]. Generally speaking, $N_i > n$ owing to the possible compensation of the impurities (at low compensation $\tau_{ee} \sim \tau_p$, and consequently $\tau_{ee} \ll \tau_e$. In this case the symmetrical part of the electron distribution function in a strong field is close to Maxwellian with a certain temperature T larger than the temperature of the crystal lattice T_0.

The conclusion of the presence of a decreasing section of the CVC in a quantizing magnetic field is not connected with the presence of electron-electron collisions and with the Maxwellian form of the energy distribution function[13,17].

S-shaped CVC were observed in the purest samples of n-InSb ($n \sim 10^{13}$–10^{14} cm^{-3}) at helium temperatures. At $H = 0$ the carrier density does not depend on T. Therefore the S-shape observed at $H = 0$ and $T_0 < 2.5\,^\circ$K[20,21] is apparently connected not with the breakdown of the impurities but with the superheat mechanism, namely with scattering of the momentum by the impurity and the scattering of the energy by the piezoelectric potential of the acoustic phonon[10]. In a magnetic field, the S-shape is much more strongly pronounced[22-24]. It can be connected not only with the action of the above-described mechanism of Kazarinov and Skobov[13,17], but also with breakdown, i.e., with ionization of neutral donors, the levels of which are split from the continuous spectrum by the strong magnetic field. The effect of transition from the S-shape characteristic to the N-shaped characteristic when the measurement regime in a transverse field is changed, which was observed in[19], is evidence that the negative resistance is due to the superheating mechanism. In[120], the nonlinearity of the CVC of n-InSb at helium temperatures is connected with shock excitation of the electrons from the impurity band into the conduction band, and the dependence of the resistance of the electric and magnetic fields at 53°K is connected with the mechanism of Kazarinov and Skobov.

1.2. The Ridley-Watkins-Hilsum mechanism[2,25,26]

In a number of semiconductors (GaAs, InP) the conduction band has a minimum at the center of the Brillouin zone, and in addition several minima located symmetrically at a certain distance from the center and having a higher energy compared with the central one (Fig. 2). It is significant that the effective mass of the electrons in the upper valleys is much larger than in the lower valley. The large value of the effective mass leads to a small mobility. This is due not only to the explicit dependence of μ on m, $\mu = e\tau_p/m$: the momentum scattering time decreases with increasing mass as a result of an increase in the density of state.

We denote by $n^{(0)}$ and $\mu^{(0)}$ the concentration and the mobility of the electrons in the lower valley, and by $n^{(1)}$ and $\mu^{(1)}$ the same quantities for each of the ν upper valleys. Let Δ be the difference between the energies of the upper and lower minima. If the lattice temperature is $T_0 \ll \Delta$, then in weak electric fields almost all of the electrons are concentrated in the lower valley, $n^{(0)} = n$, and $n^{(1)} = 0$. With increasing electric field and electron temperature, an ever increasing number of electrons goes over into the upper valleys with $\mu^{(1)} \ll \mu^{(0)}$, the specific electric conductivity

$$\sigma = e(\mu^{(0)}n^{(0)} + \nu\mu^{(1)}n^{(1)}) = en\mu^{(0)}\, 1 - \frac{\mu^{(0)} - \mu^{(1)}}{\mu^{(0)}n}\nu n^{(1)} \qquad (1.8)$$

decreases rapidly with increasing field, so that starting with a certain field the current density also decreases with the field ($\sigma_d < 0$). Ultimately, in very strong fields, the decrease of σ with increasing field should slow down or should stop completely; the current will again increase with increasing field. On the whole, the CVC should have an N-shape (Fig. 2). Semiconductors in which the CVC is connected with the mechanism described above are called for brevity two-valley semiconductors.

The most investigated from this point of view is GaAs. In this material the upper minima are located on the [100] axis. Their number is not known exactly ($\nu = 3$ or 6), $\Delta = 0.35$ eV. The effective mass in the lower valley $m^{(0)} = 0.07m_0$, and in the upper valleys $\nu^{2/3}m^{(1)} = 1.2m_0$. The mobilities in the customarily employed materials are $\mu^{(0)} = 5 \times 10^3$ cm^2/V-sec and $\mu^{(1)}$ 10^2cm^2/V-sec[26,27]. The current begins to decrease with increasing field at $E \cong 3 \times 10^3$ V/cm[30-33].

Let us make a rough estimate of the characteristic dimensions, starting from the following model. We assume that the electrons have Maxwellian energy distributions in all the valleys. The temperature T of the light electrons is larger than the lattice temperature T_0, and that of the heavy ones coincides with T_0 because of the large frequency of collision with the phonons. Then, when $T \ll \Delta$, we have

$$\frac{\nu n^{(1)}}{n^{(0)}} \simeq \frac{\nu N^{(1)}}{N^{(0)}} \exp\left(-\frac{\Delta}{T}\right), \qquad (1.9)$$

where $N^{(1)}$ and $N^{(0)}$ are the effective numbers of states, the ratio of which is $(m^{(1)}/m^{(0)})^{3/2}$ and is large. Substituting (1.9) in (1.8) and calculating the differential conductivity, we find that σ_d reverses sign when

$$\frac{N^0}{\nu N^{(1)}} \exp\left(\frac{\Delta}{T}\right) = \frac{\Delta}{T}\frac{d\ln T}{d\ln E} - 1. \qquad (1.10)$$

We have put here $\mu^{(0)} \gg \mu^{(1)}$. As shown by calculation[33], d ln T/d ln E ~ 1. When the parameters are suitably chosen, Eq. (1.10) has two roots T/Δ, corresponding to the fields E_{c1} (smaller root) and E_{c2} (larger root) on the N-shaped CVC (Fig. 2). At a large value of $\nu N^{(1)}/N^{(0)}$, the smaller root T/Δ is small. This means that the electron temperature in a field E_{c1} equal to the threshold field is lower than Δ, and the number of heavy electrons is smaller than the number of light electrons by a factor $\sim T/\Delta$. In GaAs we have $\nu N^{(1)} + N^{(0)} \sim 60$, and therefore $T/\Delta \sim 1/6$. It is possible that this is cause why the concentration of the light electrons, measured by means of the hole coefficient near the threshold field[34,124] does not change noticeably.

As seen from (1.10), when Δ decreases the electron temperature corresponding to the threshold field decreases, and hence also the value of E_{c1} itself. The decrease of E_{c1} was observed experimentally. The value of Δ was varied in[35] by applying pressure, and in[36] by replacing the arsenic atoms in the GaAs by phosphorus.

A theoretical calculation of the CVC of two-valley semiconductors is a difficult task, because many parameters characterizing the scattering of the electrons are in essence unknown. Concrete calculations for GaAs under certain simplifying assumptions were performed in[33,37]. The parameters of N-shaped CVC, obtained in these investigations, did not differ greatly from each other.

Direct measurement of j(E) in constant fields above threshold (E_{c1}) is impossible by virtue of the resultant instability (see Ch. 2). The measurement time should be shorter than the time of instability development, but longer than the time of establishment of the electron energy distribution, which determines the form of j(E). Gunn and Elliott[29] applied to the sample short pulses (0.25×10^{-9} sec) of a strong field. It turned out that the CVC continued to decrease up to ~20 kV/cm, and the differential mobility σ_d/en beyond the threshold field is equal to 300 cm^2/V-sec. The latter result does not agree with the data of almost all the other investigations (see below). The slope of the decreasing section of the CVC of n-GaAs turned out to be higher (Fig. 4).

In the experiment of Ruch and Kino[32], the electrons were injected in a sample of isolating GaAs by a pulsed method with the aid of an electron gun. The form v(E) of the CVC was determined from measure-

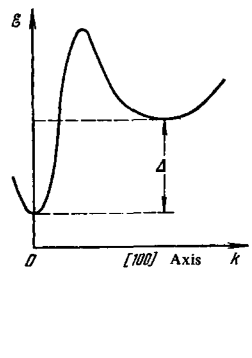

FIG. 3. Energy structure of GaAs (schematic).

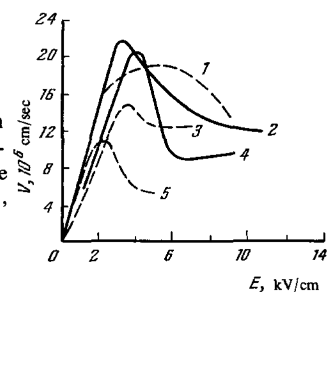

FIG. 4. Dependence of the drift velocity of the electrons in n-GaAs on the field when the latter is homogeneously distributed over the sample. The plots are taken from: 1–[38], 2–[32], 3 and 5–[39], 4–[30].

ments of the dependence of the time of passage of the pulse through the sample on the applied field.

In many experiments[38-41] the electrons were heated by a strong microwave field. The dependence of the conductivity in additional weak constant fields on the amplitude of the alternating field was measured and used to reconstruct the form of the static CVC. The CVC was measured by other methods in[30,31]. In all the indicated experiments, it was found that the CVC continued to decrease up to the strongest applied fields (~20 kV/cm), and there are no reliable observations of a second increasing section of the CVC.

In Ge under uniaxial deformation, the degeneracy of the hole bands in the center of the Brillouin zone is lifted. Two "new" hole bands are produced with ellipsoidal equal-energy surfaces. The energy gap between them is proportional to the deformation. Following compression along the [111] or [100] axis, the effective mass in the compression direction (which coincides with the direction of the electric field) is larger in the hole band that has the larger energy[111]. Therefore the CVC of compressed p-Ge can have an N-shaped form[2]. The generation of oscillations in p-Ge, connected with this effect, was observed by A. A. Kastal'skiĭ and S. M. Ryvkin[88].

1.3. Recombination and Ionization Mechanisms

In an impurity semiconductor with one type of carrier (for example, conduction electrons) the stationary carrier density is determined by the recombination equilibrium: the number of electrons excited from the impurity atoms into the conduction band per unit time is equal to the number of electrons captured by the impurity atoms from the band. By heating the electrons, a strong electric field shifts the recombination equilibrium and consequently changes the stationary carrier density. This leads in many cases to the occurrence of a decreasing branch of the CVC. The most investigated two effects are the occurrence of an N-shaped CVC in semiconductors in which the carriers are captured by repelling centers, and of an S-shaped CVC in the case of low-temperature impurity breakdown in compensated semiconductors.

Let us consider for concreteness n-Ge doped with Au and compensated with an element of group V. The electrons are captured by the Au^{2-} ions, and the thermal motion of the lattice excites the electrons from Au^{3-} into the band:

$$Au^{2-} + e^- \rightleftarrows Au^{3-}.$$

The Au^{2-} ion binds the conduction electrons (the energy of excitation into the band is 0.04 eV), but at large distances the electron is repelled by the Coulomb field of the ion. Therefore the electrons with higher energy have a greater probability coming closer to the center without being captured. Heating of the electron gas leads to an increase of the capture velocity and to a decrease of the stationary concentration. Thus, in Ge at 20°K the coefficient of capture of the electrons by Au^{2-} increases from 2×10^{-12} cm^3 sec^{-1} in weak fields to 3.5×10^{-9} cm^3 sec^{-1} in fields on the order of several kV/cm[42,43]. The dependence of the cross section of carrier capture by the repelling center on the temperature and on the field was calculated in[44].

If the carrier capture coefficient increases sufficiently rapidly with increasing field, the CVC may become N-shaped[45]. The effect was observed in Ge doped with Au[46,43] and then in Ge doped with copper (capture by Cu^{2-} ions)[47].

Boer[48] observed phenomena connected with the N-shape of the CVC in CdS. To explain the N-shape, the following model was proposed[49]. Assume that the conductivity in the semiconductor is determined by electrons that be combined with holes on deep centers. In weak fields, the holes are at shallow levels (close to the valence band). A strong field ionizes these levels, so that the holes go over to deep recombination centers. This increases the rate of recombination of the electrons and decreases their concentration.

The change of the recombination equilibrium as a result of heating of the carriers can lead to an occurrence of not only N-shaped but also a S-shape CVC. A known example is the low temperature breakdown in germanium and silicon doped with shallow impurities and compensated[50-53]. In these materials, the concentration of the carriers and the current increase with increasing field. If the sample is connected in a circle with a large load and the current increases then the electric field, reaching a certain value E_c ("breakdown initiation field"), it decreases jumpwise to E_s (Fig. 5). With further increase of the current, the field remains unchanged, so that the observed CVC is vertical. Accordingly, E_s is called "the field necessary to maintain the breakdown". The indicated features of the observed CVC are connected with the pinching of the current (see Ch. 5).

The carrier density and the electric conductivity increase with increasing field, owing to impact ionization of the impurities and to the decrease of the rate of carrier capture by the ionized impurity centers (the cross section for capture on attractive impurities decreases with increasing electron energy[54]). There is still no meeting of the minds concerning the concrete mechanism producing the decreasing branch on the characteristics of compensated semiconductors[50,55,56]). It is clear, however, that the non-unique dependence of the electron density on the field can occur only when the form of the distribution function of the electrons by

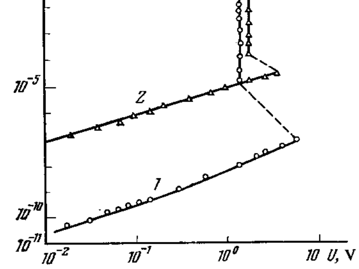

FIG. 5. CVC of Ge doped with In and strongly compensated with Sb. Sample 2 is doped more than sample 1. Temperature 4.2°K. The circles and triangles correspond to pulsed measurements. Sample length 0.048 cm; contact diameter 0.1 cm.

energies depends on the degree of ionization of the impurity (and not only on the field)[57].

2. INSTABILITY OF HOMOGENEOUS STATES WITH NEGATIVE DIFFERENTIAL CONDUCTIVITY

In the very first theoretical investigations of semiconductors with negative volume differential resistance, it was concluded that the homogeneous states with $\sigma_d < 0$ are unstable[1,2,28]. In systems with N-shaped CVC, in which the characteristic times of the processes determining the form of the CVC are smaller than the Maxwellian time $\tau_M \sim \epsilon/4\pi|\sigma_d|$, the fluctuation of the charge disappears within a time on the order of τ_M if $\sigma_d > 0$, and increases with an increment $\sim|\sigma_d|$ if $\sigma_d < 0$. However, considering the behavior of small perturbations, we can not only determine the stability of the system, but also obtain many important characteristics of the possible instability. The method used in this case for the investigation is well known (see, for example,[58]). The initial equations are linearized for perturbations having the form, for example, of plane waves, $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$, and from the condition for the existence of nonzero solutions, one obtains the dependence $\omega(\mathbf{k})$. The latter determine the characteristic wavelengths of the increasing or damped perturbations, their velocity, and the time of instability development. In particular, we are interested in the direction of the wave having the largest growth increment. Knowing the dispersion relation $\omega(\mathbf{k})$, we can establish the character of the instability—whether it is absolute or convective[58,59].

The form of $\omega(\mathbf{k})$, of course, depends on the concrete system, i.e., on the concrete equations describing the system. A rather wide class of systems can be regarded by using the effective-temperature method. According to this method, the symmetrical part of the electron distribution function has under non-equilibrium conditions the form of a Maxwellian function with a certain temperature T (see Sec. 1.1). In the case of low inelasticity of the collisions between the electrons and the phonons, the asymmetrical part of the distribution function can be expressed with the aid of the Boltzmann equations in terms of the symmetrical part[5,60]. This makes it possible to obtain expressions for the current density $\mathbf{j}$ and for the energy flux density $\mathbf{j}_\epsilon$:

$$\mathbf{j} = \sigma\mathbf{E} - \nabla\left(\frac{\sigma T}{e}\right), \tag{2.1}$$

$$\mathbf{j}_\epsilon = \left(\frac{3}{2}\right) nT\mathbf{v} + p\mathbf{v} + \alpha_{st}T\mathbf{j} - \varkappa\nabla T; \tag{2.2}$$

here $\sigma(T) = e\mu(T)n$ and $\kappa(T)$ are the specific electric conductivity and the thermal conductivity of the electrons, μ is the mobility, $p = nT$ the pressure, $\mathbf{v} = \mathbf{j}/ne$ the drift velocity, and $\alpha_{st} = e^{-1}d\ln\mu/d\ln T$ is the part of the differential thermal emf connected with the collisions. Expressions (2.1) and (2.2) are valid at frequencies ω that are small compared with the collision frequency τ_p^{-1}.

The first term in the expression for $\mathbf{j}$ is the field current, and the second comprises the diffusion and thermoelectric currents. The energy flux density $\mathbf{j}_\epsilon$ is best represented in the form (2.2), since the flux of the internal energy (first term), the flux connected with the work of the pressure forces (second term), and the heat flux (the last two terms) are then clearly separated.

The concentration of the electrons and the temperature $T(\mathbf{r}, t)$ should satisfy the continuity and the energy conservation equations

$$e\left(\frac{\partial n}{\partial t}\right) + \operatorname{div}\mathbf{j} = 0, \tag{2.3}$$

$$\frac{3}{2}\frac{\partial(nT)}{\partial t} + \operatorname{div}\mathbf{j}_\epsilon - \mathbf{j}\mathbf{E} + P = 0. \tag{2.4}$$

In this form, the equations pertain to conditions in which the recombination of the carriers and the impact ionization of the impurities and of the valence electrons can be neglected. For example, in n-GaAs under conditions when stationary domains are observed, the electron concentration is most frequently constant.

Usually the systems under consideration satisfy the inequality $k^2\delta^2 \gg 1$, where k is the reciprocal of the characteristic scale of the inhomogeneity of the electric field and $\delta = c(2\pi\sigma\omega)^{-1}$ is the depth of the skin layer at the characteristic frequency ω. In this case the solenoidal field can be neglected (curl $\mathbf{E} = 0$), and $\mathbf{E}$ can be determined from the Poisson equation

$$\operatorname{div}\mathbf{E} = \frac{4\pi e}{\varepsilon}(n - n_0); \tag{2.5}$$

Here and below, the field is assumed positive if it is directed along the electron drift.

In various concrete cases, the system (2.1)–(2.5) can be simplified.

2.1. Superheat Instability

Let us consider semiconductors in which $\tau_e \gg \tau_M$ (in n-InSb, where an S-shaped CVC is observed at helium temperature[20-24], this condition is usually satisfied). As we shall show, in this case the characteristic frequencies are $\omega \lesssim \tau_e^{-1}$, and the characteristic wavelengths k^{-1} are larger than or of the order of the length of the scattering of energy by the electrons $l \sim v_T(\tau_p\tau_e)^{1/2}$, where v_T is the thermal velocity. Therefore $\omega\tau_M \ll 1$, $kl_D \ll 1$ (here l_D is the Debye screening length), and the electron gas can be regarded as incompressible, i.e., the electron density in (2.1)–(2.4) is constant.

Let us first analyze qualitatively the stability of a semiconductor with S-shaped CVC. If the field E in the sample is fixed and is between E_{c2} and E_{c1}, the semiconductor can be in three homogeneous states with different values of the current density (for example, j_1, j_2, and j_3 in Fig. 1). The state with $\sigma_d < 0$ ($j = j_2$) is unstable against homogeneous perturbations (independent of the coordinates)[6,31], and the semiconductor goes over into one of the stable states. On the other hand, if the current in the sample is fixed (by the load resistance), then the homogeneous stationary state is unique and is stable against homogeneous perturbations even when $\sigma_d < 0$. It is, however, unstable against inhomogeneous perturbations that do not change the total resistance of the sample and are not connected with the external circuit. Let us assume that a homogeneous electron temperature fluctuation δT has been produced in the sample in the direction of the current. In one part of the cross section $\delta T > 0$ and in the other δT

< 0, so that the total resistance and the current remain unchanged. Since the field is not solenoidal, the fluctuation, which does not depend on x (the Ox axis is parallel to the current) does not perturb the field component E_x (this is precisely the "working" field component that makes a contribution to the Joule heat). The current density at each point changes by $(d\sigma/dT)E_x\delta T$, and the rate of change of the electron-gas energy changes by

$$\delta[\mathbf{jE}-P(T)]=\left(\frac{d\sigma}{dT}E_x^2-\frac{dP}{dT}\right)\delta T.$$

If the characteristic is S-shaped, the sign of the expression in the brackets is opposite the sign of σ_d (see (1.3)). When $\sigma_d < 0$, the fluctuation δT causes such a change in the difference $\mathbf{j}\cdot\mathbf{E} - P$, as to produce further growth of the fluctuation amplitude. In one part of the sample (where the initial $\delta T > 0$) the temperature increases, and in the other ($\delta T < 0$) it decreases. We note that in the case of an N-shaped CVC ($d\sigma/dT < 0$) the fluctuation that is homogeneous along the current will always attenuate.

In the opposite limiting case, when the perturbation δT is homogeneous over the cross section of the sample, but varies along the current, the current density remains unchanged: $\delta_j = 0$, by virtue of the continuity of the current. The field, however, changes by $\delta E_x = (d\sigma^{-1}/dT)j\delta T$, and therefore the rate of change of the electron-gas energy changes by

$$\delta[\mathbf{jE}-P(T)]=-\left(\frac{d\sigma}{dT}E_x^2+\frac{dP}{dT}\right)\delta T.$$

It follows from (1.3) that in the case of a S-shaped CVC such a fluctuation always attenuates, and in the case of an N-shaped characteristic it increases if $\sigma_d < 0$.

Thus, the perturbations that break up the sample into layers (pinches) stretched along the current grow in a semiconductor with an S-shaped CVC, and in the case of an N-shaped characteristic the growing fluctuations break the sample up into regions with different values of E_x, but with homogeneous cross sections. It is precisely these fluctuations which have, in each case, the largest increment. The latter can be seen also from the dispersion equation, which is obtained by linearizing Eqs. (2.1)–(2.4) with allowance for the incompressibility of the electron gas:

$$\omega(\mathbf{k})=k_xv\left[1+\left(\frac{eT}{c_e}\right)\left(\frac{d\alpha_{st}}{dT}\right)\right]-\\ -i(nc_e)^{-1}\left[\frac{dP}{dT}-E_x^2\left(\frac{d\sigma}{dT}\right)\frac{k_\perp^2-k_x^2}{k^2}\right]-i\frac{\varkappa}{nc_e}k^2, \qquad (2.6)$$

where

$$k_\perp^2=k_y^2+k_z^2,\ k^2=k_\perp^2+k_x^2.$$

The thermal conductivity (which corresponds to the last term in (2.6)) leads to a damping of the short-wave perturbations with a wavelength smaller than

$$l_c=2\pi\varkappa^{1/2}\left[E_x^2\left(\frac{d\sigma}{dT}\right)-\frac{dP}{dT}\right]^{-1/2}. \qquad (2.7)$$

In order of magnitude, l_c is the energy scattering length, $v_T(\tau_e\tau_p)^{1/2}$. According to the data of[21], this length is $\sim 10^{-2}$–10^{-3} cm in n-InSb at helium temperatures.

The largest growth increment ($\sim\tau_e^{-1}$) is possessed by long-wave perturbations. In such perturbations, in the case of an S-shaped CVC ($k_x = 0$) we have Re $\omega(\mathbf{k}) = 0$, i.e., the instability is aperiodic and absolute. In semiconductors with S-shaped CVC, the growth increment of the long-wave perturbations is of the order of τ_e^{-1} regardless of the relation between τ_M and τ_e, i.e., it is of the order of the reciprocal time of establishment of CVC.

The above-considered superheat instability is well known in a gas plasma[4]. As applied to a plasma of superconductors with various ratios of their parameters, it was investigated in[60-63,117].

2.2. Instability in Two-valley Semiconductors

In this case the experiments are performed usually on samples with not too large a conductivity (to avoid heating and breakdown), so that the largest of the characteristic times is Maxwellian. It exceeds the energy relaxation time and the time of the intervalley transitions, Therefore the electron gas cannot be regarded as incompressible.

We present first a qualitative explanation of the instability. Assume that an electron-density fluctuation has been produced in a current-carrying semiconductor, in the form of a dipole layer: the concentration increases in one region and decreases by the same amount in the neighboring region located "farther down" in the electron drift direction. Inside such a dipole layer, the field increases. If the differential conductivity is negative, the current inside the dipole layer decreases. As a result, the initial increase of the concentration of the electrons increases still further (more electrons flow into this region than out). The region in which the concentration decreased initially, loses still more electrons.

Let us obtain the dispersion equation for the growing perturbations. Owing to the insufficiently large electron concentration, the frequency of the interelectron collisions is not large enough to make the electron energy distribution function Maxwellian. The effective-temperature approximation cannot be used; to describe the considered phenomena in two-valley semiconductors it would be necessary to solve the kinetic equation for the light and heavy electrons, with allowance for the inhomogeneity of the field and of the concentration. Such a program, however, is difficult to perform. It is reasonable to consider approximate models.

In particular, we can expect the effective-electron-temperature approximation to give the correct picture of the phenomena in two-valley semiconductors also in the case when the interelectron collisions are insignificant. The model in which the electron distribution function in a two-valley semiconductor is regarded Maxwellian with a certain single effective temperature T was investigated by McCumber and Chynoweth[72]. It was assumed that the equilibrium between the valleys is established more rapidly than the electron temperature, so that the ratio of the concentrations of the heavy and light electrons is a definite (exponential) function of T at the same point. The temperature satisfies the energy conservation equation (it is incorrectly written out in[72]). The system of equations, even in these approximations, is too complicated for an analytic investigation of the nonlinear solutions. The equations in[72] were therefore solved numerically.

An analytic description of waves in a two-valley semiconductor was presented in[64-68] on the basis of model equations. The gist of the model consists in the following:

In the expression (2.1) for the current density, the mobility $\mu(x)$ is assumed to be an explicit function of the field $E(x)$ (even in the spatially-inhomogeneous case), and furthermore such that the drift velocity $\mu(E)E$ is an N-shaped function of the field E. In addition, the diffusion coefficient is assumed to be a function of the field, D(E), and the thermal current is neglected. In final analysis it is assumed that

$$j = en\mu(E)E - eD(E)\frac{dn}{dx}. \tag{2.8}$$

Actually the mobility averaged over both types of electrons (light and heavy) is more readily determined by their effective temperature. The latter, on the other hand, is not a function of only the field E at this point, since the diffusion current makes a contribution to the heating. Neglect of the diffusion current and of other gradient terms in the equation for the effective temperature (or of the corresponding terms in the equation for the energy distribution function) leads, in the calculation of the current j, to an error of the same order as the diffusion current, which is retained in (2.8). Therefore expression (2.8) can be regarded only as a model expression. Nonetheless, this expression, together with (2.3) and (2.5), describes qualitatively correctly the physics of the phenomena (this will be discussed later).

We have verified earlier that in a semiconductor with an N-shaped CVC and $\sigma_d < 0$, the largest growth increment is possessed by waves that depend only on the coordinate along the current. We shall therefore assume all the quantities as a function of only one coordinate (and of the time). In this case (see (2.3) and (2.5)), the current in the external circuit, divided by the cross section area of the sample, j_{ext}, is equal to the sum of the densities of the conduction current and of the displacement current:

$$j_{ext}(t) = j(x, t) + \frac{\varepsilon}{4\pi}\frac{\partial E}{\partial t}. \tag{2.9}$$

Linearizing Eqs. (2.5), (2.8), and (2.9), we obtain the dispersion equation

$$\omega(\mathbf{k}) = kv - i\frac{4\pi}{\varepsilon}\sigma_d - iDk^2, \tag{2.10}$$

where $\sigma_d = en_0 d(\mu E)/dE$. The meaning of the first two terms is quite simple. The charge fluctuation attenuates (when $\sigma_d > 0$) or grows (when $\sigma_d < 0$) within the Maxwellian time* and is carried away by the electron stream with the drift velocity of the unperturbed motion.

The last term in (2.10) describes the suppression of the short-wave perturbations by the diffusion. The question of the role of the diffusion is not obvious, since an inconsistency was admitted in the derivation of (2.10), namely, as noted above, in writing down the model equation (2.8), practically no account was taken of the influence of the gradients of the different quantities (including the concentration) on the effective temperature and on the mobility. However, the stability against small perturbations can be investigated also without resorting to the assumptions that lead to expression (2.8) for the current. In[62,69,117] there were considered several different models of a semiconductor with N-shaped CVC. The obtained dispersion relations $\omega(\mathbf{k})$ have the same form as (2.10), and differ only in factors on the order of unity preceding Dk^2.* This agreement makes the use of the model equation (2.8) more justified.

S. I. Anisimov, V. I. Mel'nikov, and É. I. Rashba proposed a different for an analytic investigation of the waves in a two-valley semiconductor[115]. In expression (2.8) for the current density, μ and D are assumed to be functions not of the field but of the power received on the average by one electron from the field, and equal $jE/n = evE$. The assumed expression for j, like (2.8), is not rigorous, but seems to be more justified physically.

The results obtained with all the three foregoing models coincide qualitatively.

We shall show below that as a result of the instability of the homogeneous field distribution in the sample, stationary waves are produced, with a velocity equal to the drift velocity of the electrons, just as the velocity of the perturbations under consideration. According to (2.10), the fastest to grow, within a time $\sim |\sigma_d|^{-1}$, are the long-wave perturbations; accordingly, long-wave harmonics predominate in the spectrum of the stationary waves. Since the minimum value of k is determined by the sample length l_X, the frequency of the oscillations produced in the sample can be estimated by putting $k = l_X^{-1}$ in (2.10), namely $\mathrm{Re} \cong v/l_X$. It follows also from (2.10) that no oscillations will be produced in a sample of sufficiently short length l_X, and this sample is stable against the considered perturbations even if $\sigma_d < 0$. There are two reasons for this. If $l_X < l_D(\sigma/|\sigma_d|)^{1/2}$, where $l_D = (D\epsilon/4\pi\sigma)^{1/2}$ is the Debye screening length, then the instability is suppressed by diffusion. On the other hand, if l_X is smaller than the characteristic length the fluctuation growth $l_{dr}(\sigma/|\sigma_d|)$, where the "drift length" is $l_{dr} = v(\epsilon/4\pi\sigma)$ (the drift length within the Maxwellian time), the fluctuation is carried over the system before it has a chance to grow appreciably. If l_X is smaller than the larger of the lengths l_D or l_{dr}, the system is stable†. One should expect the characteristic dimension of the stationary wave occurring as a result of the instability (domain length) to be of the order of the larger of the lengths l_D or l_{dr}.

*We note that in those cases when the Maxwellian time is much shorter than the characteristic time τ of the establishment of the CVC (for example the time of energy scattering, etc.), the growth increment of the considered perturbations is of the order of τ^{-1}.

*Blotekjaer [70] pointed out that it is incorrect to assume the relation $\mu(x) = \mu(E(x))$. By a linear analysis of the stability he found that the diffusion does not suppress the short-wave perturbations when $\sigma_d < 0$. However, the assumption $\mu(x) = \mu(v(x))$ on which he based the calculation is incorrect, and consequently the deduced role of the diffusion is in error.

†Strictly speaking, it will be stable also against the considered perturbations that are not connected with contacts of the sample. Perturbations are also possible in which the total charge is changed inside the sample (with the compensating charge on the contact). These include also perturbations that are not carried away by the electron drift.

The ratio $\alpha = l_{dr}/l_D$ of the drift and the Debye lengths determines the type of instability (absolute or convective[58]). Indeed, the evolution of a field perturbation that had a δ-like form at t = 0 and was localized at the origin, is described by the expression (t > 0)

$$\delta E(x, t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp[ikx - i\omega(k)t]\,dk. \quad (2.11)$$

Substituting here (2.10) and integrating, we find that at the point x = 0

$$\delta E(0, t) = (4\pi Dt)^{-1/2} \exp\left[-\left(\frac{4\pi\sigma t}{\varepsilon}\right)\left(\frac{\sigma_d}{\sigma} + \frac{\alpha^2}{4}\right)\right]. \quad (2.11a)$$

When $\alpha > 2|\sigma_d/\sigma|^{1/2}$ the fluctuation attenuates at the point of its initial localization even when $\sigma_d < 0$ and the instability is of the drift type. In the opposite case, $\alpha < 2|\sigma_d/\sigma|^{1/2}$, the instability is absolute.

The conclusion that a system with not too large dimensions is unstable, was confirmed also experimentally. Usually in GaAs we have $\alpha > 1$ and, if the sample length is $l_x < l_{dr}$, stationary moving waves do not have time to form in it. The latter inequality is a condition for the product of the electron concentration by the sample length $n_0 l_x$*. A numerical calculation based on the temperature model[72] gives for the critical value $n_0 l_x$ in GaAs at room temperature a value 3×10^{11} cm^{-2}.

If the condition $l_x < l_{dr}$ is not satisfied, then the instability produces in the semiconductor a stationary wave of large amplitude—domain of increased resistance, and the current decreases (the Gunn effect[73]). The motion of the domains leads to generation of current oscillations in the external circuit. In high-resistance semiconductors, in which $l_x < l_{dr}$, the current fluctuations actually are not observed[74]. Such samples are used to intensify the oscillations.

2.3. Recombination Instability

Let us consider a semiconductor in which there are impurity atoms with concentration N, capable of capturing one electron each. We denote the concentration of the impurity atoms capturing the electron by n_t, the concentration of the free electrons will again be denoted by n. The change of the concentration n_t with time is described by the kinetic equation

$$\frac{\partial n_t}{\partial t} = An(N - n_t) - Bn_t. \quad (2.12)$$

The coefficient of thermal or optical generation B is assumed to be constant; A is the coefficient of electron capture by the impurity. The charge density is

$$\rho = e(n_t + n) - eN_{d,\,\mathrm{eff}}, \quad (2.13)$$

where $N_{d,eff}$ is the "effective donor concentration." (In the case of Ge doped with Au and compensated with a donor of group V (so that $2N_{Au} < N_V < 3N_{Au}$) we have $n_t = N_{Au}^{-3}$ and $N_{d,eff} = N_V - 2N_{Au}$.) We eliminate n_t from (2.12) and (2.13). Usually $n \ll N - N_{d,eff}$. Then

$$\frac{\partial\left(\frac{\rho}{e} - n\right)}{\partial t} = \frac{n}{\tau} - \frac{\rho}{e\tau_g} - BN_{d,\,\mathrm{eff}},$$
$$\tau^{-1} \equiv A(N - N_{d,\,\mathrm{eff}}), \quad \tau_g^{-1} \equiv An + B. \quad (2.14)$$

In the stationary state $n = B n_{d,eff}$, and τ is a decreasing function of the field E. Therefore the differential conductivity

$$\sigma_d = \sigma\left(1 + \frac{d\ln\tau}{d\ln E}\right) \quad (2.15)$$

reverses sign in sufficiently strong fields.

Generally speaking, in an inhomogeneous field, the capture coefficient A is not a function of the field alone at the same point. We confine ourselves to the case when the characteristic length of the inhomogeneity $l \gg l_D$, where l_D is the length for screening by the free electrons and, in addition, l is much larger than the energy scattering length (the latter is usually the case). Then the diffusion and thermoelectric currents can be neglected, and we can assume that A = A(E). For simplicity we consider the case $\tau \gg \tau_M$,* when the displacement current in (2.9) can be neglected and it can be assumed that

$$j_{\mathrm{ext}} = en\mu E. \quad (2.16)$$

We linearize Eqs. (2.14) and (2.16) and the Poisson equation. The dispersion equation takes the form[67]

$$i\omega\tau = \left(\frac{\sigma_d}{\sigma} + \frac{ikv\varepsilon\tau}{4\pi\sigma\tau_g}\right)\left(1 + \frac{ikv\varepsilon}{4\pi\sigma}\right)^{-1}. \quad (2.17)$$

The largest growth time in a time $\sim\tau$ is possessed by the long-wave perturbations. Perturbations with a wavelength shorter than $l_c\ (\sigma/|\sigma_d|)$, where

$$l_c = v\frac{\varepsilon}{4\pi\sigma}\left(\frac{\tau}{\tau_g}\right)^{1/2}, \quad (2.18)$$

attenuate already when $\sigma_d < 0$. The phase velocity of the short waves is

$$v_{ph} = \mathrm{Re}\left(\frac{\omega}{k}\right) = v\frac{\varepsilon}{4\pi\sigma\tau_g} \quad (2.19)$$

and is much shorter than the drift velocity. Thus, for Ge doped with Au, at $T_0 = 20$°K and $n = 10^9$ cm^{-3}, we obtain $v_{ph}/v \sim 10^{-10}$ (the parameters for the calculation were taken from[43], in which recombination instability was observed). The small value of this ratio is connected with the capture of the carriers by the immobile centers. We note that the perturbation drift velocity $v_{ph}\tau$ is always smaller than l_c.

The instability considered here must not be confused with the recombination instability considered by Konstantinov and Perel'[116]. The latter can occur in semiconductors with sufficiently large concentration of minority carriers, at drift velocities exceeding certain threshold value. This instability is not connected with heating of the carriers, and consequently with the presence of a decreasing section on the CVC of the semiconductor.

*An analogous condition was first obtained from other considerations by Kroemer [71].

*A linear theory of recombination instability, for an arbitrary ratio τ/τ_M, was presented by Ridley [75].

3. STATIONARY WAVES OF FINITE AMPLITUDE IN SEMICONDUCTORS WITH N-SHAPED CVC

The growth of small perturbations in the homogeneous state with $\sigma_d < 0$ is limited by nonlinear effects. It can be expected that the new flow of the semiconductor plasma, resulting from the development of the instability, is a stationary wave, in which the effective temperature and the field are functions of the argument $x' = x - ct$, where C is the constant velocity of the wave. In the system of coordinates x' connected with the wave, the stationary waves are solutions of nonlinear but ordinary differential equations. We shall now investigate them.

3.1. Initial Equations

We rewrite Eqs. (2.8), (2.9), and (2.5), assuming all quantities to be functions of x' and t. After eliminating $n(x', t)$ and going over to dimensionless quantities, we obtain the following equation for the field:

$$\mathscr{D}\mathscr{E}_{\xi\xi}+\alpha[s-u(\mathscr{E})]\mathscr{E}_{\xi}+[1-u(\mathscr{E})]=\mathscr{E}_{\eta}(\xi,\eta); \qquad (3.1)$$

Here

$$\xi=(x-ct)/l_D(E_2),\ \eta=t/\tau_M(E_2),\ l_D(E_2)=(\varepsilon D/4\pi e\mu n_0)^{1/2}_{E_2}-$$

is the screening length for the field E_2 (see below),

$$\tau_M(E_2)=[\varepsilon/4\pi e n_0\mu(E_2)],\quad \mathscr{E}=\frac{E}{E_2},$$

$$\mathscr{D}=\frac{D(E)}{D(E_2)},\quad u=\frac{\mu(E)E}{\mu(E_2)E_2},\quad s=c/\mu(E_2)E_2,\quad \alpha=l_{dr}(E_2)/l_D(E_2).$$

The field E_2 is described by the condition $j_{ext} = j(E_2)$ (Fig. 2). The lower index denotes differentiation with respect to the corresponding argument.

For stationary waves, there is no time derivative in the right hand side of (3.1), and the equation for $E(\xi)$ takes the form

$$\mathscr{E}_{\xi\xi}+K(\mathscr{E})\mathscr{E}_{\xi}+\frac{dU}{d\mathscr{E}}=0; \qquad (3.2)$$

$$U=\int^{\mathscr{E}}\frac{d\mathscr{E}[1-u(\mathscr{E})]}{\mathscr{D}(\mathscr{E})},\quad K=\frac{\alpha[s-u(\mathscr{E})]}{\mathscr{D}}. \qquad (3.3)$$

Let us find an analogous equation for stationary waves in the case of the recombination mechanism of the N-shaped CVC. After eliminating ρ and n from (2.14) and (2.16) and from the Poisson equation, we obtain an equation in the form (3.2), in which

$$\xi=\frac{x'}{|s|^{1/2}l_C(E_2)},\quad s=\frac{c\tau_g}{\mu E_2\tau_M}, \qquad (3.4a)$$

$$U=\int^{\mathscr{E}}d\mathscr{E}\left[\frac{j_{ext}}{j(\mathscr{E})-1}\right], \qquad (3.4b)$$

$$K=\left(\frac{\beta}{|s|^{1/2}\mathscr{E}^2}\right)\left[s-\mathscr{E}^2\left(\frac{1+pj_{ext}}{j(\mathscr{E})}\right)\right]; \qquad (3.5)$$

$\beta = (\tau/\tau_g)^{1/2}$, $j = e\,EB\,N_{d,eff}$ is the current density as a function of the field in the case of homogeneous distribution (N-shaped curve, Fig. 2; see (2.14)).

Since we are considering stationary waves, we should assume that the length of the sample is much larger than the characteristic scale of variation of the field in the wave. We can then speak of waves in an unbounded medium. It is required that the solutions be bounded at infinity.

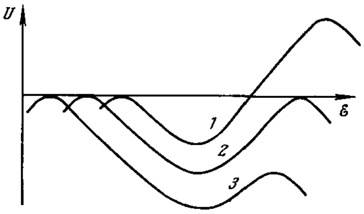

FIG. 6. The potential U(ℰ). In the case of an N-shaped CVC: 1 – $j_0 < j_{ext} < j_{c1}$; 2 – $j_{ext} = j_0$; 3 – $j_{c2} < j_{ext} < j_0$. In the case of an S-shaped characteristic, ℰ should be replaced by Θ/Θ_2: 1–$E_0 < E_x < E_{c1}$; 2–$E_x = E_0$; 3–$E_{c2} < E_x < E_0$.

3.2. Forms of Stationary Waves

Eq. (3.2) is in the form of an equation of motion of a particle in a field with potential E (which depends on the "coordinate" ℰ) under the influence of a friction force, the coefficient K of which can be either positive or negative. The potential U(ℰ) at different values of the current j_{ext} (i.e., different E_2) is shown schematically in Fig. 6. In the interval of variation of j_{ext} from j_{c2} to j_{c1}, the potential U has three extrema, since the equation $j(E) = j_{ext}$ has in this case three roots: E_1, E_2, and E_3 (see Fig. 2). When j_{ext} coincides with j_{c1} (or j_{c2}), the points E_1 and E_2 (or E_2 and E_3) coalesce. The potential U has in this case only two equilibrium points.

We denote by j_0 that value of j_{ext} at which (Fig. 6, curve 2)

$$U(\mathscr{E}_1)=U(\mathscr{E}_3),\qquad j_{ext}=j_0. \qquad (3.6)$$

When $j_{ext} > j_0$ (curve 2), we have $U(\mathscr{E}_1) < U(\mathscr{E}_3)$, and conversely when $j_{ext} < j_0$ (curve 3) we have $U(\mathscr{E}_1) > U(\mathscr{E}_3)$.

Let us investigate the motion of the "particle" as a function of the parameter s. The possible trajectories are best represented in the phase plane $(\mathscr{E}, \mathscr{E}_\xi)$ (see[76]).

At large positive values of s, the "friction coefficient" K is large and the "particle" drops from the equilibrium point $\mathscr{E}_1$ or $\mathscr{E}_3$ of the "saddle" and reaches a point $\mathscr{E} = 1$ ($E = E_2$), which in this case is a stable node. The form of the phase trajectories describing such a motion can be obtained by neglecting the "inertia of the particle" ($\mathscr{E}_{\xi\xi}$):

$$\mathscr{E}_{\xi}=-\frac{(dU/d\mathscr{E})}{K}. \qquad (3.7)$$

We note that at large values of s the coefficient K is positive throughout.

Solutions of the type (3.7) are electric-field waves of the shock type, which bring the system from the unstable state with $E = E_2$ into a stable state with $E = E_1$ or E_3 (when $j_{c2} < j_{ext} < j_{c1}$)* [67]. When $j_{ext} = j_{c1}$ (or j_{c2}), such waves transfer the particle from one stable state to another (E_1 into E_3 into E_1). A rigorous analysis of the corresponding equation can be found in[77].

With further decrease of s, the "friction" decreases and the inertia becomes appreciable. There are also values of s at which the "particle" executes several

*These waves, which exist when S has a continuum of values, are analogous to waves investigated in the theory of chain reactions.

oscillations before "settling" in $\mathscr{E}_2 = 1$. The corresponding solutions are shock waves with oscillating fronts, and transfer the system from the unstable state into one of the stable ones[67]. Far from the front of such a wave, the field is homogeneous and corresponds to the decreasing section of the characteristic ($E = E_2$). Small perturbations in this region increase, making the wave unstable.

When s decreases, it reaches a value at which the "particle," moving from one equilibrium point, falls into another equilibrium point (for example, from $\mathscr{E}_3$ into $\mathscr{E}_1$). The corresponding wave transfers the system from one stable homogeneous state to another stable homogeneous state: from E_1 into E_3 or E_3 into E_1*. It has the form of a traveling depletion layer, or respectively accumulation layer, of the electron density compared with n_0. (These solutions were obtained by Copeland in a numerical analysis of stationary waves[79].) At each value of u_{ext} in the interval from j_{c2} to j_{c1}, either wave can propagate, but their velocities are generally speaking different. When j_{ext} is close to j_{c1}, the depletion waves corresponds to a descent of the particle from the level $U(\mathscr{E}_3)$ to the level $U(\mathscr{E}_1)$ (Fig. 6, curve 1), and the accumulation wave corresponds to an upward rise of the particle. In the latter case, the "friction" and consequently also the wave velocity (see (3.3)), should be smaller (in principle s can be negative).

When j_{ext} is close to j_{c2}, the velocity of the accumulation wave, to the contrary, is larger than that of the depletion wave. There exists a unique value of j_{ext}, at which the velocities of the depletion and accumulation waves coincide. We shall show below that for the systems considered by us this value of j_{ext} coincides with j_0 (3.6).

Let us assume that the current in the sample is specified, and that on a certain segment the field is equal to E_3, and outside the segment to E_1. If $j_{ext} > j_0$, then the leading front of the field distribution will move more rapidly than the trailing edge, and the region in which the field is equal to E_3 will expand. When $j_{ext} < j_0$, to the contrary, the region in which the field is equal to E_1 will expand.

We emphasize once more that in the propagation of the stationary waves considered above, the average field in the sample varies, so that these waves cannot be realized at a fixed voltage across the sample.

There exists a value of s such that the particle moving from $\mathscr{E}_1$ (when $j_{ext} > j_0$) in a potential well under the influence of an alternating-sign "friction force" will return to the point $\mathscr{E}_1$. The "work of the friction force" in such a motion is equal to zero:

$$\oint K\mathscr{E}_\xi \, d\mathscr{E} = 0. \qquad (3.8)$$

The condition (3.8) is obtained from Eq. (3.2) by multiplying the left side of this equation by $\mathscr{E}_\xi$ and integrating over the period of the motion. The corresponding trajectory Γ on the phase plane is a closed separatrix (Fig. 7a). Near $\mathscr{E}_1$ we have $\mathscr{E}_\xi = c_{1,2}\,(\mathscr{E} - \mathscr{E}_1)$, where C_1 and C_2 are the characteristic roots of the singular point $\mathscr{E}_1$, and have opposite signs. The asymptotic form $\mathscr{E}(\xi)$ as $\xi \to \pm\infty$ is given by

$$\mathscr{E} - \mathscr{E}_1 \propto \exp(C_{1,2}\,\xi).$$

The field goes through a maximum, from which it decreases in both sides exponentially to E_1 (Fig. 8). Such waves are called solitary waves in the theory of waves on water. They can exist also in a collisionless plasma[80] and in magnets[81].

Solitary waves in semiconductors with N-shaped CVC are called in the literature domains or dipole waves. The latter is connected with the fact that the volume charge in such waves represents a steadily moving dipole layer (Fig. 8a).

It is of interest to trace the change of the form of the domain as a function of the current j_{ext} in the sample[64,65,67,68]. When $j_{ext} > j_0$, the solitary wave is a narrow strong-field domain (of increased resistance). The width of such a domain is proportional either to l_D or l_{dr} (see below), and in the case of the recombination mechanism, it is proportional to l_c. When $j_{ext} = j_0$, broad domains can exist[64,68], in which one front is a depletion layer and the other an accumulation layer, and the distance between the fronts is large compared with their effective thickness (Fig. 8b). Such a domain corresponds to the motion of a particle from $\mathscr{E}_1$ to $\mathscr{E}_3$ and back (Fig. 6). The phase trajectory (Fig. 7b) goes through two singular points. When j_{ext} deviates from j_0, the domain width decreases very sharply and the domain becomes narrow.

If $j_{ext} < j_0$, the solitary wave is a narrow domain of weak fields (of decreasing distance); the field outside the domain is equal to E_3.

We note that the domain velocity can be only positive (the domain moves in the electron drift direction). Only in this case is the "friction" K of alternating sign and condition (3.8) can be satisfied.

Let us investigate the existence of solutions in the form of a stationary field distribution (s = 0). Unlike

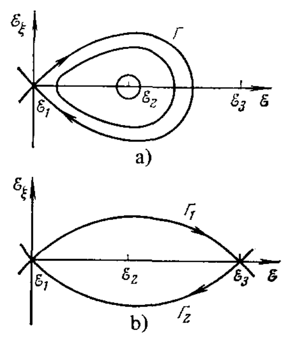

FIG. 7. Phase trajectories: a) of narrow domains and of oscillating waves, $j_0 < j_{ext} < j_{c1}$; b) of layer waves (Γ_1 or Γ_2) and a broad domain ($\Gamma_1 + \Gamma_2$), $j_{ext} = j_0$.

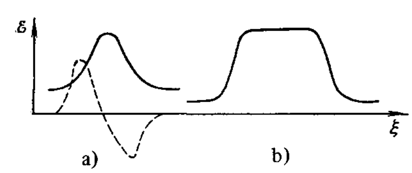

FIG. 8. Distribution of the fields (solid curve) and of the space charge (dashed curve) in the domain. a) Narrow domain, b) broad domain.

*Similar waves were considered in the theory of combustion (the thermal regime of combustion). The existence of such waves and the uniqueness of their velocity are proved in a paper by Ya. B. Zel'dovich [78].

the moving stationary layer waves ($s \neq 0$), they can be observed when a given voltage is applied across the sample. The equation describing the stationary distributions in two-valley semiconductors is of the form (3.2) with

$$K = -\frac{\alpha u(\mathscr{E})}{\mathscr{D}(\mathscr{E})}.$$

In this case the friction does not reverse sign, and as already indicated, there are no distributions in the form of domains. Only a distribution in the form of a space-charge layer with a monotonic variation of the field is possible. At large values of α, the "inertia" ($\mathscr{E}_{\xi\xi}$ in (3.2)) can be neglected and the phase trajectories given by (3.7). Then the distribution in the form of a space-charge layer separating the stable homogeneous phases (with $E = E_1$ and $E = E_3$) is possible only when $j_{ext} = j_{c1}$ (E_1 at the cathode) or $j_{ext} = j_{c2}$ (E_3 at the cathode). At very small α (the particle moves with almost no friction), for the existence of a layer solution it is necessary that j_{ext} be close to j_0. Thus, when α increases the current density at which a solution exists in the form of a stationary layer separating two stable phases either increases from j_0 to j_{c1} or decreases from j_0 to j_{c2}.

In the case of the recombination mechanism, the equation for the stationary distributions (which follow from (2.14), (2.16), and the Poisson equation) is of first order: there is no "inertial" term. Consequently, under the assumptions made above (Sec. 2.3), the stationary layers exist only at j_{c1} and j_{c2}. Theoretically, stationary layers of the distribution (with allowance for diffusion) were considered in[82,83].

3.3. Velocity and Form of Gunn Domains

The different models of a two-valley semiconductor leads to different values of the domain velocity. It is shown in[68] that Eq. (3.2) has solutions in the form of domains (or periodic waves) only for a single value of the dimensionless wave velocity $s = 1$.* This means that in the "field" model ($\mu = \mu(E)$; see Sec. 2.2) the domain velocity is equal to the drift velocity of the electrons outside the domain:

$$c = v(E_1) = v(E_2). \tag{3.9}$$

It is interesting that exactly the same result is obtained in the model in which $\mu = \mu(vE)$[115] (see Sec. 2.2). On the other hand, in the "temperature" model[72] the domain velocity is 10% larger than $v(E_1)$. It is, however, easy to establish that the equality (3.9) should take place regardless of the model, if almost complete depletion of the electron concentration takes place on the domain front. In the depletion region, the total current coincides with the displacement current $(\epsilon/4\pi)(\partial E/\partial t)$. By virtue of the stationary behavior of the wave, this current coincides with the current outside the domain $en_0v(E_1)$. On the other hand, it equals $(\epsilon/4\pi)c(\partial E/\partial x)$, which equals en_0c by virtue of the Poisson equation. From this we get (3.9).

Let us consider in greater detail the solutions (3.2), which describe the domains. When $s = 1$ (domain velocity equal to the drift velocity of the electrons outside the domain), Eq. (3.2), at which K and U are given by expressions (3.3), can be integrated and the form of the phase trajectories can be found[64,68]:

$$\alpha^{-1}\mathscr{E}_\xi - \alpha^{-2}\ln|\alpha\mathscr{E}_\xi + 1| + U(\mathscr{E}) - U(\mathscr{E}_{\min}) = 0; \tag{3.10}$$

Here $U(\mathscr{E}_{\min})$ is the integration constant, having the meaning (in the analogy with a moving particle as used by us) of the potential energy at the turning point. The phase trajectories are shown in Fig. 7. The trajectories close to Γ represent series of domains, and those close to $\mathscr{E}_2 = 1$ represent waves in which the field oscillates weakly about E_2. In this case the singular point $\mathscr{E}_2$ is a center.

We obtain the wave amplitude from (3.10), putting $\mathscr{E}_\xi = 0$, i.e, from the equation

$$U(\mathscr{E}_{\min}) = U(\mathscr{E}_{\max}), \tag{3.11}$$

which assumes in dimensional quantities the form

$$\int_{E_{\min}}^{E_{\max}} dE\,\frac{j_{ext} - j(E)}{D(E)} = 0. \tag{3.11a}$$

If D does not depend on E, then one can call Eq. (3.11a), when applied to single domains (E_{min}, E_1 for the strong-field domain, $E_{max} = E_3$ for the weak-field domain), the equal-area rule[64].

In a broad domain with a flat top, whose fronts are layer waves traveling with the same velocity, we have $E_{min} = E_1$ and $E_{max} = E_3$. The condition (3.11) coincides in this case with definition (3.6) of the current density j_0.

Let us consider now the shape of the domain, when $\alpha \ll 1$ (for example, large electron density)

$$\ln|\alpha\mathscr{E}_\xi + 1| \cong \alpha\mathscr{E}_\xi - \left(\frac{1}{2}\right)\alpha^2\mathscr{E}_\xi^2.$$

Then, as follows from (3.10), the domain is represented by a symmetrical phase trajectory

$$\mathscr{E}_\xi^2 = 2\,[U(\mathscr{E}_{\min}) - U(\mathscr{E})]. \tag{3.12}$$

The characteristic dimension of the domain is the Debye length l_D, and its form is symmetrical. If $\alpha \gg 1$ and the domain amplitude is not too small, then its form is strongly asymmetrical, and the characteristic dimensions of the leading and trailing edges differ greatly. Indeed, when $\mathscr{E}_\xi > 0$ (trailing edge), the logarithmic term in (3.10) can be neglected. We see that the characteristic width of the trailing edge is of the order of l_D/α. When $\mathscr{E}_\xi < 0$ (leading edge), Eq. (3.10) is satisfied if the logarithm is a large negative quantity. Consequently,

$$\alpha\mathscr{E}_\xi \cong -1. \tag{3.13}$$

This means that on the leading front of the domain the charge density is constant and is equal to the charge density of the impurities, and the concentration of the free electrons is close to zero.

Thus, the field on the leading front increases linearly over a length $\alpha l_D = l_{dr}$, and decreases rapidly on the trailing edge. The domain has the form of a right triangle. As expected from the linear analysis (Sec. 2.2), in the case when the instability is absolute ($\alpha \ll 1$), the characteristic scale of the field inhomogeneity coincides with l_D, and in the case of convective instability ($\alpha \gg 1$) it coincides with l_{dr}.

*See, however, next page concerning Ref. 90.

3.4. The Gunn Effect

In 1963, Gunn, in a study of the effects of hot electrons in gallium arsenide, observed that when a field of ~3 kV/cm is applied to an n-GaAs sample, coherent oscillations are produced in the sample, with a frequency inversely proportional to the sample length[73]. Later, Gunn established experimentally that these oscillations are connected with the passage of strong-field domains through the sample, and measured their shape[34]. The domain motion in two-valley semiconductors and the phenomena created by this motion are called the Gunn effect.

When a voltage V is applied to a sample of a semiconductor with N-shaped CVC, such that the average field $\overline{E} = V/l_x$ corresponds to the decreasing branch of the CVC ($E_{c1} < \overline{E} < E_{c2}$; Fig. 2), a strong-field domain is produced in the sample (if the product $n_0 l_x$ in the sample is sufficiently large; Sec. 2.2). During the time of development of the domain, the current in the circuit decreases and remains unchanged when the domain becomes stationary. Reaching the anode, the domain vanishes. The current in the circuit then increases approximately to j_{c1} (see Fig. 2). The picture then repeats. The characteristic time variation of the current is shown in Fig. 9.

In sufficiently homogeneous samples, the domain is produced at the cathode. In measuring the static field distribution in the sample prior to the occurrence of the oscillations, it was observed that a strong-field region, in which the domain is produced, exists at the cathode[84]. Since the domain velocity equals the drift velocity of the electrons outside the domain (Sec. 3.3), the frequency of the generated current oscillations is $f = v/l_x$[73]. In GaAs at room temperature we have $v \sim 10^7$ cm/sec. At $l_x = 360\ \mu$, the frequency of the current oscillations shown in Fig. 9 is 2×10^8 Hz.

The form of the domain in n-GaAs was first measured by Gunn[34,118] (see also[85]). At small domain voltage drops V_d (see (4.12a)), the form is symmetrical with increasing V_d, the length of the leading front increases compared with the length of the trailing edge, and at large V_d the ratio of these lengths is ~3–4.[118] An estimate shows that $\alpha > 1$ in the materials customarily employed in these experiments, so that one can speak of a qualitative agreement of the conclusions of the theory (see the preceding section) with experiment.

At large V_d, the domain becomes unstable, a fact usually connected with shock ionization. In Gunn's experiment[118], it began at $V_d = 477$ V. The maximum field in the domain was in this case ~130 kV/cm (he used a sample 260 μ long, with $n = 2.7 \times 10^{14}$ cm^{-3} and $\mu = 8000$ cm^2/V-sec in weak fields).

With increasing voltage on the sample (with the domain) the stationary value of the current density decreases and approaches asymptotically a certain constant value[84,118]. The latter can be apparently identified with j_0—the current density in the presence of a broad domain (Sec. 3.2). However, measurement of the domain form has shown that up to the highest attainable voltages V_d, the maximum field continues to increase with increasing V_d, and the domain does not have a flat top (no broad domain is observed in GaHs). We note that weak-field domains were likewise not observed.

When the voltage on the sample in which the stationary domain moved was reduced, it was observed that the domain vanishes at fields E smaller than E_{c1}[125,119]. With increasing sample voltage, the current, as already mentioned, decreases, meaning that the drift velocity of the electrons outside the domain and the velocity of the domain itself decreases (Sec. 3.2). This is confirmed by experiment[84,118].

Current oscillations connected with the occurrence of domains were observed, besides in GaAs, also in InP, CdTe[86], ZnSe[87], p-Ge[88] and n-Ge[89].

3.5. Form and Velocity of Recombination Domains

In the derivation of the equation for the recombination stationary waves (3.2), (3.4), (3.5)) it was assumed that $\beta \ll 1$. Therefore the wave form can be obtained by integrating (3.12). It is symmetrical, and its characteristic dimension, in accord with the conclusions of the linear theory (Sec. 2.3) is of the order of $l_c(E_2)$.

The expression for the velocity is obtained from (3.8) by substituting in it $\mathscr{E}_\xi$ from (3.12):

$$s = (1+p) \int_{\mathscr{E}_{min}}^{\mathscr{E}_{max}} \mathscr{E}_\xi \, d\mathscr{E} \left[\int_{\mathscr{E}_{min}}^{\mathscr{E}_{max}} \left(\frac{\mathscr{E}_\xi}{\mathscr{E}^2} \right) d\mathscr{E} \right]^{-1}. \qquad (3.14)$$

The velocity of the stationary wave depends both on its amplitude ($\mathscr{E}_{min}$ and $\mathscr{E}_{max}$) and on the form of $U(\mathscr{E})$ (i.e., in the form of the CVC). At $s = 1 + p$, the coefficient is $K(\mathscr{E}_2) = 0$ (see (3.5)) and the point $\mathscr{E}_2$ is a center. As seen from (3.14), the domain velocity, generally speaking, does not equal $1 + p$, and the point $\mathscr{E}_2$ (unlike in the Gunn domains) is not a center.

In the case of the recombination mechanism, series of domains and weakly oscillating waves are also possible. However, their velocity at each j_{ext} depends on the amplitude in accordance with (3.14). The dependence of the velocity of the weakly oscillating waves (closed phase trajectory close to $\mathscr{E}_2$) on the amplitude was investigated in[90] by the Van der Pol method. However, the nonlinearities in the equations for the stationary waves were not taken into account correctly. It was incorrectly concluded that periodic waves with $s \neq 1$ are possible in the assumed model of the two-valley semiconductor. The waves in the form of weak oscillations of the field about E_2 are obviously unstable, since the differential conductivity of the medium is negative for perturbations with a wavelength much larger than the period of the oscillations.

The velocity of waves in the form of a series of domains (phase trajectory close to the separatrix Γ in Fig. 6), naturally, differs little from the velocity of the single domain. This can be verified by using as an example the domains at $j_{ext} \cong j_{c1}$. In this region of currents, the CVC can be approximated by the parabola

FIG. 9. Form of the Gunn current oscillations in n-GaAs [86].

$j_{c1} - (1/2) j_{EE}(E_{c1})(E - E_1)^2$. Then

$$U(\mathscr{E}) - U(\mathscr{E}_1) = \frac{1}{6}\frac{j_{\mathscr{E}\mathscr{E}}(\mathscr{E}_{c1})}{j_{c1}}(\mathscr{E}-\mathscr{E}_1)[\mathscr{E}-\mathscr{E}_1-3(\mathscr{E}_{c1}-\mathscr{E})]. \quad (3.15)$$

Substituting (3.15) in (3.12) and (3.14), we obtain the form of a single domain of small amplitude

$$\mathscr{E} = \mathscr{E}_1 + 3(\mathscr{E}_{c1}-\mathscr{E}_1)(\operatorname{ch} a\xi)^{-2}, \quad a = \left[(\mathscr{E}_{c1}-\mathscr{E}_1)\left(\frac{j_{\mathscr{E}\mathscr{E}}}{2j_{c1}}\right)\right]^{1/2}, \quad (3.16)$$

and its velocity

$$s_1 = (1+p)\left[1 - \frac{4}{7}(\mathscr{E}_{c1}-\mathscr{E}_1)\right]. \quad (3.17)$$

The point $\mathscr{E}_2$ is in this case an unstable focus. At $\mathscr{E}_{min}$ close to $\mathscr{E}_1$, the velocity of the waves (which have the form of a series of domains) is given by the equation (see (3.14) and (3.15))

$$\frac{s-s_1}{s_1} = \frac{45}{14}(\mathscr{E}_{c1}-\mathscr{E}_1)(\mathscr{E}_{min}-\mathscr{E}_1)^{3/2}. \quad (3.18)$$

Since $\mathscr{E}_{min} - \mathscr{E}_1$ depends exponentially on the distance between the domains in the series, the velocity of such waves practically coincides with s_1.

As seen from (3.4a), (3.14), and (3.17), the velocity of the recombination domains is of the same order as the velocity of the recombination waves in the linear theory (2.19), and consequently is much smaller than the drift velocity of the electrons.

Recombination domains in CdS were observed by Boer and co-workers[91,101]. Later on, domains were thoroughly investigated in Ge: Au[43,92] and Ge: Cu[47]. They were observed in GaAs[93] and in InSb[94]. The qualitative features of the phenomena connected with the occurrence of motion of domains are the same as in two-valley semiconductors.

Experiments reveal a dependence of the velocity of the recombination domains on the free-carrier density. Thus, with increasing illumination, the velocity of the domain in Ge: Au changed from 3×10^{-5} to ~ 0.5 cm/sec (with n changing by four orders of magnitude)[43], and in Ge: Cu, from 21 to 75 cm/sec, with n changing by one order of magnitude[95]. At present, however, the experimental and theoretical values of the domain velocity do not agree. In particular, the experimentally observed dependence of the domain velocity on the free-carrier density has not been satisfactorily explained.

Many experiments have shown that moving domains are produced only when the density exceeds a certain critical value. At lower densities, a stationary distribution is observed, in the form of a space-charge layer (see Sec. 3.2). The observed CVC has in this case a horizontal section[83].

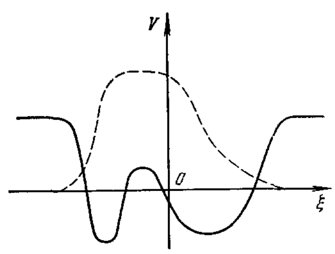

FIG. 10. Form of the potential V(ξ) for the case of a single domain. The dashed line shows the field distribution in the domain $E(x') - E_1$.

4. STABILITY OF STATIONARY WAVES. APPLICATIONS OF THE GUNN EFFECT

4.1. Stability of Stationary Waves

In the preceding section we have found that at each value of the current j_{ext} there exist different stationary waves. Not all are stable. To investigate the stability of the stationary waves, we linearized Eq. (3.1)* for small deviations from the stationary distribution

$$\delta\mathscr{E}(\xi, \eta) = \mathscr{E}(\xi, \eta) - \mathscr{E}_{cm}(\xi) = \delta\mathscr{E}(\xi)\exp(-\lambda\eta). \quad (4.1)$$

We shall assume first that the current in the external circuit remains unchanged, $\delta j_{ext} = 0$, i.e., either the impedance V_{ext} of the external circuit is large, or the perturbation is such that it does not change the resistance of the entire sample. The linear equation obtained from (3.1) for $\delta E(\xi)$ by means of the substitution

$$\psi = \delta\mathscr{E}(\xi)\exp\left\{\left(\frac{\alpha}{2}\right)\left[\int^{\xi} U_{\mathscr{E}}\, d\xi + (s-1)\xi\right]\right\} \equiv F(\xi)\,\delta\mathscr{E}(\xi) \quad (4.2)$$

is reduced to the canonical form

$$(\hat{H} - \lambda)\psi = 0, \quad (4.3)$$

$$\hat{H} = -\frac{d^2}{d\xi^2} + V(\xi), \quad (4.4a)$$

$$V(\xi) = \left(\frac{\alpha^2}{4}\right)(s - 1 + U_{\mathscr{E}\mathscr{E}})^2 - (1 + \alpha\mathscr{E}_{\xi})U_{\mathscr{E}\mathscr{E}}. \quad (4.4b)$$

To simplify the exposition we assume that D does not depend on the field. This limitation does not affect the results.†

Equation (4.3) has the form of the Schrödinger equation. To obtain the concrete form of the potential $V(\xi)$ it is necessary to substitute in (4.4) the solution (3.2) which describes that stationary wave, whose stabilities they investigated. For a layer wave, $V(\xi)$ has the form of a potential well, and for a domain it has the form of two potential wells (Fig. 10). The stationary wave is unstable if the eigenvalues of the operator $\hat{H}$ include negative ones $(\lambda < 0)$.

Differentiating (3.2) with respect to ξ, we can verify that the function $f(\xi)\mathscr{E}_{\xi}$ is an eigenfunction of $\hat{H}$, corresponding to a zero eigenvalue. The meaning of this is obvious: the perturbation $\delta\mathscr{E} \propto \mathscr{E}_{\xi}$ is a small displacement of the wave. The wave is in indifferent equilibrium with respect to such a perturbation. If the wave is of the layer type, then $\mathscr{E}_{\xi}$ does not reverse sign and consequently $F(\xi)\mathscr{E}_{\xi}$ is a function of the ground state of H, and zero is the lowest eigenvalue. Therefore the layer wave is stable in the stabilized-current regime[68].

In the case of a domain $\mathscr{E}_{\xi}$ reverses sign once, and therefore the eigenvalue $(\lambda_1 = 0)$ corresponding to the function $F(\xi)\mathscr{E}_{\xi}$ is not the smallest one. There exists one negative eigenvalue $\lambda_0 < 0$: the domain is unstable in the given-current regime[68]. We shall see below

*Up to now, only the stability of stationary waves in two-valley semiconductors was investigated. The methods used for these investigations, generally speaking, cannot by applied directly to the problem of the stability of recombination waves.

†An investigation of the stability of stationary waves in the Gunn effect (in the general case D = D(E)) is due to Knight and Peterson [68]. They used the idea of the general method developed earlier by Zel'dovich and Barenblatt [96] using a plane flame front as an example.

that in the given-voltage regime, a single domain is stable.

Waves in the form of two and more domains are unstable independently of the external circuit. Let us consider two domains, in this case the first four eigenvalues λ are obtained as a result of the "splitting" of the first two levels of the single domain ($\lambda_0 < 0$ and $\lambda_1 = 0$). A zero eigenvalue describes the shift of both domains in one direction. A near-zero negative eigenvalue describes domains that come together or move apart. The minimum $\lambda \cong \lambda_0$ corresponds to a broadening or a narrowing of both domains, and a level close to it corresponds to broadening of one domain and narrowing of the other. The latter perturbation does not change the resistance of the sample and has $\lambda \approx \lambda_0 < 0$. As a result of the growth of such a perturbation, one of the domains vanishes. Instability of two domains was observed experimentally by Gunn[103].

Let us make a remark concerning the investigation of the stability of layer waves. The condition that the perturbation be bounded was imposed on the function $\delta\mathscr{E}(\xi)$. On the other hand we investigated the spectrum of the equation for the function $\psi(\xi)$, which differs from $\delta\mathscr{E}$ by a factor $F(\xi)$ (see (4.2)). For layer waves and for domains, we have $U_{\mathscr{E}} = 0$ as $\xi \to \pm\infty$. The behavior of $F(\xi)$ at infinity is determined by the exponential $\exp(k_1(\xi)$, where $k_1 = (s-1)\alpha/2$. For domains $(s=1)$ the exponent k_1 vanishes, and the conditions for the boundedness of $\delta\mathscr{E}$ and ψ coincide, and our analysis remains in force. But for layer waves $(s \neq 1)$, the function ψ may even increase at infinity, but not more rapidly than $\exp(|k_1\xi|)$. It follows from (4.3) that at infinity, where $U_{\mathscr{E}} = 0$ and $\mathscr{E}_\xi = 0$, the function $\psi(\xi)$ increases exponentially or attenuates like $\exp(k_2\xi)$, where

$$k_2 = \pm\left[\frac{\alpha^2(s-1)^2}{4} - U_{\mathscr{E}\mathscr{E}} - \lambda\right]^{1/2}. \qquad (4.5)$$

For layer waves, which take the sample from a stable homogeneous state into another stable homogeneous state, $U_{\mathscr{E}\mathscr{E}}(\pm\infty) < 0$ and $|k_2| > |k_1|$ for all $\lambda < 0$. Therefore the solutions $\psi(\xi)$, which increase at infinity and correspond to $\lambda < 0$, must be discarded (only solutions attenuating at infinity should be retained). Thus, in this case the boundedness of ψ follows from the boundedness condition of δE. The foregoing conclusions concerns the stability of layer waves pertains precisely to such waves ($E_1 \to E_3$ or $E_3 \to E_1$).

For layer waves that change the system from an unstable state into a stable one ($E_2 \to E_1$ or E_3), we have $U_{\mathscr{E}\mathscr{E}}(\mathscr{E}_2) < 0$ and therefore, far from the layer, in the homogeneous unstable phase, we have $|k_2| < |k_1|$ in a certain interval of negative λ. Solutions of (4.3) that grow at infinity are admissible, and since they correspond to $\lambda < 0$, the waves in question are unstable*, as follows also from simple physical considerations (see Sec. 3.2). The instability of waves analogous to these considered here (but not stationary) was demonstrated by McCumber and Chyoweth by numerical calculation[72].

*The instability of such waves was proved by a quasiclassical method by V. M. Eleonskiĭ [97].

4.2. Impedance of Sample with Domain

The stability against perturbations that change the current in an external circuit ($\delta j_{ext} \neq 0$) is best investigated by calculating the differential impedance $Z(\omega)$ of the sample[68]:

$$Z(\omega) = Z(-i\lambda) = \int \frac{\delta E(x,\omega)}{S\delta j_{ext}(\omega)}dx = \frac{E_2 l_D}{S\delta j_{ext}}\int \delta\mathscr{E}(\xi,\omega)\,d\xi \qquad (4.6)$$

(S—cross section of sample). To calculate Z, we linearize Eqs. (2.5), (2.8), and (2.9). The resultant equation differs from (4.3) by having a term $(\delta j_{ext}/j_{ext})F(\xi)$ in the right-hand side. The "response" ψ is expressed simply in terms of the eigenfunctions ψ_n and eigenvalues λ_n of the operator $\hat{H}$. Substituting this expression in (4.6), we obtain

$$Z(\omega) = \sum_n i a_n (\omega + i\lambda_n)^{-1}, \qquad a_n = \frac{E_2 l_D}{S}\int d\xi\,\psi_n F^{-1}\frac{\int \psi_n F\,d\xi}{\int \psi_n^2 d\xi}. \qquad (4.7)$$

The terms in (4.7) correspond to different modes $\delta\mathscr{E}_n = F^{-1}\psi_n$. In the case of a domain, the fundamental mode $(n=0)$ represents its compression or expansion. The mode corresponding to the first excited state describes a small displacement of the domain without a change in form (see above): it makes no contribution to $Z(\omega)$. Thus, $Z(\omega)$ has one pole in the upper half plane. The impedance can be represented in the form

$$Z(\omega) = R_0\left(1 + \frac{i\omega}{|\lambda_0|}\right)^{-1} + Z_1(\omega), \qquad (4.8)$$

where $R_0 < 0$. The first term coincides with the first term of the sum (4.7) $(n=0)$; Z_1 is the contribution made to the impedance by the discrete levels with $n \geq 2$ (if they exist at all) and by the continuous spectrum. Since the region of the domain itself makes a small contribution to the terms corresponding to the states of the continuous spectrum (of the order of the width of the domain to the width of the sample), the sum over such states reduces approximately to the impedance of the homogeneous regions of the sample outside the domain

$$Z_{un} = R_{un}(1 - i\omega R_{un}C)^{-1}; \qquad (4.9)$$

Here $R_{un} > 0$ and C are the resistance and capacitance of the homogeneous regions. If we neglect the contribution from the discrete levels, then $Z_1 \cong Z_{un}$. To prove the stability of the domain, we shall use only the fact that the imaginary part of $Z(\omega)$ has a capacitive character at all frequencies ω, i.e., the sign of Im Z coincides with the sign of ω. We note that the equivalent circuit of the domain impedance (the first term in the right side of (4.8)) consists of a resistance $R_0 < 0$ connected in parallel with a capacitance $C = 1/R_0\lambda_0$.

To ascertain the stability against perturbations that change the resistance of the sample, we use the roots of the equation (Kirchhof's law)

$$f(\omega) \equiv Z(\omega) + R_{ext} = 0. \qquad (4.10)$$

We assume the load resistance to be purely active. The domain is stable if the function $f(\omega) \equiv Z(\omega) + R_{ext}$ of the complex variable ω has no zeroes in the upper half-plane. According to the argument principle (see, for example,[98]), the difference between the number of zeroes N and number of poles P of the function $f(\omega)$ in the upper half plane is equal to the increment of the argument (divided by 2π) on the function $f(\omega)$ on going along the contour C around the upper half-plane, i.e.,

$$N-P=(2\pi)^{-1}\Delta_C \arg f(\omega). \tag{4.11}$$

The number of poles P, as established above, is equal to unity.

We draw in the plane f the contour C′, which maps the contour C. The point $\omega = 0$ corresponds to the point $Z(0) + R_{ext}$ on the real axis. We consider first the case $Z(0) < 0$. At small load, $R_{ext} < |Z(0)|$, the point $\omega = 0$ corresponds to a negative real solution (Fig. 11). With increasing frequency, f goes over into the upper half plane (the reactive part of the impedance is a capacitance). When $\omega \to \infty$ we have $f(\omega) \to R_{ext} > 0$. When $\omega < 0$ we obtain a curve that is symmetrical with respect to the real axis, so that the contour C′ surrounds the point $f = 0$. Then $\Delta_C \arg f = -2\pi$, and consequently $f(\omega)$ does not have any zeroes in the upper half plane (the domain is stable). If the load is large, $R_{ext} > |Z(0)|$, then the contour C′ does not encircle the point $f = 0$ (see Fig. 11), and $f(\omega)$ has one zero in the upper half plane (the domain is unstable). Obviously, when $Z(0) > 0$ the domain is unstable regardless of the load (the contour C′ does not encircle the point $f = 0$). Thus, in the regime in which the voltage on the sample is fixed, the stability of the domain is determined by the sign of Z(0), i.e., by the slope of the static CVC of the sample with the domain.

4.3. Current-voltage Characteristic of Sample with Domain

To find the form of the static CVC of a sample with a domain, it is necessary to find the dependence of the average field in the sample $\overline{E} = V/l_x$ on j_{ext}:

$$\overline{E}=E_1(j_{ext})+\frac{V_d(j_{ext})}{l_x}, \tag{4.12}$$

where E_1 is the field on the left growing branch of CVC of the semiconductor (see Fig. 2), and

$$V_d=\int_{-\infty}^{+\infty}[E(x)-E_1]\,dx \tag{4.12a}$$

has the meaning of the excess voltage drop on the domain. The $V_d(j_{ext})$ dependence is determined, in particular, by the concrete form of CVC for a homogeneous field distribution j(E). The qualitative behavior of $j_{ext}(\overline{E})$ can, however, be explained in general form.

If the current j_{ext} is close to j_{c1}, it is possible to use (3.16) for the distribution of the field in a small-amplitude domain. Integrating (3.16) over the length of the sample and going over to dimensional quantities, we obtain

$$\overline{E}=E_{c1}\left\{1-a\left(\frac{1-j_{ext}}{j_{c1}}\right)^{1/2}+6\left(\frac{l_D}{l_x}\right)a^{3/2}\left(1-\frac{j_{ext}}{j_{c1}}\right)^{1/4}\right\}. \tag{4.13}$$

The last term on the right is the voltage of the domain, divided by l_x. It is small at small l_D/l_x, and $j_{ext}(E)$ passes barely below the rising branch of the CVC of the homogeneous sample.

In the other limiting case, when j_{ext} approaches j_0, the maximum field in the domain and its width increase, so that the average field in the sample increases with decreasing j_{ext}. On the whole, the static characteristic of the sample with the domain $j_{ext}(\overline{E})$ should have the form shown schematically in Fig. 11. When $j_{ext} < j_0$, the characteristic is constructed in analogous manner.

When the voltage (i.e., $\overline{E}$) increases up to $\overline{E} = E_{c1}$, we get the CVC of the homogeneous sample. With further increase of $\overline{E}$, a domain of finite amplitude is produced in the sample, and the current drops jumpwise (transition a → c in Fig. 12). The stability regime is hard (the difference between the regimes of the occurrence of instability in a plasma is discussed, for example, in[99]). In the entire segment bcd (Fig. 12), the characteristic $j_{ext}(\overline{E})$ is a decreasing one, i.e., $Z(0) < 0$, and the domain is stable (see above). Domains exist also when $\overline{E} < E_{c1}$ segment bc). When the voltage on the terminals of the sample with the domain is decreased, the current grows along the curve cb and increases jumpwise at the point b (generally speaking, the point b can be very close to the CVC of the homogeneous sample). Thus, the CVC of a sample with domain, $j_{ext}(\overline{E})$, exhibits hysteresis. The domains on the segment ab of the characteristic are unstable ($Z(0) > 0$).

The jumpwise decrease of the current and the occurrence of a strong-field domain was observed in many experiments. With further increase of the voltage on the sample, the current in the external circuit decreases, approaching the horizontal asymptote, and the domain broadens. A domain is observed also at $\overline{E} < E_{c1}$; it vanishes when $\overline{E}$ reaches a certain minimum value[84,100,101,125,119].

If the load is not very large, a stationary field distributions (Sec. 3.2) can also exist besides the domains. For these, the minimum eigenvalue of $\hat{H}$ is zero. The corresponding term in the impedance (see (4.7)) is proportional to i/ω. This means that the observed CVC is horizontal: with increasing voltage, the "wall" shifts without a change of the current. If an investigation of the stability is made, analogous to that made for the domain, then it turns out that the stationary distri-

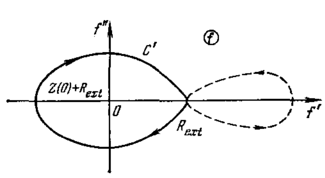

FIG. 11. Contour C′ in the complex plane f. Solid curve: $Z(0) < 0$, $R_{ext} < |Z(0)|$. Dashed curve: $Z(0) > 0$, $R_{ext} > |Z(0)|$.

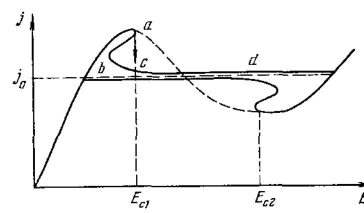

FIG. 12. CVC of a sample with a domain. The dashed line shows the decreasing section of the CVC in the case of a homogeneous field distribution.

bution is stable. In the experiment, the stationary distribution of the field and the horizontal CVC are observed only at carrier densities below a certain threshold. At higher concentrations, domains arise[95,30,102]. This regularity has not been theoretically explained.

4.4. Generation and Amplification of Oscillations in Two-valley Semiconductors

The Gunn effect is used for the generation of microwave oscillations. The frequency of the resultant oscillations is determined mainly by the time of travel of the domain from the cathode to the anode, l_x/v. It varies somewhat with the frequency of the resonant circuit to which the sample is coupled[103]. At the present time, Gunn generators with output power 340 mW under continuous operation (at frequency 8 GHz) and 615 W in the pulsed mode (at frequency 1.1 GHz) have been constructed. The largest efficiency reached in the pulsed mode is ~25%[104]. The main shortcoming of Gunn diodes is the decrease of power with increasing oscillation frequency. This is connected with the fact that to increase the frequency it is necessary to increase the length of the sample, on which the output power depends.

Copeland proposed another scheme for the generation of oscillations with the aid of a semiconductor with N-shaped CVC[105]. A strong constant field E_0, exceeding the threshold value E_{c1}, is applied to the sample, placed in a resonator. The value of E_0, the amplitude of the microwave field E_1, the oscillation frequency f, and the electron density n are chosen such that no domains are produced in the sample, but at the same time, the differential conductivity of the sample is negative during an appreciable fraction of the period of the oscillations ($\sigma_d < 0$), and consequently work is performed by the dc source on the microwave field. The total field in the sample, comprising the bias field E_0 and the microwave field, should exceed during a definite working part of the period ($\delta_n < 1$) the value of E_{c1} (in this case $\sigma_d < 0$), and should be smaller than E_{c1} during the other, idling part of the period ($\sigma_d > 0$). During the idling part of the period, the volume charges which have grown during the working part of the period, are dissipated.

The operating principle of the Copeland generator imposes definite limitations on the possible values of E_0, E_1, f, and n. Following Copeland, we present qualitative estimates explaining the limitations on f and n.

Since the instantaneous growth increment of the space-charge fluctuations is $-4\pi\sigma_d/\epsilon$, they grow during the working part of the period by

$$G_n = \exp\left(\frac{\gamma_n}{\tau_{M0} f}\right)$$

times, and decrease during the idling part by a factor $\exp(\gamma_p/\tau_{M0}f)$; here $\tau_{M0} = \epsilon/4\pi\, en\mu_0$ and μ_0 are the Maxwellian time and the mobility in weak fields,

$$\gamma_n = \int_0^{\delta_n} \frac{|\mu_d|}{\mu_0}\, d\left(\frac{t}{T}\right), \quad \gamma_p = \int_{\delta_n}^{1} \frac{\mu_d}{\mu}\, d\left(\frac{t}{T}\right),$$

$\mu_d = \sigma_d/en$ is the effective differential mobility, and $T = 1/f$ is the period of the oscillations. On the whole during the period, the space charge does not grow if $\gamma_p > \gamma_n$. Since the field in the sample is actually inhomogeneous, this inequality should be satisfied with a certain margin. In addition, G_n cannot be too large. Otherwise domains will be produced in the sample. Copeland assumed in the numerical calculations[105]

$$\frac{\gamma_p - \gamma_n}{\tau_{M0} f} > 1, \quad \frac{\gamma_n}{\tau_{M0} f} < 5. \tag{4.14}$$

These inequalities limit the permissible values of $\tau_{M_0}f$, and consequently the ratio n/f. If $\tau_{M0}f$ is too small, a domain will have time to form in the sample during the working part of the period. If it is too large, then a large value of γ_p is necessary for the operation of the device (see (4.14), i.e., a large idling part of the period is required. The efficiency of the device is then small. Copeland estimates have shown, in agreement with experiment, that in n-GaAs at room temperature the described generation regime takes place if

$$2\cdot 10^4 < \frac{n}{f} < 2\cdot 10^5 \text{ sec/cm}^3$$

The generation method proposed by Copeland is called the limited space charge accumulation (LSA) mode. The oscillation frequency of the Copeland generator is determined by the resonator and (unlike the Gunn generators) is not connected with the travel time of the domains through the sample. Therefore in the LSA mode there are no limitations on the length of the sample, and it becomes possible to obtain high power at very high frequencies. At the present time a continuous power of 30 mW was reached at frequencies 44 and 88 GHz[106], and a power of 630 W in the pulsed mode at ~10 GHz[107]. Copeland generators exceed all other solid-state devices with respect to power and frequency.

There are two known methods of amplifying oscillations with the aid of two-valley semiconductors[110]. In the first, samples with $l_x \lesssim l_{dr}$ (see Sec. 2.2) are used, in which no oscillations are generated. The real part of the impedance of such samples is negative at the frequency v/l_x[72,108,109]. The signal is either applied to the sample and is amplified upon reflection from it, or else is fed to a small near-cathode part of the sample and is picked off (in amplified form) from the near-anode part).

In amplifiers of the second type, the length of the sample is larger than the drift length. In such a sample, domains are produced. The signal fed to the sample, in which the stationary domain moves, is amplified as a result of the fact that the real part of the impedance of the sample is negative up to a certain maximum frequency (see (4.8)).

5. STATIONARY CURRENT DISTRIBUTIONS IN SEMICONDUCTORS WITH AN S-SHAPED CHARACTERISTIC[61]

5.1. Current Layers and Pinches

We have seen (Sec. 2.1) that in homogeneous states with $\sigma_d < 0$, the perturbations that increase most rapidly are those which are independent of the coordinate x in the current direction. We can therefore expect the development of the instability in the sample to give rise to stationary (the phase velocity of the perturbations with the largest increment is zero) temperature and current-density distributions that depend on the coordinates y and z. These distributions are described by the equation of heat conduction with a nonlinear source, which follows from (2.2) and (2.4):

$$\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Theta+\frac{dU}{d\Theta}=0, \tag{5.1}$$

$$\Theta=\int^{T} dT\varkappa(T), \quad U(\Theta)=\int^{\Theta}[\sigma(\Theta)E_x^2-P(\Theta)]\,d\Theta. \tag{5.2}$$

The character of the possible distributions is simplest to explain by considering the one-dimensional case, i.e., layered distribution $\Theta(y)$. Then the equation

$$\frac{d^2\Theta}{dy^2}+\frac{dU}{d\Theta}=0 \tag{5.3}$$

has the form of the equation of motion of a particle without friction in a field with potential $U(\Theta)$. In the field interval from E_{c2} to E_{c1} (see Fig. 1), the potential $U(\Theta)$ has three extrema at the points Θ_1, Θ_2, and Θ_3, which are roots of (1.1). The electron "temperatures" $\Theta_1<\Theta_2<\Theta_3$ correspond to three possible homogeneous stationary states of the semiconductor at a given field E_x, at which the currents are equal to j_1, j_2, and j_3 (see Fig. 1). The form of the potential $U(\Theta)$ is shown in Fig. 4 (the value of $\mathscr{E}$ should be replaced by Θ/Θ_2). The potentials $U(\Theta_1)$ and $U(\Theta_2)$ coincide (curve 2 in Fig. 6), i.e.,

$$\int_{\Theta_1}^{\Theta_3}[\sigma(\Theta)E_x^2-P(\Theta)]\,d\Theta=0 \tag{5.4}$$

at a single value of the field E_x, which we shall denote by E_0.

The form of the phase trajectories is given by the "energy integral" of Eq. (5.3)

$$\frac{1}{2}\left(\frac{d\Theta}{dy}\right)^2=U(\Theta_m)-U(\Theta), \tag{5.5}$$

where Θ_m is the minimal or maximal value of the "temperature" Θ in the distribution. When $E_0<E_x<E_{c1}$, the separatrix Γ (it is necessary to replace $\mathscr{E}$ by Θ and $\mathscr{E}_\xi$ by $d\Theta/dy$ in Fig. 5) represents a narrow layer with increased temperature and current density. Outside this layer, the "temperature" equals Θ_1, and the current density is j_1. The width of the layer is the order of the electron energy scattering length (2.7). Trajectories close to Γ depict the distribution in the form of a series of such narrow layers, and those close to Θ_2 depict distributions in the form of weak oscillations of the temperature about T2 and of the current about j_2.

When $E_{c2}<E_x<E_0$, the separatrix which passes in this case through Θ_3 describes a narrow cold layer in a hot phase.

Only when $E_x=E_0$ can two stable homogeneous phases exist: cold (Θ_1, j_1) and hot (Θ_3, j_3). Such a distribution corresponds to the trajectory Θ_1 or Θ_2 in Eq. 7, which passes through two singular points "saddles") Θ_1 and Θ_3. The width of the transition layer between the phases ("wall") is obviously of the order of l_c. If E_x is very close to E_0, there exists a solution in the form of a single layer with $\Theta\cong\Theta_3$ or $\cong\Theta_1$, with a width much larger than l_c.

For distributions with axial symmetry $\Theta(\rho)$ (current pinches), Eq. (5.1) takes the form

$$\frac{d^2\Theta}{d\rho^2}+\frac{1}{\rho}\frac{d\Theta}{d\rho}+\frac{dU}{d\Theta}=0. \tag{5.6}$$

This equation cannot be integrated, but it follows from it that the classification established above for the layers remains in force also for single pinches. If the radius of the pinch is very large, then we deal with the coexistence of two stable homogeneous phases; to the extent that the boundary can be regarded as practically plane, the condition for the existence of the pinch of a very large radius is $E_x=E_0$.

Eq. (5.6) differs from (5.3) in the presence of "friction" with a coefficient that depends on the "time" ρ. A narrow hot pinch surrounded by a cold plasma with "temperature" Θ_1 corresponds to motion of a particle from the point $\Theta(0)$ to Θ_1. Owing to the "friction" we should have $U[\Theta(0)]>U(\Theta_1)$, and this means that $E_x>E_0$.

Eq. (5.1) for the stationary distribution $\Theta(y, z)$ is the Euler equation for the functional

$$\Phi=\int\left\{\frac{1}{2}(\nabla_\perp\Theta)^2-U(\Theta)\right\}dy\,dz, \tag{5.7}$$

i.e., the stationary distribution $\Theta(y, z)$ corresponds to the extremum of the "action" Φ.

It was assumed in[3] that the stationary state of the sample corresponds to the minimum of entropy in it. It can be verified that in the systems under consideration the production of entropy differs from Φ (and the corresponding Euler equation differs from (5.1) and (5.2)), so that the assumption made in[3] is incorrect. It was found in[3] that $E_x=E_{c2}$ in the stationary state (see Fig. 1). However, at E_{c2} the points Θ_2 and Θ_3 coalesce, the potential $U(\Theta)$ has only two equilibrium states, and there is no solution corresponding to layers (pinches). Analogously, in the case of an N-shaped CVC at the $j_{ext}=j_{c2}$ there are no solutions in the form of domains.

5.2. Stationary Waves

In semiconductors with S-shaped characteristics, stationary waves of the electron temperature and of the current can propagate in a direction transverse to the current. In such waves, the temperature depends on the coordinate y and the time t like $T(y-ct)$, where c is the velocity of the wave. The equation for $\Theta(y-ct)$ differs from (5.3) in the term $(nc_ec/\kappa)(d\Theta/dy)$, which has the meaning of a "friction force." These waves can have the form of shock waves, which transfer the system from an unstable homogeneous state (Θ_2) into a stable state (Θ_1 or Θ_3); they are unstable, as are the analogous waves of the field in the case of an N-shaped CVC. For each value of E_x in the interval from E_{c2} to E_{c1} there exists a wave that transfers the system from a stable state into a stable state. At $E_x=E_0$, the velocity of such a wave is obviously equal to zero, and the wave reduces to a stationary distribution. Temperature and current waves with $c\neq0$ can be realized only in the given-voltage regime, and not in the given-current regime, since they changed the total current in the sample. We note that such waves transfer the system into a homogeneous state with $\Theta=\Theta_3$ when $E_x>E_0$, and into a state with $\Theta=\Theta_1$ when $E_x<E_0$. In both cases, the final states correspond to an absolute minimum of Φ.

5.3. Stability of Stationary Distributions

Let us consider the stability of stationary inhomogeneous distributions against small perturbations $\delta\Theta$ that do not depend on x. (The more general case has not been considered.) In the case of stationary layer distributions the equation

$$\delta\Theta(y, z, t) = \delta\Theta(y)\exp(ik_z z - \lambda t)$$

is obtained from (2.2) and (2.4):

$$\left(\hat{H} + k_z^2 - nc_e\frac{\lambda}{\varkappa}\right)\delta\Theta = 0, \tag{5.8}$$

where

$$\hat{H} = -\frac{d^2}{dy^2} + V(y), \tag{5.9}$$

$$V(y) = -\left.\frac{d^2U}{d\Theta^2}\right|_{\Theta=\Theta(y)}. \tag{5.9a}$$

In (5.8) it is assumed that $\delta E_x = 0$, i.e., either the voltage is fixed or the perturbation does not change the resistance of the sample (for example, $k_z \neq 0$). It is again easy to verify (cf. Ch. IV) that $d\Theta/dy$ is the eigenfunction of the Hermitian operator $\hat{H}$ with zero eigenvalue. It follows therefore that the monotonic distributions of the temperature and of the current (two stable phases at $E_x = E_0$) are stable. For a narrow layer, the minimum eigenvalue of $\hat{H}$ is negative also at small $k_z < 0$, and the narrow layer is unstable (unlike the single domain in the case of an N-shaped characteristic). The corresponding perturbations, in the forms of necks, tend to break up the layer into individual current pinches.

Current pinches, including thin ones, are stable against perturbations $\delta\Theta_m(\rho)\exp(im\varphi)$ with $m \neq 0$, which do not change the resistance of the sample. Indeed, the equation for $\delta\Theta_m(\rho)$ is

$$\left(\hat{H}^{(m)} - nc_e\frac{\lambda}{\varkappa}\right)\delta\Theta_m = 0, \tag{5.10}$$

$$\hat{H}^{(m)} = -\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d}{d\rho}\right) - \left.\frac{d^2U}{d\Theta^2}\right|_{\Theta=\Theta(\rho)} + \frac{m^2}{\rho^2}. \tag{5.10a}$$

At the same time, the derivative $d\Theta/d\rho$ is an eigenfunction of $H^{(1)}$ with zero eigenvalue. If $d\Theta/d\rho$ does not reverse sign, then $\lambda \geq 0$ at $m \geq 1$.

5.4. Differential Admittance

Let us calculate the differential admittance and use the obtained expression to investigate the stability against perturbations with $m = 0$. We assume that in the sample, besides the constant field E_x, there is also a small alternating field $\delta E_x \exp(-i\omega t)$. We denote the amplitudes of the changes of the current density and of the electron temperature by $\delta j_x(\rho, \omega)$ and $\delta T(\rho, \omega)$. The admittance is equal to

$$Z^{-1}(\omega) = \frac{2\pi}{l_x}\int_0^R \rho\frac{\delta j_x(\rho, \omega)}{\delta E_x(\omega)}d\rho = \frac{2\pi}{l_x}\int_0^R \rho\left\{\sigma(T) + E_x\frac{d\sigma}{dT}\frac{\delta T(\rho, \omega)}{\delta E_x(\omega)}\right\}d\rho. \tag{5.11}$$

The equation for $\delta T(\rho, \omega)$ differs from (5.10) ($m = 0$) in the presence of a right-hand part:

$$\left(H^0(\rho) - nc_e\frac{i\omega}{\varkappa}\right)\delta\Theta(\rho, \omega) = 2\sigma E_x \delta E_x(\omega). \tag{5.12}$$

We denote by η_n and $(nc_e/\kappa)\lambda_n$ the eigenfunctions and the eigenvalues of $H^{(0)}$ (we assume for simplicity that nc_e/κ does not depend on the temperature, meaning also on the coordinate). The functions η_n describe different modes, independent of the angle, of the perturbation of the pinch and of the surrounding plasma. The ground-state function η_0 corresponds to a thickening or to a thinning of the pinch, i.e., to a change of its radius.

Let us find δT from (5.12) in the form of an expansion in η_n, and substitute it in (5.11):

$$Z^{-1}(\omega) = \frac{2\pi}{l_x}\left\{\int_0^R d\rho\,\rho\sigma(T) + \sum_n ib_n(\omega + i\lambda_n)^{-1}\right\}; \tag{5.13}$$

here

$$b_n = 2E_x^2\left(\int_0^R \rho\eta_n\frac{d\sigma}{dT}d\rho\right)\left(\int_0^R \rho\frac{\sigma\varkappa}{nc_e}\eta_n\,d\rho\right)\left(\int_0^R \rho\eta_n^2\,d\rho\right)^{-1}.$$

Since the minimum eigenvalue of $H^{(1)}$ is equal to zero, we have $\lambda_0 < 0$. Thus, $Z^{-1}(\omega)$ has at least one pole in the upper half-plane of ω.

For a sample with a current pinch of large radius ρ_0, greatly exceeding the thickness of the transition layer between the pinch and the surrounding cold plasma ($\rho_0 \gg l_c$), the admittance can be calculated in explicit form[112]:

$$Z^{-1}(\omega) = Z_1^{-1}(\omega) + Z_3^{-1}(\omega) + R_c^{-1}\left(1 + \frac{i\omega}{|\lambda_0|}\right)^{-1}; \tag{5.14}$$

here Z_1^{-1} and Z_3^{-1} are the differential admittances of the cold plasma and of the pinch, regarded as homogeneous conductors with cross sections $\pi(R^2 - \rho_0^2)$ and $\pi\rho_0^2$ respectively, and with electric conductivities σ_{d1} and σ_{d3} (see (1.3)), and

$$R_c = -\frac{l_x l}{2\pi\rho_0^3}(\sigma_3 - \sigma_1)^1 \tag{5.15}$$

is the negative resistance connected with the change of the radius of the pinch when the current changes, i.e., with the displacement of the pinch wall. In (5.15), the thickness

$$l = \frac{\int_{\Theta_1}^{\Theta_3}\left|\frac{d\Theta}{d\rho}\right|d\Theta}{2E_0^2\int_{\Theta_1}^{\Theta_3}\sigma(\Theta)\,d\Theta} \tag{5.16}$$

is of the order of the thickness of the pinch wall ($l \sim l_c$); σ_1 and σ_3 are the electric conductivities of the cold and hot plasma at $E_x = E_0$. The frequency is

$$|\lambda_0| = \frac{\rho_0^{-2}\int_{\Theta_1}^{\Theta_3}\left|\frac{d\Theta}{d\rho}\right|d\Theta}{\int_{\Theta_1}^{\Theta_3}\frac{nc_e}{\varkappa}\left|\frac{d\Theta}{d\rho}\right|d\Theta} \sim \left(\frac{l}{\rho_0}\right)^2\tau^{-1}, \tag{5.17}$$

where τ is of the order of the time of dissipation of the energy in the homogeneous plasma. The quantity $L_c = R_c/\lambda_0$ has the meaning of the inductance connected with the inertia of motion of the pinch wall. The equivalent circuit of a sample with a large-radius pinch consists of parallel-connected impedances Z_1, Z_3, and the "wall impedance." which represents the circuit made up of $R_c < 0$ and L_c.

To investigate the stability of the pinch against perturbations that change the resistance of the sample, let us find the roots of Eq. (4.10). When $R_{ext} = 0$, i.e., In the given-voltage regime, the pinch is unstable, since

$Z(\omega)$ has a zero in the upper half plane ($Z^{-1}(\omega)$ has there a pole $-i\lambda_0$). When $R_{ext} \neq 0$, the investigated equation can be represented in the form

$$g(\omega) \equiv Z^{-1}(\omega) + R_{ext}^{-1} = 0. \quad (5.18)$$

We have found above that $Z^{-1}(\omega)$ has at least one pole in the upper half plane when a current pinch is present in the sampe. If the pinch radius is large ($\rho_0 \gg l$), such a pole is a solitary one. Indeed, in this case $|\lambda_0|$ is a small quantity (see (5.17)) and we cannot expect to find even one more negative level in the potential well $-U''[\Theta(\rho)]$ (5.10).

In the investigation of the stability of the pinch it is necessary to bear in mind that its contribution to the imaginary part of $Z^{-1}(\omega)$ has an inductive character ($\omega \operatorname{Im} Z^{-1}(\omega) > 0$). In the case when $Z^{-1}(\omega)$ has only one pole in the upper half plane, the zeroes of $g(\omega)$ (5.18) are investigated in exactly the same manner as the stability of the domain (Sec. 4.2). The conclusions are also similar. If $Z^{-1}(0) < 0$, then the current pinch is stable when $R_{ext} > |Z(0)|$, i.e., in the given-current regime. But if $Z(0) > 0$, the pinch is unstable independently of the load. Thus, the current pinch is stable only on the decreasing part of the CVC of the sample with the pinch.

5.5. Current-Voltage Characteristic

Qualitatively, the form of the CVC of a sample with a pinch can be explained in rather general form. The current density averaged over the cross section is

$$\bar{j} = \frac{I}{S} = j_1(E_x) + \frac{I_p(E_x)}{S}, \quad (5.19)$$

where $j_1(E_x)$ is the current density on the lower branch of the S-shaped CVC, and

$$I_p(E_x) = 2\pi \int_0^R \rho\,[j(\rho) - j_1(E_x)]\,d\rho \quad (5.20)$$

is the excess current in the pinch. So long as the pinch wall is far from the boundaries of the sample ($R - \rho_0 \gg l_c$), the current I_p does not depend on the sample radius R.

It follows from (5.6) that in fields E_x that are close to E_{c1}, the amplitude of the pinch $j_{max} - j_1$ is small. It can be shown that it is proportional to $(E_{c1} - E_x)^{1/2}$. But with increasing $E_{c1} - E_x$, the effective area of the pinch decreases in exactly the same manner, and therefore the current I_p does not depend on E_x or on the order of $J_{c1} l_c^2$. It follows from this and from (5.19) that in samples with a large cross section ($R^2 \gg l^2$) the CVC $\bar{j}(E_x)$ near E_{c1} lies somewhat higher and parallel to the lower branch of the S-shaped $j_1(E_x)$ curve, i.e., it is increasing, $Z(0) > 0$. (Fig. 13).

On the other hand, in fields that are sufficiently close to E_0, the statistical impedance of the sample with the pinch is negative and the CVC is decreasing. Indeed, near E_0 there can exist only a large-radius pinch $\rho_0 \gg l$. The "excess" current in such a pinch is

$$I_p = \pi\rho_0^2\,[j_3(E_x) - j_1(E_x)]. \quad (5.21)$$

Differentiating $j_1(E_x)S + I_p$ with respect to E_x and comparing with expression (5.14) for $Z^{-1}(0)$, we find that with increasing ρ_0 the field E_x approaches E_0 like

$$\frac{E_x - E_0}{E_0} = \frac{l}{\rho_0}. \quad (5.22)$$

At sufficiently small $E_x - E_0$, the main term in $Z^{-1}(0)$ is the negative conductivity of the wall

$$R_c^{-1} \propto -(E_x - E_0)^{-3},$$

so that $Z^{-1}(0) < 0$. The absolute value of $|Z(0)|$ is small and the CVC is close to vertical $E_x = E_0$.

It follows from the foregoing that the CVC of the sample with small transverse dimensions ($R \gg l$) has in the presence of a current pinch the form shown schematically in Fig. 13. Only the stationary states on the decreasing section, $Z(0) < 0$, are stable (see Sec. 5.5). With increasing current in the sample, in the regime with the large load near $j = j_{c1}$, the field decreases jumpwise. When the current is increased further, the CVC approaches the vertical line $E_x = E_0$. If the pinch produced after the field jump has initially a large radius, then the CVC is vertical ($E_x = E_0$) already when $j > j_{c1}$. On the vertical section, the current increases as a result of the increase of the pinch radius. If the current is decreased, then the jumplike increase of the field occurs at $j < j_{c1}$, i.e., hysteresis takes place (see Fig. 13).

Let us examine the most important particular case, when the radius of the sample is sufficiently large compared with l, so that

$$\left\{\frac{2\,[\sigma_3(E_0) - \sigma_1(E_0)]\,l^2}{\sigma_{d1}(E_0)\,R^2}\right\}^{1/3} \ll 1.$$

Under this condition, the impedance $Z(0)$ of the sample with the pinch passes through zero in a field E_x close to E_0, i.e., when the pinch has a large radius $\rho_0 \gg l$ (this follows from (5.14)). No thin pinches are produced, since the entire decreasing (i.e., stable) branch of the CVC corresponds to thick current pinches. The entire branch observed in the presence of the pinch is close to vertical, $E_x = E_0$.

Current-voltage characteristics with a break in the voltage and with a vertical section were observed in n-InSb[22,24], and in germanium in low-temperature breakdown[50] (Fig. 5). McWhorter has established that the collapse of the voltage in the breakdown is connected with the occurrence of a current pinch, namely, the conductivity of the sample in the direction perpendicular to the current remains unchanged during the breakdown*. Current pinches under breakdown conditions were investigated in[113].

In some semiconductors, the CVC has a complicated

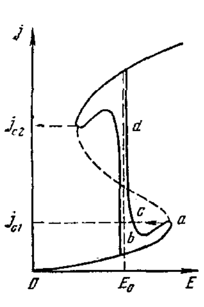

FIG. 13. CVC of a sample with a current pinch. The dashed line shows the decreasing section of the CVC in the case of uniform current distribution.

*McWhorter's work is described in [113].

form: it is N-shaped in one region of the currents (fields), and S-shaped in another. For example, in n-CdTe, when the field is increased, domain motions connected with the N-shaped characteristics is first observed, followed, at a definite average field in the sample, by a jumpwise increase of the current and a drop of the voltage. This gives rise to glowing pinches that are elongated in the direction of the current. The glow is due to the recombination of the electrons and the holes produced in the breakdown of the semiconductor[114].

5.6. Alternating-current Behavior

The properties of a sample with a current pinch when a small alternating signal is applied are described by the differential admittance (5.13). The dynamic characteristics of samples with current pinches were not investigated experimentally. Nonetheless, it is of interest to discuss the frequency dependence of the admittance $Z^{-1}(\omega)$, from which one might determine the characteristics of the pinch. We shall do so using as an example a sample with a large-radius current pinch, when the static CVC is close to vertical. In the corresponding expression (5.14), the frequency $|\lambda_0| \sim (l/\rho_0)^2\tau^{-1} \ll \tau^{-1}$, where τ is the characteristic time of energy dissipation in the homogeneous plasma. Therefore the dispersion of the admittance is much narrower at low frequencies $\omega \ll \tau^{-1}$. The motion of the pinch wall, which determines the static current-voltage characteristic, is rapidly "turned off" with increasing frequency. One can therefore speak of "inertia" of the wall, corresponding to an inductance L_c.

The real part of the impedance vanishes at a frequency

$$\omega_0 \simeq |\lambda_0|\,|R_c|^{-1/2}(Z_1^{-1}(0)+Z_3^{-1}(0))^{-1/2},$$

which is larger than $|\lambda_0|$, but still smaller than τ^{-1}. When

$$\omega \gg [Z_1^{-1}(0)+Z_3^{-1}(0)]^{-1}L_c^{-1}$$

the oscillations of the wall can be neglected in general; the electric conductivity is simply made up of the electric conductivities of the pinch and of the surrounding cold plasma.

5.7. Analogies with Phase Transitions

The result suggests a number of analogies between the behavior of systems with negative differential resistance and with phase transitions, for example in the liquid-gas system. The Van der Waals isotherm, plotted in "average density of both phases—pressure" coordinates, is similar to the S-shaped dependence of the current density on the field. The states on the decreasing branch with $\sigma_d < 0$ are unstable, similar to the states on the same branch of the isotherm on which $\partial p/\partial V > 0$. Just as in the case of the phase transitions, the long-wave perturbations are the fastest to increase. The conditions for the equilibrium of broad stable regions, $U(\Theta_1) = U(\Theta_3)$, is similar to the thermodynamic condition for phase equilibrium, which calls for equality of their chemical potentials μ. The condition for the extremum of the thermodynamic potential of the system corresponds in our case to the condition for the extremum of Φ (5.7), in which—U plays the role of μ.

Notice should also be taken of the analogy between the effects of motion of the wall of a current pinch under static or dynamic variation of the current and the sample, and the effect of motion of the domain walls is ferromagnetic and in ferroelectric substances.

6. CERTAIN PROBLEMS

In conclusion, we note certain unresolved problems in the physics of semiconductors with negative differential conductivity.

There are at present too few experimental data on semiconductors with S-shaped CVC to permit a comparison with the existing theory. In addition, almost all the observed S-shaped CVC and current pinches are connected either with breakdown or with injection from contacts. At the same time, there is no satisfactory theory of breakdown accompanied by occurrence of an S-shaped CVC, and particularly the theory of pinching of the current in breakdown and injection.

The influence of the magnetic field on the instability in semiconductors with negative differential conductivity has hardly been investigated. In particular, it is to be expected that under definite conditions magnetic striations can arise in a semiconductor, similar to the magnetic striations in a weak ionized gas plasma[121,123].

It is interesting to ascertain whether a generation similar to the Copeland generator can be based on a semiconductor with an S-shaped CVC.

No traveling layer waves, weak field domains, or (in two-valley semiconductors) broad domains with flat tops have been observed as yet. The reason why a stationary layered distribution of the field (Secs. 3.2 and 3.5) exist only at carrier densities smaller than a certain critical value, and a moving domain is produced at a higher density, remains unexplained.

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