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EXPERIMENTAL DATA ON $\mu \to e$ DECAY

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1. THE mass, magnetic moment, and the lifetime of the muon are respectively equal to

$$m = 105.659 \pm 0.002 \text{ MeV},$$
$$\mu = (1.001164 \pm 0.000003)\frac{e\hbar}{2m_\mu c},$$
$$\tau = (2.199 \pm 0.001)\cdot 10^{-6} \text{ sec}.$$

2. In the decay of a fully polarized muon, the spectrum of the electrons is given by [1-3]

$$dN(x,\vartheta) = \frac{1}{2}\left\{\frac{1+f(x)}{1+4\eta\frac{m_e}{m_\mu}}\left[12-12x+\rho\left(\frac{32}{3}x-8\right)+12\frac{m_e}{m_\mu}\cdot\frac{1-x}{x}\eta\right] \pm \xi\cos\vartheta\left[4-4x+\delta\left(\frac{32}{3}-8\right)+g(x)\right]\right\} x^2\,dx\,d(\cos\vartheta). \quad (1)$$

The signs "+" and "−" pertain to μ^+ and μ^- mesons, respectively; x is the electron momentum measured in units of the maximum value of this quantity; ρ, δ, ξ, and η are parameters which are bilinear combinations of the interaction constant (see Sec. 3); f(x) and g(x) describe the corrections to the spectrum for the radiative effects. The experimental values of the parameters ρ, δ, ξ, and η, and also of the degree of polarization of the electrons h are listed in the table.

3. Even an absolutely accurate knowledge of the six parameters to be determined from experiment (τ, h, ρ, ξ, η, and δ) will not answer the question of the form of the interaction in the $\mu \to e$ decay. The most general form of the interaction Hamiltonian of the $\mu \to e$ decay is

$$\mathscr{H} = \sum_{k=1}^{5} (\bar{e}\Gamma_k\mu)(\bar{\nu}\Gamma_k(g_k + g_k'\gamma_5)\nu) + \text{h.c.}, \quad (2)$$

where Γ_k with k = 1, ..., 5 denotes the operators 1, γ_α, $\tau_{\alpha\rho}$, $i\gamma_\alpha\gamma_5$, and γ_5.

It follows from the Hamiltonian (2) that the theory is determined by ten complex constants g_k and g_k'. Eliminating the insignificant common phase factor, we obtain 19 constants to be determined. The six experimental parameters determined in the study of the electron spectrum of the $\mu \to e$ decay yield only six relations be between the 19 interaction constants.

In the case when $h \neq 1$, it is possible to determine experimentally the parameters $\rho(h)$ and $\delta(h)$ as functions of h. Even in this case, however, the constants g_k and g_k' cannot be determined uniquely.[15]

A unique determination of the constants g_k and g_k' from experiments on $\mu \to e$ decay is possible only if the decay neutrino is registered besides the electron.[16] Such experiments, however, are not realistic at present, although they are highly desirable.

4. Experiments on the study of the electron spectrum of the $\mu \to e$ decay can be used for a unique determination of the constants of a theory that is less general than the theory determined by the Hamiltonian (2). Let us consider a theory in which a two-component neutrino is assumed. Such a theory leads[17] to experimental parameters $\rho = \delta \equiv 3/4$ and $\xi = -h$. The electron spectrum (1) is then determined by only two parameters, ξ and η. All that remain in the theory are two (complex) constants,[17] since in the case of a two-component neutrino $g_k = g_k' = 0$ for the scalar, pseudoscalar, and tensor interactions. For the vector and axial interactions we have for the constants in the Hamiltonian (2) $g_V = -g_V'$ and $g_A = -g_A'$. The equalities $\eta = 0$ and $\xi = 1$ correspond to the V–A interaction. The two complex constants g_V and g_A are uniquely determined by

Experimental values of the parameters of the electron spectrum of the $\mu \to e$ decay[7]

Parameter	Value	Reference	Parameter	Value	Reference
ρ	0.745 ± 0.025	4	h	1.05 ± 0.3	11
ρ	0.750 ± 0.003	5	h	0.94 ± 0.38	12
ρ	0.760 ± 0.009	6	h	1.04 ± 0.18	13
ρ	0.762 ± 0.008	7	h	-0.89 ± 0.28	14
ρ *)	0.750 ± 0.006	Mean value	h	$(-)1.00 \pm 0.13$	Mean value
δ	0.78 ± 0.05	4	η**)	-2.0 ± 0.9	4
δ	0.782 ± 0.031	8	η	0.05 ± 0.5	5
δ	0.752 ± 0.009	7	η	-0.7 ± 0.6	6
δ	0.754 ± 0.0085	Mean value	η	-0.7 ± 0.5	7
ξ	0.97 ± 0.05	9	η	-0.8 ± 0.4	Mean value
ξ	0.94 ± 0.07	4			
ξ	0.975 ± 0.015	10			
ξ	0.973 ± 0.014	Mean value			

*The error is increased to allow for the fact that the parameter η may differ from zero [7].

**To obtain this value of η, both parameters of the isotropic part of the spectrum ρ and η were varied. In all other cases the parameter η was calculated under the assumption that $\rho \equiv 3/4$.

three experimental parameters, ξ, η, and τ, since the phase factor common to the two constants is immaterial. The parameters ξ and η are expressed in terms of g_V and g_A as follows:[17]

$$\xi = \frac{2\,\mathrm{Re}\,(g_V g_A^*)}{|g_A|^2 + |g_V|^2}, \quad \eta = \frac{|g_A|^2 - |g_V|^2}{|g_A|^2 + |g_V|^2}. \tag{3}$$

The parameter η has so for been determined with poor accuracy, since this parameters enters in the expression for the electron spectrum (1) with a small factor m_e/m_μ (see the table). To determine the ratio $\varepsilon = g_A/g_V = |\varepsilon|\, e^{i\theta}$ in the two-component neutrino theory it is possible for the time being to use only the first equation of (3):

$$\xi = \frac{2\,\mathrm{Re}\,\varepsilon}{1 + |\varepsilon|^2} = \frac{2\,|\varepsilon|\cos\theta}{1 + |\varepsilon|^2}. \tag{4}$$

Assuming the experimental value $\xi_{min} = 0.975 - 0.015 = 0.96$,[10] we get for $|\varepsilon|$ the estimate $0.75 \le |\varepsilon| \le 1.34$. We can obtain from (4) also the estimate $\theta \le 16°$. Formula (4) is very insensitive to the values of $|\varepsilon|$ and θ, in spite of the high accuracy with which ξ is determined. It follows from (4) that the maximum deviation of $|\varepsilon|$ from unity is obtained in the T-invariant theory ($\theta = 0$).

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