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PHENOMENOLOGY OF CP VIOLATION

L. WOLFENSTEIN

Carnegie-Mellon University, Pittsburgh, Pennsylvania

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I. PHENOMENOLOGICAL PARAMETERS

We review first the standard formulation[1] of the phenomenological treatment of CP violation in the decay of the long-lived neutral K meson K_L into two pions. The two observable complex amplitude ratios are

$$\eta_{+-}=\frac{\text{Amplitude } (K_L\rightarrow\pi^+\pi^-)}{\text{Amplitude } (K_S\rightarrow\pi^+\pi^-)},$$

$$\eta_{00}=\frac{\text{Amplitude } (K_L\rightarrow\pi^0\pi^0)}{\text{Amplitude } (K_S\rightarrow\pi^0\pi^0)}.$$

These are expressed in terms of the complex parameters ϵ and ϵ'

$$\eta_{+-}=\varepsilon+\varepsilon', \tag{1a}$$

$$\eta_{00}=\varepsilon-2\varepsilon', \tag{1b}$$

where ϵ corresponds to K_L going to the $I = 0$ final state and ϵ' to the $I = 2$ final state. The significance of this parametrization lies in the fact that:

(1) $2\,\mathrm{Re}\,\epsilon = \langle K_L | K_S\rangle$ can be measured independently since it determines the relative amounts of K and $\overline{K}$ in the K_L and K_S state vectors.

(2) The phase of ϵ' is determined in terms of π-π scattering phase shifts to be $(\pi/2 + \delta_2 - \delta_0) + n\pi$.

(3) The phase of ϵ can be expressed in terms of the mass difference $\delta \equiv (m_L - m_S)$ and the amount of CP violation in other than 2π modes. In particular, if the decay of a beam which is pure K° at $t = 0$ has the time dependence for decay into channel a

$$I_a(t)=A_ae^{-\gamma_Lt}+B_ae^{-\gamma_St}+e^{-\frac{1}{2}(\gamma_L+\gamma_S)t}(C_a\cos\delta t+D_a\sin\delta t), \tag{2}$$

then

$$\mathrm{Im}\,\varepsilon\simeq(2\delta/\gamma_S)\,\mathrm{Re}\,\varepsilon-(\gamma_{La}/2\gamma_S)\sum_a D_a/A_a, \tag{3}$$

where the sum goes over channels other than 2π. The only terms that might contribute significantly to the sum are leptonic decays which violate the $\Delta Q = \Delta S$ rule and $K^\circ \rightarrow 3\pi$ decays. Unless these have a large amount of CP violation

$$\frac{\mathrm{Im}\,\varepsilon}{\mathrm{Re}\,\varepsilon}\simeq\frac{2\delta}{\gamma_S} \tag{4}$$

and the phase of ϵ is approximately 45°.

This analysis depends on the basic assumption of CPT invariance and the following approximations:

(1) Neglect of the CP-conserving decay of K_S into the $I = 2$ final state. A number of recent analyses do not make this approximation;[2,3] there is then the possibility that there may be a large violation of the $\Delta I = \frac{1}{2}$ rule in $K_S \rightarrow 2\pi$ so that the result $\mathrm{Rate}(K_S \rightarrow \pi^+\pi^-)/\mathrm{Rate}(K_S \rightarrow \pi^0\pi^0) = 2$ is accidental. With accurate measurements of η_{+-} and η_{00} it may be possible to check the $\Delta I = \frac{1}{2}$ rule.[3]

(2) Neglect of second-order terms in ϵ and ϵ'. This is a good approximation.

The only other reported experiment in which CP violation was observed measures the ratio

$$R=\frac{\text{Rate}(K_L\rightarrow\pi^-+l^++\nu)}{\text{Rate}(K_L\rightarrow\pi^++l^-+\bar{\nu})},$$

where l is a muon or electron.

The ratio R is related to Re ϵ by

$$R=1+4\,\mathrm{Re}\,\varepsilon\left(\frac{1-x^2}{1-2x\cos\varphi+x^2}\right), \tag{5}$$

where

$$xe^{i\varphi}=\frac{\text{Amplitude }(\overline{K}^0\rightarrow e^++\pi^-+\nu)}{\text{Amplitude }(K^0\rightarrow e^++\pi^-+\nu)}$$

is a measure of the violation of the $\Delta Q = \Delta S$ rule. The term in brackets in Eq. (5) is directly measurable from the time distribution of the decay of a pure K° beam into the mode $\pi^- + l^+ + \nu$ being given to a good approximation by $\frac{1}{2}(C/A)$, where A and C are defined in Eq. (2).

Reported experimental results are summarized in Table I. If we use only the result for $|\eta_{00}/\eta_{+-}|$ (liberally stretching the experimental errors) then for various values of Φ_{+-} we find the results[4] shown in Table II where ϵ and ϵ' are expressed in terms of $|\eta_{+-}|$ and phases are measured relative to Φ_ϵ. There are actually two solutions, the second of which corresponds to values of $|\epsilon|$ close to zero, which appear to be ruled out by the values of $(R - 1)$, according to Eq. (5). We shall return to the second solution shortly.

To go further, we wish to use the limited experimental evidence on Φ_{r-}, $R - 1$, and $(\delta_2 - \delta_0)$. Stretching errors and combining estimates it seems likely that[5]

$$\left.\begin{aligned}\Phi_{+-}&=65\pm20^\circ,\\ R-1&=(5{,}0\pm1{,}5)\cdot10^{-3},\\ \delta_2-\delta_0&=-45\pm20^\circ.\end{aligned}\right\} \tag{6}$$

To use this data we must make assumptions concerning $\Delta Q = -\Delta S$ decays as well as the 3π contribution to Eq. (3). We consider two cases:

(A) No $\Delta Q = -\Delta S$. No large CP violation in 3π decays. Then we can use Eq. (4) and $\Phi_\epsilon \cong 43°$ for Solution (1) with an uncertainty of the order of 5° due to the uncertainty in δ as well as the omitted $I = 2$ contribution to Eq. (3). Eq. (5) now reads $R - 1 = 4\,\mathrm{Re}\,\epsilon$ and the small $|\epsilon|$ solution is ruled out. From the values of 4 Re ϵ given in Table II we find a solution that gives

$$\begin{aligned}&|\eta_{00}|/|\eta_{+-}|\simeq2,\\ &\Phi_{+-}\simeq\Phi_\varepsilon+45^\circ\simeq90^\circ,\\ &R-1\simeq5{,}3\cdot10^{-3},\\ &|\varepsilon|/|\varepsilon'|\simeq1{,}25,\\ &\delta_2-\delta_0\simeq-120\ (\text{or } 60^\circ).\end{aligned}$$

This is essentially the solution presented by the Columbia group[6] which, however, gives what appears to be an unreasonable value of $(\delta_2 - \delta_0)$. An alterna-

Table I. Experimental Values

Parameter	Value	Reference	Parameter	Value	Reference
$\|\eta_{+-}\|$	$(1.91\pm0.06)\cdot10^{-3}$	a	$R-1$	$(8.1\mp2.7)\cdot10^{-3}$	g
$\|\eta_{00}\|$	$(3.92\pm0.30)\cdot10^{-3}$	b		$(4.5\pm0.7)\cdot10^{-3}$	h
Φ_{+-}	$60°\pm17°$	c	$2\delta/\gamma_S$	0.88 ± 0.06	f
	$70°\pm21°$	d		0.96 ± 0.05	d
	$26°\pm26°$	e		0.88 ± 0.08	a
	$81°\pm20°$	f			
	$35°\mp$?	a			

a. Fitch et al, preprint. b. Values given by J. Cronin, Princeton Conference on Decays of K Meson, November, 1967. The value of $|\eta_{00}|$ is in complete agreement with the published CERN-Rutherford results. The quoted error on $|\eta_{00}|$ is almost certainly too small. c. Rubbia, Heidelberg Conference. d. Bott-Bodenhausen et al, Physics Letters **24B**, 438 (1967). e. Mischke et al, Phys. Rev. Lett. **18**, 138 (1967). f. Rubbia, C. and J. Steinberger, Phys. Lett. **24B**, 531 (1967). g. Dorfan et al, Phys. Rev. Lett. **19**, 987 (1967). h. Bennett et al, Phys. Rev. Lett. **19**, 993 (9167).

tive that fits less well gives

$$|\eta_{00}/\eta_{+-}|\cong1.6,\quad |\varepsilon|/|\varepsilon'|\cong6,$$
$$\Phi_{+-}\simeq\Phi_\varepsilon\cong45°,\quad \delta_2-\delta_0\cong50°.$$
$$R-1\cong6{,}7\cdot10^{-3},$$

We may note in passing that Solution (1) would satisfy Eq. (6) almost perfectly if Φ_ϵ were to equal 60° instead of 43°.[7] With no $\Delta Q=-\Delta S$ this would require practically maximal CP violation in $K^\circ\to3\pi$.

(B) We take seriously present experimental indications of $\Delta Q=-\Delta S$ decays. The most detailed experiment gives the results[8]

$$x\cos\varphi=0.2\pm0.08,\quad x\sin\varphi=-0.24\pm0.10.$$

If we substitute these results into Eq. (5) using the value of $(R-1)$ in (6) we find that 4 Re ϵ may range downward from the previous value $5.0\pm1.5\times10^{-3}$ to $3.0\pm0.9\times10^{-3}$. If we now include the effect of $\Delta Q=-\Delta S$ decays in Eq. (3) still ignoring CP violation in $K\to3\pi$ decays

$$\operatorname{Im}\varepsilon\cong(2\delta/\gamma_S)\operatorname{Re}\varepsilon+\frac{\gamma_L(\text{leptonic})}{\gamma_S}\frac{2x\sin\varphi}{1-2x\cos\varphi+x^2}\tag{7}$$

and Φ_ϵ for Solution (1) goes down from 43° to between 25° and 35°. These changes make Solution (1) somewhat less satisfactory.

On the other hand solutions of the second kind become possible. (Actually the solutions form a continuum for $|\eta_{00}|\neq2|\eta_{+-}|$). We give parameters for one such solution for the purpose of illustrating the importance of $\Delta Q=-\Delta S$ decays:

$$x\sin\varphi=-0{,}24,\quad |\varepsilon|/|\eta_{+-}|\cong0{,}4,$$
$$x\cos\varphi\cong0.28,\quad |\varepsilon'|/|\varepsilon|\cong2.0,$$
$$\Phi_\varepsilon\cong-10°,\quad \delta_2-\delta_0\cong-25°,$$
$$\Phi_{+-}\cong45°,\quad R-1\cong4.5\cdot10^{-3}.$$
$$|\eta_{00}|/|\eta_{+-}|\cong1.6.$$

Table II. Solution 1 $|\eta_{00}/\eta_{+-}|=2.0\pm0.4$

$\Phi_{+-}-\Phi_\varepsilon$	$\|\varepsilon\|/\|\eta_{+-}\|$	$\|\varepsilon'\|/\|\eta_{+-}\|$	$\delta_2-\delta_0-\Phi_\varepsilon$	$4\operatorname{Re}\epsilon\times10^3$ (if $\Phi_\epsilon=43°$)
$-15°$	1.29 ± 0.15	0.41 ± 0.12	$-52\pm12°$	7.3 ± 0.9
0	1.33 ± 0.14	0.33 ± 0.14	$-90°$	7.6 ± 1.2
$+15°$	1.29 ± 0.15	0.41 ± 0.12	$-128\mp12°$	7.3 ± 0.9
$+45°$	0.93 ± 0.2	0.74 ± 0.05	$-161\mp14°$	5.3 ± 1.1
$+60°$	0,67	0,88	$-180°$	3,8

Uncertainties indicated by (±) show roughly how values change as $|\eta_{00}/\eta_{+-}|$ deviates from 2.0.

Of course, the phase Φ_ϵ could also be significantly influenced by $I=2$ states and possible CP violation in $K_L\to3\pi$ decays for this case of small $|\epsilon|$.

II. HAMILTONIAN MODEL PARAMETERS

We now consider the relation between the phenomenological parameters and possible Hamiltonian models. We consider a Hamiltonian of the form

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}_1,$$

where $\mathcal{H}_0$ is the usual (strong + electromagnetic + weak) CP-invariant Hamiltonian and $\mathcal{H}_1$ violates CP invariance. We may then use $\mathcal{H}_0$ to define the CP transformation without any phase ambiguity and define

$$|\overline{K}^0\rangle=CP|K^0\rangle,$$
$$|K_+\rangle=(|K^0\rangle+|\overline{K}^0\rangle)/\sqrt{2},$$
$$|K_-\rangle=(|K^0\rangle-|\overline{K}^0\rangle)/\sqrt{2}.$$

Letting $|I\rangle$ represent the standing wave state of two pions with isospin I, we hope to calculate with the theory the transition amplitudes

$$\left.\begin{aligned}\langle0|T|K_+\rangle&=A_0,\\ \langle2|T|K_+\rangle&=\beta A_0,\\ \langle0|T|K_-\rangle&=i\alpha A_0,\\ \langle2|T|K_-\rangle&=i\alpha\chi A_0,\end{aligned}\right\}\tag{8}$$

where A_0, α, β and χ are real by CPT. In addition we must consider the self-energy matrix $M-i\Gamma/2$, which in the K_+-K_- representation we write

$$M-i\frac{\Gamma}{2}=\begin{pmatrix}m_+ & im'\\ -im' & m_-\end{pmatrix}-\frac{i}{2}\begin{pmatrix}\gamma_+ & i\gamma'\\ -i\gamma' & \gamma_-\end{pmatrix}.\tag{9}$$

We are thus left with three parameters describing CP violation α, χ, and m'. There are really only two phenomenological CP-violating parameters describing $K^0\to2\pi$; namely, $|\epsilon|$ and $|\epsilon'|$, since the phase of ϵ' is determined by $\delta_2-\delta_0$ and the phase of ϵ is determined by Eq. (3) or, in our approximation, Eq. (4). By direct calculation, including finding the states $|K_L\rangle$ and $|K_S\rangle$, we find

$$\varepsilon'=i\alpha\frac{\chi}{\sqrt{2}}e^{i(\delta_2-\delta_0)},\quad \varepsilon=i(\alpha\delta+m')/[\delta+(i\gamma_S/2)].\tag{10}$$

To the first order in CP violation the diagonal elements m_+, γ_+, m_-, and γ_- equal the physical quantities m_S, γ_S, m_L, and γ_L. The off-diagonal elements

m' and γ' are due to the CP-violating $\mathscr{H}_1$; the antisymmetry of the matrices follows from the CPT requirement that CP-violating terms be T-violating. The matrix element γ' is associated with real intermediate states in second-order perturbation theory and is thus directly related to CP-violation in physical processes. We now assume, as is the case for the models we shall discuss later, that it is a good approximation to neglect the contribution to γ' of CP violation in other than the $I = 0$ 2π intermediate state (this is equivalent to neglecting the second term in Eq. (3)):

Comparing these results with the discussion in Sec. I we find that we must rule out any model for which $\alpha = 0$, that is, models in which CP violation is only in the mass matrix and $\epsilon' = 0$. Such models that are ruled out include the superweak interaction, the Sachs model in which only $\Delta Q = -\Delta S$ leptonic decays violate CP, and models in which CP violation occurs only as a relative phase between the parity-conserving and parity-violating non-leptonic Hamiltonians.

All other Hamiltonian models that we know of have the feature that they can make no quantitative prediction concerning either α or m'. In general α is related to some arbitrary parameter in the model that determines the amount of CP violation while m' involves a dispersive sum over (virtual) intermediate states, which is too difficult to calculate. As discussed in the next section the models can often predict χ from the isotopic spin structure, but there remain two parameters m' and α which in general can be chosen to fit the experimental results for ϵ and ϵ'. In particular, if $\rho \equiv |\epsilon'|/|\epsilon|$ then we must have

$$\frac{m'}{\delta\alpha} = \frac{\chi}{\rho} - 1. \tag{11}$$

The only models we can rule out are those for which the required value of m' is unreasonable.

From the definition (9) we have in second-order perturbation theory

$$\frac{m'}{\delta} = \frac{m'}{m_- - m_+} = \frac{\sum\limits_n \langle K_-|\mathscr{H}'|n\rangle\langle n|\mathscr{H}'|K_+\rangle/\omega_n}{\Big[\sum\limits_{n\,CP\text{-odd}} \langle K_-|\mathscr{H}'|n\rangle\langle n|\mathscr{H}'|K_-\rangle - \sum\limits_{n\,CP\text{-even}} \langle K_+|\mathscr{H}'|n\rangle\langle n|\mathscr{H}'|K_+\rangle\Big]/\omega_n},$$

where $\mathscr{H}'$ is the weak interaction Hamiltonian and the sums are principal part integrals. Since the numerator involves one CP violation we expect $|m'/\delta|$ to be of the order of magnitude α provided the denominator does not have a large cancellation and provided the same states n can be reached by CP-conserving and CP-violating virtual transitions. Since ρ is of the order of unity from the discussion in Section I, any value of χ of the order of magnitude of unity may be expected to correspond to a reasonable value of m'.

Two special classes of models are of interest: (1) $\chi \gg 1$, so that the CP violation satisfies at least approximately a $\Delta I > \frac{1}{2}$ rule. For such a model, if we consider only the 2π intermediate state (with $I = 0$ or $I = 2$), the numerator in the expression for n' must be small since those states easily reached by CP-violating transitions from K_- cannot be easily reached by CP-conserving transitions from K_+. Then we would expect in this model that ρ is much larger than unity in order to satisfy Eq. (11), as has been discussed before in the model of Truong. However, if the $I = 1$ intermediate states are important and CP-violating $\Delta I = \frac{3}{2}$ transitions (in contrast to $\Delta I = \frac{5}{2}$) are included we cannot rule out this possibility. (2) $\chi \ll 1$ so that the CP-violating transitions satisfy the $\Delta I = \frac{1}{2}$ rule perhaps with the same accuracy as do the CP-conserving transitions. Such models cannot be ruled out even though there is a large apparent violation of the $\Delta I = \frac{1}{2}$ rule (since $|\epsilon'| \sim |\epsilon|$) in $K_L \to 2\pi$. All that is necessary from Eq. (11) is that $m' \simeq -\delta\alpha$ with a precision of the order of χ. Such an accurate equality was discussed long ago by Weinberg, who pointed out that in this case a large CP violating parameter α would lead to small CP-violating effects. There is, however, no reason why such an accurate equality should hold as has emphasized by Sachs and others.[9]

Thus we consider it a reasonable conclusion that χ should be of the order of magnitude of unity, although much smaller or larger values of χ cannot be excluded.

III. CURRENT-CURRENT MODELS AND $\Delta I = \frac{1}{2}$ RULE

We consider now specific models in which CP violation occurs in the weak Hamiltonian with the aim of understanding possible relations between CP violation and violation of the $\Delta I = \frac{1}{2}$ rule.[10]

One model due to Glashow[11] uses a current-current Hamiltonian of the standard Cabibbo form

$$\mathscr{H}_W = \frac{G}{\sqrt{2}} J_\mu J_\mu^+,$$
$$J_\mu = \cos\theta\,(J_\mu^1 + iJ_\mu^2) + \sin\theta\,(J_\mu^4 + iJ_\mu^5) + l_\mu,$$

but with a modification of the phase of the axial current

$$\left.\begin{aligned} J_\mu^i &= V_\mu^i - e^{-i\varphi}A_\mu^i, \qquad i = 1, 2, \\ J_\mu^i &= V_\mu^i - e^{-i\psi}A_\mu^i, \qquad i = 4, 5, \end{aligned}\right\} \tag{12}$$

where l_μ, V_μ^i, and A_μ^i are the standard lepton, hadronic vector, and hadronic axial vector currents, respectively. The superscripts i define components of an SU_3 octet. The product $J_\mu J_\mu^+$ really means one-half the anticommutator $(J_\mu J_\mu^+ + J_\mu^+ J_\mu.)$ The strangeness-changing parity-violating Hamiltonian involves the current-current combinations

$$L = (A_\mu' + iA_\mu^2)(V_\mu^4 - iV_\mu^5),$$
$$K = (V_\mu' + iV_\mu^2)(A_\mu^4 - iA_\mu^5),$$

or

$$M = L + K, \quad N = L - K.$$

Keeping only the lowest order in φ and ψ, the CP-conserving part of $\mathscr{H}_W$ has the standard form

$$\frac{G}{\sqrt{2}}\cos\theta\sin\theta M,$$

whereas the CP-violating part is

$$\frac{G}{2\sqrt{2}}\, i\cos\theta\sin\theta\,\{(\varphi - \psi)M + (\varphi + \psi)N\}. \tag{13}$$

To investigate the validity of the $\Delta I = \frac{1}{2}$ rule we can use the standard techniques of current algebra and PCAC.[12] Reducing both pions following the technique of Weinberg[13] one finds for the $K^0 \to 2\pi$ matrix element

$$\langle\pi^a, \pi^b|\mathscr{H}_W|K^0\rangle \propto \langle 0|[F^{5a}, [F^{5b}, \mathscr{H}_W]] + [F^{5b}, [F^{5a}, \mathscr{H}_W]]|K^0\rangle \tag{14}$$

to the zeroth order in the pion four-momenta where

$$F^{5a} = \int d^3x A_4^a(\mathbf{x}, 0).$$

The combination relations of F^5 with the currents in H_W assure that the double commutators in Eq. (14) will produce a new current-current Hamiltonian but one that now may also involve neutral currents and so may have a different isospin structure. We therefore define $M^{1/2}$ and $M^{3/2}$ which are normalized $\Delta I = 1/2$ and $\Delta I = 3/2$ current-current forms constructed by adjoining to M suitable neutral current terms; similarly $N^{1/2}$ and $N^{3/2}$. We then may write

$$[F^{5a}, [F^{5b}, M^I]] + [F^{5b}, [F^{5a}, M^I]] = \sum_{I'} m_{ab}^{II'} M^{I'} \qquad (15)$$

and similarly $n_{ab}^{II'}$; M and N are not mixed because of their symmetry character. If the weak Hamiltonian term involving M is decomposed into isospin parts

$$M = \alpha M^{\frac{1}{2}} + \beta M^{\frac{3}{2}},$$

then it follows from Eqs. (14) and (15) that

$$\langle \pi^a, \pi^b | M | K^0 \rangle = (\alpha m_{ab}^{\frac{1}{2}\frac{1}{2}} + \beta m_{ab}^{\frac{3}{2}\frac{1}{2}}) \langle 0 | M^{\frac{1}{2}} | K^0 \rangle.$$

The derivation of the $\Delta I = 1/2$ rule for the CP-conserving $\mathcal{H}_W$ follows, from the fact that $m_{ab}^{3/2\ 1/2} = 0$ so that $\beta M^{3/2}$ makes no contribution; the matrices m are diagonal since it is possible to replace F^5 by F in the double commutator. This is not true for n, however, so that the $\Delta I = 1/2$ rule does not hold for $\mathcal{H}_W$ if it involves N. A direct calculation for the term N yields

$$\chi = -\frac{8}{19\sqrt{2}}.$$

For the general Glashow Hamiltonian Eq. (13) *one gets*

$$\frac{1}{\chi} = -\frac{\sqrt{2}}{8}\left[19 + 3T\frac{\varphi - \psi}{\varphi + \psi}\right],$$

where

$$T = \frac{\langle 0 | M | K^0 \rangle}{\langle 0 | N | K^0 \rangle}.$$

An explicit calculation using the intermediate-vector boson method of Glashow, Schitzer, and Weinberg[14] gives $T \simeq -1/2$. For most non-zero values of $(\varphi + \psi)/(\varphi - \psi)$, χ is of the order-of-magnitude of unity as desired.

Another "current-current" model due to Zachariasen and Zweig[15] employs neutral scalar and tensor "currents" to form the CP-violating weak Hamiltonian. It can be shown[16] that in this model $\chi = -\sqrt{2}/4$, which is also satisfactory.

IV. DISCUSSION

The most crucial piece of evidence concerning the nature of CP violation remains the result $|\eta_{00}| \neq |\eta_{+-}|$. It is this which rules out the superweak interaction as well as a number of other models. We await the results of the variety of $K_L \to 2\pi^0$ experiments now being analyzed or carried out for the confirmation of this result.

There remain three classes of theories which are, following the terminology of Okun:[17]

a. Milliweak - a small CP violation in the $|\Delta S| = 1$ non-leptonic weak interaction Hamiltonian.

b. Millistrong - a small CP violation in the $\Delta S = 0$ parity-conserving "strong" Hamiltonian.

c. Electromagnetic CP violation

From the discussion in Section II we prefer theories in which χ *is not zero. This means the matrix elements of the milliweak Hamiltonian should violate the* $\Delta I = 1/2$ rule as in the models discussed in Section III. For millistrong theories this means the CP violating term should also violate isotopic spin invariance. One possibility would be $\Delta I = 1$ which would produce no CP violation in $\eta \to 3\pi$ decay, but would make a contribution to various isospin forbidden transitions as $d + d \to He^4 + \pi^0$. An alternative would be a mixture of $\Delta I = 0$ and $\Delta I = 2$. Electromagnetic CP violation might involve only $\Delta I = 0$ since the normal current provides a $\Delta I = 1$.

An important test for any model comes from the increasingly strong limits on the electric dipole moment of the neutron. The present limits are smaller than one would expect if there were a large violation in *electromagnetism; however, quantitative calculations* are very difficult to believe. Nevertheless, the millistrong and milliweak theories now seem to be the most likely.

We must hope, however, for a deeper understanding of CP violation than provided by a small CP-violating piece of the strong or weak Hamiltonian. Our picture of weak interactions is determined almost entirely by low-momentum transfer processes. The standard weak interaction Hamiltonian may only be a good approximation for these processes and the true Hamiltonian may contain a basic CP violation that shows up only in high-energy processes. Phenomenology is at best a classification scheme for organizing the experimental data while we await a satisfactory theory.

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DISCUSSION

R. N. Faustov

What can you say concerning the Nishijima model of CP-violation, in which the usual weak interaction occurs in second order, and the CP-odd interaction in third order?

L. Wolfenstein:

Nothing. I do not understand the model.

L. Frenkel:

Dr. Wolfenstein stated that Re ϵ can be very small if $\Delta Q \neq \Delta S$. It seems to me that the experimental data do not admit of a change in the order of magnitude of Re ϵ. Please clarify the situation.

L. Wolfenstein:

It is apparently possible to change Re ϵ by a factor of two if it is recognized that the existing experiments on $\Delta Q = -\Delta S$ decays admit of Re $x \cong 0.4$ with the same probability as Re $x = 0$. In addition, for a solution with "small ϵ," it is possible to have the phase of ϵ close to zero. We thus arrive at a solution with $|\epsilon|$ apparently half as large as $|\epsilon'|$, instead of twice as large. Of course, we cannot have $\epsilon = 0$ and we must assume that $|\eta_{00}|/|\eta_{+-}|$ is somewhat smaller than two.

G. Marx:

Dr. Wolfenstein neglected the contribution of the CP-even interaction with $\Delta I > \frac{1}{2}$. What is the actual experimental contribution of $\Delta I > \frac{1}{2}$ to K_S^0 decays?

L. Wolfenstein:

There are at present no good data on the verification of the deviation from the $\Delta I = \frac{1}{2}$ rule in $K_S \to 2\pi$. New experiments are now under way on the determination of the ratio $K_S \to \pi^+\pi^-/K_S \to \pi^0\pi^0$. Together with the determination of $\delta_2 - \delta_0$, they will make it possible to find the value of Re(A_2/A_0) as defined by Wu and Yang. All that we can note at present is an agreement of the experimental data with the absence of $\Delta I = \frac{5}{2}$ and with the value Re$(A_2/A_0) \sim 0.05$. It is possible to neglect Re(A_2/A_0) and all will be in order within the limits of present-day experimental error.

L. A. Khalfin:

I wish to call attention to the need for further experimental research on the detailed form of the law of the K_L and K_S decay as a function of time in as large a time interval as possible, and with maximum possible accuracy. There are two theoretical reasons for this:

1. The usual phenomenological analysis of the entire K^0-decay problem, the unitarity relations and all their consequences are based in most essential manner on the assumption that the K_L and K_S meson decays are strongly exponential, and consequently are described by simple mass-distribution poles. At the same time, if their mass distribution has poles of higher order (in the sense of Goldberger-Watson), then it can be shown that it follows from the unitarity condition that $\langle L|S\rangle = 0$. This result is due to the fact that the decay probability per unit time is no longer constant in this case, depends on the time, and can be shown to vanish at $t = 0$.

2. I have previously proposed a mechanism whereby the $K_L \to 2\pi$ decay problem is explained as being due to a sui generis "mass filtration" (ZhETF Pis. Red. 3, 129 (1966), JETP Lett. 3, 81 (1966)). The gist of the explanation is that "filtration" causes the K_S^0 to decay not only with its own characteristic time, but a certain part decays with the characteristic time of the K_L^0. In effect, all the deductions coincide with the deductions of Wolfenstein's superweak interaction, but the interaction responsible for the $K_L \rightleftarrows K_S$ transition need not be introduced in my model.

I have recently developed a theory of dynamic mass filtration, based on exact consequences of the rigorous energy-momentum conservation law from stable particles to stable ones, i.e., accurate to weak widths. It turns out that if the unstable particles are created simultaneously, their mass distributions are "imprinted" on each other as a result of the conservation laws, and this leads to dynamic filtration of the mass distribution.

This result makes an investigation of the detailed form of the decay laws even more interesting.