SOVIET PHYSICS USPEKHI VOLUME 11, NUMBER 4 JANUARY-FEBRUARY 1969

539.12.01

NONCONSERVATION OF CP PARITY IN $K \to 3\pi$ DECAY

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Usp. Fiz. Nauk 95, 453–459 (1968)

AT the present time the nature of CP-invariance violation, discovered in the decay $K_L \to 2\pi$,[1] is not as yet clear. Consequently search for effects connected with CP nonconservation in other reactions is important. The decay $K \to 3\pi$ has been discussed in the papers[2-8] from this point of view. Below we discuss three main effects that arise in the decays $K \to 3\pi$ under CP nonconservation.

One of the more important peculiarities of the decay $K \to 3\pi$ is the comparatively small energy release. This circumstance leads one to believe that one may expand the decay amplitude in a series in the energy of the pions in the neighborhood of the midpoint on a Dalitz plot and confine oneself to a few terms only. Indeed, the decay probability is not too badly described by formulas of the form $\lambda^2[1 + 2\alpha/m_\pi^2(s_{12} - s_0)]$, where $s_{12} = (p - k_3)^2$, p being the momentum of the K meson, k_3 being the momentum of the odd meson (the π^- meson in the decay $K^+ \to \pi^+\pi^+\pi^-$, the π^+ meson in the decay $K^+ \to \pi^0\pi^0\pi^+$ and the π^0 meson in the decay $K_{20} \to \pi^+\pi^-\pi^0$), and s_0 being the value of s_{12} at the midpoint in the Dalitz plot. The $\Delta T = \frac{1}{2}$ rule requires the total isospin of the pion system to equal unity and the constants λ in the different reactions are related to each other: $\lambda^{00+} = \lambda^{+-0} = -\frac{1}{2}\lambda^{++-} = -\frac{1}{3}\lambda^{000}$. An analogous relation holds also among the coupling constants a: $a^{00+} = a^{+-0} = a^{++-}$, $a^{000} = 0$.

If the interaction between π mesons at low energies is not small (if the scattering lengths of pions are of order $0.5\,m_\pi^{-1} - m_\pi^{-1}$), then an important role may be played in the decay amplitude by terms due to rescattering of the produced particles. In that case the probability for the decay $K \to 3\pi$ is describable by a function of the type $\lambda^2[1 + 2a/m^2(s_{12} - s_0)]$, only in the middle of the Dalitz plot, and may differ at the edges of the plot since at the edges the amplitude has singularities due to the rescattering of pions (Fig. 1).

For three-particle production reactions with small kinetic energy release (of the type of the decay $K \to 3\pi$), a rigorous phenomenological theory may be developed, analogous to the Bethe-Peierls effective-radius theory for the deuteron. Within the framework of this theory the interaction of the pions produced in the decay $K \to 3\pi$ can be taken into account.[9,10] Moreover, if the pion interaction at low energies is not too large (the scattering lengths of pions are less than or approximately equal to m_π^{-1}), the amplitude can be expanded in a series, the first term of which is a constant, and the following terms are of order $E^{1/2}$, E, $E^{3/2}$ etc. (where E is the kinetic energy released in the decay). The singular terms of order $E^{1/2}$ have their origin in the diagrams of the type shown in Fig. 1a, singular terms of order E have their origin in diagrams of the type shown in Fig. 1b. The numerical coefficients multiplying these singular terms are expressed in terms of low-energy pion scattering amplitudes. The coefficients in front of the

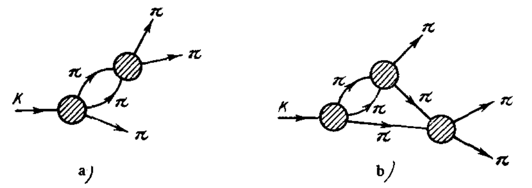


FIG. 1

analytic terms cannot be connected with other physical quantities—in such considerations these coefficients must be considered as unknown constants. Along with the singular terms one calculates in this fashion unambiguously all imaginary parts of the amplitude due to rescattering of the produced pions. It turns out that for scattering lengths of the pions of the order m_π^{-1} the singular terms do not substantially affect the spectrum of pions in the decay $K^+ \to \pi^+\pi^+\pi^-$ and the decay probability may still be described in the entire region with sufficient accuracy by a formula of the type $\lambda^2[1 + 2a/m_\pi^2(s_{12} - s_0)]$. By contrast in the spectra of $K^+ \to \pi^0\pi^0\pi^+$ and $K_{20} \to \pi^+\pi^-\pi^0$ the contribution of the singular terms is more significant and at the edges of the spectrum one may note substantial deviation from the linear character. In[11], data were collected on the decays $K^\pm \to \pi^0\pi^0\pi^\pm$ and $K_{20} \to \pi^+\pi^-\pi^0$, obtained by various authors (altogether 6000 events). Deviation from the linear character was observed. This, apparently, means that the scattering lengths of pions are not small and that in the analysis of the $K \to 3\pi$ decays it is necessary to take into account singularities due to the rescattering of the produced pions.

1. CP-PARITY NONCONSERVING INTERACTIONS IN THE DECAYS $K \to 3\pi$

An experimental study of the decays $K \to 3\pi$ makes it possible to determine whether CP is violated in milliweak interactions with conservation of space parity.

For milliweak interactions with $\Delta Y = \pm 1$ (Y being the hypercharge), that violate CP we shall use the notation $MW^P_{\Delta T}$. P = ± shows whether space parity is conserved (+) or not (–) in the given interaction. The subscript ΔT indicates nonconservation of isotopic spin. For example, $MW^+_{1/2}$ denotes a milliweak interaction with conservation of space parity and the selection rule $\Delta Y = \pm 1$, $\Delta T = \pm \frac{1}{2}$.

The milliweak interaction with space parity change and the super weak interaction of Wolfenstein[12] do not contribute to the 3π-decay of the $K^\pm$ meson, and in the decay of neutral K mesons they contribute only via processes $K_{10} \xrightarrow{\text{weak}} \text{hadrons} \xrightarrow{MW^-} K_{20} \to 3\pi$, $K_{20} \xrightarrow{MW^-} \text{hadrons} \xrightarrow{\text{weak}} K_{10} \to 3\pi$, $K_{10} \xrightarrow{\text{superweak}} K_{20} \to 3\pi$, $K_{20} \xrightarrow{\text{superweak}} K_{10} \to 3\pi$. If we define the longlived and the short-lived components by means of equations $K_L = K_{10} + \epsilon K_{20}$ and $K_S = K_{20} + \epsilon K_{10}$, then in the matrix elements describing the decays K_{10} and K_{20} it is no longer necessary to take into account the transitions $K_{10} \to K_{20}$ and $K_{20} \to K_{10}$: they are completely taken into account by the mixing parameter ϵ. In this manner the direct transitions $K_{10} \to 3\pi$ and $K_{20} \to 3\pi$ under the influence of the milliweak interaction with a change in space parity and the superweak interaction of Wolfenstein need not be considered.

The magnitude of the interactions $MW^{+}_{1/2}$ and $MW^{+}_{3/2}$ relative to the weak (in the following such ratios will be denoted by $r^{P}_{\Delta T}$) should not exceed 10^{-3}. Should that not be the case then the interactions $MW^{+}_{1/2}$ and $MW^{+}_{3/2}$ would give rise to an excessive violation of CP in the decay $K_L \to 2\pi$ via the process $K_{20} \xrightarrow{\text{weak}} \text{hadrons} \xrightarrow{MW^{+}_{1/2,\,3/2}} K_{10} \to 2\pi$. The interactions $MW^{+}_{5/2}$ and $MW^{+}_{7/2}$ do not contribute to such transitions. We shall see, however, that from experiments on the measurement of the ratios of probabilities for the decays $K^+ \to \pi^+\pi^+\pi^-$ and $K^- \to \pi^-\pi^-\pi^+$ analogous limitations follow also for the interactions $MW^{+}_{5/2}$ and $MW^{+}_{7/2}$: $r^{+}_{5/2} \lesssim 10^{-3}$, $r^{+}_{7/2} \lesssim 10^{-3}$.

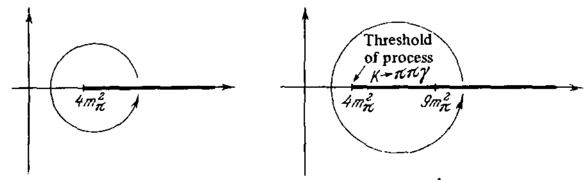
2. NONCONSERVATION OF CP PARITY IN THE DECAYS $K^{\pm} \to 3\pi$

The amplitudes for the decays $K^+ \to \pi^+\pi^+\pi^-$ and $K^+ \to \pi^0\pi^0\pi^+$ have the form

$$A^{++-} = \lambda^{++-}\left\{1 + \frac{4a^{++-}}{m_\pi^2}\left(k_{12}^2 - \frac{Em_\pi}{2}\right) + ik_{12}a_2 + i(k_{13}+k_{23})\left[\frac{2}{3}a_0 + \frac{1}{3}a_2 + \frac{1}{3}\rho(a_2 - a_0)\right] + \ldots\right\}, \tag{1}$$

$$A^{00+} = \lambda^{00+}\left\{1 + \frac{4a^{00+}}{m_\pi^2}\left(k_{12}^2 - \frac{Em_\pi}{2}\right) + ik_{12}\left[\frac{1}{3}a_0 + \frac{2}{3}a_0^2 + \frac{2}{3}\rho^{-1}(a_2 - a_0)\right] + i(k_{13}+k_{23})a_2 + \ldots\right\}; \tag{2}$$

where k_{il} is the relative momentum of the π mesons (the indices 1 and 2 refer to identical pions), $\rho = \lambda^{00+}/\lambda^{++-}$. The first two terms in the curly braces are the analytic terms in the expansion of the amplitude $(s_{12} - s_0 = 4k_{12}^2 - 2m_\pi E)$, the following terms are singular of order $E^{1/2}$, due to diagrams of the type Fig. 1a. The dots refer to further terms: singular of order E, due to diagrams of the type Fig. 1b, singular of order $E^{3/2}$ etc. There is a rule for going over from the amplitude for the decay of the K^+ meson to the amplitude for the decay of the K^- meson in the case of CP nonconservation. As was already stated, owing to the interaction of the produced pions the decay amplitude has singularities in s_{ie} (for $s_{ie} = 4m_\pi^2$). Moreover, the amplitude has also singularities in M_K^2 (due to the transitions $K \to \pi\pi\pi \to \pi\pi\pi$, $K \to \pi\pi\gamma \to \pi\pi\pi$). The physical amplitude for the decay is taken on the upper edge of the cuts due to these singularities, i.e., for $s_{ie} + i\epsilon$, $M_K^2 + i\epsilon$ ($\epsilon \to +0$) (Fig. 2). The decay amplitude has imaginary parts due to the possible real processes, as well as imaginary parts due to violation of CP. On going over from the amplitudes for the decay of the K^+ to the amplitudes for the decay of the K^- meson one must change the sign of only those imaginary parts that are due to violation of CP, and the imaginary parts due to real processes should have their sign unchanged. Therefore on going over from the decay amplitude for the K^+ meson to the decay amplitude for the K^- meson one should go from values $s_{ie} + i\epsilon$, $M_K^2 + i\epsilon$ to values $s_{ie} - i\epsilon$, $M_K^2 - i\epsilon$, as is shown in Fig. 2 (thus changing the sign of the imaginary parts connected with real processes), and then take the complex conjugate of the amplitude*.



a b

FIG. 2

It is thus seen that if the $\pi\pi\gamma$ channel is neglected the amplitudes for the decay of the K^- mesons are obtained from Eqs. (1) and (2) by complex conjugation of the constants λ and a (and, consequently, ρ).

It then follows from Eqs. (1) and (2) that the following expressions hold for the ratios of total decay probabilities:

$$\frac{W(K^+ \to \pi^+\pi^+\pi^-)}{W(K^- \to \pi^-\pi^-\pi^+)} = 1 + \frac{256}{45\pi}\sqrt{m_\pi E}\,(a_0 - a_2)\,\mathrm{Im}\,\rho, \tag{3}$$

$$\frac{W(K^+ \to \pi^0\pi^0\pi^+)}{W(K^- \to \pi^0\pi^0\pi^-)} = 1 - \frac{1024}{45\pi}\sqrt{m_\pi E}\,(a_0 - a_2)\,\mathrm{Im}\,\rho. \tag{4}$$

The quantity ρ is defined by the ratio of the two amplitudes of the τ decay at zero energy. In that case both the spatial and the charge parts of the wave function of the three pions should be completely symmetric, so that they must be in the state with T = 1 or T = 3. If the decay interaction belongs to the class $MW^{+}_{1/2,\,3/2}$, then only the first state may be formed. In that case $\rho = \frac{1}{2}$, so that $\mathrm{Im}\,\rho = 0$. In fact a nonzero $\mathrm{Im}\,\rho$ may arise here due to the virtual electromagnetic interaction, which has an additional factor of smallness $\approx 10^{-2}$. In the cases of $MW^{+}_{5/2}$ and $MW^{+}_{7/2}$ the state with T = 3 is allowed and $\mathrm{Im}\,\rho \neq 0$. The interactions $MW^{+}_{1/2,\,3/2}$ can also give rise to a nonzero $\mathrm{Im}\,a^{++-}$ and $\mathrm{Im}\,a^{00+}$. Analogous calculations show however that the contribution of the terms proportional to $\mathrm{Im}\,a^{++-}$ and $\mathrm{Im}\,a^{00+}$ is small: in the total probabilities the coefficients in front of these imaginary parts are of order 10^{-2}. The experimental data on the ratios of the probabilities of the decays

$$\frac{W(K^+ \to \pi^+\pi^+\pi^-)}{W(K^- \to \pi^-\pi^-\pi^+)} = 1.005 \pm 0.009\ [13],\quad 1.0004 \pm 0.0021\ [14]$$

show that the interactions $MW^{+}_{5/2}$ and $MW^{+}_{7/2}$ should be of order 10^{-3} or less of the weak interaction. An increase in the experimental accuracy in these experiments by an order of magnitude is of great interest.

3. CHARGE ASYMMETRY IN THE DECAY $K_L \to \pi^+\pi^-\pi^0$

The system $\pi^+\pi^-\pi^0$ may exist in states with total isospin equal to 0, 1, 2 and 3. The states with total isospin 0 and 2 have positive parity with respect to CP, and the

*This rule has a general character. The only difference in the general case has to do with the fact that in such a procedure we go over to the amplitude containing antiparticles with all spin projections replaced by their opposites.

Table I. ($r^P_{\Delta T}$ stands for the ratio of the "forces" of the interaction $MW^P_{\Delta T}$ and the weak interaction, M stands for a characteristic mass of the order $m_\pi - 5m_\pi$ (m_π is the pion mass)).

	$MW^+_{1/2}$	$MW^+_{3/2}$	$MW^+_{5/2}$	$MW^+_{7/2}$
$\tilde{\lambda}/\lambda$	$\approx r^+_{1/2}$	$\approx r^+_{3/2}$	$\approx r^+_{5/2}$	$\approx r^+_{7/2}$
B_0^-/λ	$\approx \frac{r^+_{1/2}}{M^6}$	$\approx 10^{-2}\frac{r^+_{3/2}}{M^6}$	$\approx 10^{-2}\frac{r^+_{5/2}}{M^6}$	$10^{-4}\frac{r^+_{7/2}}{M^6}$
B_2^-/λ	$\approx 10^{-2}\frac{r^+_{1/2}}{M^2}$	$\approx \frac{r^+_{3/2}}{M^2}$	$\approx \frac{r^+_{5/2}}{M^2}$	$\approx 10^{-2}\frac{r^+_{7/2}}{M^2}$

states with total isospin 1 and 3 have negative CP parity. The amplitude with T = 0 is completely antisymmetric in the energies of the π mesons and therefore the first term in the expansion contains the small factor $(K_{12}^2 - K_{13}^2)(K_{12}^2 - K_{23}^2)(K_{13}^2 - K_{23}^2)$. The amplitude with T = 2 is antisymmetric in the energies of the π^+ and the π^-, therefore the first term in its expansion has the factor $(K_{13}^2 - K_{23}^2)$ (the indices 1, 2 and 3 refer to respectively the π^+, π^- and π^0 meson). The expansion of the amplitude with T = 1 and T = 3 in a series in the energy of the pions starts with a constant term, as in the case of the decay of the K^+. This expansion for the decay $K_{20} \to \pi^+\pi^-\pi^0$ has the form

$$A(K_{20} \to \pi^+\pi^-\pi^0) = \lambda^{+-0}\Big\{1 + \frac{4a^{+-0}}{m_\pi^2}\Big(k_{12}^2 - \frac{Em_\pi}{2}\Big) + ik_{12}\Big[\frac{2}{3}a_0 + \frac{1}{3}a_2 + \frac{1}{3}\sigma(a_2 - a_0)\Big] + i(k_{13} + k_{23})a_2 + \ldots\Big\}$$

$$+ iB_2^-\Big\{k_{13}^2 - k_{23}^2 + i\frac{3}{2}a_2\Big[k_{13}\Big(k_{13}^2 - \frac{Em_\pi}{2}\Big) - k_{23}\Big(k_{23}^2 - \frac{Em_\pi}{2}\Big)\Big] + \ldots\Big\} + iB_0^-(k_{12}^2 - k_{13}^2)(k_{12}^2 - k_{23}^2)(k_{13}^2 - k_{23}^2); \tag{5}$$

here the first term (proportional to λ^{+-0}) describes transitions with conservation of CP into states with total isospin 1 and 3, and the second (proportional to B_2^-) and third (proportional to B_0^-)—transitions with a change in CP into states with isospins 2 and 0. The constants λ^{+-0}, a^{+-0}, B_2^- and B_0^- are real. The real constant $\sigma = \lambda^{000}/\lambda^{+-0}$ equals −3 in the absence of transitions with $\Delta T = 5/2$ and $\Delta T = 7/2$. In Eq. (5), as in Eqs. (1) and (2), only singular terms arising from diagrams of type Fig. 1a are taken into account, and singularities due to diagrams of the type Fig. 1b etc. are omitted.

The amplitude for the transition $K_{10} \to \pi^+\pi^-\pi^0$ is obtained from Eq. (5) by the replacement $\lambda^{+-0} \to i\tilde{\lambda}^{+-0}$, $a^{+-0} \to \tilde{a}^{+-0}$, $B_2^- \to -iA_2^+$, $B_0^- \to -iA_0^+$ where the new constants $\tilde{\lambda}^{+-0}$, $\tilde{a}^{+-0}$, A_2^+ and A_0^+ are also real. The term proportional to $\tilde{\lambda}^{+-0}$ is due to transitions into states with total isospins 1 and 3 with change in CP, the term proportional to A_2^+ describes transitions with CP conservation into states with isospin 2 ($\Delta T = 3/2$ and $\Delta T = 5/2$), and the term proportional to A_0^+—into states with isospin 0 (with conservation of CP and $\Delta T = 1/2$).

The charge asymmetry in the decay $K_L = \pi^+\pi^-\pi^0$ equals ($K_L = K_{20} + \epsilon K_{10}$)

$$\frac{W(E_+ > E_-) - W(E_+ < E_-)}{W(E_+ > E_-) + W(E_+ < E_-)} \cong$$

$$\cong -\frac{Em_\pi}{\lambda^{+-0}}\{0.7A_2^+ \operatorname{Re}\varepsilon + \sqrt{Em_\pi}(0.8a_0 - 0.1a_2)(B_2^- + A_2^+ \operatorname{Im}\varepsilon)\} - \frac{E^3m_\pi^3}{\lambda^{+-0}}\{0.04A_0^+ \operatorname{Re}\varepsilon + \sqrt{Em_\pi}(0.05a_2 + 0.04a_2)(B_0^- + A_0^+ \operatorname{Im}\varepsilon)\}. \tag{6}$$

Table II. (a_0 and a_2 are scattering lengths of pions in the states with isospin 0 and 2, m_π is the pion mass, M is a characteristic mass of the order $m_\pi - 5m_\pi$)

Interaction type	Ratio of the "forces" of the interaction violating CP and the weak interaction $r^P_{\Delta T}$	$\frac{W(K^+ \to \pi^+\pi^+\pi^-)}{W(K^- \to \pi^-\pi^-\pi^+)} - 1$	Asymmetry $\frac{W(E_+ > E_-) - W(E_+ < E_-)}{W(E_+ > E_-) + W(E_+ < E_-)}$ in the decay $K_L \to \pi^+\pi^-\pi^0$	Magnitude of amplitude α of the oscillations in the time dependence of the decay $K \to \pi^+\pi^-\pi^0$
Wolfenstein's superweak		0	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2}$	$\approx 10^{-3}$
$MW^-_{1/2}$	$\leqslant 10^{-3}$	0	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2}$	$\approx 10^{-3}$
$MW^-_{3/2}$	$\leqslant 10^{-3}$	0	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2}$	$\approx 10^{-3}$
$MW^+_{1/2}$	$\leqslant 10^{-3}$	$\approx 10^{-2}r^+_{1/2}$	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2}$	$\approx 10^{-3}$
$MW^+_{3/2}$	$\leqslant 10^{-3}$	$\approx 10^{-2}r^+_{3/2}$	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2} + r^+_{3/2}\frac{m_\pi^3 a_0}{M^2}$	$\approx 10^{-3}$
$MW^+_{5/2}$	$\leqslant 10^{-3}$	$\approx \lvert a_0 - a_2\rvert m_\pi \cdot r^+_{5/2}$	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2} + r^+_{5/2}\frac{m_\pi^3 a_0}{M^2}$	$\approx 10^{-3} \div \lvert a_0 - a_2\rvert m_\pi r^+_{5/2}$
$MW^+_{7/2}$	$\leqslant 10^{-3}$	$\approx \lvert a_0 - a_2\rvert m_\pi \cdot r^+_{7/2}$	$\approx (10^{-4} \div 10^{-5})\frac{m_\pi^2}{M^2}$	$\approx 10^{-3} \div \lvert a_0 - a_2\rvert m_\pi r^+_{7/2}$

The amplitude A_0^+ is due to transitions with CP conservation and $\Delta T = \frac{1}{2}$, therefore $A_0^+/\lambda^{+-0} \sim M^{-6}$, where M is some "characteristic" mass. No experimental estimates of the quantity A_0^+ (i.e. the quantity M) exist. It is apparently reasonable to suppose that $M \approx m_\pi - 5m_\pi$: the "characteristic" mass which makes the coefficient of $k_{12}^2 - Em_\pi/2$ in Eqs. (1) and (2) dimensionless is of order m_π, however, it is not impossible that for other coefficients this mass is substantially larger. The ratio of the amplitudes $A_2^+/\lambda^{+-0} \approx (10^{-1}-10^{-2})M^{-2}$. The factor of smallness of order $10^{-1}-10^{-2}$ is due to the fact that the amplitude A_2^+ describes transitions with violation of the $\Delta T = \frac{1}{2}$ rule. Estimates for the amplitudes B_2^- and B_0^- for violation of CP by the interactions $MW_{1/2}^+$, $MW_{3/2}^+$, $MW_{5/2}^+$ and $MW_{7/2}^+$ are given in Table I. The additional factor of smallness of order 10^{-2} in the amplitude B_2^- in the case of the interaction $MW_{1/2}^+$ is due to the fact that this interaction leads to transitions into the state with total isospin 2 only in the presence of a virtual photon. An analogous reason is responsible for the appearance of additional factors of smallness in other cases.

Estimates of the size of the charge asymmetry

$$\frac{W(E_+ > E_-) - W(E_+ < E_-)}{W(E_+ > E_-) + W(E_+ < E_-)}$$

for various types of interactions violating CP are given in Table II. It is seen that for an arbitrary variant of CP violation the charge asymmetry should be small—of order $10^{-4}-10^{-5}$.

4. THE TIME DEPENDENCE OF THE DECAY PROBABILITY OF THE K^0 OR $\overline{K}^0$ MESON INTO $\pi^+\pi^-\pi^0$

The time dependence of the decay of the K^0 or $\overline{K}^0$ meson into $\pi^+\pi^-\pi^0$ is described by the following formula (the sign "+" refers to the decays of the K^0 mesons, the sign "−" to the decays of the $\overline{K}^0$ meson):

$$W_\pm(\pi^+\pi^-\pi^0) \sim e^{-t/\tau_L} + \frac{W(K_S \to \pi^+\pi^-\pi^0)}{W(K_L \to \pi^+\pi^-\pi^0)} e^{-t/\tau_S}$$
$$\pm 2 \frac{\left| \int d\Phi A^*(K_L \to \pi^+\pi^-\pi^0) A(K_S \to \pi^+\pi^-\pi^0) \right|}{W(K_L \to \pi^+\pi^-\pi^0)} e^{-t/2\tau_L - t/2\tau_S} \cos(\Delta m t + \varphi),$$
$$\varphi = \arg \int d\Phi A^*(K_L \to \pi^+\pi^-\pi^0) A(K_S \to \pi^+\pi^-\pi^0), \tag{7}$$

where $\int d\Phi$ is the integral over the phase space of the pions, t is the time, $m = m_L - m_S$. Oscillations in the time dependence are due to the last term. The amplitude of these oscillations may be obtained starting from Eq. (5) and from the analogous formula for the decay $K_{10} \to \pi^+\pi^-\pi^0$:

$$\alpha = 2 \frac{\left| \int d\Phi A^*(K_L \to \pi^+\pi^-\pi^0) A(K_S \to \pi^+\pi^-\pi^0) \right|}{W(K_L \to \pi^+\pi^-\pi^0)}$$
$$\simeq 2\left(1 + \frac{W(K_{10} \to \pi^+\pi^-\pi^0)}{W(K_{20} \to \pi^+\pi^-\pi^0)}\right) \operatorname{Re}\varepsilon + \frac{64}{45\pi} \sqrt{m_\pi E}\,(a_0 - a_2)\left(3\frac{\widetilde{\lambda}^{+-0}}{\lambda^{+-0}} + \frac{\widetilde{\lambda}^{000}}{\lambda^{+-0}}\right). \tag{8}$$

It is seen that in an arbitrary variant of the interaction the oscillations to be expected are of order 10^{-3} (see Table II).

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Translated by A. M. Bincer