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QUANTUM THEORY OF THERMOMAGNETIC PHENOMENA IN
METALS AND SEMICONDUCTORS

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I. INTRODUCTION

THE study of transport phenomena in conductors in the presence of a magnetic field has been the subject of many theoretical and experimental papers. The classical theory of galvanomagnetic phenomena are dealt with in the review of Lifshitz and Kaganov[1], and in the monograph of Beer[2]. The classical theory and analysis of experiments on thermomagnetic phenomena are developed in the monograph of Tsidil'kovskiĭ[4], in the reviews of Delves[5], Harman et al.[6], and Zawadski and Kolodzieczak[7].

The classical theory of thermogalvanomagnetic phenomena is based on the Boltzmann kinetic equation. When the external parameters of the system, such as the magnetic field intensity H, the temperature $T = kT°$ (k—Boltzmann constant, $T°$—absolute temperature in degrees) are altered, appreciable quantum effects may appear, which can no longer be described by the Boltzmann equation. Indeed, on going to the quantum description of the motion of an electron in a plane perpendicular to the magnetic field, a discrete energy spectrum arises (Landau levels), and the electron energy becomes dependent on H. Such a discrete spectrum is the consequence of the quantization of the finite orbital classical motion of the electron. The presence of a discrete spectrum under certain conditions can lead to significant changes in the thermodynamic and kinetic characteristics of conductors. Obviously, when the magnetic field is changed, a shift of the Landau levels relative to the Fermi level takes place. And whenever one of the Landau levels coincides with the Fermi level, a sharp increase takes place in the density of the electron states near the Fermi level. This is due to the fact that the Landau levels are strongly degenerate, and the multiplicity of the degeneracy is proportional to the magnetic field intensity. Thus, the density of states at the Fermi surface is an oscillating function of H. In order for these oscillations to appear in the thermodynamic and kinetic characteristics of the conductors, it is necessary that the energy difference between the neighboring Landau levels $\hbar\Omega$ exceed the width of the thermal smearing of the Fermi level, which equals $\sim kT°$. At the same time, it is necessary to satisfy one more important condition. In all real systems there exist collisions between the electrons and various kinds of scatterers. These collisions lead to an uncertainty in the energy of the stationary states, or to a broadening of the Landau levels by an amount $\sim \hbar\tau^{-1}$ ($\hbar$—Planck constant, τ—characteristic electron relaxation time). It is obvious that the discrete energy spectrum exists only when $\hbar\Omega \gg \hbar\tau^{-1}$, i.e., the "distance" between the Landau levels greatly exceeds the width of the level. Consequently, under the conditions $\hbar\Omega \gg kT°$ and $\Omega\tau \gg 1$, all the thermodynamic and kinetic characteristics of the electrons, which depend on the density of the states at the Fermi surface, will be oscillating functions of H. Strong magnetic fields can also strongly influence the electron scattering processes. Thus, for example, with increasing magnetic field intensity the average electron momentum along the magnetic field decreases, and the de Broglie wavelength λ_z increases. In the ultraquantum limit, when all the electrons are at the lower Landau level, it may happen that $\lambda_z \gtrsim a$—the average distance between the scattering centers. In this case scattering by many centers is significant.

It becomes obvious that the description of transport phenomena in strong magnetic fields ($\Omega\tau \gg 1$) and at low temperatures ($\hbar\Omega \gg T$) must be a quantum description.

During the last decade, much progress was made in the quantum theory of galvanomagnetic phenomena, as reflected in the review by Kubo et al.[3]. There are at present no review articles devoted to the quantum theory of thermomagnetic phenomena*. This is not accidental. The point is that in the quantum theory of galvanomagnetic phenomena it is sufficient to consider

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*A recently published review by Puri and Geballe [81] analyzes the experimental research on the thermal emf, dragging, and magnetophonon oscillations of the thermal emf in n-Ge and n-InSb.

only spatially-homogeneous systems in the presence of only dynamic forces, such as the electric field intensity $\mathbf{E}$ or the magnetic field intensity $\mathbf{H}$, which are included in the usual manner in the Hamiltonian of the system, making it possible to write down directly the Schrödinger equation for the density matrix. In the construction of a quantum theory of thermomagnetic phenomena, unlike galvanomagnetic ones, we encounter a more difficult problem, since it is necessary to consider spatially-inhomogeneous systems, so as to include consistently into consideration not only dynamic forces but also forces of a static nature, due to the spatial inhomogeneities of the temperature T and of the chemical potential ζ. Further, calculation of the volume densities of the conduction current and of the heat flux, which are needed for the construction of a quantum theory of thermomagnetic phenomena, lead to new complications compared with the quantum theory of galvanomagnetic phenomena. In order to explain the foregoing, we note that the density matrix can be used to calculate the mean values of the quantum-mechanical operators corresponding to physical quantities. Thus, for example, it is possible to calculate the charge and energy flux-density operators. But by virtue of the macroscopic nature of the definitions of the conduction current and the heat flux (see Sec. II) we cannot construct quantum-mechanical operators for these quantities. And since, for instance, the conduction current in spatially-inhomogeneous systems in a quantizing magnetic field does not at all coincide with the volume charge flux density, this raises a new problem, that of separating the conduction current from the volume charge flux density and the heat flux from the volume energy flux density.

All these difficulties, and also the lack of a clear-cut understanding of the formulation of the problem (as manifest in attempts either to identify the volume charge flux density and energy flux density, calculated with the aid of the density matrix, with the conduction current and the heat flux, or else to define thermodynamic phenomena not on the basis of the conduction current and the heat flux but some other fluxes), have led to contradictory results and hindered the construction of a quantum theory of thermomagnetic phenomena.

Yet a theoretical analysis of the already available experimental data on thermomagnetic phenomena in a quantizing magnetic field uncovers new prospects and can yield very interesting information not only on the structure of the energy spectrum and on the carrier relaxation mechanisms, but also on the character of the interaction of phonons in solids, and on the absorption of sound at very high frequencies $\sim 10^{10} - 10^{12}$ Hz, where direct measurement of the sound absorption coefficient is at present practically impossible.

These prospects are uncovered because in the region of quantizing magnetic fields and at low temperatures, an appreciable contribution to the differential thermal emf of semiconductors with small conduction-electron density, such as n-Ge and n-InSb, is made by the deviation of the phonons from local equilibrium. The differential thermal emf is made up of two parts. The first is due to the deviation of the electrons from thermodynamic equilibrium as a result of the temperature gradient existing in the system. The second part is connected with the deviation of the thermal phonons from equilibrium. The first is usually called the electronic part of the thermal emf, and the second is called the dragging thermal emf. The electronic thermal emf in strong magnetic fields ($\Omega \tau \gg 1$) does not depend on the scattering in the case of conductors with unequal hole and electron densities, and is usually small. The dragging thermal emf is a result of the fact that the temperature gradient produces a phonon flux proportional both to the temperature gradient and to the phonon mean free path l_p $\sim v_s / \omega_{pp}$ (v_s = s—speed of sound, ω_{pp}—phonon damping decrement or the frequency of relaxation of phonons on phonons or defects, in short—the non-electronic phonon relaxation frequency). When this phonon flux collides with the electrons, momentum is transferred from the phonons to the electrons. The fraction of the momentum transferred is proportional to the frequency of the phonon-electron collisions ω_{ep}. Thus, the dragging current, meaning also the dragging thermal emf, is proportional to the ratio ω_{ep}/ω_{pp}. In the classical limit ($\hbar = 0$), ω_{ep} does not depend on the magnetic field and is usually much smaller than ω_{pp} for semiconductors with low carrier density. This can be easily verified with the aid of the momentum conservation law in collisions between electrons and phonons. In the case of Maxwellian statistics, the average electron momentum is $\bar{p}_e = \sqrt{mT}$. In accordance with the momentum conservation law, the electrons can interact only with phonons whose momentum is $\hbar q \lesssim \bar{p}_e$. But for $T° > 1°K$, the average thermal momentum of the phonons $\hbar q_T = T/v_s$ greatly exceeds $\bar{p}_e$. Therefore the electrons interact only with the long-wave thermal phonons, the number of which is relatively small.

In a quantizing magnetic field, the situation is different. For simplicity we consider only the quantum limit $\hbar\Omega \gg T$ and electrons obeying Maxwell's statistics. In this case the localization of the electron in a plane orthogonal to the magnetic field is determined in order of magnitude by the magnetic length $\alpha = (c\hbar/|e|H)^{1/2}$ (c—speed of light, e—electron charge) or by the Larmor quantum radius, inasmuch as the electrons populate essentially the lowest Landau level. From the uncertainty relation it follows that the order of magnitude of the transverse-motion momentum is $\sim \hbar/\alpha^{-1}$. Therefore the electron moving across the magnetic field can interact only with phonons whose momentum is $\hbar q \lesssim \hbar \alpha^{-1}$. It follows therefore that with increasing magnetic field the volume of the phase space of the phonons interacting with the electrons increases in proportion to H, and this in turn leads to a rapid increase of the frequency of phonon relaxation on the electrons, ω_{ep}. The latter is due to the exponential dependence of ω_{ep} on α (see, for example, formula (6.19)). Inasmuch (as noted above) as the dragging thermal emf is determined in terms of the ratio ω_{ep}/ω_{pp}, this ratio increases with increasing H, since ω_{pp} does not depend on H. This is precisely why, for example, in InSb the dragging thermal emf in the quantum limit ($\hbar\Omega \gg kT°$) increases by almost 100 times. With the aid of a theoretical analysis of the experimental dependence of the dragging thermal emf on T and H it is possible to obtain valuable informa-

tion on the mechanism of relaxation of long-wave phonons on short-wave thermal phonons, and to find the frequency and temperature dependences of ω_{pp} for sound waves with length $\lambda \sim \sqrt{c\hbar/|e|H}$.

It should also be noted that at least some of the thermomagnetic effects have definite experimental advantages over galvanomagnetic ones. The point is that measurements, say, of the differential thermal emf in strong magnetic fields are much less subject to the influence of random inhomogeneities in the impurity distribution than galvanomagnetic measurements, since the transverse Nernst electric field tends to zero with increasing magnetic field (in the classical limit), whereas the Hall field tends to infinity. This is precisely why it is easier to separate "true" effects, due to the band structure, degeneracy, and carrier scattering mechanism in the analysis of the measurements of certain thermomagnetic effects. Thus, it has been demonstrated experimentally that the magnetothermal emf has a clearly pronounced classical saturation region (in accordance with the predictions of the theory), whereas for magnetoresistance there is practically no such region. In spite of this fact, we have at present very limited experimental material on thermomagnetic phenomena in a quantizing magnetic field, whereas the galvanomagnetic phenomena have been thoroughly investigated in numerous experiments. This situation obviously has been due in part to the absence, until quite recently, of a consistent quantum theory of thermomagnetic phenomena.

The present review is the first attempt at a consistent exposition of such a theory.

The entire theory is based on the kinetic equation for the density matrix in the mixed Wigner representation. It was just this formalism of quantum statistical theory which turned out to be very convenient and exceedingly simple for the calculation of fluxes, and was widely used by Eleonskiĭ, Zyryanov, and Silin[8], Akhiezer, Bar'yakhtar, and Peletminskiĭ[9,10], and Zyryanov[11-14]. In the second section of the review we discuss the problem of thermogalvanomagnetic phenomena in conductors. The third section is devoted to an exposition of a procedure for separating the conduction current from the volume charge flux density and separating the heat flux from the volume energy flux density, neglecting scattering. In the fourth section we calculate the dissipative (collision) heat flux and the conduction current in the approximation of elastic scattering of the carriers. The fifth section is devoted to the calculation also of the dissipative parts of the conduction current and the heat flux, but with allowance for inelastic scattering of the electrons and phonons, and also with allowance for the nonequilibrium nature of the latter (the phonon dragging effect). In the sixth section we consider thermomagnetic effects in the case when the temperature and chemical-potential gradients are directed parallel to the magnetic field (longitudinal effects). The seventh section is devoted to a comparison of theory with experiment. This section contains a theoretical analysis of only the most significant experiments, in which various general laws, with weak dependence on the concrete details of the structure of the energy spectrum of the carriers, have been established. The appendix contains a derivation of the kinetic equations used to calculate the charge and energy fluxes.

II. FORMULATION OF PROBLEM IN THE THEORY OF THERMOMAGNETIC PHENOMENA

Thermogalvanomagnetic phenomena in conducting media, according to Landau and Lifshitz[15], are determined with the aid of formulas for two vector fluxes, namely the volume conduction-current density $j_{con}(r)$ and the volume heat flux density $Q_T(r)$. Thus, the problem of constructing a microscopic theory of thermogalvanomagnetic phenomena reduces to a calculation of $j_{con}(r)$ and $Q_T(r)$. Before we proceed to calculate these quantities, let us stop to define them. The most general definition of $j_{con}(r)$ is given in[15]. According to[15], $j_{con}(r)$ is that part of the volume density of the charge flux $j(r)$, which contributes to the transport of the charge through the cross section of the entire conductor. $Q_T(r)$ is analogously defined as that part of the volume energy flux density $Q(r)$ which makes the contribution to the energy transport through the cross section of the entire conductor at $j_{con}(r) = 0$. In other words, $Q_T(r)$ is the energy flux of the thermal random motion of the carriers. The kinetic coefficients in $j_{con}(r)$ and $Q_T(r)$ should satisfy the Einstein relation and the Onsager symmetry principle—requirements imposed on these fluxes by the thermodynamics of irreversible processes. As is well known, the Einstein relation is the consequence of the principle of maximum entropy at thermodynamic equilibrium. Maximum entropy corresponds to a state in which the temperature T and the electrochemical potential $\zeta_\varphi = \zeta + e\varphi$ (φ—electric-field potential) are constant along the entire system[16]; in this case both the conduction current and the heat flux vanish. Under deviations from the thermodynamic equilibrium, a heat flux and a conduction current are produced in the electron system, and are proportional in the case of small deviations from equilibrium to the spatial gradients: ∇T and $-\nabla\zeta_\varphi = e(E - (1/e)\nabla\zeta)$, $E = -\nabla\varphi$. Inasmuch as the conduction current and the heat flux are proportional to $\nabla\zeta_\varphi$, the kinetic coefficients preceding E and $(-(1/e)\nabla\zeta)$ in these fluxes are identical. This is the Einstein relation. Thus, the conduction current and the heat flux are of the form

$$(j_{con})_i = \sigma_{ik}(H)\left(E_k - \frac{1}{e}\nabla_k\zeta\right) - \beta_{ik}(H)\nabla_k T,$$

$$(Q_T)_i = \chi_{ik}(H)\left(E_k - \frac{1}{e}\nabla_k\zeta\right) - \varkappa_{ik}(H)\nabla_k H. \tag{2.1}$$

The Onsager symmetry principle, which is a consequence of the time reversibility of the mechanical equations of motion of particles and of the macroscopic damping of the fluctuations of the physical quantities in the system[16], leads in the presence of a magnetic field to the following relations:

$$\sigma_{ik}(H) = \sigma_{ki}(-H), \quad \varkappa_{ik}(H) = \varkappa_{ki}(-H),$$

$$\beta_{ik}(H) = \frac{1}{T}\chi_{ik}(-H). \tag{2.2}$$

Inasmuch as j_{con} and Q_T serve as the basis of the theory of thermogalvanomagnetic phenomena and are defined with the aid of volume charge and energy flux

densities, we must be able first of all to calculate these fluxes.

In classical theory, **j** and **Q** are determined with the aid of the distribution function f, which is a solution of the Boltzmann equation; this solution is linear in the electric field **E** and in the gradients of the chemical potential ζ and the temperature T. These formulas are

$$\mathbf{j}(\mathbf{r}) = \frac{2}{(2\pi\hbar)^3} \int (d\mathbf{p})\, ev f(\mathbf{p},\ \mathbf{r}), \qquad (2.3)$$

$$\mathbf{Q}(\mathbf{r}) = \frac{2}{(2\pi\hbar)^3} \int (d\mathbf{p})\, v\varepsilon(\mathbf{p}) f(\mathbf{p},\ \mathbf{r}), \qquad (2.4)$$

where e is the charge and $\varepsilon(\mathbf{p})$ the energy of the carrier. In quantum theory, the analog of these formulas are the mean values of the quantum mechanical operators corresponding to the volume charge and energy flux densities

$$\mathbf{j}(\mathbf{r}) = \mathrm{Sp}(\hat{\rho},\ \hat{\mathbf{j}}), \qquad (2.5)$$

$$\mathbf{Q}(\mathbf{r}) = \mathrm{Sp}(\hat{\rho},\ \hat{\mathbf{Q}}), \qquad (2.6)$$

where $\hat{\rho}$—density matrix, and

$$\hat{\mathbf{j}} = \frac{e}{2}(\hat{\mathbf{v}}\hat{N} + \hat{N}\hat{\mathbf{v}}), \qquad \hat{\mathbf{Q}} = \frac{1}{2e}(\hat{\mathbf{j}}\mathscr{H} + \mathscr{H}\hat{\mathbf{j}}), \qquad (2.7)$$

$\hat{\mathbf{v}}$, $\mathscr{H}$, and $\hat{N}$ are the operators of the velocity, energy, and the carrier density, respectively.

In some earlier papers, attempts were made to use in the theory of thermomagnetic phenomena another definition of the charge and energy fluxes, namely

$$\mathbf{j}' = e\,\mathrm{Sp}(\hat{\mathbf{v}},\ \hat{\rho}), \qquad \mathbf{Q}' = \frac{1}{2}\mathrm{Sp}(\hat{\rho},\ \hat{\mathbf{v}}\mathscr{H} + \mathscr{H}\hat{\mathbf{v}}).$$

It is easy to verify that $\mathbf{j}'$ and $\mathbf{Q}'$ coincide with $\mathbf{j}(\mathbf{r})$ and $\mathbf{Q}(\mathbf{r})$ defined in (2.5) and (2.6) only in the particular case of spatially-homogeneous systems and in the presence of only a homogeneous electric field. In spatially-inhomogeneous systems, when $\nabla T \neq 0$ and $\nabla\zeta \neq 0$, $\mathbf{j}'$ and $\mathbf{Q}'$ differ from **j** and **Q**. The classical limit ($\hbar = 0$) of $\mathbf{j}'$ and $\mathbf{Q}'$, unlike that of **j** and **Q**, does not coincide with the well known results that follow from the Boltzmann equation. This is precisely why attempts to use $\mathbf{j}'$ and $\mathbf{Q}'$ in the theory of thermomagnetic phenomena led to erroneous results.

We shall consider further examples from which it will be seen that $\mathbf{j}(\mathbf{r})$ does not always coincide with $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$. Nor does $\mathbf{Q}(\mathbf{r}) - (\zeta/e)\mathbf{j}_{\mathrm{con}}(\mathbf{r})$ always coincide with the heat flux $\mathbf{Q}_T(\mathbf{r})$. If the external magnetic field is H = 0 and at the same time the magnetization of the carriers **M** vanishes, then the fluxes $\mathbf{j}(\mathbf{r})$ and $\mathbf{Q}(\mathbf{r}) - (\zeta/e)\mathbf{j}(\mathbf{r})$ due to the electric field and the gradients of T and ζ satisfy both all the requirements of the thermodynamics of irreversible processes imposed on $\mathbf{j}_{\mathrm{con}}$ and $\mathbf{Q}_T$, and the definitions of these fluxes as given above. This is precisely why $\mathbf{j}(\mathbf{r})$ can be identified with $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$ and $\mathbf{Q}(\mathbf{r}) - (\zeta/e)\mathbf{j}(\mathbf{r})$ with the heat flux $\mathbf{Q}_T(\mathbf{r})$. If in the spatially-homogeneous system H $\neq$ 0 and the fluxes are due only to the homogeneous electric field **E** ($\nabla T = \nabla\zeta = 0$), then **M** will also be spatially homogeneous. In this case $\mathbf{j}(\mathbf{r})$ coincides with $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$ but $\mathbf{Q}(\mathbf{r}) - (\zeta/e)\mathbf{j}_{\mathrm{con}}(\mathbf{r})$ no longer coincides with the heat fluxes, since $\mathbf{Q}(\mathbf{r})$ contains besides $(\zeta/e)\mathbf{j}_{\mathrm{con}}$ also the magnetic-energy flux $(-c\mathbf{E}\times\mathbf{M})$, which equals the contribution made to the Poynting vector by the magnetization $(c/4\pi)\mathbf{E} \times (\mathbf{H} - \mathbf{B})$. Thus, the heat flux is determined in this

case by the formula*

$$\mathbf{Q}_T(\mathbf{r}) = \mathbf{Q}(\mathbf{r}) - \frac{\zeta}{e}\mathbf{j}_{\mathrm{con}}(\mathbf{r}) - \frac{c}{4\pi}[\mathbf{E}(\mathbf{H}-\mathbf{B})].$$

If the fluxes $\mathbf{j}(\mathbf{r})$ and $\mathbf{Q}(\mathbf{r})$ are due to both **E** and ∇T and $\nabla\zeta$, then the magnetization of the carriers $\mathbf{M}(T(\mathbf{r}),\ \zeta(\mathbf{r}))$ depends on the coordinates when H $\neq$ 0. In this case $\mathbf{j}(\mathbf{r})$ no longer coincides with $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$, since the spatial inhomogeneities of the magnetization **M** will make a contribution (equal to c curl **M**) to $\mathbf{j}(\mathbf{r})$, in addition to the contribution of $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$. Analogously, $\mathbf{Q}(\mathbf{r})$ contains, besides the heat flux $\mathbf{Q}_T(\mathbf{r})$, also the flux of magnetic energy $\mathbf{Q}_M(\mathbf{r})$ which includes, besides the term $-c\mathbf{E}\times\mathbf{M}$ which occurs also in spatially-homogeneous systems, also terms that depend on curl **M**. It will be shown below that a contribution is made to the transport of charge through the cross section of the entire conductor only by part of the volume density of the charge flux, equal to the difference

$$\mathbf{j}_{\mathrm{con}}(\mathbf{r}) = \mathrm{Sp}(\hat{\rho},\ \hat{\mathbf{j}}) - c\,\mathrm{rot}\,\mathbf{M}, \qquad (2.8)$$

and a contribution to the transport of thermal energy is also made only by the difference

$$\mathbf{Q}_T(\mathbf{r}) = \mathrm{Sp}(\hat{\rho},\ \hat{\mathbf{Q}}) - \frac{\zeta}{e}\mathbf{j}_{\mathrm{con}}(\mathbf{r}) - \mathbf{Q}_M(\mathbf{r}), \qquad (2.9)$$

$$\mathbf{Q}_M(\mathbf{r}) = -c[\mathbf{E}\mathbf{M}] + \frac{c}{e}\left\{ T\left(\frac{\partial}{\partial T}\right)_\zeta + \zeta\left(\frac{\partial}{\partial\zeta}\right)_T \right\} \int_{-\infty}^{\zeta} d\zeta'\,\mathrm{rot}\,\mathbf{M}(T,\ \zeta'). \qquad (2.10)$$

Formulas (2.8)–(2.10), which determine the volume density of the conduction current and the heat flux, serve, in accordance with[15], as the basis for the construction of the microscopic theory of thermogalvanomagnetic phenomena. The kinetic coefficients characterizing $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$ and $\mathbf{Q}_T(\mathbf{r})$ satisfy both the Einstein relation and the Onsager symmetry principle. Thus, the problem of the microscopic theory of thermogalvanomagnetic phenomena reduces:

1) To the calculation of $\mathbf{j}(\mathbf{r}) = \mathrm{Tr}(\hat{\rho}, \mathbf{j})$ and $\mathbf{Q}(\mathbf{r}) = \mathrm{Tr}(\hat{\rho}\mathbf{Q})$ and

2) to the separation of $\mathbf{j}_{\mathrm{con}}(\mathbf{r})$ from $\mathbf{j}(\mathbf{r})$ and of $\mathbf{Q}_T(\mathbf{r})$ from $\mathbf{Q}(\mathbf{r})$.

Obraztsov[17] proposes to determine the thermomagnetic coefficients for spatially-homogeneous media and for the temperature and chemical-potential gradients with the aid of the so-called total fluxes (volume plus surface) per unit area of the conductor cross section. This formulation of the problem is a particular case of the more general formulation described in[15].

In the case of spatial inhomogeneities of T and ζ, formula (2.8) is well known and is the consequence of averaging of the Lorentz equations for the microscopic electromagnetic field[15]. However, the flux of the magnetic energy $\mathbf{Q}_M(\mathbf{r})$ now no longer reduces to the contribution made to the Poynting vector by the magnetization, and is expressed by the more complicated formula (2.10), which was derived in[18] (see also[19,20]).

In concluding this section we note that the thermogalvanomagnetic coefficients determined with the aid of the fluxes (2.1) are perfectly identical for both classical and quantum systems and are given, for ex-

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*$[\mathbf{E}(\mathbf{H}-\mathbf{B})] \equiv \mathbf{E} \times (\mathbf{H} - \mathbf{B})$

ample, in[15,4]. We shall present here formulas for only some of them.

Under isothermal conditions at a specified temperature gradient $\nabla_x T \perp H$, the differential thermal emf α_{xx} the Nernst coefficient $N = \alpha_{xy}/H$ and the thermal conductivity κ_1 characterizing the heat flux along $(-\nabla_x H)$ are all determined with the aid of (2.1) under the condition $j_{con} = 0$ and $\nabla_y T = 0$. For the case of an isotropic medium, when all the tensors of the kinetic coefficients in (2.1) have the structure

$$\begin{pmatrix} A_{xx} & A_{xy} & 0 \\ -A_{xy} & A_{xx} & 0 \\ 0 & 0 & A_{zz} \end{pmatrix},$$

we get

$$\alpha_{xx} = (\sigma_{xx}\beta_{xx} + \sigma_{xy}\beta_{xy})(\sigma_{xy}^2 + \sigma_{xx}^2)^{-1}, \quad (2.11)$$

$$\alpha_{xy} = (\sigma_{xy}\beta_{xx} - \sigma_{xx}\beta_{xy})(\sigma_{xy}^2 + \sigma_{xx}^2)^{-1}, \quad (2.12)$$

$$\varkappa_{\perp} = -(\chi_{xx}\alpha_{xx} + \chi_{xy}\alpha_{xy} - \varkappa_{xx}), \quad (2.13)$$

and the electric resistance is given by

$$\rho_{xx} \equiv \rho_{\perp} = \sigma_{xx}(\sigma_{xx}^2 + \sigma_{xy}^2)^{-1},$$
$$\rho_{xy} = \sigma_{xy}(\sigma_{xx}^2 + \sigma_{xy}^2)^{-1}. \quad (2.14)$$

The Righghi-Leduc effect* is adiabatic and is obtained from (2.1) under the conditions $\nabla_x T \neq 0$, $\nabla_y T \neq 0$, $j_{con} = 0$, and $(Q_T)_y = 0$. In an isotropic medium we have

$$\nabla_y T = [\chi_{xx}\alpha_{xy} - \chi_{xy}\alpha_{xx} + \varkappa_{xy}](\varkappa'_{\perp})^{-1}\nabla_x T \quad (2.15)$$

($\kappa'_{\perp}$—thermal conductivity of the electrons and phonons). In the case of strong magnetic fields ($\Omega\tau \gg 1$) we have $\sigma_{xy} \gg \sigma_{xx}$, $\beta_{xy} \gg \beta_{xx}$, $\kappa_{xy} \gg \kappa_{xx}$, since the non-diagonal components do not depend on the scattering, whereas the diagonal ones are proportional to $1/\Omega\tau$. However, the inequality $\sigma_{xy} \gg \sigma_{xx}$ is violated in conductors for which the electron density $n_e = n_h$ (n_h—hole density). This can be easily understood by recalling that the electric field $E \perp H$ causes carrier drift with velocity $v_E = cE \times H/H^2$, and this velocity depends neither on the sign of the charge nor on its mass. Therefore both the holes and the electrons drift with the same velocity, and if $n_e = n_h$, then the electric current vanishes and $\sigma_{xy} = 0$. Consequently, when $n_e = n_h$ expansion of σ_{xy} in powers of the parameter $(1/\Omega\tau)$ begins not with the zeroth degree, as in the case $n_e = n_h$, but with the second: $(1/\Omega\tau)^2$.

III. NONDISSIPATIVE ELECTRON FLUXES IN A QUANTIZING MAGNETIC FIELD

As is well known, the force $F \perp H$ acting on the electrons leads in the zeroth approximation in the scattering only to a drift with velocity

$$v_F = \frac{c}{eH^2}[FH].$$

Such a motion of the electrons is produced by dynamic forces, and for example for the electric field E and for the gravitational field g we have respectively

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*This effect consists in the fact that the mutually perpendicular magnetic field and the heat flux in conductors lead to the appearance of a temperature gradient in a direction perpendicular to both the magnetic field and to the heat flux (in the "Hall direction").

$F_E = eE$ and $F_g = mg$. The dynamic forces can be taken into account directly in the Hamiltonian of the electrons. There exist forces of a different nature, due to the spatial inhomogeneities of the temperature T and of the chemical potential ζ, sometimes called forces of a statistical nature. These forces can be taken into account consistently only by a statistical description of the system. In the case of a strong magnetic field ($\Omega\tau \gg 1$) in the zeroth approximation in the scattering, the distribution function of the electrons over the states will depend only on the single-particle integrals of motion. Such integrals of motion can be taken, for example, to be the energy $\epsilon(p)$, the momentum p_z along the magnetic field, and the x-coordinate of the center of the Larmor orbit x_0. We shall consider henceforth distributions that depend only on $\epsilon(p)$ and x_0, for example, $f_0([\epsilon(p) - \zeta(x_0)]/T(x_0))$. Such distributions describe systems that are spatially-inhomogeneous along the axis $x \perp H$. In the case of weak spatial inhomogeneities, when $\zeta(x_0)$ and $T(x_0)$ remain practically constant on the Larmor radius, we can identify x_0 with the x-coordinate of the electron; then $\zeta(x)$ and $T(x)$ have the meaning of local values of the chemical potential and the temperature. Putting $x_0 = x - (x - x_0)$, assuming weak spatial inhomogeneities, we expand f_0 in a series

$$f_0\left(\frac{\varepsilon(p) - \zeta(x_0)}{T(x_0)}\right) = f_0\left(\frac{\varepsilon(p) - \zeta(x)}{T(x)}\right)$$
$$+ \frac{\partial f}{\partial \varepsilon}\left(\nabla_x \zeta + \frac{\varepsilon(p) - \zeta(x)}{T(x)}\nabla_x T\right)(x - x_0) + \dots \quad (3.1)$$

With the aid of (3.1) we can easily find the volume density of the charge flux. Recognizing that

$$x - x_0 = \frac{v_{\perp}}{\Omega}\cos\varphi, \quad v_y = v_{\perp}\cos\varphi$$

($\varphi = \Omega t$) and taking into account the periodicity of f_0 with respect to the variable φ, we get

$$j_y = e\frac{2}{(2\pi\hbar)^3}\int dp \frac{\partial f_0}{\partial \varepsilon}\left(\nabla_x\zeta + \frac{\varepsilon(p) - \zeta}{T}\nabla_x T\right)\frac{v_y^2}{\Omega}. \quad (3.2)$$

This is a well known result which follows, in particular, from the Boltzmann kinetic equation (neglecting scattering).

We now proceed to consider the case of a quantizing magnetic field. The single-particle states of the electron will be described in the Landau representation. The Hamiltonian of the electron has in this representation the form

$$\hat{\mathscr{H}}_0 = \left(p - \frac{e}{c}A\right)^2 / 2m, \quad A_0 = \{0, Hx, 0\}, \quad (3.3)$$

and its eigenfunctions and eigenvalues are

$$|\nu\rangle \equiv |n, p_z, x_0\rangle = (L_y L_z \alpha)^{-1/2}\exp\left(\frac{ip_y y}{\hbar} + \frac{ip_z z}{\hbar}\right)\Phi_n\left(\frac{x - x_0}{\alpha}\right),$$

$$E_\nu = E(n, p_z) = \hbar\Omega(n + 1/2) + \frac{p_z^2}{2m},$$

where $x_0 = cp_y/eH$ is the projection of the center of the Larmor orbit on the x axis, $\Phi_n(x)$ is the eigenfunction of the harmonic oscillator normalized to unity, $\alpha^2 = c\hbar/|e|H$, and finally L_y and L_z characterize the normalization volume. In the presence of an electric field E directed along the x axis, the eigenfunctions (3.3) remain the same, but

$$x_0 = \frac{c}{eH}p_y + \frac{mc^2}{eH^2}E_x,$$

and the eigenvalues are

$$E(n, p_z, x_0) = E(n, p_z) - ex_0 E_x + \frac{mc^2}{2H^2} E_x^2.$$

Neglecting scattering, the density matrix is diagonal in the Landau representation: $\rho_{\nu\nu'} = f_\nu \epsilon_{\nu\nu'}$, and plays the role of the electron distribution function over the states n, p_z, and x_0. Just as in the classical case, we shall consider distributions that depend only on two integrals of motion, $E(n, p_z)$ and x_0. Assuming weak spatial inhomogeneities, we get

$$f(n, p_z, x_0) = f\left(\frac{E(n, p_z) - \zeta(x)}{T(x)}\right) + \frac{\partial f}{\partial E}\left(\nabla_x \zeta + \frac{E(n, p_z) - \zeta}{T}\nabla_x T\right)(x - x_0) + \cdots$$

$$(3.4)$$

With the aid of (2.4) and the definitions (2.5) and (2.6), we obtain the volume densities of the charge and energy fluxes:

$$j_y(x) = -\frac{ceN}{H}E_x - \frac{c}{H}\frac{2}{(2\pi\alpha)^2\hbar}\sum_n E_n \int dp_z \frac{\partial f}{\partial E}\left[\nabla_x\zeta + \frac{E(n, p_z) - \zeta}{T}\nabla_x T\right],$$

$$(3.5)$$

$$Q_y(x) = -\frac{cN}{H}(\bar{E} + \bar{E}_\perp)E_x - \frac{c}{eH}\frac{2}{(2\pi\alpha)^2\hbar}$$

$$\times \sum_n E_n \int dp_z E(n, p_z)\frac{\partial f}{\partial E}\left[\nabla_x\zeta + \frac{E(n, p_z) - \zeta}{T}\nabla_x T\right],$$

$$N(\bar{E} + \bar{E}_\perp) = \frac{2}{(2\pi\alpha)^2\hbar}\sum_n \int dp_z\,[E(n, p_z)$$

$$+ E_n]f(n, p_z), E_n = \hbar\Omega\left(n + \frac{1}{2}\right).$$

$$(3.6)$$

From (3.5) and (3.6) we see that the Einstein relation is not satisfied for the kinetic coefficients characterizing j_y and Q_y, i.e., the coefficients preceding $(-1/e\,\nabla_x\zeta)$ and E_x are different.

This fact was first pointed out by the Japanese physicists Kasuya[21] and Nakajima[22] (see also the paper by Kubo[23]). However, the actual cause of the violation of the Einstein relation was brought to light in a paper by Zyryanov and Silin[24], where it was shown that in the case of spatially-inhomogeneous systems in the presence of a quantizing magnetic field a contribution is made to the volume density of the charge flux $j_y(x)$ by the current c curl M, due to the dependence of the Landau diamagnetic susceptibility on $\zeta(x)$. It was also shown there that the difference $j_y(x) - c\,\text{curl}_y M$ satisfies the Einstein relation. A physical interpretation of the results of[24] was given in[18], where it was shown that the volume density of the charge flux does not coincide with the conduction-current density and differs from it by an amount c curl M; in other words, they obtained from the microscopic theory the well known formula[15]

$$j = j_{con} + c\,\text{rot}\,M,$$

in which the magnetization is due to the orbital motion of the conduction electrons (the Landau diamagnetism).

Thus, the problem of separating the conduction current from $j_y(x)$ turned out to be relatively simple, whereas the problem of separating the heat flux $Q_T(x)$ from the volume density of the energy flux $Q(x)$ turned out to be much more difficult. To solve this problem in the presence of a quantizing magnetic field, we shall first separate that part of the volume densities of the charge and energy fluxes which depends on the spatial inhomogeneities of the magnetization, due to the gradients of the temperature and of the chemical potential. It is clear from general considerations that

only the nondiagonal components of the tensors of the kinetic coefficients in j and Q can depend on the magnetization vector M. Indeed, inasmuch as j and Q are polar vectors and M is an axial vector, in the approximation linear in M the most general dependence of j and Q on M can be represented in the form of a linear functional of curl M. In the case of spatial inhomogeneities of M, due only to the dependence of M on T and ζ, we have

$$\text{rot}\,M = \left[\left(\nabla_r T\left(\frac{\partial}{\partial T}\right)_\zeta + \nabla_r\zeta\left(\frac{\partial}{\partial\zeta}\right)_T\right)M\right].$$

$$(3.7)$$

It is seen from this formula that within the framework of the linear transport theory, only the non-diagonal components of the tensors of the kinetic coefficients in j and Q can depend on M. On the other hand, in the conditions $\Omega\tau \gg 1$, the non-diagonal components of the tensors of the kinetic coefficients do not depend on the scattering. It is therefore sufficient to solve the problem of separating j_{con} from j and Q_T from Q by neglecting scattering. To this end, we express the kinetic coefficients in (3.5) and (3.6) in terms of the thermodynamic functions of the electron gas. Such a possibility is obvious, since the fluxes were calculated, first, neglecting scattering and, second, in the local-equilibrium state.

We introduce the thermodynamic potential of the electron gas

$$\Phi(H, T, \zeta, V) = -\frac{2TV}{(2\pi\alpha)^2\hbar}\sum_n\int dp_z\ln[1 + \exp\{[\zeta - E(n, p_z)]/T\}],$$

where V is the volume of the system.

Since $\Phi = -\mathscr{P}V$, the pressure P is numerically equal to the thermodynamic potential per unit volume, taken with the minus sign. Therefore the entropy S, the number of particles N, and the magnetization M, all per unit volume, are expressed by the following formulas:

$$S = \left(\frac{\partial\mathscr{P}}{\partial T}\right)_{V, H, \zeta}, \quad N = \left(\frac{\partial\mathscr{P}}{\partial\zeta}\right)_{V, T, H}, \quad M = \left(\frac{\partial\mathscr{P}}{\partial H}\right)_{V, T, \zeta}. \quad (3.8)$$

The kinetic coefficients in formulas (3.5) and (3.6) can be expressed in terms of the thermodynamic functions (3.8). If we further neglect the compressibility of the electron gas, then (3.5) and (3.6) can be written in the following form:

$$j_y = -\frac{c}{H^2}\text{rot}_y((\mathscr{P} + eN\varphi)H) + c\,\text{rot}_y M, \quad (3.9)$$

$$Q_y = -\frac{c}{eH^2}\left[\zeta\left(\frac{\partial}{\partial\zeta}\right)_T + T\left(\frac{\partial}{\partial T}\right)_\zeta\right]\int_{-\infty}^{\zeta} d\zeta'\,\{\text{rot}_y((\mathscr{P} + eN\varphi)H)$$

$$- H^2\,\text{rot}_y M\} - c\,[EM]_y; \quad (3.10)$$

Here

$$E_x = -\nabla_x\varphi,$$

$$-\frac{1}{H}\text{rot}_y(\mathscr{P} + eN\varphi)H = \left(\frac{\partial\mathscr{P}}{\partial\zeta}\right)_{V, T, H}\nabla_x\zeta + \left(\frac{\partial P}{\partial T}\right)_{V, \zeta, H}\nabla_x T + eN\nabla_x\varphi.$$

$$(3.11)$$

It is seen from these formulas that the collisionless or non-dissipative fluxes are essentially solenoidal, and therefore it is impossible to use the continuity equation to separate the conduction current and the heat flux.

Formulas (3.9) and (3.10) are remarkable primarily because both j and Q are expressed in terms of the macroscopic quantities M and ($\mathscr{P} + eN\varphi$), for which

the boundary conditions on the surface of the conductor are known; in addition, these formulas can be generalized to the case of an explicit dependence of $\mathbf{M}$ and $(\mathscr{P} + eN\varphi)$ on the spatial coordinates. It is known from electrodynamics[15] that the tangential component $\mathbf{M}$ experiences a discontinuity on the surface of the magnetized conductor and vanishes outside the conductor, and $(\mathscr{P} + eN\varphi)$, neglecting compressibility, is the electrochemical potential per unit volume, which is constant everywhere in the state of thermodynamic equilibrium and is continuous everywhere, including the separation boundaries of the media, at small deviations from thermodynamic equilibrium (see[16]). Taking these boundary conditions into account, it can be noted that the terms in (3.9) and (3.10), containing curl $\mathbf{M}$, make no contribution to the charge and energy transport through the cross section of the entire conductor[15]. This can be easily verified. We integrate (3.9) over the area of the cross section of the entire conductor, and use the Stokes theorem

$$\int (\mathbf{j}\, ds) = -\frac{c}{H^2} \int \text{rot}\,((\mathscr{P} + eN\varphi)\,H)\,ds \div c \int \text{rot}\,\mathbf{M}\,ds$$
$$= -\frac{c}{H^2} \oint_L (\mathscr{P} + eN\varphi)(\mathbf{H}\,dl) \div c \oint_L (\mathbf{M}\,dl). \qquad (3.12)$$

The integration contour L should enclose the cross section area of the entire conductor. The contour L should be chosen to be outside the conductor throughout, without touching its surface anywhere. Since $\mathbf{M} = 0$ outside the conductor, meaning also on the contour L, we get $\oint_L \mathbf{M}\,dl = 0$. By virtue of the continuity of $(\mathscr{P} + eN\varphi)$ we have everywhere*

$$\oint_L (\mathscr{P} + eN\varphi)(\mathbf{H}\,dl) \neq 0,$$

if the state of the system differs from thermodynamic equilibrium characterized by the condition $(\mathscr{P} + eN\varphi)$ = const. Thus, in states differing from thermodynamic equilibrium, the contribution to the transport of charge through the cross section of the entire conductor is made only by the first term in (3.9), which in accordance with the definition is the conduction-current density:

$$\mathbf{j}_{\text{con}} = -\frac{c}{H^2} \text{rot}\,((\mathscr{P} + eN\varphi)\,\mathbf{H}). \qquad (3.13)$$

The result admits of the following illustrative interpretation. Let c curl $\mathbf{M} \neq 0$ at any point of the conductor cross section. Then c curl $\mathbf{M}$ makes a contribution to the charge transport, but $\mathbf{M}$ experiences a finite jump on the surface of the conductor and curl $\mathbf{M} = \infty$. The latter circumstance leads to the occurrence of a surface current which offsets completely the contribution made to the charge transport through the cross section of the conductor by the volume current c curl $\mathbf{M}$. This is precisely why $\int$ curl $\mathbf{M}\,ds = 0$. The function $(\mathscr{P} + eN\varphi)$ has no finite jumps anywhere, including the surface of the conductor, so that the derivatives are bounded everywhere. This leads, in

particular, to the absence of a surface conduction current.

We proceed to separate the heat flux from (3.10). Just as in the preceding case, we integrate (3.10) over the cross section are of the entire conductor:

$$\int Q\,ds = \int ds\,\left\{ -\frac{c}{eH}\left[\zeta\frac{\partial}{\partial\zeta} + T\left(\frac{\partial}{\partial T}\right)_\zeta \right] \int_{-\infty}^{\zeta} d\zeta'\,(\text{rot}\,(\mathscr{P}$$
$$+ eN\varphi)\,\mathbf{H} - H^2\,\text{rot}\,\mathbf{M}) \right\} - c\int ds\,[\mathbf{EM}]. \qquad (3.14)$$

In the case of weak spatial inhomogeneities (i.e., in the transport theory that is linear in ∇T and $\nabla \zeta$) it is necessary to replace ζ and T in the operator

$$\left(\zeta\frac{\partial}{\partial\zeta} + T\left(\frac{\partial}{\partial T}\right)_\zeta\right)\int_{-\infty}^{\zeta} d\zeta'\,\ldots$$

by their mean values over the cross section of the conductor. Allowance for the deviation of ζ and T from their mean values in this operator leads to terms of second order in ∇T and $\nabla \zeta$, which will be neglected throughout. Taking this into account, we changed the order of integration with respect to ds and $d\zeta$; then the analysis of the obtained expression will not differ at all from the analysis of formula (3.12), presented above. The contribution made to the energy transport through the cross section of the conductor by the term $- c\mathbf{E} \times \mathbf{M}$ is completely cancelled by the surface flux due to the jump of the tangential component of $\mathbf{M}$. Consequently, a contribution to the energy transport through the cross section of the entire conductor is made only by the term proportional to $\mathscr{P} + eN\varphi$. Using the definition of the heat flux, we obtain for it the following formula:

$$(Q_T)_y = Q_y - \frac{\zeta}{e}(\mathbf{j}_{\text{пр}})_y - (Q_M)_y = -\frac{c}{eH^2}T\left(\frac{\partial}{\partial T}\right)_\zeta$$
$$\times \int_{-\infty}^{\zeta} d\zeta'\,\text{rot}_y\,(\mathbf{H}\,(\mathscr{P} + eN\varphi)), \qquad (3.15)$$

and the magnetic-energy flux $\mathbf{Q_M}$ contained in this formula is described by formula (2.10).

With the aid of (3.8), (3.11) we can rewrite (3.13) in (3.15) in a different form, namely:

$$(\mathbf{j}_{\text{con}})_y = \frac{c}{H^2}Ne\left[\left(\mathbf{E} - \frac{1}{e}\nabla\zeta\right)\mathbf{H} \right]_y + \frac{c}{H^2}S\,[\mathbf{H}\nabla T]_y, \qquad (3.16)$$

$$(Q_T)_y = \frac{c}{H^2}TS\left[\left(\mathbf{E} - \frac{1}{e}\nabla\zeta\right)\mathbf{H} \right]_y$$
$$+ \frac{c}{eH^2}T\left(\frac{\partial}{\partial T}\right)_\zeta \int_{-\infty}^{\zeta} d\zeta'S\,(\zeta',\,T)\,[\mathbf{H}\nabla T]_y. \qquad (3.17)$$

From these expressions for $\mathbf{j}_{\text{con}}$ and $\mathbf{Q_T}$ we see that these fluxes satisfy both the Einstein relation and the Onsager symmetry principle. In the case of a strongly degenerate electron gas, the corresponding coefficients in (3.16) and (3.17) are connected by the Wiedemann-Franz law[18] (see also[25]).

A formula similar to (3.16), and the resultant differential thermal emf neglecting scattering, $\alpha_{xx} \approx \beta_{xy}/\sigma_{xy} = S/(eN)$ (see (2.11)), were first obtained by Obraztsov[17]. Peletminskiĭ and Bar'yakhtar[25] calculated all the coefficients in $\mathbf{j}_{\text{con}}$ and $\mathbf{Q_T}$, and obtained results that differed from (3.16) and (3.17) only in form. Formulas (3.16) and (3.17) remain in force

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*On the surface of the conductor, both $\mathscr{P}$ and $eN\varphi$ are discontinuous, but their sum is continuous, since the jump of $\mathscr{P}$ is compensated by the jump of φ due to the double electric layer on the surface of the conductor.

when spin is taken into account[13], and also for an arbitrary isotropic carrier dispersion law[20]. Recently Tséndin and Éfros[26] confirmed the validity of (3.16) when account is taken of the spin-orbit interaction of the carriers in the two-band model of the crystal *with isotropic dispersion law.* In conductors with anisotropic dispersion of the carriers, the spatial inhomogeneities of the temperature and of the chemical potential in a plane perpendicular to the magnetic field cause fluxes along the magnetic field. The calculation of the heat flux and of the conduction current in such conductors neglecting scattering, was carried out by Okulov[27]. Peletminskiĭ[28] described a method of separating the conduction current and the heat flux in spatially-homogeneous conductors with arbitrary relation between the Larmor frequency and the carrier-relaxation frequency.

We note that formulas (2.8)–(2.10), which determine the density of the conduction current and of the heat flux, have a wider region of applicability, beyond the limits of those model limitations under which they were derived from the microscopic theory in[18-20]. In particular, it can be assumed that these formulas are valid for arbitrary magnets and for an arbitrary value of $\Omega\tau$. Indeed, the current entering in (2.8) and connected with the magnetization of the carriers, is given by the formula of[15] for all magnets and, of course, for all $\Omega\tau$. This formula is the consequence of the macroscopic boundary conditions for **M** on the surface of the conductor, and the magnetic-energy flux which enters in (2.9) can also be expressed in terms of c curl **M** (see (2.10)).

In concluding this section, let us stop to justify rigorously the procedure used to calculate the nondissipative fluxes. In the derivation of formulas (3.5), (3.6) and (3.13), (3.15) it was tacitly assumed that the distribution function $f_0([E(n, p_z) - \zeta(x_0)]/T(x_0))$ is a Fermi function and describes the spatially-inhomogeneous distributions of the electrons along the x axis, and serves in the case of weak inhomogeneities as a definition of the local thermodynamic characteristics of the electron gas, such as the temperature $T(x)$ and the chemical potential $\zeta(x)$. Let us prove these premises.

In a sufficiently dense electron gas, thermodynamic equilibrium is established as a result of interelectron collisions. In the case of weak spatial inhomogeneities, the electron collision integral (see (A.14) of the appendix) can be expanded in the small parameter

$$\alpha^2 q_y/L = \left(\frac{r_L}{L}\right)\left(\frac{\hbar q_y}{p_e}\right) \ll 1;$$

where

$$L \sim f\,|\partial f/\partial x_0|^{-1}, \quad r_L = \bar{p}_e/m\Omega,$$

$\bar{p}_e$—average electron momentum, and $\hbar q_y$—change of the electron momentum upon collision. The zeroth-order term in this parameter leads to the kinetic equation

$$\frac{\partial f_\varkappa}{\partial t} = \sum_{v, v', v''} W_{vv''}^{\varkappa v'}\{f_v(1-f_\varkappa)f_{v'}(1-f_{v''})$$

$$-f_\varkappa(1-f_v)f_{v''}(1-f_{v'})\}, \qquad (3.18)$$

which is the ordinary equation for the balance of the particles in cell $\varkappa$. The first term in the curly bracket describes the arrival of the particles in the cell $\varkappa$, and the second the departure. The probability of transition per unit time is

$$W_{vv'}^{\varkappa v'} = \sum_q \frac{2\pi}{\hbar}\,|C_q|^2\, F_{n_v n_\varkappa}(\alpha^2 q_\perp^2/2)\, F_{n_{v'} n_{v''}}(\alpha^2 q_\perp^2/2)$$

$$\times\, \delta\,[E_{v'} + E_\varkappa - E_v - E_{v''}]\,\delta\,(p_z^{(v)} - p_z^{(\varkappa)} - \hbar q)\,\delta\,(p_z^{(v')} - p_z^{(v'')} - \hbar q_z)$$

$$\times\, \delta\,(x_{0\varkappa} - x_{0v''})\,\delta\,(x_{0v} - x_{0\varkappa})\,\delta\,(x_{0v'} - x_{0\varkappa}).$$

It follows from (3.18) that the function

$$f(n, p_z, x_0) = \left\{\exp\left(\frac{E(n, p_z) - \zeta(x_0)}{T(x_0)}\right) + 1\right\}^{-1} \qquad (3.19)$$

causes the collision integral to vanish. At low conduction-electron concentrations, the collisions between the electrons and the phonons play an essential role in the establishment of the thermodynamic equilibrium. Let us assume that the interaction of the phonons with the thermostat, whose temperature is specified by the function $T(x)$, is sufficiently strong and that their collisions with the electrons do not violate the local equilibrium of the phonons with the thermostat. Under these assumptions, a local-equilibrium phonon distribution function can be assumed in the kinetic equation for the electrons

$$N_q(x) = N_q^0(x) = \left\{\exp\left(\frac{\hbar\omega_q}{T(x)}\right) - 1\right\}^{-1}. \qquad (3.20)$$

This function is an approximate solution of the kinetic equation (A.29), (A.30) under the condition that the frequency of collision of the phonons with electrons is small compared with the frequency of the collision of the phonons with the thermostat $\omega_{pp}(q)$ (A.30). Substituting (3.20) in the kinetic equation for the electron density matrix (A.28) and then expanding the electron-phonon collision integral $I_{ep}[f]$ in a series in $(r_L/L)\,(\hbar q_y/\bar{p}_e) \ll 1$, we obtain in zeroth order the equation

$$\frac{\partial f_\varkappa}{\partial t} = \sum_{(v, q)} (1 - \hat{\mathscr{P}}_{v\varkappa})\,W_{v\varkappa}(q)\,\{f_v(1-f_\varkappa)(N_q^0(x_{0\varkappa}) + 1) - f_\varkappa(1-f_v)N_q^0(x_{0\varkappa})\}$$

$$(3.21)$$

which takes into account the arrival and the departure of the electrons in the $\varkappa$ state following their interaction with the phonons. The transition probability per unit time is here:

$$W_{v, \varkappa}(q) = \frac{2\pi}{\hbar}\,|C_q|^2\, F_{n_\varkappa n_v}(\alpha^2 q_\perp^2/2)\,\delta\,[E_\varkappa - E_v + \hbar\omega_q]$$

$$\times\, \delta\,(p_z^{(\varkappa)} + \hbar q_z - p_z^{(v)})\,\delta\,(p_y^{(\varkappa)} - p_y^{(v)}); \qquad (3.22)$$

$\hat{\mathscr{P}}$—operator effecting the replacement $v \rightleftarrows \varkappa$. The collision integral in (3.21) is made to vanish also by the function (3.19). We shall show that (3.19) determines the local values of the temperature and of the chemical potential of the electron gas. The number of particles in the state n, p_z at the point x is by definition

$$N_{n, p_z}(x) = \int dx_0 f\left(\frac{E(n, p_z) - \zeta(x_0)}{T(x_0)}\right) \Phi_n^2\left(\frac{x - x_0}{\alpha}\right).$$

The assumption of weak spatial inhomogeneities makes it possible to take f outside the integral sign at the point $x_0 = x$, inasmuch as $\Phi_n^2(x - x_0/\alpha)$ differs from

zero when $|x - x_0| \lesssim r_L$, and f is practically constant at distances $\sim r_L$. Taking account of the normalization of Φ_n^2, we get

$$N_{n, p_z}(x) = f_0\left(\frac{E(n, p_z) - \zeta(x)}{T(x)}\right).$$

It follows therefore that $\zeta(x)$ and $T(x)$ are the local values of the chemical potential and of the temperature.

IV. DISSIPATIVE ELECTRON FLUXES IN THE ELASTIC-SCATTERING APPROXIMATION

When account is taken of only elastic scattering of the electrons in the first Born approximation, the problem of calculating the conduction current and the heat flux due to the spatial inhomogeneities of the temperature T and of the chemical potential ζ is particularly simple. It is precisely with this case that we shall begin the calculation of the dissipative electron fluxes.

We first stop to determine the dissipative heat fluxes and the conduction current. These fluxes, in the case of an isotropic carrier dispersion law, are characterized only by diagonal components of the tensors of the kinetic coefficients (i.e., they are potentials); they are determined with the aid of the continuity equations. The conduction current j_{con} must be determined from the equations for the conservation of the charge $eN(r)$ and the energy $W(r, t)$

$$\frac{\partial}{\partial t}(eN(r, t)) + \operatorname{div} j_{con} = 0, \quad \frac{\partial}{\partial t}W(r, t) + \operatorname{div} Q_T = 0. \quad (4.1)$$

If we confine ourselves to the study of spatial inhomogeneities only along the x axis, which is orthogonal to the quantizing magnetic field, then it is sufficient to consider in this case only the diagonal elements of the density matrix in the Landau representation (energy representation) $f(n, p_z, x_0)$. The charge density at the point x equals by definition the diagonal element of the density matrix in the coordinate representation (x-representation), multiplied by the charge of the carrier:

$$eN(x) = e\frac{2}{(2\pi\alpha)^2 \hbar}\sum_n \int dp_z \int dx_0 f(n, p_z, x_0)\Phi_n^2\left(\frac{x - x_0}{\alpha}\right). \quad (4.2)$$

As seen from (4.2), $\partial/\partial t N$ is expressed in terms of $\partial/\partial t f(n, p_z, x_0)$. The latter quantity can be determined from the kinetic equation for f, which can be written down immediately, taking into account the arrival of the electrons in the cell ν and their departure from this cell. This leads to the following kinetic equation:

$$\frac{\partial f_\nu}{\partial t} = \sum_{\nu'} W_{\nu\nu'}\{f_{\nu'}(1 - f_\nu) - f_\nu(1 - f_{\nu'})\}, \quad (4.3)$$

where the probability of transition per unit time of an electron from the state ν' into the state ν upon scattering by an immobile center equals, in the Born approximation,

$$W_{\nu\nu'} = \frac{2\pi}{\hbar}\sum_q |V_q|^2 F_{nn'}(\alpha^2 q_\perp^2/2)\delta(E(n', p_z + \hbar q_z) - E(n, p_z))$$

$$\times \delta(x_{0\nu'} - x_{0\nu} - c\hbar q_y/eH);$$

$|V_q|^2 = N_{imp}|C_q|^2$, N_{imp}—number of impurities, and C_q—Fourier component of the potential energy of the interaction of the electron with the scatterer. We note that (4.3) can be obtained from the kinetic equation

(A.28) by taking into account the collisions of the electrons with the phonons in the elastic-scattering approximation. To this end, it is necessary to neglect processes of phonon emission, i.e., to assume in (A.28) that $N_q \gg 1$ and to neglect the change in the electron energy upon scattering, putting $\hbar\omega_q \sim 0$ in $\delta(E(n', p_z + \hbar q_z) - E(n, p_z) + \hbar\omega_q)$, and finally, replace $|C_q|^2 N_q$ by $|C_q|^2 N_{imp}$. With the aid of (4.3) it is easy to obtain an expression for the conduction current. Differentiating (4.2) with respect to time and substituting (4.3) in the obtained expression, we get

$$\frac{\partial}{\partial t}(eN(x)) = \frac{2eN_{imp}}{(2\pi\alpha)^2 \hbar}\sum_{(n', q)}\int dp_z \int dx_0 \Phi_n^2\left(\frac{x - x_0}{\alpha}\right)\frac{2\pi}{\hbar}|C_q|^2 F_{nn'}(\alpha^2 q_\perp^2/2)$$

$$\times \delta[E(n', p_z + \hbar q_z) - E(n, p_z)][f(n', p_z + \hbar q_z, x_0 + \gamma\alpha^2 q_y)$$

$$- f(n, p_z, x_0)](\gamma = \operatorname{sign} e). \quad (4.4)$$

Further, it is necessary to substitute in (4.4) the solution of the kinetic equation for the function $f(n, p_z, x_0)$. Such a solution, in the zeroth approximation in the scattering, is the local-equilibrium distribution (3.19). It must be substituted in (4.4) to find the conduction current with accuracy not exceeding the accuracy of the kinetic equation. In the case of weak spatial inhomogeneities, the function f is practically constant over the length of the Larmor radius, where the function $\Phi_n^2(x - x_0)/\alpha$ is noticeably different from zero, so that f can be taken outside the sign of the integral with respect to x_0 at values $x_0 = x$. Taking this into account, substituting (3.19) in (4.4), and expanding the results in a series in $\alpha^2 q_y$, we get[13,14]

$$\frac{\partial}{\partial t}(eN(x)) = -\frac{\partial}{\partial x}\left\{-\frac{2eN_n}{(2\pi\alpha)^2 \hbar}\sum_{n, n', q}\int dp_z \frac{2\pi}{\hbar}|C_q|^2 F_{n, n'}(\alpha^2 q_\perp^2/2)\right.$$

$$\times \delta[E(n', p_z + \hbar q_z) - E(n, p_z)]\frac{(\alpha^2 q_y)^2}{2!}$$

$$\left.\times \frac{\partial f_0}{\partial E}\left[\nabla_x\zeta + \frac{E(n, p_z) - \zeta}{T}\nabla_x T\right] + \dots\right\}. \quad (4.5)$$

The expression in the curly brackets in (4.5) determines the conduction current in terms of the gradients of the spatial inhomogeneities of T and ζ. The coefficient in front of $(-(1/e)\nabla_x\zeta)$ coincides with the electric-conductivity coefficient obtained by Adams and and Holstein[29]. This indicates that the Einstein relation, connecting the diffusion coefficient and the electric conductivity coefficient, holds. In complete analogy with the derivation of (4.5), we obtain a continuity equation for the thermal energy[13,14]

$$\frac{\partial}{\partial t}\left\{\frac{2}{(2\pi\alpha)^2 \hbar}\sum_n \int dx_0 dp_z \Phi_n^2\left(\frac{x - x_0}{\alpha}\right)[E(n, p_z) - \zeta]f(n, p_z, x_0)\right\}$$

$$= \frac{\partial}{\partial x}\left\{-\frac{2N_n}{(2\pi\alpha)^2 \hbar}\sum_{n, n', q}\frac{2\pi}{\hbar}\int dp_z |C_q|^2 F_{n', n}\left(\frac{\alpha^2 q_\perp^2}{2}\right)\left(\frac{\alpha^2 q_y}{2!}\right)^2 [E(n, p_z - \zeta)]\right.$$

$$\times \delta[E(n', p_z + \hbar q_z) - E(n, p_z)]\frac{\partial f_0}{\partial E}\left[\nabla_x\zeta + \frac{E(n, p_z) - \zeta}{T}\nabla_x T\right] + \dots\right\}. \quad (4.6)$$

Thus, the diagonal components of the tensors of the kinetic coefficients in j_{con} and Q_T (see (2.1)) can be represented in the form

$$\begin{pmatrix}\sigma_{xx} \\ \beta_{xx} \\ \varkappa_{xx}\end{pmatrix} = \frac{-2N_n}{(2\pi\alpha)^2 \hbar}\sum_{(n, n', q)}\frac{2\pi}{\hbar}|C_q|^2 F_{n, n'}\left(\frac{\alpha^2 q_\perp^2}{2}\right)\frac{\alpha^4 q_y^2}{2!}\frac{\partial f_0}{\partial E}$$

$$\times \, \delta\,[E\,(n',\,p_z + \hbar q_z) - E\,(n,\,p_z)] \begin{pmatrix} e^2 \\ e\,[E\,(n,\,p_z) - \zeta]/T \\ [E\,(n,\,p_z) - \zeta]^2/T \end{pmatrix}, \quad T\beta_{xx} = \chi_{xx}. \tag{4.7}$$

The dependence of $|C_q|^2$ on q is determined by the scattering mechanism and is given in Table I, taken from the paper of Adams and Holstein[29]. The dependence of σ, β, and κ on T and H turns out to be the most sensitive to the electron scattering mechanism only in the quantum limit.

Table I. Dependence of $|C_q|^2$ on q and values of the numbers m and p which enter in the tables

Scattering mechanism	$|C_q|^2$	m	p
Acoustic phonons:			
low temperatures	$A_1 q$	0	1
high temperatures	$A_1\,(2T/\hbar s)$	1	0
Piezoelectric phonons			
low temperatures	$A_3 q$	0	−1
high temperatures	$A_3\,(q^2 + q_s^2)^{-1}\,(2T/\hbar s)$	1	−2
Optical phonons			
high temperatures	$A_4\,(q^2 + q_s^2)^{-1}\,(2T/\hbar \omega_D)$	1	−2
		0	0
Pointlike defects	A_2	0	0
Ionized impurities	$A_5\,(q^2 + q_s^2)^{-2}$	0	−4

When H varies in the range $\zeta \gg \hbar\Omega \gtrsim kT^\circ$, the quantities σ, β, and κ experience quantum oscillations. As shown by Adams and Holstein[29], the dependence of these oscillations on the scattering mechanism, at least for σ_{xx}, turns out to be not very important. This is precisely why it is of interest to study in the quantum limit the dependence of σ, β, and κ on T and H at different mechanisms of carrier scattering. We present here such dependences, obtained by Zyryanov and Kalashnikov[30] for two limiting cases.

a) In the case of nondegenerate electrons in the quantum limit $\hbar\Omega \gg kT^\circ$ we get from (4.7)

$$\left.\begin{aligned} \varkappa_{xx} &= \frac{1}{e^2}\left(\frac{\partial}{\partial T}\,T^2\,\frac{\partial}{\partial T}\,\sigma_{xx}\right)_\zeta, \quad \beta_{xx} = \frac{1}{e}\left(\frac{\partial}{\partial T}\,T\sigma_{xx}\right)_\zeta, \\ \sigma_{xx} &= \frac{e^2\alpha^2}{2\pi}\,\frac{m}{T}\,e^{\zeta^*/T}\int\frac{dq_x dq_y}{(2\pi\hbar)^3}\,N_n\,|C_q q_y|^2 \\ &\quad \times F_{0,\,0}\left(\frac{\alpha^2 q_\perp^2}{2}\right)\int_0^\infty\frac{dE}{E}\,e^{-E/T}, \\ \zeta^* &= \zeta - \frac{\hbar\Omega}{2}. \end{aligned}\right\} \tag{4.8}$$

In the latter formula for σ_{xx}, a divergence appears at the lower limit. The nature of this divergence and different methods of its elimination are discussed in the paper of Adams and Holstein[29]. Following that paper, we can introduce a cutoff energy, connected for example with the broadening of the Landau energy levels as a result of the collisions of the electrons with the scatterers. A detailed discussion of the question is contained in the review of Kubo et al.[3] In the quantum limit $\hbar\Omega > T$, the dependence of the diagonal components of the tensors of the kinetic coefficients on the magnetic field and on the temperature for different scattering mechanisms is shown in Table II. In the

Table II. Dependence of diagonal components of the tensors σ, β, χ, κ on T and H for nondegenerate electrons in the quantum limit

Tensor	a)	b)	Tensor	a)	b)
σ_{xx}	$T^{m-3/2}H^{p/2}$	$T^{p+5/2}H^{-2}$	χ_{xx}	$T^{m-1/2}H^{p/2}$	$T^{p+7/2}H^{-2}$
β_{xx}	$T^{m-3/2}H^{p/2}$	$T^{p+5/2}H^{-2}$	$\varkappa_{xx}$	$T^{m-1/2}H^{p/2}$	$T^{p+7/2}H^{-2}$

case of acoustic and piezoelectric phonons, the results given in column a) (Tables II—V) are valid for $H < cT(\hbar s^2 e)^{-1}$ (s—speed of sound). If the opposite inequality holds, then it is necessary to use for these two scattering mechanisms the results given in column b) (for acoustic phonons p = 1, and for piezoelectric phonons p = − 1).

These results, together with formulas (3.23) and (3.24) for the nondissipative fluxes, suffice to determine the dependence of any thermomagnetic effect on T and H. For example, in Table III we present these relations in the quantum limit for the isothermal Nernst coefficients Q_N and for the electronic thermal conductivity κ_e at zero conduction current.

Table III. Dependence of the isothermal Nernst coefficients Q_N and of electronic thermal conductivity κ_e at zero current in the case of nondegenerate electrons at $\hbar\Omega \gg T$.

Thermo-magnetic coefficient	a)	b)
Q_N	$T^{m-3/2}H^{p/2}$	$T^{p+5/2}H^{-2}$
$\varkappa_e$	$T^{m-1/2}H^{p/2}$	$T^{p+7/2}H^{-2}$

b) In the other limiting case—strongly degenerate electron gas $(T \ll \zeta)$ we have

$$\beta_{xx} = \frac{\pi^2}{3e}\,T\left(\frac{\partial\sigma_{xx}}{\partial\zeta}\right), \quad \varkappa_{xx} = \frac{\pi^2}{3e^2}\,T\sigma_{xx}, \quad \sigma_{xx} = \frac{Ne^2\tau}{m}, \tag{4.9}$$

where for the ultraquantum limit $T \ll \zeta < \hbar\Omega$ we have

$$\tau = \frac{8}{\pi}\,\frac{\alpha^4}{\hbar^2}\left(\frac{2m}{\zeta^*}\right)^{3/2}\frac{N_n}{(2\pi)^3}\int dq\,|C_q|^2\exp\left(-\frac{\alpha^2 q_\perp^2}{2}\right)$$
$$\times\left\{\delta(q_z) + \delta\left(q_z + 2\sqrt{\frac{2m\zeta^*}{\hbar}}\right)\right\}. \tag{4.10}$$

It follows from these formulas that both the diagonal components of σ and κ, and the nondiagonal ones, are connected by the Wiedermann-Franz law. In other words, the Wiedermann-Franz law is valid also in the ultraquantum limit.

Table IV gives the dependences of the kinetic coefficients on T and H in the ultraquantum limit $\hbar\Omega > \zeta$.

Table IV. Dependence of the diagonal components of the tensor σ, β, χ, κ on T and H for strongly degenerate electrons in the ultraquantum limit $(T \ll \xi < \hbar\Omega)$.

Tensor	a)	b)	Tensor	a)	b)
σ_{xx}	$T^m H^{3+p/2}$	$T^{p+4}H$	χ_{xx}	$T^{m+2}H^{5+p/2}$	$T^{p+6}H^3$
β_{xx}	$T^{m+1}H^{5+p/2}$	$T^{p+5}H^3$	$\varkappa_{xx}$	$T^{m+1}H^{3+p/2}$	$T^{p+5}H$

Table V. Dependence of isothermal Nernst coefficients Q_N and of the electronic thermal conductivity κ_e at zero current in the case of strongly degenerate electrons in the ultraquantum limit $(T \ll \xi < \hbar\Omega)$

Thermo-magnetic coefficient	a	b
Q_N	$T^{m+1}H^{5+p/2}$	$T^{p+5}H^3$
κ_e	$T^{m+1}H^{3+p/2}$	$T^{p+5}H$

For the same limiting case, Table V lists the dependence of the isothermal Nernst coefficients Q_N and of the electronic thermal conductivity κ_e on T and H.

Bar'yakhtar and Peletminskii[31] constructed a more general theory, in which the diagonal components of the tensors of the kinetic coefficients in the conduction current and in the heat flux are expressed in terms of the exact electron-impurity scattering amplitude. The procedure for calculating the conduction current and the heat flux used in[31] is fully equivalent, as shown by Peletminskii[32], to the kinetic-equation method. The kinetic equation obtained in[32] differs in this case from the approximate equation (A.41) in that it contains the exact scattering amplitude instead of the Born amplitude. However, the kinetic coefficients can be expressed in terms of the amplitude of scattering by a single center, provided the de Broglie wavelength of the carrier is $\lambdabar \ll a_n$—the average distance between the scattering centers. If at the same time $\lambdabar \gg r_0$—the force radius of the scatterer, then it is possible to obtain an asymptotically exact scattering amplitude for the electron with zero energy. Skobov[34] obtained for this case the scattering amplitude in the following form:

$$t_{vv'}(\mathbf{q}) = a \frac{\langle v | e^{i\mathbf{q}\mathbf{r}} | v' \rangle}{1 + iaK(E_v)}. \qquad (4.11)$$

Here

$$K(E) = K'(E) + iK''(E), \qquad K''(E) = (2\eta\alpha^2)^{-1/2},$$

$$K'(E) = (2\alpha^2)^{-1/2} \sum_{m=0}^{N-1} \left(\frac{E}{\hbar\Omega} - m - \frac{1}{4} \right),$$

a—amplitude for scattering of an electron with zero energy by an impurity in the absence of a magnetic field, and $0 \leq \eta < 1$ is defined by the equation $E = (N + \frac{1}{2} - \eta)\hbar\Omega$ (N—positive integer). If the transition probability $W_{vv'}$ in (4.3) is defined in terms of the Skobov amplitude of scattering $t_{vv'}(\mathbf{q})$, then we get for the kinetic coefficients formulas analogous to (4.7), the only difference being that in (4.7) $|C_q|^2 F_{nn'}(\alpha^2 q_\perp^2/2)$ is replaced by $|C_0|^2 F_{nn'}(\alpha^2 q_\parallel^2/2)$ $|1 + iaK(E_v)|^{-2}$, where $|C_0|^2 = (N_{imp}/V)(2\pi\hbar^2 a/m)^2$. The kinetic coefficients obtained in this manner no longer contain divergences in the case of the Maxwell statistics, inasmuch as in the Bar'yakhtar-Peletminskii theory[31] the Skobov scattering amplitude (4.15) takes into account the broadening of the Landau levels due to the scattering of the carriers. The dependence of the kinetic coefficients on T and H in this case is analyzed in[31].

In concluding this section, let us stop to discuss other works devoted to transport phenomena in a strong magnetic field in the elastic-scattering approximation. Jajdu[35] and Hajdu and Fischer[36] considered transport phenomena in systems with spatially-inhomogeneous temperature in the elastic-scattering approximation. The diagonal components of the kinetic-coefficient tensors obtained by them coincide with formulas (4.7), the only difference being in the form. As to the nondiagonal components, they satisfy neither the Einstein relation nor the Onsager symmetry principle. In the case of strong degeneracy of the electron gas, as noted in[36], the nondiagonal components, unlike the diagonal ones, are not connected by the Wiedermann-Franz law. These paradoxes are the consequence of an erroneous identification of the volume density of the nondissipative charge flux with the conduction current, and of $(\mathbf{Q} - \zeta\mathbf{j}/e)$ with the heat flux (see Sec. II). The justification of the Wiedermann-Franz law in quantizing magnetic fields is the subject also of a paper by Zak[37]. However, this paper contains no valid proof, since the initial equation of this paper is postulated, and its connection with the Schrödinger equation for the density matrix is not clear and is not discussed at all. The coefficient β_{xx} (see (4.7)) was first calculated by Ansel'm and Askerov[77,78]. Recently Ansel'm, Obraztsov, and Tarkhanyan[33] refined their results.

V. DISSIPATIVE FLUXES WITH ALLOWANCE FOR INELASTIC SCATTERING OF ELECTRONS AND PHONONS

The elastic-scattering approximation used in the calculation of the conduction current and the heat flux in the preceding section is not always convenient. Allowance for inelastic scattering in the collisions of electrons and phonons leads to dragging effects. These effects are most important in semiconductors with Maxwellian statistics of the carriers. For example, as noted in Sec. I, the thermal emf in n-Ge and n-InsB at low temperatures in quantizing magnetic fields is due essentially to the effect of electron dragging by phonons, i.e., to inelastic scattering.

When electrons interact with optical phonons, the inelasticity of the scattering leads to the so-called magnetophonon resonance predicted independently in[38]. The gist of this resonance consists in the fact that when the optical-phonon frequency is a multiple of the cyclotron frequency of the carriers, i.e., $\omega_D = N\Omega$ (N—positive integer), resonant transitions of the carriers between the Landau levels are possible, accompanied by absorption or emission of an optical phonon. Thus, the inelastic scattering of electrons by optical phonons leads to oscillations of the electric conductivity and of other kinetic coefficients when the magnetic field is varied. We shall not stop here to discuss these questions.

In other cases, allowance for inelasticity in electron scattering apparently does not lead to qualitatively new phenomena. We shall henceforth concentrate our attention on inelastic scattering of electrons by acoustic phonons and proceed to calculate the conduction current and the heat flux in such systems. If we take into

account the deviation of the phonons from local equilibrium, then in the plane orthogonal to the magnetic field it is necessary to consider already two-dimensional inhomogeneities. Such spatial inhomogeneities are described by a density matrix which is nondiagonal in $p_y = eHx_0/c$. However, it is more convenient in what follows to use the mixed representation of Wigner

$$f(n, \ p_z, \ p_y, \ y) = \sum_{p'_y} f_{n, \ p_z, \ p_y; \ n, \ p_z, \ p'_y} \exp\left(i\left(p_y - p'_y\right) y/\hbar\right), \quad (5.1)$$

since, just as in the preceding section, the conduction current and the heat flux will be determined with the aid of the continuity equation. Using the approximation of weak spatial inhomogeneities, we obtain with the aid of the kinetic equation (A.32), in full analogy with the derivation of (4.5), the following formula:

$$-\frac{\partial}{\partial t} eN(x, y) = 2e \sum_{n, \ p_z, \ p_y} I_{ep} [f(n, p_z, p_y, y)] \Phi_n^2 \left(\frac{x-x_0}{\alpha}\right) \quad (5.2)$$

The rate of time variation of the density of the thermal energy of the electrons at the point x, y can also be expressed with the aid of (A.32) by means of the formula

$$\frac{\partial}{\partial t} \left\{ 2 \sum_{(n, \ p_z, \ x_0)} [E(n, p_z) - \zeta] \Phi_n^2 \left(\frac{x-x_0}{\alpha}\right) f(n, p_z, p_y, y) \right\}$$

$$\equiv \frac{\partial}{\partial t} W^{(e)}(x, y) = 2 \sum_{n, \ p_z, \ x_0} [E(n, p_z) - \zeta] \Phi_n^2 \left(\frac{x-x_0}{\alpha}\right) I_{ep} [f(n, p_z, p_y, y)], \quad (5.3)$$

and finally, using the kinetic equation for the phonons (A.33), we obtain the rate of change of the phonon energy density with time, in the form

$$\frac{\partial}{\partial t} \sum_q N_q(x, y) \hbar\omega_q \equiv \frac{\partial}{\partial t} W^{(p)} = -\frac{\partial}{\partial x} \left(\sum_q \hbar\omega_q \frac{\partial\omega_q}{\partial q_x} N_q(x, y)\right)$$

$$-\frac{\partial}{\partial y} \left(\sum_q \hbar\omega_q \frac{\partial\omega_q}{\partial q_y} N_q(x, y)\right) + \sum_q \hbar\omega_q I_{pe} [N_q] + \sum_q \hbar\omega_q I_{pd} [N_q]. \quad (5.4)$$

We shall apply the general formulas (5.2)—(5.4) to some particular cases.

We first calculate the conduction current and the heat flux, assuming a local-equilibrium phonon distribution. Such a situation is realized when the frequency of the collisions between the phonons and the "thermostat" at temperature T(x, y) greatly exceeds the frequency of their collision with the electrons. The collisions of the electrons with the phonons do not violate in this case the local equilibrium of the phonons with the thermostat, and their distribution function takes the form

$$N_q^0(x, y) = \left\{\exp\frac{\hbar\omega_q}{T} - 1\right\}^{-1}. \quad (5.5)$$

In the approximation under consideration, Eq. (5.5) is the solution of the kinetic equation (A.35). Owing to the collisions with the phonons, there is also established in the electron system a local-equilibrium distribution with a Fermi function

$$f_0(n, p_z, x, y) = \left[\exp\left(\frac{E(n, p_z) - \zeta(x, y)}{T(x, y)}\right) + 1\right]^{-1}. \quad (5.6)$$

This function is a solution of the kinetic equation (A.32) in the zeroth approximation in $(\Omega\tau)^{-1} \ll 1$. Substituting (5.5) and (5.6) in (5.2)—(5.4) and then expanding the right sides in $\alpha^2 q_x$ and $\alpha^2 q_y$, we find the continuity equations for the charge and the energy[39] of both the

electrons and the phonons:

$$-\frac{\partial}{\partial t}\left(\begin{array}{c} eN(x, y) \\ W^{(e)}(x, y) \\ W^{(p)}(x, y) \end{array}\right) = -\operatorname{div}\left(\begin{array}{c} \mathbf{j}_{\text{con}}(x, y) \\ \mathbf{Q}_T^{(e)}(x, y) \\ \mathbf{Q}_T^{(p)}(x, y) \end{array}\right), \quad (5.7)$$

where

$$\left(\begin{array}{c} (j_{np})_x \\ y \\ (Q_T^{(e)})_x \\ y \\ (Q_T^{(p)})_x \end{array}\right) = \left(\frac{\alpha^2}{4\pi}\right)^2 \int \left\{ \left(\begin{array}{c} e\omega_{ep}\langle 1\rangle \\ \omega_{ep}\langle(E + 2\hbar\omega - \zeta)\rangle \\ \hbar\omega_q\omega_{ep}\langle 1\rangle \end{array}\right)\left(-\frac{1}{T}\nabla_x\zeta\right) \right.$$

$$\left. + \left(\begin{array}{c} e\omega_{ep}\langle(E + \hbar\omega - \zeta)\rangle \\ \omega_{ep}\langle(E + \hbar\omega - \zeta)(E - \zeta)\rangle \\ \hbar\omega_q\omega_{ep}\langle(E + \hbar\omega - \zeta)\rangle \end{array}\right)\nabla_x T^{-1} \right\} q_\perp^3 \left[\operatorname{ch}\frac{\hbar\omega_q}{T} - 1\right]^{-1} dq_z \, dq_\perp, \quad (5.8)$$

$$\omega_{ep}\langle(E + \hbar\omega - \zeta)^{\overline{m}}(E - \zeta)^{\overline{m}'}\rangle = \frac{2}{(2\pi\alpha)^2\hbar} \sum_{(n, \ n')} \frac{2\pi}{\hbar} |C_q|^2 F_{n, \ n'}\left(\frac{\alpha^2 q_\perp^2}{2}\right)$$

$$\times \int dp_z \delta\left[E(n', \ p_z + \hbar q_z) - E(n, \ p_z) - \hbar\omega_q\right]\langle(E(n, \ p_z) + \hbar\omega_q - \zeta)^{\overline{m}}$$

$$\times (E(n, \ p_z) - \zeta)^{\overline{m}'}\rangle\left[f_0\left(\frac{E(n, \ p_z) - \zeta}{T}\right) - f_0\left(\frac{E(n, \ p_z) + \hbar\omega_q - \zeta}{T}\right)\right], \quad (5.9)$$

$\overline{m}$ and $\overline{m}'$ assume the values 0, 1, 2; the symbol $\omega_{ep}\langle 1\rangle$ is used when $\overline{m} = \overline{m}' = 0$. Summing $Q_T^{(e)}$ and $Q_T^{(p)}$, we obtain the total heat flux of the electrons and phonons[39]

$$(Q_T)_x = \left(\frac{\alpha^2}{4\pi}\right)^2 \int dq_z \, dq_\perp q_\perp^3 \left[\operatorname{ch}\frac{\hbar\omega_q}{T} - 1\right]^{-1}$$

$$\times \left\{\omega_{ep}\langle(E + \hbar\omega - \zeta)^2\rangle \nabla_x T^{-1} - \omega_{ep}\langle(E + \hbar\omega - \zeta)\rangle T^{-1}\nabla_x\zeta\right\}. \quad (5.10)$$

The kinetic coefficients are connected by the Onsager symmetry relation only in the total heat flux Q_T (5.10) and in the conduction current j_{con} (5.8). These calculations were performed with such detail only because in some papers an error has crept into the determination of the phonon energy flux density in spatially-inhomogeneous systems. Thus, for example, Fischer[40] gives for the heat flux carried by the phonons the formula

$$\sum_q \hbar\omega_q \left(\frac{\partial\omega}{\partial q}\right) N_q. \quad (5.11)$$

This formula describes, as can be seen from (5.4), only that part of the phonon-energy flux which is due to the deviation of the distribution function N_q from the local equilibrium (5.5). In other words, the energy flux (5.11) is due only to the phonon drift. In the state of local equilibrium, the flux (5.11) vanishes, but in this case the spatial inhomogeneities of the temperature of the electrons and phonons lead to a diffusion heat flux described by formula (5.8) for $Q_T^{(p)}$. If the total heat flux in the system is determined by the formula $\left\{Q_T^{(e)} + \sum_q \hbar\omega_q \times (\partial\omega/\partial q) N_q\right\} = Q^*$, then the Onsager relation for j_{con} (5.8) and for Q^* will not be satisfied.

We now proceed to consider effects connected with allowance for the deviation of the phonon distribution function from the local equilibrium function (4.6). In pure metals and semiconductors, at sufficiently low temperatures, it may turn out that the non-electronic phonon relaxation frequency (the relaxation of the phonons on the "thermostat") $\omega_{pp}(q)$ is comparable

with or even smaller than their relaxation frequency on the electrons. Under these conditions, the phonon distribution function will depend not only on the state of the thermostat, but also on the state of the electrons. A correct theory of the kinetic phenomena calls for a consistent description of the electron-phonon systems with the aid of two coupled integro-differential equations for the nonequilibrium additions to the phonon and electron distribution functions. Neglect of one of these additions in the equation for the other leads, as shown by calculations in the absence of a magnetic field (see[41]) to violation of the Kelvin relations for the thermoelectric coefficients. These relations, or more accurately the second one, are the consequence of the symmetry principle of the kinetic coefficients of Onsager. Consequently, the violation of the second Kelvin relation is the consequence of violation of the Onsager symmetry principle. As shown by Sondheimer[42], the correct theory, based on a system of integro-differential equations for the nonequilibrium additions to the electron and phonon distribution functions, ensures satisfaction of the Onsager symmetry principle. However, so far no exact solution of such a system of equations in a weak or zero magnetic field could be found. In the case of strong magnetic fields, when the small parameter $(\Omega\tau)^{-1} \ll 1$ appears (τ—electron relaxation time), it becomes possible to find the solution of such a system of integro-differential equations in the form of a series in powers of this parameter. The Onsager symmetry principle is then satisfied; consequently, the Kelvin relations are also satisfied.

We shall henceforth assume, as is usually done in linear transport theory, that the phonon distribution function differs little from the local-equilibrium function (5.5). Accordingly we put

$$N_q(x, y) = N_q^0(x, y) + g_q(x, y), \quad |g_q| \ll N_q^0. \tag{5.12}$$

Substituting (5.12) in the kinetic equation (A.33), we obtain an equation for the determination of $g_q(x, y)$. This equation includes the electron distribution function $f(n, p_z, x, y)$. In order to find g_q in the first approximation in $(\Omega\tau)^{-1}$, it is sufficient to take for f the solution of Eq. (A.32) in the zeroth approximation in $(\Omega\tau)^{-1}$. Such a solution is the local-equilibrium electron distribution function (5.6). Substituting (5.6) in the equation for the function g_q, we obtain in the linear approximation in the gradients T and ζ the following formula[11,12]:

$$g_q = \left\{ 2\left[\mathrm{ch}\,\frac{\hbar\omega_q}{T} - 1\right][\omega_{ep}\langle 1\rangle + \omega_{pp}(q)] \right\}^{-1}$$

$$\times \{\hbar s^2(q\nabla)T^{-1} - \gamma\alpha^2 q_y [\omega_{ep}\langle(E+\hbar\omega-\zeta)\rangle \nabla_x T^{-1} + \omega_{ep}\langle 1\rangle \nabla_x\zeta]$$

$$+ \gamma\alpha^2 q_x [\omega_{ep}\langle(E+\hbar\omega-\zeta)\rangle \nabla_y T^{-1} + \omega_{ep}\langle 1\rangle \nabla_y\zeta]\}. \tag{5.13}$$

Substituting (5.12) and (5.6) in (5.2), we obtain in the linear approximation in ∇T and $\nabla\zeta$ an expression for the conduction current with allowance for the phonon dragging[11,12,43]

$$(j_{con})_x = e\left(\frac{\alpha^2}{4\pi}\right)^2 \int dq_z\,dq_\perp\,q_\perp^3 \left[\mathrm{ch}\,\frac{\hbar\omega_q}{T} - 1\right]^{-1} (\omega_{ep}\langle 1\rangle + \omega_{pp}(q))^{-1}$$

$$\times \left\{ \omega_{ep}\langle(E+\hbar\omega-\zeta)\rangle\,\omega_{pp}\,\nabla_x T^{-1} - \omega_{ep}\langle 1\rangle\,\omega_{pp}(q)\,T^{-1}\nabla_x\zeta \mp \gamma\frac{\hbar s^2}{\alpha^2}\,\omega_{ep}\langle 1\rangle\,\nabla_y T^{-1} \right\}$$

$$\tag{5.14}$$

Substituting (5.12) and (5.6) in (5.3) and (5.4), we obtain, again in the linear approximation, a formula for the total heat flux[11,12,43]

$$(Q_T)_x = \left(\frac{\alpha^2}{4\pi}\right)^2 \int dq_z\,dq_\perp\,q_\perp^3 \left[\mathrm{ch}\,\frac{\hbar\omega_q}{T} - 1\right]^{-1} (\omega_{ep}\langle 1\rangle + \omega_{pp}(q))^{-1}$$

$$\times \left\{ \left[\omega_{ep}\langle(E+\hbar\omega-\zeta)^2\rangle\,(\omega_{ep}\langle 1\rangle + \omega_{pp}(q)) \right. \right.$$

$$\left. + \left(\left(\frac{\hbar s^2}{\alpha^2}\right)^2 - (\omega_{ep}\langle(E+\hbar\omega-\zeta)\rangle)^2\right) \right] \nabla_x T^{-1}$$

$$- \omega_{ep}\langle(E+\hbar\omega-\zeta)\rangle\,\omega_{pp}(q)\,\nabla_x\zeta$$

$$\mp \gamma\frac{\hbar s^2}{\alpha^2}\,\omega_{ep}\langle(E+\hbar\omega-\zeta)\rangle\,\nabla_y T^{-1} \pm \gamma\frac{\hbar s^2}{\alpha^2}\,\omega_{ep}\langle 1\rangle\,\nabla_y\zeta \right\}. \tag{5.15}$$

Comparing the kinetic coefficients in front of the temperature gradient in (5.14) and the gradient of the chemical potential (5.15), we verify that they satisfy the Onsager symmetry principle. The coefficient preceding $(-\nabla\zeta)/e$ in (5.14) coincides with the electric conductivity coefficient obtained earlier in the paper by Gurevich and Nedlin[44] by the method of Konstantinov and Perel'[45]. This confirms once more the satisfaction of the Einstein relation.

Formulas (5.14) and (5.15) were obtained also by Akhiezer, Bar'yakhtar, and Peletminskiĭ[9]. The difference lies in the notation. In this interesting paper, the total heat flux and the conduction current in an electron-phonon system were determined in terms of the entropy production. The entropy production was calculated with the aid of kinetic equations for the electron and phonon single-particle density matrices.

A recently published paper by Fischer[40] is also devoted to the calculation of the conduction current and the heat flux with allowance for phonon dragging. The kinetic coefficients in the conduction current, obtained in[40], coincide with the corresponding coefficients in our formula (5.14). The coefficients in the electronic part of the heat flux also coincide. As already noted above, the total heat flux was not determined correctly in[40].

VI. FLUXES ALONG THE MAGNETIC FIELD AND LONGITUDINAL THERMOMAGNETIC PHENOMENA

In preceding sections we considered fluxes orthogonal to the magnetic field, and the associated transverse thermomagnetic phenomena. In this section we shall calculate the fluxes along the magnetic field. Attention will be paid principally to effects connected with the non-equilibrium behavior of the phonons, i.e., with dragging effects. These effects are of greatest interest since the differential thermal emf due to the non-equilibrium behavior of the phonons (the dragging thermal emf) increases with increasing magnetic field intensity in the quantum region as shown in[48].

We shall be interested in what follows in sufficiently strong magnetic fields $(\Omega\tau \gg 1)$ and with distributions that are spatially inhomogeneous along the magnetic field H ǁ Oz. In this case, to describe the system it is necessary to retain in the Landau representation the density matrices that are diagonal in the quantum number n, do not depend on p_y, and are nondiagonal in p_z. In the case of weak spatial inhomogeneities, it is convenient to use the density matrix in the Wigner representation

$$f(n, p_z, z) = \sum_{p_z'} f_{n, p_z; n, p_z'} \exp\{i(p_z - p_z')z/\hbar\}. \quad (6.1)$$

Indeed, if the spatial inhomogeneities are small compared with the de Broglie wavelength, the left side (the Liouville operator) of the kinetic equation can be written in the usual form

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} + eE_z \frac{\partial}{\partial p_z}\right) f(n, p_z, z) = I\{f(n, p_z, z)\}. \quad (6.2)$$

The electron-phonon collision integral contain in this equation can be represented in the form

$$I\{f(n, p_z, z)\} = \frac{2\pi}{\hbar} \sum_{(n', q, p_z')} |C_q|^2 \left\{ F_{n', n}\left(\frac{\alpha^2 q_\perp^2}{2}\right) \right.$$
$$\times \delta[E(n', p_z') - E(n, p_z) - \hbar\omega_q]\, \delta[p_z' - \hbar q_z - p_z]$$
$$\times [(f(n', p_z', z) - f(n, p_z, z))N_q(z) + f(n', p_z', z)(1 - f(n, p_z, z))]$$
$$+ F_{n, n'}\left(\frac{\alpha^2 q_\perp^2}{2}\right) \delta[E(n', p_z') - E(n, p_z) + \hbar\omega_q]\, \delta[p_z' + \hbar q_z - p_z]$$
$$\left. \times [(f(n', p_z', z) - f(n, p_z, z))N_q(z) - f(n, p_z, z)(1 - f(n', p_z', z))]\right\}. \quad (6.3)$$

The phonon distribution function is obtained also from the kinetic equation

$$\left(\frac{\partial}{\partial t} + \frac{\partial\omega}{\partial q_z}\frac{\partial}{\partial z}\right) N_q(z) = I\{N_q(z)\}, \quad (6.4)$$

where

$$I\{N_q(z)\} = \frac{2\pi}{\hbar} \sum_{n, n'} \frac{2}{(2\pi\alpha)^2\hbar} \int dp_z dp_z' |C_q|^2 F_{n, n'}\left(\frac{\alpha^2 q_\perp^2}{2}\right) \delta(p_z' - \hbar q_z - p_z)$$
$$\times \delta[E(n', p_z') - E(n, p_z) - \hbar\omega_q]\,[(f(n', p_z', z) - f(n, p_z, z))N_q(z)$$
$$+ f(n', p_z', z)(1 - f(n, p_z, z))] + [N_q^0(z) - N_q(z)]\omega_{pp}(q). \quad (6.5)$$

In (6.5) we took into account the non-electronic mechanisms of phonon relaxation by introducing the corresponding relaxation frequency $\omega_{pp}(q)$, and $N_q^0(z)$ is the local-equilibrium phonon distribution function, equal to

$$\left\{\exp\left(\frac{\hbar\omega_q}{T(z)}\right) - 1\right\}^{-1}, \quad \omega_q = v_s q. \quad (6.6)$$

Eqs. (6.2) and (6.4) are sufficient for the calculation of the fluxes along the magnetic field. When obtaining the dragging effect, it is necessary to take into account the deviation of both the electronic and phonon distribution functions from the local equilibrium. In other words, it is necessary to find the nonequilibrium additions of u_2 and g_q to the local-equilibrium distribution functions of the electrons and phonons

$$f(n, p_z, z) = f_0\left(\frac{E(n, p_z) - \zeta(z)}{T(z)}\right) - \frac{\partial f_0}{\partial E} p_z u_z, \quad (6.7)$$

$$N_q(z) = N_q^0(z) + g_q, \quad |g_q| \ll N_q^0(z). \quad (6.8)$$

With the aid of (6.2) and (6.4) we can find a system of integro-differential equations for the functions u_z and g_q. Inasmuch as the exact solution of these equations is difficult, we confine ourselves only to the approximate analysis of the problem, namely: we assume that u_z—the velocity of the ordered motion of electrons under the influence of the temperature gradient and the electric field—is much smaller than the average velocity of their random motion. We can then replace $u_z(n, p_z)$ approximately by the mean value of u_z which does not depend on n or p_z.

In this approximation, it is possible to analyze the problem to conclusion. Indeed, substituting (6.7) in

(6.4) we obtain in the linear approximation in $\nabla_z T$ and u_z

$$g_q = \hbar q_z \left[2(\omega_{ep}\langle 1\rangle + \omega_{pp}(q))\left(\text{ch}\left(\frac{\hbar\omega_q}{T}\right) - 1\right)\right]^{-1}(v_s^2\nabla_z T^{-1} + T^{-1}\omega_{ep}\langle 1\rangle u_z). \quad (6.9)$$

Using (6.7)—(6.9), we obtain with the aid of (6.2) for stationary conditions the formula

$$u_z = \frac{(\sigma_{zz}E_z^* + \beta_{zz}\nabla_z T^{-1})}{eN}, \quad (6.10)$$

where

$$E_z^* = E_z - e^{-1}\nabla_z\zeta, \quad \sigma_{zz} = \frac{Ne^2}{m}\tau,$$

$$\tau^{-1} = N^{-1}\int \frac{dq}{(2\pi)^3}(\hbar q_z)^2 \left[2mT(\omega_{ep}\langle 1\rangle + \omega_{pp}(q))\left(\text{ch}\left(\frac{\hbar\omega_q}{T}\right) - 1\right)\right]^{-1}$$
$$\times \omega_{pp}(q)\omega_{ep}\langle 1\rangle, \quad (6.11)$$

$$\beta_{zz} = +\frac{e\tau}{m}\left\{T^2 S - \int\frac{dq}{(2\pi)^3}(v_s\hbar q_z)^2\right.$$
$$\left. \times \left[2(\omega_{ep}\langle 1\rangle + \omega_{pp}(q))\left(\text{ch}\left(\frac{\hbar\omega_q}{T}\right) - 1\right)\right]^{-1}\omega_{ep}\langle 1\rangle\right\}, \quad (6.12)$$

S—entropy of the electron gas (see (3.18)). Substituting (6.10) and (6.9) in (6.7) and (6.8), we obtain non-equilibrium electron and phonon distribution functions. With the aid of these functions it is easy to write down formulas for the conduction current and for the total heat flux transported both by the electrons and the phonons:

$$(j_{con})_z = \sigma_{zz}E_z^* + \beta_{zz}\nabla_z T^{-1}, \quad (6.13)$$

$$(Q_T)_z = T^{-1}\beta_{zz}E_z^* + (\beta_{zz}^2(T\sigma_{zz})^{-1} + \varkappa_p)\nabla_z T^{-1}, \quad (6.14)$$

where

$$\varkappa_p = \int\frac{dq}{(2\pi)^3}(v_s\hbar q_z)^2\left[2(\omega_{ep}\langle 1\rangle + \omega_{pp}(q))\left(\text{ch}\left(\frac{\hbar\omega}{T}\right) - 1\right)\right]^{-1}. \quad (6.15)$$

The kinetic coefficients in (6.13) and (6.14) satisfy both the Onsager symmetry principle and the Einstein relation.

Let us consider first of all the differential thermal emf. From the equation $j_z = 0$ we get for it

$$\alpha_{zz}(H) = \beta_{zz}(T^2\sigma_{zz})^{-1} = \frac{1}{e}\frac{S}{N} + \alpha_{zz}^{(p)}(H), \quad (6.16)$$

where $\alpha_{zz}^{(p)}(H)$ is the contribution made to thermal emf by dragging; this contribution vanishes when $\omega_{pp} \to \infty$. In this case the electronic part of the longitudinal thermal emf($E^* \parallel \nabla T \parallel H$), as well as the electronic part of the transverse emf($E^* \parallel \nabla T \perp H$), are expressed by the same formula

$$\alpha^{(e)}H = \frac{1}{e}\frac{S}{N}. \quad (6.17)$$

This formula is approximate and corresponds to equality of the mean value of the force exerted on the electrons by the applied temperature gradient to the force exerted by the electric field, i.e.,

$$\frac{2}{(2\pi\alpha)^2\hbar}\sum_n\int dp_z\left(eE_z + \frac{E(n, p_z) - \zeta}{T}\nabla_z T\right)\frac{p_z^2}{2m}\frac{\partial f_0}{\partial E} = 0. \quad (6.18)$$

It follows from (6.16) that the thermal emf is made up of an "electron" and "phonon" parts. The former is due to the deviation of only the electrons from equilibrium, whereas the latter is due to deviation of the phonon system from equilibrium, too. The former was investigated in the relaxation-time approximation by

Ansel'm and Askerov[49]. The greatest interest in the quantum region attaches to the second part of the thermal emf, since it increases with increasing magnetic field intensity.

Let us consider the case of practical interest, namely $\omega_{ep}\langle 1 \rangle \ll \omega_{pp}(q)$. In this case when $\hbar\Omega \gg kT^0$ we have for nondegenerate electrons, in accordance with (5.13),

$$\omega_{ep}\langle 1\rangle = \frac{2\pi E_0^2 N}{\hbar v_s\rho_0 \sqrt{2\pi mT}}\frac{q}{|q_z|}\left[1-\exp\left(-\frac{\hbar\omega_q}{T}\right)\right]^{-1}$$

$$\times\exp\left\{-\frac{1}{2m}\left(\frac{m\omega_q}{q_z}-\frac{\hbar q_z}{2}\right)^2-\frac{1}{2}\alpha^2 q_\perp^2\right\}, \qquad (6.19)$$

where it is assumed that $|C_q|^2 = E_0^2\hbar q/(v_s\rho_0 V)$, ρ_0—crystal density, E_0—constant of the deformation potential, v_s—speed of sound, and V—volume of the system. In the case of not too strong magnetic fields $H < c\hbar(T/\hbar v_s)^2/|e|$, the only interaction of importance is that of electrons with phonons whose momentum is $\hbar q < \hbar/\alpha$. In this approximation we have

$$\alpha_{zz}^{(p)}(H) = \frac{1}{e}B\left(\frac{\hbar\Omega}{T}\right)^{\frac{3-t}{2}} \qquad (t\leqslant 2), \qquad (6.20)$$

where

$$B = \frac{2}{\hbar}\left(\frac{m}{2\pi}\right)^{3/2}\left(\frac{E_0^2}{\rho_0 T\lambda}\right)\left(\frac{mT}{\hbar^2}\right)^{\frac{3-t}{2}}\Gamma(3-t)D_{t-3}(0)$$

with $\omega_{pp}(q) = \lambda q^t$, $\Gamma(t)$—Gamma function, and $D_t(x)$—parabolic-cylinder function. For the most important phonon relaxation mechanism, with $q \lesssim 1/\alpha$, under conditions when the dragging effects make a noticeable contribution to the thermal emf, $\omega_{pp} \sim q$—the Simons relaxation mechanism[50], i.e., t = 1, and when $\hbar\Omega \gg T$

$$\frac{\alpha_{zz}^{(p)}(H)-\alpha(0)}{\alpha(0)} \cong \frac{\Delta\alpha_{zz}^{(p)}(H)}{\alpha(0)} \sim H; \qquad (6.21)$$

$\alpha(0)$—thermal emf at H = 0. In the region of low temperatures, when scattering of phonons by the boundaries of the sample predominates (L—sample dimension), t = 0, and

$$\frac{\Delta\alpha_{zz}^{(p)}(H)}{\alpha(0)} \sim H^{3/2}. \qquad (6.22)$$

Recently Gluzman and Tsidil'kovskii[79] investigated experimentally the dependence of $\alpha_{zz}^{(p)}(H)/\alpha(0)$ on the temperature and on the quantizing magnetic field in n-Ge samples. Their results agree well with formula (6.21), which was obtained under the assumption that $\omega_{pp} \sim qT^4$.

The longitudinal thermal emf in a quantizing magnetic field was calculated independently by another method by Ohta[51]. He considered only the case $\omega_{pp} \sim q^2$, and obtained the formula

$$\frac{\Delta\alpha_{zz}^{(p)}(H)}{\alpha(0)} \sim H^{1/2}. \qquad (6.23)$$

We note that when t = 2 this formula follows from (6.20).

Thus, all the most significant phonon relaxation mechanisms lead to a growth of the dragging thermal emf with increasing magnetic field intensity. We note that an analysis of the dependence of $\Delta\alpha_{zz}^{(p)}(H)/\alpha(0)$ on the magnetic field and on the temperature makes it

possible to establish the frequency and the temperature dependences of the attenuation of sound in the frequency region $\omega \sim v_s/\alpha$.

The charge and heat fluxes along the magnetic field were calculated, in the approximation of elastic scattering by a short-range potential by Peletminskii[52], who also analyzed the different limiting cases. In the quantum limit, with the relaxation time introduced phenomenologically, the heat and charge fluxes were calculated also by Ansel'm and Askerov[49].

VII. COMPARISON OF THEORY WITH EXPERIMENT

In this section we shall use the theory developed above to interpret the available experimental data on thermomagnetic phenomena. However, before we proceed to do so, it is necessary to stop and discuss the influence of microinhomogeneities in the distribution of the impurities on the measurements of the thermomagnetic and galvanomagnetic effects. The first to investigate this question was Herring[53]. He studied theoretically the influence exerted on the galvanomagnetic measurements by random inhomogeneities in the impurity distribution, and showed that in strong magnetic fields the inhomogeneities lead to a quadratic increase of the magneto-resistance with increasing field.

The question of the influence of microinhomogeneities on the measurements of thermomagnetic effects in strong magnetic fields was investigated theoretically by Kudinov and Moyzhes[54]. Using the general method, developed by Herring[53], for expanding fluctuations quantities in a Fourier series, the authors of[54] have shown that the relative influence of the random inhomogeneities increases with increasing magnetic field in measurements of the Nernst coefficient, but does not increase in measurements of the longitudinal and transverse thermal emf. This conclusion was satisfactorily confirmed experimentally by Drichko and Mochan[55] who have shown that the thermal emf in a strong field is not very sensitive to the influence of microinhomogeneities. However, this influence affects strongly measurements of the Nernst coefficient at large values of H. The effective (experimentally measured) Nernst coefficient Q_N^{eff} varies like H^m, where $-1 < m < -0.2$. According to the predictions of the theory of Kudinov and Moyzhes[54], $Q_N^{eff} \sim H^{-1}$, and in some cases Q_N^{eff} will tend to saturation when the field H increases without limit, whereas according to the theory that does not take into account the corrections for the microinhomogeneities, the Nernst coefficient Q_N is, as is well known, of the order of H^{-2}.

Thus, in a quantizing magnetic field a comparison of the experimental data on the Nernst effect with theory must be carried out with great caution, and the influence of the microinhomogeneities must be specially investigated. On the other hand, measurements of the thermal emf are perfectly reliable. The microinhomogeneities introduce only insignificant corrections (within the limits of the measurement errors), which do not depend on H. We shall therefore compare in detail with the theory only the measurements of the thermal emf.

Simpler formulas, and therefore more convenient for a theoretical analysis, are those obtained for

Table VI. Experimental study of thermomagnetic phenomena in semiconductors and semi-metals in a quantizing magnetic field

Material	Literature	Investigated effect	Carrier density, cm⁻³	Temperature interval, °K	H_{max}, kG	$\left(\frac{\hbar\Omega}{T}\right)_{max}$	Obtained information on the physical properties of the conductors
Bi	56	Temperature oscillations	10^{18}	1.3	12	13	Determination of the form of the Fermi surface
	57	Oscillations of $\alpha_\perp$, $\varkappa_\perp$, $\rho_\perp$	10^{18}	1.47—4.33	13	13	Coincidence of the periods and phases of the oscillations of $\alpha_\perp$, $\varkappa_\perp$ and $\rho_\perp$
n-InSb	58	Oscillations of $\dfrac{\Delta\alpha_\perp}{\alpha(0)}$	$1.3 \cdot 10^{16}$	4,2	30	70	Coincidence of the periods and phases of the oscillations of $\dfrac{\Delta\alpha_\perp}{\alpha(0)}$ and $\dfrac{\Delta\rho_\perp}{\rho(0)}$
n-InAs	59		$2.4 \cdot 10^{16}$	4	28	48	
n-InSb	60	Magnetophonon oscillations of $\dfrac{\Delta\alpha_{\|}}{\alpha(0)}$	$1.2 \cdot 10^{4}$	20 — 110	80	40	Coincidence of the periods of the oscillations of $\dfrac{\Delta\alpha_{\|}}{\alpha(0)}$ and $\dfrac{\Delta\rho_{\|}}{\rho(0)}$
	61		$5.5 \cdot 10^{13} \div 3.9 \cdot 10^{17}$	120 — 140	33	2.8	Determination of the relative role of different carrier mechanisms
n-Ge	64	$\dfrac{\Delta\alpha_\perp}{\alpha(0)}$	$(1-2)\cdot 10^{3}$	15 — 85	90	6	Mechanism of relaxation of long wave phonons on the thermal ones
n-InSb	72, 73	$\alpha_\perp (H) -$ $-\alpha_\perp$ sat	$3 \cdot 10^{13} - 4 \cdot 10^{14}$	6,5 — 82	100	150	Character of the electron-phonon and phonon-phonon interaction
	75		$6.3 \cdot 10^{13}$ $1.3 \cdot 10^{14}$	92 — 101	20	2,2	Verification of the validity of the theory for the electronic part of the thermal emf

thermomagnetic coefficients under isothermal conditions. However, it is quite difficult to realize isothermal conditions in a direction perpendicular to the primary gradient of the temperature and of the electrochemical potential, owing to the occurring transverse thermomagnetic effects. Reduction of the appearing temperature gradients entails considerable difficulties, since it calls for the supply and removal of heat from different sides of the sample. It is much easier to realize adiabatic conditions, by thermally insulating, for example, the side walls of the sample and placing it in vacuum. An estimate of the adiabatic corrections to the isothermal coefficients in a quantizing magnetic field shows that for both a degenerate and a nondegenerate electron gas these corrections are negligibly small.*

For easy visualization, the available experimental data on thermal magnetic phenoma in a quantizing magnetic field are summarized in Table VI. The next to

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*For the scattering of electrons, for example, by point defects or by acoustic phonons, in the case of limiting degeneracy, the difference between the isothermal and adiabatic thermal emf depends on the predominance of the phonon component of the thermal conductivity $\varkappa_{xx}^{(p)}$ on the electronic component $\varkappa_{xx}^{(e)}$

$$\alpha_{xx}^{ad} \approx \alpha_{xx}^{is}\,(5\varkappa_{xx}^{(e)} + \varkappa_{xx}^{(p)})/(\varkappa_{xx}^{(e)} + \varkappa_{xx}^{(p)}).$$

The coefficient of electronic thermal conductivity transversely to a strong magnetic field is smaller by a factor $(\Omega\tau)^{-2}$ than in the absence of the field. Therefore when $\Omega\tau \gg 1$ the electronic part of the thermal conductivity becomes small compared with the phonon part, and $\alpha_{xx}^{ad} \sim \alpha_{xx}^{is}$

the last column of this table gives the maximal values of $(\hbar\Omega)/T$ obtained in experiments, which show the margin with which the conditions for quantization of the energy levels of the carriers are realized. The last column indicates what information of general nature can be obtained from the given experiment.

The entire aggregate of the experimental data on thermomagnetic phenomena, presented in the table, can be separated into two groups: a) oscillatory phenomena and b) phenomena in the quantum limit. Let us consider both groups in detail.

a) Oscillatory Effects

A special position in the investigation of oscillatory effects is occupied by the paper of Kunzler et al.[56], who investigated the so-called magnetothermal oscillations in bismuth, i.e., the reversible changes of the temperature of an adiabatically isolated sample, occurring when the magnitude or direction of an applied magnetic field is changed. The oscillations of the temperature in a quantizing magnetic field are connected with oscillations of the entropy of the electron gas S, and have the same physical origin as the well known low-temperature magnetization oscillations. Inasmuch as

$$S = -\left(\frac{\partial\Phi}{\partial T}\right)_\zeta, \quad \text{a} \quad M = -\left(\frac{\partial\Phi}{\partial H}\right)_\zeta,$$

the oscillations of both S and M are due to oscillations of the thermodynamic potential Φ, and in final analysis they are connected with the nonmonotonic character of the dependence of the density of states on the energy.
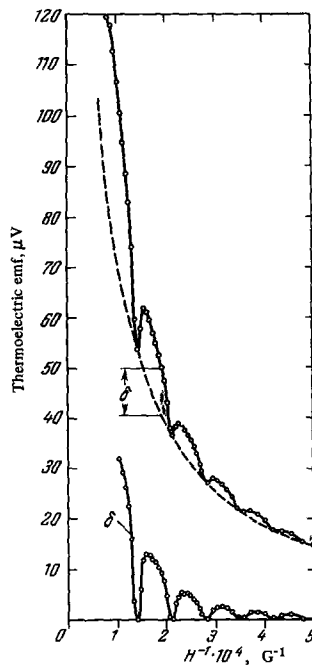
FIG. 1.

FIG. 2.

With increasing entropy of the electron gas, the thermal energy necessary to maintin the initial temperature is also increased. However, for an adiabatically isolated sample, the only source of thermal electron energy is the lattice. With the aid of the thermometer, peaks of cooling are observed with changing H. The magneto-thermal oscillations have a high resolving power and make it possible to determine with great accuracy the parameters of the Fermi surface.

Oscillations of the thermomagnetic coefficients in a quantizing magnetic field (thermal conductivity, differential thermal emf) were first discovered experimentally by Steele and Babiskin[57] in very pure single crystals of bismuth. They established very general laws relating the oscillations of the coefficients of thermal emf, thermal conductivity, and electric resistance upon variation of the magnetic field, namely, that the periods and phases of the oscillations of these coefficients coincide.

Figures 1 and 2 illustrate the oscillations of the thermoelectric potential difference and the coefficient of thermal conductivity transversely to the magnetic field, as observed by Steele and Babiskin. In these figures, the solid curves correspond to the experimental results, and the dashed ones are the envelopes of the minima of the oscillations; δ is the difference between the ordinates of the experimental curve and the envelope of the minima. The quantity δ was introduced in order to separate the oscillating components. The plot of δ is shown in the lower part of the figures. This curve is periodic in H^{-1}. In order to be able to compare the period and the phase of the oscillations of the thermomagnetic coefficients with the galvanomagnetic ones, the electric resistance of the same single crystal was measured. Comparison shows (Fig. 3) that the periods of the oscillations with change of H^{-1} are the same for all three effects. The difference in the phases of the oscillations is quite small. The relative
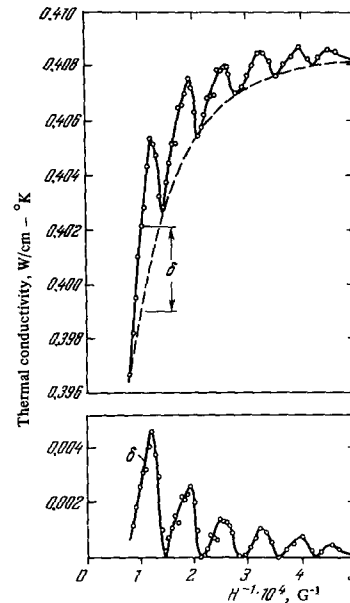
phase shift of the oscillations of the thermal conductivity and of the electric resistance amounts to 0.16 rad, and in the case of the thermal emf and the electric resistance it is 0.08 rad. This difference in the phases of the oscillations is due principally to the errors in the construction of the envelope of the minima. In[57] they investigated also the temperature dependence of the thermal emf, and it was shown that the monotonic part of the thermal emf decreases with decreasing temperature like T^n, where $n \approx 1$. It follows from the experiments of[57] that the amplitudes of the oscillations of the thermal emf are proportional to $H^{5/2}$. When the temperature changes, this dependence remains unchanged. The envelope of the minima of the thermal emf is proportional to H.

Before we discuss the possible theoretical interpretation of the experimental results of[57], let us find the consequence ensuing from the simplest single-band model of a conductor with an isotropic carrier dispersion law. We assume that the electrons are strongly
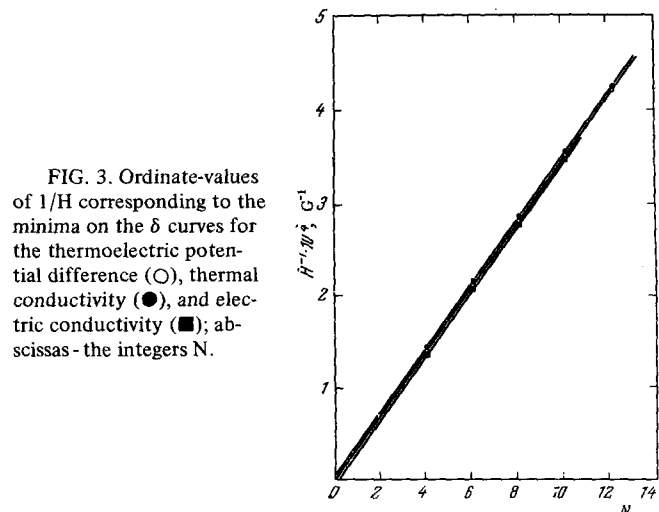

FIG. 3. Ordinate-values of 1/H corresponding to the minima on the δ curves for the thermoelectric potential difference (O), thermal conductivity (●), and electric conductivity (■); abscissas - the integers N.

degenerate and that the elastic scattering of the electrons with the impurities and defects is dominating in the transport phenomena.

To calculate the thermal emf α_{xx}, the electronic part of the thermal conductivity $\kappa_\perp$, and the magnetoresistance $\rho_\perp$, we need formulas for the kinetic coefficients σ_{ik}, β_{ik}, κ_{ik}, and χ_{ik}. In the case of strong degeneracy, we get from (3.16), (3.17), and (4.7) the Wiedemann-Franz law

$$\varkappa_{ik} = \frac{\pi^2}{3} \left(\frac{k}{e} \right)^2 T^\circ \sigma_{ik}. \tag{7.1}$$

The remaining kinetic coefficients can also be expressed in terms of σ_{ik}. The corresponding formulas are

$$\beta_{ik} = \frac{1}{e} \left(\frac{\partial}{\partial T} \right)_\zeta \int_{-\infty}^{\zeta} d\zeta' \sigma_{ik}(\zeta'), \tag{7.2}$$

$$e\chi_{ik} = -T\beta_{ik}. \tag{7.3}$$

In a strong magnetic field, in the presence of carriers of the same sign

$$\sigma_{xx} \ll \sigma_{xy}, \quad \beta_{xx} \ll \beta_{xy}; \tag{7.4}$$

Here, in accordance with (2.11) and (2.14),

$$\alpha_{xx} \equiv \alpha_\perp = \frac{\beta_{xy}}{\sigma_{xy}}, \tag{7.5}$$

$$\rho_\perp = \frac{\sigma_{xx}}{\sigma_{xy}^2}. \tag{7.6}$$

According to (4.7), in the approximation that is nonvanishing in the degeneracy, the following formulas hold:

$$\beta_{ik} \sim T, \quad \chi_{ik} \sim T^2, \quad \varkappa_{ik} \sim T. \tag{7.7}$$

Therefore the first two terms of (2.13) are proportional to T^3. Neglecting in $\kappa_\perp$ the terms proportional to T^3, we get in the approximation linear in T

$$\varkappa_\perp = \frac{\pi^2}{3} \left(\frac{k}{e} \right)^2 T^\circ \sigma_{xx}. \tag{7.8}$$

Adams and Holstein[29] have shown that for sufficiently large values of the quantum number n, the character of the oscillations σ_{xx} is essentially independent of the scattering mechanism, and that the oscillating part of the electric conductivity can be represented in the form

$$\sigma_{\text{osc}} \sim \sigma_{\text{cl}} \left(\frac{\hbar\Omega}{\zeta} \right)^{1/2} \delta^{-1/2}, \tag{7.9}$$

where σ_{cl} is the classical (non-quantum) electric conductivity in a strong magnetic field, and

$$\delta^{-1/2} = 2\sqrt{\pi} \sum_{M>0} (-1)^M (2\pi M)^{-1/2} \cos\left(\frac{2\pi M\zeta}{\hbar\Omega} - \frac{\pi}{4} \right). \tag{7.10}$$

The oscillating part of the differential thermal emf $(\alpha_{xx})_{\text{osc}}$ can also be represented, at not too small quantum numbers n, by the formula

$$(\alpha_\perp)_{\text{osc}} = (\alpha_{xx})_{\text{osc}} = \alpha_{\text{cl}} \left(\frac{\hbar\Omega}{\zeta} \right)^{1/2} \delta^{-1/2}, \tag{7.11}$$

where

$$\alpha_{\text{cl}} = \frac{1}{2} \frac{\pi^2}{e} \left(\frac{kT^\circ}{\zeta} \right)$$

is the thermal emf in the classical limit for a strongly degenerate electron gas. Since the oscillating part of the magnetoresistance, thermal conductivity, and thermal emf are proportional to the same oscillating function

$$(\rho_\perp)_{\text{osc}} \sim (\varkappa_\perp)_{\text{osc}} \sim (\alpha_\perp)_{\text{osc}} \sim \delta^{-1/2},$$

the periods and the phases of the oscillations of $\rho_\perp$, $\kappa_\perp$, and $\alpha_\perp$ are identical. The maximum values of the amplitudes of the oscillations at low temperatures are determined essentially by the broadening of the energy levels, due to the collision of the electrons with the scatterers. According to[29], the maximum values of $\delta^{-1/2}$ are equal in order of magnitude $\sqrt{\Omega\tau}$ (τ—carrier free path time, which enters in the electric conductivity). If the dominating role in the broadening of the energy levels is played by elastic scattering by impurities, then τ is independent of the temperature. The dependence of the amplitudes of the oscillations $\delta^{-1/2}$ on the magnetic field is the same at different temperatures. Finally, a linear dependence of the thermal emf on T is due to strong degeneracy. These conclusions of the single-band model are in good qualitative agreement with the experiments of Steele and Babiskin[57]. However, we have no justification for transferring the results obtained in the single-band model to the case of bismuth, in which the electron density is equal to the hole density, and the first inequality of (7.4) is violated, inasmuch as in the zeroth approximation in the scattering $\sigma_{xy}^{(0)} = 0$. In this case, a nonzero contribution to σ_{xy} will arise only in the approximation $(\Omega\tau)^{-2}$. Under the conditions of the experiments of[57] (a large number of filled Landau levels) it can be shown that the quantity $\sigma_{xy}^{(2)} \sim (1/\Omega\tau)^2$ can also be represented in the form of a slowly-varying classical part $(\sigma_{xy})_{\text{cl}}$ and an oscillating part proportional to $\delta^{-1/2}$. Therefore, when the amplitudes of the oscillations of the kinetic coefficients are small compared with the classical part that depends smoothly on H, α_{xx} as well as β_{xx} and ρ_{xy} can be represented in the form

$$\alpha_{xx} = (\alpha_{xx})_{\text{cl}} + (\alpha_{xx})_{\text{osc}},$$
$$\rho_{xx} = (\rho_{xx})_{\text{cl}} + (\rho_{xx})_{\text{osc}},$$
$$\rho_{xy} = (\rho_{xy})_{\text{cl}} + (\rho_{xy})_{\text{osc}},$$

where

$$(\alpha_{xx})_{\text{osc}} \sim (\rho_{xx})_{\text{osc}} \sim (\rho_{xy})_{\text{osc}} \sim \delta^{-1/2}.$$

Thus, in not too strong magnetic fields, if a sufficiently large number of Landau levels are filled, the character of the oscillations of the thermogalvanomagnetic coefficients turns out to be the same in both the single-band and the two-band models of the conductor as well as when the hole and electron densities are equal. On the other hand, if the fields are so strong that not too many levels are filled, then the character of the oscillations of the different thermogalvanomagnetic coefficients is different. Thus, for example, the experiments of Antcliffe and Stradling[80], made on n-InSb samples, have shown that the oscillations of ρ_{xx} and ρ_{xy} differ in phase by $\pi/4$.

In semiconductors, the quantum low-temperature oscillations of the thermal emf in a transverse magnetic field ($H \perp \nabla T$) were observed in n-InSb[58] and n-InAs[59]. Comparison of the experimental curves for the transverse magnetothermal emf $\Delta\alpha_\perp/\alpha(0)$ and

the transverse magnetoresistance $\Delta\rho_\perp/\rho(0)$ reveals,
just as in the case of bismuth, agreement of the
periods and good agreement of the phases. In[58], a
distinct spin splitting of the first ($n = 1$) Landau level
was observed. However, an estimate of the g-factor
from the magnitude of the splitting leads to a value
$|g| = 34$, which is smaller than the g-factor predicted
by the theory and determined from the spin resonance
of the electron conductivity in InSb ($|g| = 50$).
Another physical nature (compared with the low-tem-
perature oscillations considered above) is possessed
by the so-called magnetophonon oscillations of the
thermal emf, first observed in n-InsB by Puri and
Geballe[60] and investigated in greater detail by
Muzhdaba, Parfen'ev, and Shalyt[61].

Both with respect to the observation conditions
(higher temperatures) and with respect to the depend-
ence of the period on the parameters of the conductor,
this type of oscillations differs from the low-tempera-
ture quantum oscillations of the Shubnikov—de Haas
type considered above.

The period of low-temperature oscillations is deter-
mined only by the electron density:

$$\Delta\left(\frac{1}{H}\right) = \frac{2e}{\hbar c}(3\pi^2 n)^{-2/3}, \tag{7.12}$$

and the period of the magnetophonon oscillations on the
effective mass of the electrons m^* and the limiting
frequency of the optical phonons:

$$\Delta\left(\frac{1}{H}\right) = \frac{e}{m^*\omega_D c}. \tag{7.13}$$

In this review we shall not analyze in detail the mag-
netophonon oscillations of the thermal emf, referring
the reader to the special literature[62,81]; we note only
the following two circumstances.

First, no oscillations were observed on the curve
of the transverse magnetothermal emf in[61], in accord
with the theoretical result concerning the independence
of the transverse electronic part of the thermal emf on
the scattering. In a longitudinal magnetic field
($H \parallel \nabla T$), the magnetothermal emf oscillates. The
maxima that can be observed in the experimental
curves have a periodicity that agrees with (7.13).

Second, the oscillation maxima of the longitudinal
thermal emf $(\Delta\alpha_\parallel)/\alpha(0)$ are shifted relative to the
resonant values of the magnetic field determined by
condition (7.13). A comparison of the magnitude of this
shift with the predictions of the theory of Pavlov and
Firsov[62] yields information on the relative role of
the different carrier scattering mechanisms in n-InSb.

b) Thermomagnetic Effects in the Quantum Limit

The dependence of the thermomagnetic effects in
the quantum limit on the temperature and intensity of
the magnetic field for different mechanisms of elastic
scattering is shown in Tables II—IV. At the present
time, however, we still do not have sufficiently reliable
experimental data to be able to judge the validity of the
theoretical dependences of the thermomagnetic effects
on T and H, given in Tables II—IV. Thus, for example,
measurements of the Nernst effect, quoted by Amirk-
hanov et al.[63], cannot be interpreted theoretically,
because the character of the dependence of the Nernst
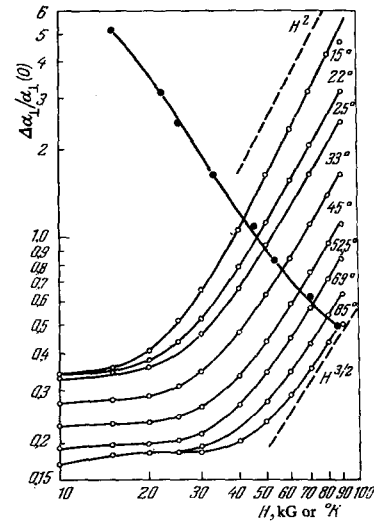constant on the magnetic field changes from sample to

FIG. 4.

sample. This is apparently connected with the influence
of the inhomogeneities referred to in the beginning of
this section.

The most reliable experimental information about
the dependence on the temperature and on the magnetic
field intensity in the quantum limit were obtained for
measurements of the differential thermal emf. These
experimental data are of greatest interest, because
their theoretical analysis makes it possible, on the
one hand, to judge the validity of the premises of the
theory and, on the other, obtain information on the
mechanisms of electron-phonon and even phonon-
phonon relaxation.

In the experiments made by Puri and Geballe[64] on
pure samples of n-Ge with impurity density n_D
$\sim 10^{13}$ cm^{-3}, they measured the ratio

$$\frac{\alpha_\perp(H) - \alpha_\perp(0)}{\alpha_\perp(0)} = \frac{\Delta\alpha_\perp}{\alpha_\perp(0)}. \tag{7.14}$$

Figure 4 shows the experimental results obtained in[64]
for the dependence of $\Delta\alpha_\perp/\alpha_\perp(0)$ on the magnetic field
at different temperatures for $H \parallel [100]$ and $\nabla T \parallel [010]$.
The same figure shows the dependence of $\Delta\alpha_\perp/\alpha_\perp(0)$
on the temperature, obtained in a field $H = 88$ kG. The
experimental data lead to the following empirical for-
mulas: in the temperature interval 20—80°K

$$\frac{\Delta\alpha_\perp}{\alpha_\perp(0)} = D_1\left(\frac{H}{T}\right)^{3/2}, \tag{7.15}$$

and for $T < 20°$K

$$\frac{\Delta\alpha_\perp}{\alpha_\perp(0)} = D_2\left(\frac{H}{T}\right)^2. \tag{7.16}$$

D_1 and D_2 vary little with the temperature and with the
field.

In semiconductors with an anisotropic constant-
energy surface (such as n-Ge or n-Si), the coupling of
the carriers with the longitudinal and transverse pho-
nons is of the same order of magnitude. Therefore an
analysis of these experiments is impossible within the
framework of the simplest model.

In[65,66], the main premises of the quantum theory
of thermomagnetic phenomena, developed in Secs. III—
IV as applied to a quadratic isotropic dispersion law,
were extended to a multivalley anisotropic energy

spectrum of the electrons. The anisotropy of the carrier scattering was taken into account within the framework of the deformation-potential method. The isothermal thermal emf was investigated in greatest detail in different limiting cases, as functions of the magnitude of the magnetic field and the degree of dragging of the electrons by the phonons.

The isothermal differential thermal emf is made up of two parts: electronic $\alpha^{(e)}(H)$, due only to the non-equilibrium nature of the electrons, and phonon $\alpha^{(p)}(H)$, due to the deviation of the phonons from the equilibrium distribution as a result of electron-phonon collisions: $\alpha(H) = \alpha^{(e)}(H) + \alpha^{(p)}(H)$. Under the conditions of the experiments of[64], $\alpha^{(p)}$ turns out to be small compared with $\alpha^{(p)}$ [sic!]. In a quantizing magnetic field, the main contribution to the interaction with the electrons is made by phonons with $q \sim q_H$ = $\sqrt{|e|H/c\hbar}$. The magnetic field and temperatures used in[64] were such that $q_H^2 < q_\lambda^2 = (T/\hbar s_\lambda)^2$. In this limit, for the case when the magnetic field is directed along the fourfold axis and the dragging of the electrons by the λ-branch phonons is weak $(\omega_{pp}^{(\lambda)} \gg \omega_{pe}^{(\lambda)})$, the following expression was obtained for the dragging thermal emf[65]:

$$\alpha_\lambda(H) = \frac{\Gamma\left(\frac{2-r_\lambda}{2}\right)B}{e\pi^{3/2}2^{(7+r_\lambda)/2}} \left(\frac{\hbar\Omega}{T}\right)^{\frac{2-r_\lambda}{2}} \left(\frac{ms_\lambda^2}{T}\right)^{\frac{3-r_\lambda}{2}} \left(\frac{C_2}{T}\right)^2 \frac{T^\gamma F_\lambda}{P^{\gamma/5}c_\lambda}, \quad (7.17)$$

where s_λ—phase velocity of the λ-branch phonons, C_2—deformation-potential constant, c_λ—mean value of the elastic constant for the λ-brach, F_λ contains a weak logarithmic dependence on the magnetic field,

$$P = \rho_0 \hbar^{(2-r_\lambda)} s_\lambda^{(4-r_\lambda)} L^{-1},$$ ρ_0—density of the substance, L—dimension of the crystal, B is determined by a combination of the components of the reciprocal-effective-mass tensor in the coordinate frame connected with the external fields[65]; r_L and γ are determined by the phonon damping decrement

$$\tau_\lambda^{-1}(q, T) = A_\lambda T^{3-r_\lambda - \gamma} q^{2+r_\lambda}. \quad (7.18)$$

and A_λ depends on the parameters of the material. The ratio $\alpha_\lambda^{(p)}(H)$ to the dragging thermal emf in the absence of a magnetic field $\alpha_\lambda^{(p)}(0)$ is expressed by means of the simple formula

$$\frac{\alpha_\lambda^{(p)}(H)}{\alpha_\lambda^{(p)}(0)} = B_1 \left(\frac{\hbar\Omega}{T}\right)^{\frac{2-r_\lambda}{2}}. \quad (7.19)$$

B_1 contains a weak logarithmic dependence on H and T. In n-Ge, the experimentally investigated quantity

$$\frac{\Delta\alpha}{\alpha(0)} = \frac{\sum_\lambda \Delta\alpha_\lambda^{(p)}(H)}{\sum_\lambda \alpha_\lambda^{(p)}(0)} \quad (7.20)$$

contains a contribution from different branches of the phonon spectrum and is more complicated for analysis.

It follows from (7.17)–(7.20) that the dependence on T and H of the measured quantity $\Delta\alpha/\alpha(0)$ is determined essentially by the damping decrement of the phonons with $q \sim q_H = \sqrt{|e|H/c\hbar}$. Usually one uses in this frequency region, for the damping decrement of the transverse sound waves, the Landau-Rumer formula[67]

$$\tau_t^{-1} = A_t q T^4, \quad (7.21)$$

and for longitudinal sound the Herring formula[68]

$$\tau_l^{-1} = A_l q^2 T^3. \quad (7.22)$$

These formulas were obtained in the approximation involving three-phonon processes in which the energy and momentum conservation laws are exactly satisfied.

For sufficiently low temperatures, the phonon relaxation on the boundaries of the sample, which does not depend on T and q, becomes important; in this case the damping decrement is given by

$$(\gamma_b)_\lambda = (\tau_b)_\lambda^{-1} \sim \frac{s_\lambda}{L} \quad (7.23)$$

(L—characteristic dimension of the sample).

In the experiments of[64], the magnetic field intensity and the temperature satisfied the inequality

$$H < \frac{c\hbar}{|e|}\left(\frac{T}{\hbar s_\lambda}\right)^2.$$

In this case, as follows from (7.21) and (7.22), we have

$$\frac{\tau_l}{\tau_t} = \left(\frac{s_l}{s_t}\right)^3 \left(\frac{T}{ms_t^2}\frac{T}{\hbar\Omega}\right)^{1/2} \gg 1. \quad (7.24)$$

The inequality (7.24) leads to the conclusion that the most significant contribution to the dragging thermal emf is made by longitudinal phonons, and in accordance with (7.19)

$$\frac{\alpha_l^{(p)}(H)}{\alpha_l^{(p)}(0)} \sim \frac{H}{T}.$$

This formula gives a different dependence than the experimentally observed (7.15). The latter formula would be obtained only if an inequality opposite to (7.24) were to apply, thus contradicting the experimental conditions of[64]. Consequently, τ_l and τ_t, calculated in the approximation of three-phonon processes with satisfaction of the energy and momentum conservation laws, cannot be reconciled with the experiment[64] performed in the presence of a quantizing magnetic field. A similar situation took place also earlier in an investigation by Herring et al.[69] in an analysis of experiments on thermomagnetic phenomena in the classical region of fields. There exist also other experiments[70] on the study of the temperature dependence of τ_l and τ_t, which also contradict formula (7.22).

Thus, it becomes necessary to review our concepts concerning the relaxation of the longitudinal long-wave sound in solids. So far we disregarded the width of the energy levels of the short-wave thermal phonons that take part in the three-phonon processes. Allowance for this factor in the absorption of the longitudinal long-wave sound leads to new possibilities. In order to explain the foregoing, let us consider the scattering of sound by short-wave thermal phonons. Neglecting the level width (the damping decrement) of the short-wave thermal phonons, there should be exactly satisfied in three-phonon processes only the energy and momentum conservation laws. If the dispersion of the speed of longitudinal sound is zero, then a process is possible in which three phonons of the longitudinal branch, with three wave numbers, take part; then the momentum conservation law is given by $q_1 = q_2 - q_3$, and the energy conservation law is obtained by multiplying this equality by the speed of longitudinal sound. Allowance for an arbitrarily small dispersion of the short-wave thermal phonons already excludes these processes,

since the law of energy conservation will not be satis-
fied if all three phonons are longitudinal. However, the
situation can be radically altered if account is taken of
the damping decrement of the shortwave thermal pho-
nons. Indeed, allowance for the damping decrement of
the thermal phonons leads to an uncertainty in the
energy conservation law, and if this uncertainty is
larger than the magnitude of the dispersion, then
processes in which three phonons belonging to one
branch take part, become possible. It is precisely
these processes which make an appreciable contribu-
tion to the absorption of the long-wave sound in solids
by short-wave thermal phonons and lead, as shown by
Simons[50] (see also[71]), in place of (7.22), to the
following damping decrement of the longitudinal long-
wave sound:

$$\tau_l^{-1} = A_l q T^4. \qquad (7.25)$$

The damping decrement of the transverse long-wave
sound, with allowance for the final width of the level
of the thermal phonons, is described as before by
formula (7.21).

Formulas (7.25) and (7.21), together with (7.19),
lead to an experimental observation of relation (7.15)
in a temperature interval in which phonon-phonon re-
laxation predominates. This confirms the notions con-
cerning the mechanism of relaxation of long-wave pho-
nons by short-wave phonons, on which formulas (7.25)
were based. It must be emphasized that the results of
experiments[70] on the study of the temperature depend-
ence of absorption of longitudinal and transverse sound
in dielectrics also agree well with formulas (7.21) and
(7.25). At $T < 20°K$, formulas (7.23) and (7.19) lead
to

$$\frac{\alpha(H)}{\alpha(0)} \sim \left(\frac{H}{T}\right)^2,$$

which agrees well with the experimental result[84].

Of great interest for the clarification of the mecha-
nism of phonon-phonon relaxation in semiconductors
with cubic lattice is also another paper by Puri and
Geballe[72]. The authors observed that in n-InSb. in
the temperature interval 6—40°K, the dragging thermal
emf $\alpha^{(p)}(H)$ increases very strongly with increasing
magnetic field exceeding $\alpha^{(e)}$ by dozens of times, al-
though in the classical region of strong fields, i.e.,
when $\Omega\tau \gg 1$, $\hbar\Omega < T$, it is practically nonexistent.
Since the constant-energy surfaces of n-InSb are iso-
tropic, unlike n-Ge, only longitudinal phonons should
make a contribution to the dragging effects. This cir-
cumstance greatly simplifies the comparison of theory
with experiment.

In[72] they measured the change of the thermal
emf $\Delta\alpha_{exp} = \alpha(H) - \alpha_{sat}$ compared with the classical
saturation value α_{sat}, as a function of the magnetic
field and of the temperature. Neglecting $\alpha_{sat}^{(p)}$, the
dragging thermal emf $\alpha^{(p)}(H)$ was determined from
the following relation:

$$\alpha^{(p)}(H) = \Delta\alpha_{exp} - \Delta\alpha_{theor}^{(e)}, \qquad (7.26)$$

where

$$\Delta\alpha_{theor}^{(e)} = \alpha^{(e)}(H) - \alpha_{sat}^{(e)} = \frac{1}{e}\left(\frac{S}{N} - \frac{5}{2} + \frac{\zeta_0}{T}\right). \qquad (7.27)$$

Here $\alpha^{(e)}(H)$—electronic part of the thermal emf in

the quantizing magnetic field, determined by the rela-
tion (7.4); $\alpha_{sat}^{(e)}$—electronic part of the thermal emf in
the classical region of strong magnetic fields, ζ_0—
chemical potential at $H = 0$. In a subsequent investiga-
tion[74], Puri compared the experimental data for
$\alpha^{(p)}(H)$ with the formula obtained for the dragging
thermal emf by extending the Herring theory[74]
("π" approach) to the quantum region. In the limit
$q_H^2 < (T/\hbar S_\lambda)^2$, this formula leads to the same depend-
ence on T and H as the formula determined by the
conduction current (5.18).

A study of the dependence of $\alpha^{(p)}$ on the magnetic
field at low temperatures ($T < 15°K$), when the pho-
nons are scattered principally from the boundaries of
the sample, has made it possible to determine uniquely
whether electron scattering in n-InSb by the deforma-
tion potential predominates absolutely over their scat-
tering by piezoelectric oscillations.

At higher temperatures ($15 < T < 40°K$), using an
analysis of the field and temperature dependences of
the dragging thermal emf $\alpha^{(p)}$ for the damping decre-
ment of the longitudinal long-wave phonons, Puri ob-
tained the following relation:

$$\tau_l^{-1}(q, T) \sim q T^3. \qquad (7.28)$$

This expression contradicts the results of a theory,
which takes into account three-phonon processes and
leads, as noted by Herring[68], to a sum equal to five
for the exponents of q and T in the damping decre-
ment. It seems to us that the result (7.28) is the con-
sequence of an insufficiently correct analysis of the
experimental data.

First, Puri ignores those serious difficulties which
arise in the separation of the change of the electronic
part of the thermal emf $\Delta\alpha_{theor}^{(e)}$ from the total ex-
perimentally-measured change of the thermal
emf $\Delta\alpha_{exp}$. He calculates $\Delta\alpha_{theor}^{(e)}$ by means of for-
mula (7.27), taking into account, as follows from gen-
eral considerations, the spin splitting of the Landau
levels and the nonquadratic nature of the conduction
band in n-InSb. However, in this region of temperatures
and magnetic field where the dragging effect is prac-
tically missing, and consequently $\Delta\alpha_{exp} = \Delta\alpha^{(e)}$, this
formula does not describe adequately the experimental
electronic part of the thermal emf. As was observed
in a number of papers, better agreement with experi-
ment takes place only if the spin splitting of the Landau
levels is not taken into account in (7.27). For a quad-
ratic isotropic carrier dispersion law without allow-
ance for the spin, formula (7.27) takes the form

$$e\Delta\alpha^{(e)} = -1 + \ln\left(\frac{x_\nu}{sh\, x_\nu}\right) + x_\nu\, cth\, x_\nu, \qquad (7.29)$$

where $x_\nu = \hbar\Omega/T$. Good agreement between the experi-
mental data and formula (7.29) was observed by Drichko
and Mochan[75], who measured $\Delta\alpha^{(e)}$ for not very
strong magnetic fields and high temperatures (x_ν
~ 1, $T = 100°K$), and also by Puri and Geballe[75].
This is evidenced by Fig. 5 of[72], in which measure-
ments of $\Delta\alpha_{exp}$ and $T = 82°K$, when there is no
dragging, are compared with the theoretical curve cal-
culated by formula (7.29).

If the carrier spin and the effective value of the g-
factor are taken into account in formula (7.27) for a
quadratic isotropic law, then the agreement between

theory and experiment, as noted in[72], becomes worse. The corrections for the non-quadratic nature of the conduction band in n-InSb, with account of the spin splitting of the energy levels, do not improve the agreement with experiment[76]. This disparity between theory and experiment for $\Delta\alpha^{(e)}$ remains unexplained to this day. However, in the reduction of the experimental data, the existing discrepancy between theory and experiment for $\Delta\alpha^{(e)}$ should be kept in mind.

In[73] there is a table of the experimental values for $\Delta\alpha_{exp}$ in a wide range of temperatures (6.5—82°K) and magnetic fields (10—100 kG). If $\Delta\alpha^{(e)}_{theor}$ is calculated not from a formula that takes into account both the spin splitting of the levels and the non-quadratic band, as is done by Puri in[73], but use is made, for example, of formula (7.29), which agrees better with experiment, then the following dependence on T and on K is obtained for $\alpha^{(p)}$ in the temperature interval 25—40°K:

$$\alpha^{(p)} \sim H^{1.6}T^{-5.05} \qquad (7.30)$$

in place of the relation $\alpha^{(p)}_{Puri} \sim H^{1.3}T^{-4.46}$ obtained by Puri. This yields for the damping decrement of the longitudinal long-wave phonons the formula

$$\gamma_l = \tau_l^{-1}(q, T) \sim qT^{3.55}, \qquad (7.31)$$

in lieu of formula (7.28) obtain in[73].

Another significant circumstance to which Puri paid no attention in the reduction of the experimental data and in the derivation of formula (7.28) is allowance for the scattering of the phonons at the boundaries of the sample. In the temperature interval 27—43°K, Puri analyzed the dependence of the dragging thermal emf $\alpha^{(p)}$ on the magnetic field and the temperature with allowance for only phonon-phonon relaxation, neglecting completely the scattering of the phonons at the boundaries of the sample. However, an earlier paper[75] presents convincing experimental proof to the contribution made to the phonon relaxation by their scattering from the boundaries in the indicated temperature interval. In particular, in[75] they investigated the ratio of the thermal emf's of two samples with different cross section area at a fixed value of the field (80 kG) as a function of the temperature (the so-called size effect). In the interval 27—43°K this ratio differs from unity and equals respectively 1.45—1.15. The maximum value of this ratio equals 1.55 at T = 6°K. The presented values of the size effect offer evidence of the role of the boundary scattering of the phonons in the investigated temperature interval.

Theoretical estimates of the field and temperature dependences of the dragging thermal emf $\alpha^{(p)}$ by means of formula '7.17) for the relaxation of longitudinal long-wave phonons on short-wave phonons (in accordance with (7.25)) and on the boundaries of the sample (in accordance with (7.23)) lead respectively to the following expressions:

$$\alpha^{(p)} \sim H^{1.5}T^{-5.5}, \qquad (7.32)$$

$$\alpha^{(p)}_b \sim H^2 T^{-1.5}. \qquad (7.33)$$

Comparison of these relations with the experimental data (7.30) indicates that in the interval 27—43°K scattering of phonons by phonons predominates, but scatter-

ing of the phonons at the boundaries cannot be neglected completely.

Thus, we can hope that a description of $\Delta\alpha^{(e)}_{theor}$ which is in better agreement with experiment, and also the introduction of corrections for the boundary scattering of the phonons, will confirm the validity of the relation for the damping decrement of longitudinal long-wave sound $\gamma_l \sim qT^4$, which agrees with the experimental data for the dragging thermal emf in n-Ge[64], as well as with experiment[70] on the absorption of ultrasound in solids, and finally, with the theoretical concepts[50,71].

APPENDIX

a) KINETIC EQUATION FOR ELECTRONS

Let us consider the system of interacting electrons with Hamiltonian

$$\hat{H} = \sum_\nu E_\nu a_\nu^+ a_\nu + \sum_{(\mu, \mu', \nu, \nu')} G^{\mu'\mu}_{\nu'\nu} a_\nu^+ a_\mu^+ a_{\nu'} a_{\mu'}, \qquad (A.1)$$

where E_ν—eigenvalue of the energy of the single-particle Hamiltonian, and $G^{\mu'\mu}_{\nu'\nu} \equiv \langle \nu, \mu \mid G(r - r') \mid \nu', \mu' \rangle$ —matrix element of the interelectron interaction in terms of the eigenfunctions of the single-particle Hamiltonian, a_ν^+ and a_μ—second-quantization operators obeying the Fermi statistics. We introduce, following Bogolyubov and Gurov[46], the density matrices of one, two, etc. particles with the aid of the formulas

$$\mathrm{Sp}\,(\hat{\rho}a_\varkappa^+ a_{\varkappa'}) \equiv \langle a_\varkappa^+ a_{\varkappa'}\rangle \equiv f_{\varkappa\varkappa'},$$

$$\mathrm{Sp}\,(\hat{\rho}a_\varkappa^+ a_{\varkappa'}^+ a_\nu a_{\nu'}) \equiv \langle a_\varkappa^+ a_{\varkappa'}^+ a_\nu a_{\nu'}\rangle \equiv h\,(\varkappa\varkappa'\nu\nu'),$$

$$\cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \qquad (A.2)$$

The averaging in (A.2) is over the Gibbs ensemble.

Using the definitions (A.2) and the equation of motion of the operators

$$i\hbar\,\frac{\partial}{\partial t}\,\hat{F} = [\hat{H}, \hat{F}]_- \equiv \hat{H}\hat{F} - \hat{F}\hat{H}, \qquad (A.3)$$

we get

$$\left(i\hbar\,\frac{\partial}{\partial t} + E_{\varkappa'} - E_\varkappa\right) f_{\varkappa\varkappa'} = \sum_{(\nu, \nu', \mu, \mu')} G^{\mu'\mu}_{\nu'\nu} \{h\,(\nu\mu\varkappa'\nu')\,\delta_{\varkappa\mu'}$$

$$- h\,(\nu\mu\mu'\varkappa')\,\delta_{\nu'\varkappa} + h\,(\varkappa\mu\mu'\nu')\,\delta_{\varkappa'\nu} - h\,(\varkappa\nu\mu'\nu')\,\delta_{\mu\varkappa'}\}, \qquad (A.4)$$

$$\left(i\hbar\,\frac{\partial}{\partial t} + E_{\varkappa'} + E_\varkappa - E_\gamma - E_{\gamma'}\right) h\,(\varkappa\varkappa'\gamma\gamma')$$

$$= \sum_{(\mu\mu', \nu, \nu')} G^{\mu'\mu}_{\nu'\nu}\langle[a_\nu^+ a_\mu^+ a_{\nu'}a_{\mu'}, a_\varkappa^+ a_{\varkappa'}^+ a_\gamma a_{\gamma'}]_-\rangle. \qquad (A.5)$$

To obtain the equations of motion of the single-particle density matrix, without taking into account the effects of dynamic screening of the interelectron interactions in the Born approximation, it is necessary to express the mean values contained in the right side of (A.5), of the product of six operators, in terms of the product of the single-particle matrices. Thus, for example

$$\langle a_\nu^+ a_\mu^+ a_{\nu'} a_{\varkappa'}^+ a_\gamma a_{\gamma'}\rangle = f_{\nu\gamma'}f_{\mu\nu}f_{\varkappa'\gamma} + f_{\nu\nu}f_{\mu\gamma'}f_{\varkappa'\gamma} + f_{\nu\gamma'}f_{\mu\nu'}(\delta_{\nu'\varkappa'} - f_{\varkappa'\nu'}),$$

$$\langle a_\varkappa^+ a_\nu^+ a_\mu^+ a_{\mu'} a_\gamma a_{\gamma'}\rangle = f_{\mu\mu'}f_{\nu\gamma}f_{\varkappa\gamma'} + f_{\varkappa\mu'}f_{\nu\gamma'}f_{\mu\gamma} + f_{\varkappa\gamma}f_{\nu\mu'}f_{\mu\gamma'}. \qquad (A.6)$$

Substituting (A.6) in (A.5) we obtain an equation relating h with f, in the following form:

$$\left(i\hbar\,\frac{\partial}{\partial t} + E_\varkappa + E_{\varkappa'} - E_\gamma - E_{\gamma'}\right) h\,(\varkappa\varkappa'\gamma\gamma') = \hat{M}\,\{f\}. \qquad (A.7)$$

Here $M[f]$ denotes the right-hand side of (A.5) following substitution of (A.6). In the case of slow processes, h does not depend explicitly on the time t, and depends implicitly in terms of the single-particle matrix $f_{\nu\nu'}$. Therefore $i\hbar\,\partial/\partial t\,h = 0$. However, it is customary to introduce in perturbation theory an adiabatic parameter $\epsilon \to 0$, that includes the interaction between the electrons at $t \to -\infty$. Consequently, it is necessary to make in (A.7) the substitution $i\hbar\,\partial/\partial t \to i\epsilon$. We then get from (A.7)

$$h\,(\varkappa\varkappa'\gamma\gamma') = \lim_{\varepsilon \to 0} \frac{\dot{M}\,[f]}{E_{\varkappa'} \cdot E_{\varkappa} - E_{\gamma} - E_{\gamma'} + i\varepsilon}\,. \qquad (A.8)$$

Substituting (A.8) in (A.3) we obtain a kinetic equation for the single-particle density matrix $f_{\nu\nu'}$.

Let us consider the particular case of the kinetic equation for electrons in a homogeneous quantizing magnetic field. We are interested in distributions that are not homogeneous along the x axis, which is perpendicular to the magnetic field, which in turn is parallel to the z axis. Such inhomogeneities, in the Landau representation $|\varkappa\rangle \equiv |n, p_2, x_0\rangle$ are described by the diagonal elements of the density matrix $f_{\varkappa\varkappa'}$ $= f_\varkappa \delta_{\varkappa\varkappa'}$, which depends on $p_y = eHx_0/c$.

Taking this into account and substituting (A.8) and (A.4), we get

$$\frac{\partial f_\varkappa}{\partial t} = \sum_{(\nu,\,\nu',\,\nu'')} W\,(\varkappa\nu\nu'\nu'')\,\{f_\nu\,[1-f_\varkappa]\,f_{\nu'}\,[1-f_{\nu''}] - f_\varkappa\,[1-f_\nu]\,f_{\nu''}\,[1-f_{\nu'}]\},$$
$$(A.9)$$

where

$$W\,(\varkappa\nu\nu'\nu'') = \frac{2\pi}{\hbar}\,|\,G^{\varkappa\nu''}_{\nu\nu'}\,|^2\,\delta\,[E_\nu + E_{\nu'} - E_\varkappa - E_{\nu''}],$$

$$|\,G^{\varkappa\nu''}_{\nu\nu'}\,|^2$$
$$= \int \frac{dq V}{(2\pi)^3}\int \frac{dq' V}{(2\pi)^3}\,G_q G^*_{q'}\,\langle\nu\,|\,e^{iqr}\,|\,\varkappa\rangle\,\langle\varkappa\,|\,e^{-iq'r}\,|\,\nu\rangle\,\langle\nu'\,|\,e^{-iqr}\,|\,\nu''\rangle\,\langle\nu''\,|\,e^{iq'r}\,|\,\nu'\rangle.$$
$$(A.10)$$

The products of the matrix elements contained in this equation can be written in the Landau representation in the form:

$$\langle\varkappa\,|\,e^{-iq'r}\,|\,\nu\rangle\,\langle\nu\,|\,e^{iqr}\,|\,\varkappa\rangle$$
$$= \delta\,(-p^{(\varkappa)}_y - p^{(\nu)}_y - \hbar q_y)\,\delta\,(q_y - q'_y)\,\delta\,(-p^{(\varkappa)}_z + p^{(\nu)}_z - \hbar q_z)\,\delta\,(q_z - q'_z)$$

$$\times \exp\{-i\,[q'_x - q_x]\,x_{0\varkappa}\}$$

$$\exp\{-i\gamma\alpha^2 q_y\,(q'_x - q_x)/2\}\,I_{n_\varkappa n_\nu}\,(-q'_x,\ q_y)\,I_{n_\varkappa n_\nu}\,(q'_x,\ q_y). \qquad (A.11)$$

Here

$$I_{n_\varkappa n_\nu}\,(\pm\,q'_x,\ q_y) = \sqrt{\frac{n!}{n_\nu!\,n_\varkappa!}}\,\left(\frac{\alpha}{\sqrt{2}}\,\{[\text{sign}\,(n_\varkappa - n_\nu)\,e]\,q_y - iq'_x\}\right)^{|n_\varkappa - n_\nu|}$$

$$\times\exp\left(-\frac{\alpha^2 q^2_\perp}{4}\right)L^{|n_\varkappa - n_\nu|}_{\bar{n}}\left(\frac{\alpha^2 q^2_\perp}{2}\right),\ \ q^2_\perp = q^2_x + q^2_y,\ \bar{n} = \min\,\{n_\varkappa,\ n_\nu\},\ \ (A.12)$$

$$L^s_n\,(t) = \sum_{m=0}^{n}\,(-1)^m\,\binom{n+s}{n-m}\,\frac{t^m}{m!}\,, \qquad (A.13)$$

$L^s_n(t)$ — generalized Laguerre polynomial.

The product of the matrix elements in (A.11) decreases exponentially if $|q'_x|$ or $|q_x|$ is larger than $1/\alpha$. Consequently, only the factor $\exp[\,ix_{0\varkappa}\,(q_x - q'_x)]$ can change noticeably in the region where this product differs greatly from zero, i.e., when $|q'_x|$ or $|q_x|$ is smaller than $1/\alpha$. Consequently, following substitution of (A.11) in (A.10) and integration with respect to q', the difference between q'_x and q_x can be neglected in the slowly-varying parts of the products of the matrix

elements; then the integral with respect to q'_x reduces to the form

$$\int_{-\infty}^{+\infty} dq'_x\,\exp\,[-iq'_x\,(x_{0\varkappa} - x_{0\nu''})] = 2\pi\delta\,(x_{0\varkappa} - x_{0\nu''}).$$

Taking this formula into account, we can represent (A.9) in the form

$$\frac{\partial f_\varkappa}{\partial t} = \sum_{(\nu,\,\nu',\,\nu'',\,q)} \frac{2\pi}{\hbar}\,|G_q\,|^2\,|\langle\nu\,|\,e^{iqr}\,|\,\varkappa\rangle\,|^2\,|\langle\nu'\,|\,e^{iqr}\,|\,\nu''\rangle\,|^2$$

$$\times\delta\,[E_\nu + E_{\nu'} - E_\varkappa - E_{\nu''}]\,\delta\,(x_{0\varkappa} - x_{0\nu''})$$

$$\times\{f_\nu\,[1-f_\varkappa]\,f_{\nu'}\,[1-f_{\nu''}] - f_\varkappa\,[1-f_\nu]\,f_{\nu''}\,[1-f_{\nu'}]\}. \qquad (A.14)$$

Taking into account the equality $x^\nu_0 = (c/eH)p^\nu_y$ and (A.11), we can easily verify that the matrix elements $\langle\nu'\,|\,e^{iq\cdot r}\,|\,\nu''\rangle$ and $\langle\nu\,|\,e^{iqr}\,|\,\varkappa\rangle$, which enter in (A.14), contain the factor

$$\delta\,(x_{0\varkappa} - x_{0\nu} + \gamma\alpha^2 q_y)\,\delta\,(x_{0\nu''} - x_{0\nu'} + \gamma\alpha^2 q_y) \qquad (\gamma \equiv \text{sign}\,e).$$

In the case of weak spatial inhomogeneities, when f as a function of x_0 remains practically constant on the Larmor radius, this factor can be expanded in a series in $\alpha^2 q_y$. The zeroth-order term in the collision integral (A.14), using the notation of (A.11) and (A.12) as well as

$$F_{n_\nu n_\varkappa}\left(\frac{\alpha^2 q^2_\perp}{2}\right) = |\,I_{n_\nu n_\varkappa}\,(\pm\,q_x,\ q_y)\,|^2, \qquad (A.14a)$$

is given by (3.3).

b) KINETIC EQUATIONS FOR ELECTRON-PHONON SYSTEMS

To calculate the charge and energy volume flux densities it is necessary to know the equations of motion of the single-particle density matrices of the electron and the phonon. In the Born approximation in the amplitude of electron-phonon scattering, the kinetic equations for the single-particle matrices can be obtained by using the already described procedure of Bogolyubov and Gurov[46]. Such calculations were performed in the paper of Bar'yakhtar and Peletminskii[10].

The Hamiltonian of the electron-phonon system is

$$\hat{H} = \hat{H}_{0e} + \hat{H}_{0p} + \hat{H}_{ep}, \qquad (A.15)$$

where

$$\hat{H}_{0e} = \sum_\nu E_\nu a^+_\nu a_\nu,\ \ \hat{H}_{0p} = \sum_q \hbar\omega_q b^+_q b_q,$$

$$\hat{H}_{ep} = \sum_{(\nu,\,\nu',\,q)}\,\{A\,(\nu'\nu q)\,b_q + A^*\,(\nu'\nu q)\,b^+_q\}\,a^+_\nu a_{\nu'},$$

$$A\,(\nu'\nu q) = C_q\,\langle\nu'\,|\,e^{iqr}\,|\,\nu\rangle;\ A^*\,(\nu'\nu q) = C^*_q\,\langle\nu'\,|\,e^{-iqr}\,|\,\nu\rangle,$$

C_q — Fourier component of the electron-phonon interaction energy.

We define the single-particle matrices

$$f_{\varkappa\varkappa'} = \text{Sp}\,(\hat{\rho}a^+_\varkappa a_{\varkappa'}),\ \ \ N_{qq'} = \text{Sp}\,(\hat{\rho}b^+_q b_{q'}), \qquad (A.16)$$

and the correlative matrices

$$h\,(\varkappa\varkappa'q) = \text{Sp}\,(\hat{\rho}a^+_\varkappa a_{\varkappa'}\,b^+_q),\ \ h^*\,(\varkappa\varkappa'q) = \text{Sp}\,(\hat{\rho}a^+_\varkappa a_{\varkappa'}\,b_q). \qquad (A.17)$$

With the aid of (A.3) and the definitions (A.16) and (A.17) we get

$$\left(i\hbar\frac{\partial}{\partial t}-E_\varkappa+E_{\varkappa'}\right)f_{\varkappa\varkappa'}=\sum_{(\nu,\nu',q)}\{A\,(\nu'\nu q)\,[\delta_{\varkappa\nu}h^*\,(\nu\varkappa'q)-h^*\,(\varkappa\nu'q)\,\delta_{\nu\varkappa'}]$$

$$-A^*\,(\nu'\nu q)\,[\delta_{\varkappa\nu}h\,(\nu\varkappa'q)-h\,(\varkappa\nu'q)\,\delta_{\nu\varkappa'}]\},\qquad(A.18)$$

$$\left(i\hbar\frac{\partial}{\partial t}+\hbar\omega_{q'}-\hbar\omega_q\right)N_{qq'}=\sum_{(\nu,\nu')}\{A\,(\nu'\nu q)\,h^*\,(\nu\nu'q')-A^*\,(\nu'\nu q)\,h\,(\nu\nu'q)\}.$$
$$(A.19)$$

In the same manner we can obtain an equation for $h(\nu'\nu q)$ and $h^*(\nu'\nu q)$, which contains the mean values of the products of four operators

$$\mathrm{Sp}\,(\hat\rho a_\varkappa^+a_{\varkappa'}a_\nu^+a_{\nu'}),\qquad \mathrm{Sp}\,(\hat\rho a_\nu^+a_{\nu'}b_q^+b_q).\qquad(A.20)$$

In order to obtain a closed system of equations for the single-particle matrices $N_{qq'}$ and $f_{\varkappa\varkappa'}$, it is necessary to express the mean values of the product of four operators in terms of $N_{qq'}$ and $f_{\varkappa\varkappa'}$. This can be done, as in the preceding section, only approximately, neglecting in the relations

$$\mathrm{Sp}\,(\hat\rho a_\varkappa^+a_{\varkappa'}a_\nu^+a_{\nu'})=f_{\varkappa\varkappa'}f_{\nu\nu'}+f_{\varkappa\nu'}\,(\delta_{\varkappa'\nu}-f_{\nu\varkappa'})+g\,(\varkappa\varkappa'\nu\nu'),$$

$$\mathrm{Sp}\,(\hat\rho a_\nu^+a_{\nu'}b_q^+b_q)=f_{\nu\nu'}N_{q'q}+g'\,(\nu\nu'q'q)\qquad(A.21)$$

the correlative functions g and g'. In this approximation we get

$$\left(i\hbar\frac{\partial}{\partial t}+E_{\varkappa'}-E_\varkappa-\hbar\omega_q\right)h\,(\varkappa\varkappa'q)=\sum_{(\nu,\nu',q)}A\,(\nu'\nu q')\,\{[f_{\nu\varkappa}\delta_{\varkappa\nu'}$$

$$-\delta_{\nu\varkappa}f_{\varkappa\nu'}]\,(\delta_{qq'}+N_{qq'})+[f_{\varkappa\varkappa'}f_{\nu\nu'}+f_{\varkappa\nu'}\,(\delta_{\nu\varkappa}-f_{\nu\varkappa'})]\,\delta_{qq'}\}\qquad(A.22)$$

and a similar equation for $h^*(\varkappa\varkappa'q)$. In the case of slow processes, h and h^* depend on the time implicitly, via $f_{\nu\varkappa}$ and $N_{qq'}$. Therefore the derivatives of h with respect to the time in the left-hand sides can be neglected. Then, introducing the adiabatic parameter $\epsilon\to 0$, we get

$$h\,(\varkappa\varkappa'q)=\lim_{\varepsilon\to+0}\frac{1}{E_{\varkappa'}-E_\varkappa-\hbar\omega_q+i\varepsilon}\sum_{(\gamma,\gamma',q)}A\,(\gamma'\gamma q')\,\{(\delta_{qq'}+N_{qq'})$$

$$\times[f_{\gamma\varkappa}\delta_{\gamma'\varkappa}-f_{\varkappa\gamma'}\delta_{\gamma\varkappa}]+\delta_{qq'}\,[f_{\varkappa\varkappa'}f_{\gamma\gamma'}+f_{\varkappa\gamma'}\,(\delta_{\gamma\varkappa'}-f_{\gamma\varkappa'})]\},\qquad(A.23)$$

$$h^*\,(\varkappa\varkappa'q)=\lim_{\varepsilon\to+0}\frac{1}{E_{\varkappa'}-E_\varkappa+\hbar\omega_q+i\varepsilon}\sum_{(\gamma,\gamma',q)}A^*\,(\gamma'\gamma q')\,\{f_{\varkappa\gamma'}$$

$$\times[f_{\gamma\varkappa}\delta_{\gamma'\varkappa}N_{qq'}-\delta_{\gamma\varkappa}\,(N_{qq'}+\delta_{qq'})]+[f_{\gamma\varkappa'}f_{\varkappa\gamma'}-f_{\varkappa\varkappa'}f_{\gamma\gamma'}]\,\delta_{qq'}\}.\qquad(A.24)$$

Substituting (A.23) and (A.24) in (A.18) and (A.19), we obtain a system of kinetic equations for the single-particle electron and phonon density matrices.

Let us proceed to consider the particular cases of interest to us. We find first the kinetic equation for the electronic density matrix in the presence of a homogeneous quantizing magnetic field. The dissipative charge and energy fluxes due to the one-dimensional spatial inhomogeneities in the plane perpendicular to the magnetic field are expressed in terms of the equation of motion of the diagonal matrix elements of the density matrix $f_{\varkappa\varkappa'}=f_\varkappa\delta_{\varkappa\varkappa'}$ in the Landau representation. With the aid of (A.18), (A.23), and (A.24) we get

$$\frac{\partial}{\partial t}f_\varkappa=\frac{2\pi}{\hbar}\sum_{(\nu,q,q')}\{[A\,(\varkappa\nu q)\,A^*\,(\nu\varkappa q')\,\delta\,[E_\varkappa-E_\nu+\hbar\omega_q]$$

$$\times(f_\nu\,(1-f_\varkappa)\,(N_{qq'}+\delta_{qq'})-f_\varkappa\,(1-f_\nu)\,N_{qq'})]+[A\,(\nu\varkappa q)\,A^*\,(\varkappa\nu q')\,\delta\,[E_\varkappa-E_\nu-\hbar\omega_q]$$

$$\times(f_\nu\,(1-f_\varkappa)\,N_{qq'}-f_\varkappa\,(1-f_\nu)\,(N_{qq'}+\delta_{qq'}))]\}.\qquad(A.25)$$

It is seen from this formula that the second curly bracket is obtained from the first by making the substitution $\varkappa\rightleftarrows\nu$ and multiplying by -1.

The products of the matrix elements of the electron-phonon interaction energy, which are contained in (A.25), are of the form given in (A.11). Just as in the derivation of (A.14), we shall neglect the difference between q_x and q_x' in the slowly-varying part of the product of the matrix elements; then the integral with respect to q_x' takes the form

$$\int_{-\infty}^{+\infty}dq_x'\exp\,[i\,(q_x-q_x')\,x_{0\varkappa}]\,\{f_\nu\,(1-f_\varkappa)\,(N_{q_xq_x'}+\delta_{qq'})$$

$$-f_\varkappa\,(1-f_\nu)\,N_{qq'}\}=f_\nu\,(1-f_\varkappa)\,(N_q\,(x_{0\varkappa})+1)-f_\varkappa\,(1-f_\nu)\,N_q\,(x_{0\varkappa}),\qquad(A.26)$$

in which the phonon matrix $N_q(x_{0\varkappa})$ is introduced in a mixed Wigner representation, or the quantum function of the phonon distribution with the aid of the relation

$$N_q\,(x_{0\varkappa})=\int_{-\infty}^{\infty}dq_x'\exp\,[i\,(q_x-q_x')\,x_{0\varkappa}]\,N_{qq'}.\qquad(A.27)$$

Making in (A.26) the substitutions $\varkappa\to\nu$ and $\nu\to\varkappa$, and multiplying everything by -1, we obtain the result of the integration with respect to q_x' in the second curly bracket of (A.25). The final result is the following kinetic equation:

$$\frac{\partial f_\varkappa}{\partial t}=\frac{2\pi}{\hbar}\sum_{\nu,q}|C_q|^2\,(1-\hat{\mathscr{P}}_{\nu\varkappa})\,F_{n_\varkappa n_\nu}\left(\frac{\alpha^2q_\perp^2}{2}\right)$$

$$\times\delta[-p_y^{(\varkappa)}-\hbar q_y+p_y^{(\nu)}]\delta[-p_z^{(\varkappa)}-\hbar q_z+p_z^{(\nu)}]\,\delta\,[E_\varkappa-E_\nu+\hbar\omega_q]$$

$$\times\{f_\nu\,(1-f_\varkappa)\,(N_q\,(x_{0\varkappa})+1)-f_\varkappa\,(1-f_\nu)\,N_q\,(x_{0\varkappa})\}\equiv I\,[f_\varkappa\,(n,\,p_z,\,x_0)].\qquad(A.28)$$

In this equation we introduce, to abbreviate the notation, the operator $\hat{\mathscr{P}}_{\nu\varkappa}$, which makes the substitutions $\nu\to\varkappa$ and $\varkappa\to\nu$. Similarly we obtain from (A.19), with the aid of (A.23), (A.24), and (A.27), a kinetic equation for the quantum distribution function of the phonons in the form:

$$\left(\frac{\partial}{\partial t}+\frac{\partial\omega}{\partial q_x}\frac{\partial}{\partial x}\right)N_q\,(x)=2\sum_{\nu,\varkappa}\frac{2\pi}{\hbar}|C_q|^2$$

$$\times F_{n_\varkappa n_\nu}\left(\frac{\alpha^2q_\perp^2}{2}\right)\delta\,[E_\varkappa-E_\nu+\hbar\omega_q]\,\delta\,[-p_y^{(\varkappa)}-\hbar q_y+p_y^{(\nu)}]$$

$$\times\delta\,[-p_z^{(\varkappa)}-\hbar q_z+p_z^{(\nu)}]\,\{f_\nu\,(1-f_\varkappa)\,(N_q\,(x_{0\varkappa})+1)$$

$$-f_\varkappa\,(1-f_\nu)\,N_q\,(x_{0\varkappa})\}\,\delta\,(x_{0\varkappa}-x)\equiv I_{pe}\,[N_q].\qquad(A.29)$$

Allowance for the non-electronic relaxation of the phonons, i.e., the relaxation of phonons on phonons, on the boundaries of the sample, etc., can be effected by adding to the right side of (A.29) the term

$$[N_q^0\,(x)-N_q\,(x)]\,\omega_{pp}\,(q),\qquad(A.30)$$

where N_q^0 is the local-equilibrium distribution function of the phonons, and $\omega_{pp}(q)$ is the effective phonon relaxation frequency. In this form, the kinetic equations (A.28) and (A.29) have been used in[11,12,47]. To study the effect of dragging of phonons by electrons, it is necessary to consider two-dimensional spatial inhomogeneities of the distribution in a plane perpendicular to the magnetic field. Such inhomogeneities are described by the non-diagonal matrix elements of the density matrix only with respect to the quantum number p_y, i.e., $f_{n,p_z,p_y;n,p_z,p_y'}$. Calculation of the charge and energy fluxes in such systems is best carried out with the aid of the Wigner representation of the density matrix, namely

$$f(n, p_z, p_y, y) = \sum_{p'_y} f_{n, p_z, p_y; n, p_z, p'_y} \exp\left[i (p'_y - p_y) \frac{y}{\hbar} \right],$$

$$N_q(x, y) = \sum_{q'} N_{qq'} \delta_{q_z q'_z} \exp[i (q_x - q'_x) x + i (q_y - q'_y) y]. \quad (A.31)$$

For convenience in writing down the kinetic equation, it is expedient to introduce in lieu of the function

$$f(n, p_z, p_y, y) \equiv f_\kappa(y)$$

a new function

$$\tilde{f}_\kappa = f_\kappa (y_\kappa) = \int f_\kappa (y) \delta (y - y_\kappa) dy.$$

From (A.18) and (A.19) we get with the aid of (A.23) and (A.24), for weak spatial inhomogeneities[11,12], kinetic equations for $\tilde{f}_\kappa$ and $N_q(x, y)$ similar to (A.28) and (A.29):

$$\frac{\partial \tilde{f}_\kappa}{\partial t} = \frac{2\pi}{\hbar} \sum_{(\nu, q)} |C_q|^2 (1 - \hat{\mathscr{P}}_{\nu \kappa}) \left\{ F_{n_\kappa n_\nu} \left(\frac{\alpha^2 q_\perp^2}{2} \right) \delta (E_\kappa - E_\nu + \hbar \omega_q) \right.$$

$$\times \delta (y_\nu - y_\kappa + \gamma \alpha^2 q_x) \delta (-p_y^{(\kappa)} - \hbar q_y + p_y^{(\nu)}) \delta (-p_z^{(\kappa)} - \hbar q_z + p_z^{(\nu)})$$

$$\left. \times [\tilde{f}_\nu (1 - \tilde{f}_\kappa) (N_q (x_{0\kappa}, y_\kappa) + 1) - \tilde{f}_\kappa (1 - \tilde{f}_\nu) N_q(x_{0\kappa}, y_\kappa)] \right\} \equiv I_{ep} [\tilde{f}_\kappa], \quad (A.32)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial \omega}{\partial q_x} \frac{\partial}{\partial x} + \frac{\partial \omega}{\partial q_y} \frac{\partial}{\partial y} \right) N_q(x, y) = \frac{2\pi}{\hbar} \sum_{\nu, \kappa} |C_q|^2 F_{n_\kappa n_\nu} \left(\frac{\alpha^2 q_\perp^2}{2} \right)$$

$$\times \delta (E_\kappa - E_\nu + \hbar \omega_q) \delta (y_\nu - y_\kappa + \gamma \alpha^2 q_x) \delta (-p_y^{(\kappa)} - \hbar q_y + p_y^{(\nu)})$$

$$\times \delta (-p_z^{(\kappa)} - \hbar q_z + p_z^{(\nu)}) [\tilde{f}_\nu (1 - \tilde{f}_\kappa) (N_q (x_{0\kappa}, y_\kappa) + 1) - \tilde{f}_\kappa (1 - \tilde{f}_\nu) N_q (x_{0\kappa}, y_\kappa)]$$

$$\times \delta (x_{0\kappa} - x) \delta (y - y_\kappa) + (N_q^0 (x, y) - N_q (x, y)) \omega_{pp} (q) \equiv I_{pe} [N_q] + I_{pd} [N_q],$$

$$F_{nn'} (t) = |I_{n, n'} (t)|^2. \quad (A.33)$$

The kinetic equations (A.28), (A.29), and (A.32), (A.33) could be written out immediately by taking into account the arrival of the electrons in the cell with quantum number κ and their departure from this cell. Since the electron distribution is spatially inhomogeneous, the phonon distribution function should also depend on the spatial coordinates (local equilibrium!) and, in accordance with the Bose statistics of the phonons in the processes of their emission, the factor $(N_q + 1)$ should be taken at the point corresponding to the final state of the electron, whereas in absorption processes the factor N_q should be referred to the point characterizing the initial state of the electron. Such an interpretation of the equations (A.32) and (A.33) is possible in the case of weak spatial inhomogeneities, when $f(n, p_z, p_y, y)$ changes little over the Larmor radius. It is precisely in this case that it is possible to specify simultaneously the quantum numbers n, p_z and the coordinates y and $p_y = eHx_0/c$, without contradicting the uncertainty principle. Then $f(n, p_z, x_0, y)$ can be identified with the probability that the electron in the state n, p_z is at the point x_0, y (more accurately, in the center of the Larmor orbit of the electron). Of course, $N_q(x, y)$ can also be identified with the usual probability, when the characteristic scale of the spatial inhomogeneities is large compared with the wavelength of the phonons that play an important role in the problem.

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