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*MAGNETIC SURFACE LEVELS**

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I. INTRODUCTION. PHYSICS OF CONDUCTION ELECTRONS WITH LONG MEAN FREE PATH

The remarkable progress in metal physics has been attained recently mainly in the study of metallic single crystals, in which the conduction carriers have a long mean free path comparable with the dimensions of the single crystal or with some other dimension characteristic of the experiment. The variety of the phenomena that arise under such specific conditions, the volume of the research performed in this region, the quantity and scientific value of the obtained information are so great that there is every reason for speaking of the development of a new branch of physics—the physics of electrons with large mean free paths. This region includes, in particular, "fermiology" (this term is widely used in foreign literature), i.e., an extensive complex of various experimental and theoretical investigations of the energy spectra of the carriers in metals, the main purpose of which is to study the Fermi surfaces of metals.

Let us recall certain aspects and phenomena characteristic of experimental investigations of metals whose conduction electrons have large mean free paths.

Modern methods of chemical purification of metals and of production of their single crystals yield samples with an electron mean free path on the order of several millimeters at liquid-helium temperature. The corresponding free path time is of the order of nanoseconds. This makes it advantageous to use single crystals with dimensions on the order of millimeters in the experiments. Under such conditions it is possible to investigate size effects observed at all frequencies (from zero to several GHz) of the measuring current flowing through the sample. In the GHz region, when the free path time of the electrons exceeds the period of the measuring current, resonance effects are observed.

Application of a constant magnetic field of sufficient strength to set the electrons to revolve on closed orbits leads to quantization of their periodic motion. Therefore, owing to the change of the magnetic field in measurements at all frequencies, oscillations of various characteristics of the metal are observed, namely, oscillations of the magnetic susceptibility (the de Haas–van Alphen effect), of the conductivity (the Shubnikov–de Haas effect), and of the surface impedance. One more large group of phenomena observed and investigated in recent years takes place in metals placed in a magnetic field, namely the propagation of weakly-damped electromagnetic waves of various types and frequencies in the metal.

In this article we consider the oscillations of the surface impedance measured at microwave frequencies as functions of the weak magnetic field applied to the metallic sample[1]. The region of magnetic fields in which this effect is considered—from several hundredths to several Oersteds (at a frequency 10 GHz)—is the main phenomenological feature distinguishing it from all the other phenomena.

II. HIGH-FREQUENCY PHENOMENA IN METALS WITH LARGE ELECTRON MEAN FREE PATHS

Let us consider the following experiment: a single crystal of a metal in the form of a plane-parallel plate of thickness D is placed in a constant magnetic field H directed along the y axis and parallel to the surface of the single crystal. The measuring current I with frequency $\omega = 2\pi/T$ flows along the x axis over the surface of the sample in a skin layer of depth δ (Fig. 1). As is well known, in this case a classical cyclotron resonance is observed[2], namely the decrease of the surface resistance of the metal when the period t of the revolution of a definite group of electrons is a multiple of the period of the current: $t = nT$, $n = 1, 2, 3, \ldots$ This group should consist of electrons having practically identical dynamic properties, and the number of the electrons

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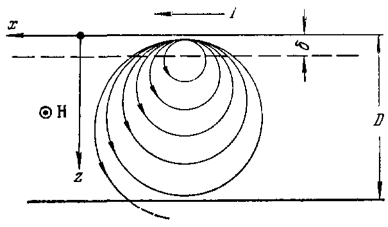


FIG. 1. Diagram of motion of the electron in cyclotron resonance in a flat sample of thickness D.

must be sufficiently large, as is the case for definite sections of the Fermi surface, for example, those adjacent to its extremal sections. From the foregoing resonance condition it follows that

$$\omega = n\Omega = \frac{neH}{m^*c}, \quad (1a)$$

where Ω is the Larmor frequency for electrons with effective mass m*. It follows therefore that when the field H changes, the minima of the surface resistance—resonances of order n—will follow periodically as a function of H^{-1} in intervals of $\Delta H^{-1} = e/m^*c\omega$. A plot of such an experiment[3] is shown in Fig. 2; its direct result is the measurement of m*.

With decreasing field H, the dimensions of the electron trajectories and the order n of the observed resonance increase, up to a value H_{CO}, at which the diameter of the trajectory becomes equal to the thickness D of the sample. Collisions with the surface (see Fig. 1) disturb the periodicity of the electron motion and consequently violates the resonance condition, leading to a vanishing of cyclotron resonances of order higher than n_{CO} from the experimental plots. This phenomenon of cutting off the cyclotron resonances is demonstrated by the upper curve 3 of Fig. 2. The jump of the surface impedance at the field value at which the diameter of the trajectories

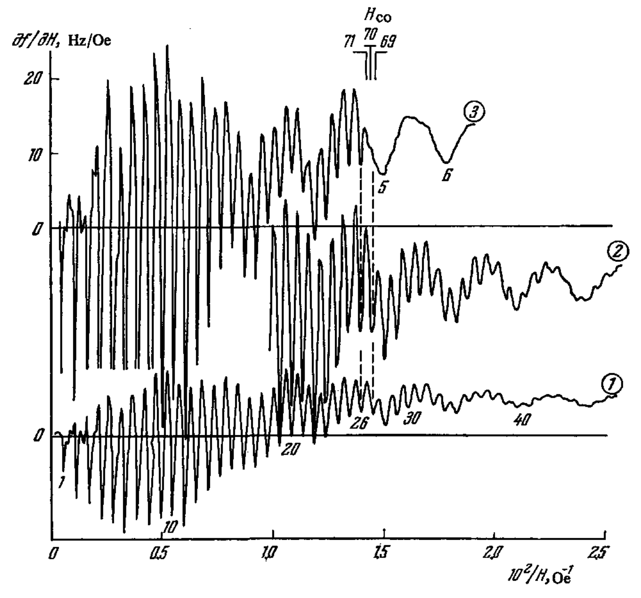


FIG. 2. Plot of cyclotron resonances in single-crystal tin of thickness 2 mm (1 and 2) and 0.982 mm (3) [3]; resonance of order 5 and 6 of the electrons with a smaller mass remain visible. It is seen that resonances of order 27 and higher are missing from curve 3. The orders of the resonance are indicated under curves 1 and 3 (Sn; 3.75°K; 9.4 GHz, $H \parallel C_4 \perp N \parallel C_2$).

of a definite group of electrons becomes equal to the thickness of the sample is observed also in the case if the resonance condition (1a) is not satisfied (cutoff of nonresonant trajectories[4]). Size effects of this kind make it possible to measure the diameter of the Fermi-section cross section $2p = (e/c)HD$, on which the orbits of the investigated group of electrons lie; the presented formula was obtained by direct integration of the electron equation of motion $\dot{\mathbf{p}} = (e/c)\dot{\mathbf{r}} \times \mathbf{H}$ (p is the electron momentum and r its radius vector).

The increase of the trajectory dimensions with increasing field H leads to a drop in the amplitude of the cyclotron resonances with increasing n. This is the result of the scattering of the electrons, as they move inside the metal, by the thermal phonons and defects of the crystal lattice of the sample. If the sample thickness is sufficiently large, then the experimental plots (1 and 2 in Fig. 2) show how the resonances broaden with increasing n, cease to be resolved, acquire the form of sinusoidal oscillations with decreasing amplitude, and finally vanish in the noise.

III. EXPERIMENTAL PROCEDURE

To study the foregoing high-frequency phenomena and oscillations of the surface resistance of a metal in a weak magnetic field, which are the subject of this article, it is necessary to use a measurement method sensitive enough to be able to register very small changes of the surface resistance of the sample. In microwave radiospectroscopes, the direct measurement object is usually not the sample but a cavity resonator, in which the sample is placed in such a way that the changes of its surface resistance influence as strongly as possible the properties of the resonator. Most frequently one measures the changes of the Q of the resonator, which depends on the active surface resistance of the sample.

In[1,9-11] we have used a frequency-modulation method[5], which makes it possible to measure small changes of the natural frequency of the resonator, due to the changes of the surface impedance of the sample. Usually the measuring circuit is tuned in such a way as to register changes of the reactive component of the surface impedance, but it is also possible to register changes of its active component.

The measurement setup, a diagram of which is shown in Fig. 3, operates in the following manner. The investigated metal sample serves as part of a microwave resonator cooled with liquid helium and placed in a magnetic field. If the change of the field influences the surface impedance of the sample, this leads to a change in the electric characteristics of the resonator. The resonator is connected in a broadband feedback circuit of a traveling-wave-tube oscillator, and its characteristics determine the frequency of the generated signal. Modulation of the magnetic field with frequency ~ 10 Hz leads to frequency modulation of the microwave signal of the oscillator. The electronic circuitry which measures the frequency deviation generates a voltage proportional to the logarithmic derivative of the surface impedance of the sample with respect to the magnetic field. This voltage (which can be calibrated in units of Hz/Oe), as a function of the magnetic field, is recorded by an automatic-plotting electronic potentiometer. The field is produced by a Helmholtz system with compensa-

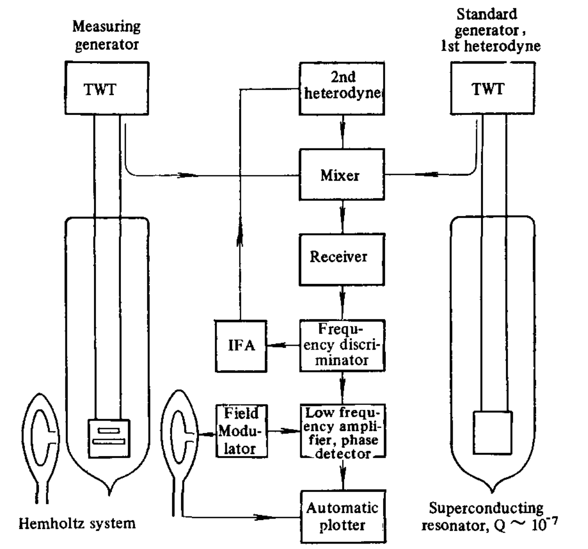
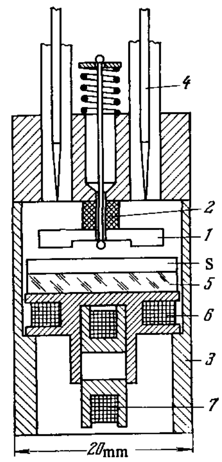


FIG. 3. Block diagram for measurement setup by the frequency modulation method [5]. The relative frequency stability of the standard generator, stabilized by a superconducting high-Q resonator, is $\sim 10^{-9}$.



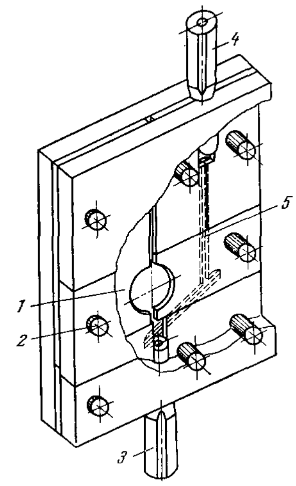


FIG. 4. Section through strip resonator. TEM mode. S – sample, 1 – resonating strip, 2 – foamed polystyrene, 3 – housing, 4 – coaxial communication lines, 5 – disc of fused quartz, 6 and 7 – coils used to determine the reference point for the measurement of the orientation angles of the magnetic field by the vanishing of the emf induced in these coils by the field modulation.

FIG. 5. Mold of optical fused quartz for growing metallic single crystals. 1 – Cavity in which the sample crystallizes, 2 – ground dowels which secure the parts of the mold, 3 – capillary with priming crystal, having a reflecting spot for orientation; 4 – end of funnel through which the metal is poured, 5 – channel milled in the rear wall of the mold.

tion of the earth's magnetic field, is determined by the current flowing through the coils, and is calibrated by nuclear magnetic resonance.

The plots shown in Figs. 2, 6, 8, and 17 were obtained by the described frequency-modulation method[5]. The sensitivity of the method to relative changes of the surface impedance Z of the sample is $\sim 10^{-6}$, and when converted into absolute changes of the depth of penetration δ of the microwave electromagnetic field into the metal it amounts to $\sim 10^{-11}$ cm.

The construction of the strip resonator used in these experiments is shown in Fig. 4. Sample S in the form of a flat disc serves as the bottom of a cylindrical volume, in which the resonant element is contained. This element is a rectangular strip 1, the electric length of which equals to half the wavelength of the generated microwave signal. The microwave currents induced in the sample flow over the section of its sample situated under the strip, in the direction of the strip length. Rotation of the lower part of the housing 3 of the resonator together with the sample makes it possible to change the polarization of the currents relative to the crystallographic axes of the sample. The single-crystal sample S lies freely on the quartz disc 5 on which it was grown; the absence of any mechanical mounting of the sample protects it against damage that might be produced by stresses due to temperature changes.

The metallic single-crystal samples were grown from the melt in optically polished dismountable molds made of fused quartz. The construction of one such mold is shown in Fig. 5; the single crystal grows in a round cavity 1 located in the center of the mold and made up of thick lateral walls of the mold with inserts compressed between them, which are secured with ground through dowels 2. The capillary 3 inserted in the mold from the bottom contains a suitably oriented priming crystal; the metal is poured through a funnel terminated with a ground capillary 4 and inserted from above. The mold, with the lightly soot-covered internal surfaces, is placed inside a vacuum setup with an oven that heats the mold, thus ensuring a vertical temperature gradient. After the molten metal passing through the capillary 4 into channel 5 fills the molds, the mold is cooled from below, and the single crystal filling the cavity grows from the primer. The thickness of the cavity is determined usually beforehand by means of a quartz disc inserted in the cavity. After the mold is opened, the pouring end and the primer are cut off and the single crystal is moved into the resonator on the quartz disc (5 in figure) located in the cavity of the mold.

The directions of the crystallographic axes in the sample are determined from the anisotropy of the investigated effects and are monitored by x-ray measurements.

IV. EFFECT OF OSCILLATIONS OF THE SURFACE IMPEDANCE OF A METAL IN A WEAK MAGNETIC FIELD

Figure 6a shows an experimental plot that demonstrates the appearance of the new effect—oscillations of the surface impedance of a metal in a weak magnetic field[1], in a field region lying much lower than the value at which the traces of cyclotron resonances of higher orders, seen in Fig. 6b, disappear. The amplitude of these new oscillations is quite appreciable, approximately of the same order of magnitude as the amplitude of the cyclotron resonances of the first orders.

The new effect was observed in[1] by the frequency-modulation method[5] in tin, indium, and cadmium, and a number of empirical characteristics of the effect were explained. Its existence was confirmed by observations made in[6] on tin and in[7] on tungsten. The most detailed experimental investigations of the effect on tin, indium, and aluminum were performed in[8] in the frequency region 28—70 GHz.

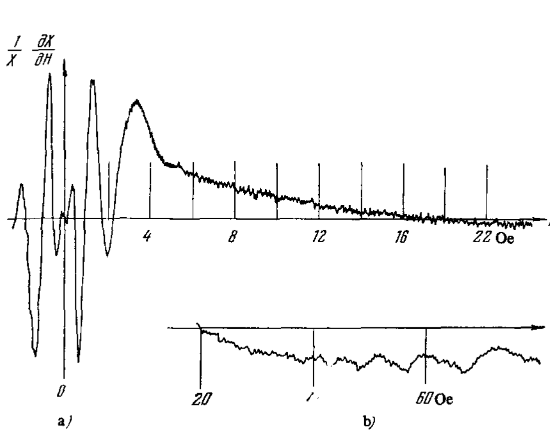


FIG. 6. a) Oscillations of the surface impedance of tin in a weak magnetic field; b) plot of higher-order cyclotron resonances. Sn; 3.75°K, 9.4 GHz, $H \| C_4 \perp N$, $I \| C_2$.

To ascertain the physical causes of the occurrence of the new effect, which remained unexplained for a long time, it was necessary to investigate it in a metal whose Fermi surface is well known, so as to be able to set in correspondence the main characteristics of the effect with definite singularities of the Fermi surface. Such a metal is bismuth; its relatively simple Fermi surface has been investigated in detail, and many of its numerical characteristics have been measured with a very high accuracy. For this reason, experimental investigations were undertaken of the oscillations of the surface impedance of bismuth[9-11], which are used in the present article for a comparison with the experimental calculations.

A plot of an experiment with single-crystal bismuth is shown in Fig. 7. We see that when the magnetic field decreases, the cyclotron resonances weaken rapidly, vanishing in the noise, and give way to surface-impedance oscillations of appreciable amplitudes, the location of which as a function of the field turns out to be entirely different.

Let us consider the empirical characteristics of the effect of oscillations of the surface impedance of a metal in a weak magnetic field.

1) The effect is observed only in very thin and physically perfect single crystals, having an optically smooth surface. Mechanical damage (bending) of the crystal or etching of the surface destroys the effect. In the case of tin, it suffices to etch the surface with hydrochloric acid vapor, which leads to appearance of roughnesses of magnitude on the order of several microns[12]. In the case of bismuth, a rougher treatment by a liquid etchant is necessary, until roughnesses of several dozen microns in magnitude appear; even then, very weak oscillations are noticeable in fields <0.1 Oe, whereas the stronger oscillations in fields >1 Oe vanish completely[11].

2) The effect is observed at helium temperatures; the amplitude of oscillations in bismuth increases with decreasing temperature like $A(T^\circ) \cong A(0)\,[1 - (T^\circ/6)]$ in the interval 4.2–1.6°K.

3) Oscillations of both components of the complex surface impedance of the metal take place; changes of the active surface resistance, as functions of the magnetic field, are proportional to the derivative with respect to the field of the changes of the reactive component[1].

4) The field value H_n, corresponding to some characteristic position (say, a maximum) of the oscillation of the number n, is anisotropic and increases like $H_n(\varphi) = H_n(0) \sec \varphi$ when the vector H is rotated through an angle φ relative to a certain crystallographic direction in the sample, regardless of the angle of inclination of the field to its surface[1]. At small values of φ, this law is always valid; at φ close to the limit of observation of the oscillations in the given sample, the field H_n sometimes grows more rapidly[8]. Figure 8 shows a plot of the oscillations of the surface impedance of bismuth at different directions of the field H. The polar diagram $H_n(\varphi)$ obtained as a result of such experiments for the basal plane of the bismuth crystal is shown in Fig. 9; we see that the law $H_n(\varphi) = H_n(0) \sec \varphi$, which is represented by straight lines, holds in the entire region of observation of the oscillations[10,11].

5) The amplitude of the oscillations depends on the polarization of the microwave currents, following the law $A(\vartheta) = A(0) \cos \vartheta$, where ϑ is the angle between the direction of the vector I and the crystallographic direction $\varphi = 90°$. The corresponding plot (circle) and the experimental points obtained in investigations of bismuth

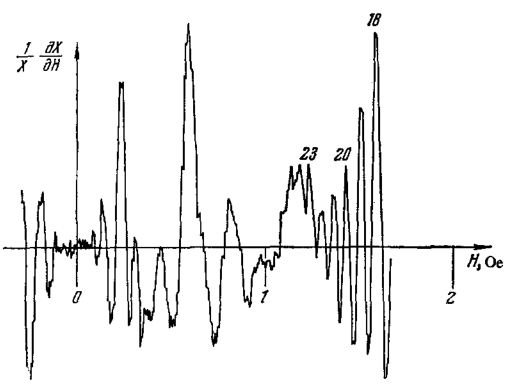


FIG. 7. Oscillations of surface impedance of bismuth in a weak magnetic field and cyclotron resonances of higher orders (18-23) observed in a field > 1 Oe. Bi; 2mm; 1.7°K; 9.4 GHz; $C_3 \| N$.

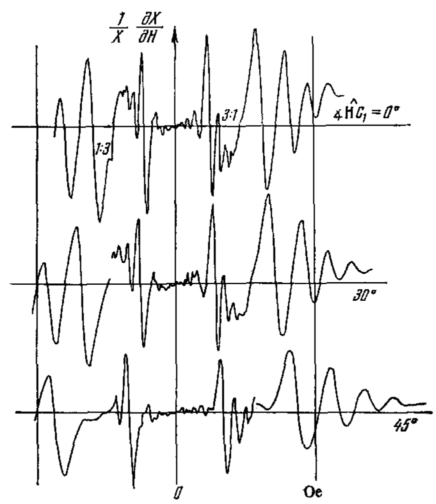


FIG. 8. Oscillations of the surface impedance of bismuth, recorded for different directions of the field vector H, indicated on the right sides of the curves. 3:1 – place where the gain of the system changes by a factor of 3 (Bi; 1.7°K; 9.4 GHz; $H \perp C_3 \| N$).

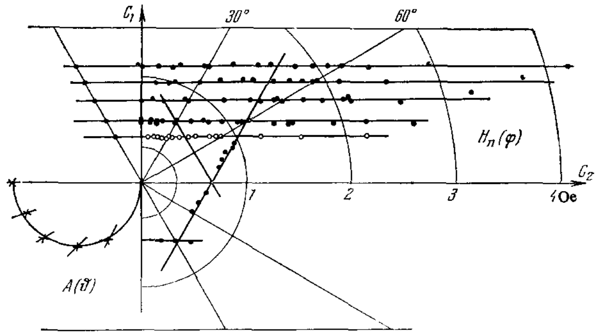


FIG. 9. $H_n(\varphi)$ – dependence of the values of the field corresponding to different oscillation maxima shown in Fig. 8 on the direction of the field vector. $\varphi = \sphericalangle HC_1$; $A(\vartheta)$ – dependence of the amplitude of the highest maximum on the current direction, $\vartheta = \sphericalangle C_2 I$ (Bi; 9.4 GHz; $H \perp C_3 \| N$).

are shown in Fig. 9. Thus, the absolute maximum of the amplitude of the oscillations is observed at $\varphi = 0°$ [8,10,11].

6) The increase of the frequency ω of the measuring current leads to a stretching of the oscillations in the direction of large fields in accordance with the law $H_n \sim \omega^{3/2}$ [8,9].

7) The effect could not be observed in copper, silver, sodium, or potassium[6].

V. CLASSICAL HYPOTHESES CONCERNING THE ORIGIN OF THE EFFECT

1. We consider first certain results of empirical investigations of the effect. The clearest and most direct conclusion follows from the character of its anisotropy, which is particularly clearly pronounced for bismuth (Fig. 9): the effect is determined by the projection of the magnetic field $H_n(0) = H_n(\varphi) \cos \varphi$ on the major axis of the strongly elongated electronic "ellipsoid" of the Fermi surface, the central part of which (within the limits of experimental accuracy) is cylindrical[13,14].

The polar diagram of Fig. 9 corresponds to three cylindrical Fermi surfaces, the axes of which are perpendicular to the binary axes C_2 of the bismuth crystal (these cylinders are the central parts of the three electronic "ellipsoids"). The motion of the electrons belonging to the cylindrical Fermi surface is determined by the projection of the magnetic field on the cylinder axis; the other component of the field has no influence at all on the motion. The trajectories, and with them the velocities of the electrons, lie in planes perpendicular to the axis of the cylinder, in full agreement with the dependence of the amplitude of the effect on the polarization of the microwave currents.

We note that the Fermi surfaces of polyvalent metals, in which the effect is observed, have parts whose shape is very close to cylindrical (the barrel of the hole surface of zone IV of tin[4], the tubes of the electronic surfaces of zone III of indium[15] and of aluminum[16]) and the anisotropy of H_n makes it possible in all cases to relate the effect with these "cylinders." At the same time, monovalent metals, on which the effect has not yet been observed to date, have practically spherical Fermi surfaces.

Separating in this manner the relatively numerous extremal group of conduction electrons which should cause in a weak magnetic field oscillations of the surface impedance of the metal, let us turn to a further analysis of their motion in order to clarify the possible physical causes of the effect.

2. In a magnetic field $H \sim 1$ Oe, the radius of curvature of the electron trajectories $R = Pc/eH$ is $R(Bi) \sim 0.1$ cm for bismuth, $R(Sn) \sim 10$ cm for tin, i.e., $R \gtrsim l$, D, where the electron mean free path is $l \sim 0.1$ cm and the sample thickness is $D \lesssim 0.1$ cm (the corresponding values for tin and other polyvalent metals have the same order of magnitude). It follows from this estimate, first, that an electron moving on a "glancing" trajectory, the turning point of which O is near the surface of the metal (Fig. 10), can stay in the skin layer $\delta(Bi) \cong 10^{-4}$ cm or $\delta(Sn) \cong 10^{-5}$ cm only once during its free path time. Second, the arc of the trajectory lying in the skin layer subtends a small angle $\alpha \cong \sqrt{2\delta/R} \sim 10^{-1}-10^{-3}$, so that it can be regarded as a circular arc; its radius R is determined by the radius of curvature of the Fermi surface P at the point near which the representative part of the electron moving in the skin layer remains.

The duration of the motion of the electron in the skin layer is $t = 2\alpha R/v_F \sim 10^{-10}$ sec (the Fermi velocity of the electrons is $v_F \sim 10^8$ cm/sec), i.e., of the order of the period T of the currents of frequency $\omega/2\pi \sim 10$ GHz, at which the experiment described in Sec. IV were performed. This comparison immediately suggests that the dependence of the surface impedance on the magnetic field, Z(H), should have a singularity in a field H_t in which $t = T/2$. Under this condition, the energy transferred to the electron from the electric field in the skin layer will be maximal. Using the foregoing formulas for t, α, and R we get

$$H_t = \frac{8c}{\pi^2 e} \frac{P}{v_F^2} \delta\omega^2. \quad (1)$$

An estimate yields $H_t(Bi) \sim 1$ Oe and $H_t(Sn) \sim 10$ Oe, which falls in the region of fields in which the oscillations are observed. A more accurate calculation, performed in[6], leads to practically the same results.

However, the considered situation justifies the appearance of only a certain single singularity of Z(H), without explaining the occurrence of oscillations. We can attempt to make use of an additional consideration, namely that a series of singularities of Z(H) should occur if the condition $t = [n + (1/2)]T$ is satisfied (n = 0, 1, 2, 3, ...), which undoubtedly would have to be taken into account when the electron moves in a homogeneous alternating electric field[8]. In this case the singularities of Z(H) should occur at field values

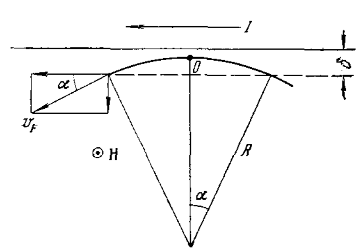


FIG. 10. Trajectory, glancing over the metal surface of an electron penetrating through the skin layer, in a weak magnetic field.

$$H_{tn}=\frac{H_t}{(2n+1)^2},$$

which are located relatively far away from each other in accordance with a law that does not agree with experiment. Incidentally, such an approach can hardly be justified, for it is necessary to take into consideration the fact that the electric field in the skin layer decreases approximately exponentially with depth (in the case of the anomalous skin effect), as a result of which the predominant fraction of the energy acquired by the electron, in any magnetic field, will occur in that section of the trajectory which is closest to the surface of the metal.

3. To determine the conditions under which the energy transfer from the alternating electromagnetic field to the electron is most effective, it is necessary to take into consideration the motion of the electromagnetic wave in the skin layer in the direction of the inward normal z to the surface of the metal with velocity $v_B = \omega\delta \sim 10^6$ cm/sec. If the wave were not to attenuate, then the maximum energy would be acquired by those electrons that move deep in the metal with velocity v_B in a constant-phase field (the Landau condition); in the absence of a constant magnetic field these are the electrons moving in an angle $\alpha_{eff} \cong v_B/v_F \sim 10^{-2}$ to the surface of the metal. If we take into account the damping of the wave, then the maximum energy will be acquired by the electrons moving at a smaller but nonvanishing angle α. This means that the strip a of the Fermi surface (Fig. 11) representing the effective electrons is that region of the Fermi surface on which the electron velocity vector is inclined to the plane of the boundary of the metal, inside the boundary, by a small angle α, and is not parallel to the boundary as is assumed in accordance with Pippard's "ineffectiveness concept."[17] Such a shift of the strip of the ineffective electrons over the Fermi surface from its central section in the direction of z is relatively not small, since $\alpha_{eff} \gg \alpha_P$, where $\alpha_P = \delta/l \sim 10^{-4}$ is the angle width of the Pippard "effectiveness strip" (c of Fig. 11).

Application of a magnetic field, which bends the electron trajectories, leads to rotation of the effectiveness strip relative to the field vector, if it is assumed that the effective electrons will be those whose average

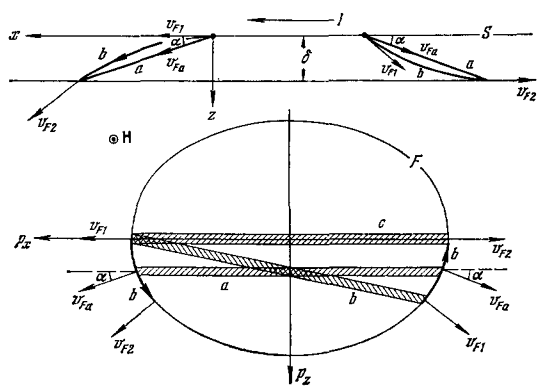


FIG. 11. Lower part of the figure—momentum space (F – section of Fermi surface); upper part of figure – coordinate space (S – section of metal surface). a – electron trajectory at H = 0, on both parts of the figure l-trajectory of electrons at $H \neq 0$; the corresponding vectors of the Fermi velocity v_F are designated by identical indices. Strips a and b are represented as linear to simplify the figure.

velocity in the skin layer on the path from the surface to the interior of the metal equals the wave velocity. When the field is increased, the effectiveness strip is rotated (Fig. 11) and at a certain value of the field H_L it falls on the central cross section c of the Fermi surface, so that the optimal Landau conditions are satisfied for the extremal group of electrons belonging to the central section and beginning their motion with a velocity tangent to the boundary of the metal (see Fig. 10). Assuming as an estimate that $\alpha \cong \alpha_{eff}$, we get

$$H_L \simeq \frac{2c}{e}\frac{P}{v_F^2}\delta\omega^2, \quad (2)$$

i.e., a value practically coinciding with the value of H_t calculated above. Thus, the mechanism considered here can in principle also cause the appearance of a single singularity of the function Z(H) in the region of weak magnetic fields, but there are apparently no grounds whatever for indicating the appearance of oscillations here.

4. So far we have considered electrons moving along arc orbits whose centers lie deep in the metal (see Fig. 10). We now proceed to consider the role of electrons moving along arcs whose centers lie outside the metal, over its surface (Fig. 12). As will be shown subsequently, this is a very important step in the direction of explaining the nature of the investigated effect[9,10]. These electrons belong to Fermi-surface points that are diametrally opposite to the points to which the previously-considered electrons belong. (Since these points are located symmetrically relative to the center of the reciprocal-lattice cell, they are equivalent.)

These electrons begin their motion at the point O on the surface of the metal (Fig. 12), reach a depth h in the interior of the metal, and after a time t_0 they return to its surface. After executing a jump over the surface of the metal, such electron is reflected from the surface in a random direction, i.e., is scattered by it (the possibility of specular reflection will be discussed later). All the electrons belonging to the cylindrical Fermi surface move with identical velocities v_F along arcs of equal radius R = Pc/eH. The centers of the different arcs lie at different distances from the surfaces of the metal, and the lengths of these arcs, which terminate on the surface of the metal, are different. Out of the entire set of arcs we chose those for which motion over the initial sections satisfies the Landau condition of optimal acceleration of the field E(t) of the microwave in the skin layer δ. This condition (as already mentioned) is that the wave velocity v_B and the electron velocity in the direction of the inward normal Oz to the surface of the metal be equal: it defines the angle $\alpha = v_B/v_F = \omega/kv_F \sim 10^{-2}$ (k—wave number). Thus, by considering only electrons moving in the skin layer in phase with the

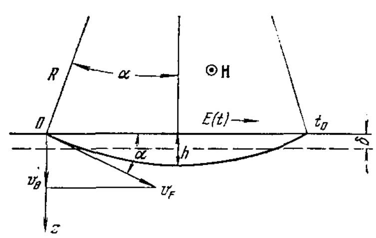


FIG. 12. "Jumping" trajectory of an electron in a weak magnetic field, starting and terminating on the surface of the metal.

wave, the attenuation of which is not taken into consideration, we can separate the extremal group of synchronously-moving electrons receiving the maximum energy from the electromagnetic wave.

The time that the electrons of this group move over their arc trajectories is $t_0 = 2\alpha/\Omega$, where $\Omega = ev_F H/Pc$ is the Larmor frequency. It is clear that at certain values of the magnetic field H_n there may be a simple integer ratio between t_0 and the microwave field period T; whenever this occurs during the variation of the field H, a singularity of the function Z(H) should be observed, and this leads to its oscillations.

In the case of a sufficiently weak field H, the condition $h > \delta$ is satisfied (see Fig. 12). The interaction of the electron with the field E(t) then proceeds principally at the beginning and at the end of its trajectory, while the central part of the trajectory lies inside the metal. As a result, the maximum contribution of this group of electrons to the microwave current, and consequently the singularity of Z(H), will occur when the condition $t_0 = nT$ (n = 1, 2, 3, ...) is satisfied, from which we get the relation $(\alpha/\pi)\omega = n\Omega$.

We note that the latter equality goes over in the particular case $\alpha = \pi$ into the well known condition (1a) of classical cyclotron resonance $\omega = n\Omega$. This is in full agreement with the fact that in this particular situation the proposed cause of the oscillations of the surface impedance is analogous to the cause of cyclotron resonance, namely, the microwave current in the skin layer increases when the magnetic field is such that it ensures the return to the skin layer of a definitely singled-out extremal group of electrons in synchronism with the microwave field.

For the values of the magnetic field H_n at which the foregoing condition is satisfied we get

$$H_n^{-1} = n\frac{\pi e k v_F^2}{cP\omega^2} = n\Delta H^{-1} \qquad (n = 1, 2, 3, \ldots). \tag{3}$$

Thus, in our case when $h > \delta$, periodic oscillations of the function $Z(H^{-1})$ should occur, with a period

$$\Delta H^{-1} = \frac{\pi e v_F^2}{cP\omega^2\delta}. \tag{4}$$

In this equation we used the relation $k = 1/\delta$, which is approximately correct under the conditions of the anomalous skin effect.

In the case of a stronger field, when $h < \delta$, the trajectory of the electron lies entirely in the skin layer, and the amplitude E(t) of the microwave field can be assumed to be the same on the entire path of the electron. The contribution of the electrons moving under such conditions to the microwave current is maximal when $t_0 = [n + (1/2)]T$, inasmuch as the drift velocity acquired in a time nT by an electron in the homogeneous microwave field is equal to zero. In the case under consideration the conditions of electron motion are analogous to the conditions in the relaxation region[18], namely, the contribution of the electron to the microwave current is determined by the difference between its free path time and an arbitrary integer number of periods of the microwave field. Consequently, the singularities of the function $Z(H^{-1})$ should occur at field values

$$H_n^{-1} = \left(n + \tfrac{1}{2}\right)\Delta H^{-1} \qquad (n = 0, 1, 2, \ldots), \tag{5}$$

i.e., the oscillations of $Z(H^{-1})$ should have the same period (4) as in the case when $h > \delta$, but with a shift by $\Delta H^{-1}/2$.

Let us see, by finding the connection between h and δ, which of the two considered cases can actually be realized. Since the electron trajectory can be regarded as circular arcs, we have $h = R\alpha^2/2$ and a simple calculation leads to the ratio $h/\delta = 3t_0/2T$. It follows therefore that the case $h < \delta$ can be realized only when $t_0 = T/2$, i.e., when n = 0; all other singularities of $Z(H^{-1})$, occurring in weaker fields H_n, can pertain only to the case $h > \delta$ when $n \gg 1$, or else to transient conditions of the else to transient conditions of the electron motion when $n \sim 1$.

Let us turn to a comparison with experiment. Estimate on the basis of formula (3) yields $H_1(\text{Bi}) \cong 0.3$ Oe and $H_1(\text{Sn}) \cong 3$ Oe, which is in very good agreement with observation. The situation is worse with the periodicity of the oscillations: the experimentally obtained spectra are much more complicated than predicted by formulas (3) and (5), and by suitable choice of the parameters contained in them it is possible to obtain satisfactory agreement in the position of only some of the observed oscillations.

There is one more point in which the formulas (1)—(5) derived on the basis of the classical hypotheses contradict the experiment—the dependence of H_n on the frequency. As is well known, in the anomalous skin effect $\delta \sim \omega^{-1/3}$, so that formulas (1)—(5) lead to the same dependence of the field of the singularity of Z(H) on the frequency, namely $H_n \sim \omega^{5/3}$, whereas experiments[8,9] yield $H_n \sim \omega^{3/2}$.

Thus, attempts at explaining the oscillations of the surface impedance of the metal by a classical analysis of the electron motion lead only to partial success: it is possible to find physical mechanisms capable of leading to the appearance of isolated singularities and oscillations of the function Z(H) in the correct range of frequencies and fields, in agreement with a number of empirical characteristics of the effect. However, comparison with experimental results shows that certain important features of the effect contradict the conclusions based on the considered hypotheses.

The complicated character of the observed oscillation spectra do not exclude the possibility of occurrence of the described singularities of classical origin, but it is nevertheless perfectly clear that the main contribution to the effect is made by another mechanism, which, as will be shown later, has a purely quantum nature.

VI. QUANTUM THEORY OF THE EFFECT. MAGNETIC BOUND SURFACE STATES OF ELECTRONS

1. The quantization of "glancing" trajectories of the type shown in Fig. 10, performed in[19], has shown that the quantum approach to the calculation of the surface impedance of metals in a weak magnetic field may prove to be fruitful, but no explanation of the effect was obtained.

The decisive step in the clarification of the physical causes of the effect were made by Nee and Prange[20,21], who applied quantization to the analysis of the "jumping" trajectories[9,10], similar to that shown in Fig. 12. They developed the theory of stationary quantum states of the conduction electrons, which occur near the surface of the metal under the influence of a weak constant magnetic field. In order for such states to occur, the electrons must be specularly reflected from the surface of the metal.

In the classical analysis of the electron motion on jumping trajectories (see Sec. 4 of Ch. V) it was assumed that the electrons are diffusely scattered by the surface. Allowance for the possibility of specular reflection greatly changes the situation, for in this case periodic motion of the electrons in momentum space appears (along the z axis—in coordinate space), and must be quantized. Let us see what grounds there are for regarding specular reflection of the electrons by the surface of the metal as probable. First, we can advance the simple consideration that at a sufficiently small angle α the encounters between the electron and the surface, the de Broglie wavelength λ_B of the electron in the normal direction $\lambda_{B\perp} \cong \lambda_B/\alpha$ (this relation follows from the formulas $\lambda_B = h/p$, $p_\perp = \alpha p$) can be sufficiently large compared with the roughnesses of the optically smooth surface, which have dimensions not exceeding the light wavelength $\lambda_c \sim 10^{-4}$ cm. Since $\lambda_B(\text{Bi}) \sim 10^{-5}$ cm and $\lambda_B(\text{Sn}) \sim 10^{-7}$ cm, we get for the maximum angle α_{max} at which $\lambda_{B\perp} \cong \lambda_c$ the values $\alpha_{max}(\text{Bi}) \sim 10^{-1}$ and $\alpha_{max}(\text{Sn}) \sim 10^{-3}$, in agreement with the estimate of α for the jumping trajectories in Sec. 4 of Ch. V. In such a situation, specular reflection of the electrons can occur, similar to the reflection of optical waves from a rough surface under oblique incidence. However, experiments directly confirming the existence of specular reflection of the electrons by the surface of the metal are more convincing.

In the experiments of[22] we investigated the cut-off of cyclotron resonances in single-crystal bismuth ~0.2 mm thick; in a magnetic field weaker than the cutoff field, resonance is observed on electrons experiencing specular reflection from the surface of the sample. The amplitude of this "resonance with reflection" differs little from the amplitude of ordinary resonances, from which it follows that the probability of reflection under the experimental conditions was of the order unity. Investigations of the conductivity of thin layers and whiskers of different metals, performed in recent years (see, e.g.[23-25]) have established that the probability of reflection of the conduction electrons by the surface lies in the range ~0.5—1. Thus, the assumed specular reflection of the electrons by the surface of the metal as the electrons move along jumping orbits is fully justified.

2. We present a simple calculation of the spectrum of the stationary states of electrons in a weak magnetic field, using the Bohr-Sommerfeld quantization rule

$$\oint \mathbf{p}\, d\mathbf{r} = n_1 h,$$

where $n_1 = n + \gamma$ ($n = 1, 2, 3, \ldots$; $0 < \gamma < 1$). From the Bohr-Sommerfeld rule follows the condition for the quantization of the area of the closed orbit of the electron in momentum space

$$S_p = n_1 ehH/c = n_1 S_1^0, \qquad (6)$$

when the electron moves in a homogeneous constant magnetic field H. A consequence of this condition is the formation of Landau levels, which determine the allowed stationary states of the electrons. The number n of the Landau levels within the limits of the Fermi energy ϵ_F in a field $H \sim 1$ Oe is $n(\text{Bi}) \sim 10^4$ or $n(\text{Sn}) \sim 10^8$, and their energy splitting is

$$\Delta\varepsilon(\text{Bi}) \sim \varepsilon_F/n \sim 10^{-2}\,^\circ\text{K}, \quad \Delta\varepsilon(\text{Sn}) \sim 10^{-4}\,^\circ\text{K}.$$

It is obvious that at a sample temperature ~1°K the neighboring Landau levels are equally populated and no transitions between them can be observed.

This is the situation if the orbit is the total intersection 1 (Fig. 13) of the Fermi surface by a plane perpendicular to the field H. On the other hand, if the electron trajectory in coordinate space turns out to be truncated as a result of collision between the electron and the surface of the metal and specular reflection from the latter, then the quantization should be applied to the corresponding part of the area of the orbit in momentum space. For example, the orbit delineated by the dashed line 2 in Fig. 13 corresponds to motion of an electron from one surface of a thin sample to the other—such a situation is considered in[26] from the point of view of an investigation of the de Haas—van Alphen effect. A particular case of truncated trajectories of this kind arises upon reflection of electrons from one surface of the sample; this gives rise to jumping trajectories, corresponding in momentum space to the orbit delineated by the thick line 3 in Fig. 13. In such a case, it is necessary to quantize the segment area shaded in Fig. 13; at small segment dimensions, the orbit 3 bounding it can be regarded as a circular arc with radius R equal to the radius of curvature of the intersection of the Fermi surface at the vertex of the segment.

Let us proceed from the orbit in momentum space to the trajectory in coordinate space (Fig. 14). The linear elements of the spaces are connected by the replation $d\mathbf{p} = (e/c)d\mathbf{r} \times \mathbf{H}$, from which we get the following connection between the areas

$$S_p = \left(\frac{eH}{c}\right)^2 S_r. \qquad (7)$$

The area of the small segment enclosed by the trajectory in coordinate space is

$$S_r = \frac{4}{3}\sqrt{2Rz_n^3}, \qquad (8)$$

where $R = Pc/eH$ is the curvature radius of the arc, and z_n is the height of the segment corresponding to the n-th stationary orbit of the electron, allowed by the quantization rule (6). Substituting (7) and (8) in (6) we obtain the quantized values

$$z_n = \left(\frac{3h}{4\sqrt{2}}\right)^{2/3}\left(\frac{c}{eHP}\right)^{1/3} n_1^{2/3}, \qquad (9)$$

which determine uniquely the allowed electron trajectories for n = 1, 2, 3, ...

We note that the quantization rule (6) and Eq. (7) lead to an equation for the magnetic field flux Φ enclosed by the electron trajectory:

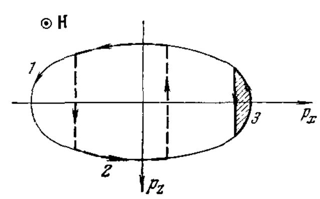


FIG. 13. Orbits of electron in momentum space: 1 — complete orbit, 2 and 3 — orbits truncated by reflections from the surface of the metal.

$$\Phi = HS_r = n_1 \frac{ch}{e} = n_1 \Phi^0, \tag{10}$$

where Φ^0 is the quantum of the flux; this relation can be used for the quantization of the electron trajectories in coordinate space.

We now find the energy levels of the allowed states. An electron moving along a trajectory with velocity v_F experiences, upon collision with the surface of the metal, a specular reflection and acquires in the direction of the normal z an initial velocity $v_\perp = v_F\alpha$. On the path to the turning point, which is located at a depth z_n, the electron loses an energy $\Delta E_n = mv_\perp^2/2$. Using the formula $z_n = R\alpha^2/2$, which is valid for a small segment, and the expression given above for R, we get the quantity

$$\Delta E_n = \frac{e}{c} v_F H z_n, \tag{11}$$

which characterizes the n-th stationary state of the electron. The system of energy levels (11) can be regarded as a set of allowed states of electrons in a potential well made up of an infinitely high potential wall on the metal boundary z = 0 and a potential V(z) = $(e/c)v_F Hz$, which increases with increasing depth in the metal.

The calculation of formula (11) contains an approximation, whereby the electron trajectory in the magnetic field is replaced by a parabolic arc. The accuracy of this approximation is quite high, in view of the smallness of the angular dimension α of the arc. This approximation consists in effect of replacement of the Lorentz force acting on the electron by an equivalent constant force permanently directed along the normal to the surface of the metal. As a result we get the picture of an electron moving in a field of a potential V(z) that increases linearly with the distance z. The correctness of this approximation is confirmed by the fact that the solution of the Schrödinger equation for this problem, given in Sec. 3 of this chapter, leads to the same potential V(z).

The motion of an electron jumping on the surface of a metal in a force field whose potential increases linearly with increasing distance from the surface is perfectly analogous to the motion of an elastic sphere jumping on a rigid horizontal surface in a gravitational field; the equivalent of the solutions of these two problems is obvious.

The frequencies of the resonance absorption or emission lines corresponding to the transitions between the different energy levels (11) are determined by Einstein's formula

$$\nu_{nk} = \frac{\Delta E_n - \Delta E_k}{h} = \left(\frac{3e}{4\sqrt{2}\,c}\right)^{2/3} \left(\frac{H^2}{Ph}\right)^{1/3} v_F (n_1^{2/3} - k_1^{2/3})^{3/2}. \tag{12}$$

This formula, similar to Balmer's formula for the spectrum of the hydrogen atom, describes a spectrum whose lines form series that differ in the quantum number k (analogous to the series of Lyman, Balmer, Paschen, etc.). The line frequencies depend on the magnetic field; on the other hand, if the frequency ν is fixed and the resonant values of the magnetic field are calculated, then we obtain

$$H_{nk} = \frac{4\sqrt{2h}\,c}{3e} \nu^{3/2} \left(\frac{P}{v_F^3}\right)^{1/2} (n_1^{2/3} - k_1^{2/3})^{-3/2}. \tag{13}$$

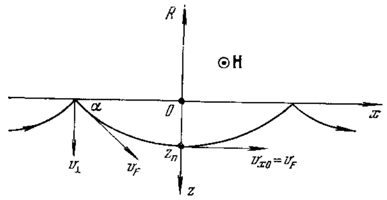


FIG. 14. Trajectory of electron jumping on a reflecting metal surface under the influence of a weak magnetic field.

At the obtained values of the field, there should be observed maxima of the absorption of the electromagnetic radiation of frequency ν in the skin layer of the metal; these maxima appear as oscillations of the dependence of the surface impedance of the metal on the magnetic field.

We note that the dependence of the resonant field of the frequency H_{nk} ll $\nu^{3/2}$, which is contained in formula (13), agrees with the experimental results; a detailed comparison of theory and experiment will be presented after solving the problem by quantum-mechanical methods. Such a solution is necessary because the quasiclassical analysis presented above leaves the quantum addition γ unknown, and gives a definite result only for large n and k.

3. The quantum theory of electronic quantum levels produced in a weak magnetic field was developed by Nee and Prange[20,21] and consists in the following.

We consider the motion of electrons belonging to a cylindrical Fermi surface, in the coordinate system of Fig. 14. The magnetic field has coordinates H(0, H, 0) and the vector potential is A(Hz, 0, 0). The electrons moving on the jumping orbits will have a momentum $p_x < 0$. Assuming the quantities H, z, p_z, and δp_x to be small, we calculate the Hamiltonian of the electron, retaining small quantities of second order:

$$\mathcal{H} = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 \simeq \frac{p_x^2}{2m} + \frac{p_x e}{mc} Hz + \frac{p_z^2}{2m}.$$

We denote the various quantities pertaining to the turning point by the index zero. Bearing in mind that $p_x = p_{x0} + \delta p_x$, we expand in a series near the turning point:

$$\frac{p_x^2}{2m} \simeq \varepsilon(p_{x0}, 0) + \frac{p_{x0}}{m}\delta p_x = \varepsilon_F + v_{x0}\delta p_x.$$

Using this expression, we obtain the approximate value of the Hamiltonian

$$\mathcal{H} \simeq \varepsilon_F + v_{x0}\delta p_x + v_{x0}\frac{e}{c} Hz + \frac{p_z^2}{2m}.$$

The Schrödinger equation, which determines the stationary states, has the following form for our problem:

$$(\mathcal{H} - E)\,\psi(z) = \left[\varepsilon_F + v_{x0}\delta p_x + v_{x0}\frac{e}{c}Hz - E - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\right]\psi(z) = 0.$$

This equation reduces by means of the substitution

$$\zeta = az, \quad a = \left(\frac{2p_{x0}eH}{c\hbar^2}\right)^{1/3}, \quad z_0 = \frac{E - \varepsilon_F - v_{x0}\delta p_x}{(e/c)\,v_{x0}H},$$

to the Airy equation

$$\psi''(\zeta) - (\zeta - \zeta_0)\,\psi(\zeta) = 0,$$

which has for arbitrary ζ a finite solution expressed by the Airy function

$$\psi(\zeta) = v(\zeta - \zeta_0).$$

The function $v(\zeta)$ was determined and tabulated by V. A. Fock in[27]. A plot and tables of the function $\mathrm{Ai}(\zeta) = v(\zeta)/\sqrt{\pi}$ are given also in the handbook[28]. The boundary condition of the problem is that the electrons be specularly reflected by the surface of the metal; this means that the wave function on the surface of the metal should vanish:

$$\psi(0) = v(-\zeta_0) = 0.$$

Consequently, the sought stationary states can occur under the condition $-\zeta_0 = \zeta_n$ (n = 1, 2, 3, ...), where ζ_n are the roots of the Airy function $v(\zeta)$. The values of ζ_n can be taken from the tables, but it is more convenient to use their asymptotic expression

$$\zeta_n = \left[\frac{3\pi}{2}\left(n - \frac{1}{4}\right)\right]^{2/3} = \left(\frac{3\pi}{2} n_1\right)^{2/3}, \qquad (14)$$

which is accurate within 1% even for n = 1. Inserting the obtained ζ_n in the substitution formula $\zeta_0 = az_0$ with the indicated values of a and z_0, we obtain the energy levels

$$E_n = \left(\frac{3he}{4\sqrt{2}\,c}\right)^{2/3} \frac{v_{x0}}{p_{x0}^{1/3}} (Hn_1)^{2/3} + v_{x0}\delta p_x + \varepsilon_F. \qquad (15)$$

The energy level scheme of the stationary surface states of the electrons and plots of their wave functions $\psi_n(z)$ are shown in Fig. 15; V(z) is the potential energy, the form of which, which follows from the Hamiltonian calculated above, coincides with the expression for V(z) obtained by the quasiclassical method in Sec. 2 of this chapter.

The frequencies ν_{nk} of the transitions between the levels n and k, determined by Einstein's formula from the energy levels (15), and the resonant values of the magnetic field H_{nk}, coincide exactly (with allowance for the equalities $p_{x_0} = P$ and $v_{x_0} = v_F$) with those obtained above with the aid of the Bohr-Sommerfeld quantization rule, but in addition we now know the correction $\gamma = 1/4$ to the quantum numbers n, k = 1, 2, 3, ... This makes it possible to calculate the quantum factor in (13) and to obtain in this manner the positions of the magnetic-spectrum lines in relative units. The result of the calculation of the first five series of the spectrum n, k is shown in Table I.

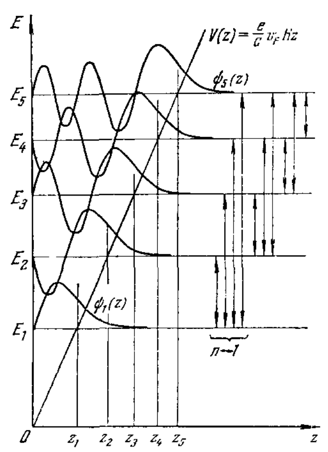


FIG. 15. Energy level scheme and plots of wave functions of magnetic surface states of electrons.

VII. COMPARISON OF THEORETICAL CALCULATIONS WITH THE EXPERIMENTAL RESULTS

1. A rigorous verification of the quantum theory of the effect, developed in Ch. VI, should consist of a calculation of the absolute values of the resonant magnetic fields, H_{nk}, and a comparison of these values with the oscillation spectrum of the experimentally obtained surface impedance of the metal. Such a procedure can be realized with bismuth, since its Fermi surface has been investigated quantitatively with sufficient detail and accuracy, and the spectrum of the oscillations of its surface impedance was investigated with good resolution[11].

As seen from formula (13), to calculate H_{nk} it is necessary to know the curvature radius P of the cylindrical part of the Fermi surface on which the orbits of the oscillation-causing electrons are located, as well as the Fermi velocity v_F of these electrons. Figure 16 shows schematically one of the three "ellipsoids" of the electronic Fermi surface of bismuth. The electrons causing the oscillations in the experiment for which the orientations of the normal to the surface of the sample of the field, and of the current are indicated in Fig. 16, are represented by the points lying along the generatrix passing through the point 2, of the central cylindrical part of the "ellipsoid." To compare the results of this experiment with theory it is therefore necessary to know the electron velocity v_{F_2} at the point 2, and the radius of curvature P_{23} of section 2—3 at the point 2. The

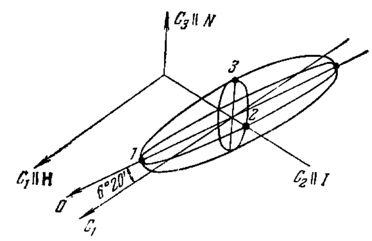


FIG. 16. Location of one of the three "ellipsoids" of the electronic Fermi surface of bismuth relative to its crystallographic axes. The directions of the magnetic field, the microwave current, and the normal to the surface of the sample are indicated.

Table I. Calculation of spectrum of the magnetic surface levels

	ζ_n	$(\zeta_n - \zeta_k)^{-3/2}$				
n	k	1	2	3	4	5
1	2.3381					
2	4.0879	0.4320				
3	5.5206	0,1761	0.5831			
4	6.7867	0,1066	0.2255	0,7019		
5	7.9442	0.07534	0.1321	0,2650	0,8030	
6	9.021	0,05788	0,09127	0,1527	0,2994	0,8949
7	10.039	0,04679	0,06888	0.1041	0,1705	0,3298
8	11,008	0.03917	0,05493	0.07779	0,1153	0,1865
9	11,935	0.03364	0.04549	0.06156	0,08561	0.1254
10	12.828	0,02943	0.03870	0,05062	0,06734	0,09265

Fermi velocity of the electrons was determined by measuring the Doppler damping of the magnetoplasma waves in[29]:

$$v_{F2} = (11.0 \pm 0.5) \cdot 10^7 \text{ cm/sec}$$

The curvature radii can be calculated by assuming the sections of the Fermi surface to be ellipses, from the values of the semiaxes p_2 and p_3 which are parallel to C_2 and C_3 and are obtained by measuring the dimensions of the electron trajectories[22], and from the semiaxes p_1 parallel to C_1, which is determined from the semiaxes ratio 1:1.46:15 as measured by the de Haas–van Alphen effect in[30]:

$$p_2 = (5.4 \pm 0.15) \cdot 10^{-22} \text{ g-cm/sec},$$
$$p_3 = (7.9 \pm 0.3) \cdot 10^{-22} \text{ g-cm/sec},$$
$$p_1 = (81 \pm 3) \cdot 10^{-22} \text{ g-cm/sec},$$

(more accurately speaking, the semiaxes p_1 and p_3 make angles of $6°20'$ each with the axes C_1 and C_3, respectively (see Fig. 16), but this is of no significance in this case).

Calculation of the curvature radii yields

$$P_{23} = \frac{p_3^2}{p_2} = (11.5 \pm 0.7) \cdot 10^{-22} \text{ g-cm/sec},$$
$$P_{21} = \frac{p_1^2}{p_2} = (1200 \pm 70) \cdot 10^{-22} \text{ g-cm/sec}.$$

2. Figure 17 shows a plot of the surface-impedance oscillations in bismuth, obtained in an experiment[11] corresponding to Fig. 16; the sample was chosen to be sufficiently thin (~0.2 mm), so as to eliminate, by the cut-off method[3], the higher-order cyclotron resonances that are observed in a thick sample (see Fig. 7). In the lower part of Fig. 17 are plotted the positions of the lines of the magnetic spectrum H_{nk}, calculated by means of formula (13) with a value of the parameter (P/v_F^3) ensuring the best agreement with experiment for several of the sharpest and most intense resonances designated by the thin vertical lines. We see that for about ten lines lying above 0.5 Oe it is possible to obtain practically perfect agreement between the calculation and experiment; in stronger fields the situation is unclear, and will be discussed somewhat later.

Substituting the Fermi velocity v_{F_2} into the observed value of the parameter (P/v_F^3), we obtain the radius of

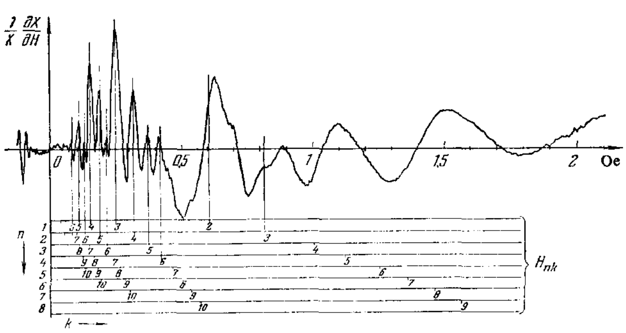


FIG. 17. Plot of surface-resistance oscillation of bismuth. The lower part of the figure shows the calculated positions of several series of the resonance spectral lines of the magnetic surface levels in bismuth (0.2 mm; 1.7°K, 9.7 GHz, H ∥ C ⊥ N ∥ C_3, I ⊥ C_2).

curvature

$$P'_{23} = (13.5 \pm 2.5) \cdot 10^{-22} \text{ g-cm/sec},$$

the value of which coincides, within the measurement accuracy, with the value of P_{23} given above.

We next use the effective mass of the electrons of the limiting point 2 (see Fig. 16), measured in[13] by the cyclotron resonance method:

$$m_2^* = (v_{F2}\sqrt{K_2})^{-1} = (0.137 \pm 0.003)\, m_e,$$

where $K_2 = (P_{23}P_{21})^{-1}$ is the Gaussian curvature of the Fermi surface at the point 2, m_e is the electron mass. From this, again using v_{F_2} and the obtained value of P'_{23} we get

$$P'_{21} = (1400 \pm 200) \cdot 10^{-22} \text{ g-cm/sec},$$

i.e., a value practically coinciding with P_{21}.

It should be noted, however, that a somewhat smaller Fermi-surface curvature, compared with that calculated from the semiaxes of its cross sections, follows from investigations of the cyclotron resonance[13]. In view of this, a certain increase of P'_{23} and P'_{21} over P_{23} and P_{21} apparently has real causes.

In the field region H > 0.5 Oe, where the first lines of the spectral series are located, owing to the insufficient resolution, it is difficult for the time being to see any correspondence between the experimental results and the calculated spectrum. In this connection it should be recalled that the lines of the spectrum of Fig. 17 are calculated for electrons of only one "ellipsoid" of the Fermi surface of bismuth, located, as indicated in Fig. 16, in the most convenient manner for the observation of surface-impedance oscillations. The two other "ellipsoids," which are rotated 60° relative to the first, give lines of double the width and half the amplitude at a doubled field value $H'_{nk} = 2H_{nk}$ (see Fig. 9). The presence in the spectrum of these lines, which furthermore should be broadened still more by the splitting as a result of possible inaccuracies in the orientations of the crystal and of the field, can greatly reduce the resolution of the spectrum.

One must not forget likewise that certain relatively broad singularities of Z(H), observed in the region of weak magnetic fields can be attributed to the presence of the classical effects considered in Ch. V. These effects have that advantage over the quantum surface levels that no specular reflection of the electrons by the surface of the metal is necessary for their occurrence. In addition, they should be less sensitive to a decrease of the mean free path of the electrons and to an increase of the sample temperature. The indicated differences, and also the different dependence of the field of the singularity on the frequency, can serve as symptoms whereby classical and quantum effects can be distinguished experimentally.

Thus, the foregoing comparison shows that the spectrum of the resonant values of the field H_{nk}, calculated from formula (13) with allowance for independently measured characteristics of the bismuth electrons, agrees within the limits of the experimental and computational accuracy, in the region $H_{nk} < 0.5$ Oe, with the spectrum of the surface-impedance oscillations observed in the experiment.

3. With the aid of the formulas presented above it is

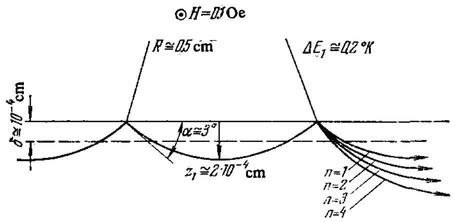


FIG. 18. Numerical characteristics of the trajectory of an electron situated at the first magnetic surface level in bismuth (n = 1). The length of the jump over the surface is $x \cong 0.05$ cm.

easy to obtain expressions for all the parameters of the electronic surface states, particularly for the geometric characteristics of the trajectories of the jumping electrons. Figure 18 shows the results of a numerical calculation for the first surface level in bismuth. The corresponding quantities for tin in a field of 1 Oe are

$$\Delta E_1 \simeq 1.8^\circ\,\mathrm{K}, \quad R \simeq 5\ \mathrm{cm},$$
$$x \simeq 0.05\ \mathrm{cm}, \quad z_1 \simeq 2\cdot 10^{-5}\ \mathrm{cm}, \quad \alpha \simeq 20', \quad \delta \simeq 10^{-5}\ \mathrm{cm}.$$

The trajectories of electrons in higher quantum states have correspondingly larger dimensions. The obtained values of the angle α coincide in order of magnitude with the estimates for α_{max} given in Sec. 1 of Ch. VI; it must therefore be assumed that the considerations advanced there with respect to the large probability of specular reflection of electrons from the surface of the metal, are valid for the effect under consideration.

Analyzing the scattering of electrons moving on jumping trajectories, it is necessary to distinguish between two parts of this scattering: scattering inside the metal and scattering upon collision with the surface of the metal. In turn, the former should be subdivided into scattering by phonons and by impurities, and the latter into scattering by surface phonons and by surface roughnesses. Both phonon parts of the scattering, which decrease rapidly with temperature, owing to the low energy and momentum of the phonons at low temperature, as is well known, can cause electron scattering only through small angles, of the order of the ratio of the phonon momentum to the electron momentum. However, the existence of the spectrum of allowed magnetic surface levels should bring about a situation wherein the small-angle scattering enables the electron only to go from one level to another, or else to be specularly reflected and remain at the previous level, if the energy conditions exclude the possibility of a transition, say, to a higher level. For this reason, the collisions of the electrons with the surface should not violate the system of magnetic surface levels, and the specular reflection of the jumping electrons should be regarded as predominating over diffuse scattering.

4. The experimentally observed growth of the oscillation amplitude with decreasing temperature (Sec. IV) in accordance with the law $A(T^\circ) \cong A(0)\,[1 - (T^\circ/6)]$ is apparently due to the improvement of the resoultion of the levels with decreasing temperature as a result of the increase of the level lifetime due to the increase of the electron mean free path in the volume of the metal, and also due to the change in the level population. The latter should be quite appreciable, since the energy of the surface-state excitation is close to the temperature of the sample in the experiment. Thus, for example, in the described experiments there was observed a magnetic spectrum of resonant absorption of quanta with energy $\sim 0.5^\circ$ K at a sample temperature 4.2–1.6° K (the quantum energy in[8] reached $\sim 3.5^\circ$ K).

Experiment shows (Fig. 17) that the resolution of the resonances and the amplitude of the oscillations decrease when the magnetic field exceeds 0.5 Oe. This may be due in part to the fact that with increasing field the trajectories of the jumping electrons are crowded towards the surface of the metal, and the frequency of their collision with the surface of the metal increases, so that the probability of scattering of the electron through a large angle, i.e., scattering that knocks out the electron from the system of the surface states, increases. Increased scattering results also from an increase of the angle α with increasing field.

Table II shows the values of the quantum numbers of the transitions $n \to k$, identified and noted on the magnetic spectrum of Fig. 17 by thin vertical lines. The observed intensities of the absorption spectrum lines in each series increase with decreasing difference $k - n$ to $k - n = 2$, at which the most intense line of the series takes place (bold face numbers in the table); the first lines of the series are weaker than the others. The clearly nonomonotonic dependence of the line intensity on the field and the regular variation of the intensity within the limits of each series indicate that the intensity depends principally on the probability of the transitions between the levels, which is determined by the density of states and their population.

The decrease of the absorption line intensity with increasing number n of the initial state, i.e., on going from series to series to the right in Table II, is obvious. This is due to the fact that the population of the thermally excited surface states decreases with increasing excitation energy (15), which increases with the number of the level n.

A quantitative analysis of the data on the width and amplitude of the spectral lines should be postponed until a detailed theory of the effect is developed, in which the foregoing singularities of the effects are taken into account.

VIII. ACCOMPANYING PHENOMENA AND CERTAIN POSSIBILITIES OF INVESTIGATING THE EFFECT

The oscillations of the surface impedance of a metal, resulting from the resonant absorption of microwave radiation quanta in transitions of the electrons between stationary surface states excited by a weak magnetic field, undoubtedly serve as the most convincing proof of

Table II. Quantum transitions in the spectrum of the magnetic surface levels identified in Fig. 17.

"Lyman series"	"Balmer series"	"Paschen series"	"Brackett series"
$1 \to 2$	—	—	—
$1 \to 3$	$2 \to 3$	—	—
$1 \to 4$	$2 \to 4$	—	—
$1 \to 5$	$2 \to 5$	$3 \to 5$	—
$1 \to 6$	$2 \to 6$	$3 \to 6$	$4 \to 6$

their existence and a direct method of their quantitative study. There are, however, also other phenomena accompanying the effect considered in this article and observed in experiments of a different kind. An investigation of these phenomena is of great interest both for the study of the main effect itself, and for the purpose of using it as a tool for the investigation of certain properties of metals. No less attractive are also the possibilities of searching for new phenomena, the existence of which can be predicted[11].

1. Mention should be made first of the excitation of magnetic surface level of normal electrons in a superconductor situated in a magnetic field weaker than critical. This phenomenon was observed in[31] in aluminum, and its study in tin[12] was briefly reported in[32], where the results of experiments with indium are also reported. In this phenomenon a peak of resonant absorption is observed in measurements of the dependence of the surface impedance of a superconductor on the magnetic field, under the condition that the microwave radiation quantum is smaller than the pair excitation threshold: $h\nu < 2\Delta(T^\circ)$, where $\Delta(T^\circ)$ is the width of the gap in the superconductor spectrum.

The theory of magnetic surface states in superconductors of the first kind is given in[33]. The reason for their appearance can be explained simply on the basis of the concept developed in Ch. V and VI. Some of the normal electrons present in the superconductors move over its surface along jumping trajectories under the influence of the constant magnetic field that penetrates in the superconductor in accordance with the law $H(z) = He^{-z/\lambda}$. Of course, the only electrons returning to the surface are those that penetrate inside the metal not deeper than $z \sim \lambda$. As a result of the homogeneity of the field H(z), the electron trajectories are not circular arcs as in a normal metal, but are strongly elongated along the surface, and a small change of the depth of the trajectory leads to a strong elongation of the trajectory.

The motion of such electrons can be regarded as motion in a well bounded by the potentials

$$V(0) = \infty, \quad V_s(z) = -\frac{e}{c} v_F H \lambda e^{-z/\lambda}.$$

In this well there is only one magnetic surface level at a depth $E_1 \sim (e/c)V_F H\lambda < \Delta$ under the lower boundary of the continuum of the normal states N of the energy spectrum of the superconductor electrons (Fig. 19). When $T^\circ \neq 0$, a certain number of thermally excited electrons is at the level 1; after absorbing microwave radiation quanta $h\nu \geq E_1$, they can go over into the states N. If the frequency ν is specified, then as the field H increases and E_1 grows, the absorption also increases as a result of an increase of the density of the states N to which the transitions take place. The maximum of absorption will take place at $E_1 = h\nu$, after which it again decreases. The described picture is in full agreement with experiment not only qualitatively but also quantitatively[32].

We note that excitation of the magnetic surface level of normal electrons was observed for metals in the superconducting state, whose Fermi surfaces have almost cylindrical parts, i.e., for metals in which the main effect of formation of surface levels in a magnetic field was previously observed.

2. The wave functions $\psi_n(z)$ of the magnetic surface states were obtained in Sec. 3 of Ch. VI, and their plots are shown in Fig. 15. The density distribution for electrons in the n-th state is given by the function $|\psi_n(z)|^2$,

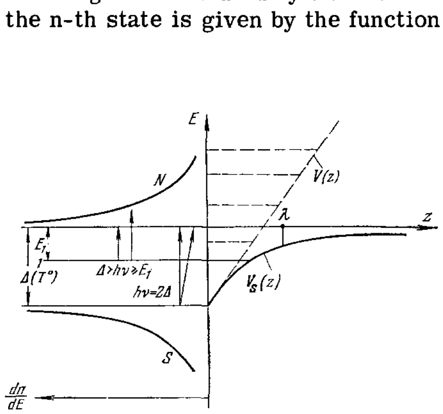


FIG. 19. Left-scheme of formation of magnetic surface level 1 in the energy spectrum of the electrons of a superconductor and of the quantum transitions in it (dn/dE – density of state, $\Delta(T^\circ)$ – energy gap), Right – potential as a function of the distance z from the surface in a normal metal, V(z), and in a superconductor, $V_s(z)$ (λ – depth of penetration of the magnetic field; the dashed lines show schematicaly the magnetic surface levels in the normal metal).

which oscillates with increasing depth z at distances on the order of 10^5-10^3 cm, with a period that depends on the metal, on the magnetic field intensity, and on the number of the states. As a result, the application of the magnetic field leads to inhomogeneity of the distribution of the electrons in the surface layer of the metal, and influences the conditions of their motion, particularly their interaction with the surface, increasing the probability of specular reflection. An inevitable consequence of the considered influence of the magnetic field on the behavior of the electrons in the metal should be a dependence of the surface impedance of the metal on a weak magnetic field. This dependence, which is not connected with the condition of resonant absorption of the quanta, should be observed at any frequency at which the skin layer depth δ is comparable in order of magnitude with the period of the oscillating part of the wave function $\psi_n(z)$, i.e., $\delta \sim z_n \sim 10^{-3}-10^{-5}$ cm. In the limiting case of zero frequency, this phenomenon should be observed in thin single crystals, for example in whiskers and in epitaxial layers of thickness of the order of z_n. It should be noted here that the spectrum of the magnetic surface states in thin samples should differ greatly from the spectrum in thick samples of the same metal. Thus, in a thin plate there should take place interference of the ψ functions of the states connected with opposite surfaces of the sample; a cutoff of the higher states by the thickness of the sample is also possible.

The phenomenon under consideration has a well known analog, the Shubnikov–de Haas effect, which consists in a dependence of the bulk resistance of a metal on a strong magnetic field causing a splitting of the Fermi surface into Landau levels, which is appreciable near its extremal sections. In analogy with this effect, the surface impedance of the metal should depend on a weak magnetic field that excites surface electronic levels localized near the effectiveness strip of the Fermi surface. There is no doubt that this is precisely the cause of the previously unexplained nonlinear dependence of the surface impedance of tin, bismuth, and gallium on a weak magnetic field at frequencies on the order of several MHz, which was observed in experiments[34-36].

The result of investigations of gallium is particularly interesting, because the oscillations of its surface impedance at microwave frequencies has not yet been observed, and the phenomenon observed in[36] is so far the only consequence of the fact that magnetic surface levels are excited in gallium. An analogous situation takes place with potassium, in which a nonlinear dependence of the surface impedance on a weak magnetic field was recently observed at a frequency of several MHz[37].

The anomaly of the behavior of the surface impedance of copper in a weak field, observed in[38] at microwave frequencies, obviously also indicates the existence of surface levels. To be sure, the cause of this anomaly may be quantum resonant absorption, but the nature of this phenomenon does not change: in any case, the nonlinear dependence of the surface impedance on the weak magnetic field is evidence that magnetic surface levels are excited in the metal.

Let us return to the conclusion formulated above, that the resistance of thin single crystals depends on the weak magnetic field at zero frequency, i.e., in direct current. Strictly speaking, this effect has not yet been investigated experimentally. In the measurements of the resistance of thin metal samples, discussed in Sec. 1 of Ch. VI, the investigated samples were in the magnetic field of the earth and of the measuring currents, each on the order of 1 Oe; the dependence of the resistance on the low-intensity field was not investigated. For this reason, these experiments were not performed under the conditions needed for the study of the effect under consideration. The only indirect conclusion that can be drawn from their results is that the unexpectedly large probability of specular reflection of the electrons from the surface of the sample, observed in these experiments, may be due to the surface levels excited on the sample by the uncontrolled magnetic field.

It obviously follows from the foregoing that it is necessary to perform experiments with thin samples for the purpose of observing and investigating the new phenomenon, the dependence of the resistance of conductors on the magnetic field in the region of low intensities, at zero or low frequency.

3. The dependence of the surface impedance of a metal on the weak magnetic field at a frequency on the order of 1 MHz was considered in the preceding chapter as a manifestation of the field dependence of the conductivity of the surface layer of a metal. However, the surface impedance is determined not only by the conductivity but also by the dielectric constant and by the magnetic susceptibility of the conductor, and therefore its dependence on the magnetic field can be attributed in principle to the influence of the field on any of these three characteristics of the conductor. The dielectric constant of the surface layer of the conductor may change under the influence of the field as a result of the dependence of the electron concentration on the depth when magnetic surface levels are excited.

The magnetic susceptibility of the conductor must inevitably change when surface levels are excited by a weak magnetic field, in the same way as a strong magnetic field, which gives rise to Landau levels, leads to the de Haas—van Alphen effect, which consists in a dependence of the volume static magnetic susceptibility of the metal on the magnetic field. The gist of the effect is that the excitation of the surface states leads to the appearance of infinite trajectories of the electrons jumping over the surface of the metal. These electrons form a layered surface current whose magnetic moment is directed parallel to the external magnetic field (see Fig. 18). Thus, the paramagnetism characteristic of the field should arise in the surface layer of the conductor. This paramanngtism is manifest by a violation of the usual linear dependence of the magnetic moment of the metal (diamagnetic or paramagnetic) on the field. We note that this paramagnetism should have an anisotropy connected with the shape of the Fermi surface; this anisotropy is particularly appreciable for the cylindrical parts of the surface, and it greatly affects the organization of the experiments.

The magnetic properties of metals were investigated in a number of studies by Dingle; in particular, he considered the influence of the surface of a metal on the Landau diamagnetism and on the de Haas—van Alphen effect[39]. However, the features of the magnetic susceptibility of the metals, which can result from the excitation of surface states of electrons by a weak magnetic field, have not been investigated so far at all. The importance of experimental searches for this new effect and its study is obvious, but the great difficulties facing the realization of such experiments are no less evident. We are referring to the measurement of small changes of the magnetic susceptibility of weakly-magnetic metals in very weak magnetic fields.

4. The principles of the theory of magnetic surface levels, expounded in Ch. VI, are of quite general importance and can be applied to the analysis of the behavior of any charged quasiparticle in any conductor placed in a magnetic field of suitable magnitude and direction. The conductor under consideration should be cooled to a sufficiently low temperature, should have a large charged-quasiparticle mean free path, and a smooth surface; these requirements can be readily satisfied in experiments. At the same time, the occurrence of surface states is not connected in principle with any limitations regarding the energy spectrum of the quasiparticles—the shape of the Fermi surface or even its very existence, and the quasiparticle distribution may also be Maxwellian. To an equal degree, there is no need for the sample to be a single crystal, provided, of course, that this does not lower the quasiparticle mean free path to unacceptable limits.

In microwave experiments aimed at observing and investigating the oscillations of the surface impedance of a metal, well-resolved lines were observed in the spectrum of the resonant absorption of the quanta for electron transitions between various magnetic surface levels. To resolve the spectral lines, the metal should contain a group of electrons having identical spectra and surface states. The existence of such a group of identical quasiparticles is ensured by a cylindrical Fermi surface, in agreement with experiments performed on tin, cadmium, indium, tungsten, aluminum, and bismuth, whose Fermi surfaces have almost-cylindrical parts[1,6-10]. The spectra of the magnetic surface levels of the electrons belonging to other non-cylindrical parts of the Fermi surfaces were not resolved in these experiments and could be observed only as a broad region (compared with the resolvable lines) of an irregular dependence of the surface impedance of the metal on the magnetic field. Such a phenomenon was apparently observed in copper[38].

The spectrum of the magnetic surface levels of electrons belonging to non-cylindrical Fermi surfaces can be resolved in principle if the width of the resonance lines is greatly decreased by lowering the temperature and improving the quality of the samples. Under these conditions there should be observed spectra of the magnetic surface levels of the electrons of the extremal cross sections of the Fermi surface, similar to what occurs in cyclotron resonance. In such experiment it would be possible to study the spectra of several extermal groups of carriers in each polyvalent metal, and the spectra of the electrons of monovalent metals, which have not been observed as yet.

Until the conditions of high resolution of the spectra of magnetic surface levels are attained, the existence of these levels in various conductors can be established with the aid of the nonresonant phenomena considered above. The simplest method of observing surface states is to measure the surface impedance at medium frequencies. This method has already been tried and led to results offering convincing evidence of the excitation of surface levels in gallium[36], copper[38], and potassium[37]. We note that the Fermi surface of gallium[40] is complicated and apparently has almost-cylindrical parts, whereas the Fermi surface of copper[41] has no cylindrical parts, and the Fermi surface of potassium[42] is an almost regular sphere. It must be borne in mind, however, that owing to the uncertainty of the cause of the dependence of the surface impedance on the weak magnetic field (conductivity, magnetic susceptibility, dielectric constant), connected with insufficient knowledge of the effect, this method must be regarded as qualitative and ill suited for the study of the effect.

5. The prospects of investigating magnetic surface levels become attractive primarily because of the possibility of observing a number of new phenomena, which should occur in conductors in the region of weak magnetic fields that excite these states. At microwave frequencies, the next problem is to attain a high resolution of the spectra of the magnetic surface levels, which would greatly enhance the capabilities and results of investigations of already known spectra, and will make it possible to study the spectra of extremal groups of carriers.

At low and zero frequency, it is necessary above all to attempt to observe the dependence of the conductivity and of the magnetic susceptibility of the conductor on the weak magnetic field. These two effects should be regarded as the main manifestations of the existence of magnetic surface levels. Although these effects are undoubtedly weak, their value for the investigation of magnetic states is very large, particularly in those cases when the microwave spectra cannot be resolved.

An entire group of new phenomena will be revealed by the use of energy pumping at the resonant frequency of any of the transitions between the surface magnetic levels. We can propose the following experiment, the very simplest: pumping is produced by irradiating the sample with quanta equal to the excitation energy of one of the upper level, and a study is made of the radiation emitted by the sample at lower frequencies, corresponding to transitions between the lower levels (the ratio of the pump can change the distribution of the electrons over the surface level, and thus influence the conductivity and the magnetic susceptibility, and thereby also the surface impedance of the metal. Finally, energy pumping can produce a potential difference between different points of the sample, as is the case in plasma phenomena.

Another group of new phenomena consists of geometrical and size effects. It must be indicated, first, that the spectrum of the magnetic surface levels depends on the curvature of the metal surface. From the classical point of view, owing to the great length of the jumps of the electrons over the surface of the metal, the spectrum of a sample having a low-curvature surface should differ from the spectrum of a flat sample. In turn, the spectrum of a thin plate, whose thickness is of the order of the height of the jumping trajectories, should differ from the spectrum of a thick sample, primarily because of the interference of the states connected with its different surfaces. In addition, in thin samples, the upper quantum states should be cut off by the dimensions of the sample, a phenomenon quite similar to the classical size effect—the cutoff of the electron trajectory by the thickness of the sample[3,4].

The principal connection between the considered effect and the surface of the conductor creates new possibilities of studying the properties of the surface or, as a more general program, of the boundary between two media, from which carriers can be reflected. The distinguishing feature of this approach lies in the possibility of investigating the surface from the interior of the conductor and to investigate the internal interface between two media, one lying at a macroscopic depth. Thus, the problem of investigating the boundary of a conductor acquires a new base in a new direction, raising hopes of appreciable progress. The presently available experimental data on the physical properties of conductor surfaces, particularly the character of the interaction between the carriers and the surface, are quite scanty. The problem of surface states is dealt with in only a few theoretical studies (for example,[43]), which are based on a paper by Tamm[44]; the role of the magnetic field was not considered in these works.

Concluding this brief review of the prospects of new investigations, we note that for the surface layer of a conductor, or for a thin conductor sample, magnetic surface levels play the same role as the Landau levels for the volume of a bulky metal. The considerations advanced here far from exhaust the large number of phenomena that should occur as a result of the excitation of the magnetic surface levels. Any property of a surface or thin layer of a conductor, connected with the state of the carrier energy spectrum in the layer, should reveal a dependence on a weak magnetic field. The foregoing should be regarded only as a tentative program for searches to be carried out in the near future.

IX. CONCLUSION

The oscillatory dependence of the surface impedance of a metal on a weak magnetic field at microwave frequencies was observed by purely empirical means in the course of investigations of the properties of the conduction electrons of metals[1]. Only after seven years and as a result of many investigations was this new effect explained and its theory developed.

The theory of the effect is based on the idea that it is essential to take into account the role of the electrons that jump along the surface of the metal as they move inside the metal. The trajectories of these electrons are arcs with end points on the surface of the metal[9]. The quantization of the periodic motion of such electrons, which are specularly reflected from the surface of the metal, has made it possible to construct for the magnetic surface levels a theory that agrees with experiment[21].

Thus, the new effect turned out to be a fundamental quantum phenomenon, consisting of the occurrence of stationary bound states of electrons in the surface layer of the metal under the influence of a weak magnetic field. The reason for the appearance of a system of discrete magnetic surface quantum levels of the electrons is the same that causes Landau levels to exist in the volume of the metal under the influence of a strong magnetic field, namely the quantization of the closed phase trajectories of the electrons in accordance with the Bohr-Sommerfeld condition. The effect should be quite widespread. The point is that the only requirements for the occurrence of magnetic surface levels are a low temperature, a large mean free path of the charged quasiparticles, and smoothness of the sample surface. There are no limitations whatever on the character of the energy spectrum of the quasiparticles. Therefore the magnetic bound surface states of the charged particles can arise not only in a normal or superconducting metal, but also in any nonmetallic conductor.

The geometrical dimensions of the quantum system of magnetic surface states, determined by the period of the oscillating part of the wave function of the bound state, are fully macroscopic in order of magnitude. Therefore the new effect should be classified as a macroscopic quantum effect; the presently known two macroscopic quantum effects, connected with superconductivity and with superfluidity, are described in the review[45].

The new quantum effect, to which this paper is devoted, is not merely one more new effect, no matter how interesting it may be in itself. This fundamental effect will create a base for an entire field of physical research, both experimental and theoretical, dealing with a number of new phenomena still to be discovered, but whose existence, as a consequence of the main effect, can be predicted. In fact, the excitation of a system of magnetic surface levels denotes a change of the energy spectrum of the charged quasiparticles under the influence of a weak magnetic field. A consequence of this change should be a dependence of all the properties of a surface layer of the conductor, or a thin sample of the conductor, on the weak magnetic field; these properties are determined by the state of the carrier spectrum in the sample. Examples are the surface impedance, the electric conductivity, the magnetic susceptibility, thermal properties, etc.

A distinguishing feature of the new research trend is that both the main effect and its consequences take place in weak magnetic fields. It must be emphasized that this region of weak magnetic field has heretofore not attracted any attention of the physicists, since it is widely thought that the action of a weak field on matter cannot be significant.

Of course, we can hardly predict the significance and the value of the results that can be obtained from the study of phenomena that are as yet to be discovered and made tools of physical experiment. But it is quite clear that the new field of research on conductors in weak magnetic fields deserves persistent attention and intense development.

Several recent papers[46-52] are devoted to the study of magnetic surface levels. The lecture[46] is a review of the investigations of the effect. It should be noted that it is reported in that lecture that the microwave spectrum of magnetic surface levels was observed in gallium[49]. This fact confirms the explanation presented in Sec. 2 of Ch. VIII for the nonlinear dependence of the surface impedance of metals on a weak magnetic field at frequencies on the order of several MHz[34-36]. Reference 47 considers several aspects of the theory; calculations were made in particular, of the sample surface-impedance oscillations resulting from resonant absorption of microwave energy by electrons in their transitions between magnetic surface levels. The dissertation[48] contains a theoretical investigation of the effect, and the dissertation[49] an experimental one; their results are included in a publication by the Maryland physicists. Thus, ref. 50 is devoted to a detailed exposition of the theory of the spectrum of the magnetic surface levels, to a calculation of the surface impedance of the sample, and to the question of the width of the spectral line, in connection with the lifetime of the magnetic surface levels.

The changes produced in the spectrum of the magnetic surface levels by geometrical singularities of the metal, namely the spectrum of a sample with cylindrical surface and the spectrum of a sample whose surface has periodic roughnesses (a replica of an optical diffraction grating), are considered in[51]. We note that the existence of the first two geometric effects is predicted in Secs. 2 and 5 of Ch. VIII.

The theoretical calculations of the shapes of the spectral lines and are analyzed in[52], where a comparison is made with the results on indium. The characteristics of the metal electrons determined in this manner agree with the results of independent experiments (within the accuracy limits of the experiment and calculation). It is interesting to note that the probability of specular reflection of the electrons from the surface of the sample turned out to be ~0.9.

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