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K^0 THREE BODY DECAYS

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THE three body decays of the K meson allow us to test the important hypothesis of the weak interactions: selection rules, form of the interaction, etc., but their study is more difficult than the $K \to 2\pi$ decays.*

These last years, there has been a great amount of experimental results reported in the very good surveys by Chuvilo (1964),[1] Trilling (1965),[2] Cabibbo (1966)[3] and Willis (1967).[4] In this report I will refer to the results of these authors, brought up to date if necessary, and to the September version of the UCRL-8030.[5]

Unfortunately it is still to early for me to give you definite answers for many fundamental problems such as the $\Delta I = 1/2$ non-leptonic rule, or the values of the form-factor ratio.

I will discuss first the $K \to 3\pi$ decays then the leptonic decays; I will not deal with rare decay modes such as $\pi\pi\gamma$ or $\pi\gamma\gamma$.

1. THE $K \to 3\pi$ DECAY

1. I will briefly consider the problem of the violation of PC in $K^0 \to 3\pi$ because there are practically no results published. The lower published value is:[6]

$$R=\frac{\Gamma(K_S^0 \to \pi^+\pi^-\pi^0)}{\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)} \leqslant 0.45$$

with 90% confidence level.

But these authors suppose the validity of the $\Delta I = 1/2$ rule. If this rule is violated in $K^0 \to 2\pi$ then this hypothesis must be disregarded, as a result from what R becomes an upper limit of an order of about 1. Looking for the charged asymmetry the most precise experiments (5–10) give a precision of a few percent (with a statistic of the order of 2000 $K^0 \to \pi^+\pi^-\pi^0$ decays). The expected theoretical asymmetry is 1 per thousand.

2. Branching ratio. After the new K^+ rates measurement presented in 1966 at Berkeley by Auerbach et al.,[7] a determination of the K_2^0 mean life[8]

$$\tau_{K_2^0} = (5.15 \pm 0.14)\cdot 10^{-8}\ \text{sec}$$

and of the ratio[9]

$$\frac{\Gamma(K_L^0 \to \pi^0\pi^0\pi^0)}{\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)} = 1.69 \pm 0.12$$

allow us to improve the provision of the branching ratio (Table I); with these new rates we can remake the usual tests. We then use the classical space phase factors:

$$\Phi_{++-} = 1.000,$$
$$\Phi_{+00} = 1.248,$$
$$\Phi_{+-0} = 1.225,$$
$$\Phi_{000} = 1.495.$$

Table I. $K \to 3\pi$ decays

	Mean Life 10^{-8} sec^{-1}	Width: Absolute Value 10^6 sec^{-1}	Width: % of total
K^+	1.236±0.003	80.9±0.2	100
$K^+ \to \pi^+\pi^+\pi^-$		4.50±0.02	5.57±0.03
$K^+ \to \pi^+\pi^0\pi^0$		1.38±0.05	1.71±0.07
K^0	5.37±0.12	18.6±0.4	100
$K^0 \to \pi^0\pi^0\pi^0$		4.09±0.19	22.0±1.0
$K^0 \to \pi^+\pi^-\pi^0$		2.34±0.07	12.6±0.3

To understand better the evolution of this test, we have reported the 1965 values in the tables.

Table II. $\Delta I = 5/2$ test

	Trilling [2]	Willis [4]
$\frac{\gamma(++-)}{4\gamma(++00)}$	1.03±0.04	1.01±0.05
$\frac{\gamma(000)}{\frac{3}{2}\gamma(+-0)}$	1.07±0.13	0.96±0.06

The rate comparison between the K^+ themselves and K^0 themselves allow to verify the absence of $\Delta I = 5/2$. If the symmetric final state I = 3 exists then:*

$$\frac{\gamma(\pi^+\pi^+\pi^-)}{4\gamma(\pi^+\pi^0\pi^0)} \neq 1 \text{ and } \frac{\gamma(\pi^0\pi^0\pi^0)}{3/2\gamma(\pi^+\pi^-\pi^0)} \neq 1.$$

As the test (Table II) is satisfactory, we can try to check the $\Delta I = 1/2$ comparing the K^0 and the K^+:

$$\frac{\gamma(\pi^+\pi^-\pi^0)}{2\gamma(\pi^+\pi^0\pi^0)} = 1, \quad \frac{\gamma(\pi^0\pi^0\pi^0)}{\gamma(\pi^+\pi^+\pi^-) - \gamma(\pi^+\pi^0\pi^0)} = 1.$$

The two ratios are not completely independent. This (Table III) seems to indicate a possible violation of the $\Delta I = 1/2$ rule.

Table III. $\Delta I = 3/2$ test

	Trilling [2]	Willis [4]
$\frac{\gamma(+-0)}{2\gamma(+00)}$	0.89±0.07	0.86±0.05
$\frac{\gamma(000)}{\gamma(++-)-\gamma(+00)}$	0.91±0.13	0.81±0.04

*Detailed experimental data on $K \to 3\pi$ decays are given in the tables compiled by V. V. Anisovich (p. 000).

*γ is the reduced decay width, obtained dividing the width by the corresponding correction to the phase volume.

3. We can look for a confirmation comparing the energy spectra. It is well known that the simplified matrix element can be written

$$|M| = 1 - \frac{a}{m_\pi^2}(S_3 - S_0) = 1 - aY,$$

where

$$S_i = (p_K - p_{\pi_i})^2, \quad S_0 = 1/2\,(S_1 + S_2 + S_3), \quad a = \text{const};$$

The spectrum is then the product of the space phase by the square of the matrix element. We suppose a linear approximation

$$|M|^2 = 1 - \frac{2a}{m_\pi^2}(S_3 - S_0).$$

As for the ratios, we have the possibility to test the absence of $\Delta I = 5/2$:

$$a(+00) = -2a\,(++-),$$

and to test the absence of $\Delta I = 3/2$

$$a(+-0) = a(+00).$$

Table IV shows the actual state; there are new values for $K^+ \to \pi^+\pi^+\pi^-$ and for $K^0 \to \pi^+\pi^-\pi^0$ but nothing new for the $K^+ \to \pi^+\pi^0\pi^0$. The uncertainties in the ratios have been purposely increased to take into account the systematic effect. Some of these results have not yet been published. There is no violation evidence in the spectra. We have supposed a linear development of the matrix element in $(S_3 - S_0)$. The presence of a quadratic term has not been demonstrated. The experimental measure is in $K^+ \to \pi^+\pi^+\pi^-$ and it gives for b from $|M|^2 = 1 - 2aY + bY^2$, the values[11]

$$b = -0.068 \pm 0.058,$$
$$b = +0.05 \pm 0.07.$$

The only indication of a possible $\Delta I = 1/2$ violation remains in the rate $K^0 \to \pi^+\pi^-\pi^0$ and $K^+ \to \pi^+\pi^0\pi^0$ comparisons, but it seems to me that experimental work is still necessary on this subject. In particular, as W. J. Willis mentioned at the Heidelberg Conference, a slight modification of the mean life of K_L^0 would be sufficient to make everything normal again.

4. Current algebra. Now I will discuss the prediction of Callan and Treiman[12] concerning the extrapolation of the $K \to 3\pi$ spectra to $(E_\pi = 0)$. This prediction comes from PCAC and the current algebra. Hara and Nambu have deduced the relation between the $K \to 3\pi$ amplitudes and $K \to 2\pi$ amplitudes

Table IV. $K \to 3\pi$ spectra

	$a(++-)$	$a(+00)$	$a(+-0)$
Trilling (1965)	0,093±0.011	−0,25±0,02	−0,24±0.02
New	0,095±0,015 [17] 0,102±0,015 (Rutgers) (in press)		−0.21±0,02 [10] −0,20±0.04 [9] −0.18±0.02 [11]
Mean value (1968)	0.096±0.007	−0,25±0.02	−0.21±0,015
	Predicted if $\Delta T = 1/2$	1965	1968
$\frac{a(+-0)}{2a(++-)}$	1	1.29±0,25	1,09±0.13
$\frac{a(+00)}{2a(++-)}$	−1	−1.34±0,24	−1.30±0,19
$\frac{a(+-0)}{a(+00)}$	1	0.96±0.15	0.84±0,12

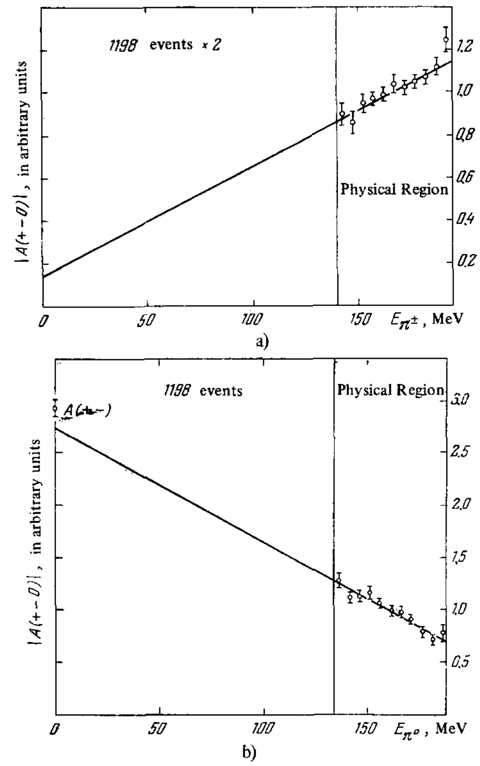


FIG. 1. Spectra extrapolation to $E_\pi = 0$ for test of current algebra; result of the Illinois group. Phys. Rev. 157 (1967) 1233. $K_2^0 \to \pi^+\pi^-\pi^0$.

$$\lim_{q_{\pi^-}\to 0} A(++-) = 0, \quad \lim_{q_{\pi^+}\to 0} A(++-) = \frac{1}{2f_\pi} A(+-),$$
$$\lim_{q_{\pi^0}\to 0} A(+00) = 0, \quad \lim_{q_{\pi^+}\to 0} A(+00) = \frac{1}{2f_\pi} A(00),$$
$$\lim_{q_{\pi^0}\to 0} A(000) = \frac{1}{2f_\pi} A(00),$$
$$\lim_{q_{\pi^\pm}\to 0} A(+-0) = 0, \quad \lim_{q_{\pi^0}\to 0} A(+-0) = \frac{1}{2f_\pi} A(+-).$$

Figures 1 and 2 represent the spectra of the K^0 experiments at Illinois and Saclay; Figures 3 and 4 the extrapolation for K^+ given by Nefkens.[13] All these linear extrapolations, even though they are not justified, seem to correspond with the predictions.

For $K^+ \to \pi^+\pi^0\pi^0$ (Fig. 4), the π^0 line is obtained with the hypothesis $a^0 = -1/2\,a_+$. There is no experimental spectra for $\pi^0\pi^0\pi^0$; supposing it is flat and using the rate, Callan and Treiman found also a good agreement for the $3\pi^0$ mode.

Bouchiat and Meyer,[14] using current algebra, which seems to be so effective for the slopes, have given the predictions to connect $K \to 3\pi$ and $K \to 2\pi$ rates:

$$\frac{\gamma_2(+-0)}{2\gamma(+00)} - 1 = \frac{\gamma_2(000)}{\gamma(++-) - \gamma(+00)} - 1 = \frac{2\gamma(00)}{\gamma(+-)} - 1.$$

The experimental status of $K \to 2\pi$ is not clear enough* to conclude this test which allows us to connect the $\Delta I = 1/2$ deviation in $K(3\pi)$ and $K(2\pi)$.

*Available data $\frac{\Gamma(oo)}{\text{all}}$ = 0.335 ± 0.014 (Brown PR **130** 769)
0.288 ± 0.021 (Chrétien PR **131** 2208)
0.260 ± 0.023 (Anderson CERN Conference 1962).

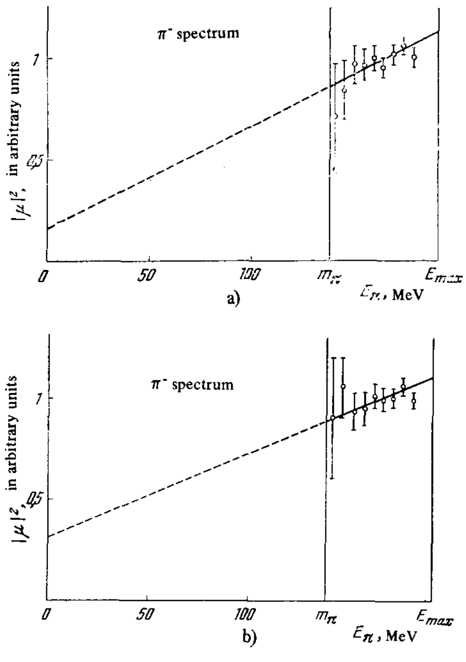


FIG. 2. Same in Fig 1 for Saclay Group.

II. LEPTONIC DECAYS

I will not talk about the $K^0 \to \pi^+ l^- \nu / \bar{K}^0 \to \pi^- l^+ \nu$ asymmetry observed in the Columbia and Stanford experiments, which are shown elsewhere. We shall first consider the selection rule and secondly the form factor determination.

1. The first question about leptonic decay concerns certainly the presence of a $\Delta S = -\Delta Q$ amplitude. There is nothing new on this subject. We call X the ratio between the two amplitudes

$$x = \frac{g}{f} = \frac{A\,(\Delta S' = -\Delta Q)}{A\,(\Delta S' = +\Delta Q)} = |x|\, e^{i\varphi}.$$

g and f are usually defined as follows

$$A\,(K^0 \to \pi^- e^+ \nu) = f,$$
$$A\,(\bar{K}^0 \to \pi^- e^+ \nu) = g.$$

If we suppose CPT, the time distribution of leptonic

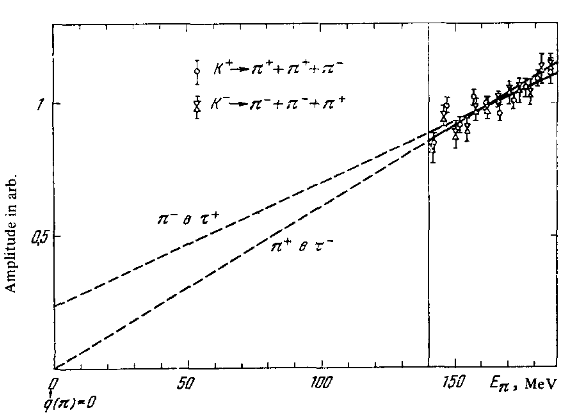


FIG. 3. Same for $K^+ \to \pi^+\pi^+\pi^-$ following Nefkens [13]

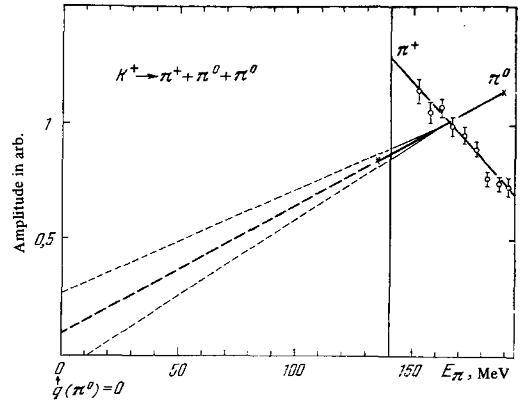


FIG. 4. Same for $K^+ \to \pi^+\pi^0\pi^0$ following Nefkens [13]

decays can be written

$$N^{\pm}(t) = (1+x)^2 e^{-\gamma_1 t} + (1-x)^2 e^{-\gamma_2 t} \pm 2\,(1-x^2) \cos \Delta m t\, e^{-\frac{\gamma_1+\gamma_2}{2} t} + 4x \sin \varphi \sin \Delta m t\, e^{-\frac{\gamma_1+\gamma_2}{2} t};$$

X sin φ or Im X is the CP violating part.

The status is shown in Fig. 5 as presented by W. J. Willis at the Heidelberg Conference. But the Im x sign is determined by the Δm sign. Due to the last determination of the Δm sign, $(M_{K_S} - M_{K_L})$ is negative. To be consistent, we have to reverse the Im x axis or make Im x negative.

Let me remind you that if the upper limit at 90% confidence limit is 0.5 in K^0 decay, it is 0.13 for Σ decay and 0.16 for K_{e4} decay. At present some experiments are in progress and promise us some thousands of K^0_{e3} events in the first K_1^0 mean life.

2. $\Delta I = 1/2$ leptonic. As for $K \to 3\pi$, we can test $\Delta I = 1/2$ comparing the K^0 and K^+ leptonic decay.

The greatest difficulties arise when choosing among experimental results. In UCRL-8030[5] you can find all kinds of absolute and relative measured rates and if, for example, you look at the $K^+ \to \pi^0 e^+ \nu / \tau$ rate you have results from 0.50 ± 0.03 to 0.90 ± 0.16. So the test of $\Delta I = 1/2$ depends strongly on the result that you accept or reject.

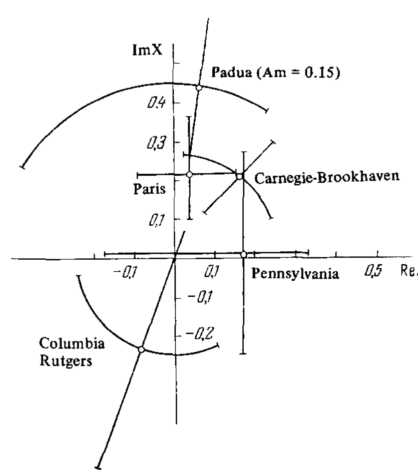


FIG. 5. Experimental results for $x = \frac{\Delta S = -\Delta Q}{\Delta S = +\Delta Q}$.

Table V. $\Delta T = 1/2$ leptonic

	Trilling [2]	Willis [4]
$\Gamma(K^0 \to \pi/\nu)/\Gamma(K^+ \to \pi/\nu)$	1.06 ± 0.06	0.91 ± 0.04
$\dfrac{\Gamma(K^0 \to \pi\mu\nu)}{\Gamma(K^0 \to \pi l\nu)} \Big/ \dfrac{\Gamma(K^+ \to \pi\mu\nu)}{\Gamma(K^+ \to \pi l\nu)}$	1.07 ± 0.14	1.14 ± 0.09

Table V shows the compilation given by Willis at Heidelberg, which is not too bad.

It is of course trivial to point out that if the $\Delta S = -\Delta Q$ amplitude exists the $\Delta I = 1/2$ selection rule is not to be considered.

3. Structure of decay amplitude. In the $K \to \pi l \nu$ decay, the form of the covariant general amplitude is, supposing a pure vector interaction:

$$M = \frac{G}{\sqrt{2}} [f_+(p_K + p_\pi) + f_-(p_K - p_\pi)] J^l,$$

where p_K and p_π are the K and π quadrimoments, J^l the leptonic current, G the universal weak constant, f_+ and f_- are the form factor functions of the momentum transfer.

$$q = (p_K - p_\pi)$$

The f_- terms lead to a factor proportional to m_l; so we can use the K_{e3} decay to study the f_+ and, assuming the μ-e universality, use this result to evaluate in $K_{\mu 3}$ decay: $\varepsilon = f_-/f_+$.

4. K_{e3} decay. Table VI shows the results of f_+. These results are presented in the form of a λ_+ parameter and with a linear dependence in q^2 of f_+:

$$f_+(q^2) = f_+(0)(1 + \lambda_+ q^2/m_\pi^2).$$

The most precise K^0 experiment is that of Basile et al.[17] This experiment, with 7000 electronic decays, gives:

$$\lambda_+ = 0.022 \pm 0.012.$$

In the error evaluation, any possible systematic effect has been included by the authors.

Table VI. Form factor of K_{e3} decay (after [5] and [17])

	Experiment	Technique	Number of cases	λ_+
K^0	Luers	HBC	153	0.07 ± 0.06
	Fisher	SpC		0.15 ± 0.08
	Firestone	HBC	762	-0.01 ± 0.02
	Lowys	FBC	240	$+0.08 \pm 0.10$
	Kadyk	HBC	531	$+0.01 \pm 0.15$
	Basile [17]	SpC	7000	0.022 ± 0.012
K^+	Brown	XeBC	217	0.038 ± 0.045
	Borreani	HBC	230	-0.04 ± 0.05
	Jensen	XeBC	407	-0.01 ± 0.029
	Bellotti	FBC	953	0.045 ± 0.018
	Imlay	SpC	1393	$+0.016 \pm 0.016$
	Kalmus	FBC	515	$+0.028 \pm 0.013$
	Average	$(\lambda_+)_{K^0} = 0.013 \pm 0.009$ $(\lambda_+)_{K^+} = 0.023 \pm 0.008$		$\lambda_+ = 0.02 \pm 0.005$

Technique symbols: HBC – hydrogen bubble chamber, SpC – spark chamber, FBC – freon bubble chamber, XeBC – xenon bubble chamber

Table VII. μ^+ polarization (Component normal to the plane $\mathbf{p}_\pi \times \mathbf{p}_\mu$)

	Experiment	Technique*	P_N	Im ξ
K^+	Callahan et al	FBC		0.8 ± 0.6 -0.9
	X_2 col.	FBC	-0.08 ± 0.10	$0.1 + 0.4$ $- 0.3$
K^0	Bartlett [19b]	Counters	0.02 ± 0.07	0.11 ± 0.35
	Abrams [19c]	SpC	-0.05 ± 0.18	0.1 ± 0.5
	Longo [19a]	Counts	0.002 ± 0.012	0.014 ± 0.066

*For symbols see Table VI.

The agreement between K^0 and K^+ is also very good. This encourages us to rely on the $\Delta I = 1/2$ rule previously discussed. The $K^0 K^+$ mean values of the λ_+ parameter will be taken

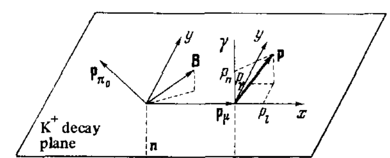
$$\lambda_+ = 0.02 \pm 0.006.$$

5. $K_{\mu 3}$ decay. Great efforts have been made on this disintegration in connection with T violation tests: as a matter of fact, the μ is completely polarized and the T invariance forbids a component normal to the disintegration plane (Fig. 6). One of the PC interpretations[18] predicts a normal component of the order of 20%, the electromagnetic interaction in the final state can lead to a component of the order of 1%.

FIG. 6. Polarization in the decay $K^0_{\mu 3}$. (γ – z axis)

$$\mu = \frac{p_\mu}{|p_\mu|}, \quad n = \frac{[p_\pi p_\mu]}{|[p_\pi p_\mu]|},$$

$$t = [\mu n].$$



The most precise experiment deals with K^0.[19] The layout of the experiment is shown in Fig. 7. The authors chose configurations where the decay plan is horizontal and they measured the component parallel to the magnetic field. Their result is

$$P_n = 0.02 \pm 0.012.$$

From this result Im ξ = 0.014 ± 0.066. Thus we can

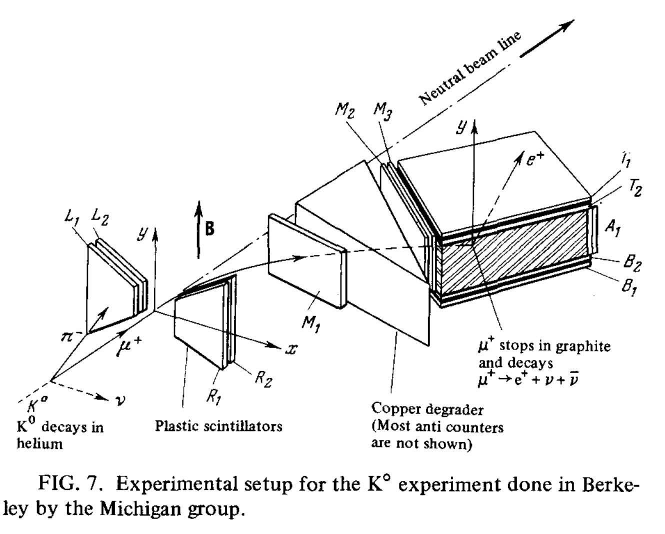


FIG. 7. Experimental setup for the K^0 experiment done in Berkeley by the Michigan group.

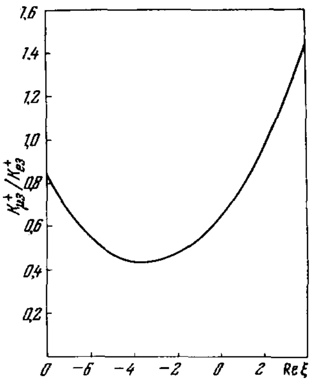


FIG. 8. $K_{\mu3}/K_{e3}$ versus Re ξ

consider ξ real. We have many possibilities to study it:

a) Study of the branching ratio. As we know from the K^0_{e3} decay, λ_+ is small and we can use the first terms of the development given by Cabibbo at the Berkeley Conference.[3]

$$K_{\mu3}/K_{e3}=0.648+0.126\,\mathrm{Re}\,\xi+0.019\xi^2+1.41\lambda_++0.47\lambda_-\mathrm{Re}\,\xi.$$

Figure 8 shows $K_{\mu3}/K_{e3}$ versus Re ξ; there are 2ξ values corresponding to a given branching ratio. The rule is to use either the spectra or the Dalitz plot to be able to choose between the two solutions.

b) Dalitz plot study of $K_{\mu3}$ decay. The density of the Dalitz plot is written:

$$\rho(\xi,\ E_\mu,\ E_\pi)=\frac{f_+^2}{2\pi^3M_K^2}\left[A(E_\pi,\ E_\mu)+B(E_\pi,\ E_\mu)\,\mathrm{Re}\,\xi+C(E_\pi)\,|\xi|^2\right].$$

Figure 9 shows the Dalitz diagram with the equal-density lines for different ξ values. When we superpose on this diagram a detection function $f(E_\pi,\ E_\mu)$ we realize the difficulties of such a measure.

The two Illinois and Saclay great experiments[20] have studied this Dalitz plot and their results are presented by means of a χ^2 curve, in Figs. 10 and 11. The first one gives two possible solutions −4 and +1.2 with a greater probability at 1.2; the second one gives about the same values but with the reverse probabilities.

When they cannot use the diagram's density, several groups have studied the E_μ projection or the angular correlations and their results are shown in Table VIII as for the branching ratio.

The results given in this table are taken from UCRL 8030,[5] published before the Heidelberg conference,

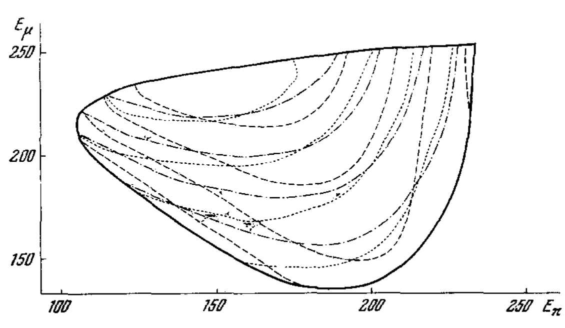


FIG. 9. Dalitz plot for the $K^0_{\mu3}$ decay: dotted lines are lines of equal density for ξ = 0, −2 and −4

Table VIII. Data on the ratio $\xi = f_-(0)/f_+(0)$ (from [5])

	Spectra and angular correlation		Branding ratio data	
K^+	Brown	1.8 ± 1.6	Shaklee	−0.17 ± 0.75
	Giacomelli	0.7 ± 0.5	Bisi	+0.6 ± 0.5
	Jensen	−0.1 ± 0.7	Callahan	+0.4 ± 0.4
	Callahan	0.72 ± 0.37	Auerbach	+0.75 ± 0.5
		0.0 ± 1.0		
			Garland	+1.3 ± 0.5
	Mean value ξ^+ = +0.6 ± 0.16			
K^0	Carpenter	1.2 ± 0.8	Adair	1.1 ± 0.9
	Kulyukina	−0.2 ± 1.0	Luers	0.66 ± 0.9
			De Bouard	0.9 ± 0.9
	Mean value ξ^0 = +0.86 ± 0.46			

and the results were:

$$K^+:\quad \xi=0.60\pm0.16,$$
$$K^0:\quad \xi=0.86\pm0.46.$$

We must not forget the actual problems involved in the measurement of the branching ratios, as previously reported. At the Heidelberg conference, two new branching ratio measurements were presented:

$$K^+:\quad K_{\mu3}/K_{e3}=0.65\pm0.05\ ^{21},$$
$$K^0:\quad K_{\mu3}/K_{e3}=0.71\pm0.07.$$

The K^+ experiment (which is the X_2 collaboration) should allow us to fairly improve the K_μ/K_e situation, and also help in choosing ξ.

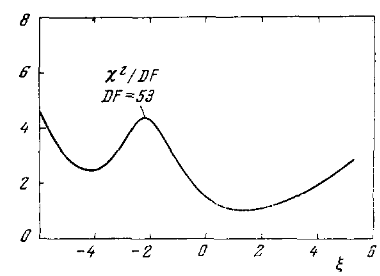


FIG. 10. Result of Dalitz plot analysis for the Illinois Group.

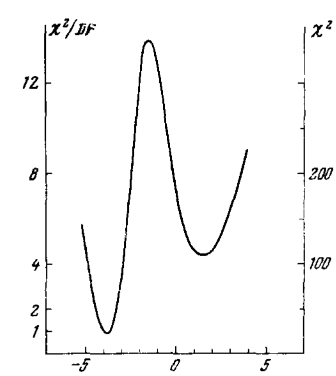


FIG. 11. Same for the Saclay group.

Taking into account the new branching ratios and the spectra result, the ξ mean value is therefore:

$$\xi=0.6\pm0.3. \qquad \text{(I)}$$

6. <u>Total Polarization.</u> The μ polarization is given by Cabibbo and Maksymowicz.[22] *

$$\mathbf{P}=\frac{\mathbf{A}}{|\mathbf{A}|},\quad \mathbf{A}=A(\xi,\ E_\pi,\ E_\mu)\,\mathbf{p}_\pi+B(\xi,\ E_\pi,\ E_\mu)\,\mathbf{p}_\mu+m_K\,\mathrm{Im}\,\xi\,[\mathbf{p}_\pi\mathbf{p}_\mu].$$

*$[\mathbf{p}\pi\ \mathbf{p}\mu] \equiv \mathbf{p}\pi \times \mathbf{p}\mu$

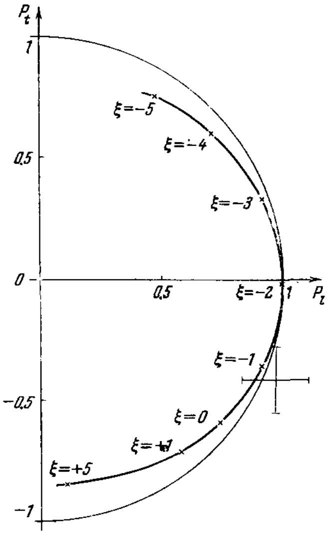


FIG. 12. Variation of the polarization with ξ.

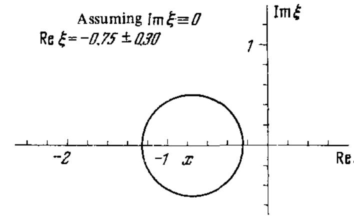


FIG. 13. Result of the polarization measurement for the X_2 collaboration.

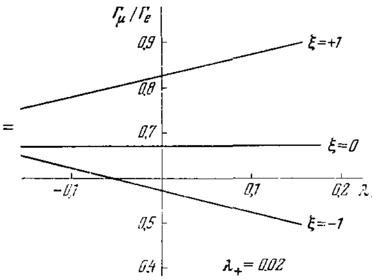


FIG. 14. Variation of $K^0_{\mu 3}/K^0_{e3}$ with λ_- from $f_-(q^2) = f_-(0)\ 1 + \lambda_- \frac{q^2}{m_\pi^2}$ for different values of ξ.

The last term shows the component normal to the decay plan as we have already discussed. Figure 12 shows the variation of ξ as a function of the polarization components in the decay plan. The measure of the transverse component is more sensible to ξ than the longitudinal component and this determination has an advantage: it depends weakly on the q^2 variation.

Table IX. Data on the ratio $\xi = f_-(0)/f_+(0)$ (determined from μ^+ polarization)

		P_T	$Re\,\xi$
K^0	Abrams [24]	-0.29 ± 0.29	-1.1 ± 0.5
	Auerbach [25]	-0.28 ± 0.12	-1.2 ± 0.5
K^+	X_2 [23]	-0.40 ± 0.12	-0.75 ± 0.3

Table IX shows the available results up to date. The better K^+ determination is in the X_2 experiment, realized by a European collaboration which studies 5×10^6 stopped K^+'s. More than 10 000 $K_{\mu 3}$ have been measured. Their result is shown in Fig. 13. If we accept the K^0 and K^+ mixture, the average will be

$$\xi = -1.0 \pm 0.2. \qquad \text{(II)}$$

To try to explain the difference between (I) and value (II) we can try to imagine a variation of the factor f_- in function of q^2. Figure 14 shows for different values the branching ratio variation in function of λ_-. It is clear that we need $\lambda_- \sim 0.4$ to obtain compatibility between (I) and (II). Such a value seems to be excluded by the X_2 collaboration.[16]

If we improve the determination of the branching ratio we could increase substantially the knowledge of ξ. And if really the difference between the two results is confirmed we ask the theoreticians to try to explain this.

I apologize for not having given you the experimental references which appeared in Trilling's report (Argonne Conference 1965) and in the Cabibbo's report (Berkeley Conference 1966).

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DISCUSSION

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A note concerning the verification of $\Delta Q = \Delta S$: an improvement of the accuracy with which the rule $\Delta Q = \Delta S$ is checked is needed for two "practical" reasons:

1) To interpret the experiments on lepton asymmetry in K^0-meson decays. In this experiment it is necessary to know the value of $|1 - x|^2/[1 - |x|^2]$. For small x, this quantity is determined by Re x:

$$\frac{|1-x|^2}{1-|x|^2} \approx 1-2\,\mathrm{Re}\,x+(\text{terms} \propto x^2).$$

2) For the connection of the possible CP-noninvariant amplitudes in the unitarity condition. The largest term is a CP-odd lepton decay with $\Delta Q = -\Delta S$. For small x, its contribution is determined by Im x.

At the present time, the following is known concerning the quantity x:

$$|\mathrm{Im}\,x| \leqslant 0.4 \quad |\mathrm{Re}\,x| \leqslant 0.2,$$

for example, from Willis's paper at the Heidelberg Conference.

We can propose a regeneration experiment for the measurement of Im x with the aid of the "null measurement" technique. We can expect a 10- or even 100-fold increase of the sensitivity compared with the world data on this question.

We define the following amplitudes:

$$\begin{aligned}
\Delta Q = \Delta S \quad &\text{for} \quad K^0 \longrightarrow \pi^- e^+ \nu: \ -1,\\
&\phantom{\text{for}} \quad \overline{K}^0 \longrightarrow \pi^+ e^- \bar{\nu}: \ -1,\\
\Delta Q = -\Delta S \quad &\text{for} \quad K^0 \rightarrow \pi^+ e^- \bar{\nu}: -x^*,\\
&\phantom{\text{for}} \quad \overline{K}^0 \rightarrow \pi^- e^+ \nu: -x.
\end{aligned}$$

The state $|t\rangle$ can be expanded in the eigenstates of the CP operator $|K_1\rangle$ and $|K_2\rangle$ as follows:

$$|t\rangle = |K_2\rangle + \rho\,|K_1\rangle = \frac{1+\rho}{\sqrt{2}}|K^0\rangle - \frac{1-\rho}{\sqrt{2}}|\overline{K}^0\rangle,$$

where ρ is the measure of the (CP = +1) admixture in the long-lived state; then (the signs + and − pertain to the lepton charge)

$$A^+ \sim (1+x)\,\rho + (1-x),$$
$$A^- \sim (1+x^*)\,\rho + (1-x^*),$$

and consequently we get for the decay probabilities

$$N^+ = |1-x|^2 + |1+x|^2\,|\rho|^2 + 2\,\mathrm{Re}\,[\rho\,(1+x)\,(1-x^*)],$$
$$N^- = |1-x|^2 + |1+x|^2\,|\rho|^2 - 2\,\mathrm{Re}\,[\rho\,(1-x)\,(1+x^*)].$$

After passing through a thick regenerator $\varepsilon \ll \rho$, and consequently

$$\rho = \rho_0 \exp\left\{ i\left(\Delta m + \frac{i\Gamma_{S'}}{2}\right) t + i\varphi_\rho \right\}.$$

As a result we get

$$N^\pm = |1-x|^2 + |1+x|^2 e^{-\Gamma_S t}|\rho_0|^2 \pm \{2\,(1-|x|^2)\cos(\Delta m t+\varphi_\rho)\}\,|\rho_0|\,e^{-\Gamma_S t}/2$$
$$-4\,\mathrm{Im}\,x\cdot\sin(\Delta m t+\varphi_\rho)\,|\rho_0|\,e^{-\Gamma_S t}/2.$$

The numbers of the decays $N_+ + N^-$ and $N^+ - N^-$ are expressed in the following manner ($\rho \ll 1$):

$$N^+ + N^- \sim 2\,|1-x|^2 + 8\,\mathrm{Im}\,x\cdot\sin(\Delta m t+\varphi_\rho)\,|\rho_0|\,e^{-\Gamma_S t/2},$$
$$N^+ - N^- \sim 4\,(1-|x|^2)\cos(\Delta m t+\varphi_\rho)\,|\rho_0|\,e^{-\Gamma_S t/2}.$$

It is proposed to compare these decays with the decays in the absence of a regenerator, normalized in such a way as to give the same number of decays when $t \gg 1/\Gamma_S$. The number of decays in the absence of a regenerator is

$$N_0^+ + N_0^- \sim 2\,|1-x|^2,$$
$$N_0^+ - N_0^- \sim 4\,\mathrm{Re}\,\varepsilon\,(1-|x|^2).$$

The quantity of interest to us, in the case of the contribution of the amplitude with $\Delta Q = -\Delta S$, is obviously equal to

$$\chi = \frac{N^+ + N^-}{N_0^+ + N_0^-} = 1 + 4\,\mathrm{Im}\,x\cdot\sin(\Delta m t+\varphi_\rho)\,e^{-\Gamma_S t/2}\,|\rho_0|.$$

In practice $\rho_0 \approx 0.07$ and $\sin(\Delta mt + \varphi_\rho) \approx 0.71$ at $t = 0$. We expect at $t \approx 0$

$$(\chi - 1) \approx 0.2\,\mathrm{Im}\,x.$$

Recognizing that $\mathrm{Im}\,x \leq 0.4$ we get $(\chi - 1) \lesssim 0.08$, which is a large number.

If we can attain an accuracy of $\sim 10^{-2}$ in the measurement of χ, then we can impose on the value of Im x a limitation which is approximately ten times more accurate than presently known (Im x < 0.05!).

It is necessary only to know approximately the regeneration amplitude, if $\mathrm{Im}\,x \approx 0$. No difficulties whatever arise as a result of the absorption, except that the effect decreases somewhat.

The magnitude of the expected effect can be related directly to the difference $N^+ - N^0$, which behaves like

$$\cos(\Delta m t+\varphi_\rho)\,|\rho|\,e^{-\Gamma_S t/2},$$

since $\left|1 - |x|^2\right| \approx 1$ if $|x|$ is small.

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