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HYDRODYNAMIC THEORY OF SHORT-RANGE WEATHER FORECASTS

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1. BRIEF HISTORY OF THE PROBLEM

WITHOUT aiming at any detailed exposition of the history of scientific methods of weather forecasting (and, all the more, nonscientific methods, such as almanacs), we confine ourselves to a mention of only five events which, in our opinion, played the most important role in the formation of modern theory of weather forecasting (more detailed historical information can be found, for example, in the book by Khrgian[1]).

The first event was the organization of the weather service by the Director of the Paris Astronomical Observatory Urban Leverier, who, under orders of the French government following the catastrophic storm of 14 November 1854 in Balaklava, was the first to compile, on 19 February 1855, the weather map of that day.

The second event was an article by Vilhelm Bjerknes "The problem of Weather Forecasting, Considered from the Point of View of Mathematics and Mechanics" (1904), in which this problem was first formulated as an initial-value problem for the equations of the hydromechanics of a baroclinic liquid* (the works of Jacob Bjerknes, 1917–1919, and subsequent work of the so-called Bergen school, in which the concept of atmospheric fronts was developed, i.e., surfaces separating the different air masses, and concerning the formation of cyclones as a result of loss of wave stability on the interface, were the basis of modern synoptic methods of short-range weather forecasts, but these concepts so far have not acquired great importance for numerical forecasting methods, to be sure only for purely technical reasons: the space grids used in numerical forecasting, with a longitudinal spacing of several hundred kilometers, do not make it possible to take into account narrow zones with large hydrodynamic-field gradients, especially such as atmospheric fronts; to take them into account in numerical forecasts it is necessary to develop other methods).

The third event was the book of Lewis Richardson "Weather Forecasting by a Numerical Process" (1922), which contained the first attempt to calculate the future weather by numerically solving the hydromechanics equations—in the same manner as astronomers predict the positions of the planets by solving the equations of dynamics of a planetary system. The attempt was unsuccessful: a weather forecast prepared after many very long calculations for one day (20 May 1920 for the region Nurnberg—Augsburg) was unsatisfactory. From the present-day point of view, the reasons for Richardson's failure are clear: 1) incompleteness of the initial data (at that time there were only surface data and furthermore from a skimpy grid of stations in Europe), 2) imperfection of the finite-difference schemes (e.g., the Courant-Friedrichs-Lewy criterion for the ratio of the spatial and time intervals, which was established later, in 1928, was not fulfilled), 3) the excessive complexity of the integrated equations, which describe not only the motions of importance for the weather (the so-called synoptic processes) also all possible "noise" such as acoustic waves, breathing of vegetation, etc.

This latter difficulty was overcome in 1940 by I. A. Kibel'[2], whose paper is probably the most important event in the history considered by us; he proposed a fundamental principle for simplifying the hydromechanics equations, namely the asymptotic "quasigeostrophic approximation," which made it possible to "filter out" from the solutions of the equations the "meteorological noise" which is of no importance to the weather. This principle was subsequently the basis for the creation of the hydrodynamic theory of short-range weather forecasts.

In 1940, striving to limit the calculation only to the lower layer of the atmosphere (the troposphere), Kibel' imposed on its upper limit (the tropopause) an artificial condition, which does not follow from the laws of hydrodynamics. "Quasigeostrophic approximation" equations free of these limitations were introduced only in the post-war years, in the almost-simultaneously published papers of A. M. Obukhov[3] and J. Charney[4,5]. These equations were immediately used for practical calculations: weather forecasting was one of the problems which John von Neumann had in mind in the development of high-speed computers; he was the co-author of one of the first papers[6] on the realization of "quasigeostrophic approximation" equations with a computer. The development of a hydrodynamic theory of short-range forecasting of meteorological fields was the greatest accomplishment of the physics of the atmosphere in the post-war years. This theory has now already been described in a number of books[7-10] specially devoted to numerical methods of weather forecasting; the first of them, by I. A. Kibel'[7], was in our opinion the best.

As regards the physical principles of long-range weather forecasting, their history is much shorter; it dates back from a paper published by E. N. Blinova in 1943[11]. The main difference between the short-range and long-range forecasts (causing the difference in the states of their theory) lies, in our opinion[12] in the fact that for short-range forecasts it is sufficient to use the

*A liquid is called barotropic if its density ρ is a function of the pressure p only, and baroclinic in the opposite case. A real atmosphere is baroclinic, its ρ depends not only on p but also on the temperature T and, neglecting the humidity, it satisfies the Clapeyron equation $p = \rho RT$ (R $\approx$ 0.287 J/g-deg is the gas constant for dry air). A liquid in the field of potential forces and in the absence of viscosity satisfies the theorem of W. Bjerknes: $\frac{d\Gamma}{dt} = -\oint_L \frac{dp}{\rho}$ (where $\Gamma = \oint_L V dS$ is the circulation of the velocity along the closed contour L), from which it follows that in a baroclinic liquid the intersection of the surfaces p = const and ρ = const (or p = const and T = const) leads to vorticity production.

hydrodynamic equations in the adiabatic approximation, whereas the very nature of the long-range weather changes is due to nonadiabatic processes.

2. SCALES OF WEATHER PROCESSES

We present a number of numerical characteristics of the atmosphere—the object to which we shall apply the equations of hydromechanics. The atmosphere contains $M \approx 5.3 \times 10^{21}$ g of air. The total kinetic energy E of its motion is of the order of 10^{21} J (the energy of an individual cyclone, roughly speaking, is smaller by two orders of magnitude; for comparison, we indicate that 1 megaton of TNT is approximately equivalent to 4×10^{15} J). Thus, according to empirical estimates by Borisenkov[13], in the northern hemisphere $E = 4 \times 10^{20}$ J in the winter and 1.9×10^{20} J in the summer, whereas in the southern hemisphere 7.1×10^{20} J in the winter and 3.9×10^{20} J in the summer; similar estimates are given by Pisharoty[14] and Gruza[15] (the latter indicates that on the average, more than 70% of the kinetic energy in the troposphere goes into zonal flow, i.e., flow parallel to the latitude circles, and less than 30% to meridional flow; approximately half the energy is connected with the average zonal circulation and half with the deviations from it). The kinetic energy per unit mass E/M is of the order of 10^6 erg/g = $(10 \text{ m/sec})^2$; therefore U = 10 m/sec is taken to be the typical air velocity in synoptic processes.

The primary source of the energy of the atmospheric processes is solar heat. The power of this source on earth is 1.8×10^{14} kW, but approximately 40% of the solar radiation is immediately reflected back to the outer space, so that the initial figure should be taken to be 1×10^{14} kW, or on the average per unit area of the earth's surface, 20 mW/cm^2. Only a small fraction of this energy is transformed into the kinetic energy of the atmospheric motion; according to Palmen's empirical estimates[16,17], the rate $\partial E/\partial t$ of transformation of the potential energy into kinetic energy in the atmosphere is on the whole of the order of 2×10^{12} kW, so that the efficiency of the "atmospheric engine" amounts to only about 2% (in individual cyclones, $\partial E/\partial t \sim (1-2) \times 10^{11}$ kW, and outside the cyclones, on the average, there is a slow inverse conversion of kinetic energy into potential energy). The average rate of generation of kinetic energy per unit mass, $(1/M)(\partial E/\partial t)$, is according to these data 4 cm^2/sec^3. The average per unit rate ϵ of the dissipation of kinetic energy into heat as a result of friction should be of the same order of magnitude; indeed, Brent[18] obtained by an independent method, back in 1926, a value $\epsilon \sim 5$ cm^2/sec^3 for the troposphere.

The typical time of energy conversion

$$\tau = \left(\frac{1}{E}\frac{\partial E}{\partial t}\right)^{-1}$$

turns out to be 10^{21} J/2×10^{12} kW = 5×10^5 sec, i.e., on the order of one week. The typical time of degeneracy of the energy of the synoptic processes under the influence of viscosity is of the same order. Indeed, in the interval of scales L in which there is a cascade process of energy transfer from the large-scale motions to the small-scale motions, occurring with a constant rate ϵ (i.e., a rate independent of L), the effective "viscosity" is of the form $\nu(L) \sim \epsilon^{1/3}L^{4/3}$ (the so-called "four-thirds" law of Richardson[19], which is valid practically in the entire spectrum of atmospheric-motion scales from millimeters to thousands of kilometers—Fig. 1), and the time of degeneracy of the energy under the influence of this "viscosity" is $\tau(L) \sim L^2/\nu(L) \sim \epsilon^{-1/3}L^{2/3}$. A typical length scale for synoptic processes, according to Obukhov[3], is $L_0 = c/l$, where c is the speed of sound and $l = 2 \cos\theta$ is the so-called Coriolis parameter ($\omega = 7.29 \times 10^{-5}$ 1/sec is the angular velocity of rotation of the earth, θ is the complement of the latitude); in moderate latitudes, $L_0 \sim 3000$ km. Using this L_0 and the value of ϵ given above, we obtain for synoptic processes $\tau(L_0) \sim \epsilon^{-1/3}L_0^{2/3} \sim 3 \times 10^5$ sec. We note that the Euler scale of time for synoptic processes $\tau_1 = L_0/U$ is of the same order of magnitude (inasmuch as in the west-east transport the average layers of the atmosphere in middle latitudes complete their revolution around the earth in several weeks, the Euler time scale for the atmosphere is on the whole of the order of a month).

We present also some data on the energy role played by the humidity of the air. According to Rudloff[20] (similar data are given by Neik[21]), the atmosphere contains on the average 1.24×10^{19} g of moisture, which is equivalent to a layer of precipitated water of 24 mm (the ocean contains 1.37×10^{24} g of water, and the icebergs 2.9×10^{22} g of ice, the melting of which would raise the sea level by 80 m). The average annual precipitation on earth is 3.96×10^{20} g (of which 2.97×10^{20} g falls on the oceans and 0.99×10^{20} g on dry land), which is equivalent to a layer of water of 780 mm (thus, the water vapor in the atmosphere is replenished on the average 780/24 = 32 times annually, or every 11 days). This is also the annual amount of evaporated moisture, but the relative shares of the ocean and the dry land are 3.34 and 0.62; the runoff from land amounts to 0.37×10^{20} g. If we take for the latent heat of evaporation a value 2.4×10^3 J/g, then the heat consumed in the evaporation amounts to 3×10^{13} kW, i.e., 30% of the solar heat absorbed by the earth—this influx of heat in the atmosphere turns out to be 15 times larger than the rate of generation of kinetic energy!

3. SPECTRUM OF ATMOSPHERIC PROCESSES

The time oscillations of the meteorologic elements, namely the rates of motion of the air, temperature,

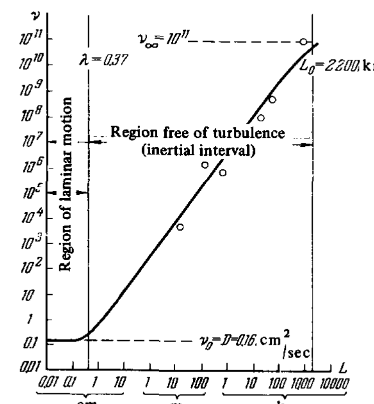

FIG. 1. Virtual diffusion coefficient $\nu(L)$ as a function of the turbulence scale L (empirical points – after Richardson [19].

pressure, humidity, etc. contain components with periods from a fraction of a second and at least up to tens of thousands of years. The entire spectrum of the oscillation periods can be broken up into the following nine intervals:

1) Micrometeorological oscillations with periods from a fraction of a second to several minutes. The largest contribution is made to them by the small-scale turbulence. Its energy spectrum fS(f) ($f = 1/\tau$—frequency, τ—period of the oscillations, S(f)—spectral energy density) in the surface layer of the air has a maximum at the period $\tau_{max} \sim 1$ min, corresponding to a horizontal turbulent inhomogeneity scale $L = U\tau_{max} \sim 600$ m. When $f \gg 1/\tau_{max}$, the wind-velocity spectra satisfy the Kolmogorov-Obukhov "five-thirds law"[22]

$$S(f) \sim \frac{\varepsilon^{2/3}}{U}\left(\frac{f}{U}\right)^{-5/3};$$

the temperature spectra have a similar form[23]

$$S_T(f) \sim \frac{N\varepsilon^{-1/3}}{U}\left(\frac{f}{U}\right)^{-5/3}$$

(where $N = \chi(\overline{\nabla T})^2$ is the rate of equalization of the temperature inhomogeneities, T is the temperature, and χ is the molecular temperature conductivity). In the region of the maximum frequencies of turbulent fluctuations $f \sim U\epsilon^{1/4}\nu^{-3/4}$ (ν—molecular viscosity), the turbulence spectrum is sharply cut off.

In addition to the turbulence, the micrometeorological oscillations include (with relatively small amplitudes) also acoustic and short-duration gravitational waves. According to the theory (see below), the periods of the gravitational waves are predominantly larger than 330 sec, and those of the acoustic ones are shorter than 300 sec; this explains the minimum near $\tau \sim 300$ sec in the spectrum $fS_p(f)$ of the pressure pulsations, constructed according to microbarogram data by Golitsyn[24] (Fig. 2, where the scale unit is the dispersion of the micropulsations of the pressure $\sigma_p \sim 10^{-2}$ mb).

2) The mesometeorological interval with periods from minutes to hours, in which intense oscillations of the meteorological elements (including, e.g., their oscillations during thunderstorms or in large-amplitude gravitational waves) are relatively rare. Therefore the spectra fS(f) usually have a broad and deep minimum in this interval (a summary of data concerning this interval can be found in the paper of Kolesnikova and Monin[25]). The mesometeorological minimum is clearly seen, for example, in the spectrum of the horizontal wind velocity, shown in Fig. 3, which was constructed by Van der Hoven[26] from data obtained on the 125-m meteorological tower in Brookhaven. This minimum corresponds to a period τ on the order of 20 min and to a scale $L = U\tau$ on the order of the effective thickness of the atmosphere, $H \sim 10$ km (the lower 10-km layer con-

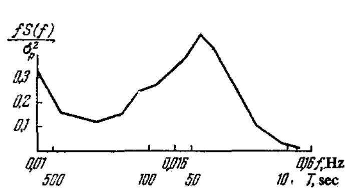

FIG. 2. Spectrum of micropulsations of pressure, after Golitsyn [26].

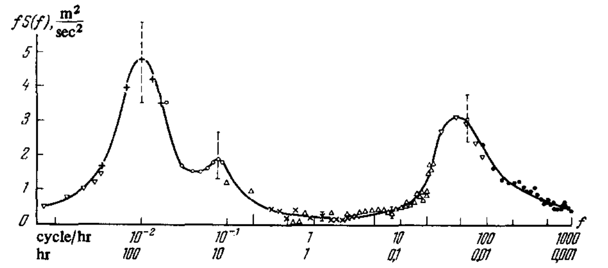

FIG. 3. Spectrum of horizontal wind velocity after Van der Hoven [26].

tains 80% of the mass of the atmosphere); it separates the quasi-two-dimensional (quasi-horizontal) synoptic inhomogeneities with scales $L \gg H$ from the essentially three-dimensional (quasi-isotropic) micrometeorological inhomogeneities with scales $L < H$. The presence of this minimum makes it possible to obtain in micrometeorology relatively stable mean values for the wind velocity, temperature, etc., by using values averaged over the periods from the mesometeorological interval (in practice one chooses τ = 10—20 min).

3) Synoptic oscillations with periods from many hours to several days, with an energy-spectrum maximum at about τ = 4 days (see Fig. 3; the particular maximum at τ = 12 hrs is considered by Van der Hoven to be insignificant). This interval includes also the diurnal oscillations which are manifest, for example, in the temperature oscillation spectrum in the form of the diurnal line, and in the pressure oscillation spectrum in the form of the diurnal and semi-diurnal line. In the high-frequency half of the synoptic interval, there takes place a cascade transfer of energy over the spectrum, from the large-scale motions to the small-scale ones, as a result of the hydrodynamic instability of the quasi-horizontal synoptic motions, which have large Reynolds numbers $Re = UL/\nu$ (on the low-frequency end of the synoptic interval, apparently, there is a transfer of energy in the opposite direction, from the synoptic motions to the larger-scale flows of the general circulation of the atmosphere[27,28]). In addition, any motion on a synoptic scale generates a microturbulence directly (i.e., bypassing any motion of the intermediate scales) and continuously, as a result of the hydrodynamic instability of the vertical inhomogeneities of the wind field, especially near the earth's surface in the so-called jet streams, where the vertical gradients of the wind velocity are the largest.

By regarding the microturbulence as a dissipative factor for the synoptic motions, it can be characterized by an effective viscosity coefficient ν_{turb}. The minimal scale of synoptic motion, capable of overcoming this viscosity, is $L_{min} \sim \epsilon^{-1/4}\nu_{turb}^{3/4}$. The presence of the mesometeorological minimum denotes[25] that $L_{min} \gtrsim H$.

4) Global oscillations with periods from weeks to months, which are of greatest interest for the problems of long-range weather forecasting, but have still been little investigated. So far, only the so-called "index cycle," i.e., the cycle of oscillations of the planetary circulation between the states of intense zonal flow (the west-east transfer) with weak meridional mixing and weakened zonal flow with intense meridional mixing, is the only more or less pronounced phenomenon in this

range. It was traced, for example[29-31], by means of the oscillations of the circulation index $\alpha = u/a \sin\theta$, i.e., the average angular velocity of rotation of the atmosphere in moderate latitudes relative to the earth's surface (u—zonal velocity averaged along the latitude circle, a—earth's radius). The period of the "index cycle" is close to two weeks (Fig. 4); the spectral density of the oscillations of the circulation index[30-31] has a sharp maximum at $\tau = 12$ days (we note that the oscillations of the circulation index form, generally speaking, not a stationary but a periodic random process[31] with a period of one year).

5) Seasonal oscillations—the annual period and its harmonics.

6) Inter-annual oscillations with periods on the order of several years, the spectrum of which has not yet been investigated (we mention the 26-month rhythm of oscillations in the equatorial stratosphere, observed by several authors, and also the hypothesis that the earth's weather is subject to an 11-year cycle of oscillations of solar activity, which in our opinion has not been convincingly proven). According to Kolesnikova and Monin[32], the swing of the inter-annual oscillations of the average annual values of the temperature and of many other meteorological elements usually amounts to 15—30% of the swing of their seasonal and irregular oscillations within the year.

Inter-annual oscillations must still not be classified as oscillations of the climate. If all the oscillations in 1)—6) are called short-period oscillations, then the climate is a statistical regime of the short-period oscillations of the meteorological fields, which itself experiences long-period oscillations. The spectrum of the latter can be subdivided into the following three intervals:

7) Intrasecular oscillations, a clear-cut example of which is the rising temperature which occurred in the first-half of the 20th century (and which is apparently now terminating), a connection being observed between the changes of the climate and the character of the general circulation of the atmosphere: According to Dzerdzeyevskiĭ[33], in 1900—1930 zonal types of circulation in the northern hemisphere were observed less frequently and meridional ones more frequently than in 1930—1950. The genesis of the climatic heating of the 20th century is a timely problem of the physical theory of the climate.

8) Intersecular oscillations, among which are known[34] the heating following the end of the glacier period (65th century BC), leading to the so-called "climatic optimum" in the 40—20th centuries BC, followed by a deterioration of the climate in the so-called "sub-atlantic period" (10th century BC to 3rd century AD), its improvement in the 4th—9th centuries AD,* the subsequent deterioration in the 13th—14th centuries, improvement in the 15th—16th centuries, and deterioration in the 17th—19th centuries (the so-called "small glacier period").

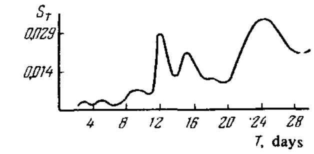

FIG. 4. Spectral density of the oscillations of the circulation index, after Monin [30].

9) The glacier periods of the Pleistocene: Günz (500—475 thousand years BC), Mindel (425—325 thousand years BC), Riss (200—125 thousand years BC), Würm (60—25 thousand years BC), during which the temperature of the surface layer of the air (now +15° C) dropped approximately by 10°. So many causes have been proposed for the icing of the earth (including, e.g., Simpson's hypothesis[35] that the solar radiation has increased, thereby leading to an increase of the evaporation, cloudiness, and snowfall), that it is possible that not the appearance of icing but the absence of glaciers during 90% of the post-Cambrian time calls more readily for an explanation.

4. ADIABATIC INVARIANTS

It was shown in Ch. 2 that the typical time of generation of the kinetic energy of synoptic processes

$$\tau = \left(\frac{1}{E}\frac{\partial E}{\partial t}\right)^{-1}$$

is of the order of a week, and that the typical time of the dissipation of the kinetic energy of the synoptic processes $\tau(L_0) \sim \epsilon^{-1/3} L_0^{2/3}$ is of the same order of magnitude. The periods $t - t_0 < \tau$ are naturally called short, and $t - t_0 > \tau$ long[12]. Thus, in the theory of short-range forecasts, by the very definition, we can disregard the influx and dissipation of energy, i.e., we can use the adiabatic approximation: obviously, this approximation is utterly unacceptable for long-range forecasts.

For the adiabatic processes, which are the only ones considered in this article, there are two conservation laws: when any volume of air V moves, there are conserved in it the entropy $\int S\rho dV$ (S—entropy per unit mass, ρ—density) and the "vortex charge" $\int (\mathbf{\Omega}_a \nabla S) dV$ ($\mathbf{\Omega}_a$—the absolute vorticity), i.e.,

$$\frac{dS}{dt} = 0, \quad \frac{d}{dt}\frac{(\mathbf{\Omega}_a \nabla S)}{\rho} = 0. \tag{4.1}$$

The "specific vorticity" $\Omega = (\mathbf{\Omega}_a \nabla S)/\rho$ is also called the potential vorticity[36]. The conservation law $d\Omega/dt = 0$ was first introduced by Ertel[37] (see also[4,38,39-40]).

Hollman[41] indicated three other independent combinations of hydrodynamic fields which are conserved in adiabatic processes:

$$\psi_3 = \left(\frac{[\nabla S \nabla \Omega]}{\rho}\mathbf{w}\right), \; \psi_4 = \left(\frac{[\nabla S \nabla \psi_3]}{\rho}\mathbf{w}\right), \; \psi_5 = \left(\frac{[\nabla S \nabla \Omega]}{\rho}\nabla\psi_3\right), \tag{4.2*}$$

where $\mathbf{w} = \mathbf{v}_a - \nabla W$, with $\mathbf{v}_a$ the absolute velocity and $W = \int_0^t \Lambda dt$ is the so-called action (with Lagrangian $\Lambda = (v_a^2/2) - \varphi - \eta$, where φ is the gravitational potential and η is the specific enthalpy).

*During the period of the "small climatic optimum" in the 8th - 10th centuries BC, when the Vikings colonized Iceland and Greenland, the Arctic was not very icy according to their chronicles. This offers evidence against the hypothesis that if the Arctic increases, it no longer freezes; this hypothesis is due to an overistimate of the role of the local albedo (reflectance of the underlying surface) for the Arctic conditions. Recognizing that within the 72° northern latitude circle the Arctic amounts to only 2.5% of the surface of the earth, it is more natural to assume, conversly, that the local conditions in the Arctic (including its iciness and consequently its albedo) are not the cause but the consequence (indicator) of the state of the general circulation of the atmosphere.

*$[\nabla S \nabla \Omega] \equiv \nabla S \times \nabla \Omega$

The entropy increments dS in dry air are defined by the formula

$$dS = c_p d \ln \frac{p^{\frac{1}{\varkappa}}}{\rho} = c_p d \ln \Theta, \quad \Theta = T \left(\frac{p_0}{p}\right)^{\frac{\varkappa-1}{\varkappa}}, \tag{4.3}$$

where p and T are the pressure and temperature; $\kappa = c_p/c_v$ is the ratio of the specific heats at constant pressure and constant volume ($c_p = 1.003$ and $c_v = 0.717$ J/g-deg; $c_p/c_v = 1.41$); Θ is the so-called potential temperature, p_0 is the standard pressure (usually $p_0 = 1000$ mb $= 10^6$ dyne/cm^2). The moisture content of the air is always small*; therefore the entropy of unsaturated moist air can be determined in many calculations (but not always) by the same formula (4.3), and in the case of saturated air it suffices to add to the expression (4.3) for dS the term $\mathscr{L} dq_m/T$, where $q_m \approx (R/R_v)(e_m/p)$ is the specific saturation humidity. On the other hand, if the influx of heat due to the phase transitions of the moisture is neglected, then it is possible to disregard completely the humidity of the air; so far, this is the most frequent practice in operational numerical short-range weather forecasting. Accordingly, we shall use for the time being formula (4.3) for dS.

According to (4.1), any function of S and Ω is an adiabatic invariant, i.e., a conservative characteristic of the moving particles of air in adiabatic processes. It is convenient to choose two such independent functions as the Lagrangian coordinates of the air particles. The corresponding coordinate surfaces cut up the atmosphere into tubes, and air does not flow through the walls of these tubes, so that the adiabatic evolution of the atmosphere consists only in the deformation of such tubes. The prediction of these deformations is the basis of the short-range weather forecasting.

As one of the Lagrangian coordinates, it is convenient to choose the potential temperature Θ. Recognizing that its changes in the vertical direction are much larger than in the horizontal direction, so that the vector $\nabla\Theta$ is directed approximately vertically, we can put

$$\Omega_a \nabla\Theta \approx \Omega_{az} \frac{\partial\Theta}{\partial z} = (\Omega_z + l) \frac{\partial\Theta}{\partial z},$$

where l is the aforementioned Coriolis parameter and Ω_z is the vertical component of the relative vorticity. Using the quasistatic approximation, i.e., the equation $\partial p/\partial z = -\rho g$ (g is the acceleration due to gravity; the meaning of this approximation will be explained later), we obtain

$$\frac{\partial\Theta}{\partial z} = \frac{\Theta}{T}(\gamma_a - \gamma),$$

where $\gamma_a = (\kappa - 1)/\kappa(g/R) \approx 10$ deg/km is the adiabatic temperature gradient, and $\gamma = -\partial T/\partial z$ is the actual temperature gradient. Using these results, it is convenient to choose as the second Lagrangian coordinate, following Obukhov[40], the function

$$\tilde{\Omega} = \frac{\Omega}{c_p R} \frac{p^*(\Theta)}{\gamma_a - \gamma^*(\Theta)} \approx (\Omega_z + l) \frac{\gamma_a - \gamma}{\gamma_a - \gamma^*(\Theta)} \frac{p^*(\Theta)}{p}, \tag{4.5}$$

where $p^*(\Theta)$ and $\gamma^*(\Theta)$ are the standard values of p and γ on the surfaces Θ = const (i.e., the characteristics of the so-called standard atmosphere). Inasmuch as Θ varies most rapidly in a vertical direction and $\tilde{\Omega}$ along the meridian (since usually $|\Omega_z| \ll l$ and $\Omega_z + l \approx 2\omega \cos\theta$), Θ and $\tilde{\Omega}$ can replace the vertical coordinate and the latitude, and the most lucid picture of the invariant $(\Theta, \tilde{\Omega})$ tubes is given by their meridional section. An example of such a section is shown in Fig. 5, while Fig. 6 shows for the same example the isolines $\tilde{\Omega}$ = const on one of the surfaces Θ = const. If we denote by $\mu(\Theta, \tilde{\Omega}) d\Theta d\tilde{\Omega}$ the fraction of the atmosphere mass contained in an infinitesimally thin $(\Theta, \tilde{\Omega})$ tube, then $\mu(\Theta, \tilde{\Omega})$ can be interpreted[40] as the probability density for the values of the Lagrangian coordinates Θ and $\tilde{\Omega}$ of a randomly chosen air particle. In Fig. 7 we present an example of the probability distribution $\mu(\Theta, \tilde{\Omega})$ from data for the period from 1—10 April 1962, kindly supplied to us by A. B. Karunin.

In addition to the differential invariants S and Ω (or any other two functions of these invariants, say Θ and $\tilde{\Omega}$), adiabatic processes, as is well known, have also an integral invariant, namely the total energy E = K + N, where

$$K = \int \frac{\rho v^2}{2} dV = \int \frac{v^2}{2} dm$$

is the kinetic energy (v—is the modulus of the velocity, dV the volume element, $dm = \rho dV$ the mass element, and the integral extends over the entire atmosphere), and N is the labile energy, i.e., the sum of the potential energy $\mathscr{P} = \int \Phi dm$ ($\Phi = gz$ is the potential of the force of gravity) and the internal energy $J = \int c_v T dm$. In the quasistatic approximation, the potential energy of a vertical column of air with unity cross section is $\int gz\rho dz = -\int z dp = \int p dz$, so that by virtue of the Clayperon equation $p = \rho RT$ we get $\mathscr{P} = \int RT dm$, and since $c_v + R = c_p$, we have

$$N = \int c_p T \, dm = c_p \int \left(\frac{p}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}} \Theta \, dm. \tag{4.6}$$

In adiabatic processes, the total mass of the air over any isentropic surface Θ = const remains unchanged, so

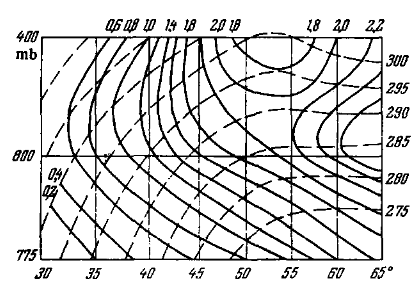

FIG. 5. Meridional section of $(\Theta, \tilde{\Omega})$ tubes at longitude 100° E for 1 April 1962 (from [40]). Abscissas – latitudes, ordinates,– atmospheric pressure: Ω = const – solid lines, Θ = const – dashed lines; $\tilde{\Omega}$ is measured in units of 10^{-4} sec^{-1}.

*The partial pressure of the saturating water vapor $e_m(T)$ is determined by the Clausius-Clapeyron formula

$$\frac{1}{e_m} \frac{de_m}{dT} = \frac{\mathscr{L}}{R_v T^2}, \quad \mathscr{L} = \mathscr{L}_0 - (c_w - c_{pv})(T - T_0)$$

(where $\mathscr{L}$ is the latent heat of evaporation, $c_{pv} = 1.81$ and $R_v = 0.461$ J/g-deg are the specific heat at constant pressure and the gas constant of water vapor; $c_w = 4.19$ J/g-deg is the specific heat of water) and ranges from 0.509 to 42.47 mb when the temperature increases from – 30° to + 30°C. The specific humidity q, i.e., the ratio of the densities of water vapor and moist air, usually does not exceed 3 - 4%. The content of liquid water and ice in clouds is usually much smaller than the vapor content.

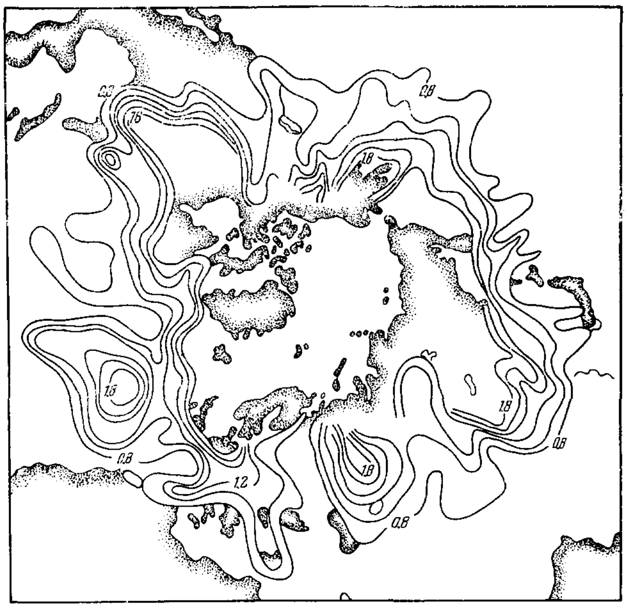

FIG. 6. Configuration of the lines $\tilde\Omega$ = const on the surface $\Theta = 300°$ for 1 April 1962 (from [40]).

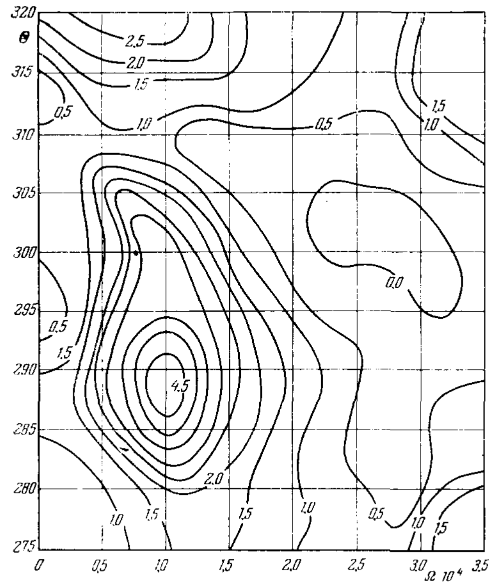

FIG. 7. Probability density $\mu(\Theta, \tilde\Omega)$ from the data for the period 1 - 10 April 1962.

that the average value $p^*(\Theta)$ on any isentropic surface likewise remains unchanged, and consequently also the quantity

$$N^* = c_p \int \left[\frac{p^*(\Theta)}{p_0}\right]^{\frac{\varkappa-1}{\varkappa}} \Theta\, dm, \tag{4.7}$$

i.e., that labile energy of the atmosphere, which remains if the atmosphere is brought adiabatically to a state with a constant pressure along any isentropic surface (and with stable stratification). It is clear that not all the labile energy can be converted into kinetic energy in adiabatic processes, but at the most only a part of it $A = N - N^*$, called the available potential energy[42-43] (A turns out to equal the weighted average value of the dispersion of Θ on the isobaric surfaces). Consequently, the sum K + A is an adiabatic invariant.

If the atmosphere is brought adiabatically to a state with indifferent stratification (in which the isentropic surfaces are vertical, i.e., Θ is independent of p), then the labile energy $\bar N$ in this state can be calculated by replacing in (4.6) the factor $p^{(\kappa-1)/\kappa}$ under the integral sign by its value averaged over the mass of the vertical air column:

$$\frac{\int p^{\frac{\varkappa-1}{\varkappa}}\, dm}{\int dm} = \frac{\varkappa}{2\varkappa-1}(\bar p)^{\frac{\varkappa-1}{\varkappa}},$$

where $\bar p$ is the pressure at the earth's surface; this yields

$$\bar N = \frac{\varkappa}{2\varkappa-1} c_p \int \left(\frac{\bar p}{p_0}\right)^{\frac{\varkappa-1}{\varkappa}} \Theta\, dm. \tag{4.8}$$

The difference $\xi = \bar N - N$ is called the macrostability parameter[44]. The quantity ξ turns out to equal the weighted average value of the vertical gradient of the potential temperature $(-\partial\Theta/\partial p)$ in the entire thickness of the atmosphere. It equals the amount of kinetic energy released or lost in the adiabatic transition from a given stratification to the indifferent stratification. The difference $K - \xi$ is an adiabatic invariant.

5. CLASSIFICATION OF ATMOSPHERIC MOTIONS

To construct a hydrodynamic theory of short-range weather forecasting it is important to clarify first of all what are the possible types of atmospheric motions in adiabatic processes. All these motions have the character of waves, and for their classification it is sufficient to consider the case of small-amplitude waves, i.e., small oscillations of the atmosphere relative to the state of rest (in which the pressure $\bar p$, the density $\bar\rho$, and the temperature $\bar T$ are functions of only the altitude z, connected with the static equations $\partial\bar p/\partial z = -g\bar\rho$ and the Clapeyron equation $\bar p = \bar\rho R\bar T$). In the flat-earth approximation, the equations of motion for small oscillations are of the form

$$\bar\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + l\bar\rho v, \quad \bar\rho\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - l\bar\rho u, \quad \bar\rho\frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} - g\rho, \tag{5.1}$$

where u, v, and w are the components of the velocity vector **v**, p and ρ are the perturbations of the pressure and of the density (these five functions characterize completely adiabatic small oscillations), and l, as above, is the Coriolis parameter. It is necessary to add to these equations the continuity and adiabaticity equations, in the form

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\bar\rho\mathbf{v} = 0, \quad \frac{\partial p}{\partial t} + b\bar\rho w = c^2\frac{\partial\rho}{\partial t}, \tag{5.2}$$

where $c^2 = \kappa R\bar T$ is the square of the speed of sound and

$$b = (\varkappa-1)g + \frac{\partial c^2}{\partial z}$$

is the stratification parameter. The natural boundary conditions with respect to z for Eqs. (5.1) and (5.2) are the requirement that the vertical mass flux $\bar\rho w$ vanish on the boundaries of the atmosphere (at z = 0 and $z \to \infty$). The system (5.1)—(5.2) is of fifth order in the time; to solve it uniquely it is necessary to specify at t = 0 the initial values $\mathbf{v}_0$, p_0, and ρ_0 of all the unknown functions.

Let us assume for the time being[45] that l = const (this is valid for territories spread not too widely in latitude). Then Eqs. (5.1)—(5.2) will have a family of stationary solutions v_S, p_S, ρ_S, describing motions of the first kind which are: (1) quasistatic, i.e., $\partial p_S/\partial z = -g\rho_S$, (2) horizontal, i.e., $w_S = 0$, and (3) geostrophic, i.e.,

$$u_s = -\frac{1}{l\bar\rho}\frac{\partial p_s}{\partial y}, \quad v_s = \frac{1}{l\bar\rho}\frac{\partial p_s}{\partial x}$$

(the latter denotes that the divergence of the velocity field is zero, and that its current function ψ_S equals $p_S/l\bar\rho$). We note that our equations have two invariants (stationary combinations of unknown functions)

$$J_1 = (p - c^2\rho)_{z=0}, \quad J_2 = \bar\rho\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + l\left(\frac{\partial}{\partial z}\frac{p - c^2\rho}{b} - \rho\right). \quad (5.3)$$

The first of them is the linearized form of the entropy (at z = 0), and the second is that of the potential vortex. We can indicate one integral invariant, from which follows invariance of both quantities J_1 and J_2; for the quasistatic case it was found in[46].

We indicate also an energy invariant

$$J_3 = \int\left[\frac{\bar\rho V^2}{2} + \frac{p^2}{2\varkappa\bar p} + \frac{g(p - c^2\rho)^2}{2\varkappa\bar p b}\right]dV, \quad (5.4)$$

where the first, second, and third terms in the square brackets correspond to the kinetic, elastic, and the so-called thermobaric[47] energy (connected with the buoyancy forces acting on a particle that is displaced vertically from the equilibrium state).

An arbitrary solution of Eqs. (5.1)—(5.2) will be stationary if and only if the initial data have the properties (1)—(3). In the opposite case, it will be a sum of a stationary solution with invariance (5.3) determined from the initial data (the invariants J_1 and J_2 and the conditions (1)—(3) determine the stationary solution completely), and a certain nonstationary solution, for which $J_1 \equiv J_2 \equiv 0$. Such nonstationary solutions describe motions of the second kind. These solutions are superpositions of waves of the form $\Phi(z)\exp\{i(k_1x + k_2y - \sigma t)\}$; owing to the adiabaticity and linearity, their frequencies σ are real. There are two types of waves: 1) two-dimensional waves, in which there are no vertical oscillations of the air particles, i.e., w = 0, 2) internal waves, in which $w \neq 0$.

It can be verified that the frequencies of the two-dimensional waves are determined by the formula $\sigma^2 = l^2 + k^2c^2$ (where $k = \sqrt{k_1^2 + k_2^2}$ is the horizontal wave number), so that the wave-front velocity equals the velocity of sound c; their amplitudes decrease monotonically with altitude like $\Phi \sim (p)^{-1/\kappa}$. Only such waves are possible in a quasistatic barotropic atmosphere; in the case of a quasistatic baroclinic atmosphere, they are singled out in[46]. If c^2 = const and b = const (this is an exact condition for an isothermal atmosphere), the amplitudes of the internal waves depend on the altitude like $\exp\{-[(b + g)/2c^2]z + imz\}$, where $m \neq 0$ is the vertical wave number, and their frequencies are determined from the equation

$$(\sigma^2 - l^2)\left[\sigma^2 - \frac{(b+g)^2}{4c^2} - m^2c^2\right] = k^2c^2\left(\sigma^2 - \frac{bg}{c^2}\right) \quad (5.5)$$

and at arbitrary k and m they fill the intervals $l^2 \leq \sigma^2 < bg/c^2$ and $\sigma^2 > (b + g)^2/4c^2$, corresponding to two different types of internal waves. Considering the case $l = 0$ in an isothermal atmosphere ($c^2 = \kappa gH$, $b = (\kappa - 1)g$, where H is the thickness of the homogeneous atmosphere), we can verify that on going to the limit as $\kappa \to \infty$ to isopycnic processes (i.e., to an incompressible liquid), the second interval goes off to infinity, and the first takes the form $\sigma^2 < g/H$ and corresponds to internal gravitational waves. On going to the limit as $\kappa \to 1$ to isothermal processes (in which the isothermal stratification becomes indifferent), the first interval vanishes at zero, and the second takes the form $\sigma^2 > g/4H$ and corresponds to acoustic waves. Inasmuch as $bg < (b + g)^2/4$, the frequency spectra of the acoustic and gravitational waves do not overlap in an isothermal atmosphere. Dikiĭ[48], who investigated the wave spectra already not in an isothermal but in a temperature-stratified atmosphere, established that only a very small overlap of the spectra of the acoustic and gravitational waves occurs in such an atmosphere. These results explain the presence of the minimum in the spectrum of Fig. 2.

In the quasistatic approximation, i.e., neglecting the left part of the third equation of (5.2), and consequently using the static equation $\partial p/\partial z = -\rho g$, all the frequencies of the internal acoustic waves become infinite, i.e., the latter are completely "filtered out." The frequencies of the gravitational waves are in this case slightly overestimated, but the smaller k (i.e., the longer the waves), the less the overestimate. The frequencies of two-dimensional waves, the stationary solutions, and the invariants remain unchanged, and this justifies the use of the quasistatic approximation for the description of synoptic processes.

The main change introduced in the foregoing results by allowance for the curvature of the earth's surface reduces to a transformation of the stationary solutions (motions of the first kind) into slow gyroscopic waves. The earth's curvature can be approximately accounted for by constructing with the aid of the first two equations of (5.1) equations for the vorticity $\partial v/\partial x - \partial u/\partial y$ and the divergence $\partial u/\partial x + \partial v/\partial y$, with allowance for the dependence of the Coriolis parameter l on the coordinate along the meridian y, and then replacing the obtained equations l and $\beta = \partial l/\partial y = (2\omega \sin\theta)/a$ by constants (i.e., going over to the so-called β-plane). Then, in the case of a barotropic atmosphere and neglecting its horizontal compressibility (i.e., assuming $\partial u/\partial x + \partial v/\partial y = 0$) we obtain for the frequencies of the gyroscopic waves the formula $\sigma = -\beta k_1/k^2$[49] (the minus sign denotes that the waves move westward).

With exact account taken of the sphericity of the earth, the distinction in the barotropic atmosphere between the slow gyroscopic waves, which are important for weather forecasting, and the fast two-dimensional waves, which are important for the description of tides, was established already at the end of the 19th century by Hough[50] (see also the works on tide theory by Love[51], Kochin[52], Pekeris[53], Kertz[54], and Siebert[55]). The effect of weak horizontal compressibility of the atmosphere can be described here with the aid of series in powers of the parameter $\gamma = (2\omega a/c)^2$[56]. As $\gamma \to 0$, the fast waves vanish, and the frequencies of the gyroscopic waves are given by the formula

$$\sigma = \alpha m - \frac{2(\alpha + \omega)m}{n(n+1)}, \quad (5.6)$$

where m and n are integers, and α is the already-mentioned circulation index[57,11]. For the earth's atmosphere ($\gamma \approx 10$) for small values of n and m this formula is rough[56], and its accuracy increases rapidly with increasing n and m.

In the case of a baroclinic atmosphere on the earth's surface, the description of all the possible types of waves was given by Dikiĭ[58,59]—in the first of these papers for the isothermal atmosphere, and in the second for the so-called standard atmosphere CIRA-1961. Seeking the waves

$$\Phi(\zeta)\,\Psi(\cos\theta)\,e^{i(m\lambda+\sigma t)}$$

(where λ is the longitude, θ the complement of the latitude, and $\zeta = \ln(p/p_0)$ the vertical coordinate), Dikiĭ obtained for Ψ the so-called Laplace equation of the theory of tides, containing m, σ, and the constant h arising upon separation of the variables (the so-called depth of the dynamically equivalent ocean), but not containing the stratification characteristics, and obtained for Φ a "vertical" equation containing σ, h, and the stratification characteristics, but not containing the horizontal wave number m. Each of these equations makes it possible to determine a family of proper $\sigma(h)$ curves; the intersections of the curves of different families determine the possible eigenvalues of σ and h. Figure 8 shows the "horizontal" proper curves $\sigma(h)$ for m = 2; the lower curves correspond to fast waves and the upper ones to slow waves. Figure 9 shows the "vertical" proper curves $\sigma(h)$: on the lower right they correspond to acoustic waves and on the upper left to gravitational waves; the numbers at the curves indicate the number of the nodes of the corresponding eigenfunctions.

6. ADAPTATION OF METEOROLOGICAL FIELDS

Let us return for a time to the previously considered simplified "flat" model of the atmosphere with l = const. As already noted, if the initial data v_0, p_0, and ρ_0 have properties (1)—(3), i.e., if only motions of the first kind are present at the initial instant, then only such motions will remain also in the future (since the solutions describing them are stationary). On the other hand, if at the initial instant of time the conditions (1)—(3) are violated in some region of space V, then motions of the second kind—fast waves—are also produced in this region. These, however, scatter away in all directions, and when they leave the region V, then conditions (1)—(3) are established in this region, i.e., only certain motions of the first kind are left (defined by the invariant fields J_1 and J_2, which can be constructed from the initial data). This process of restoration of the consistency conditions (1)—(3) of the meteorological fields v, p, and ρ is called the adaptation of the meteorological fields. The problem of adaptation of meteorological fields in the case of a quasistatic barotropic atmosphere was first formulated by Rossby[60] and Cahn[61] and solved by Obukhov[3]; in the case of a quasistatic baroclinic atmosphere, this problem was dealt with by Bolin[62], Kibel'[63] (without taking into account two-dimensional waves), Veronis[64], Fjelstad[65], Monin[46]; see also the outstanding review by Phillips[66], devoted to geostrophic motions (or, in our terminology, motions of the first kind) in the atmosphere and in the ocean.

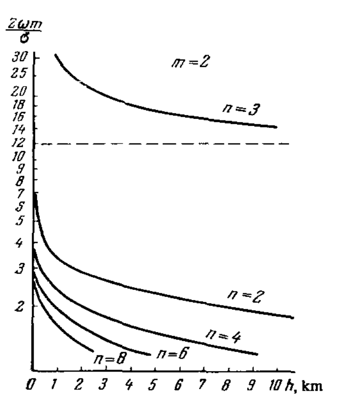

FIG. 8. Proper curves $\sigma(h)$ of the Laplace equation of the tidal theory at m = 2 (from[58]).

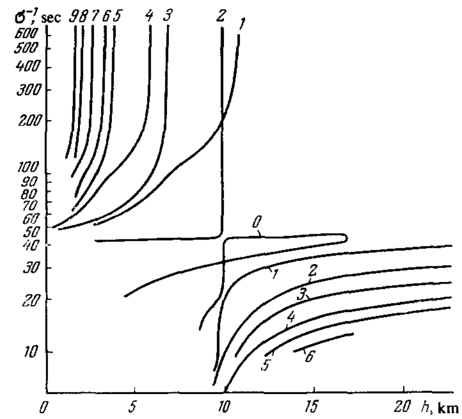

FIG. 9. Proper curves $\sigma(h)$ of the "vertical" equation for the standard atmosphere CIRA-1961 (from [58]).

The adaptation to the state of the static equilibrium (1) is effected by generation and spreading of internal acoustic waves, and its duration is approximately equal to the time during which the front of the internal acoustic waves traverses (with the speed of sound $c \sim 20$ km/min) the main thickness of the atmosphere, for which only several minutes are needed. After this, the adaptation of the atmosphere to the state of the geostrophic equilibrium (1)—(3) continues, and this state is reached, on the average over the thickness of the atmosphere, after the two-dimensional waves escape from the region V (the fronts of these waves move with the same velocity c), and is established at all altitudes even later, after the slower internal gravitational waves escape (their velocities depend on the thermal stratification of the atmosphere[63,46,67]—the fronts move with a velocity $2\sqrt{(1 - 1/\kappa)(1 - \gamma/\gamma_a)RT}$, and behind the fronts, as in the case of two-dimensional waves, there is a continuous "wake" in which damped oscillations take place).

By way of an example of the adaptation of the meteorological fields we show in Fig. 10 a case[46] in which at the initial instant of time there were no pressure perturbations, and the velocity field corresponded to a plane-parallel flow of the type of tangential discontinuity along the ordinate axis (the initial distribution of the surface velocity $v_0(x)$ is shown in the figure dotted). As a result of adaptation, the velocity field changed little—see the limiting distribution of the surface veloc-

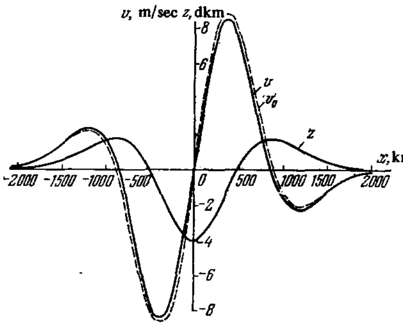

FIG. 10. Example of adaptation of meteorological fields in a baroclinic atmosphere (from [46].

ity v(x) (the kinetic energy decreased by 3%—loss to the generation of fast waves and to the formation of inhomogeneities of the pressure field), and the pressure "adapted" actively to the velocity field—a distinct dip was produced in it (see the limiting distribution of the heights of the surface isobaric surface z(x); it dropped by 4 dkm along the ordinate axis).

So far we have spoken only of adaptation of the meteorological field in the "flat" model of the atmosphere with l = const. Allowance for the sphericity of the earth's surface introduces into this process two not very important changes. First, the fast waves escaping from the perturbed region will now propagate not in an infinite but in a horizontally-bounded space, producing an interference pattern, which will attenuate with time as a result of the dissipative processes (which were not taken into account above). Second, on the sphere, motions of the first kind will not be stationary—there will be a superposition of slow gyroscopic waves, the motions of which will lead at all times to changes of the configuration of the fields ⊛ and Ω, which in turn upset the consistency conditions (1)—(3) of the meteorological fields. Thus, two competing processes occur continuously on a sphere: 1) disturbance of the consistency of the velocity, pressure, and density fields as a result of an evolution of spatial distribution of the entropy and of the potential vortex, and 2) adaptation of the meteorological fields as a result of generation, escape, and damping of the fast waves.

So far we considered in this chapter, as in the preceding one, only waves of small amplitudes, which can be described by linearized dynamic equations. In a real atmosphere, motions of the second kind—acoustic and gravitational waves—have small amplitude practically at all times (and therefore produce "meteorological noise" which is of low importance for the weather), but the motions of the first kind no longer have, generally speaking, small amplitudes, and are described by the nonlinear equations (4.1). This nonlinearity (as well as allowance for the sphericity of the earth's surface) leads to a nonstationary character of the motions of the first kind—an evolution of the spatial distribution of the quantities ⊛ and Ω as a result of their transport by the air currents and, as a consequence, to a continuous competition between the disturbance of the consistency and the adaptation of the meteorological fields. As a result of this competition, the disturbances of the consistency conditions (1)—(3) are as a rule only small, and the motions of the first kind still satisfy these conditions, albeit approximately. We shall make this statement more precise in the next chapter.

7. QUASIGEOSTROPHIC APPROXIMATION

It is desirable to simplify the hydrodynamic equations in such a way that the simplified equations describe with sufficient accuracy the motions of the first kind, which are important for the weather, but do not contain among their solutions the inessential motions of the second kind (i.e., the latter are "filtered out"). We have already mentioned that the quasistationarity condition (1) for the motions of the first kind is satisfied practically exactly, and that its use in lieu of the complete equation of motion in the vertical direction leads to a "filtering out" of the acoustic waves from the solutions of the hydrodynamic equations. We shall henceforth use throughout such a "quasistatic approximation," and transform with its aid to the vertical pressure coordinate p in lieu of the altitude z; then the pressure field p(x, y, z, t) will be replaced in the hydrodynamic equations by another unknown function z(x, y, p, t), the values of which at fixed values of p are the heights of the isobaric surfaces p = const.*

The hydrodynamic equations (more accurately, the Euler equations) contain two-dimensional parameters—the acceleration due to gravity g and the Coriolis parameter l (on a sphere it is better to use in lieu of l the earth's rotation 2ω; equations on a sphere include also the earth's radius a). In the boundary condition on the earth's surface one adds also the average surface pressure p_0 and density ρ_0 (with the aid of which it is possible to define the height of the homogeneous atmosphere $H = p_0/\rho_0 g$ and the isothermal speed of sound $c_0 = \sqrt{gH}$). Finally, we introduce typical length and velocity scales L and U for the synoptic processes. Out of the foregoing dimensional quantities it is possible to set up the following four dimensionless parameters [68-70]: 1) the sphericity parameter L/a; 2) the quasistatic parameter H/L; 3) the Kibel' number $\mathrm{Ki} = U/Ll$ (in some foreign papers it is also called sometimes the Rossby number); 4) the Mach number $\mathrm{Ma} = U/c_0$ (or the parameter of the horizontal compressibility of the atmosphere $L/L_0 = \mathrm{Ma}/\mathrm{Ki}$, where $L_0 = c_0/l$ is the already mentioned scale of the oscillations of the two-dimensional compressible atmosphere in the field of the Coriolis force, introduced by Obukhov [3]).

The Kibel' number Ki can be interpreted as the ratio of the typical value of U/L of the relative vorticity Ω_z to the value of the earth's rotation $2\omega_z = l$ (or as the ratio of the typical relative acceleration U^2/L to the

*On going over from the coordinates (x, y, z) to (x, y, p), the horizontal pressure gradients $\nabla_h p$ is replaced by $\rho g \nabla_h z$ (and $\partial p/\partial t$ by $\rho g \partial z/\partial t$), and the static equation $\partial p/\partial z = -\rho g$ is rewritten in the form $\partial z/\partial p = -1/g\rho$ (by determining ρ from this equation, we can reduce the Clapeyron equation $p = \rho RT$ to the form $T = -(g/R)p\partial z/\partial p$); the individual derivative d/dt assumes the form $d_h/dt + w^*\partial/\partial p$, where d_h/dt is the derivative with respect to the horizontal motion (on the isobaric surfaces), and $w^* = dp/dt$ replaces the vertical velocity. One of the advantages of the coordinates (x, y, p) is the particularly simple form of the continuity equation: $D + \partial w^*/\partial p = 0$, where $D = \partial u/\partial x + \partial v/\partial y$ is the horizontal divergence of the velocity. On the other hand, the boundary conditions w = 0 at z = 0 assumes the more complicated form $w^* = \rho g\, d_h z/dt$ (it is customary to require satisfaction of this condition, for the sake of simplicity, not at z = 0 but at $p = p_0$, where p_0 is the average surface pressure).

typical Coriolis acceleration Ul). With the exception of the tropical zone, this number is as a rule small: thus, according to calculations by Chaplygina[71], who determined the values of Ω_z/l from actual data, the modulus of this number is almost always smaller than 0.4, and in 75% of the cases it is smaller than 0.2. This means that the rotation of the air in large-scale atmospheric vortices at moderate and high latitudes (cyclones and anticyclones) is much more slower than the rotation of the earth. Consequently, in the scales of the synoptic processes, the horizontal gradient of the pressure is approximately balanced by the Coriolis force, i.e., the geostrophy conditions (3) are approximately satisfied (accurate to terms of order U Ki), and assume in the coordinates (x, y, p) the form

$$u = -\frac{g}{l}\frac{\partial z}{\partial y}, \quad v = \frac{g}{l}\frac{\partial z}{\partial x}. \tag{7.1}$$

It follows from (7.1) that in scales of synoptic processes; the variations of the quantity z in the horizontal direction are of the order of lLU/g (the variations of z in time are of the same order). Then we see from the adiabaticity equation $dS/dT = 0$, which with the aid of (4.3) and the static equation $1/\rho = -g\,\partial z/\partial p$ can be reduced to the form

$$gp^2 \frac{d_h}{dt}\frac{\partial z}{\partial p} + \alpha_0^2 c_0^2 w^* = 0, \tag{7.2}$$

where $\alpha_0^2 = -(T/T_0)(p/c_p)\partial S/\partial p$ is the dimensionless parameter of the static stability (the quantity $Ri = (1/Ma^2)(T_0^2/T^2)\,\alpha_0^2$ is sometimes called[68-69] the Richardson number), that the values of w* are of the order of

$$\frac{p_0 U}{L}\frac{L^2}{L_0^2\alpha_0^2}\,Ki \approx \frac{p_0 U}{L}\,Ki$$

(in the troposphere usually $\alpha_0^2 \sim L^2/L_0^2$). Finally, the continuity equation $D + \partial w^*/\partial p = 0$ shows that the values of the horizontal divergence of the velocity are of the order of $w^*/p_0 \sim (U/L)Ki$. Thus, for motions of the first kind (synoptic processes) the following conditions are satisfied

$$\Omega_z \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{g}{l}\Delta z + O\left(\frac{U}{L}Ki\right), \quad D \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = O\left(\frac{U}{L}Ki\right), \tag{7.3}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$. For motions of the second kind, these conditions, to the contrary, are not satisfied. Therefore, it is possible to "filter out" the fast waves by finding solutions of the hydrodynamic equations in the form of asymptotic series in powers of Ki with principal terms satisfying the conditions (7.2) (the so-called quasigeostrophic expansion, first proposed by Kibel'[2]). The equations for these principal terms describe the synoptic processes with sufficient accuracy (with a relative error only of the order of Ki), and do not include the fast waves among their solutions. Such equations are the relations (7.1) and the equation obtained from the conservation law $d_h\Omega/dt + w^*(\partial\Omega/\partial p) = 0$ of the potential vorticity $\Omega \approx -\Omega_{az} g\,\partial s/\partial p$ after eliminating from it, with the aid of (7.2), the quantity w* and after taking into account only terms of zeroth order in Ki. The latter equation is reduced to the elegant form*

$$\mathscr{F}\frac{\partial z}{\partial t} = -\frac{g}{l}[z, \mathscr{F}z + l], \tag{7.4}$$

where $\mathscr{F}$ is an elliptic linear operator (the analog of the three-dimensional Laplace operator), defined by the formula

$$\mathscr{F}z = \frac{g}{l}\Delta z + \frac{gl}{c_0^2}\frac{\partial}{\partial p}\frac{p^2}{\alpha_0^2}\frac{\partial z}{\partial p}, \tag{7.5}$$

and the square brackets [A, B] will henceforth denote the Jacobian $\partial(A, B)/\partial(x, y)$. The method of asymptotic expansions, which makes it possible to derive Eq. (7.4) from the initial hydromechanic equations, is a particular case of the general asymptotic methods developed by N. N. Bogolyubov and N. M. Krylov for the description of slow oscillations in nonlinear mechanical systems, in which rapid oscillations occur besides these slow oscillations (Van der Pol was the first to develop one such method for the description of current oscillations in an electric circuit containing a vacuum tube with feedback). From the purely mathematical point of view, we are dealing here with equations (describing the oscillations) that contain a small parameter (the Kibel' number) at the higher derivatives (terms in the equations of motion describing the relative accelerations).

Equation (7.4) contains only one unknown function z; it describes synoptic changes of the three-dimensional field of atmospheric pressure in the quasigeostrophic approximation, and is of the first order in the time; this is natural, since the initial system of hydrodynamic equations was of fifth order, but the "filtered out" two families of waves (acoustic and gravitational) "carried away" two orders each. Thus, in order to calculate beforehand the field of the atmospheric pressure in the quasigeostrophic approximation it is sufficient to know the initial values of only the pressure field itself, and the initial values of the velocity field (which would be needed to solve the complete hydrodynamic equations) need no longer be known. This simplification is very important in practice, since the wind field is presently measured rather crudely, and an organization of its exact measurements would be a very cumbersome and expensive matter.

The differential equation (7.4) is of second order in p, and for its solution it is necessary to specify the boundary conditions on the upper and lower limits of the atmosphere $p = 0$ and $p = p_0$. For $p \to 0$ we require that the kinetic energy be bounded, i.e., $p\,|\nabla_h z|^2 < \infty$, and for $p = p_0$ we use the condition $w = 0$, which with the aid of Eq. (7.2) at $w^* = \rho_0 g\, d_h z/dt$ (see the footnote on the preceding page) reduces to the form

$$\frac{d_h}{dt}\left(p\frac{\partial z}{\partial p} + \alpha_0^2 z\right) = 0.$$

Equation (7.4) with such boundary conditions can be rewritten in integral form

$$\left(\mathscr{G}\Delta - \frac{1}{L_0^2}\right)\frac{\partial z}{\partial t} = -\mathscr{G}\left[z, \frac{g}{l}\Delta z + l\right] - \frac{1}{L_0^2}\int_p^{p_0}\left[z, \frac{\partial z}{\partial p}\right]dp, \tag{7.6}$$

where $\mathscr{G}$ is the operator of integration with respect to p, defined by the formula

$$\mathscr{G}z = \frac{1}{p_0}\int_0^{p_0} z\,dp + \int_p^{p_0}\frac{\alpha_0^2\,dp}{p^2}\int_0^p z\,dp. \tag{7.7}$$

Equation (7.6) is particularly convenient for the transition in the limit as $\alpha_0^2 \to 0$ to the case of the barotropic

*Obtained from the more general formula

$$\frac{d_h}{dt}\left(\ln\Omega_{az} + \frac{g}{c_0^2}\frac{\partial}{\partial p}\frac{p^2}{\alpha_0^2}\frac{\partial z}{\partial p}\right) = 0, \tag{7.4'}$$

which expresses a certain approximate conservation law for the horizontal motions.

atmosphere. For synoptic processes in a barotropic atmosphere, $\nabla_h z$ and $\partial z/\partial t$ are proportional to a certain function of p (see[72]), the second term in the right side of (7.6) drops out, the operator $\mathcal{G}$ becomes equal to unity, and Eq. (7.6) takes the simple form

$$\left(\Delta - \frac{1}{L_0^2}\right)\frac{\partial z}{\partial t} = -\left[z, \frac{g}{l}\Delta z + l\right]. \qquad (7.6')$$

Such an equation was proposed for weather forecasting purposes by Obukhov[3] and Charney[4-5] (a similar equation, but without the term with $1/L_0^2$, was previously obtained by another method by Ertel[73], but the significance of that equation was not understood then). This is precisely the equation which was first used in the post-war years (e.g., in[6]) for numerical forecasts of the pressure field (at a certain average level in the troposphere).

It follows from (7.6′) that the value of $\partial z/\partial t$ at a fixed point M is obtained by integrating the "advection of the vortex" $[z, (g/l)\Delta z + l]$ over all the points of the plane M′ with weights $(1/2\pi)K_0(r/L_0)$, where r is the distance from M′ to M and K_0 is the symbol of the cylindrical Macdonald function. The influence function $K_0(r/L_0)$ decreases with increasing r; thus, changes of the pressure $\partial z/\partial t$ at each fixed point M are determined by the entire pressure field z(M′), but the influence of the individual points M′ turns out to be small (the "influence radius" is the distance L_0). We note that retention of the subtrahend $1/L_0^2$ in the operator $\Delta - (1/L_0^2)$ in the left side of (7.6′) (and analogously in the operator $\mathcal{G}\Delta - 1/L_0^2$ in the left side of Eq. (7.6)) is essential: if this subtrahend is neglected (it describes the effect of the "horizontal compressibility" of the atmosphere), the influence function $K_0(r/L_0)$ is replaced by the function ln r, which increases with increasing r, i.e., corresponds to an increase of the influence of the remote points M′ with increasing distance r, which of course is not natural. If we represent the horizontal field z(x, y) in the form of a superposition of elementary harmonic waves, then it becomes clear that for waves with length much shorter than L_0 allowance for the term $-1/L_0^2$ in the dynamic operator is insignificant, but it becomes quite appreciable for the description of the evolution of long waves (with length $L \gtrsim L_0$).

In analogy with the foregoing, it follows from (7.6) that the value of $\partial z/\partial t$ at a fixed point M = (x, y, p) of a baroclinic atmosphere is obtained by integration over all points M′ = (x′, y′, p′) of the sum of the "vorticity advection" $[z, (g/l)\Delta z + l]$ with a certain weight $G(r/L_0; p, p')$ and "heat advection" $-(g/l)[z, p(\partial z/\partial p)]$ with weight $(1/L_0^2\alpha_0^2)p'(\partial/\partial p')G(r/L_0; p, p')$ where r is the horizontal distance from M to M′. The influence functions G and $p'(\partial G/\partial p')$ were first determined (using coordinates x, y, z, and Fourier transforms in x and y) in 1951—1952 by Obukhov and Chaplygina[74] and almost simultaneously (in coordinates x, y, p) by Buleev and Marchuk[75], and later by Hinkelmann[76] and Kuo[77]. The simplest derivation of these functions is given in[46], where the role of these influence functions in the problem of adaptation of meteorological fields is also established. Figures 11—12 show plots of these functions from[74] (in dimensionless form, following Fourier transformations with respect to x and y, and at a value of the dimensionless wave number $kL_0\sqrt{[(\gamma_a - \gamma)/(\kappa - 1)](\kappa R/g)} = 4$), showing clearly the

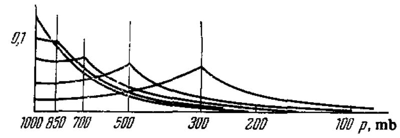

FIG. 11. Influence functions of vorticity advection in various layers of the air on the values of $\partial z/\partial t$ at the levels p = 1000, 850, 700, 500, and 300 mb (from [74]).

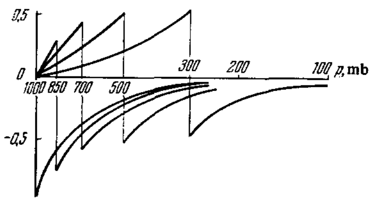

FIG. 12. Influence functions of heat advection in various layers of the air on the values of $\partial z/\partial t$ at the levels p = 1000, 850, 700, 500, and 300 mb (from [74]).

relative weights with which the "dynamic" and "thermal" contributions of different layers of air enter in the values of $\partial z/\partial t$ at various levels.

8. QUASISOLENOIDAL APPROXIMATION

As the equator is approached, the Coriolis parameter $l = 2\omega\cos\theta$ decreases, the Kibel′ number $Ki = U/Ll$ ceases to be small, and consequently the quasigeostrophic approximation is no longer valid. In addition, experience with numerical forecasts of the pressure field has shown that even outside the tropical zone the description of the synoptic processes with the aid of the quasigeostrophic approximation turns out in some cases to be insufficiently accurate. It may therefore be useful to find in lieu of the conditions (7.3) other "consistency" conditions for the synoptic fields of the velocity and pressure, making it possible to distinguish the synoptic processes from the fast wave motions.

Such conditions (which are suitable not only near the equator but everywhere) can be obtained by starting from the fact that the potential component of the field of the horizontal velocity of the slow synoptic motions is small compared with the solenoidal component, or in other words, the horizontal divergence of the velocity $D = \partial u/\partial x + \partial v/\partial y$ is small (in absolute magnitude) compared with the vorticity $\Omega_z = \partial v/\partial x - \partial u/\partial y$. As a result, the principal terms in the equation for D, obtained by applying to the equations of motion the divergence operation, will be those containing neither D (or $w^* \sim p_0 D$) nor the Coriolis parameter l (which can be small). These principal terms are of the form $2[u, v] - g\Delta z$; from a comparison of these terms it follows that the variations of z are of the order of $U^2/g = H\,Ma^2$, where $Ma = U/c_0$ is the Mach number, which is quite small for synoptic processes. We then obtain from (7.2) $w^* \sim p_0 U Ma^2/L\alpha_0^2$, and therefore also $D \sim U Ma^2/L\alpha_0^2$. In other words, the condition for the smallness of D compared with Ω_z for motions of the first kind (synoptic processes) can be written in the form

$$\Omega_z = O\left(\frac{U}{L}\right), \quad D = O\left(\frac{U}{L}\frac{Ma^2}{\alpha_0^2}\right). \qquad (8.1)$$

For motions of the second kind (fast waves), these conditions, to the contrary, are not satisfied. We can therefore "filter out" the fast waves, finding solutions of the hydrodynamic equations in the form of asymptotic series of powers of $Ma^2 = U^2/c_0^2$ with principal terms satisfying the conditions (8.1), and with a principal term for z on the order of HMa^2. The principal term of the wind velocity field (u, v) will then be its solenoidal component, and $u = -\partial\psi/\partial y$ and $v = \partial\psi/\partial x$, where ψ is the current function; for this reason, the indicated asymptotic series are sometimes called the quasisolenoidal expansion. One of the equations for the principal terms of the quasisolenoidal expansion is obtained with account taken of only the zeroth-order terms in Ma in the approximate conservation law (7.4′):

$$\frac{\partial F}{\partial t} = -[\psi, F], \quad F = \ln(\Delta\psi + l) + \frac{g}{c_0^2}\frac{\partial}{\partial p}\frac{p^2}{\alpha_0^2}\frac{\partial z}{\partial p}. \tag{8.2}$$

Unlike the quasigeostrophic approximation equation (7.4), which contains only one unknown function, z, Eq. (8.2) contains two unknown functions, z and ψ. The connection between them (i.e., the connection between the velocity and pressure fields, which was given in the geostrophic approximation by formulas (7.1)), will be given by an equation obtained from the aforementioned equation for D in which only terms of zeroth order in Ma are retained. This equation, called the balance equation, is of the form

$$g\Delta z = (\nabla l \nabla)\psi + 2\left[\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}\right]. \tag{8.3}$$

Equations (8.2) and (8.3) describe the synoptic changes of the velocity and pressure fields in the quasisolenoidal approximation. They are of the first order in the time (since the "filtered out" two families of the fast waves, the acoustic and gravitational, have "carried away" two orders of magnitude each). Thus, for the forecasting of synoptic processes in the quasisolenoidal approximation it suffices to have the initial values of only the pressure field z, and the initial values of the velocity field (the current functions ψ) can be determined from the field z by means of (8.3).

The quasisolenoidal approximation equations (8.2)—(8.3) are suitable both near the equator, where l is small and the quasigeostrophic approximation is not valid, and outside the tropical zone, i.e., in the region where the Kibel' number Ki is small. In the latter region, the quasigeostrophic approximation is valid accurate to terms of order Ki, and the quasisolenoidal approximation is valid with great accuracy up to terms of order Ki^2 (here the second term in the right side of (8.3) is smaller than the remaining terms by one order of magnitude relative to term Ki, and if it is neglected and the variations of l with latitude are neglected, then the geostrophic relation $l\psi \approx gz$ is obtained, whereby (8.2) is transformed into the quasigeostrophic approximation (7.4)).

The balance equation (8.3) was obtained as the second approximation in the quasigeostrophic expansion in[72] (see also the papers of Bolin[78], Thompson[79], and also the later paper[46]). It became popular after it was pointed out in a paper by Charney[80] with reference to an unpublished paper by Fjortoft. A justification of the quasisolenoidal approximation by asymptotic expansion in powers of Ma was proposed in[67], and in a very similar form by Charney[68]; Gavrilin[70] derived the equations of the quasisolenoidal approximation for non-adiabatic synoptic processes on a spherical earth.

Actually, the quasisolenoidal approximation was used for the description of synoptic processes long ago: the theory of synoptic waves developed by Blinova[11] (gyroscopic waves in the terminology of Ch. 5) was based on consideration of the quasisolenoidal approximation for the vortex transport equation in an adiabatic barotropic atmosphere on a spherical earth (in the case of a barotropic atmosphere, it is possible to put $\partial z/\partial p = 0$ in (8.2), and then F can be simply replaced by $\Delta\psi + l$). In the indicated theory this equation was linearized relative to a state in which the atmosphere rotates around the earth as a rigid body, with an angular velocity α (the aforementioned circulation index), and took the form

$$\left(\frac{\partial}{\partial t} + \alpha\frac{\partial}{\partial\lambda}\right)\Delta\psi + 2\frac{\alpha+\omega}{a^2}\frac{\partial\psi}{\partial\lambda} = 0, \tag{8.2′}$$

where λ is the longitude and a is the earth's radius (an analog of this equation in Cartesian coordinates was proposed earlier for the description of the gyroscopic waves by Rossby[49], who obtained from this analog a formula for the velocity of motion of baric depressions, i.e., planetary pressure waves; see Ch. 5). The elementary wave solutions of (8.2′) were indicated by Haurwitz[57]. Blinova[11] constructed with the aid of this equation a general solution of the initial-value problem for the field of the atmospheric pressure, relating the latter with the field ψ by an actually linearized balance equation.

The solution of the quasisolenoidal-approximation equations (8.2)—(8.3) entails considerable mathematical difficulties (see[80,68]). They are connected, first, with the need for determining from the balance equation (8.3) the initial field ψ for a specified initial field z, and second, with the need for determining, during each step of the integration of (8.2) with respect to time, the field ψ from the field F, which in turn is determined by the second formula of (8.2) using the same balance equation (8.3). The latter, regarded as an equation with respect to ψ, is among the so-called Monge-Ampere equations. In practice, owing to the incompleteness or even absence of synoptic information over a considerable fraction of the earth's surface, it becomes necessary to solve Eq. (8.3) with respect to ψ only inside a certain bounded territory, specifying in some manner the values of ψ on its boundary. Such a boundary-value problem for Eq. (8.3) will be correct only if it is elliptic. The condition for its ellipticity reduces to the form $g\Delta z + (l^2/2) > 0$. One can be sure that such a condition is satisfied only for small values of Ki, but if Ki is not small (e.g., in the tropics), this condition may not be satisfied. On the other hand, in the case of small Ki Eq. (8.3) can be rewritten (in the simplest scheme with l = const) in the form[46]

$$l\Delta\psi = g\Delta z - 2\frac{g^2}{l^2}\left[\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right]. \tag{8.3′}$$

9. PRIMITIVE EQUATIONS

Recently in many papers on hydrodynamic theory of short-range weather forecasts (e.g., Smagorinsky[81], Hinkelmann[82], Phillips[83], Charney[68], Buleev and Marchuk[84]), there appeared a tendency to forego "filtered out" fast waves and to return to the use of complete hydrodynamic equations (albeit in the quasistatic approximation), which were called "primitive" equations, i.e., the initial equations. This tendency is probably due to the following causes: 1) the quasigeostrophic approximation is insufficiently accurate in many cases; 2) the solution of the equations of the quasisolenoidal approximation entails mathematical difficulties; 3) owing to the development of computational mathematics and computer techniques, numerical solution of primitive equations is now perhaps no more difficult or only slightly more difficult than the solution of the "filtered out" equations (see, e.g., the differences schemes proposed by Marchuk[85-87,10] for the numerical solution of the primitive equations).

Just as in the balance equation considered above, the primitive equations must be integrated within the limits of confined territories. This raises the question of the correct formulation of the corresponding boundary conditions: whether in the case of insufficient or in the case of redundant boundary conditions, the solutions of the equations are unstable, and the resultant errors will extend more and more, following each step of integration with respect to time, away from the boundaries to the interior of the territories under consideration. Charney[68] has shown that the correct boundary value problem for the primitive equations is obtained by specifying the normal component of the velocity on the entire boundary of the territory under consideration, and the values of the potential vorticity on those sections of the boundary, where the motion of the air is directed to the interior of the territory (analogous boundary conditions were previously formulated by Charney, Fjortoft, and von Neumann[6] for the equation of the quasigeostrophic approximation in a barotropic atmosphere). The use of such boundary conditions makes it possible, on going over from the differential to the difference equations, to avoid calculations on boundaries of unilateral normal derivatives of the velocity components, the appearance of which would lead to a computational instability of the solutions of the difference equations.

The attempt to increase the accuracy of the description of the synoptic processes (compared with the quasigeostrophic approximation) by returning to the primitive "unfiltered" equations is obtained at the cost, first, of retaining the high order in time in the employed system of equations, and consequently with the need for specifying a large number of initial data (namely the initial values not only of the pressure field but also of the wind-velocity field), and second, retaining the gravitational waves along the solutions of the forecasting equations.

We recall that the gravitational waves can be generated, first, as a result of the initial "inconsistency" of the pressure and velocity fields, and, second, because the nonstationary character of the synoptic processes (due to their nonlinearity, and also to the influence of the sphericity of the earth) continuously causes the consistency of these fields to be disturbed. When the primitive equations are used, the second of these factors remains in force, but the first can be eliminated by specifying using actual data, only the initial pressure field, and choosing the initial velocity field from the conditions that it match the pressure field (this eliminates once and for all the errors connected with the inaccuracy of measuring the initial wind field; see[88]). These conditions can be written, in accordance with the foregoing, in terms of the quasigeostrophic or quasisolenoidal expansion (see[83] and[67]). Accurate to terms of order Ki^2, these conditions reduce to the fact that the current function ψ should be connected with z by the balance equation (8.3), and the velocity divergence $D = \partial u/\partial x + \partial v/\partial y$ should be connected by the formula

$$D = -\frac{g}{l^2}\left\{\Delta\frac{\partial z}{\partial t} + \left[z, \frac{g}{l}\Delta z + l\right]\right\}, \qquad (9.1)$$

in which it is necessary also, with the aid of (7.4) or (7.6), to express $\partial z/\partial t$ in terms of the values of the field z at the same instant of time.

The advantage of returning to the primitive equations cannot be connected with allowance for the gravitational waves; these, to the contrary, must be "filtered out" (at least in part, by making approximately "consistent" the initial velocity and pressure fields). But the usefulness may also lie in allowance for the actual boundedness of the "interaction radius" of the baric field, which follows from the fact that the system of the primitive equations is hyperbolic: the baric tendency, i.e., the derivative $\partial z/\partial t$, enters in the principal linear part of these equations under the sign of the hyperbolic operator

$$\mathcal{G}\Delta - \frac{1}{L_0^2} - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}$$

(where $\mathcal{G}$ is defined by formula (7.7)). Therefore the value of $\partial z/\partial t$ at a fixed point M at the instant of time t is determined by the values of the initial pressure and velocity fields in a vicinity of the point M which is bounded horizontally, with a radius $c_0 t$, and cannot in fact depend on the values of the initial fields outside this vicinity. In the "filtered out" equation, on the other hand, for example (7.6), this hyperbolic operator is replaced by the elliptic operator $\mathcal{G}\Delta - (1/L_0^2)$, as a result of which the value of $\partial z/\partial t$ at the point M at any instant of time t becomes dependent on the values of the initial pressure field in all of space, including the values of this field at points located at a distance larger than $c_0 t$ from M. The contributions of these points have no physical nature and introduce into the values of $\partial z/\partial t$ distortions, which are the price that must be paid for the "filtering out" of the gravitational waves.

However, to correct this shortcoming of the "filtered out" equations it is not necessary to return fully to the primitive equations, and it is sufficient, for example, to replace in the quasigeostrophic-approximation equation (7.6) the left side $(\mathcal{G}\Delta - 1/L_0^2)(\partial z/\partial t)$ by

$$\left(\mathcal{G}\Delta - \frac{1}{L_0^2} - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\frac{\partial z}{\partial t}$$

and to use for the obtained equation the approximately "consistent" initial values of the velocity and pressure fields; such a procedure was recommended in[67].

Incidentally, the turning from the "filtered out" equations to the primitive equations (or the restoration of the hyperbolic operator in the "filtered out" equations) for the purpose of increasing the accuracy of the forecasts of synoptic processes is so far not inevitable, since the accuracy that can be provided in principle by the "filtered-out" equations, particularly the quasisolenoidal approximation equation, is in practice not yet reached in concrete calculations, primarily because of errors in the numerical calculation connected with the approximation of the continuous pressure and velocity fields by their values on a finite set of points of a certain space-time grid and with the corresponding replacement of the differential operators contained in the dynamics equations by difference operators (and also as a result of the errors contained in the initial data and caused by the inaccurate formulation of the boundary conditions on the underlying surface and with the curvature of the relief).

10. VERTICAL STRUCTURE OF SYNOPTIC PROCESSES

One of the first problems arising in the integration of the forecasting equations is the description of the vertical structure of the synoptic processes, which is needed because these equations contain double differentiation *with respect to the vertical coordinate* p (which enters in Eq. (7.4) in the operator $\mathscr{F}$, and in (8.2) in the invariant F) or the equivalent double integration (the operator $\mathscr{G}$ in (7.6)).

The vertical structure of the synoptic processes turns out to be simplest in the case of a barotropic atmosphere (see[72]). The deviations z(x, y, p, t) of the heights of the isobaric surfaces from their values Z(p) in the standard atmosphere take on here the form

$$z(x, y, p, t) = z_0(x, y, t)\,\psi_0(p), \tag{10.1}$$

and the function z_0(x, y, t), which are the only "parameter" of the barotropic model of the atmosphere, can be assigned the meaning of the height of the isobaric surface at a certain average level in the troposphere (approximately 500 mb). In the general case of a baroclinic atmosphere, the function z(x, y, p, t) can be approximated by the expression

$$z(x, y, p, t) = \sum_{n=0}^{N-1} z_n(x, y, t)\,\psi_n(p), \tag{10.2}$$

where ψ_n(p) are certain fixed functions, and z_n(x, y, t) are "parameters" which can always be expressed in terms of the values of z(x, y, p, t) on specified levels p = p_n (so that the models of vertical structure of the synoptic processes with several "parameters" are equivalent to models with several levels). For no finite number of terms N can expression (10.2) serve as an exact solution of the forecasting equations, but the latter can be replaced approximately by the corresponding equations for the "parameters" of the given model. In the practice of numerical forecasts, use was made of models with two or three parameters, and in experiments also with a larger number of parameters (or levels). The functions ψ_n(p) were specified in this case *either by starting from some qualitative assumptions* regarding the vertical structure of the synoptic processes, or from considerations of convenience of interpolation between the specified levels. But this raises naturally the question of the optimal choice of these functions.

An optimality criterion can be introduced with the aid of statistical considerations, by regarding, for a fixed value of t, the values of z(x, y, p, t) at different points (x, y) as individual realizations of a certain random function ψ(p), characterized by the correlation function $\beta(p_1, p_2) = \overline{\psi(p_1)\psi(p_2)}$ (the bar denotes the mathematical expectation or averaging over (x, y); the mean value $\overline{\psi(p)}$ is assumed here equal to zero without loss of generality). From the general theory of random functions it follows[89,90] that the mean square

$$\sigma_N^2 = \int \overline{\left|\psi - \sum_{n=0}^{N-1} z_n\,\psi_n\right|^2}\, dp$$

of the error of the approximation of the function ψ(p) by the sum of the first N terms of the expansion in the complete orthonormal system of functions ψ(p) will be minimal at a fixed N, if one chooses for ψ_n(p) the eigenfunctions of the "dispersion operator" $\beta(p_1, p_2)$ in the integral equation

$$\int \beta(p_1, p_2)\,\psi(p_2)\,dp_2 = \mu\psi(p_1). \tag{10.3}$$

Such a choice of the functions ψ_n(p) will be optimal from the statistical point of view. The eigenvalues μ of the operator $\beta(p_1, p_2)$ will have then the meaning of the dispersions of the expansion coefficients $z_n = \int\psi\psi_n dp$ and these coefficients will be pairwise uncorrelated. Obukhov[90] used such a statistically-optimal representation for the description of the vertical structure of the field $\psi(p) = \partial z/\partial t$. In two real examples of such fields, considered by him on the discrete set of levels p = 1000, 850, 700, 500, and 300 mb, it turned out that the first term of the optimal expansion (corresponding to the barotropic model of the atmosphere) accounts for approximately 70% of the total dispersion of the field $\partial z/\partial t$, the sum of the first two terms accounts for more than 90%, and the sum of the first three for 97%; the subsequent terms contain already a very small fraction of the total dispersion and are therefore determined very inaccurately. It follows therefore that in the models *of the vertical structure of synoptic processes we can* confine ourselves to two—three parameters or levels (a larger number of levels can be useful in the planetary boundary layer of the atmosphere for a detailed allowance for the friction effects, and in the stratosphere if it is necessary to forecast its state). We note that in Obukhov's two examples the orthonormal optimal functions ψ_n(p) turn out to be very close. The small variability of such functions from territory to territory and during the year, both for the field $\psi(p) = \partial z/\partial t$ and for the vertical structure of the fields of the zonal and meridional components of the wind velocity and a few other synoptic fields, was established by a special investigation by Rukhovets[91]; by way of an example, Fig. 13 shows the first five functions ψ_n(p) for the field $\partial z/\partial t$ during different seasons of the year.

The foregoing method of statistically-optimal expansions was employed earlier by Fukuoka[92], Lorenz[93], White and co-authors[94], and Bagrov[95] for a description of the horizontal structure of the meteorological fields, for the purpose of their typization and statistical forecasting. Following Obukhov's work[90] this work found very wide application (see, e.g.,[96]).

Another method of naturally choosing the function ψ_n(p) in the expansion (10.2) is to use the eigenfunctions of the "dynamic operator" $\mathscr{H}$, which enters in the principal linear part of the forecasting equation, which we write in the form $\partial\psi/\partial t = \mathscr{H}\psi$. Under certain general conditions, such functions ψ_n(p) will coincide with the just-considered eigenfunctions of the "statistical operator" $\beta = \overline{\psi(p_1)\psi^*(p_2)}$ (for the sake of generality we admit here complex functions ψ, and the asterisk denotes the complex conjugate). Indeed, differentiating this expression for β with respect to the time t, replacing $\partial\psi/\partial t$ by

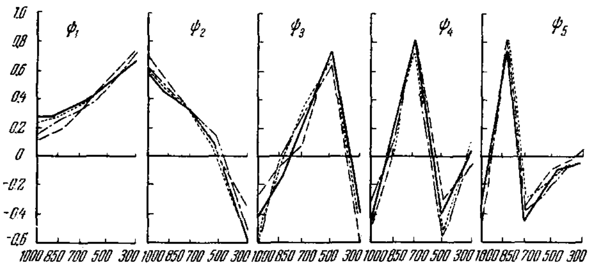

FIG. 13. Eigenfunctions $\psi_n(p)$ of the dispersion operator $\beta(p_1, p_2)$ of the field $\psi(p) = \partial z/\partial t$ during different seasons of the year (from [96]). —— – January 1958; – – – – – April 1959; – · – · – July 1959; · · · · – October 1959.

$\mathcal{H}\psi$, and; using the condition $\mathcal{H}^* = -\mathcal{H}$ (which ensures energy conservation) we obtain

$$\frac{\partial\beta}{\partial t} = \mathcal{H}\beta - \beta\mathcal{H}, \tag{10.4}$$

so that under the condition of statistical stationarity, when the dispersion operator β does not depend on the time t and the left part of (10.4) vanishes, the operators β and H turn out to be commutative and consequently have identical eigenfunctions[97]. The connection between these eigenfunctions and the time-independent "dynamic operator" $\mathcal{H}$ can explain their statistical stability (low variability from season to season and from territory to territory), which was noted above. But this does not pertain to the dispersions μ_n of the coefficients z_n, for within the framework of the linear theory their values are in general arbitrary, and they are established actually as a result of weak nonlinear interactions, so that the dispersions turn out to be statistically much less stable than the eigenfunctions of the operator β and $\mathcal{H}$.

A similar method of choosing the functions $\psi_n(p)$ in the expansion (10.2) was proposed by Gavrilin[98], where $\psi_n(p)$ were chosen to be the eigenfunctions of the "vertical operator" $\mathcal{H} = (\partial/\partial p)(p^2/\alpha_0^2)(\partial/\partial p)$ (with α_0^2 = const), which enters in the three-dimensional elliptic operator $\mathcal{F}$ of formula (7.5), which in turn enters in the Eq. (7.4) of the quasigeostrophic approximation. These eigenfunctions were obtained earlier in[67] as solutions of the equation $\mathcal{H}\psi = -\mu\psi$ under the boundary conditions $p(\partial\psi/\partial p) + \alpha_0^2\psi = 0$ at $p = p_0$ (corresponding to the vanishing of the vertical velocity w on the underlying surface) and $p\,|\psi|^2 < \infty$ as $p \to 0$ (corresponding to boundedness of the kinetic energy on the upper boundary of the atmosphere); such boundary conditions were used above to obtain the integral form (7.6) of the equation of the quasigeostrophic approximation. The spectrum of the eigenvalues μ of the operator $\mathcal{H}$ contains an isolated point $\mu = 1 - \alpha_0^2$ and the straight line $1/4\,\alpha_0^2 < \mu < \infty$ (in the limit as $\alpha_0^2 \to 0$, i.e., on going over to the barotropic atmosphere, only a single isolated point remains in the spectrum). The isolated point corresponds to the eigenfunction $\psi_0(p) = (p_0/p)^{\alpha_0^2\nu}$, and the remaining points of the spectrum correspond to the functions

$$\psi_\nu(p) = \sqrt{\frac{p_0}{p}}\left[2\nu\cos\left(\nu\ln\frac{p_0}{p}\right) - (1-2\alpha_0^2)\sin\left(\nu\ln\frac{p_0}{p}\right)\right], \tag{10.5}$$

where $\nu = \sqrt{\alpha_0^2\mu - 1/4}$. The indicated eigenfunctions are not normalized; apart from this, they are quite similar to the statistically-optimal functions of Fig. 13. In[98] Gavrilin considered a bounded layer of the atmosphere $p_0 \geq p \geq p_h$ (with boundary condition $p(\partial\psi/\partial p) + \alpha^2\psi = 0$ on the upper boundary); the continuous part of the spectrum breaks up in this case into a denumerable set of points $\nu = \pi n/\ln(p_0/p_h)$, n = 1, 2, ..., and the form of the eigenfunctions remains unchanged. For the "parameters" $z_n(x, y, t)$ of the expansion (10.2), the resultant equations are

$$\mathcal{F}_n\frac{\partial z_n}{\partial t} = -\frac{g}{l}\left\{\frac{1}{N_n^2}\sum_{p,q} a_{npq}[z_p, \mathcal{F}_q z_q] + \frac{\partial l}{\partial y}\frac{\partial z_n}{\partial x}\right\}, \tag{10.6}$$

where $\mathcal{F}_n = (g/l)(\Delta - \mu_n/L_0^2)$, where μ_n are the eigenvalues of the operator $\mathcal{H}$ ($L_0/\sqrt{\mu_n}$ plays the role of the scale of the horizontal inhomogeneities of the field z_n), N_n are the norms of the eigenfunctions ψ_n, and finally $a_{npq} = \int\psi_n\psi_p\psi_q dp$. These equations show that the time variations of each component $z_n\psi_n$ of the field z are determined by the interaction between all such components. Any three components $z_n\psi_n$, $z_p\psi_p$, and $z_q\psi_q$ interact both directly (this direct interaction is described by the coefficients a_{npq}, which are symmetrical in their three indices), and also via all the remaining components. Such a structure of the pressure-field variation as the result of direct interactions between the triads of the eigenvalues is the consequence of the quadratic form of the nonlinearity in the hydromechanics equations. In practice it is necessary to confine oneself to only a finite number of "parameters" z_n. The equation for z_0 then describes the "barotropic" changes of the pressure. If we put $z_0\psi_0 = z^*$, then this equation takes the form

$$\mathcal{F}\frac{\partial z^*}{\partial t} = -\frac{g}{l}\left\{\frac{\psi_0}{N_0^2}\int[z, \mathcal{F}z]\,\psi_0\,dp + \frac{\partial l}{\partial y}\frac{\partial z^*}{\partial x}\right\}. \tag{10.6'}$$

As $\alpha_0^2 \to 0$, it goes over into Eq. (7.6) of the barotropic model. For the "baroclinic" component $z' = z - z^*$ we obtain the equation

$$\mathcal{F}\frac{\partial z'}{\partial t} = -\frac{g}{l}\left\{[z, \mathcal{F}z] - \frac{\psi_0}{N_0^2}\int[z, \mathcal{F}z]\,\psi_0\,dp + \frac{\partial l}{\partial y}\frac{\partial z'}{\partial x}\right\}. \tag{10.6''}$$

From the system of two equations (10.6) and (10.6′) we can determine both components z* and z′ of the variation of the pressure field, and by the same token estimate directly the role of the baroclinic effects. One might think that this role will be the largest in the frontal zones (between the air masses with different properties), and outside these zones the main fraction of the changes of the pressure will be accounted for by the "barotropic" component z*).

11. WEATHER FORECASTING

In the quasigeostrophic approximation, we deal with Eqs. (7.4) or (7.6), which describe the synoptic oscillations of the field of the heights of the isobaric surfaces z(x, y, p, t), i.e., of the pressure field in the atmosphere (in the quasisolenoidal approximation the field of the current function ψ is added also to the field z in Eqs. (8.2)–(8.3)). This naturally raises the question of the extent to which it is possible to judge from the predicted changes of the atmospheric-pressure field (which themselves can hardly be sensed by humans), those weather features to which humans are very sensitive, i.e., primarily the air temperature, the wind, and finally the variable cloudiness and precipitation, which are the main characteristics of the weather in our planet. This question can be answered with sufficient optimism.

1) Indeed, knowledge of the field z(x, y, p, t) permits, first, to calculate the air temperature T by means of the formula $T = -(gp/R)\partial z/\partial p$ (given in the footnote of p. 754).

It is important here that only the synoptic oscillations of the field z are predicted, which have rather large spatial and temporal scales, and that they are predicted only in the adiabatic approximation. Therefore one can calculate from the predicted field z only the smoothed temperature field—the "synoptic" temperature background, on which there are superimposed in nature small-scale oscillations and the diurnal variation produced by non-adiabatic factors, for example in the boundary layer of the atmosphere at the underlying surface. The forecasting of the daily course of the surface temperature can be carried out separately, as is indeed done in forecasting practice.

2) Second, from the field z it is possible to calculate the horizontal components u and v of the wind field at any level, using in the quasigeostrophic approximation formulas (7.1) (in the quasisolenoidal approximation, they are determined from the current function ψ, which is calculated simultaneously with z). Knowledge of the force and direction of the wind at different altitudes is very important, for example, in aviation.

3) Third, it is possible to calculate from the field z the field of the quantity $w^* = dp/dt$, which in coordinates (x, y, p) replaces the vertical velocity (from w^* we can determine the divergence $D = -\partial w^*/\partial p$). Namely, we can obtain for w^* a diagnostic equation (i.e., containing no derivatives with respect to time), by differentiating the balance equation (8.3) with respect to p and with respect to t, and then eliminating from it the derivative $\partial\psi/\partial t$ with the aid of the equation for the transfer of the potential vorticity (8.2), and the derivative $\partial z/\partial t$ with the aid of the adiabaticity equation (7.2). We then obtain for w^* an elliptic equation with a right-hand side

$$\frac{\alpha_0^2 c_0^2}{p^2}\Delta w^* + \mathcal{A} l \frac{\partial^2 w^*}{\partial p^2} = -g\Delta\left[\psi, \frac{\partial z}{\partial p}\right] + \mathcal{A}\frac{\partial}{\partial p}[\psi, \Delta\psi + l], \quad (11.1)$$

where $\mathcal{A}$ is the operator $(\nabla \cdot l\nabla)\Delta^{-1}$ (we have left out here small terms resulting from the term $2[\partial\psi/\partial x, \partial\psi/\partial y]$ in the balance equation). In the quasigeostrophic approximation, it is sufficient to replace in (11.1) the current function ψ by gz/l. In this approximation (and at $l \approx$ const, when $\mathcal{A} \equiv l$), examples of the construction of the field w^* from the field z by solving Eq. (11.1) were given, for example, by Knighting[99]. He used actual data on the values of the field z at nine levels p = 1000, 900, 800, ..., 200 mb on a grid of 480 points spaced approximately 100 miles apart, covering a considerable portion of the northern Atlantic and western Europe, and solved the difference analog of Eq. (11.1) under zero boundary conditions for w^* on the boundaries of the region under consideration (including the lower limit p = 1000 mb and the upper limit p = 200 mb).

One of Knighting's examples (for 01^h of 2 December 1958) is shown in Fig. 14, where the profile of $w^*(p)$ (in mb/hr) is given in each of the 16×12 internal points of the horizontal grid (negative w^*, corresponding to rising air motion, are plotted to the right, and positive w^*, corresponding to descending motions, to the left of the origin). The solid lines in Fig. 14 show the isolines of the altitudes z (in meters) of the isobaric surface p = 1000 mb, and the dashed lines show the atmospheric fronts. These and other examples demonstrate that the profiles $w^*(p)$ calculated from (11.1) vary sufficiently smoothly and regularly from point to point, and agree well with the customary notions of the synopticians, for example, concerning the rise of the air ahead of baric troughs and the drop of the air behind them. If the values of w^* are large, then their maximum (and therefore also the zero value of the divergence $D = -\partial w^*/\partial p$) is reached at an average level in the troposphere near 500–600 mb; the profiles $w^*(p)$ have on the average an approximately parabolic form.

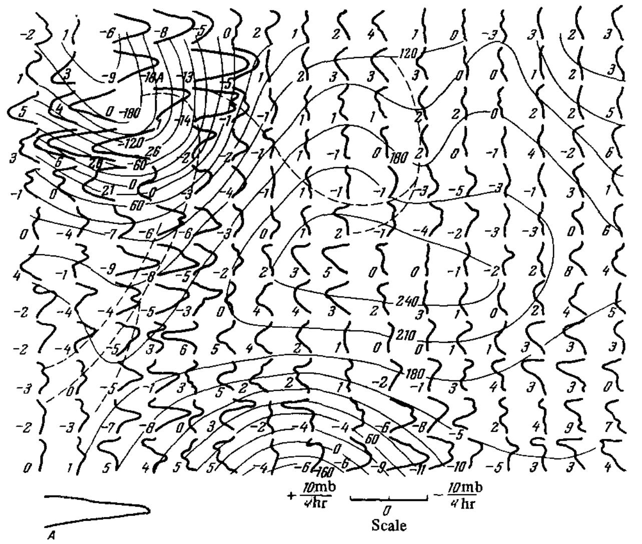

FIG. 14. Profiles of $w^*(p)$ in the layer 1000 – 200 mb and the isohypses of the heights of the isobaric surface 1000 mb over the Northern Atlantic and Western Europe at 01^h on 2 December 1958 (from [99]).

Incidentally, it must be borne in mind that compared with the pressure field the horizontal wind field, the field of the vertical velocity w (or $w^* = pg/RT[d_h z/dt) - w)]$ is much more sensitive to factors which we have so far not taken into account, and primarily to the curvature of the earth's surface and to nonadiabatic factors. The former can be taken into account by specifying the shape of the earth's surface by the functions $z = \zeta(x, y)$, so that in the case of smooth flow over the relief we get $w = u(\partial\zeta/\partial x) + v(\partial\zeta/\partial y) \approx [\psi, \zeta]$ (this condition must be written for $z = \zeta$, but at small ζ it can be approximately referred to the level z = 0 or $p = p_0$).

From among the non-adiabatic factors, foremost is the friction against the earth's surface, characterized by a horizontal vector τ of the stress of the friction against the earth's surface; when this friction is taken into account, the value of w on the upper boundary of the friction layer (or approximately again at $p = p_0$) can be determined by the formula $w = 1/l\rho \cdot |\mathrm{curl}\ \tau|$; when the friction layer is described by the so-called Eckman model, with a turbulent-viscosity coefficient $\mathcal{K}$ which is constant in altitude, we get $w = g/l\sqrt{\mathcal{K}/2l}\,\Delta z$.

Both indicated effects—the presence of the relief and the friction against the earth—should be taken into account in the boundary condition for (11.1), which should be written in the form

$$w^* = \frac{p_0 g}{RT_0}\left(\frac{d_h z}{dt} - [\psi, \zeta] - \frac{g}{l}\sqrt{\frac{\mathscr{K}}{2l}}\Delta z\right) \text{ for } p = p_0. \quad (11.2)$$

We can then calculate with the aid of (11.1) and (11.2) the field w* from the specified fields z and ψ. In a more complete formulation of the problem, it is necessary to take into account the influence of the relief and of the friction on the fields z and ψ, using the boundary condition (11.2) when solving the quasigeostrophic-approximation for it (7.4) (which introduces additional terms in the right side of formula (7.6) or of the equations of the quasisolenoidal approximation (8.2)—(8.3)). Incidentally, it is necessary here to take into account also the remaining non-adiabatic effects, among which the release of the latent heat of the condensation in the clouds is particularly important for the field.

4) Knowledge of the field ψ (which equals gz/l in the quasigeostrophic approximation) and of the field w* makes it possible to calculate the displacements of various impurities in the atmosphere. In the case of conservative impurities, it is possible to use for this purpose the transport equation in the form

$$\frac{dq}{dt} = \mathscr{D}\{q\}, \quad (11.3)$$

where q is the specific concentration of the impurity (i.e., the ratio of its mass in an elementary volume of the air to the total mass of the air with the impurity in this volume), with

$$\left.\begin{aligned} \frac{dq}{dt} &= \frac{\partial q}{\partial t} + [\psi, q] + w^* \frac{\partial q}{\partial p}, \\ \mathscr{D}\{q\} &= -\frac{1}{\rho}\operatorname{div}\mathbf{Q}, \end{aligned}\right\} \quad (11.3')$$

where **Q** is the density of the impurity diffusion flux produced principally by turbulent diffusion, which is usually assumed to depend linearly on the gradient ∇q of the field q. Incidentally, in the free atmosphere (i.e., above the planetary boundary layer of the atmosphere), during the course of time intervals which are not too large, the turbulent diffusion of the impurity (i.e., the right side of (11.3)) is frequently neglected.

If the water vapor in the air is in the unsaturated state, then it is a conservative admixture, and Eq. (11.3) is suitable for a description of the evolution of the field of the specific humidity q (in this case, incidentally, knowledge of w* makes it possible to estimate the adiabatic cooling of the rising air particles, and the heating of the descending air particles, with the aid of the simple formula $dT/dt = [(\kappa - 1)/\kappa]T/pw^*$). It is sometimes more convenient to use for the description of the humidity of the air not q but the so-called dew point T_m—the temperature at which the air with a fixed specific humidity q and a pressure p becomes saturated (over a plane surface of water). The dew point is determined from the relation $q = R/R_v \cdot e_m(T_m)/p$, where R and R_v are the gas constants of dry air and water vapor, and $e = e_m(T)$ is the partial pressure of the saturated water vapor. Substituting this formula for q in (11.3) (neglecting its right-hand side) and using for $e_m(T)$ the Clausius-Clapeyron equation (4.4), we obtain for the deficit of the dew point $\Delta = T - T_m$ the equation

$$\frac{d\Delta}{dt} = \frac{\varkappa - 1}{\varkappa}\frac{T}{p}\left(1 - \frac{\varkappa}{\varkappa - 1}\frac{R_v}{\mathscr{L}}\frac{T_m^2}{T}\right) w^*. \quad (11.4)$$

Lewis[100] proposed a very simple method for predicting the amount of cloudiness and the presence of precipitation, based on the use of the empirical connection between these phenomena and the values of w* and Δ, which can be calculated with the aid of Eqs. (11.1) and (11.4). Such an empirical connection is shown in Fig. 15, where the abscissas show the values of Δ at the level 700 mb, and the ordinates show the values of w in the central troposphere (approximately $w^* \approx -\rho g w$); different symbols on the nomogram represent the actually observed weather phenomena (in the period 15—28 March 1960 in Japan, after[101]). Analogous methods of forecasting the cloudiness and the precipitation were developed or used by Shvets[102], Dushkin, Lomonosov, and Lunin[103], Vederman[104], Ovsyannikov[105-106], Kuznetsov[107], Pedersen[108], Uspenskiĭ[109], Bagrov[110], and many other authors.

Methods of predicting cloudiness and precipitation from their empirical connected with the values of Δ and w* (or with the relative humidity at some level in the troposphere) are, of course, very crude. Thus, according to Antonov[111], the probabilities of precipitation, total cloudiness, or, to the contrary, of clear weather when the point (Δ, w*) falls in the corresponding region on a nomogram such as in Fig. 15, amounts to 70—80%, and for partial cloudiness only approximately 50%. Obviously, it is necessary to develop another forecasting procedure, which would start from a concrete description of the physical processes of cloud and precipitation formation. The water vapor can no longer be regarded as a conservative and passive impurity, as was done outside the clouds when using Eq. (11.2); when clouds appear, it is necessary to take into account the phase transitions of the moisture, the non-adiabatic dynamic effects of the release or absorption of the latent heat of the phase transitions, and also the dropping out of the moisture with the precipitation.

One such scheme was developed in a series of papers by Matveev[112-116] (see also Chap. 21, Sec. 3 of his book[117] and the papers of Lushev and Matveev[118] with examples of forecasting of large-scale cloudiness, and of Bychkova and Matveev[119] with a description of a numerical experiment on the evolution of cloudiness in a cyclone, and of Feĭgel'son and Frolova[120] with certain methodological improvements). In this scheme there

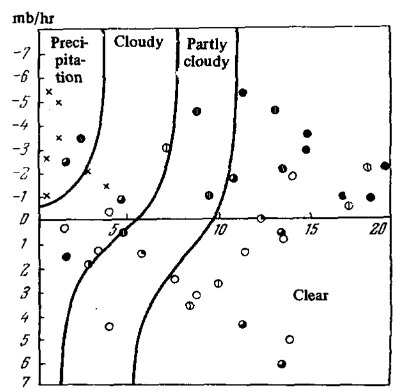

FIG. 15. Empirical connection between the amount of large-scale cloudiness and the presence of precipitation with the values of the dew-point deficit Δ at the level 700 mb and the velocity w of the large-scale motions in the central troposphere (from [100]). Actually observed weather phenomena – from [101] (shaded circles – cloud contents, cross – precipitation).

are considered two phases of moisture in the clouds—water vapor with a specific concentration q, and water droplets (plus ice) with specific concentration q_w (called the specific water content), and the following equations are assumed for them

$$\left.\begin{aligned}\frac{dq}{dt} &= \mathscr{D}\{q\} - m,\\ \frac{dq_w}{dt} &= \mathscr{D}\{q_w\} + m - n,\end{aligned}\right\} \tag{11.5}$$

where m is the specific rate of condensation (plus sublimation) of the water vapor (i.e., the mass of the water vapor condensed or sublimated per unit mass of air in a unit time), and $n = (1/\rho)(\partial Q_w/\partial z)$ is the rate of precipitation, where Q_w is the density of the flux of the mass of the water drops and ice crystals, produced by their gravitational settling; the latter can be represented in the form $Q_w = -\rho_w \tilde{w}$, where $\tilde{w}$ is the average rate of the precipitation, weighted with weights $r^3 f(r)$, with f(r) the probability density for the radii r of the cloud elements, which in this scheme is assumed to be known, for example, logarithmically normal with parameters connected by some empirical formulas with q_w. To Eqs. (11.5) one adds the equation for the influx of heat, which is conveniently written here in the form

$$\frac{dT}{dt} - \frac{\varkappa-1}{\varkappa}\frac{T}{p}w^* = \mathscr{D}\{T\} + \frac{\varepsilon}{c_p}, \tag{11.6}$$

where $\epsilon = \epsilon_r + \epsilon_q$ is the rate of heat influx (per unit mass) produced by the radiant heat exchange (ϵ_r) and by the phase transitions of the moisture (ϵ_q), while $\mathscr{D}\{T\} = -1/c_p\rho \cdot \operatorname{div} \mathbf{Q}_T$, where $\mathbf{Q}_T$ is the density of the turbulent heat flow.

According to Zilitinkevich and Laĭkhtman[121], the vertical component of the turbulence of the flow of water vapor in clouds can be written in the form $Q_z = -\rho K(\partial q/\partial z + \beta)$, where $\beta = c_p/\mathscr{L}\,(\gamma_a - \gamma_w)$ is the equilibrium gradient of the humidity, K is the coefficient of turbulent diffusion, $\gamma_a = [(\kappa-1)/\kappa](g/R)$ is the adiabatic temperature gradient, and γ_w is the so-called wet-adiabatic temperature gradient, defined by the formula

$$\gamma_B = \gamma_a \frac{1 + \frac{\mathscr{L}}{R}\frac{q_m}{T}}{1 + \frac{\mathscr{L}}{c_p}\frac{\partial q_m}{\partial T}}. \tag{11.7}$$

The vertical component of the turbulent heat flow should be written in this case in the form $Q_{Tz} = -c_p\rho\mathscr{K}(\partial T/\partial z + \gamma_w)$.

The influx of heat ϵ_q in the atmosphere differs from zero only in clouds, where phase transformations of the moisture takes place; there it is equal to $\mathscr{L}m$, where L is the latent heat of evaporation (or the latent heat of sublimation, which is close to it in magnitude). Matveev's scheme is based on the fact that equations for the "equivalent temperature" $\Pi = T + (\mathscr{L}/c_p)q$ and the "total specific moisture content" $\tilde{q} = q + q_m$, derived (under certain simplifications) from (11.5)—(11.6), do not contain m and have the same form in clouds and outside the clouds. Finding Π and $\tilde{q}$ from these equations, we can obtain from the relation

$$\Pi = T + \frac{\mathscr{L}Re_m(T)}{c_p R_v p}$$

the value of T and define clouds as regions in which the difference $\tilde{q} - (Re_m(T)/R_v p)$ is positive; then this quantity is the specific humidity q_w). To avoid an unreliable numerical calculation of this difference, it is proposed in[120] to find q_w from the second equation of (11.5), in which m is defined by the formula

$$m = \left(1 + \frac{\mathscr{L}}{c_p}\frac{\partial q_m}{\partial T}\right)^{-1} \times \left[w^*\frac{q_m}{p}\left(1 - \frac{\varkappa-1}{\varkappa}\frac{T}{q_m}\frac{\partial q_m}{\partial T}\right) + \mathscr{D}\{q_m\} - \frac{\partial q_m}{\partial T}\mathscr{D}\{T\} - \frac{\varepsilon_r}{c_p}\frac{\partial q_m}{\partial T}\right], \tag{11.8}$$

which is obtained after eliminating dT/dt with the aid of (11.6) from the first equation of (11.5), written out for a cloud, i.e., at $q = q_m(T, p)$. A formula of this type was apparently first derived for m by Shvets[122]. In the construction of concrete physico-mathematical models of clouds, a similar formula for m was used by Lebedev[123-124].

A more detailed scheme is proposed in the article by Marchuk[125] (see also his book[10]), where, besides q and q_w, he introduces separately the specific concentration of the ice crystals q_{ic}, the phase transitions of the moisture are described by the terms $\sum_i \alpha_{ij}q_i$ in the expressions for dq_j/dt (with the coefficients α_{ij}, which satisfy the condition $\sum_i \alpha_{ij} = 0$, being specified by certain semi-empirical formulas), and the precipitation in the liquid and solid phases is described by very simple empirical formulas. Even more detailed schemes include the calculation of the characteristics of the microstructure of the cloud—primarily the already-mentioned probability density f(r) for the radii r of the cloud elements (which, generally speaking, depends on the spatial coordinates and on the time). By the same token, it is necessary to synthesize the dynamics and microphysics of the clouds.

The microphysics of the clouds was actively developed in the post-war years, principally in connection with experiments on artificial scattering of clouds (e.g., by seeding them with crystals of dry ice, silver iodide, or other coagulants), and reached considerable progress (a review of which, e.g., is found in Fletcher's monograph[126]). Whereas experiments on raid production still do not yield reliable results, the dispersal (albeit temporary) of certain types of clouds has already been attained with good reliability (the failure to use the developed methods, e.g., on the part of large airports, is apparently only manifestation of organizational inertia). At the same time, the microphysics of clouds has not been sufficiently well oriented towards the problem of weather forecasting, and so far only very little research has been done in this direction. By way of examples, we mention papers by Buĭkov[127] and Shulepov and Buĭkov[128-129], in which it is assumed that $m = 4\pi N\tilde{r}\chi(q - q_m)$ (where N and $\tilde{r}$ are the average number of cloud elements per unit volume and their average radius, and χ is the diffusion coefficient of water vapor), and the following kinetic equation is used for the probability density f(r)

$$\frac{df}{dt} - ar^2\frac{\partial f}{\partial z} + \chi\frac{q - q_m}{q_w}\frac{\partial}{\partial r}\frac{f}{r} = \mathscr{D}\{f\}, \tag{11.9}$$

where ar^2 is the velocity of descent of a water drop in stationary air.

5) The synthesis of dynamics and microphysics of the clouds is still a matter for the future. Another still unsolved problem is the forecasting of convective clouds and the precipitation from them: individual convective

clouds turn out to be in their spatial scales not synoptic but mesometeorological phenomena, so that the theory of Ch. 7–8 is not applicable for their description. At the same time, the amount of precipitation from convective clouds is comparable with the amount of precipitation from large-scale cloud systems, and consequently should be taken into account in the description of synoptic processes. So far, attempts are being made to obtain empirical connections of convective cloudiness and precipitation with the large-scale synoptic background; it is possible that the establishment of such connections will be aided by the physical and mathematical models of convective cells (of the type constructed by Lebedev[124]).

In searches for these connections, apparently, it will be necessary to distinguish between free cumulus convection, observed for example in moderate latitudes over areas occupied by cold air masses (intramass showers and thunderstorms, determined primarily by the humidity field and by the energy of the instability of the lower troposphere, but probably not strongly connected with the large-scale vertical velocity w*, which, incidentally, corresponds to settling of the air in the anticyclone regions, typical of summer convection and induced cumulus convection, observed on the lines of horizontal convergence (on the intratropical line of convergence and on cold fronts in middle latitudes), and at convergence points (tropical cyclones), and determined primarily by the field of the humidity and by the magnitude of the divergence $\mathscr{D}$ (or the corresponding vertical velocity w*). According to Charney's idea, cumulus convection itself produces, to some degree, the horizontal convection that causes it, so that we have here a bilateral interaction between large-scale and meso-scale processes.

The theory of Ch. 7–8, based on the adiabatic approximation and on "filtered-out" equations, is insufficient for the forecasting not only of such mesometeorological phenomena as convective clouds, but also phenomena with scales on the borderline between mesometeorological and synoptic regions, such as tropical cyclones—hurricanes or typhoons. Tropical cyclones are the most intense weather phenomena on earth (see, e.g., the article by Riehl[130] and the books by Riehl[131] and Tiron[132]). Thus, in their centers one observes the lowest air pressure at sea level—the three-times record is 890 mb (the normal is 1013 mb), and the rate of decrease of the pressure when the hurricane approaches sometimes reaches 40 mb/20 min (see the example in Fig. 16), while the horizontal pressure drops frequently exceed 50 mb/50 miles—the wind velocity reaches in this case 100 m/sec. In the centers of the hurricanes one observes "the eye of the storm"—a quiet region (of course, not including the sea waviness) and partial and full clearing of the sky (and consequently, descending air motions) with an average diameter of 30 km. The trajectory of hurricanes in the northern hemisphere is almost always directed from the equator into the middle latitudes, and many hurricanes first move to the northwest and then veer to the northeast (Fig. 17). Much information, in many respects unexpected, concerning the structure of the hurricanes has been obtained recently by heroic experiments in which the "eye of the storm" and its vicinity have been sounded by airplanes (see,

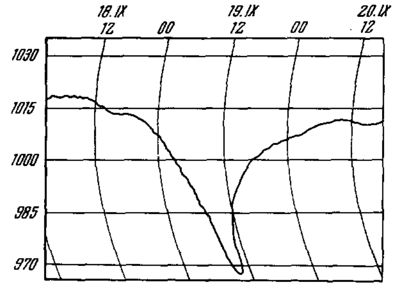

FIG. 16. Barogram of the passage of the hurricane of 18-20 September 1947 in New Orleans, La. (from [131]).

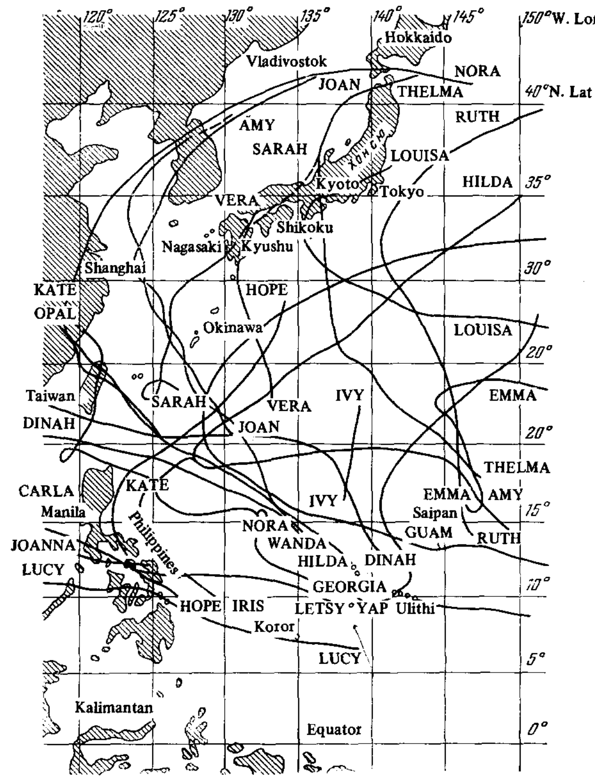

FIG. 17. Paths of typhoons in the western part of the Pacific in the 1962 season.

e.g.,[133]) and by photography from rockets and satellites. For example, Fett[134] reports observation along the boundary of the cloud system of a hurricane a narrow cloudless zone of settling with an exceedingly low humidity (dew-point deficit Δ = 15–20°), in front of which there is an external zone of intense convection with thick cumulus clouds; over the settling zone, at a level of approximately 200 mb, there is a strong jet flow, which envelopes the hurricane anticyclonically from the north, and then splits into two branches.

Synoptic methods and for the time being also hydrodynamic methods, do not offer a complete explanation or an accurate forecast of the motions and evolution of hurricanes; in particular, the sharp turns in the motion of hurricanes have not yet been physically explained. According to Hill[135], the daily forecasts of the positions of the centers of hurricanes, made up by the USA Weather Bureau, which are so far regarded as the best, have an average error exceeding 200 km, whereas only forecasts with an error not exceeding 80 km can be regarded as satisfactory. Apparently, the presently employed simplified hydrodynamic equations do not take into account many factors which are important for the evolution of hurricanes—the release of heat of condensation, friction, and possibly vertical acceleration.

Incidentally, some published physical-mathematical models of typhoons gave hopeful results. Thus, for example, Estoque[136] describes calculations by means of two models—quasigradient and one based on primitive equations, in which account is taken of the vertical and horizontal turbulent exchange and the release of latent heat of condensation in descending motions, and the nonstationary axially-symmetrical problem with time intervals of 90 and 15 sec (!) is solved. The results of the calculations duplicate the descent of the air in the "eye of the storm" and the rise at the "wall of the eye," the inflow of air at the lower levels and the outflow at the higher levels.

Morikawa[137-138] constructed a hurricane model in which, taking into account its small (mesometeorological) scales, the hurricane is treated as a pointlike vortex, interacting with the "leading stream." Namely, he proposed to use for the description of the hurricane the solution of the quasigeostrophic barotropic equation (7.6′) in the form $z = l/g \cdot (\psi_0 + \psi_1)$, where

$$\psi_0 = \frac{\gamma}{2\pi} K_0 \left[\frac{|\mathbf{r} - \mathbf{r}_0(t)|}{L_0}\right]$$

($l\psi_0/g$ is the solution of Eq. (7.6′) corresponding to a pointlike vortex at the point $\mathbf{r}_0(t) = \{x_0(t), y_0(t)\}$), and the trajectory of the vortex is determined by the relation

$$\frac{dx_0}{dt} = -\frac{\partial\psi_1}{\partial y}\bigg|_{\mathbf{r}_0}, \quad \frac{dy_0}{dt} = \frac{\partial\psi_1}{\partial x}\bigg|_{\mathbf{r}_0}.$$

This solution was applied successfully to describe the motion of the hurricane "Betsy" on 14–17 August 1956; in addition, a model was calculated with a quasiuniform leading stream

$$\psi_1 = UL_0(e^{-\frac{y}{L_0}} - 1) \approx Uy.$$

The approximate representation of the continuous vorticity field (or better, the potential vorticity) in the cited papers of Morikawa by a finite number of pointlike vortices, the motion of which is described by ordinary differential equations, is analogous to replacing the continuous distribution of a mass by a discrete set of pointlike masses, and is a method of approximately describing continuous fields, capable of competing with the use of expansion in orthogonal functions (Ch. 10) or discrete spatial grids. In hydrodynamics, such a procedure was used for example by Onsager[139], Fermi, Pasta, and Ulam[140], Pasta and Ulam[141], and Ulam[142]. For meteorology problems, it was publicized by Charney[143].

Summarizing, it can be admitted that the hydrodynamic theory is already capable, to some degree, of coping with short-range forecasts not only of the pressure field, but also of such singularities of the weather as the temperature and the wind, and makes important steps towards forecasting cloudiness and precipitation. However, announcements by weather services that they are already using the hydrodynamic theory of short-range weather forecasts in practice can be accepted, of course, only when objective hydrodynamic forecasts are compiled not together with the subjective synoptic forecasts, but in place of them.

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