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KINETIC THEORY OF DRIFT-DISSIPATIVE INSTABILITIES OF A PLASMA

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1. INTRODUCTION

THE theory of oscillations and stability of a spatially-inhomogeneous plasma contained by an external magnetic field has been extensively developed in recent papers. Interest in this theory is due to the efforts to solve the problem of controlled thermonuclear fusion, persistent efforts to realize phenomena occurring in the ionospheric and interplanetary plasma, and finally research in an old "classical" branch of physics, namely gas-discharge physics. Owing to the inhomogeneous plasma contained by a magnetic field, and also owing to the curvature of the magnetic-field force lines, different drift motions are possible in the plasma. The latter lead to the occurrence of instabilities in the plasma, called drift instabilities. The frequencies and growth increments of the unstable drift oscillations lie in the range

$$\omega \lesssim \omega_{dr} \sim \frac{k_y v_T^2}{\Omega L_0} \gtrsim \frac{v_T^2}{\Omega L_0^2},$$

where $v_t = \sqrt{T/m}$ is the thermal velocity of the particles, $\Omega = eB_0/mc$—their gyroscopic frequency, and L_0—the characteristic dimension of the plasma inhomogeneity. Under conditions when $\omega_{dr} \gg \nu_\alpha$, where ν_α (α = e, i)—effective collision frequencies of the charged particles (electrons and ions), or when the wavelengths of the drift oscillations are smaller than their mean free path, $k_z v_{T\alpha} \gg \nu_\alpha$, it is possible to neglect particle collisions in first approximation in the investigation of the drift instability of an inhomogeneous plasma. A survey of the results of the theory of drift instability of a collisionless plasma was presented in an article by the authors [1] (see also [2-15]). A number of instabilities are connected with Cerenkov emission and absorption of plasma oscillations by plasma particles. In this sense, we can speak of a dissipative effect leading to a growth of the plasma oscillations.

The subsequent development of the theory of stability of magnetic plasma containment followed the path of the study of the influence of other dissipative effects on the stability. A large number of investigations have been devoted to a study of the role of plasma-particle collisions [16-28]. It can be said, that the theory of plasma stability has been subdivided here into two parts. The first is connected with the need for a consistent utilization of kinetic theory and the solution of the kinetic equations, in which it actually becomes necessary to consider in detail the particle collision integrals. To the contrary, the second part does not require the kinetic theory, and all its results can be obtained with the aid of the hydrodynamics of two fluids [29] (electrons and ions). It is obvious that the second part is simpler. However, it is only in the initial stage of development (see, for example [30,31]). In particular, we shall therefore attempt insofar as possible, not to touch upon questions of hydrodynamic theory of plasma stability in the present article*. To the contrary, we shall focus our attention on the kinetic theory of plasma stability. We shall make an attempt below to create a definite summary of the theory of the drift-dissipative instability of a fully ionized plasma, taking into account the Coulomb collisions of the particles in the plasma. Such a theory is not at all of academic interest. The point is that even in a thermonuclear plasma ($N \sim 10^{14}$–10^{15} cm^{-3}, $T \sim 1$–10–keV, $L_0 \sim 10$ cm, $B_0 \sim 10^4$–10^5 Oe) the inequalities for the most dangerous fundamental modes of the drift oscillations are $\nu_e \sim 10^4$–10^5 sec$^{-1} \gtrsim \omega_{dr} \sim v_T^2/\Omega L_0^2 \sim 10^3$–$10^4$ sec$^{-1} \gtrsim \nu_i \sim 10^2$–$10^{-3}$ sec^{-1}. As a rule, a similar relation is satisfied for the frequency of the drift oscillations in a discharge plasma, and also in ionospheric and interplanetary plasma. The construction of a theory of drift oscillations, as well as of a theory of stability of magnetic containment of the plasma under such conditions, is possible only on the basis of kinetic equations that take consistent account of the particle collisions.

In a series of papers [22-27], the kinetic theory of the stability of magnetic containment of a plasma was based on the kinetic equation with a Bathnagar-Gross-Krook (BGK) model collision integral [32]. However, as shown by Pitaevskiĭ [33,34], such a collision integral, as applied to a fully ionized plasma leads to results that differ qualitatively from the true ones. Thus, he has shown that in the case of short-wave oscillations with a wavelength shorter than the gyroscopic radius of the particles, the effective collision frequency obtained with the aid of the Landau collision integral [35] differs from that obtained with the aid of the BGK model integral by the square of the ratio of the gyroscopic radius to the wavelength of the oscillation across the magnetic field. This result was used by Kadomtsev and Pogutse [28]. However, in general it does not suffice to replace the frequency of the ion-ion collisions by such an effective frequency. The authors of the present article [36] (see also [37]) have shown that for the investigations of oscillations and stability of a plasma with an inhomogeneous temperature it is necessary to use the exact Boltzmann or Landau collision integrals, for otherwise

* Description of the oscillations with the aid of hydrodynamics is possible only for frequencies ω that are much smaller than the ion and electron effective collision frequencies υ_i and υ_e. In [16-21] where a theory of plasma oscillations was constructed under conditions $\upsilon_e > \omega > \upsilon_i$, use was made of approximations based on modifications of the equations of two-fluid hydrodynamics. Such an approach can allow only a qualitative description, suitable within relatively narrow limits which can be established only with the aid of the kinetic method. Bearing in mind the latter remark, and also the fact that the consistency and greater degree of development of the kinetic approach not only yields more reliable concrete results, but also yields them more rapidly, we shall not employ semi-quantitative methods anywhere.

it is impossible to observe the essential qualitative effects that appear in both short-wave and long-wave oscillations. In that short paper, the authors presented several results of the kinetic theory of oscillations of a weakly-inhomogeneous plasma with allowance for the Coulomb collisions of the particles, making it possible to draw a number of conclusions concerning the stability of plasma containment by a magnetic field with straight force lines. Simple approximate methods for solving the kinetic equation, which are treated in appendices to the present article were used. In some of the solutions it is actually shown that integration of the kinetic equation with an exact collision integral does not make the theory more complicated than when the 3GK method is used. Following publication of[36,37] it became clear which of the results obtained with the aid of the BGK model integral were qualitatively correct and which were in error. In addition, these papers have made it possible to determine the limits of applicability of the different hydrodynamic approximate methods used in[16-21] to describe drift oscillations of an inhomogeneous plasma in the kinetic region of frequencies (i.e., when $\omega > \nu_i$), used in[16-21].

We develop below a kinetic theory of drift-dissipative instabilities of magnetic containment of a low pressure plasma, when the thermal motion of the plasma is negligibly small compared with the pressure of the containing magnetic field. We take into account here both the curvature and the shear of the magnetic force lines. To take into account the curvature of the force lines we introduce an effective gravity field g, the vector of which is oriented along the inhomogeneity of the plasma (along the x axis). In order of magnitude

$$|g| \approx \frac{v^2_{Ti} + v_s^2}{R},$$

where $v_s = \sqrt{T_e/M}$—velocity of the long-wave ion sound and R is the curvature of the magnetic-field force.*

The instability conditions are analyzed within the framework of the method of geometrical optics. As is well known[1], the concept of the dielectric constant of a weakly-inhomogeneous plasma is productive in this case. Expressions for such dielectric constants are given in Sec. 2 for a number of limiting cases; these expressions are necessary for the study of the influence of Coulomb collisions of charged particles on the plasma drift oscillation. In Secs. 3 and 4 we present the results of the investigation of low-frequency and respectively long-wave and short-wave oscillations, and in Sec. 5 we study the spectra of the drift-cyclotron oscillations of an inhomogeneous plasma. We determine the frequencies and increments of the growing oscillations, and discuss the regions of their localization. We clarify the possibility of stabilizing drift-dissipative instabilities of the plasma by means of shear of the force lines of the magnetic field, and indicate criteria for such a stabilization. Attention is paid to presently fashionable minimum-B systems[38-40]*.

*The introduction of this gravity-field effect to take into account the curvature of the magnetic-field force lines can be explained as follows. The particle moving along the force lines of the curved magnetic field with thermal velocity v_T is acted upon by a centrifugal force (directed along the principal normal to the force lines) equal to mv_T^2/R. This force causes in turn a centrifugal drift of the particle with velocity $u = v_T^2/R\Omega$. The velocity of the relative drift of the electrons and ions is then equal to $u_{rel} = u_e - u_i = -(v_{Ti}^2 + v_s^2)/R\Omega_i$. It is easy to see that the gravity field introduced by us $|g| = (v_{Ti}^2 + v_s^2)/R$ leads to the same relative particle drift. It should be noted that such an allowance for the curvature of the magnetic-field force lines is, strictly speaking, correct only for oscillations with $\omega \gg ku_{rel}$, for in the opposite case the thermal velocity scatter of the particles becomes significant. We shall therefore assume in all that follows that this inequality is satisfied.

2. DIELECTRIC CONSTANT OF A WEAKLY INHOMOGENEOUS PLASMA

A fundamental role in the theory of stability of an inhomogeneous plasma is played by the dielectric constant. We devote this section to an exposition of the main premises underlying the kinetic theory of the dielectric constant of a weakly inhomogeneous plasma contained by a strong magnetic field, and also present the results of such a theory, with account taken of the particle collisions in the plasma.

With respect to the real situation taking place under conditions of magnetic plasma containment, we shall assume that the frequency of the gyroscopic rotation of the particles Ω is large compared with their characteristic collision frequencies. In this connection, we neglect the influence of particle collisions on the equilibrium velocity distribution. The z axis is directed along a magnetic field $\mathbf{B}_0$, and the plasma is assumed inhomogeneous in the direction of the x axis. The vector of the effective gravity field g is directed along the same axis. Then the kinetic equation for the equilibrium distribution function can be written in the form

$$v_x \frac{\partial f_0}{\partial x} + g \frac{\partial f_0}{\partial v_x} + \frac{e}{mc} [\mathbf{vB}_0] \frac{\partial f_0}{\partial \mathbf{v}} = 0. \quad (2.1)†$$

Limiting ourselves to the case presently of greatest interest, when the pressure of the magnetic field greatly exceeds the thermal pressure of the plasma $\beta = 8\pi P_0/B_0^2 \ll 1$, we can neglect the inhomogeneity of the field $\mathbf{B}_0$ when integrating (2.1). Finally, we also take into account the fact that in a real plasma the particle distribution changes little over distances on the order of the gyroscopic ion radius $\rho_i = v_{Ti}/\Omega_i$. As a result, the solution of Eq. (2.1) for the equilibrium distribution function can be written in the form

$$f_0 = \frac{N(C)}{[2\pi m T(C)]^{3/2}} \exp\left\{-\frac{m[v_x^2 + v_z^2 + (v_y - u)^2]}{2T(C)}\right\}, \quad (2.2)$$

where $u = -g/\Omega$ is the velocity of the gravitational drift of the particle in the crossed fields g and $\mathbf{B}_0$, and $C = x + (v_y - u)/\Omega$ is a characteristic of (2.1) determining the dependence of the function of Eq. (2.2) on the spatial coordinates.

To study the stability of a plasma having a particle distribution (2.2), let us consider small deviations from such distributions, accompanying small disturbances of the fields. In view of the smallness of the thermal pressure of the plasma, we can neglect the perturbation of the magnetic field and confine ourselves only to an allowance for the perturbed electric field $\delta\mathbf{E} = -\nabla\Phi$.

* In minimum-B systems, the curvature of force lines is negative, since the principal normal to the force lines is directed towards the regions of increased density. This means that $g(\partial n/\partial x) > 0$, i.e., the effective gravity field is parallel to the direction of increasing plasma density.

† $[\mathbf{vB}_0] \equiv \mathbf{N} \times \mathbf{B}_0$.

Bearing in mind that the equilibrium distribution does not depend on the y and z coordinates, we shall seek the non-equilibrium quantities in the form

$$\delta f(x) \exp\{-i\omega t + ik_y y + ik_z z\}.$$

Then the kinetic equation for the non-equilibrium addition to the distribution function can be written as follows:

$$-i(\omega - k_y u - k_y v_y - k_z v_z)\,\delta f + v_x \frac{\partial \delta f}{\partial x} - \Omega \frac{\partial \delta f}{\partial \varphi} = \frac{e}{mC} \nabla \Phi \frac{\partial f_0}{\partial \mathbf{v}} + \sum I_{\alpha\beta}. \tag{2.3}$$

Here φ—azimuthal angle in momentum space (the polar axis along the z axis), and $I_{\alpha\beta}$—linearized collision integral of charged particles of type α with particles of type β[35]:

$$I_{\alpha\beta} = 2\pi e_\alpha^2 e_\beta^2 L \frac{\partial}{\partial p_i} \int d\mathbf{p}' \frac{w^2\delta_{ij} - w_i w_j}{w^3} \times \left\{ f_{0\beta} \frac{\partial \delta f_\alpha}{\partial p_j} + \delta f_\beta \frac{\partial f_{0\alpha}}{\partial p_j} - f_{\alpha 0} \frac{\partial \delta f_\beta}{\partial p'_j} - \delta f_\alpha \frac{\partial f_{0\beta}}{\partial p'_j} \right\}, \tag{2.4}$$

where $L = \ln(r_D/r_{min})$ is the Coulomb logarithm and $\mathbf{w} = \mathbf{v}_\alpha - \mathbf{v}_\beta$. The summation in the right side of (2.3) is over all types of charged particles of the plasma. It is convenient to solve Eq. (2.3) in a coordinate system in which there is no gravitational drift of particles of a given type. In such a reference frame, the distribution function has the form (2.2) with u = 0. Confining ourselves to perturbations with a wavelength along the x direction much smaller than the characteristic dimension of the plasma inhomogeneity L_0, we can assume the dependence of the perturbed quantities on the coordinate x in the form $\exp\left(i \int_0^x k_x dx\right)$. Then, solving in the geometrical optics approximation the system of kinetic equations for electrons and ions, we obtain the charge density induced in the plasma by a potential non-equilibrium field:

$$\delta\rho = \sum e \int d\mathbf{p}\,\delta f = \frac{k^2}{4\pi}(1-\varepsilon)\Phi. \tag{2.5}$$

The longitudinal dielectric constant determined in this manner

$$\varepsilon(\omega, \mathbf{k}, x) = 1 + \delta\varepsilon_e + \delta\varepsilon_i \tag{2.6}$$

will be used in the subsequent sections to study the potential plasma oscillations. In the present section we shall subsequently write out, for a large number of limiting cases, expressions for $\delta\epsilon_e$ and $\delta\epsilon_i$, which correspond to the contributions of the electrons and ions in the dielectric constant of the plasma.

We neglect throughout the gyroscopic radius of the electrons compared with the wavelength of the investigated plasma oscillations. This causes the frequencies of the investigated drift oscillations to be regarded as lower than the gyroscopic frequency of the electrons, $\omega \ll \Omega_e$. In addition, we shall neglect completely the gravitational drift of the electrons, inasmuch as the electron drift velocity u_e is smaller by M/m times than the ion drift velocity $u_i = -g/\Omega_i$. As a result, the number of possible limiting cases for the electron contribution to the dielectric constant is relatively small.

If the longitudinal wavelength of the oscillations is small compared with the electron mean free path, as well as compared with the distance traversed by the thermal electron during the period of the considered oscillations (ω, $\nu_e \ll k_z v_{Te}$), then the collisions of the electrons can be neglected, and we can use the results of the theory of a collisionless plasma[1]:

$$\delta\varepsilon_e = \frac{1}{k^2 r_{De}^2}\left\{1 + i\sqrt{\frac{\pi}{2}} \frac{\omega}{|k_z| v_{Te}} \left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{Te}}\right)\right\}. \tag{2.7}$$

The dissipative contribution is due here to the Cerenkov effect on the electrons.

To the contrary, if the longitudinal wavelength of the oscillations greatly exceeds the mean free path of the electrons, then the Cerenkov effect can be neglected, for in this case the dissipative effects are due to electron collisions. If, in addition, the frequency of the considered oscillations greatly exceeds the collision frequency, $\omega \gg \nu_e > k_z v_{Te}$, then it is easy to obtain a solution of the kinetic equation for the electrons with the aid of expansion in powers of ν_e/ω. As a result we get[36]

$$\delta\varepsilon_e = \frac{1}{k^2 r_{De}^2}\left\{\frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln N}{\partial x} - \frac{k_z^2 v_{Te}^2}{\omega^2}\left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln NT_e}{\partial x}\right) + i \frac{\nu_{\text{eff}}}{\omega} \frac{k_z^2 v_{Te}^2}{\omega^2}\left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_e}}\right)\right\}, \tag{2.8}$$

where

$$\nu_{\text{eff}} = \frac{4}{3}\sqrt{\frac{2\pi}{m}} \frac{e^4 NL}{T_e^{3/2}}.$$

The quantity ν_{eff} characterizes the collisions of the electrons with the ions, which for simplicity are assumed to be singly charged: $e_i = -e$.

Finally, under conditions when the mean free path is smaller than either the longitudinal wavelength or the distance traversed by the thermal electron during the period of the oscillations ($\nu_e \gg \omega$, $k_z v_{Te}$), the term of largest order in the kinetic equation (2.3) turns out to be the collision integral. The solution of the kinetic equation for the electrons can then be obtained with the aid of the Chapman-Enskog method (see Appendix I). This yields

$$\delta\varepsilon_e = \frac{1}{k^2 r_{De}^2}\left\{\frac{k_y v_{Te}^2}{\omega T_e} \frac{\partial \ln N}{\partial x} + i1.96 \frac{k_z^2 v_{Te}^2}{\omega \nu_{\text{eff}}}\left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln NT_e^{1,71}}{\partial x}\right)\right\} \tag{2.9}$$

when $\omega\nu_{\text{eff}} \gg k_z^2 v_{Te}^2$ (see also[36]), and

$$\delta\varepsilon_e = \frac{1}{k^2 r_{De}^2}\left\{1 + i \cdot 1.44 \frac{\omega\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln NT_e^{-0,56}}{\partial x}\right)\right\}. \tag{2.10}$$

We now proceed to consider the ionic contribution to the dielectric constant. Allowance for the gravitational ion drift leads to the appearance in the kinetic equation for the ions of a Doppler frequency shift $\omega' = \omega - k_y u_i$. With the aid of a simple method of successive approximations we can readily obtain the solution of the kinetic equation corresponding to the limit $\omega' - s\Omega_i \gg \nu_i$, where s = 0, 1, 2, ... For the ionic contribution to the dielectric constant we obtain in this limit

$$\delta\varepsilon_i = \frac{1}{k^2 r_{Di}^2}\left\{1 - \sum_s \frac{\omega'}{\omega' - s\Omega_i}\left(1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\left[\frac{\partial \ln N}{\partial x} + \frac{\partial T_i}{\partial x} \frac{\partial}{\partial T_i}\right]\right) \times A_s(k_\perp^2 \rho_i^2) J_+\left(\frac{\omega' - s\Omega_i}{k_z v_{Ti}}\right)\right\} + \delta\varepsilon_{ii} + \delta\varepsilon_{ie}, \tag{2.11}$$

where

$$A_s(x) = e^{-x} I_s(x),$$

and

$$J_+(x) = x e^{-x^2/2} \int_{i\infty}^{x} d\tau\, e^{\tau^2/2}$$

(see[42] concerning $J_+(x)$). The first term corresponds here to the one obtained in the theory of collisionless plasma[1]. The two others are due to ion-ion and ion-electron collisions. These terms are insignificant if $\omega' - s\Omega_i \ll k_z v_{Ti}$. In the opposite case ($\omega' - s\Omega_i \gg k_z v_{Ti}$) we readily obtain simple expressions for $\delta\epsilon_{ii}$ for both long waves $k_\perp \rho_i \ll 1$:

$$\delta\varepsilon_{ii} = \frac{i}{10}\frac{\nu_{ii}}{\omega'}\frac{v_{Ti}^4}{k^2 r_{Di}^2}\left\{\left(16\frac{k_z^4}{\omega'^4} + 28\frac{k_z^2 k_\perp^2}{\omega'^2\Omega_i^2} + 7\frac{k_\perp^4}{\Omega_i^4}\right)\left(1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{d\ln N}{\partial x}\right)\right.$$
$$\left. - \left(24\frac{k_z^4}{\omega'^4} + \frac{33}{2}\frac{k_z^2 k_\perp^2}{\omega'^2\Omega_i^2} - \frac{3}{4}\frac{k_\perp^4}{\Omega_i^4}\right)\frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln T_i}{\partial x}\right\}, \qquad (2.12)$$

and short waves $k_\perp \rho_i \gg 1$ (see Appendix II and also[36]):

$$\delta\varepsilon_{ii} = i\sum_s \frac{\nu_{ii}\omega'}{(\omega' - s\Omega_i)^2}\frac{3(\pi+1)}{8\sqrt{\pi}}\frac{k_\perp \rho_i}{k^2 r_{Di}^2}\left[1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln N}{\partial x}\right.$$
$$\left. + \left(1 - \frac{3\pi+2}{4\pi+4}\frac{\partial \ln T_i}{\partial \ln N}\right)\right], \qquad (2.13)$$

where ν_{ii} is the frequency of the ion-ion collisions:

$$\nu_{ii} = \frac{4}{3}\sqrt{\frac{\pi}{M}}\frac{e^4 N L_0}{T_i^{3/2}}.$$

In obtaining formula (2.12) we also assumed that $\omega' \ll \Omega_i$, since it is only in this case that long-wave drift oscillations in the plasma are possible.

Ion-electron collisions are significant only in the long-wave region $k_\perp \rho_i \ll 1$ and under the condition $\nu_e \gg \omega$, $k_z v_{Te}$ and $\omega' \ll \Omega_i$. Then

$$\delta\varepsilon_{ie} = \frac{k_z^2 v_{Te}^2}{\omega'^2 k^2 r_{Di}^2}\left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e}\frac{\partial \ln NT_e}{\partial x}\right). \qquad (2.14)$$

In the formulas obtained above, the ion collisions were assumed to be relatively rare, as is characteristic of the kinetic (but not hydrodynamic) theory of rarefied gases. As already noted above, the solution of the kinetic equation is obtained in this case by the perturbation-theory method. A real example of the expansion, as can be seen from formulas (2.12) and (2.13), is $\nu_{ii} k_\perp^2 \rho_i^2/\omega'$ or $\nu_{ii} k_\perp^2 \rho_i^2/(\omega' - s\Omega_i)$. However, there is still one more case which calls for the use of the kinetic theory, in which this parameter is not small, namely:

$$\nu_{ii} k_\perp^2 \rho_i^2 \gg (\omega' - s\Omega_i),\ k_z v_{Ti},$$

where s = 0, 1, 2, ... This case is realized for short wavelengths ($k_\perp \rho_i \gg 1$) when[36] (see Appendix II)

$$\delta\varepsilon_i = \frac{1}{k^2 r_{Di}^2}\left\{1 + \frac{i\omega'}{\nu_{ii} k_\perp^3 \rho_i^3}\frac{2C_0}{3\pi}\left[1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln N}{\partial x}\left(1 - C_1\frac{\partial \ln T_i}{\partial \ln N}\right)\right]\right\}, \qquad (2.15)$$

where $C_0 = 0.914$ and $C_1 = 0.225$. In this region, the contribution of ion-electron collisions can be neglected.

Formulas (2.7)—(2.15) encompass all the limiting cases of the kinetic theory of drift oscillations of an inhomogeneous low-pressure plasma. Remaining outside the slope of these formulas are only the hydrodynamic oscillations satisfying the conditions $k_\perp \rho_i \ll 1$, $\Omega_\alpha \gg \nu_\alpha \gg \omega$, $k_z v_{T\alpha}$ for both electrons and ions (i.e., α = e, i). As already noted above, the hydrodynamic drift oscillations of an inhomogeneous plasma were investigated in[30,31].

3. SPECTRA OF LONG-WAVE DRIFT-DISSIPATIVE OSCILLATIONS OF A WEAKLY-INHOMOGENEOUS PLASMA

According to the method of geometrical optics, to determine the potential-field oscillation spectrum in an inhomogeneous plasma it is necessary to use the eikonal equation[1]

$$\varepsilon(\omega, \mathbf{k}, x) = 0. \qquad (3.1)$$

Substituting in this equation the previously obtained expressions for the longitudinal dielectric constant, we determine $k_x(\omega, x)$—the component of the wave vector $\mathbf{k}(\omega, x)$—as a complex function of the coordinates and of the complex frequency ω. Finally, the quasiclassical quantization rules[9,43]

$$\int_{x_\mu}^{x_\nu} d_x(\omega, x)\, dx = \pi n \qquad (3.2)$$

(where n is an integer much larger than unity) determine the spectrum of the plasma oscillations and, in particular, resolve the question of plasma instability. In formula (3.2), the integration is carried out between the complex turning points* x_μ and x_ν, in which $k_x(\omega, x) = 0$, or else between the turning point and the boundary of the plasma, on which nondissipative boundary conditions can be specified. In the case of weakly damped (or growing) oscillations, this region is called the transparency region of the plasma.

Among the possible oscillations described by geometrical optics, there may be such for which, owing to the smooth dependence of the dielectric constant on the coordinates, the correct order of magnitude of the frequencies and the correct information on the plasma stability are obtained from the so-called local spectra[8]. When the local spectra† are obtained with the aid of the eikonal equation (3.1), the frequency is determined as a complex function of the coordinates and of the real components of the wave vector. Recalling these general premises of the geometrical optics method, we proceed now to study the oscillation spectra of weakly inhomogeneous plasma.

In this section we consider long-wave drift-dissipative oscillations of a plasma, for which $k_\perp \rho_i \ll 1$ and consequently, $\omega' \ll \Omega_i$.

a) In the frequency region $|\omega + i\nu_e| \ll k_z v_{Te}$, $\omega' \gg \nu_i$, $k_z v_{Ti}$ (3.1), the substitution of expressions (2.7), (2.11) and (2.12) leads to the following spectra of the long wave drift-dissipative oscillations:

$$\omega_1 \approx -\frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln N}{\partial x},$$

$$\gamma_1 \approx \sqrt{\frac{\pi}{2}}\frac{\omega_1^2}{|k_z| v_{Te}}\left\{k^2 r_{De}^2 + \left(1 + \frac{T_i}{T_e}\frac{\partial \ln NT_i}{\partial \ln N}\right)\frac{k_\perp^2 v_s^2}{\Omega_i^2} - \frac{1}{2}\frac{\partial \ln T_e}{\partial \ln N} - \frac{g}{v_s^2 \frac{\partial \ln N}{\partial x}}\right\}$$
$$- \frac{7}{10} k_\perp^4 \rho_i^4 \left(1 + \frac{T_e}{T_i} - \frac{3}{28}\frac{\partial \ln T_i}{\partial \ln N}\right)\nu_{ii}, \qquad (3.3)$$

$$\omega_2 = \frac{k_z^2 \Omega_i}{k_y \frac{\partial \ln N}{\partial x}},$$

$$\gamma_2 = -\sqrt{\frac{\pi}{2}}\frac{\omega_2^2}{|k_z| v_{Te}}\left(1 - \frac{1}{2}\frac{\partial \ln T_e}{\partial \ln N} - \frac{g}{v_s^2 \frac{\partial \ln N}{\partial x}}\right) - \frac{8}{5}\nu_{ii}\frac{k_z^2 v_{Ti}^2}{\omega^2}, \qquad (3.4)$$

$$\omega_3^2 = -k_z^2 v_{Ti}^2 \frac{\partial \ln T_i}{\partial \ln N}, \qquad (3.5)$$

$$\omega_4^3 = -k_z^2 v_s^2 \frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}, \qquad (3.6)$$

* According to Dnestrovskiĭ and Kostomarov [43] a complex of kind II should take place here. This means that two unconnected regions, bounded by the lines Im $k_x(\omega, x) = 0$ and containing the respectively remote points of the real axis x = +∞ and x = −∞ emerge from each of the two turning points.

† Turning points may be real points of "localization" of oscillations.

which appear under a variety of conditions. The last branch, first discovered in[2] and called the drift-temperature branch, corresponds to hydrodynamically unstable oscillations in the frequency region

$$1 \ll \omega_4 \Big/ \frac{k_y v_s^2}{\Omega_i} \frac{\partial \ln N}{\partial x} \ll \frac{T_i}{T_e} \frac{\partial \ln T_i}{\partial \ln N}$$

and is possible only if

$$\frac{T_i}{T_e} \frac{\partial \ln T_i}{\partial \ln N} \gg 1.$$

The third branch[44], also corresponding to hydrodynamically unstable oscillations of an inhomogeneous plasma, exists in the region of lower frequencies, when

$$\omega_3 \ll \frac{k_y v_s^2}{\Omega_i} \frac{\partial \ln N}{\partial x};$$

It is also possible only if

$$\frac{T_i}{T_e} \frac{\partial \ln T_i}{\partial \ln N} \gg 1.$$

The second branch[2] corresponds to weakly growing oscillations ($\gamma_2 \ll \omega_2$) and exists only in nonisothermal plasma with $T_e \gg T_i$ (since $k_z v_{Ti} \ll \omega_2 \ll k_z v_s$); we see that in such oscillations the ion-ion collisions play a stabilizing role, whereas the Cerenkov effect on the electrons can lead to their buildup if $\partial \ln T_i/\partial \ln N > 2$. The situation is reversed for the first oscillation branch (when $k_\perp^2 \omega_i^2 > k_z^2 \Omega_i^2$), where the Cerenkov dissipation on the electrons at large values of

$$\frac{\partial \ln T_e}{\partial \ln N} > 2\left[k^2 r_{De}^2 + \frac{k_\perp^2 v_s^2}{\Omega_i^2}\left(1 + \frac{T_i}{T_e} \frac{\partial \ln N T_i}{\partial \ln N}\right)\right]$$

stabilizes the oscillations, and the ion-ion collisions can lead to buildup of the oscillations if

$$\frac{\partial \ln T_i}{\partial \ln N} > \frac{28}{3}\left(1 + \frac{T_e}{T_i}\right).$$

A characteristic influence is exerted by the curvature of the force lines of the magnetic field on the oscillations in question. As can be readily seen from (3.5) and (3.6), for the third and fourth branches of the oscillations the curvature of the force lines of the magnetic field does not influence the spectrum at all, and consequently it has no effect on the stability of the oscillations. For the first and second branches of the oscillations, the curvature of the force lines of the magnetic field influences the stability, and in systems with positive curvature ($g\, \partial \ln N/\partial x < 0$) in the first branch it leads to a buildup of the oscillations, while in systems with negative curvature (minimum-**B** systems) it plays a stabilizing role. From the expression for γ_1 it is seen that the influence of the curvature becomes appreciable when

$$\frac{T_i}{T_e} \gtrsim k_\perp^2 \rho_i^2 \frac{R}{L_0}.$$

The curvature of the magnetic-field force lines has the opposite effect on the second branch of the oscillations. It should be noted that the influence of the curvature of the force lines on the oscillations of the second branch can become appreciable only under conditions when the velocity of the gravitational drift of the ions is comparable with the velocity of the Larmor drift of the electrons, i.e., when $T_i/T_e \gtrsim R/L_0 \gg 1$. In this limit, however, the introduction of an effective gravity field to take into account the curvature of the force lines of the magnetic field, as already noted above, is incorrect; we can speak rigorously only of a weak influence of the curvature on the character of the plasma oscillations.

So far we have said nothing of the shear of the force lines of the magnetic field, due to the small but strongly inhomogeneous transverse component of the field $B_{0y}(x) \ll B_{0z}$. The shear of the force lines is accounted for by making the simple substitution[45] (see also[1,8,20])

$$k_z \to k_z^*(x) = k_z + k_y \theta(x), \tag{3.7}$$

where $\theta(x) = B_{0y}/B_{0z}$. For concreteness we assume that $\theta = Sx$, corresponding to a linear change of the transverse component of the magnetic field with coordinate[20], i.e. $B_{0y} = SxB_{0z}$, where x varies in the region of localization of the oscillations and consequently assumes values on the order of the dimension of the region of localization*, while S characterizes the twisting of the magnetic field force lines per unit length.

A nonzero shear of the force lines of the magnetic field can lead to two effects. First, it can influence the region of localization of the drift oscillations, which is obviously, near the surface of the plasma (in the region of its inhomogeneity). It is precisely with increasing θ, according to (3.7), that the effective wave number k_z^* increases, and the condition for the existence of drift oscillations may be violated. For oscillations considered above it is necessary to have $\omega_{dr} \gg k_z v_s, k_z v_{Ti}$; $\omega \gg k_z v_{Ti}$ (when $\omega^2 \ll k_z^2 v_{Ti}^2$ we have Debye screening of the field and oscillations are impossible). Therefore, such a violation occurs when[45]

$$\theta = Sx \gtrsim \sqrt{\frac{T_e + T_i}{M}} \frac{1}{\Omega_i L_0}. \tag{3.8}$$

If this inequality holds true in the entire inhomogeneity region of a plasma, at values of x on the order of the region of the localization of the oscillations, which exceeds the Larmor radius of the ions, (i.e., $x \sim 1/k_\perp > \rho_i$), the branches of the drift-dissipative oscillations considered above are impossible, in other words, the oscillations are stabilized by the shear of the force lines of the magnetic field.† It should be noted that inequality (3.8) is sufficient (and therefore too stringent), but far from necessary for the stabilization of the considered drift oscillations. For individual concrete

* For a cylindrically inhomogeneous plasma $\theta = r\Delta r\, \delta/\delta r\, (B_{0\varphi}/rB_{0z})$ where Δz – dimension of the region of localization of the oscillations. Consequently, in a cylindrically inhomogeneous plasma, S should be taken to be the quantity

$$S = r \frac{\partial}{\partial r}\left(\frac{B_{0\varphi}}{rB_{0z}}\right)$$

(here $S = \theta \Delta r^{-1}$). The minimum value of Δr (x in the plane case) is of the order of the wavelength of the oscillations in question in the direction of the inhomogeneity of the plasma, and the maximum value does not exceed the dimensions of the inhomogeneity region.

† The smaller the dimension x for which the inequality (3.8) is satisfied, the smaller the wavelength of the possible unstable oscillations and consequently the less dangerous the instability is from the point of view of magnetic containment of the plasma. This remark pertains to all types of the unstable drift oscillations considered by us, stabilized by the shear of the force lines of the magnetic field. The most dangerous of them, obviously, are the oscillations with greatest wavelength, $\lambda_\perp \sim L_0$, which are excited under conditions when an inequality of type (3.8) is not satisfied for dimensions $x \gtrsim L_0$.

branches of the oscillations, the necessary and sufficient conditions of stabilization can be readily obtained from the inequality $k_z^* v_{Ti} > \omega$. These conditions differ little from (3.8) and will therefore not be written out here.

Second, the shear of the force lines of the magnetic field can greatly influence the spectrum of the drift-oscillation frequencies. In the presence of strongly inhomogeneous shearing of the force lines of the magnetic field, local expressions for the oscillation spectra, obtained directly from the eikonal equations, may differ qualitatively, as shown below, from those obtained with the aid of the quantization rules.

For the oscillations considered above, in the frequency region $\omega \gg \nu_i$, $|\omega + i\nu_e| \ll k_z v_{Te}$, taking into account the shear of the force lines of the magnetic field, the dispersion equation obtained from the eikonal equation (3.1) and equation (3.2) is

$$\int d\xi \left\{ -k_y^2 + k_{\perp 0}^2 - i\sqrt{\frac{\pi}{2}}\frac{1}{r_{De}^2}\frac{1}{|k_y S\xi|\, v_{Te}}\left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e}\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_e}}\right) - i\cdot\frac{7}{10}\frac{k_{\perp 0}^4 v_{Ti}^4}{\Omega_i^4}\frac{\nu_{ii}}{r_{Di}^2}\left[1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln N}{\partial x} + \frac{3}{28}\frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln T_i}{\partial x}\right]\right\}^{1/2} = \pi n, \tag{3.9}$$

where

$$k_{\perp 0}^2 = -\left\{\frac{1}{r_{De}^2}\left(1 + \frac{k_y v_s^2}{\omega'\Omega_i}\frac{\partial \ln N}{\partial x}\right) - \frac{\omega_{Li}^2}{\omega'^2}k_y^2 S^2\xi^2\left(1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln NT_i}{\partial x}\right)\right\}\left[1 + \frac{\omega_{ii}^2}{\Omega_i^2}\left(1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\frac{\partial \ln NT_i}{\partial x}\right)\right]^{-1}. \tag{3.10}$$

In the derivation of these formulas we used the relation (3.7) and made the substitution $\xi = x + k_z/k_y S$. The integration in the dispersion equation (3.9) is over the transparency region of the plasma, in which $-k_y^2 + k_{\perp 0}^2 > 0$ (in the case under consideration, the dissipative terms are small and we can speak of the transparency region of the plasma). If there are no turning points in the inhomogeneous plasma, then the limits of integration coincide with the surfaces of the inhomogeneous plasma layer (in a homogeneous plasma, these oscillations do not exist and consequently, the surface of the inhomogeneous layer is the turning point for them), or else are determined from an inequality which is the inverse of (3.8) and which serves as the condition for the existence of oscillations.*

Examining relation (3.9) in the limiting cases in which the local spectra (3.3)–(3.5) were obtained, we obtain analogous formulas, in which $k_y S\xi_0$ should be taken for k_z, where ξ_0 is the point in the vicinity of the surface of the plasma (it is determined by inequality (3.8)). Thus, for these oscillations the conditions for existence and the instability criteria of the plasma are correctly described qualitatively by the local spectra. We note that this is the consequence of the fact that such oscillations turn out to be surface oscillations, locked-in between the turning point and the surface of the inhomogeneous plasma layer.

The situation is different for the fourth branch of the oscillations, described by formula (3.6). Relation (3.9) for this branch of oscillations takes the form

$$\int d\xi \sqrt{\frac{T_e}{T_i\rho_i^2}}\left\{\frac{\omega}{\frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}} + \frac{k_y^2 S^2\xi^2 v_s^2}{\omega^2}\right\}^{1/2} = \pi n. \tag{3.11}$$

In the presence of shear of the force lines of the magnetic field, these oscillations turn out to be locked inside the plasma between two turning points. Neglecting the inhomogeneity of the ion temperature compared with the inhomogeneity of the shear of the force lines of the magnetic field, we obtain from (3.11)[44]

$$\omega^2 = in\Omega_i\,|k_y S|\,\rho_i^2\frac{T_e}{T_i}\frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}. \tag{3.12}$$

The local spectrum of the oscillations, determined by formula (3.6) differs qualitatively from the spectrum (3.12). This is precisely the manifestation of the aforementioned strong influence of the shear of the force lines of the magnetic field on the oscillation spectrum of the inhomogeneous plasma.

With increasing shear of the force lines of the magnetic field as seen from (3.12), the growth increment of the examined oscillations increases. However, it follows from the condition $\omega < \omega_{dz}$ that when

$$S \gg \frac{T_i}{T_e}\frac{1}{L_0} \tag{3.13}$$

no such oscillations are possible in the plasma or, in other words, they are stabilized by the shear of the force lines of the magnetic field. When $x \sim \rho_i\sqrt{T_e/T_i}$ the inequality (3.13) corresponds to the condition (3.8).

It should be noted that the drift-dissipative oscillations with spectra (3.3)–(3.6) and (3.12), which we investigated above, can exist both in a collisionless plasma and in a plasma with a rather high collision frequency. In particular, they are possible under conditions when $\omega \lesssim \omega_{dz} < \nu_e$. Then, however, it is necessary that the wavelength of the oscillations along the magnetic field be smaller than the mean free path of the electrons in the plasma, i.e., $k_z v_{Te} > \nu_e$. As to the ion collisions, they should be sufficiently rare, in order to satisfy the condition $\omega_{dz} \gtrsim \omega \gg \nu_i \sim \nu_{ii} k_\perp^2 \rho_i^2$.

We note also that the use of the model BGK integral leads to the conclusion that ion-ion collisions have a stabilizing influence on these oscillations[22–25,28]. This conclusion, however, as shown above, is valid only in the case of a homogeneous ion temperature. In addition, since the particle collision frequencies are not strictly defined in the BGK model integral, it follows that even in those cases when its results are qualitatively correct, they are quantitatively inaccurate.

b) We now consider long-wave oscillations in the frequency region $\omega \gg \nu_e$, $k_z v_{Te}$ (and consequently $\omega \gg \nu_i$). Using expressions (2.8), (2.11), and (2.12) we get from the eikonal equation (3.1) in the frequency region under consideration, under the condition $\omega \gg \omega_{dz}$, the well known spectrum of the hydrodynamically unstable flute oscillations ($k_z \approx 0$) in systems with positive

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* We note that in a collisionless plasma this circumstance is automatically taken into account in the dispersion equation

$$\int dx \left\{ -k_y^2 - \left[\frac{\rho_i^2}{r_{Di}^2}\left(1 - \frac{k_y v_{Ti}^2}{\omega'\Omega_i}\left\{\frac{\partial \ln N}{\partial x} + \frac{\partial T_i}{\partial x}\frac{\partial}{\partial T_i}\right\}\right)J_+\left(\frac{\omega'}{k_z^* v_{Ti}}\right) + 1\right]^{-1} \times\left[\frac{1}{r_{De}^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{|k_z^*|\, v_{Te}}\left\{1 - \frac{k_y v_{Te}^2}{\omega\Omega_e}\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_e}}\right\}\right) + \frac{1}{r_{Di}^2}\left(1 - J_+\left(\frac{\omega'}{k_z^* v_{Ti}}\right)\right) + \frac{1}{k^2 r_{Di}^2}\frac{k_y v_{Ti}^2}{\omega'\Omega_i}\left\{\frac{\partial \ln N}{\partial x} + \frac{\partial T_i}{\partial x}\frac{\partial}{\partial T_i}\right\}J_+\left(\frac{\omega'}{k_z^* v_{Ti}}\right)\right]\right\}^{1/2} = \pi n,$$

which includes the relation (3.9) and is generalized to the frequency region $\omega \leqslant k_z^* v_{Ti}$. It is seen from this relation that in the limit $\omega' \ll k_z^* v_{Ti}$ the oscillations are impossible, and Debye screening of the field in the plasma takes place. This denotes in turn that there is a turning point in the region $\omega' \sim k_z^* v_{Ti}$.

curvature of the force lines of the containing magnetic field[13]*:

$$\omega^2 = \frac{k_y^2 g}{k_\perp^2 (1+v_A^2/c^2)} \frac{\partial \ln N}{\partial x}. \tag{3.14}$$

As shown in[3], allowance for the finite ion Larmor radius in the collisionless plasma stabilizes the flute instability if

$$k_\perp^2 \rho_i^2 \left(\frac{\partial \ln NT_i}{\partial x}\right)^2 > 4g \frac{\partial \ln N}{\partial x}. \tag{3.15}$$

In a plasma with collisions, when this condition is satisfied, the flute branch of the oscillations goes over into the drift-dissipative branch with $k_z \approx 0$[36]:

$$\omega = \frac{1}{1+v_A^2/c^2} \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln NT_i}{\partial x},$$

$$\gamma = \frac{\nu_{ii}}{40} \frac{k_\perp^2 \rho_i^2}{1+v_A^2/c^2} \frac{\partial \ln N}{\partial \ln NT_i} \left[28 \frac{v_A^2}{c^2} - \left(31 + 3\frac{v_A^2}{c^2}\right) \frac{\partial \ln T_i}{\partial \ln N}\right]. \tag{3.16}$$

Consequently, when

$$\left(31 + 3\frac{v_A^2}{c^2}\right) \frac{\partial \ln T_i}{\partial \ln N} < 28 \frac{v_A^2}{c^2}$$

the flute oscillations are unstable even when the Rosenbluth condition (3.15) is satisfied. Here, however, the instability has a kinetic character and the buildup of the oscillations is due to the ion-ion collisions. This conclusion also shows the incomplete nature of the theory which uses a model BGK integral, and according to which the ion collisions always play a stabilizing role[25]. We note that such an instability is possible in systems with both positive and negative curvatures.

The oscillations considered above can be relatively easily stabilized by the shear of the force lines of the magnetic field. Indeed, if we recall that such oscillations are possible only when $\omega \gg k_z v_{Te}$, and take into account the transformation (3.7), then the stabilization conditions can be written in the form

$$\theta = Sx > \sqrt{\left|\frac{g}{k_\perp^2 v_{Te}^2} \frac{\partial \ln N}{\partial x}\right|} \sim \sqrt{\frac{m}{M} \frac{L_0}{R}} \tag{3.17}$$

for flute oscillations with spectrum (3.17), and

$$\theta = Sx > \sqrt{\frac{m}{M} \frac{T_i}{T_e}} \frac{\rho_i}{L_0} \tag{3.18}$$

for drift-dissipative oscillations with spectrum (3.18). In deriving condition (3.18) we also took into account the inequality

$$\frac{k_z^2}{k_\perp^2} < \frac{m}{M} \frac{T_i}{T_e} \frac{\omega^2}{\Omega_i^2},$$

satisfaction of which is necessary in order for the spectrum (3.16) to be valid. If these inequalities are not violated at dimensions exceeding the region of localization of the oscillations $x \gtrsim \rho_i$, then the oscillations considered above will be stabilized.

We shall now show that the stabilization condition (3.17) remains in force also when the spectrum of the oscillations is determined with the aid of the quantization rules. Indeed, if we take into account the fact that $\omega \gg \omega_{dz}$, the quantization rule (3.2) leads to the following dispersion equation:

$$\int d\xi \left\{ -k_y^2 + k_y^2 \frac{g \frac{\partial \ln N}{\partial x} + \frac{M}{m} S^2 \xi^2 \Omega_i^2}{\omega^2 (1+v_A^2/c^2)} \right\}^{1/2} = \pi n, \tag{3.19}$$

where $\xi = x + k_z/k_y S$. Neglecting the inhomogeneity of the density compared with the inhomogeneity of the shear of the force lines of the magnetic field, we get from relation (3.19)[20]

$$k_y^2 \left(1 - \frac{g \frac{\partial \ln N}{\partial x}}{\omega^2 (1+v_A^2/c^2)}\right) + 2n \sqrt{\frac{M}{m} \frac{\Omega_i^2}{\omega^2} \frac{k_y^2 S^2}{1+v_A^2/c^2}} = 0. \tag{3.20}$$

We see that the considered flute oscillations become stabilized by the shear force lines of the magnetic field under the condition

$$S > \sqrt{\frac{m}{M} \frac{k_y^2}{\Omega_i^2} \left|g \frac{\partial \ln N}{\partial x}\right|} \sim \frac{k_y \rho_i}{L_0} \sqrt{\frac{m}{M} \frac{L_0}{R}}. \tag{3.21}$$

When $x \sim 1/k_y \gtrsim \rho_i$ this condition coincides with (3.17).

As already noted above, for the local spectrum (3.16) to be valid, it is necessary to satisfy, in addition to condition (3.15), the inequality

$$\frac{k_z^2}{k_\perp^2} < \frac{m}{M} \frac{T_i}{T_e} \frac{\omega^2}{\Omega_i^2}.$$

In the opposite limit, the local spectrum of the drift-dissipative oscillations is determined by the expressions (when $c^2 \gg v_A^2$)

$$\omega = -\frac{k_y v_s^2}{\Omega_i} \frac{\partial \ln NT_e}{\partial x}, \quad \gamma = \nu_{\text{eff}} \frac{\partial \ln T_e^{3/2}}{\partial \ln NT_e}. \tag{3.22}$$

Unlike the oscillations with the spectrum (3.16), these oscillations are unstable in the presence of a temperature gradient, namely when $\partial \ln T_e/\partial \ln N > 0$ or $\partial \ln T_e/\partial \ln N < -1$, and the buildup of the oscillations is then due to the collisions between the electrons and the ions. In the presence of shear of the magnetic-field force lines, these oscillations become relatively easily stabilized as a result of violation of the inequality $\omega > k_z v_{Te}$. To stabilize the oscillations it is necessary that in the region of the plasma inhomogeneity, at dimensions $x \sim 1/k_\perp \gtrsim \rho_i$, the following condition remains unviolated:

$$\theta = Sx > \sqrt{\frac{m}{M} \frac{T_e}{T_i}} \frac{\rho_i}{L_0}. \tag{3.23}$$

Finally, in the region of frequencies $\omega \ll \omega_{dr}$, when the inequality (3.15) is satisfied, there exist also two branches of aperiodically (hydrodynamically) unstable drift oscillations with local spectra*[15] (at $c^2 \gg v_A^2$)

$$\omega^2 = -\frac{k_z^2}{k_\perp^2} \frac{M}{m} \frac{T_e}{T_i} \Omega_i^2 \frac{\partial \ln NT_e}{\partial \ln NT_i}, \tag{3.24}$$

$$\omega^3 = \frac{k_z^2}{k_\perp^2} \frac{M}{m} \frac{T_e}{T_i} \Omega_i^2 \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N}{\partial x}. \tag{3.25}$$

The first of these branches is possible in a plasma with

* The branches of oscillations with $k_z \neq 0$, which arise as a result of the presence of a gravity field, are frequently termed in the literature "balloon" modes. The spectrum (3.14) is in force also for oscillations with $k_z \neq 0$, provided only

$$\frac{k_z}{k_\perp} < \sqrt{\frac{m}{M} \frac{g}{\Omega_i^2} \frac{\partial \ln N}{\partial x}}.$$

In particular, it follows from this that a system in which the section with positive curvature of the magnetic force lines has a finite length $L_\parallel$ such as the system considered here, the high frequency balloon instability is possible, as was demonstrated in [12], only under the condition

$$L_\parallel > L_0 \sqrt{\frac{M}{m} \frac{R}{L_0}}.$$

* When $T_c = T_i$ the spectrum (3.24) goes over into the spectrum obtained earlier in [4].

an arbitrary ratio T_e/T_i, and the second only in a nonisothermal plasma, in which $T_e \gg T_i$. The obtained spectra do not depend at all on the dissipative processes in the plasma, and it is therefore perfectly natural that they are correctly described by any theory that does not take particle collisions into account. These oscillations can be easily stabilized by the shear of the magnetic force lines; to this end it is sufficient to satisfy one of the conditions (3.18) or (3.23) in the entire region of the plasma inhomogeneity.

We now write out the dispersion equation obtained with the aid of the quantization rule, for oscillations described by local spectra (3.16), (3.22), (3.24), and (3.25). Confining ourselves for simplicity to consideration of only the spectrum of frequencies ω, we neglect in the eikonal equation (3.1) the small dissipative terms, and in order to take into account the shear of the magnetic-field force lines we make the substitution $\xi = x + k_z/k_y S$. As a result we get from the eikonal equation (31), using the quantization rule (3.2), the sought-for dispersion equation

$$\int d\xi \left\{ -k_y^2 + \frac{k_y^2 S^2 \xi^2}{\omega^2} \frac{\omega_{Le}^2 \left(1 + \frac{k_y v_s^2}{\omega \Omega_i} \frac{\partial \ln N T_e}{\partial x}\right)}{1 + \frac{c^2}{v_A^2}\left(1 - \frac{k_y v_{Ti}^2}{\omega \Omega_i} \frac{\partial \ln N T_i}{\partial x}\right)} \right\}^{1/2} = \pi n. \quad (3.26)$$

Allowance for the dissipative terms leads to an oscillation growth increment which goes over, within the appropriate limits, into expressions (3.16) and (3.22), which are averaged over the region of the transparency of the plasma (in the limit corresponding to the spectra (3.24) and (3.25), the oscillations are aperiodically unstable and there is no need to take into account the dissipative terms). The conditions for the buildup of the oscillations and their stabilization by the shear of the magnetic force lines turns out in this case to be the same as for the corresponding local oscillations.

The unstable drift oscillations considered in this section, with $\omega \gg \nu_e, k_z v_{Te}$, with the exception of the hydrodynamic flute oscillations with spectrum (3.14), are possible only when the drift frequencies of the particles are larger than the collision frequencies $\omega_{dr} \gg \nu_e$. In addition, the condition $\omega \gg k_z v_{Te}$ means that they can be excited only in sufficiently long installations, in which the longitudinal plasma dimension is larger than the transverse one by at least $\sqrt{M/m}$ times, and can be easily stabilized by a weak shear of the force lines of the magnetic field. However, in those cases when such instabilities are possible, they are more dangerous than those considered in the preceding section at $\omega \lesssim \omega_{dr} \ll k_z v_{Te}$, since the density disturbances in them encompass, generally speaking, a large region of plasma in the longitudinal direction. As to the flute instability with spectrum (3.14), the increment of its development is larger than the drift frequencies, and the instability can therefore develop also when $\omega_{dr} \ll \nu_e$. Moreover, it will be shown below that hydrodynamic flute instability is developed in systems with positive curvature of the magnetic-field force lines not only when $\omega > \nu_e$, but also when $\omega < \nu_e$.

c) We now proceed to investigate long-wave oscillations in the frequency region $\nu_e \gg \omega, k_z v_{Te}$; $\omega \gg \nu_i, k_z v_{Ti}$. The drift oscillations in this region of frequencies are of interest because they can be excited also in a relatively dense and low-temperature plasma, in which $\omega_{dr} \ll \nu_e$. Substituting expressions (2.9) (or else (2.10), (2.11), and (2.12)) in the eikonal equation (3.1), we see that the dissipative ionic term in this region of frequencies is always small. On the other hand, the electronic dissipative term in the limit $\omega\nu_e \gg k_z^2 v_{Te}^2$ (when the diffusion and thermal conductivity of the electrons can be neglected in the oscillation process) is small only under the condition $\omega \gg \omega_s$, where

$$\omega_s = \frac{k_z^2}{k_\perp^2} \frac{M}{m} \frac{\Omega_i^2}{\nu_{eff}}.$$

Under these conditions, the eikonal equation (3.1) leads to the well known flute instability (at $k_z = 0$) with spectrum (3.14).* Just as in the case considered above ($\omega \gg \nu_{eff}$), when inequality (3.15) is satisfied, the flute branch of the oscillations goes over into the drift-dissipative branch with $k_z = 0$, the spectrum of which is determined by expressions (3.16). In the considered case, however, the conditions of stabilization of such oscillations by the shear of the magnetic field lines assume a different form. Namely, the flute oscillations with spectrum (3.14) are stabilized under the condition

$$\theta = Sx \gg \sqrt{\frac{m}{M} \frac{\nu_{eff}}{\Omega_i} \frac{\rho_i}{\sqrt{L_0 R}}}, \quad (3.27)$$

whereas for the stabilization of the drift-dissipative oscillations with spectrum (3.16) it is necessary to have

$$\theta = Sx > \sqrt{\frac{m}{M} \frac{\nu_{eff}}{\Omega_i} \frac{\rho_i}{L_0}}. \quad (3.28)$$

If these inequalities are not violated for dimensions $x \gtrsim \rho_i$, then the indicated oscillations are impossible in a plasma, or in other words, the corresponding plasma instabilities become stabilized.

When inequality (3.28) is satisfied, the condition $\omega > \omega_s$ is violated. In this case the dissipative terms in the eikonal equation (3.1), due to the electron collisions, become comparable with the nondissipative real terms. As a result, in the region of frequencies that are larger than the drift frequencies of the particles, there appears a dissipative oscillation branch, determined by the finite conductivity of the plasma[17,20]. The eikonal equation (3.1) for such oscillations, in the presence of weak shear of the magnetic-field force lines, is written in the form†

$$\left(1 + \frac{v_A^2}{c^2}\right) \frac{\eta}{4\pi} k_\perp^2 \omega^2 + i\omega k_z^{*2} v_A^2 - \frac{\eta}{4\pi} k_y^2 g \frac{\partial \ln N}{\partial x} = 0, \quad (3.29)$$

where $\eta = c^2/\sigma$ and $\sigma = 1.96\,\omega_{ie}^2/4\pi\nu_{eff}$ is the conductivity of the plasma. Making the substitution $\xi = x + k_z/k_y S$ and using the quantization rule (3.2), we obtain the dispersion equation of the oscillations

$$\int d\xi \left\{ -k_y^2 - \frac{4\pi}{\eta\omega^2} \frac{k_y^2}{1 + v_A^2/c^2} \left(\frac{\eta}{4\pi} g \frac{\partial \ln N}{\partial x} - i\omega \xi^2 S^2 v_A^2\right) \right\}^{1/2} = \pi n. \quad (3.30)$$

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* This spectrum is retained also when $k_z \neq 0$, but

$$\frac{k_z^2}{k_\perp^2} < \frac{m}{M} \frac{\nu_{\text{эфф}}}{\Omega_i^2} \left| g \frac{\partial \ln N}{\partial x} \right|^{1/2}.$$

From this we get, in particular, that in systems with length

$$L_\parallel < L_0 \left(\frac{M}{m} \frac{\Omega_i^2}{\nu_{\text{эфф}}} \sqrt{\frac{RL_0}{v_{Ti}^2}} \right)^{1/2}$$

such an instability, which can also be called balloon instability, is impossible.

† Such an equation is obtained also in the model of the single-fluid hydrodynamics [20] under the condition $c \gg v_A^2$ and $k_\perp^2 \gg k_z^{*2}$.

Neglecting the inhomogeneity of the plasma density compared with the inhomogeneity of the shear of the magnetic field force lines in the transparency region, we get therefore

$$k_y^2\left(1-\frac{g\frac{\partial\ln N}{\partial x}}{\omega^2(1+v_A^2/c^2)}\right)+2n\sqrt{\frac{k_y^2S^2 4\pi v_A^2}{-i\omega\eta\,(1+v_A^2/c^2)}}=0. \qquad (3.31)$$

In the case of a weak shear of the magnetic field force lines, $S \to 0$, when the second term in the left side of (3.31) can be neglected, the equation describes ordinary flute oscillations (see (3.14)). In the case of the opposite limit (when $\omega^2 \ll g[\partial \ln N/\partial x]$) we get the spectrum of the unstable dissipative branch of the oscillations[17,20]

$$\gamma=-i\omega=\left\{\frac{\eta k_y^2 g^2}{(4\pi)^2 n^2 S^2 v_A^2\,(1+v_A^2/c^2)}\left(\frac{\partial\ln N}{\partial x}\right)^2\right\}^{1/3}. \qquad (3.32)$$

One should not think that the considered dissipative branch of the oscillations exists only in the case of a sufficiently large shear of the magnetic field force lines. With decreasing shear, this instability still remains. In this case, however, its increment changes. The point is that the region of localization of the oscillations with spectrum (3.2) increases like $S^{-4/3}$ with decreasing shear of the force lines, and can become larger than the dimensions of the inhomogeneous plasma layer. In this case, the region of transparency of the plasma is determined by the surfaces of this layer and the plasma instability increment can be obtained directly from the eikonal equation (3.29) in the form of the local spectrum. We have[20] *

$$\omega=-i\frac{\eta}{4\pi}\frac{k_y^2}{k_z^2}\frac{g}{v_A^2}\frac{\partial\ln N}{\partial x}. \qquad (3.33)$$

From this we see, incidentally, that the considered dissipative instability can develop only in systems with positive curvature of the force lines of the magnetic field; it does not occur in minimum-B systems.

The local spectrum (3.33) differs qualitatively from the oscillation spectrum (3.32). In this example we see the already mentioned qualitative influence of the shear of the magnetic field force lines on the plasma oscillation spectrum.

Finally, we note that at sufficiently large shear, the considered dissipative plasma instability due to the narrowing of the region of localization of the unstable oscillations to dimensions on the order of the Larmor radius of the ions, can be stabilized. To this end it is necessary to have

$$\theta=Sx\gg\sqrt{\frac{m}{M}\frac{\rho_i}{L_0}\sqrt{\frac{L_0}{R}}}. \qquad (3.34)$$

The dissipative branch of the oscillations, just as the flute branch, is due to the curvature of the force lines of the containing magnetic field and disappears when the Rosenbluth conditions (3.15) are satisfied. More accurately speaking, in this case the frequency of the flute oscillations (3.14) becomes smaller than that of the drift oscillations, and the formulas obtained above for the spectra of the flute and dissipative instabilities of the plasma are no longer applicable. We have seen above that the flute branch of the oscillations ($\omega \gg \omega_s$) goes over into the drift branch with spectrum (3.16) when condition (3.15) is satisfied.

Let us trace now the transition of the dissipative branch ($\omega \ll \omega_s$) into the drift-dissipative branch with increasing transverse wave number $k_\perp$, when the inequality (3.15) is satisfied. From the eikonal equation (3.1) in the frequency region $\omega \lesssim \omega_s$, $\omega \lesssim \omega_{dz}$ and $\omega^2 \ll (k_z^2/k_\perp^2)\sigma\omega_{dr}$ we obtain the following three local spectra[16,19,36]:

$$\omega_1=-\frac{k_y v_s^2}{\Omega_i}\frac{\partial\ln NT_e^{1,71}}{\partial x},$$

$$\gamma_1=\frac{\omega_1^2}{1.96\omega_s}\left\{1+\frac{v_A^2}{c^2}+\frac{T_i}{T_s}\frac{\partial\ln NT_i}{\partial\ln NT_e^{1,71}}\left[1-\frac{k_z^2}{k^2}\frac{\Omega_i^2}{\omega_1^2}\left(1+\frac{T_e}{T_i}\frac{\partial\ln NT_e}{\partial\ln NT_i}\right)\right]\right\}, \qquad (3.35)$$

which is valid when $\omega_1^2 \ll \omega_s^2$,

$$\omega_2=i\cdot 1.96\omega_s\frac{T_e}{T_i}\frac{\partial\ln NT_e^{1,71}}{\partial\ln NT_i}, \qquad (3.36)$$

which is valid when $\omega_2 \sim \omega_s \ll \omega_{dr}$, and finally

$$\omega_3=\frac{i}{1.96}\frac{m}{M}\frac{T_i}{T_e}\nu_{\mathrm{eff}}\frac{\partial\ln NT_i}{\partial\ln NT_e^{1,71}}, \qquad (3.37)$$

which is valid when

$$\frac{k_z^2}{k_\perp^2}\Omega_i^2 \gg \omega_3^2 \ll \omega_{dr}^2.$$

The oscillations in the second and third branches are practically always unstable, whereas those in the first branch are unstable only if $\omega_1^2 \sim \omega_{dr}^2 \gg (k_z^2/k_\perp^2)\Omega_i^2$. From the condition for the applicability of formula (3.36) it follows that the latter inequality should be satisfied also for the second branch of the oscillations. On the other hand, the third unstable branch of the oscillations is valid when the inverse inequality is satisfied. However, it follows from the condition $\omega \gg \nu_i$ that such oscillations are possible only in a non-isothermal plasma with hot ions, when

$$\frac{T_i}{T_e} \gg \left(\frac{M}{m}\right)^{2/5}.$$

It should be noted that in the limit when $\omega^2 \gg (k_z^2/k_\perp^2)\Omega_i^2$ (which is equivalent to neglecting the longitudinal motion of the ions) the spectra of the drift-dissipative oscillations (3.35) and (3.36) coincide with those obtained in[16,19] under the condition $\omega\nu_e \gg k_z^2 v_{Te}^2$, i.e., in the case of weak electronic thermal conductivity*. As to the spectrum (3.37), it is due entirely to longitudinal motion of the ions and cannot be obtained within the framework of the approximate method proposed in these papers.

In the presence of shear, the oscillations under consideration can be fully stabilized under conditions when the inequality (3.8) is satisfied in the region of plasma inhomogeneity, for dimensions on the order of the oscillation-localization length, which can be compared with the gyroscopic radius of the ions.

Finally, we write out the "quantization rule" for oscillations corresponding to the local spectra (3.35)–(3.37). By determining from the eikonal equation (3.1) the complex function $k_x(\omega, x)$ and substituting in (3.2), we get

$$\int d\xi\left\{-k_y^2-\left[1+\frac{c^2}{v_A^2}\left(1-\frac{k_y v_{Ti}^2}{\omega\Omega_i}\frac{\partial\ln NT_i}{\partial x}\right)\right]^{-1}\right.$$

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* *We note that the dissipative branches of the oscillations (3.32) and (3.33) are frequently also called dissipative balloon modes.*

*We *note that these papers contain annoying errors, and only correction of these errors can yield formulas (3.35) and (3.36).*

$$\times\left[i\cdot 1{,}96\frac{\omega_{Le}^2 k_y^2 S^2\xi^2}{\omega\nu_{\text{eff}}}\left(1-\frac{k_y v_{Te}^2}{\omega\Omega_e}\frac{\partial\ln NT_e^{1,71}}{\partial x}\right)\right.$$
$$\left.\left.+\frac{\omega_{Li}^2 k_y^2 S^2\xi^2}{\omega^2}\frac{k_y v_{Ti}^2}{\omega\Omega_i}\frac{\partial\ln NT_i}{\partial x}\left(1+\frac{T_e}{T_i}\frac{\partial\ln NT_e}{\partial\ln NT_i}\right)\right]\right\}^{1/2}=\pi n.$$

From this we can easily show that the spectrum of the frequencies, the buildup conditions, and the conditions for stabilization of the oscillations under consideration by the shear of the magnetic field force lines are described correctly by the local formulas (3.35)–(3.37).

d) In conclusion, let us consider long-wave drift oscillations of an inhomogeneous plasma in the frequency region $\nu_e \gg \omega, k_z v_{Te}$; $\omega \gg \nu_i, k_z v_{Ti}$, and under the condition $\omega\nu_e \ll k_z^2 v_{Te}^2$. The electronic contribution to the dielectric constant of the plasma is determined in this case by (2.10), which is similar to (2.7), although it differs fundamentally from it. The point is that in (2.7) the dissipative term is due to collisionless Cerenkov absorption of the waves by the plasma electrons, whereas in (2.10) the dissipative term is connected with particle collisions, namely with the diffusion and thermal conductivity of the electrons. As a result, the spectra of the frequencies of the drift oscillations of the plasma in the region under consideration coincide with the spectra (3.3)–(3.6). Only the expressions for the growth increments γ_1 and γ_2, which are determined by the dissipative processes in the plasma[41], are changed:

$$\gamma_1=1.44\frac{\omega_1^2\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\left\{k^2 r_{De}^2+\frac{k_\perp^2 v_s^2}{\Omega_i^2}\left(1+\frac{T_i}{T_e}\frac{\partial\ln NT_i}{\partial\ln N}\right)-\frac{\partial\ln T_e^{0,56}}{\partial\ln N}-\frac{g}{v_s^2\frac{\partial\ln N}{\partial x}}\right\}$$
$$-\frac{7}{10}\nu_{ii}k_\perp^4\rho_i^4\left(1+\frac{T_e}{T_i}-\frac{3}{28}\frac{\partial\ln T_i}{\partial\ln N}\right),\quad(3.39)$$
$$\gamma_2=-1.44\frac{\omega_2^2\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\left(1-\frac{\partial\ln T_e^{0,56}}{\partial\ln N}-\frac{g}{v_s^2\frac{\partial\ln N}{\partial x}}\right)-\frac{8}{5}\nu_{ii}\frac{k_z^2 v_{Ti}^2}{\omega_2^2}.$$

This leads to a change in the role of the dissipation on the electrons in the buildup of the oscillations. We see that for the first branch of the oscillations, the electron collisions exert an unstabilizing effect when

$$\frac{\partial\ln T_e}{\partial\ln N}<1.8\left[k^2 r_{De}^2+\frac{k_\perp^2 v_s^2}{\Omega_i^2}\left(1+\frac{T_i}{T_e}\frac{\partial\ln NT_i}{\partial\ln N}\right)\right],$$

and for the second when $\partial\ln T_e/\partial\ln N > 1.8$. In all other respects, the entire foregoing analysis, concerning the influence of the curvature of the force lines of the containing field on the plasma oscillation spectrum (see the analysis of formulas (3.3)–(3.6) and (3.9)–(3.12)) remains in force also in the region under consideration.

In spite of the analogy with the spectra (3.3)–(3.6), oscillations in the region under consideration are more dangerous (even for a thermonuclear plasma). Indeed, from the condition $\nu_{\text{eff}} \gg k_z v_{Te}$ it follows that the longitudinal wavelength of the oscillations under consideration is larger than the electron mean free path, whereas for oscillations investigated in Sec. a) the inverse inequality should be satisfied (since for these oscillations $k_z v_{Te} \gg \nu_{\text{eff}}$). This means in turn that the oscillations under consideration can lead to larger-scale (and consequently more dangerous) plasma instabilities. It should be noted, however, that such oscillations are easier to stabilize by the shear of the magnetic field, namely, for their stabilization it is sufficient to satisfy the inequality

$$\theta>\min\left(\frac{\rho_i}{L_0},\ \frac{L_0}{v_{Te}}\nu_{\text{eff}}\right),\quad(3.40)$$

which, generally speaking, is weaker than the inequality (3.8).

Finally, we note that the spectra of the considered oscillations cannot be obtained from the general formulas given in[16]. It is easy to show that in the limit when $\omega\nu_{\text{eff}} \ll k_z^2 v_{Te}^2$ these formulas yield incorrect results. This circumstance indicates that the region of applicability of the quasihydrodynamic method used in[16] is limited, and is the consequence of the already noted neglect of the longitudinal ion motion.

4. SHORT-WAVE LOW-FREQUENCY DRIFT-DISSIPATIVE PLASMA OSCILLATIONS

In this section we shall deal with low-frequency ($\omega \ll \Omega_i$) drift oscillations with a transverse wavelength much lower than the gyroscopic radius of the ions (but at the same time larger than the electron gyroscopic radius). It is precisely in this sense that we shall use the term "short-wave oscillations." It appears at first glance that short-wave drift instabilities are not so dangerous for magnetic containment of the plasma as long-wave oscillations, since they lead only to very small-scale perturbations of the density, and not to a disintegration of the plasma as a whole. It must be borne in mind, however, that such oscillations can be localized in very narrow regions near the surface of the plasma, with dimensions smaller than the gyroscopic radius of the ions, and this is precisely why it is practically impossible to stabilize them by the shear of the force lines of the magnetic field. In this sense, short-wave drift oscillations are quite dangerous. We note that the theory of such oscillations is always kinetic. Finally, since the characteristic dimension of the inhomogeneity of the plasma greatly exceeds the gyroscopic radius of the ions in the experimental situation considered by us, the conditions under which the excitation of the short-wave oscillations takes place can be determined by merely analyzing the local spectra. Just as in the preceding section, the analysis of the short-wave drift oscillations will be carried out for different regions of frequencies and longitudinal waves.

a) In the region of frequencies $\omega, \nu_e \ll k_z v_{Te}$, the short-wave oscillations ($\rho_e \ll 1/k_\perp \ll \rho_i$) are possible only if $\omega \gg \nu_{ii}k_\perp^2\rho_i^2$, and their frequencies are much lower than the drift frequencies of the particles. Taking into account the smallness of the dissipative terms compared with the nondissipative ones, we obtain from the eikonal equation (3.1) in the frequency region under consideration the following spectrum of the short-wave plasma oscillations:

$$\omega=-\frac{T_e}{T_e+T_i(1+k^2 r_{De}^2)}\frac{k_y v_{Ti}}{\sqrt{2\pi}k_\perp}\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_i}},$$
$$\gamma=\frac{T_e^2}{[T_e+T_i(1+k^2 r_{De}^2)]^2}\frac{k_y^2 v_{Ti}^3}{2|k_z|k_\perp v_{Te}\Omega_i}\left(\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_e}}\right)\left(\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_i}}\right)\times$$
$$\times\left(1-\frac{g}{v_s^2\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_i}}}\right)-\nu_{ii}k_\perp^2\rho_i^2\frac{3(\pi+1)}{4\sqrt{2}}\frac{\partial\ln NT_i^{-0.69}}{\partial\ln NT_i^{-0.5}}.\quad(4.1)$$

The expression obtained for the increment shows that the Cerenkov effect on the electrons, just as in a collisionless plasma (see[1,8]) leads practically always to a buildup of the oscillations; on the other hand, the ion-ion collisions exert a stabilizing influence, with the exception of the region where $2 > \partial\ln T_i/\partial\ln N > 1.45$, in which, to the contrary, they lead to a buildup of oscillations and by the same token they broaden the regions of plasma instability[36]. The effect of the curvature of the force lines of the containing magnetic field is peculiar: when $\partial \ln T_i/\partial \ln N < 2$, in systems with positive curvature, it plays an unstabilizing role, and when $\partial \ln T_i/\partial \ln N > 2$ it stabilizes the oscillations; in systems with negative curvature (minimum-B systems), the curvature of the force lines exerts an opposite effect. It should be noted that the influence of the curvature of the force lines can become considerable only in a plasma with hot ions, when $T_i/T_e \gtrsim R/L_0 \gg 1$. But in this case, as already noted above, the introduction of the effective gravity field is no longer valid and we can speak only of a weak influence of the curvature of the force lines on the character of the plasma oscillations.

The local spectrum (4.1) is retained also in the presence of shear of the magnetic field. The shear of the field lines only narrows down the region of localization of the oscillations, and under the condition

$$\theta = Sx > \frac{1}{\sqrt{2\pi} k_\perp \rho_i} \frac{\rho_i}{L_0} \tag{4.2}$$

it stabilizes the oscillations completely. Such a stabilization, however, is quite difficult under real conditions, since the region of localization of the oscillations can be of the order of or even smaller than the gyroscopic radius of the ions* and it is necessary that the inequality (4.2) not be violated for such dimensions.

b) The spectrum (4.1), with a slight modification, remains valid also in the region $\nu_e \gg \omega, k_z v_{Te}$ under the condition $\omega, \nu_e \ll k_z^2 v_{Te}^2$. The change concerns the growth increment γ of the oscillations, and is due to the change of the dissipative contribution to $\delta\epsilon_e$, which in the region under consideration is determined by the expression (2.10) in lieu of (2.7). Taking this circumstance into account, we get[41] (see also[46])

$$\gamma = \frac{\sqrt{2\pi}\cdot 1{,}44 k_y^2 v_{Ti}^3 T_e^2 \nu_{\text{eff}}}{(T_e + T_i(1+k^2 r_{Di}^2))^2 k_z^2 v_{Te}^2} \frac{1}{k_\perp \Omega_i} \left(\frac{\partial \ln NT_e^{-0{,}56}}{\partial x}\right)\left(\frac{\partial \ln NT_i^{-0{,}5}}{\partial x}\right) \times \left(1 - g/v_s^2 \frac{\partial \ln NT_i^{-0{,}5}}{\partial x}\right) - \frac{3(\pi+1)}{2\sqrt{2}} \nu_{ii} k_\perp^2 \rho_i^2 \frac{\partial \ln NT_i^{-0.69}}{\partial \ln NT_i^{-0.5}}. \tag{4.3}$$

The analysis presented above for the conditions of the buildup of the oscillations and stabilization of the shear of the force lines of the magnetic field (4.2) remains in this case unchanged.

c) Let us consider now short-wave oscillations in the frequency region $\omega \gg \nu_e, k_z v_{Te}$. By substituting (2.8), (2.11), and (2.13) (or (2.15)) in the eikonal equation (3.1) we get the following relations for the local spectra of such oscillations[36]:

$$\omega = \frac{1}{1+k^2 r_{Di}^2} \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N}{\partial x},$$

$$\gamma = -\nu_{\text{eff}} \frac{T_i}{T_e} \frac{k_z^2 v_{Te}^2}{\omega^2} \left(\frac{1}{1+k^2 r_{Di}^2} + \frac{T_e}{T_i} \frac{\partial \ln NT_e^{-0{,}5}}{\partial \ln N}\right) + \gamma_i, \tag{4.4}$$

where

$$\gamma_i = \begin{cases} -\nu_{ii} k_\perp \rho_i \dfrac{3(3\pi+2)}{32\sqrt{\pi}} \left[\dfrac{\partial \ln T_i}{\partial \ln N} - \dfrac{1.45 k^2 r_{Di}^2}{1+k^2 r_{Di}^2} + \dfrac{1.45 g (1+k^2 r_{Di}^2)}{v_{Ti}^2 \frac{\partial \ln N}{\partial x}}\right] & \text{if } \omega > \nu_{ii} k_\perp^2 \rho_i^2, \quad (4.5a) \\ -\dfrac{2 C_0 C_1 \omega^2}{3\pi \nu_{ii} k_\perp^3 \rho_i^3} \left[\dfrac{\partial \ln T_i}{\partial \ln N} - \dfrac{4.45 k^2 r_{Di}^2}{1+k^2 r_{Di}^2} + \dfrac{4.45 g (1+k^2 r_{Di}^2)}{v_{Ti}^2 \frac{\partial \ln N}{\partial x}}\right] & \text{if } \omega \ll \nu_{ii} k_\perp^2 \rho_i^2. \quad (4.5b) \end{cases}$$

It is seen from these formulas that the collisions of the electrons with the ions, under the condition

$$\frac{\partial \ln T_e}{\partial \ln N} > 2\left(1 + \frac{T_i}{T_e}\right)$$

contribute to the buildup of the oscillations; on the other hand, collisions of ions with ions can lead to buildup only if

$$\frac{\partial \ln T_i}{\partial \ln N} < 4.45 k^2 r_{Di}^2.$$

The influence of the curvature of the force lines of the magnetic field on the considered oscillations is negligibly small practically always, with the exception of the case of a nonisothermal plasma, in which

$$\frac{T_e}{T_i} \gg \frac{R}{L_0} \frac{k^2 r_{Di}^2}{(1+k^2 r_{Di}^2)^2}.$$

In systems with positive curvature, the curvature plays an unstabilizing role, whereas in systems with minimum B it can lead to stabilization of the unstable oscillations (provided only $k^2 r_{Di}^2 \ll 1$).

Finally, we note that the short-wave oscillations under consideration are stabilized by relatively small shear, determined by the inequality (3.18). The real difficulty in stabilizing the instability, however, just as above, lies in the fact that this inequality must not be violated when the dimensions are on the order of the region of the oscillation localization, which can be comparable with the gyroscopic radius of the ions or even smaller.

d) In conclusion, let us consider short-wave oscillations in the frequency region $\nu_e \gg k_z v_{Te}$ under the condition $\omega \nu_e \gg k_z^2 v_{Te}^2$. Substituting expressions (2.9), (2.11), and (2.13) (or (2.15)) in the eikonal equation (3.1), we can show that the frequency spectrum of such oscillations coincides with the spectrum (4.4), and the growth increment is determined by the formula[36]

$$\gamma = -1.96 \frac{T_i}{T_e} \frac{k_z^2 v_{Te}^2}{\nu_{\text{eff}}} \left\{\frac{1}{1+k^2 r_{Di}^2} + \frac{T_e}{T_i}\left(1 + \frac{\partial \ln T_e^{1{,}71}}{\partial \ln N}\right)\right\} + \gamma_i, \tag{4.6}$$

where γ_i is given by formulas (4.5). The spectrum (4.6) is a continuation of the spectrum (4.4) from the frequency region $\omega > \nu_{\text{eff}}$ into the frequency region $\omega < \nu_{\text{eff}}$. This leads to a modification of the condition for the stabilization of the oscillations by the shear of the magnetic field, which is written for the oscillations in question in the form

$$\theta = Sx > \sqrt{\frac{\nu_{\text{eff}}}{\Omega_e}} \tag{4.7}$$

and is due to the violation of the inequality $\omega \nu_{\text{eff}} > k_z^2 v_{Te}^2$ in systems with sufficiently large shear. We note that satisfaction of this condition, generally speaking, is necessary only for oscillations in the frequency region

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*Actually, for example, for wavelengths much larger than the Debye radius of electrons, the quantization rule corresponding to the oscillation frequency (4.1) is given by.

$$\int dx \left\{-1 + \left(\frac{T_e}{T_e + T_i} \frac{v_{Ti}}{\sqrt{2\pi}\omega} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}}\right)^2\right\}^{1/2} = \frac{\pi n}{|k_y|}.$$

It is obvious that when $n \sim k_y \rho_i \gg 1$, the region of localization of the oscillations turns out to be of the order of the gyroscopic radius of the ions. Oscillations are also possible with a larger region of localization. For them it is easier to determine the conditions under which the shear of the force lines may turn out to be appreciable.

$\omega \ll \nu_{ii} k_\perp^2 \rho_i^2$. The oscillations in the frequency region $\omega \gg \nu_{ii} k_\perp^2 \rho_i^2$ can be stabilized by a shear determined by the inequality (3.8). Under real conditions, however, this inequality is frequently more difficult to satisfy than inequality (4.7). The role of the ion collisions for the oscillations in question is the same as for the oscillations described by formulas (4.4); on the other hand, the electron collisions always play a stabilizing role.

The entire analysis performed in the present section of the short-wave drift-dissipative oscillations of inhomogeneous plasma shows that the conditions of their instability are significantly determined by the character of the inhomogeneity of the plasma-particle temperatures (particularly the ion temperatures). This is precisely why such oscillations cannot be described by the BGK model collision integral, since it does not make it possible to take into account the inhomogeneity of the plasma temperature. The analysis of the low-frequency short-wave oscillations performed in[22] with the aid of such a model collision integral is, unfortunately, inaccurate both qualitatively and quantitatively.

5. DRIFT-CYCLOTRON OSCILLATIONS

We now proceed to investigate drift oscillations in the region of frequencies of ion cyclotron resonance $\omega \sim \omega_{dr} \gtrsim s\Omega_i$. We note immediately that an analysis of the ion-cyclotron oscillations, as well as of the short-wave drift oscillations, with the aid of the BGK model collision integral, as was performed in[23], leads to qualitatively incorrect conclusions.

We shall therefore not compare the results obtained below with the results of[23]. Under conditions when the characteristic dimension of the plasma inhomogeneity is much larger than the gyroscopic radius of the ions, the drift-cyclotron oscillations are short-wave ($k_\perp \rho_i \gg 1$). In the investigation of such oscillations, as already noted above, we can confine ourselves to an analysis of the local spectra. In our case, such an approximation describes sufficiently well the oscillation spectra of the inhomogeneous plasma not only qualitatively but also quantitatively.

Bearing in mind a low-pressure plasma in which

$$\beta = \frac{8\pi P_0}{B_0^2} \ll 1,$$

we confine ourselves here to an analysis of longitudinal oscillations only*.

a) Under conditions when $|\omega + i\nu_e| \ll k_z v_{Te}$ (i.e., when the longitudinal wavelength of the oscillations is small both compared with the electron mean free path and compared with the distance traversed by the electron during the field oscillation period), the drift-cyclotron oscillations are possible only in the frequency region $|\omega - s\Omega_i| \gg \nu_{ii} k_\perp^2 \rho_i^2$, $k_z v_{Ti}$, i.e., far from the line of resonant cyclotron absorption of the waves in the plasma. Neglecting the gravitational drift of the ions (it can be shown that the effects of curvature of the magnetic force lines in the region of cyclotron frequencies are always negligibly small) and taking into account the smallness of the dissipative terms, we get from the eikonal equation (3.1) the following local spectrum of the drift-cyclotron plasma oscillations ($\Delta = \omega - s\Omega_i$) in the frequency region under consideration

$$\operatorname{Re}\Delta = \frac{T_e}{T_e + T_i(1 + k^2 r_{De}^2)} \frac{1}{\sqrt{2\pi} k_\perp \rho_i} \left(s\Omega_i - \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}} \right),$$

$$\gamma = \operatorname{Im}\Delta = -\frac{T_i}{T_e} \frac{\sqrt{2\pi} k_\perp \rho_i}{s\Omega_i - \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}}} \left\{ \sqrt{\frac{\pi}{2}} \frac{\operatorname{Re}\Delta^2}{|k_z| v_{Te}} \right. \tag{5.1}$$

$$\left. \times \left(s\Omega_i + \frac{k_y v_s^2}{\Omega_i} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_e}} \right) + \frac{T_e}{T_i} \nu_{ii} k_\perp \rho_i \frac{3(\pi + 1)}{8\sqrt{\pi}} \left(s\Omega_i - \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N T_i^{-0,69}}{\partial x} \right) \right\}.$$

It is seen from these formulas that when $\omega \sim s\Omega_i \gg \omega_{dr}$ the oscillations are stable ($\gamma < 0$); instability ($\gamma > 0$) is possible only if $\omega_{dr} \gtrsim s\Omega_i$. In the limit $\omega_{dr} \gg s\Omega_i$, the spectrum (5.1) coincides with that obtained in[36], and when $\nu_{ii} \to 0$ it goes over into the results of[11]. It should be noted that the Cerenkov effect on the electrons (the first term in the curly brackets of the expression for γ), under conditions when the considered drift-cyclotron instability can develop in the plasma, always plays an unstabilizing role, whereas the ion-ion collisions in a plasma with uniform particle temperature stabilizes the instability. On the other hand, if the temperature of the particles (ions) is inhomogeneous, then the ion collisions, to the contrary, may even be the cause of the instability. Thus, it follows from (5.1) that when $\omega_{dr} \gg s\Omega_i$ the collisions of the ions with the ions lead to a buildup of oscillations if

$$2 > \frac{\partial \ln T_i}{\partial \ln N} > 1.45.$$

This instability is stabilized by the shear of the magnetic field, determined by the condition (4.2); stabilization is due in this case to the violation of the inequality $\operatorname{Re}\Delta \gg k_z v_{Ti}$. We note that for real stabilization of the instability it is necessary to have inequality (4.2) not violated over dimensions on the order of the gyroscopic radius of the ions, which determines in order of magnitude the region of localization of the oscillations.

b) Let us consider now drift-cyclotron oscillations in the frequency region $|\omega + i\nu_e| \gg k_z v_{Te}$. We note first that in the region inside the resonant absorption line, when $|\omega - s\Omega_i| \ll \nu_{ii} k_\perp^2 \rho_i^2$ and $\nu_{ii} k_\perp^2 \rho^2 \gg k_z v_{Ti}$, the spectra of the drift-cyclotron oscillations are determined by expressions (4.4) and (4.6) respectively in the cases $\omega > \nu_e$ and $\omega < \nu_e$ (when $\omega\nu_e \gg k_z^2 v_{Te}^2$); the quantity ν_i is given in this case by formula (4.5b). This means that the spectra of the low-frequency drift oscillations (4.4) and (4.6) are continued without change into the region of cyclotron frequencies. Obviously, the conditions for stabilization of the oscillations by the shear, which are given by (3.18) and (4.7) in the cases $\omega > \nu_e$ and $\omega < \nu_e$ respectively, remain likewise unchanged.

New characteristic singularities in the spectra of drift-cyclotron oscillations in the frequency region $|\omega + i\nu_e| \gg k_z v_{Te}$ appear far from the resonant absorption line, when $|\omega - s\Omega_i| \gg \nu_{ii} k_\perp^2 \rho_i^2$, $k_z v_{Ti}$. Neglecting small dissipative terms in the eikonal equation (3.1), we find under the condition $\omega > \nu_e$ that, at the intersection of the drift and cyclotron oscillation branches, in a narrow frequency region

$$k^2 r_{Di}^2 - \frac{k_y v_{Ti}^2}{s\Omega_i^2} \frac{\partial \ln N}{\partial x} \approx -1$$

* Arbitrary nonpotential drift-cyclotron oscillations of an inhomogeneous plasma, with allowance for Coulomb particle collisions, were investigated in [37].

the development of a resonant purely hydrodynamic (nondissipative) instability is possible, with a spectrum given by[11]

$$(\omega - s\Omega_i)^2 - \frac{1}{\sqrt{2\pi}k_\perp\rho_i}(\omega - s\Omega_i)\,\omega = \frac{1}{\sqrt{2\pi}k_\perp\rho_i}\,\omega^2\left[(1+k^2r_{Di}^2)\frac{\partial\ln\sqrt{T_i}}{\partial\ln N} - k^2r_{Di}^2\right]. \tag{5.2}$$

It follows therefore that when

$$\frac{\partial\ln T_i}{\partial\ln N} < \frac{2k^2r_{Di}^2}{1+k^2r_{Di}^2}$$

and

$$k_\perp^3\rho_i^3 > \frac{c^2}{v_A^2}$$

there is developed in the plasma an instability with a maximum increment $\gamma_{max} \sim \sqrt[4]{m/M}\,\Omega_i$. This instability is stabilized by a relatively small shear of the force lines, determined by the inequality

$$\theta = sx > \sqrt[4]{\frac{m}{M}}\,\frac{\rho_i}{L_0}, \tag{5.3}$$

which should not be violated at dimensions on the order of the Larmor radius of the ions.

As noted above, the resonant frequency region in which the hydrodynamic drift-cyclotron instability develops is quite narrow, and even rare collisions can lead to stabilization of such an instability. Indeed, simple analysis of (5.2) shows[37] that when $\nu_{ii} \gtrsim (m/M)^{5/4}\Omega_i$, the resonant region is completely smeared out as a result of the ion-ion collisions, and the instability in question does not arise. However, a nonresonant dissipative instability can develop in the plasma, with a spectrum determined by the expressions ($\Delta = \omega - s\Omega_i$)

$$\mathrm{Re}\,\Delta = \frac{s\Omega_i\left(1 - \dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial}{\partial x}\ln\dfrac{N}{\sqrt{T_i}}\right)}{\sqrt{2\pi}k_\perp\rho_i\left[1+k^2r_{Di}^2 - \dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial\ln N}{\partial x}\right]}, \tag{5.4}$$

$$\gamma = \mathrm{Im}\,\Delta = -\nu_{ii}k_\perp^2\rho_i^2\,\frac{3(\pi-1)}{4\sqrt{2}}\,\frac{\left(1 - \dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial\ln NT_i^{-0,69}}{\partial x}\right)}{\left(1 - \dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial\ln NT_i^{-0,5}}{\partial x}\right)} + \gamma_e,$$

where

$$\gamma_e = -\sqrt{2\pi}\,\frac{T_i}{T_e}\,k_\perp\rho_i\,\frac{\nu_{\mathrm{eff}}\,k_z^2v_{Te}^2\,\mathrm{Re}\,\Delta^2}{s^4\Omega_i^4}\,\frac{1+\dfrac{k_y v_s^2}{s\Omega_i^2}\dfrac{\partial}{\partial x}\ln\dfrac{N}{\sqrt{T_e}}}{1-\dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial}{\partial x}\ln\dfrac{N}{\sqrt{T_i}}}. \tag{5.5}$$

c) It is easy to show that the oscillation spectrum (5.4) remains, with slight modification, also in the case when $\omega < \nu_e$, provided only $\omega, \nu_e \gg k_z^2v_{Te}^2$. All that changes is the electronic contribution γ_e to the oscillation damping decrement γ, which in this case takes the form

$$\gamma_e = -1.96\sqrt{2\pi}\,\frac{T_i}{T_e}\,k_\perp\rho_i\,\frac{k_z^2v_{Te}^2\,\mathrm{Re}\,\Delta^2}{\nu_{\mathrm{eff}}\,s^2\Omega_i^2}\,\frac{1+\dfrac{k_y v_s^2}{s\Omega_i^2}\dfrac{\partial\ln NT_e^{1,71}}{\partial x}}{1-\dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial}{\partial x}\ln\dfrac{N}{\sqrt{T_i}}}. \tag{5.6}$$

(This is a consequence of the difference between the dissipative parts of the expressions (2.8) and (2.9), due to the electron collisions.) From (5.4)—(5.6) we see that the drift-dissipative instability in the region of ion-cyclotron frequencies can develop only under the condition[36] $\omega_{dr} > s\Omega_i$. The electron collisions then lead practically always to a buildup of oscillations if $s\Omega_i > \nu_{eff}$; on the other hand, if $s\Omega_i < \nu_{eff}$, oscillations are built up only when $\partial\ln T_e/\partial\ln N < 2$. As to the ion collisions, they always play a stabilizing role, with the exception of the region

$$2 > \frac{\partial\ln T_i}{\partial\ln N} > 1.45,$$

where they unstabilize the oscillations by broadening the region of instability of the plasma.

In order to stabilize the considered nonresonant drift-dissipative instability near the cyclotron frequencies, it is necessary to have a shear satisfying the condition

$$\theta = sx > \frac{1}{\sqrt{2\pi}k_\perp^2\rho_i^2} \sim \frac{\rho_i^2}{L_0^2}. \tag{5.7}$$

This condition should not be violated for dimensions on the order of the oscillation-localization region, which can be comparable with the Larmor radius of the ions.

d) Finally, in the region $|\omega + i\nu_e| \gg k_zv_{Te}$, under the condition $\omega\nu_e \ll k_z^2v_{Te}^2$, the drift-cyclotron oscillations are possible only far from the resonant line, when $|\omega - s\Omega_i| \gg \nu_{ii}k_\perp^2\rho_i^2$. The oscillation spectrum then coincides with the spectrum (5.1), and the growth increment γ differs from (5.1) and takes the form

$$\gamma = -\frac{T_i}{T_e}\,\frac{\sqrt{2\pi}\,k_\perp\rho_i}{1-\dfrac{k_y v_{Ti}^2}{s\Omega_i^2}\dfrac{\partial}{\partial x}\ln\dfrac{N}{\sqrt{T_i}}}\left\{1.44\,\frac{\nu_{\mathrm{eff}}\,\mathrm{Re}\,\Delta^2}{k_z^2v_{Te}^2}\left(1-\frac{k_y v_s^2}{s\Omega_i^2}\frac{\partial\ln NT_e^{-0,56}}{\partial x}\right) + \frac{T_e}{T_i}\,\nu_{ii}k_\perp\rho_i\,\frac{3(\pi-1)}{8\sqrt{\pi}}\left(1-\frac{k_y v_{Ti}^2}{s\Omega_i^2}\frac{\partial\ln NT_i^{-0,69}}{\partial x}\right)\right\}. \tag{5.8}$$

The change of the expression for γ is due to the change of the dissipative contribution to $\delta\epsilon_e$, which in the frequency region under consideration is determined by expression (2.10) and not (2.7), as in the case investigated above. It is seen from (5.8) that electron collisions lead to a buildup of oscillations, particularly when $s\Omega_i < \omega_{dr}$. The role of the ion collisions and the condition for stabilization of the instability by the shear then remain unchanged (see the analysis following formulas (5.1)).

6. CONCLUSION

We have considered above practically all the possible spectra of the unstable oscillations of an inhomogeneous plasma contained by a strong magnetic field, with the exception of long-wave oscillations, for which the conditions $k_\perp\rho_i \ll 1$ and $\nu_e \gg \omega$, k_zv_{Te} are satisfied, and for which $\nu_i \gg \omega$, k_zv_{Ti}. (To describe such oscillations, we can use the equations of two-fluid hydrodynamics[29]; an investigation of the instability of an inhomogeneous plasma with respect to such hydrodynamic oscillations was initiated in[30,31]). Summarizing the analysis, we can draw the following conclusions:

1. Ion-ion collisions far from always stabilize the drift instabilities of an inhomogeneous plasma (as follows from the theory using the model collision integral) and, to the contrary, even lead in a number of cases to the broadening of the plasma instability region, particularly when it comes to a plasma with an inhomogeneous temperature or when it comes to short-wave plasma oscillations.

Electron collisions (friction of the electrons against the ions) lead as a rule to a buildup of drift-dissipative plasma oscillations, but for some short-wave oscillation branches they also play a stabilizing role (see (4.4) and (4.6)).

2. The nonvanishing but small curvature of the force lines of the confining magnetic field ($R \gg L_0$) exerts a strong influence only on the relatively high-frequency oscillations of an inhomogeneous plasma, with a frequency larger than the particle drift frequencies. Namely, in systems with positive curvature of the force lines of the magnetic field, there exist the known flute and dissipative instabilities. To the contrary, the influence of the curvature of force lines of the field on the low-frequency drift-dissipative instabilities is generally speaking insignificant. Only in the case of a strongly nonisothermal plasma ($T_e/T_i \gtrsim R/L_0 \gg 1$ or $T_i/T_e \gtrsim R/L_0 \gg 1$) can this influence become appreciable, and in systems with positive curvature it reduces as a rule to a broadening of the region of instability of the plasma, while in minimum-B systems it has a stabilizing effect.

3. An analysis shows that an effective means of stabilizing drift-dissipative instabilities of an inhomogeneous plasma may be the shear of the force lines of the confining magnetic field. In this case, the most difficult to stabilize are the short-wave oscillations, since their stabilization requires a shear which is appreciable for dimensions on the order of the oscillation-localization region, which is comparable with the gyroscopic radius of the ions. When dealing with the most dangerous long-wave oscillations (flute or drift-dissipative), their stabilization requires a smaller shear than short-wave oscillations.

Table I

Existence Conditions	Spectra of long wave oscillations $k_\perp \rho_i \ll 1$, $k_z v_{Te} \gg \omega, \nu_e$; $\omega \gg \nu_i, k_z v_{Ti}$	Role of electron collisions	Role of ion collisions	Stabilization by shear
$\omega_1 \gg k_z v_s$, $\frac{k_z^2}{k_\perp^2} < \frac{\omega_1^2}{\Omega_i^2}$	$\omega_1 \approx -\frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln N}{\partial x}$, $\gamma_1 \approx \sqrt{\frac{\pi}{2}}\frac{\omega_1^2}{\|k_z\| v_{Te}}\left\{\frac{k_\perp^2 v_s^2}{\Omega_i^2}\left(1+\frac{T_i}{T_e}\frac{\partial \ln NT_i}{\partial \ln N}\right) - \frac{1}{2}\frac{\partial \ln T_e}{\partial \ln N} - \frac{g}{v_s^2 \frac{\partial \ln N}{\partial x}}\right\} - \frac{7}{10}\nu_{ii} k_\perp^4 \rho_i^4 \left(1+\frac{T_e}{T_i} - \frac{3}{28}\frac{\partial \ln T_i}{\partial \ln N}\right)$	Insignificant	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} > \frac{28}{3}\left(1+\frac{T_e}{T_i}\right)$	$\theta > \sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}$
$\omega_2 \ll k_z v_s$, $T_e \gg T_i$	$\omega_2 = k_z^2 \Omega_i / k_y \frac{\partial \ln N}{\partial x}$, $\gamma_2 \approx -\sqrt{\frac{\pi}{2}}\frac{\omega_2^2}{\|k_z\| v_{Te}}\left(1-\frac{1}{2}\frac{\partial \ln T_e}{\partial \ln N} - \frac{g}{v_s^2 \frac{\partial \ln N}{\partial x}}\right) - \frac{8}{5}\nu_{ii}\frac{k_z^2 v_{Ti}^2}{\omega_2^2}$	The same	Always stabilizing	$\theta > \sqrt{\frac{T_e}{T_i}}\frac{\rho_i}{L_0}$
$\omega \ll \omega_{dr}$, $\frac{\partial \ln T_i}{\partial \ln N} \gg 1$	$\omega_3^2 = -k_z^2 v_{Ti}^2 \frac{\partial \ln T_i}{\partial \ln N}$,	The same	Insignificant	$\theta > \sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}$
	$\omega_4^3 = -k_z^2 v_s^2 \frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}$,	» »	»	$\theta > \sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}$
	$\omega_5^2 = in\Omega_i \|k_y S\| \rho_i^2 \frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}$	» »	»	$S > \frac{T_i}{T_e}\frac{1}{L_0}$

Table II

Existence Conditions	Spectra of long wave oscillations $k_\perp \rho_i \ll 1$, $\omega \gg \nu_{e,i}, k_z v_{Te,i}$	Role of electron collisions	Role of ion collisions	Stabilization by shear
$k_\perp^2 \rho_i^2 < 4\frac{L_0}{R}$	$\omega_1^2 \approx \frac{1}{1+(v_A^2/c^2)} g \frac{\partial \ln N}{\partial x}$	Insignificant	Insignificant	$\theta > \sqrt{\frac{m}{M}\frac{L_0}{R}}$
$k_\perp^2 \rho_i^2 > 4\frac{L_0}{R}$	$\omega_2 = \frac{k_y v_{Ti}^2}{\Omega_i[1+(v_A^2/c^2)]}\frac{\partial \ln NT_i}{\partial x}$, $\gamma_2 = \frac{7}{10}\frac{\nu_{ii} k_\perp^2 \rho_i^2}{1+(v_A^2/c^2)}\frac{\partial \ln N}{\partial \ln NT_i}\left[\frac{v_A^2}{c^2} - \left(\frac{31}{28}+\frac{3}{28}\frac{v_A^2}{c^2}\right)\frac{\partial \ln T_i}{\partial \ln N}\right]$	The same	Destabilizing when $-1 < \frac{\partial \ln T_i}{\partial \ln N} < \frac{28}{31}\frac{v_A^2}{c^2}$	$\theta > \sqrt{\frac{m}{M}\frac{T_i}{T_e}}\frac{\rho_i}{L_0}$
$k_\perp^2 \rho_i^2 > 4\frac{L_0}{R}$	$\omega_3 \approx -\frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln T_e}{\partial x}$, $\gamma_3 \approx \nu_{eff}\frac{\partial \ln T_e^{3/2}}{\partial \ln NT_e}$	Destabilizing when $\frac{\partial \ln T_e}{\partial \ln N} > 0$, $\frac{\partial \ln T_e}{\partial \ln N} < -1$	Insignificant	$\theta > \sqrt{\frac{m}{M}\frac{T_e}{T_i}}\frac{\rho_i}{L_0}$
$\omega_4 \ll \omega_{dr}$	$\omega_4^2 = -\frac{k_z^2}{k_\perp^2}\frac{M}{m}\frac{T_e}{T_i}\Omega_i^2 \frac{\partial \ln NT_e}{\partial \ln NT_i}$	Insignificant	The same	$\theta > \sqrt{\frac{m}{M}}\frac{\rho_i}{L_0}$
$\omega_5 \ll \omega_{dr}$, $T_e \gg T_i$	$\omega_5^2 = \frac{k_z^2}{k_\perp^2}\frac{M}{m}\frac{T_e}{T_i}\Omega_i^2 \frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln NT_e}{\partial x}$	The same	The same	$\theta > \sqrt{\frac{m}{M}\frac{T_i}{T_e}}\frac{\rho_i}{L_0}$

Table III

Existence Conditions	Spectra of long wave oscillations $k_\perp\rho_i \ll 1$, $\nu_e \gg \omega$, $k_z v_{Te}$, $\omega\nu_e \gg k_z^2 v_{Te}^2$, $\omega \gg \nu_i$, $k_z v_{Ti}$	Role of electron collisions	Role of ion collisions	Stabilization by shear
$k_\perp^2\rho_i^2 < 4\frac{L_0}{R}$	$\omega_1^2 \approx g\frac{\partial \ln N}{\partial x}$	Insignificant	Insignificant	$\theta > \sqrt{\frac{m}{M}\frac{\nu_{\text{eff}}}{\Omega_i}\frac{\rho_i}{\sqrt{RL_0}}}$
$k_\perp^2\rho_i^2 > 4\frac{L_0}{R}$	$\omega_2 \approx \frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln NT_i}{\partial x}$, $\gamma_2 \approx \frac{7}{10}\nu_{ii}k_\perp^2\rho_i^2\frac{\partial \ln N}{\partial \ln NT_i}\left(\frac{v_A^2}{c^2} - \frac{31}{28}\frac{\partial \ln T_i}{\partial \ln N}\right)$	The same	Destabilizing when $-1 < \frac{\partial \ln T_i}{\partial \ln N} < \frac{28}{31}\frac{v_A^2}{c^2}$	$\theta > \sqrt{\frac{m}{M}\frac{\nu_{\text{eff}}}{\Omega_i}\frac{\rho_i}{L_0}}$
$k_\perp^2\rho_i^2 < 4\frac{L_0}{R}$	$\gamma_3 = -i\omega_3 = \left\lvert g\frac{\partial \ln N}{\partial x}\right\rvert^{2/3}\left(\frac{k_y^2 c^2}{4\pi S^2\sigma v_A^2}\right)^{1/3}$	Always destabilizing	Insignificant	$\theta > \sqrt{\frac{m}{M}\frac{\rho_i}{L_0}\sqrt{\frac{L_0}{R}}}$
$k_\perp^2\rho_i^2 > 4\frac{L_0}{R}$	$\gamma_4 = -i\omega_4 = -\frac{c^2}{4\pi\sigma}\frac{k_y^2 g}{k_z^2 v_A^2}\frac{\partial \ln N}{\partial x}$	The same	The same	$\theta > \sqrt{\frac{m}{M}\frac{\rho_i}{L_0}\sqrt{\frac{L_0}{R}}}$
$\omega_{\text{dr}} \ll \omega_s$	$\omega_5 = -\frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln NT_e^{1,71}}{\partial x}$, $\gamma_5 = \frac{\omega_5^2}{1{,}96\,\omega_s}\left(1 + \frac{T_i}{T_e}\frac{\partial \ln NT_i}{\partial \ln NT_e^{1,71}}\right)$	The same	The same	$\theta > \min\left\{\begin{array}{l}\sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}\\ \sqrt{\frac{\nu_{\text{eff}}}{\Omega_e}}\end{array}\right.$
$\omega_s \ll \omega_{\text{dr}}$	$\gamma_6 = -i\omega_6 = 1{,}96\,\omega_s\frac{T_e}{T_i}\frac{\partial \ln NT_e^{1,71}}{\partial \ln NT_i}$	The same	The same	$\theta > \min\left\{\begin{array}{l}\sqrt{\frac{T_e}{T_i}}\frac{\rho_i}{L_0}\\ \sqrt{\frac{\nu_{\text{eff}}}{\Omega_e}}\frac{\rho_i'}{L_0}\end{array}\right.$
$\frac{k_z^2}{k_\perp^2}\Omega_i \ll \omega_{\text{dr}}^2$, $T_i \gg T_e$	$\gamma_7 = -i\omega_7 = \frac{m}{M}\frac{\nu_{\text{eff}}\,T_i}{1.96\,T_e}\frac{\partial \ln NT_i}{\partial \ln NT_e^{1,71}}$	The same	The same	$\theta > \min\left\{\begin{array}{l}\rho_i/L_0\\ \frac{\nu_{\text{eff}}}{\Omega_e}\frac{T_i}{T_e}\end{array}\right.$

Table IV

Existence Conditions	Spectra of long wave oscillations $k_\perp\rho_i \ll 1$, $\nu_e \gg \omega$, $k_z v_{Te}$, $\omega\nu_e \ll k_z^2 v_{Te}^2$, $\omega \gg \nu_i$, $k_z v_{Ti}$	Role of electron collisions	Role of ion collisions	Stabilization by shear
$\omega_1 \gg k_z v_s$, $\frac{k_z^2}{k_\perp^2} < \frac{\omega_1^2}{\Omega_i^2}$	$\omega_1 \approx -\frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln N}{\partial x}$, $\gamma_1 \approx 1{,}44\frac{\omega_1^2\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\left\{\frac{k_\perp^2 v_s^2}{\Omega_i^2}\left(1+\frac{T_i}{T_e}\frac{\partial \ln NT_i}{\partial \ln N}\right) - 0.56\frac{\partial \ln T_e}{\partial \ln N} - \frac{g}{v_s^2\frac{\partial \ln N}{\partial x}}\right\} - \frac{7}{10}\nu_{ii}k_\perp^4\rho_i^4\left(1+\frac{T_e}{T_i}-\frac{3}{28}\frac{\partial \ln T_i}{\partial \ln N}\right)$	Destabilizing when $\frac{\partial \ln T_e}{\partial \ln N} < 1.8\frac{k_\perp^2 v_s^2}{\Omega_i^2}\left(1+\frac{T_i}{T_e}\frac{\partial \ln NT_i}{\partial \ln N}\right)$	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} > \frac{28}{3}\left(1+\frac{T_e}{T_i}\right)$	$\theta > \min\left\{\begin{array}{l}\sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}\\ \frac{L_0\nu_{\text{eff}}}{v_{Te}}\end{array}\right.$
$\omega_2 \ll k_z v_s$, $T_e \gg T_i$	$\omega_2 = \frac{k_z^2\Omega_i}{k_y\frac{\partial \ln N}{\partial x}}$, $\gamma_2 \approx -1{,}44\frac{\omega_2^2\nu_{\text{eff}}}{k_z^2 v_{Te}^2}\left(1 - \frac{\partial \ln T_e^{0,56}}{\partial \ln N}\right) - \frac{8}{5}\nu_{ii}\frac{k_z^2 v_{Ti}^2}{\omega_2^2}$	Destabilizing when $\frac{\partial \ln T_e}{\partial \ln N} > 1.8$	Always destabilizing	$\theta > \min\left\{\begin{array}{l}\sqrt{\frac{T_e}{T_i}}\frac{\rho_i}{L_0}\\ \frac{L_0\nu_{\text{eff}}}{v_{Te}}\end{array}\right.$
$\omega \ll \omega_{\text{dr}}$, $\frac{\partial \ln T_i}{\partial \ln N} \gg 1$	$\omega_3^2 = -k_z^2 v_{Ti}^2\frac{\partial \ln T_i}{\partial \ln N}$,	Insignificant	Insignificant	$\theta > \min\left\{\begin{array}{l}\sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}\\ \frac{L_0\nu_{\text{eff}}}{v_{Te}}\end{array}\right.$
	$\omega_4^3 = -k_z^2 v_s^2\frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}$,	»	»	$\theta > \min\left\{\begin{array}{l}\sqrt{1+\frac{T_e}{T_i}}\frac{\rho_i}{L_0}\\ \frac{L_0\nu_{\text{eff}}}{v_{Te}}\end{array}\right.$
	$\omega_5^2 = in\,\Omega_i\,\lvert k_y S\rvert\,\rho_i^2\frac{k_y v_s^2}{\Omega_i}\frac{\partial \ln T_i}{\partial x}$	»	»	$\theta > \min\left\{\begin{array}{l}\frac{T_i}{T_e}\frac{1}{L_0}\\ \frac{L_0\nu_{\text{eff}}}{v_{Te}}\end{array}\right.$

APPENDICES

I. CALCULATION OF $\delta\epsilon_e$

Equation (2.3) for the electrons can be written in integral form (we neglect the gravitational drift of the electrons):

$$\delta f(x) = -\frac{1}{\Omega_e}\int_{\infty}^{\varphi} d\varphi'\left(\frac{e}{mc}\nabla\Phi\frac{\partial f_0}{\partial \mathbf{v}} + I_{ee} + I_{ei}\right) \times \exp\left\{\frac{i}{\Omega_e}\int_{\varphi}^{\varphi'} d\varphi''\left(\omega - k_z v_z - k_y v_\perp \sin\varphi''\right)\right\}. \tag{I.1}$$

Integration in this equation is carried out along the characteristic

$$x + \frac{v_\perp \sin\varphi}{\Omega_e} = x' + \frac{v_\perp \sin\varphi'}{\Omega_e} = \text{const.}$$

This makes it possible to write under the integral sign

Table V

Existence Conditions	Spectra of long wave oscillations $\omega \ll \Omega_i,\ k_\perp \rho_i \gg 1$	Role of electron collisions	Role of ion collisions	Stabilization by shear
$\omega,\ \nu_e \ll k_z v_{Te}$, $\omega \gg k_z v_{Ti}$, $\omega \gg \nu_{ii} k_\perp^2 \rho_i^2$	$\omega \approx -\frac{T_e}{T_e+T_i(1+k^2 r_{De}^2)} \frac{v_{Ti}}{\sqrt{2\pi}} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}}$, $\gamma \approx \frac{\pi\omega^2}{\lvert k_z \rvert v_{Te}} k_\perp \rho_i \frac{\partial \ln N/\sqrt{T_e}}{\partial \ln N/\sqrt{T_i}} \left(1 - \frac{g}{v_s^2 \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}}}\right) - \nu_{ii} k_\perp^2 \rho_i^2 \frac{3(\pi+1)}{4\sqrt{2}} \frac{\partial \ln N T_i^{-0.69}}{\partial \ln N T_i^{-0.5}}$	Insignificant	Destabilizing when $1.45 < \frac{\partial \ln T_i}{\partial \ln N} < 2$	$\theta > \frac{1}{\sqrt{2\pi}} \frac{1}{k_\perp \rho_i} \frac{\rho_i}{L_0}$
$\omega \gg \nu_e k_z v_{Te}$, $\omega \gtrless \nu_{ii} k_\perp^2 \rho_0^2$	$\omega \approx \frac{1}{1+k^2 r_{Di}^2} \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N}{\partial x}$, $\gamma = -\frac{T_i}{T_e} \nu_{\text{eff}} \frac{k_z^2 v_{Te}^2}{\omega^2} \left(\frac{1}{1+k^2 r_{Di}^2} + \frac{T_e}{T_i} \frac{\partial \ln N T_e^{-0.5}}{\partial \ln N}\right) + \gamma_i$ *)	Destabilizing when $\frac{\partial \ln T_e}{\partial \ln N} > 2\left(1+\frac{T_i}{T_e}\right)$	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} < 4.45\, k^2 r_{Di}^2$	$\theta > \sqrt{\frac{m}{M} \frac{T_e}{T_i}} \frac{\rho_i}{L_0}$
$\nu_e \gg \omega,\ k_z v_{Te}$, $\omega \nu_e \gg k_z^2 v_{Te}^2$, $\omega \gtrless \nu_{ii} k_\perp^2 \rho_i^2$	$\omega \approx \frac{1}{1+k^2 r_{Di}^2} \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N}{\partial x}$, $\gamma = -1.96 \frac{T_i}{T_e} \frac{k_z^2 v_{Te}^2}{\nu_{\text{eff}}} \left[\frac{1}{1+k^2 r_{Di}^2} + \frac{T_e}{T_i}\left(1 + \frac{\partial \ln T_e^{1.71}}{\partial \ln N}\right)\right] + \gamma_i$ *)	Always destabilizing	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} < 4.45\, k^2 r_{Di}^2$	$\theta > \min \times \left\{ \begin{array}{l} \sqrt{\frac{M}{m} \frac{T_i}{T_e} \frac{\nu_{\text{eff}}}{\Omega_e k_\perp \rho_i}} \\ \sqrt{\frac{\nu_{\text{eff}}}{\Omega_e} \frac{T_i}{T_e} \frac{1}{k_\perp \rho_i}} \frac{\rho_i}{L_0} \end{array} \right.$
$\nu_e \gg \omega,\ k_z v_{Te}$, $\omega \nu_e \ll k_z^2 v_{Te}^2$, $\omega \gg \nu_{ii} k_\perp^2 \rho_i^2$	$\omega \approx -\frac{T_e}{T_e+T_i(1+k^2 r_{De}^2)} \frac{v_{Ti}}{\sqrt{2\pi}} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}}$, $\gamma \approx 1.44 \frac{\sqrt{2\pi}\,\omega^2 k_\perp \rho_i}{k_z^2 v_{Te}^2} \nu_{\text{eff}} \frac{\partial \ln N T_e^{-0.56}}{\partial \ln N T_i^{-0.5}} \times \left(1 - \frac{g}{v_s^2 \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}}}\right) - \frac{3\pi+3}{4\sqrt{2}} \frac{\nu_{ii} k_\perp^2 \rho_i^2\, \partial \ln N T_i^{-0.69}}{\partial \ln N T_i^{-0.5}}$	Always destabilizing	Destabilizing when $1.45 < \frac{\partial \ln T_i}{\partial \ln N} < 2$	$0 > \min \left\{ \begin{array}{l} \sqrt{\frac{M}{m} \frac{T_i}{T_e} \frac{\nu_{\text{eff}}}{\Omega_e k_\perp \rho_i}} \\ \frac{1}{\sqrt{2\pi}} \frac{1}{k_\perp \rho_i} \frac{\rho_i}{L_0} \end{array} \right.$

*) Here $\gamma_i = -\frac{\omega^2}{\nu_{ii}} \frac{2C_0 C_1}{3\pi k_\perp^3 \rho_i^3} \left(\frac{\partial \ln T_i}{\partial \ln N} - \frac{4.45 k^2 r_{Di}^2}{1+k^2 r_{Di}^2}\right)$ if $\omega \ll \nu_{ii} k_\perp^2 \rho_i^2$,

$\gamma_i = -\frac{3(3\pi+2)}{32\sqrt{\pi}} \nu_{ii} k_\perp \rho_i \left(\frac{\partial \ln T_i}{\partial \ln N} - \frac{1.45 k^2 r_{Di}^2}{1+k^2 r_{Di}^2}\right)$ if $\omega \gg \nu_{ii} k_\perp^2 \rho_i^2$.

$$\frac{\partial f_{0e}}{\partial v_i} = \left(-\frac{v_i}{v_{Te}^2} + \frac{\delta_{yi}}{\Omega_e} a_e\right) f_{0e},$$

$$a_e = \frac{\partial \ln N}{\partial x} + \frac{\partial \ln T_e}{\partial x}\left(-\frac{3}{2} + \frac{v^2}{2v_{Te}^2}\right). \tag{I.2}$$

The integral equation (I.1), will be solved in the geometric-optics approximation, writing the functions Φ and δf_e in the form $\exp i\int_0^x k_x d_x$. Taking into account the inequalities $\omega \ll \Omega_e$ and $kv_{Te} \ll \Omega_e$, we get from (I.1)

$$(\omega - k_z v_z)\,\delta f_e = \frac{e\Phi}{T_e}\left(k_z v_z - \frac{k_y v_{Te}^2}{\Omega_e} a_e\right) f_{0e} + i\,(I_{ee} + I_{ei}). \tag{I.3}$$

In the solution of this equation it is necessary to distinguish between three limiting cases.

a) $|\omega + i\nu_e \ll k_z v_{Te}$; in this case the collision integral in (I.3) can be neglected. As a result we have

$$\delta f_e = -\frac{e}{T_e}\Phi f_{0e} + \frac{e}{T_e}\Phi \frac{f_{0e}}{\omega - k_z v_z}\left(\omega - \frac{k_y v_{Te}^2}{\Omega_e} a_e\right). \tag{I.4}$$

This expression leads to (2.7). It can be shown that allowance for the collisions yields corrections on the order of $\nu_e^2/k_z^2 v_{Te}^2$.

b) $|\omega + i\nu_e| \gg k_z v_{Te}$, $\omega \gg \nu_e$; in this case the collision integral in (I.3) is also small, and in first approximation we get expression (I.4), in which $\omega \gg k_z v_z$. However, precisely because of this inequality, allowance for the particle collisions becomes essential, for when $\omega \gg k_z v_z$ expression (I.4) leads to an exponentially small dissipative term in $\delta\epsilon_e$. The correction to (I.4), due to the collisions, is

$$\delta f_e^{(1)} = \frac{i}{\omega}(I_{ee} + I_{ei}). \tag{I.5}$$

Table VI

Existence Conditions	Spectra of long wave oscillations $\omega \approx s\Omega_i,\ k_\perp\rho_i \gg 1$	Role of electron collisions	Role of ion collisions	Stabilization by shear
$k_z v_{Te} \gg \omega,\ \nu_e,$ $\Delta \gg \nu_{ii} k_\perp^2 \rho_i^2$	$\mathrm{Re}\,\Delta = -\frac{T_e}{T_e + T_i(1 + k^2 r_{De}^2)} \frac{v_{Ti}}{\sqrt{2\pi}} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}},$ $\gamma = \frac{\pi k_\perp \rho_i \,\mathrm{Re}\,\Delta^2}{\lvert k_z \rvert v_{Te}} \frac{\partial \ln N/\sqrt{T_e}}{\partial \ln N/\sqrt{T_i}} - \frac{3(\pi+1)}{4\sqrt{2}} \nu_{ii} k_\perp^2 \rho_i^2 \frac{\partial \ln N T_i^{-0.69}}{\partial \ln N/\sqrt{T_i}}$	Insignificant	Destabilizing when $1.45 < \frac{\partial \ln T_i}{\partial \ln N} < 2$	$\theta > \frac{1}{\sqrt{2\pi}} \frac{\rho_i^2}{L_0^2}$
$\omega \gg \nu_e,\ k_z v_{Te},$ $\Delta \ll \nu_{ii} k_\perp^2 \rho_i^2$	$\omega = \frac{1}{1 + k^2 r_{Di}^2} \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N}{\partial x},$ $\gamma = -\frac{\omega^2}{\nu_{ii}} \frac{2C_0 C_1}{3\pi k_\perp^3 \rho_i^3} \left(\frac{\partial \ln T_i}{\partial \ln N} - \frac{4{,}45 k^2 r_{Di}^2}{1 + k^2 r_{Di}^2}\right) - \nu_{\text{eff}} \frac{T_i}{T_e} \frac{k_z^2 v_{Te}^2}{\omega^2} \left(\frac{1}{1 + k^2 r_{Di}^2} + \frac{T_e}{T_i} \frac{\partial \ln N/\sqrt{T_e}}{\partial \ln N}\right)$	Destabilizing when $\frac{\partial \ln T_e}{\partial \ln N} > 2\left(1 + \frac{T_i}{T_e}\right)$	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} < 4.45 k^2 r_{Di}^2$	$\theta > \sqrt{\frac{m}{M} \frac{T_e}{T_i}} \frac{\rho_i}{L_0}$
$\nu_e \gg \omega,\ k_z v_{Te},$ $\Delta \ll \nu_{ii} k_\perp^2 \rho_i^2$	$\omega = \frac{1}{1 + k^2 r_{Di}^2} \frac{k_y v_{Ti}^2}{\Omega_i} \frac{\partial \ln N}{\partial x},$ $\gamma = -\frac{\omega^2}{\nu_{ii}} \frac{2C_0 C_1}{3\pi k_\perp^3 \rho_i^3} \left(\frac{\partial \ln T_i}{\partial \ln N} - \frac{4.45 k^2 r_{Di}^2}{1 + k^2 r_{Di}^2}\right) - 1.96 \frac{T_i}{T_e} \frac{k_z^2 v_{Te}^2}{\nu_{\text{eff}}} \left(\frac{1}{1 + k^2 r_{Di}^2} + \frac{T_e}{T_i} \frac{\partial \ln N T_e^{1.71}}{\partial \ln N}\right)$	Always destabilizing	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} < 4.45 k^2 r_{Di}^2$	$\theta > \frac{\nu_{\text{eff}}}{\Omega_e} \sqrt{\frac{M}{m} \frac{T_i}{T_e}} \frac{\rho_i}{L_0}$
$\omega > \left(\frac{M}{m}\right)^{5/4} \nu_{ii},\ k_z v_{Te},$ $\Delta \gg \nu_{ii} k_\perp^2 \rho_i^2$	$\mathrm{Re}\,\Delta \approx \frac{s\Omega_i}{2\sqrt{2\pi} k_\perp \rho_i},\ \gamma \leqslant \left(\frac{m}{M}\right)^{1/4} \Omega_i$	Insignificant	Insignificant	$\theta > \left(\frac{m}{M}\right)^{1/4} \frac{\rho_i}{L_0}$
$\Delta \gg \nu_{ii} k_\perp^2 \rho_i^2,$ $k_z v_{Te} \ll \omega < \left(\frac{M}{m}\right)^{5/4} \nu_{ii}$	$\mathrm{Re}\,\Delta = \frac{s\Omega_i}{\sqrt{2\pi} k_\perp \rho_i} \frac{\partial \ln N/\sqrt{T_i}}{\partial \ln N},$ $\gamma = -\frac{3(\pi+1)}{4\sqrt{2}} \nu_{ii} k_\perp^2 \rho_i^2 \frac{\partial \ln N T_i^{-0.69}}{\partial \ln N/\sqrt{T_i}} + \frac{\nu_{\text{eff}}\ \mathrm{Re}\,\Delta^2 k_z^2 v_{Te}^2}{\omega^4} \sqrt{2\pi} k_\perp \rho_i \frac{\partial \ln N/\sqrt{T_e}}{\partial \ln N/\sqrt{T_i}}$	Always destabilizing	Destabilizing when $1.45 < \frac{\partial \ln T_i}{\partial \ln N} < 2$	$\theta > \frac{1}{\sqrt{2\pi}} \frac{\rho_i^2}{L_0^2}$
$\nu_e \gg \omega,\ k_z v_{Te},$ $\omega \nu_e \gg k_z^2 v_{Te}^2,$ $\Delta \gg \nu_{ii} k_\perp^2 \rho_i^2$	$\mathrm{Re}\,\Delta = \frac{s\Omega_i}{\sqrt{2\pi} k_\perp \rho_i} \frac{\partial \ln N/\sqrt{T_i}}{\partial \ln N},$ $\gamma = -\frac{3(\pi+1)}{4\sqrt{2}} \nu_{ii} k_\perp^2 \rho_i^2 \frac{\partial \ln N T_i^{-0.69}}{\partial \ln N/\sqrt{T_i}} + 1.96 \frac{\mathrm{Re}\,\Delta^2 k_z^2 v_{Te}^2}{\omega^2 \nu_{\text{eff}}} \sqrt{2\pi} k_\perp \rho_i \frac{\partial \ln T_e^{1.71}}{\partial \ln N/\sqrt{T_i}}$	Destabilizing when $\frac{\partial \ln T_i}{\partial \ln N} > 2$	Destabilizing when $1.45 < \frac{\partial \ln T_i}{\partial \ln N} < 2$	$\theta > \frac{1}{\sqrt{2\pi}} \frac{\rho_i^2}{L_0^2}$
$\nu_e \gg \omega,\ k_z v_{Te},$ $\omega \nu_e \ll k_z^2 v_{Te}^2,$ $\Delta \gg \nu_{ii} k_\perp^2 \rho_i^2$	$\mathrm{Re}\,\Delta = -\frac{T_e}{T_e + T_i(1 + k^2 r_{De}^2)} \frac{v_{Ti}}{\sqrt{2\pi}} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}},$ $\gamma = 1.44 \frac{\sqrt{2\pi}\,\mathrm{Re}\,\Delta^2 \nu_{\text{eff}}}{k_z^2 v_{Te}^2} k_\perp \rho_i \frac{\partial \ln N T_e^{-0.56}}{\partial \ln N/\sqrt{T_i}} - \frac{3(\pi+1)}{4\sqrt{2}} \nu_{ii} k_\perp^2 \rho_i^2 \frac{\partial \ln N T_i^{-0.69}}{\partial \ln N/\sqrt{T_i}}$	Always destabilizing	Destabilizing when $1.45 < \frac{\partial \ln T_i}{\partial \ln N} < 2$	$\theta > \frac{1}{\sqrt{2\pi}} \frac{\rho_i^2}{L_0^2}$

It is necessary here to substitute (I.4) in the collision integral. When calculating the corrections to the charge density induced in the plasma with the aid of (1.5), the only contributing term is the one due to the electron-ion collisions; the electron-electron collisions make no contribution, owing to momentum conservation. We obtain ultimately for $\delta\epsilon_e$ formula (2.8).

c) $|\omega + i\nu_e| \gg k_z v_{Te}$, $\nu_e \gg \omega$; in this case the collision term in (I.3) becomes the principal one and the equation must be solved by the Chapman-Enskog method. We introduce the function

$$\delta f_e = -\frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{e}{T_e} a_e \Phi f_{0e} + F_e. \tag{I.6}$$

From (I.3) we get

$$\frac{ie}{T_e} k_z v_z \Phi f_{0e} \left[1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln NT_e}{\partial x} + \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln T_e}{\partial x} \left(\frac{5}{2} - \frac{v^2}{2v_{Te}^2}\right)\right] = I_{ee} + I_{ei}. \tag{I.7}$$

Expanding F_e in Sonine-Laguerre polynomials and confining ourselves to two terms of the expansion (for details see[47]):

$$F_e = v_z f_{0e} \left[a_0 + a_1 \left(\frac{5}{2} - \frac{v^2}{2v_{Te}^2}\right)\right], \tag{I.8}$$

we obtain from (I.7) the following system of algebraic equations for the determination of the coefficients a_0 and a_1:

$$\frac{e}{T_e} k_z \Phi \left(1 - \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln NT_e}{\partial x}\right) = i\nu_{\text{eff}} \left(a_0 + \frac{3}{2} a_1\right),$$
$$\frac{e}{T_e} k_z \Phi \cdot \frac{5}{2} \frac{k_y v_{Te}^2}{\omega\Omega_e} \frac{\partial \ln T_e}{\partial x} = i\nu_{\text{eff}} \left(\frac{3}{2} a_0 + \frac{13 + 4\sqrt{2}}{4} a_1\right). \tag{I.9}$$

From the expression obtained in this manner for F_e we get the current due to F_e, and then with the aid of the continuity equation we determine the corresponding contribution to the electron charge density induced in the plasma. This is precisely how expression (2.9) was derived under the condition $\omega\nu_e \gg k_z^2 v_{Te}^2$. We note that, unlike case b), in the present case both the electron-ion and the electron-electron collisions are important.

To calculate $\delta\epsilon_e$ in the frequency region under consideration, a very convenient method is proposed in[41]. We start from the equations of continuity and heat balance for the electrons

$$\begin{gathered}\frac{\partial eN_e}{\partial t}+\operatorname{div}\mathbf{j}_e=0,\\ \frac{3}{2}\frac{\partial N_eT_e}{\partial t}+\operatorname{div}\mathbf{q}_e=\mathbf{E}\mathbf{j}_e+Q,\end{gathered}\tag{I.10}$$

where Q is the heat transferred from the electrons to the ions. Confining ourselves to oscillations with $\omega \gg m\nu_{eff}/M$, we can put $Q = 0$. The expressions for q_e and j_e under the conditions $\nu_{eff} \gg \omega$, $\nu_{eff} \gg k_z v_{Te}$, and $\Omega_e \gg \nu_{eff}$, $k_\perp v_{Te}$ take the form[47,48]

$$\left.\begin{aligned}\mathbf{j}_{\perp e}&=e\int f_e v_\perp\,dp=-\frac{eNT_e}{m\Omega_e}\frac{1}{B}\left[\mathbf{B}\left(\frac{e\mathbf{E}}{T_e}-\frac{\partial\ln NT_e}{\partial\mathbf{r}}\right)\right],\\ j_{ze}&=e\int f_e v_z\,dp=1.96\frac{eNT_e}{m\nu_{eff}}\left(\frac{eE_z}{T_e}-\frac{\partial\ln NT_e^{1\cdot 71}}{\partial z}\right),\\ \mathbf{q}_{\perp e}&=\frac{m}{2}\int f_e v^2\mathbf{v}_\perp\,dp=-\frac{5}{2}\frac{NT_e^2}{m\Omega_e}\frac{1}{B}\left[\mathbf{B}\left(\frac{e\mathbf{E}}{T_e}-\frac{\partial\ln NT_e^2}{\partial\mathbf{r}}\right)\right],\\ q_{ze}&=\frac{m}{2}\int f_e v^2 v_z\,d\mathbf{p}=\frac{5}{2}\cdot 2.52\frac{NT_e^2}{m\nu_{eff}}\left(\frac{eE_z}{T_e}-\frac{\partial\ln NT_e^{2\cdot 21}}{\partial z}\right).\end{aligned}\right\}\tag{I.11}$$

Varying these expressions with respect to small deviations from equilibrium

$$N_e\to N_e+\delta N_e,\quad T_e\to T_e+\delta T_e,\quad \mathbf{E}\to\delta\mathbf{E}=-\nabla\Phi\tag{I.12}$$

and using (I.10), we easily obtain δN_e in the geometrical optics approximation, and consequently

$$\delta\varepsilon_e=-\frac{4\pi e\delta N_e}{k^2\Phi}=\frac{\omega_L^2}{k^2v_{Te}^2}\times$$
$$\times\frac{\left(\frac{k_yv_{Te}^2}{\omega\Omega_e}\frac{\partial\ln N}{\partial x}+i\cdot 1.96\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\right)\left(1+i\cdot 9.28\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\right)-i\cdot 3.35\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\left(\frac{k_yv_{Te}^2}{\omega\Omega_e}\frac{\partial\ln NT_e}{\partial x}+i\cdot 4.2\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\right)}{\left(1+i\cdot 1.96\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\right)\left(1+i\cdot 9.28\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\right)-i\cdot 3.35\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\left(1+i\cdot 4.2\frac{k_z^2v_{Te}^2}{\omega\nu_{eff}}\right)}.\tag{I.13}$$

Within the limits $\omega\nu_{eff} \gg k_z^2v_{Te}^2$ and $\omega\nu_{eff} \ll k_z^2v_{Te}^2$, this expression goes over into (2.9) and (2.10) respectively.

II. CALCULATION OF $\delta\epsilon_i$

Equation (2.3) for ions has, in perfect analogy, the following integral form (with allowance for the gravitational drift of the ions)

$$\delta f_i(x)=-\frac{1}{\Omega_i}\int_\infty^\varphi d\varphi'\left(-\frac{e}{Mc}\nabla\Phi\frac{\partial f_{0i}}{\partial\mathbf{v}}+I_{ie}+I_{ii}\right)$$
$$\times\exp\left\{\frac{i}{\Omega_i}\int_\varphi^{\varphi'}d\varphi''(\omega-k_yu_i-k_zv_z-k_yv_\perp\sin\varphi'')\right\}.\tag{II.1}$$

Here, too, the integration is carried out along the characteristic

$$x+\frac{v_\perp\sin\varphi}{\Omega}=\text{const}$$

and therefore $\partial f_{0i}/\partial\mathbf{v}$ is determined by formulas similar to (I.2), but only for the ions. The difference between (II.1) and (I.1) lies in the fact that the former contains not only the ion function δf_i, but also the electron function δf_e, namely

$$I_{ie}\approx\frac{1}{N_i}\frac{\partial f_{0i}}{\partial\mathbf{p}}\int d\mathbf{p}\,\mathbf{p}I_{ei}=-\frac{4\pi e^4L}{m}\frac{\partial f_{0i}}{\partial\mathbf{p}}\int d\mathbf{p}\frac{\mathbf{v}\delta f_e}{v^3}.\tag{II.2}$$

Therefore Eq. (II.1) is solved by using for δf_e the expressions obtained in Appendix I. Introducing the function F_i:

$$\delta f_i=\int\frac{e}{T_i}\Phi f_{0i}+F_i,\tag{II.3}$$

we obtain from (II.1) in the zeroth approximation of geometrical optics

$$(\omega'-\mathbf{kv})F_i-\Omega_i\frac{\partial F_i}{\partial\varphi}=-\frac{e}{T_i}\left(\omega'-\frac{k_yv_{Ti}^2}{\Omega_i}a_i\right)f_{0i}+i(I_{ie}+I_{ii}).\tag{II.4}$$

a) When $|\omega' + i\nu_i| \ll k_zv_{Ti} \ll \Omega_i$, the collision integral in (II.1) and (II.4) can be neglected. We then obtain for δf_i, and consequently $\delta\epsilon_i$, the well known[1] expression of the theory of an inhomogeneous plasma without collisions (expression (2.11) without the last two terms). Allowance for collisions leads to negligibly small corrections.

b) If $\omega' \gg \nu_i$, k_zv_{Ti} or $|\omega' - s\Omega_i| \gg \nu_i$, k_zv_{Ti}, then the collision integral in (II.1) and (II.4) is also a small term, but it must be taken into account for a correct description of the dissipative effects connected with the ion collisions. Using expansions in powers of ν_i/ω' or $\nu_i/|\omega' - s\Omega_i|$, we obtain by direct calculation the corrections (2.12), (2.13), and (2.14) necessitated by the particle collisions in the plasma. We note that in the derivation of (2.13) for the short-wave oscillations ($k_\perp\rho_i \gg 1$) it is convenient to use the simplified ion-ion collision integral obtained in[33] (see also[49]):

$$I_{ii}=\frac{2\pi e^4LN}{M^2}\frac{f_{0i}}{v}\left\{\left(A-B\frac{v_\perp^2}{v^2}\right)\frac{\partial^2\delta f_i}{\partial v_\perp^2}+\frac{A}{v_\perp^2}\frac{\partial^2\delta f_i}{\partial\varphi^2}\right\},\tag{II.5}$$

where

$$\begin{aligned}A&=\frac{1}{\sqrt{\pi}}\left\{\sqrt{\pi}\,\Phi(t)\left(1-\frac{1}{2t^2}\right)+\frac{e^{-t^2}}{t}\right\},\\ B&=\frac{1}{\sqrt{\pi}}\left\{\sqrt{\pi}\,\Phi(t)\left(1-\frac{3}{2t^2}\right)+3\frac{e^{-t^2}}{t}\right\},\\ \Phi(t)&=\frac{2}{\sqrt{\pi}}\int_0^t dx\,e^{-x^2},\quad t=\frac{v}{\sqrt{2}\,v_{Ti}}.\end{aligned}\tag{II.6}$$

c) Particularly convenient is the use of the collision integral (II.5) for the derivation of formula (2.15), which is valid when $\nu_{ii}k_\perp^2\rho_i^2 \gg k_zv_{Ti}$, $(\omega' - s\Omega_i)$. In this case the term I_{ie} in (II.4) can be neglected and this solution can be written in the form (with allowance for the fact that $\Omega_i \gg \nu_{ii}k_\perp^2\rho_i^2$)

$$F_i=\frac{ie}{T_i}\Phi f_{0i}\frac{\left(\omega'-\frac{k_yv_{Ti}^2}{\Omega_i}a_i\right)J_s\left(\frac{k_\perp v_\perp}{\Omega_i}\right)\exp\left[i\frac{k_\perp v_\perp}{\Omega_i}\sin(\varphi-\alpha)-is\varphi\right]}{\frac{4\pi e^4N}{M^2v}\frac{k_\perp^2}{\Omega_i^2}\left(A-\frac{3}{4}\frac{v_\perp^2}{v^2}B\right)},\tag{II.7}$$

where α is the polar angle of the vector $\mathbf{k}$ (i.e., $k_x = k_\perp\cos\alpha$, $k_y = k_\perp\sin\alpha$). By determining with the aid of formulas (II.3) and (II.7) the charge density induced by the ions in the plasma, we obtain ultimately expression (2.15) for the ionic contribution to the dielectric constant.

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