# VIOLATION OF CP INVARIANCE IN $K^{0}-\mathrm{MESON}$ DECAYS 

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IN this talk we shall review the experimental data on the effects of CP-violation in $\mathrm{K}^{0}$ decays. We shall as sume that the CPT theorem is rigorously valid. There is no experimental evidence to the contrary at the present time. This question is discussed in detail in the talks of V. Ya. Faĭnberg and L. I. Lapidus. We would nevertheless like to note that one of the predictions of this theorem for weak interactions is the equality of lifetimes of particles and antiparticles. This equality is tested experimentally for individual decays to an accuracy of one in one thousand (Table I).

We now introduce the definitions of shortlived and longlived states. These can be described as superpositions of states with definite $C P$-parity, $\left|K_{1}\right\rangle$ and $\left|K_{2}\right\rangle$, defined as $\left|\mathrm{K}_{1}\right\rangle=\mathrm{CP}\left|\mathrm{K}_{1}\right\rangle,\left|\mathrm{K}_{2}\right\rangle=-\mathrm{CP}\left|\mathrm{K}_{2}\right\rangle$ :

$$
\begin{align*}
& \left|K_{S}\right\rangle=\frac{\left|K_{1}\right\rangle+\varepsilon\left|K_{2}\right\rangle}{\sqrt{1+|\varepsilon|^{2}}}  \tag{1}\\
& \left|K_{L}\right\rangle=\frac{\left|K_{2}\right\rangle+\varepsilon\left|K_{1}\right\rangle}{\sqrt{1+|\varepsilon|^{2}}} \tag{2}
\end{align*}
$$

As a consequence of CPT the complex parameter $\varepsilon$ is the same in both expressions. If the CPT theorem is not assumed, the equations (1) and (2) will contain two different parameters $\varepsilon_{S}$ and $\varepsilon_{L}$. The difference between these two numbers can be represented as consisting of two parts, one being related to the difference in lifetimes of the $\overline{\mathrm{K}}^{\circ}$ and $\mathrm{K}^{\circ}$ and the other being related to the difference in mass of the $K^{0}$ and $\bar{K}^{0}$ states. Consequently the equality of lifetimes is not yet sufficient proof of the validity of CPT invariance to the degree which is required in order to write Eqs. (1) and (2).

We now consider the CP violation in the two-pion decay channel. In this case there are two possible states of the two pions with isospin $I=0$ and $I=2$. Denote by $A_{0}$ and $A_{2}$ the corresponding decay amplitudes from an initial $\mathrm{K}^{0}$-meson state (strangeness +1 ). The only method of introducing CP violation into this decay channel consists in making the ratio $A_{2} / A_{0}$ complex. More precisely, we introduce a new parameter $\varepsilon^{\prime}$ defined as

$$
\begin{equation*}
\varepsilon^{\prime}=\frac{i}{\sqrt{2}} \operatorname{Im}\left(\frac{A_{2}}{A_{0}}\right) e^{i\left\{\left(\delta_{2}-\delta_{0}\right)\right.} \tag{3}
\end{equation*}
$$

where the parameters $\delta_{2}$ and $\delta_{0}$ are the pion-pion scattering phaseshifts in states with $I=2$ and $I=0$. The term involving $\left(\delta_{2}-\delta_{0}\right)$ takes into account the final state pion-pion interaction.

One of the predictions of the $\Delta I=1 / 2$ rule is obvious ly $\mathrm{A}_{2}=0$. It is known that the decay $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{0}$ exists with a mplitude $A_{+}$. We may consider that $\left|A_{2}\right| \sim\left|A_{+}\right|$, $\left|A_{2}\right| \sim 0.05\left|A_{0}\right|$ and consequently $\left|A_{2}\right| \ll\left|A_{0}\right|$. This is an additional assumption, and one could well think of
situations in which $\left|A_{2}\right| \sim\left|A_{0}\right|$ and consequently $\left|A_{+}\right|$ $<\left|A_{2}\right|$, owing to cancellations of the amplitudes for decays with $\Delta I=3 / 2$ and $\Delta I=5 / 2$.

We now consider the experimentally measurable quantities. The amplitudes of two-pion decays with CP violation will be divided by the corresponding CPconserving amplitudes

$$
\begin{align*}
& \frac{1\left(K_{L} \rightarrow \pi^{+} T^{-}\right)}{1\left(K_{S} \rightarrow \pi^{+} \pi^{-}\right)}=\eta_{+-}=\left|\eta_{+-}\right| e^{i \Phi+-}  \tag{4}\\
& \frac{1\left(K_{L} \rightarrow \pi^{0} \pi^{0} \pi^{0}\right)}{A\left(K_{S} \rightarrow \pi^{0} \pi^{0}\right)}=\eta_{00}=\left|\eta_{00}\right| e^{i \Phi_{00}} . \tag{5}
\end{align*}
$$

The observable CP-violations are produced both by a violation in the decay process, described by the parameter $\varepsilon^{\prime}$, and by the presence of a $C P$-violating admixture in the initial long-lived state, characterized by the parameter $\varepsilon(|\varepsilon| \ll 1)$.

Following $W u$ and Yang ${ }^{[5]}$ we have

$$
\begin{align*}
& \eta_{+-}=\varepsilon+\varepsilon^{\prime}  \tag{6}\\
& \eta_{00}=\varepsilon-2 \varepsilon^{\prime} \tag{7}
\end{align*}
$$

This leads to the formula for the so-called Wu-Yang triangle

$$
\begin{equation*}
3 \varepsilon=2 \eta_{+-}+\eta_{100} \tag{8}
\end{equation*}
$$

It is important to note that the expressions (6) and (7), and hence (8) are approximate in nature and follow from the assumption that $\left|A_{2}\right| \ll\left|A_{0}\right|$ (for more details, cf., e.g., $\left.{ }^{[6]}\right)$. In the sequel we mainly discuss experiments

Table I. Verification of CPTa comparison of particle and antiparticle lifetimes

| Particle | Ratio | Reference |
| :---: | :---: | :---: |
| $\mu^{+}, \mu^{-}$ | $1.009+0.001$ | 1 |
| $\pi^{+}, \pi^{-}$ | $1.0023 \pm 0.0040$ | 2 |
|  | $1.0096 \pm 0.0028$ | 3 |
| $K^{+}, K^{-}$ | $1.004 \pm 0.0070$ | 4 |

Table II. The experimental data for the branching ratio

| $\substack{K_{L} \rightarrow 2 \pi^{ \pm} \\ K_{L} \rightarrow \text { all charged products } \\ \text { Branching ratio } \\ K_{L} \rightarrow \pi+\pi-\\ K_{L} \rightarrow \text { all charged products }}$ | Ref. |
| :---: | :---: |
| $(2.0 \pm 0.4) \cdot 10^{-3}$ | 7 |
| $(2.08 \pm 0.35) \cdot 10^{-3}$ | 8 |
| $\left(1.93 \pm 0.26 \cdot 10^{-3}\right.$ | 9 |
| $(1.993 \pm 0.08) \cdot 10^{-3}$ | 10 |
| $(1.97 \pm 0.16) \cdot 10^{-3}$ | 11 |
| $(2.12 \pm 0.18) \cdot 10^{-3}$ | 12 |

testing Eq. (8). We consider the experimental information about $\eta_{+-}, \eta_{\infty}$ and $\operatorname{Re} \varepsilon$.

## I. THE ABSOLUTE VALUE OF $\boldsymbol{\eta}_{\boldsymbol{+}}$

After Christenson et al. ${ }^{[7]}$ discovered the $\mathrm{K}_{\mathrm{L}}$ $\rightarrow \pi^{+} \pi^{-}$decay several experiments have determined the branching ratio

$$
\xrightarrow[K_{L} \longrightarrow \text { all charged products }]{K_{L} \rightarrow \pi^{+}-}
$$

The results are summarized in Table II.
The quantity $\left|\eta_{+-}\right|$can be determined knowing this ratio, the ratio of the probability of decay of charged $\mathrm{K}_{\mathrm{L}}$ mesons to the total width of the $\mathrm{K}_{\mathrm{L}}$ meson and also the probability of the decay $\mathrm{K}_{\mathrm{S}} \rightarrow \pi^{+} \pi^{-}$. From the average of Table II it follows that

$$
\frac{K_{L} \rightarrow \pi^{+} \pi^{-}}{K_{L} \rightarrow \text { all charged products }}=(2.00 \pm 0.062) \cdot 10^{-3} .
$$

Using this result and also the values

$$
\begin{gathered}
\Gamma_{L}=(1.94 \pm 0.05) \cdot 10^{7} \mathrm{sec}^{-1}, \\
\begin{array}{c}
K_{L} \rightarrow \\
K_{L} \rightarrow \text { charged products }
\end{array}=0.78 \pm 0.03, \\
\Gamma\left(K_{\mathrm{S}} \rightarrow \pi^{+} \pi^{-}\right)=(0,792 \pm 0.04) \cdot 10^{10} \mathrm{sec}^{-1},
\end{gathered}
$$

we obtain

$$
|\eta-|=(1.95 \pm 0.072) \cdot 10^{-3} .
$$

## II. THE PHASE OF $\boldsymbol{\eta}_{+-}$

The phase angle $\Phi_{+-}$of $\eta_{+-}$has been determined in the study of two-pion decays of a coherent mixture of $\left|\mathrm{K}_{\mathrm{S}}\right\rangle$ and $\left|\mathrm{K}_{\mathrm{L}}\right\rangle$ states:

$$
\begin{equation*}
|K\rangle=\left|K_{L}\right\rangle+\rho\left|K_{S}\right\rangle . \tag{9}
\end{equation*}
$$

Here $\rho$ is a known complex parameter. Until now such measurements were performed by means of two methods. The first consists in producing the state (9) in a slab of matter, called the regenerator. Then as a result of measurements one determines the quantity $\alpha$ $=\Phi_{+-}-\Phi_{\rho}$ where $\Phi_{\rho}$ is related to the regeneration process and must be determined separately. In the other method, called "vacuum regeneration" one starts with an initial state $\left|\mathrm{K}^{0}\right\rangle$, having the form

$$
\left|K^{0}\right\rangle=\frac{1}{\sqrt{2}}\left[\left|K_{L}\right\rangle+\left|K_{S}\right\rangle\right] .
$$

However, in this case interference can be practically observed only when the contribution from the decay amplitude $\mathrm{K}_{\mathrm{S}} \rightarrow 2 \pi$ is of the same order as that from the amplitude for the decay $\mathrm{K}_{\mathrm{L}} \rightarrow 2 \pi$, i.e., in the region where $\mathrm{t} \approx 2 \tau_{\mathrm{S}} \ln \left|\eta_{+-}\right| \approx 12 \tau_{\mathrm{S}}$, where $\tau_{\mathrm{S}}$ is the lifetime of the shortlived kaon. Consequently, one of the disadvantages of this kind of experiments is the fact that the quantity $\Phi_{+-}$depends in a decisive manner on an exact measurement of the $\mathrm{K}_{\mathrm{L}}-\mathrm{K}_{\mathrm{S}}$ mass difference: $\delta \mathrm{m}=\mathrm{m}_{\mathrm{L}}-\mathrm{m}_{\mathrm{S}}$. Therefore this method is of interest if one can determine the quantity $12 \delta \mathrm{~m} / \hbar \Gamma_{\mathrm{S}}$ with better accuracy than $\Phi_{\rho}$.

At the same time, if $\Phi_{+}$. would be known exactly from other methods, this experiment would yield an exact determination of the mass difference $\delta \mathrm{m}$. The presently accepted value is

$$
\delta m=(0.467 \pm 0.019) \hbar \Gamma_{S} .
$$

This yields an indeterminacy of the phase related only to the error in the mass difference

$$
\Delta \Phi_{+-}=-0.2 \mathrm{rad}
$$

There has been a recent communication ${ }^{[13]}$ about the determination of $\Phi_{+-}$by means of the vacuum regeneration method:

$$
\Psi_{+_{-}}=-(1.05 \pm 0.3) \mathrm{rad}
$$

Earlier results, based on the determination of the regeneration phase from total cross sections of $\mathrm{K}^{+}$and $\mathrm{K}^{-}$(cf. the table of N. D. Galanina on p. 520) are collected in Table III. The values of the total cross sections used in these experiments have been determined by Cool et al. An unpublished measurement of the phase of regeneration by Bennett and others ${ }^{[17]}$ has allowed to clarify to some extent the reason for the slightly too large value $\Phi_{+-}=(1.47 \pm 0.3) \mathrm{rad}$, obtained in ${ }^{[14]}$. The experiment is based on the measurement of the charge asymmetry behind a thick regenerator and yields a noticeable regeneration phase, such that the magnitude of $\Phi_{+-}$decreases by about 1.5 standard deviations. The result (preliminary and unpublished!) is:

$$
\Phi_{+-}=(1.05 \pm 0.3) \mathrm{rad}
$$

It is assumed that the charge asymmetry in leptonic decays

$$
\begin{equation*}
A=\frac{N\left(\pi^{-} l^{+} v\right)-N\left(\pi^{+} l^{-}-\bar{v}\right)}{N\left(\pi^{-} l^{+} v\right)+N\left(\pi^{+} l^{-}-\bar{v}\right)} \tag{10}
\end{equation*}
$$

behind the thick regenerator in the exact forward direction is of the form

$$
\begin{equation*}
A(t)=\frac{2\left(1-|x|^{2}\right)}{|1-x|^{2}} e^{-\Gamma_{S^{t / 2}}} \cos \left(\Delta m t+\Phi_{\rho}\right), \tag{11}
\end{equation*}
$$

where $\mathrm{x}=\mathrm{g} / \mathrm{f}$ is, as usual, the measure of violation of the $\Delta Q=\Delta S$ rule in the decay, $t$ is time in units of the proper time of the K-meson, $\Gamma_{S}$ is the width of the shortlived kaon (inverse lifetime!) and $\Delta \mathrm{m}$ is the mass difference.

It is proper to make the following remarks:

1. The magnitude of the observed asymmetry depends on the magnitude of CP violation in the decay. However, the time-dependence allows one to determine $\Phi_{\rho}$ (if $\Delta \mathrm{m}$ and $\Gamma_{\mathrm{S}}$ are known) independently of the $\Delta \mathrm{Q}$ $=\Delta S$ rule.
2. If one detects the $K^{0}$ mesons emitted in a sufficiently narrow forward cone, as was done in the exper-

Table III. The experimental data on $\Phi_{+-}$, obtained from experiments on interference after regeneration and data on the total cross sections of $\mathrm{K}^{+}$and $\mathrm{K}^{-}$
mesons

| $\Phi_{+-}$, rad | References |
| :---: | :---: |
| $+(1.47 \pm 0.3)$ | 14 |
| $+(1.22 \pm 0.36)$ | 15 |
| $+(0.44 \pm 0.44)$ | 16 |

iment of Bennett et al., ${ }^{[17]}$ one will also register events coming from diffraction regeneration. It is assumed that these events should diminish the effect by a noticeable amount. Moreover, the phase of the time-dependent asymmetry is shifted by the amount $\arg \mathrm{i}[(f(\theta)$ $+\overline{\mathrm{f}}(\theta))]$ relative to $\Phi_{\rho}$. The quantities $\mathrm{f}(\theta)$ and $\overline{\mathrm{f}}(\theta)$ are respectively the amplitudes for forward scattering of the states $\mathrm{K}^{0}$ and $\overline{\mathrm{K}}^{0}$, respectively, in the laboratory system. The authors have assumed that arg $i[(f(\theta)$ $+\overline{\mathrm{f}}(\theta))]=0$.

## III. THE ABSOLUTE VALUE OF $\eta_{\infty}$

The results available on $\left|\eta_{\text {oo }}\right|$ at the time of the Heidelberg conference are collected in Table IV. ${ }^{[6]} \mathrm{Sev}-$ eral new, not yet published results have been presented at a conference in Princeton in November 1967, and some have appeared since then as private communications. It is important to stress that all this material has a completely preliminary character and should be accepted as such; therefore the Table IV contains only the published results. Having this in mind we would like to mention here the following planned experiments:

1. CERN-Orsay-École Polytechnique collaboration. This experiment is being done at CERN with a freon bubble chamber. So far only the results of the CERN group have been communicated. All four photons convert in the liquid and their energies and angles are measured. For a value $\left|\eta_{00}\right|=4 \times 10^{-3}$ one would expect 25 genuine events in the K -meson mass interval, and five background events. In place of this only 12 events have been observed. The number 25 is a result of a Monte Carlo calculation, given the flux of K mesons. It is assumed that the efficiency of detection is 0.3 , including a coefficient 0.6 , related to losses in the selection of photon events. In order to underscore the correctness of the predictions based on a Monte Carlo calculation, the authors indicate the following facts:
a) As communicated by the same group at Heidelberg, they have correctly estimated the registration efficiency for $\mathrm{K}_{\mathrm{L}} \rightarrow 3 \pi^{\circ}$ decays. It should still be noted that such an estimate of efficiency does not contain measurements and is based only on scanning.
b) The accuracy of measurements is checked by means of reconstructing the mass of the $\pi^{0}$ meson from the measured angles and energies of the two photons appearing in its decay.
c) The decay $\mathrm{K}_{\mathrm{L}} \rightarrow \gamma \gamma$ has been clearly registered. For a fraction $10^{-3}$ of such decays one sees 10 cases, which is close to the result obtained by Cronin et al. ${ }^{\text {[19] }}$

Unfortunately, no data on regeneration are available.
2. The Princeton experiment. The article by Cronin and coworkers ${ }^{[19]}$ lists the figure

Table IV. Experimental data on | $\eta_{00} \mid$

| $\left(\eta_{00} 1\right.$ | References |
| :---: | :---: |
| $(4.3+1.1) \cdot 10^{-3}$ | 18 |
| $(4.9 \pm 0.5) \cdot 10^{-3}$ | 19 |
| $(4.17 \pm 0.3) \cdot 10^{-3}$ | 20 |

$$
\frac{R\left(K_{L} \rightarrow 2 \pi^{0}\right)}{R\left(K_{L} \rightarrow 3 \pi^{0}\right)}=(1.89 \pm 0.31) \cdot 10^{-3} .
$$

At the Princeton conference the same group quoted a new value ${ }^{[21]}$

$$
\frac{R\left(K_{L}^{0} \rightarrow 2 \pi^{0}\right)}{R\left(\overline{K_{L}^{0}} \rightarrow 3 \pi^{0}\right)}=(1.36 \pm 0,18) \cdot 10^{-3}
$$

which is by about 1.5 standard deviations smaller than the one originally published. This yields for the corresponding value of $\left|\eta_{00}\right|$

$$
\left|\eta_{00}\right|=(3.92 \pm 0.30) \cdot 10^{-3}
$$

However the fundamental new peculiarity of this analysis based on improved statistics consists in the clear presence of an additional shoulder in the energy distribution of the photons. The presence of such a shoulder which is spurious could in the final count be attributed to a "tail" in the energy distribution function.
3. The Berkeley-Hawaii collaboration. This is a spark-chamber experiment in which all four photons are registered. An interesting characteristic is that the momentum of the $K^{0}$ meson is given: $p_{K}=510 \mathrm{MeV} /$ $\mathrm{c} \pm 10 \%$. In order to eliminate events with more than four photons it was necessary to have two or more sparks. As a control experiment events with 7 photons were looked for. Only approximately $3 \%$ of the events with six photons would have corresponded to the presence of a false seventh photon. The events which were discussed at the Princeton conference contained 464 cases with four photons and comprised approximately $30 \%$ of the total number of events. Taking into account the requirements imposed on the angles of the photons reduces the number of events to three. Starting from a value of $\left|\eta_{00}\right|=4 \times 10^{-3}$ one should expect 13 events. Measurements with a carbon regenerator yielded 51 events with 51 expected, thus proving the correctness of the selection criteria.
4. The CERN-Aachen-Rutherford Lab. collaboration. The analysis we discuss here differs from the one reported to the Heidelberg conference. ${ }^{[22]}$ Basically the improvement is due to the selection of events according to transverse momentum rather than angle, and to a significant improvement of the knowledge of the background. Figure 1 shows the distribution in invariant mass for 28000 events observed by the CERN and Aachen groups. The effect contains $240 \pm 37$ events. This number should be compared with the $84 \pm 22$ events obtained on the basis of 7200 events observed in the experiment published earlier. ${ }^{[181}$

Figure 2 shows the same data as a function of transverse momentum for different intervals of invariant mass. Applying the old selection criterion to the new events, the number of events in the peak turns out to be $228 \pm 34$. Consequently the new result confirms the old one. A new number for the ratio of probabilities will be obtained after an analysis of the regeneration data.

## IV. DETERMINATION OF Re $\varepsilon$

Recently two experiments have been published on the detection of charge asymmetry in the leptonic modes of the longlived state:

$$
\begin{equation*}
\delta=\frac{N(\pi-l+v)-N\left(\pi^{+}+l-\bar{v}\right)}{N\left(\pi^{-l}+v\right)+N(\pi+l-\bar{v})}=\frac{1-|x|^{2}}{|1-x|^{2}}: 2 \operatorname{Re} \varepsilon . \tag{12}
\end{equation*}
$$

Equation (12) was derived under the assumption that the CPT theorem is valid. The two experiments have yielded the following results

$$
\begin{aligned}
& \delta=\frac{N\left(\pi^{-l+v)-N\left(\pi^{+}-\bar{v}\right)}\right.}{N\left(\pi^{-}+l^{+}\right)++N\left(\pi^{+}+\bar{v}\right)}=(2.24 \pm 0.36) \cdot 10^{-3}[23] \\
& R=\frac{N\left(\pi^{-} \mu^{+}+v\right)}{N\left(\pi^{+} \mu^{-}-\bar{v}\right)}=(1.0081 \pm 0.0027)[24]
\end{aligned}
$$

They show that $\varepsilon \neq 0$ independently of the $\Delta S=\Delta Q$ rule. If one assumes the validity of this rule, one can obtain a quantitative estimate for $\operatorname{Re} \varepsilon$ :

$$
\begin{align*}
\operatorname{Re} \varepsilon & =(1.12 \pm 0.18) \cdot 10^{-3}[23]  \tag{13a}\\
\operatorname{Re} \varepsilon & =(2.0 \pm 0.7) \cdot 10^{-3} \quad[24] \tag{13b}
\end{align*}
$$

We now consider the consequences of experiments for the Wu-Yang triangle. We shall assume that the CPT theorem holds, and in addition we make the follow ing assumptions:
a) There is no amplitude with $\Delta S=-\Delta$ Q, i.e., $x=0$;


FIG. 1. The distribution with respect to the invariant mass $\mathrm{M}_{\mathrm{K}}$ of four-photon events from [ ${ }^{22}$ ]. Only events with $p_{K} \sin \theta_{K} \leqslant 0.1 \mathrm{GeV} / \mathrm{c}$ are shown. The curve represents a formula for the background from $\mathrm{K} \rightarrow 3 \pi$ decays, computed by means of a Monte Carlo method.


FIG. 2. The distribution of events in the square of the transverse momentum $\mathrm{p}_{\mathrm{K}}^{2} \sin ^{2} \theta_{\mathrm{K}}$ for different mass intervals $\mathrm{M}_{\mathrm{K}}$. The curve representing the difference of the spectra was obtained from the first two and also from Monte Carlo calculations.
b) There is no CP violation in the three-pion decays $\mathrm{K}^{0} \rightarrow \pi^{+} \pi^{-} \pi^{0}$ and $\mathrm{K}^{0} \rightarrow \pi^{0} \pi^{0} \pi^{0}$. The assumptions a) and b) imply that $\varepsilon_{r}=0^{[6]}$ (cf. infra).

The experimental verification of these assertions is not as good as one would like. (A more detailed discus-


FIG. 3. The Wu-Yang triangle for the best values, obtained by treating the experimental data with $\epsilon_{\mathrm{r}}=0\left(\chi^{2}=6.69\right.$ for one degree of freedom).

FIG. 4. "Predicted" value for $\left|\eta_{00}\right|^{30}$ derived from the data on $\eta_{+-}, \phi_{+-} \delta$ and the condition $\varphi_{\epsilon}=\arctan 2 \mathrm{~m} / \Gamma_{S}$



FIG. 5. The Wu-Yang triangle for $\epsilon_{\mathbf{r}} \neq 0$. Two solutions are possible. Both are close to the limits determined from unitarity and the magnitude of $C P$ violation admitted by the experimental data on other channels.
sion is given, e.g., in the talk by Aubert at this seminar.)

From the assumption a) it follows that no additional CP violation is possible in the leptonic decay channels, i.e., $\operatorname{Im} x=0$. In addition, measuring the charge asymmetry in leptonic decays allows one to determine $\operatorname{Re} \varepsilon$, as indicated in Eqs. (13).

The quantities which can be determined experimentally are $\eta_{+-}, \Phi_{+-}, \eta_{00}$ and $\delta$, i.e., in all, four quantities. The Wu-Yang triangle constructed on the basis of the experimental data for these quantities is drawn in Fig. 3.

Considering that the agreement is clearly unsatisfactory ( $\chi^{2}=6.69$ for one degree of freedom) one can admit several alternatives:

1. The value of $\left|\eta_{00}\right|$ is to some extent controversial, as shown by the majority of the new results discussed above. If, in treating the data we take $\left|\eta_{00}\right|$ as a free parameter, one can compute the best value and error for $\left|\eta_{00}\right|$ from the determinations of $\delta, \Phi_{+-}, \eta_{+-}$and the relation (12). It turns out that the predicted value is

$$
\left|\eta_{00}\right| \text { predicted }=(1.87 \pm 0.8) \cdot 10^{-3}
$$

The $\chi^{2}$ curve is represented in Fig. 4. In this approach, which yields $\left|\varepsilon^{\prime}\right| \ll|\varepsilon|$ the $\pi \pi$ scattering amplitude remains undetermined.
2. If the conditions a) and b) are violated, then $\arg \varepsilon \neq \arctan \left(2 \Delta \mathrm{~m} / \Gamma_{\mathrm{S}}\right)$. One can define $\varepsilon=\varepsilon_{v}+\varepsilon_{r}$, where $\arg \varepsilon_{\mathrm{V}}=\arctan \left(2 \Delta \mathrm{~m} / \Gamma_{\mathrm{S}}\right)$ and $\varepsilon_{\mathrm{r}}$ and $\varepsilon_{\mathrm{V}}$ are orthogonal in the complex plane. The parameter $\varepsilon_{r}$ is a measure of the violation of $C P$ in the channels differing from the decay into two pions. The experimental data allow one to establish that ${ }^{[6]}$

$$
\left.\begin{array}{c}
\left|\varepsilon_{r}(K \rightarrow \pi l v)\right| \leqslant 7,7 \cdot 10^{-4},  \tag{14}\\
\left|\varepsilon_{r}(K \rightarrow \pi \pi \pi)\right| \leqslant 4 \cdot 10^{-4}, \\
\left|\varepsilon_{r}(K \rightarrow 2 \pi, T=2)\right| \leqslant 10^{-4} / \cos \left(\delta_{2}-\delta_{0}\right) .
\end{array}\right\}
$$

A Wu-Yang triangle with $\varepsilon_{r} \neq 0$ is shown in Fig. 5. For $\varepsilon_{r} \neq 0$ there are two possible values of $\varepsilon_{r}$, both close to the upper limit indicated by the relations (14). Consequently the experimental data admit the possibility of CP violation in channels other than $K \rightarrow 2$. Searches for such violations are of great interest.

[^0][^1]
## DISCUSSION

## L, B. Okun':

What is known about new experiments on the decays $\mathrm{K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \gamma$ and on the interference of the decays $\mathrm{K}_{\mathrm{L}, \mathrm{S}}^{0}$ $\rightarrow \pi^{+} \pi^{-} \gamma$ ?

## C. Rubbia:

Several experiments are planned on the decay of the charged kaons $\mathrm{K}^{ \pm} \rightarrow \pi^{ \pm} \pi^{0} \gamma$. As far as I know, two experiments are proposed at Brookhaven and one at CERN by the Oxford group. In these experiments the decays are detected in flight, with several thousand events expected in each of them.

The observation of the mode $\mathrm{K}_{\mathrm{L}}^{0} \rightarrow \pi^{+} \pi^{-} \gamma$ is possible as long as the ratio of decay rates $R$ $=\left(\mathrm{K}_{\mathrm{L}} \rightarrow \pi^{+} \pi^{-} \gamma\right) /\left(\mathrm{K}_{\mathrm{L}}-\right.$ all charged products $\left.)\right) \gtrsim 10^{-4}$. Here one has to measure the momenta of both charged particles and the direction of the photon. In my opinion the major difficulty if $\mathrm{R} \ll 10^{-4}$ will be related more to the small number of events than to the background.

## I. V. Chuvilo:

Let me say several words about what Prof. Rubbia said. An expected effect of the order of several percent
in comparing the decay probabilities for $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{0} \gamma$ and $\mathrm{K}^{-} \rightarrow \pi^{-} \pi^{0} \gamma$ is based on the concept that the amplitudes of the mechanisms of direct emission and bremsstrahlung of photons are of comparable magnitude. The experimental situation in this problem is not yet clear. The reason for this is the fact that so far the experimentalists have worked in the region of the spectrum of the $\pi^{+}$in the decay $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{0} \gamma$ where the pion energy is higher than 53 MeV , in order to avoid background from decays $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{0} \pi^{0}$. In these conditions one can only see that the contribution of the bremsstrahlung mechanism to the decay $\mathrm{K}^{+} \rightarrow \pi^{+} \pi^{0} \Gamma$ exists, and one can say nothing about the contribution to this decay of the direct emission mechanism. My remark boils down to saying that there is only hope to detect a CP violation in the decays $\mathrm{K}^{\mp} \rightarrow \pi^{\mp} \pi^{0} \gamma$ if one succeeds to work experimentally in a region where there is an interference effect from the two mechanisms. We could say something if we knew the probability of the decay $\mathrm{K}_{\mathrm{L}}^{0}$ $\rightarrow \pi^{+} \pi^{-} \gamma$, where only the direct emission mechanism works. But so far this rate has not been measured. Therefore we do not have quantitative estimates of the expected effect, if we base ourselves only on the considerations given above.

## K. A. Ter-Martirosyan:

What is the advantage of difficult experiments with radiative decays, compared, for instance, with interference experiments involving $\mathrm{K}_{\mathrm{L}, \mathrm{S}} \rightarrow 2 \pi^{\circ}$ ? What new information on CP violation could a study of these decays yield?

## L. Wolfenstein:

The interference effect in the decays $\mathrm{K} \rightarrow \pi \pi \gamma$ is of interest because this would be a CP-violating transition different from $K \rightarrow 2 \pi$, the only one observed so far. In particular, if the violation of $C P$ is related to the electromagnetic or electromagnetic-weak interaction, i.e., to the emission of a photon, we could expect a larger CP violation in the decay $\mathrm{K} \rightarrow \pi \pi \gamma$. Under different as sumptions one would expect that this effect is considerably smaller.

## A. A. Komar:

Among the values of $\Phi_{+}$shown in your slide the later figures were considerably smaller than the others. Could you comment on this circumstance?

## C. Rubbia:

The numbers differ, but they are affected by large errors. The spread between the results is within the statistical expectation. In particular, the value 0.44 $\pm 0.44$ differs by about 1.8 standard deviations from the value of 1.05 rad , which is the world average.

## L. Frenkel:

a) What is the value of the experimental result for $m_{L}-m_{S}$ and what happens if the sign of this difference changes?
b) Why is the result $\Phi_{+-}=1.05 \pm 0.3 \mathrm{rad}$ considered more reliable than $\Phi_{+-}=1.47 \pm 0.3 \mathrm{rad}$ ?

## C. Rubbia:

a) A change in sign of the mass difference leads to a reflection of the Wu-Yang triangle in the real axis
(i.e., a sign change if the imaginary axis). The only direct consequence of this would be a change in the pionpion scattering phaseshifts. This would be desirable if one takes into account the existing contradictions which I mentioned, but one cannot do it if one takes into account the mounting quantity of experimental data favoring $m_{L}>m_{S}$.
b) The quantity $\Phi_{+-}=1.47 \pm 0.3 \mathrm{rad}$ was derived from the optical model, whereas $\Phi_{ \pm}=1.05 \pm 0.3 \mathrm{rad}$ is an experimental number. I trust the experiments more than the optical model. In particular, there is no explicit contradiction between the two numbers.

## B. Aubert:

You mentioned that the Wu -Yang triangle does not close, but if you forget about the Steinberger experiment on charge asymmetry or if you increase the error (which is purely statistical) can you make it close?

## C. Rubbia:

In order to close the triangle it would be desirable to have a value for $\delta$ which exceeds the mentioned values by several standard deviations.

## G. Marx:

You mentioned a predicted value for $\left|\eta_{00}\right|$ based on the data for $\eta_{+-}, \Phi_{+-}$, and $\operatorname{Re} \varepsilon$. What is the corresponding figure for $\delta_{2}-\delta_{0}$ ?

## C. Rubbia:

The result is such that $\left|\varepsilon^{\prime}\right| \ll|\varepsilon|$. Consequently, the quantity $\delta_{2}-\delta_{0}$ is undetermined. It can be anything you like.

## K. A. Ter-Martirosyan:

What is known about the possible values of $\Phi_{00}$ ?

## C. Rubbia:

The experiment by Gaillard and coworkers is devoted to this question. At present it is not finished.

## K. A. Ter-Martirosyan:

What is the final value of $\delta_{2}^{0}-\delta_{0}^{0}$ ?

## G. A. Leksin:

To date $\left|\delta_{2}^{0}-\delta_{0}^{0}\right|=55 \pm 15^{\circ}$. According to Walker's analysis the sign is negative, but according to some other authors, e.g., V. V. Anisovich, it is positive.

## C. Rubbia:

Budker has recently found that the width of the $\rho-$ meson is considerably smaller than was previously known. How strongly would that influence the Walker analysis?

## G. A. Leksin:

I do not know.

## I. Yu. Kobzarev:

Walker asserted that all experimental data agree with his analysis. Could you comment on this?

## G. A. Leksin:

In his analysis Walker used essentially data in the $\rho$-meson region and a normalization of the number of
events to the unitarity limit of the $\pi \pi$ cross section at the $\rho$-meson mass. The phaseshifts found by Walker describe a wide variety of results of work with track chambers. However (and this has been emphasized in my talk, cf. Yad. Fiz. 8, 165 (1968) [Sov. J. Nucl. Phys. 8, 000 (1969)]) essentially the same data led other authors (cf., e.g., F. Malamud and P. E. Schlein, LRCIA-

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Translated by M. E. Mayer


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