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CONSEQUENCES OF CPT INVARIANCE AND EXPERIMENT*

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I. INTRODUCTION

It is very interesting to follow the fate of such a fundamental proposition as the CPT theorem. For a long time it was not noticed. Then, as symmetry arguments came more and more into physics, there appeared the first indications of a connection between the requirements of the discrete symmetries. For many of us this happened in the lectures of L. D. Landau in 1954. In these lectures Landau pointed out that he had not been able to find any example of a selection rule which was a new one when compared with the rules that already follow from the requirements of P and C invariance. Here we essentially encountered for the first time one of the formulations of CPT invariance: if a theory is P and C invariant, then it is, so to say, automatically also T invariant.

As can be seen from the literature, in those years approaches were made to the formulation of what is now called the CPT theorem by Bell,[1] Schwinger,[2] Lüders,[3] and Pauli.[4] Our present understanding of the importance and depth of the CPT theorem has been achieved since the discovery of violations of first the P and C invariances and subsequently also of CP invariance, in the weak interactions.

At present, at the beginning of 1968, when we have already said farewell to many previously undoubted symmetry properties, it seems almost antiscientific to utter audible doubts concerning the possibility that the CPT theorem is not valid. Its fundamental nature, and also the fact that so far no one has been able to construct even a model example of a field theory in which the CPT theorem is violated, make us regard the CPT theorem as a sort of last stronghold which must not be surrendered.

It seems that the CPT transformation is more fundamental than other discrete transformations (the C, P, and T transformations). The requirement of CPT invariance makes much weaker demands on a theory than does invariance with respect to C, P, and T taken separately. The depth of the CPT theorem makes it especially important to test the consequences that can be derived from its requirements. It is an essential fact that any local Lagrangian for a field theory is invariant under the combined operation CPT (provided only that the theory is invariant with respect to proper orthochronous Lorentz transformations).

It is usually assumed that CPT invariance is an absolute symmetry principle. There are no indications to the contrary; for example, a test of CP invariance is associated with a test of T invariance. This is particularly clearly seen in the treatment of the $K^0 - \overline{K^0}$ system. If the CPT theorem holds, the existence of the decay $K_L^0 \to 2\pi$, which contradicts CP invariance, also indicates a violation of T invariance. In this way CPT invariance allows us to make indirect tests of the C, P, or T invariances in cases in which direct tests are not accurate enough or are beyond the bounds of present experimental technique. At present only partial aspects of CPT invariance have been tested.

A phenomenological approach allows us to develop the apparatus of the analysis in such a way that proton-antiproton scattering, nonleptonic decays of hyperons and antihyperons, and decays of charged and neutral K mesons and of charged pions can (if the measurements are highly accurate) be a source of experimental information about the validity of the CPT theorem. The best known results of the application of the CPT theorem are conclusions about such properties of particles and their antiparticles as the equality of their rest masses, lifetimes, and magnetic moments. We note that it is precisely the violation of other discrete symmetries that allows us to approach the experimental test of the CPT theorem itself.

II. THE CPT TRANSFORMATION

Since the CPT transformation includes the operation of time reversal, it is clear that it is an antiunitary operator, which we denote by CPT. Its action on state vectors can be obtained from the transformation properties under the separate transformations C, P, and T. We note for the present that the state of one particle with momentum p and helicity λ is transformed by the action of CPT into the antiparticle with momentum p and helicity λ. Accordingly, the spin direction of a particle is changed to its opposite. Moreover, an in state is transformed into an out state (a diverging wave is converted into a converging wave).

Let us consider the process $|i;\ \text{in}\rangle \Longrightarrow |f;\ \text{out}\rangle$, where i and f are arbitrary initial and final states characterized by the momenta and helicities of the individual particles. We denote by $|\bar{i};\ \text{in}\rangle$ and $|\bar{f};\ \text{out}\rangle$ the corresponding states with particles replaced by antiparticles. Finally, we denote by $|\tilde{i};\ \text{in}\rangle$ and $|\tilde{f};\ \text{out}\rangle$ the original states with opposite directions of the helicities.

Let us now consider a Hermitian operator Ω, for example the weak-interaction Hamiltonian, which is assumed to be CPT invariant, i.e.,

$$CPT\Omega(CPT)^{-1} = \Omega. \tag{1}$$

Considering the matrix elements of Ω, taken between eigenstates of the strong-interaction Hamiltonian H, we then get the following relation:

$$\langle \bar{f};\ \text{out}|\Omega|\bar{i};\ \text{in}\rangle = \langle \bar{f};\ \text{out}|(CPT)^{-1}(CPT)\Omega(CPT)^{-1}(CPT)|i;\ \text{in}\rangle = \langle \tilde{f};\ \text{in}|\Omega|\tilde{i};\ \text{out}\rangle^*. \tag{2}$$

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We can introduce the S operator and rewrite (2) in the following form:

$$\langle \bar{f};\ \text{out} \,|\, \Omega \,|\, i;\ \text{in}\rangle = \langle \tilde{f};\ \text{out} \,|\, S^{-1}\Omega S \,|\, \tilde{i};\ \text{in}\rangle^{*}. \qquad (3)$$

If the states $|\tilde{f};\ \text{out}\rangle$ and $|\tilde{i};\ \text{in}\rangle$ are chosen to be eigenstates of the strong Hamiltonian, and consequently of the S operator, we get

$$\langle \bar{f};\ \text{out} \,|\, \Omega \,|\, \bar{i};\ \text{in}\rangle = e^{2i(\delta_i+\delta_f)} \langle \tilde{f};\ \text{out} \,|\, \Omega \,|\, \tilde{i};\ \text{in}\rangle^{*}. \qquad (4)$$

This relation connects matrix elements of the Hermitian operator between states of particles and between the corresponding states of the antiparticles. From this important relation we can get a number of predictions which can be subjected to experimental tests.

III. EQUALITY OF THE MASSES OF PARTICLES AND ANTIPARTICLES

Let the states $|i;\ \text{in}\rangle$ and $|f;\ \text{out}\rangle$ describe one particle at rest, so that in this case there is no difference between the in and out signs. If we then consider $\Omega = H$, where H is the total Hamiltonian, Eq. (4) gives

$$\overline{m}\delta_{\bar{i},\,\bar{f}} = m^{*}\delta_{\tilde{i},\,\tilde{f}}, \qquad (5)$$

where m and $\overline{m}$ are the respective masses of the particle and the antiparticle. Since the operator H is Hermitian and m is a diagonal matrix element of H, m is real, and we conclude that the rigorous relation

$$\overline{m} = m, \qquad (6)$$

holds, i.e., the masses of particle and antiparticle are equal.

The same result is obtained from the requirement of C invariance or of CP invariance. Since there are violations of the C and CP invariances in the weak interactions, it is very important that (6) follows from the (very weak) assumption that there is CPT invariance.

Some of the most exact experimental results on the masses of particles and their antiparticles are shown in Table I.

The most exact test of CPT invariance involves the properties of the $K^0 - \overline{K}^0$ system.[13] Since in this case a 2×2 mass matrix is required for the description of the behavior of the system, it is extremely important that we can formulate all of the necessary propositions without the requirements of the discrete symmetries.

The requirement of CPT invariance has the consequence that the diagonal matrix elements of the mass matrix are equal. Without the requirement of CPT invariance the difference between M_{11} and M_{22} is

$$M_{11} - M_{22} = (sp+qr)^{-1}(sp-qr)(M_L - M_S); \qquad (7)$$

here p, q, r, and s define the superpositions $|K_L\rangle$ and $|K_S\rangle$:

$$|K_L\rangle = p\,|K\rangle - q\,|\overline{K}\rangle, \quad |K_S\rangle = r\,|K\rangle + s\,|\overline{K}\rangle. \qquad (8)$$

During the characteristic time τ

$$\begin{gathered} |K_L\rangle \Rightarrow e^{-iM_L\tau}\,|K_L\rangle, \quad |K_S\rangle \Rightarrow e^{-iM_S\tau}\,|K_S\rangle, \\ M_L = m_L - \frac{i}{2}\Gamma_L, \quad M_S = m_S - \frac{i}{2}\Gamma_S, \\ |p|^2 + |q|^2 = 1, \quad |r|^2 + |s|^2 = 1. \end{gathered} \qquad (9)$$

The coefficients are related by the identity

$$|sp+qr|^2 + |pr^{*} - qs^{*}|^2 = (|p|^2 + |q|^2)(|r|^2 + |s|^2) = 1. \qquad (10)$$

The requirement of unitarity for the K^0 meson decays allows us to give an estimate of the degree of orthogonality of the states $|K_L^0\rangle$ and $|K_S^0\rangle$ without assuming CPT invariance. Using the experimental data, we can show that

$$\rho(U) = \text{const}\cdot\exp 2\sqrt{aAU},$$

$$|\langle K_L \,|\, K_S\rangle| = |pr^{*} - qs^{*}| \leqslant 0.06$$

and

$$|sp+qr| \cong 1.$$

For the quantity $M_{11} - M_{22}$ we have

$$|M_{11} - M_{22}| \leqslant 2\,|M_L - M_S|,$$

where as an upper bound we have used the relation

$$|sp - qr| \leqslant 2.$$

Also without assuming CPT invariance it can be shown that

$$M_{11} + M_{22} = M_L + M_S.$$

Consequently

$$\left|\frac{M_{11} - M_{22}}{M_{11} + M_{22}}\right| \leqslant \frac{2\,|M_L - M_S|}{|M_L + M_S|}. \qquad (11)$$

If in (11) we substitute[13] the experimental data, we have

$$\frac{2\,|M_L - M_S|}{|M_L + M_S|} \sim 6.5\cdot 10^{-15}.$$

To the extent that it is reasonable to break the total Hamiltonian H up into the sum $H = H_S + H_{EM} + H_W$, this result means that the ratio of the amplitude which violates CPT invariance to the amplitude which preserves this invariance is smaller than 10^{-14} for H_S, less than 10^{-12} for H_{EM} and less than 10^{-8} for the $\Delta S = 0$ part of H_W.

Table I. Equality of the masses of particles and their antiparticles (literature references in last column)

μ^{+-}	$\frac{m_+}{m_-} - 1 = 10^{-4}$	7
	$m_- = 105.659 \pm 2$ MeV	8
π^{+-}	$m_+ = 139.60 \pm 0.05$ MeV	9
	$m_- = 139.578 \pm 0.017$ MeV	10
	$m_- = 139.584 \pm 0.020$ MeV	10
	Mean $m_- = 139.580 \pm 0.015$ MeV	10
K^{+-}	$m_- = 493.7 \pm 0.3$ MeV	11
	$m_+ = 493.78 \pm 0.17$ MeV	12
$K^0 - \overline{K}^0$	$\frac{\|M_{11} - M_{22}\|}{\|M_{11} + M_{22}\|} \leqslant \frac{2\,\|M_L - M_S\|}{\|M_L + M_S\|} \cong 6.5\cdot 10^{-15}$	13
$p - \bar{p}$	$m_p = 938.256 \pm 0.005$ MeV	14
	$m_{\bar{p}} = (1.008 \pm 0.005)\ m_p$	15
	$m_{\bar{p}} = (1.004 \pm 0.025)\ m_p$	16
	$m_{\bar{p}} = (0.998 \pm 0.015)\ m_p$	17
$d - \bar{d}$	$\frac{m_+}{m_-} - 1 = \pm 3\%$	18
$\Lambda^0 - \overline{\Lambda}^0$	$m_\Lambda = 1115.44 \pm 0.12$ MeV	19
	$Q_\Lambda = 37.60 \pm 0.12$ MeV	19
	$Q_{\overline{\Lambda}} = 35^{+2.6}_{-0.9}$ MeV	20

IV. EQUALITY OF THE LIFETIMES OF PARTICLES AND THEIR ANTIPARTICLES

1. Let us now consider the state $|i;\ \text{in}\rangle$ corresponding to a particle which can decay only through the action of weak interactions. Under the strong and elec-

tromagnetic interactions it is stable. To begin with we consider only the case in which the interaction in the final state is negligibly small, so that for $\Omega = H_W$ Eq. (4) takes the form

$$\langle \bar{f};\ \mathrm{out}\,|\,H_W\,|\,i;\ \mathrm{in}\rangle = \langle \widetilde{f};\ \mathrm{out}\,|\,H_W\,|\,\widetilde{i};\ \mathrm{in}\rangle^*. \qquad (12)$$

From (12) we at once get for the rate of partial decay the relation $\bar{\Gamma} = \widetilde{\Gamma}$, where $\bar{\Gamma}$ is for the channel with antiparticles and $\widetilde{\Gamma}$ for that with particles with all spins reversed in direction. If we sum over all possible spin orientations and thus concern ourselves with the rates of partial decay without measurement of the spins in the initial and final states, we get

$$\Gamma(i \to f) = \Gamma(\bar{i} \to \bar{f}). \qquad (13)$$

Among the processes for which this holds we may mention the π_{l2}, K_{l2}, and K_{l3} decays. From (13) we conclude that for these processes the rates of the partial decays (summed over spins) are equal for the decays of K^+ and K^-

$$\left.\begin{aligned} \Gamma(K^+ \to \mu^+ + \nu_\mu) &= \Gamma(K^- \to \mu^- + \bar{\nu}_\mu),\\ \Gamma(K^+ \to \mu^+ + \nu_\mu + \pi^0) &= \Gamma(K^- \to \mu^- + \bar{\nu}_\mu + \pi^0),\\ \Gamma(\pi^+ \to \mu^+ + \nu_\mu) &= \Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu). \end{aligned}\right\} \qquad (14)$$

These predictions of CPT invariance have not yet been tested with the necessary accuracy.

2. For processes such as K^+_{l3} decay Eq. (12) allows us to derive a number of results relating to the energy spectra of the particles, the angular distributions, and the polarizations of fermions (l is a general notation for electron and muon: $l = \mu$, e). For example, for given values of the two kinematic variables—α, the angle between the directions of the pion and lepton (in the rest system of the $l\nu_l$ system) and the square of the four-momentum transfer,

$$-q^2 = m_k^2 + m_\pi^2 - 2m_k E_\pi,$$

if there is CPT invariance the results of summing the differential cross sections over the lepton polarizations are equal:

$$\frac{d^2N}{dq^2}\,\frac{(K^+_{l3})}{d\cos\alpha} = \frac{d^2N\ (K^-_{l3})}{dq^2 d\cos\alpha}. \qquad (15)$$

For the polarization of the muons the requirement of CPT invariance leads to the equation

$$\mathbf{s}(l^+) = -\mathbf{s}(l^-). \qquad (16)$$

Accordingly, the polarizations of the muons from the decays $K^+_{\mu 3}$ and $K^-_{\mu 3}$ are equal in magnitude and opposite in direction. To see the sensitivity of the result (16) of CPT invariance, let us examine the predictions obtained from requiring T invariance and CP invariance.

For CP invariance the relation (16) holds only for the components of the polarization which lie in the plane formed by the vectors $\mathbf{k}_\pi$ and $\mathbf{k}_\mu$. For the component of the polarization vector perpendicular to this plane the requirement of CP invariance leads to the equation

$$s_\perp(\mu^+) = + s_\perp(\mu^-). \qquad (17)$$

It can be seen from (16) and (17) that a nonvanishing value of $S_\perp$ contradicts T invariance. The present situation with the measurement of $S_\perp$ is that $S_\perp(\mu) = 0.04 \pm 0.35$; consequently, for the level of accuracy that has been reached there are no indications that CP invariance is violated in $K_{\mu 3}$ decay.

3. If there is interaction in the final state Eq. (12) is replaced by

$$\langle \bar{f};\ \mathrm{out}\,|\,H_W\,|\,\bar{i};\ \mathrm{in}\rangle = e^{2i\delta_f}\langle \widetilde{f};\ \mathrm{out}\,|\,H_W\,|\,\widetilde{i};\ \mathrm{in}\rangle^*, \qquad (18)$$

on the assumption that $|\bar{f};\ \mathrm{out}\rangle$ is an eigenstate of the strong (and electromagnetic) interactions. In the more general case in which this is not so, we can always expand the final state in terms of such eigenstates and get a sum of terms with different phase factors in the right member. Thus for the decays of K^0 and $\overline{K^0}$ mesons we get

$$\langle f_{CPT}\,|\,T\,|\,\bar{k}\rangle = \sum_\beta \langle\beta\,|\,T\,|\,K\rangle^*\langle\beta\,|\,S\,|\,f\rangle, \qquad (19)$$

where the sum goes over a complete set of states $|\beta\rangle$ having nonvanishing matrix elements with the states $|f\rangle$ and $|K\rangle$. The meaning of this last remark can be explained easily with an example. Let us consider the decays

$$\begin{aligned}\Lambda^0 &\to p + \pi^-,\\ &\to n + \pi^0,\end{aligned}$$

which both occur owing to the weak interaction. Since both particles in the final state are hadrons, we cannot neglect the interaction in the final state. Since the strong interaction conserves isospin and P parity, we must deal with states with definite isospin values I and L. If electromagnetic interactions are neglected there must be four eigenstates of the strong interactions and four phase shifts $\delta_{2J}(L, I)$, namely $\delta_1(0, \tfrac{1}{2})$, $\delta_1(0, \tfrac{3}{2})$, $\delta_1(1, \tfrac{1}{2})$, and $\delta_1(1, \tfrac{3}{2})$. We shall not carry the analysis further, but point out that the existence of several terms with different phase factors in (4) does not allow us to deduce that the partial decay rates are equal for charge-conjugate channels.[22] For example, it does not follow from CPT invariance that $B = \bar{B}$, where B and $\bar{B}$ are defined as

$$B = \frac{\Gamma(\Lambda^0 \to n + \pi^0)}{\Gamma(\Lambda^0 \to p + \pi^-)}, \qquad \bar{B} = \frac{\Gamma(\bar{\Lambda}^0 \to \bar{n} + \pi^0)}{\Gamma(\bar{\Lambda}^0 \to \bar{p} + \pi^+)}.$$

Equality of the rates is obtained, however, if the further assumption of C or T invariance is made.

4. In some cases the final states are eigenstates of the strong-interaction operator S owing to different independent selection rules. In this case the phase factor in (4) does not lead to any complications. As an example we consider the two charge-conjugate processes

$$K^{+-} \to \pi^{+-} + \pi^0.$$

From the generalized Pauli principle (Bose statistics) it follows that the final 2π states must be eigenstates of the isospin with I = 2. Moreover, conservation of angular momentum requires that L = 0 (s state). Since isospin and P parity are conserved in strong interactions, there are no other possible final states which can be reached owing to the strong interaction in the final state. In this case the final 2π states are eigenstates of the strong-interaction operator S. Neglecting the influence of electromagnetic interactions, we can derive the following equation from CPT invariance:

$$\Gamma(K^+ \to \pi^+ + \pi^0) = \Gamma(K^- \to \pi^- + \pi^0).$$

The corrections owing to electromagnetic interaction must be of the order 10^{-4}.

5. In the framework of isotopic invariance the decays $K^{\pm} \to 2\pi$ are connected with the decays $K^0 \to 2\pi$. It can be shown[23] that in the absence of electromagnetic interactions

$$2A\,(K^0 \longrightarrow 2\pi) + \sqrt{2}\,A\,(K^0 \longrightarrow \pi^+\pi^-) = 3\alpha A\,(K^+ \longrightarrow \pi^+\pi^0) \tag{20}$$

and

$$2A\,(\bar{K}^0 \longrightarrow 2\pi^0) + \sqrt{2}\,A\,(\bar{K}^0 \longrightarrow \pi^+\pi^-) = 3\beta A\,(K^- \longrightarrow \pi - \pi^0), \tag{21}$$

where

$$\alpha = \frac{A_3 - A_5}{A_5 + \frac{2}{3} A_3}, \qquad \beta = \frac{\bar{A}_3 - \bar{A}_5}{\bar{A}_5 + \frac{3}{2} \bar{A}_3}, \tag{22}$$

and $A_{5,3} = A_{5/2}$, $A_{3/2}$ are the respective amplitudes for the $K^0 \to 2\pi$ decays with change of the isospin I by 5/2 and 3/2; $\bar{A}_{5,3}$ are the same amplitudes for the 2π decays of $\bar{K}^0$ particles.

If we introduce A_I, $\bar{A}_I$, the reduced amplitudes of the $K \to 2\pi$ decays, we have

$$A_I = e^{-i\delta_I} \langle I \,|\, T \,|\, K^0 \rangle, \qquad \bar{A}_{\mp} = e^{-i\delta_I} \langle I \,|\, T \,|\, \bar{K}^0 \rangle; \tag{23}$$

δ_I (I = 0, 2) are the phases of the pion-pion interaction in the s state with total energy equal to the mass of the K meson.

From (20) and (21), without any requirements as to discrete symmetries, it can be shown that

$$\frac{\beta}{\alpha} \frac{A\,(K^- \longrightarrow \pi^-\pi^0)}{A\,(K^+ \longrightarrow \pi^+\pi^0)} = \frac{\bar{A}_2}{A_2} \cong 1 + 2\,\frac{\bar{A}_2 - A_2}{\bar{A}_2 + A_2}. \tag{24}$$

On the assumption of T invariance we have

$$\beta = \alpha^*, \qquad \bar{A}_2 = A_2^*,$$

$$|\,A\,(K^- \longrightarrow \pi^-\pi^0)\,| = |\,A\,(K^+ \longrightarrow \pi^+\pi^0)\,|. \tag{25}$$

If there is T invariance A_2 and $\bar{A}_2$ are real, but $|A_2| \neq |\bar{A}_2|$, and in general $|\beta| \neq |\underline{\alpha}|$.

The possible deviation of $\bar{A}_2/A_2$ from unity is evidently not larger than 10^{-3}, since it occurs in the well known[24] expression for the parameter $\gamma \equiv \epsilon'/\omega$ which characterizes the fractional violation of CP invariance in $K_L \to 2\pi$ decays:

$$\gamma \equiv \frac{(2\,|\,T\,|\,K_L)}{(0\,|\,T\,|\,K_L)} \simeq \frac{p-q}{p+q} + \frac{\bar{A}_2 - A_2}{\bar{A}_2 + A_2}. \tag{26}$$

6. Unlike the $\Lambda \to \pi N$ decay, the decay $\Sigma^- \to n + \pi^-$ brings the πN system into a state with a definite value of the isospin I = 3/2, $I_3 = -3/2$. In this case CPT invariance makes the widths of the decays $\Sigma^- \to n + \pi^-$ and $\bar{\Sigma}^- \to \bar{n} + \pi^+$ equal.

In 1959 Chou Huang-Chao pointed out a relation[25] between the asymmetry coefficients α and $\bar{\alpha}$ for the decays of Σ^- and $\bar{\Sigma}^-$,

$$\frac{\alpha}{\bar{\alpha}} = -\frac{\cos\,[\delta_s - \delta_p + \Delta_s - \Delta_p]}{\cos\,[\delta_s - \delta_p - \Delta_s + \Delta_p]}, \tag{27}$$

where δ_s and δ_p are the πN phase shifts in the s and p states, and Δ_s and Δ_p are the phase shifts associated with violation of T invariance.

Although to the accuracy so far achieved $\alpha \cong 0$ (the decay $\Sigma^- \to n + \pi^-$ decay leads to a πN system mainly in the s state), the relation (27) indicates that it is useful to compare the polarization properties of hyperons and antihyperons.

7. $K^{\pm}_{\pi 3}$ decay. In the absence of electromagnetic interaction it follows from the requirement of CPT invariance that

$$\Gamma\,(K^+ \longrightarrow \pi^+\pi^+\pi^-) + \Gamma\,(K^+ \longrightarrow \pi^+\pi^0\pi^0)$$

$$= \Gamma\,(K^- \longrightarrow \pi^-\pi^+\pi^-) + \Gamma\,(K^- \longrightarrow \pi^-\pi^0\pi^0).$$

If we exclude from the final states those with isospin $I_{3\pi} = 3$, then

$$\Gamma\,(K^+ \longrightarrow \pi^+\pi^+\pi^-) = \Gamma\,(K^- \longrightarrow \pi^-\pi^+\pi^-),$$
$$\Gamma\,(K^+ \longrightarrow \pi^+\pi^0\pi^0) = \Gamma\,(K^- \longrightarrow \pi^-\pi^0\pi^0).$$

8. We have so far been considering partial decay rates corresponding to individual decay channels. If we sum over all possible channels, we get the total decay rate (the lifetime).

We can approach this formally by means of the expression (19), if we take the square of the absolute value and use the unitarity of the S matrix.

Table II. Lifetimes of particles and their antiparticles, partial widths (literature references in last column)

$\mu^{\pm}$	$\frac{\tau^+}{\tau^-} - 1 = 0.0 \pm 0.1\%$	26
π^{+-}	$\frac{\tau^+}{\tau^-} - 1 = 0.56 \pm 0.28\%$	27
	$0.4 \pm 0.7\%$	28
	$0.23 \pm 0.40\%$	29
K^{+-}	$\frac{\tau^+}{\tau^-} - 1 =$	
Total widths:	$0.049 \pm 0.097\%$	29
	$0.47 \pm 0.30\%$	30
Widths $K \to \pi\nu$	$-0.54 \pm 0.41\%$	30
Widths $K \to 3\pi$	$-0.04 \pm 0.21\%$	30
Widths of $K \to 2\pi$ decays	$-050 \pm 0.90\%$	31
$\Gamma(K^+ \to \pi^+\pi^0)$	$(21{,}4 \pm 0.8)\ \%$	32
	$(21.0 \pm 0.56)\ \%$	33
Mean $\Gamma(K^+ \to \pi^+\pi^0)$	$(21.2 \pm 0.5)\ \%$	34
Single measurement of $\Gamma(K^- \to \pi^+\pi^0)$	$(25.0 \pm 3.3)\ \%$	35

Magnetic moments of particles and their antiparticles

e^{+-}	$\mu_- = (1.001159622 \pm 27)\ e/2m_e$	36
	$\mu_+ = (1.001168 \quad \pm 11)\ e/2m_e$	37
$\mu^{\pm}$	$\mu_+ = (1.001162 \quad \pm 5)\ e/2m_\mu$	38
	$\mu_- = (1.001165 \quad \pm 3)\ e/2m_\mu$	7

Accordingly, we arrive at the conclusion that if the theory is CPT invariant the lifetimes of particles are equal to those of their antiparticles. This prediction has been verified experimentally, and we give the most accurate results in Table II.

V. EQUALITY OF THE MAGNETIC MOMENTS OF PARTICLES AND THEIR ANTIPARTICLES

Let us consider the explicit expression for the matrix element of the electromagnetic current $J_\mu(x)$.[39] It follows from Lorentz invariance and charge conservation that for the proton the matrix element is of the form (we omit some normalizing factors and do not write the P-odd terms out explicitly)

$$\langle f;\ \text{out}\,|\,J_\mu(0)\,|\,i;\ \text{in}\rangle \equiv \langle p',\ s'\,|\,J_\mu(0)\,|\,p,\ s\rangle = = e\bar{u}(p',\ s')\,[F_1(q^2)\,\gamma_\mu + iF_2(q^2)\,\sigma_{\mu\nu}q_\nu]\,u(p,\ s), \tag{28}$$

where $q = p - p'$. Considering the static limit, i.e., letting $q \to 0$, one can easily connect $eF_1(0)$ with the charge of the proton and $eF_2(0)$ with its anomalous magnetic moment. From the fact that $J_\mu(0)$ is a Hermitian operator it follows that $F_1(q^2)$ and $F_2(q^2)$ are real functions.

To examine the consequences of CPT invariance for the electromagnetic interactions, we first note the transformation properties of the current operator which are necessary in order for the condition

$$(CPT)\,J_\mu\,|\,0)\,(CPT)^{-1} = -J_\mu(0). \tag{29}$$

to hold. From this we get in analogy with (4)

$$\langle \bar{f};\ \text{out}\,|\,J_\mu(0)\,|\,\bar{i};\ \text{in}\rangle = -\langle \bar{f};\ \text{out}\,|\,\bar{J}_\mu(0)\,|\,i;\ \text{in}\rangle^*. \tag{30}$$

We can calculate the right member, starting from Eq. (28):

$$\langle \bar{f}:\ \text{out}\,|\,J_\mu(0)\,|\,\bar{i};\ \text{in}\rangle = = -e\,\{\bar{u}(p',\ -s')\,[F_1(q^2)\,\gamma_\mu + iF_2(q^2)\,\sigma_{\mu\nu}q_\nu]\ u\ (p',\ -s')\}^*. \tag{31}$$

If we use the following spinor relations:

$$u(p',\ -s') = \gamma_0 T\bar{u}^T(p,\ s'), \qquad \bar{u}(p,\ -s) = u^T(p,\ s)\,T^{-T}\gamma_0, \tag{32}$$

we get

$$\langle \bar{f},\ \text{out}\,|\,J_\mu(0)\,|\,\bar{i};\ \text{in}\rangle = -e\bar{u}(p',\ s')\,[F_1(q^2)\,\gamma_\mu + iF_2(q^2)\,\sigma_{\mu\nu}q_\nu]\,u(p,\ s). \tag{33}$$

The change of sign in (33) as compared with (28) reflects the fact that the charges of particle and antiparticle are opposite. From (28) and (33) we conclude that CPT invariance leads to equality of the form-factors of particle and antiparticle. In particular, their anomalous magnetic moments must also be equal.

We can arrive at the same conclusion if we consider the expression for the energy of interaction of the magnetic moment μ with a magnetic field H: $(\mu \cdot H')$. Since under the CPT transformation $H' \to +H'$ and the interaction is CPT invariant, the magnetic moments of particle and antiparticle must also be equal. This last assertion has been verified experimentally only for electrons and muons.

One remark must be made about the exact measurements of $(g-2)$ for e^{+-} and μ^{+-}. The measured value is in good agreement with the results of calculations in which only the electromagnetic interactions are taken into account. The anomalous magnetic moments are also equal in virtue of the C or CP invariance of the electromagnetic interactions, and there is no indication of any deviation from such invariance. Consequently, the agreements between the anomalous magnetic moments of leptons may not be sensitive to violations of CPT.

Another remark relates to such a purely leptonic decay as $\mu \to e\nu\bar{\nu}$, and is extremely instructive. All of the experimental data about $\mu \to e\nu\bar{\nu}$ decay are well described by an effective local Lagrangian. Consequently, they cannot fail to correspond to the CPT theorem.

VI. CPT AND THE $K^0 - \bar{K}^0$ SYSTEM

Unique possibilities for testing CPT invariance are offered by the study of the decays of K^0 mesons, in particular the $K^0 \to 2\pi$ decays, in which violation of CP invariance has been shown. We indicate two possibilities for such tests, which have been discussed in the literature.[40,13,41,24,42]

1. The expression for the time dependence of the $K^0 \to 2\pi$ decays in beams which originally consisted only of K^0 or only of $\bar{K}^0$ is of the form

$$\frac{dN_\pm(\tau)}{d\tau} = \Gamma_S(2\pi)\,[A_\pm e^{-\Gamma_S\tau} + B_\pm\,|\,\eta\,|^2\,e^{-\Gamma_L\tau} + (D_\pm\eta e^{i(m_S-m_L)\tau} + \text{c. c})\,e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)\tau}], \tag{34}$$

where the sign + (−) relates to the case in which the beam was originally pure K^0 ($\bar{K}^0$) mesons. The expressions for $A_\pm$, $B_\pm$, $D_\pm$ (with squares of small quantities neglected) are[13]

$$\left.\begin{aligned} A_\pm &= \tfrac{1}{2} \mp \operatorname{Re}(\varepsilon - \delta), \\ B_\pm &= \tfrac{1}{2} \mp \operatorname{Re}(\varepsilon + \delta), \\ D_\pm &= \pm\tfrac{1}{2} - \operatorname{Re}\varepsilon - i\operatorname{Im}\delta, \end{aligned}\right\} \tag{35}$$

where if CPT invariance holds $\delta = 0$. To test CPT invariance it is necessary to measure the relative course of the time dependence and determine $A_\pm$, $B_\pm$, $D_\pm$ with an accuracy better than 10^{-3}.

The actual formulas connecting the parameters ϵ, ϵ', and ω with the amplitudes for the decays of K^0 and $\bar{K}^0$ and the expressions for the superpositions $|K_L^0\rangle$ and $|K_S^0\rangle$ in terms of $|K^0\rangle$ and $|\bar{K}^0\rangle$ depend on whether or not the requirements of CPT invariance are satisfied.

The most sensitive test is that of the phase relation[42]

$$\operatorname{Arg}(\eta_{+-} + R\eta_{00}) \equiv \operatorname{Arg}\varepsilon \equiv \varphi_\varepsilon = \operatorname{arctg}\frac{2(m_L - m_S)}{\Gamma_S + \Gamma_L} - \operatorname{arctg}\frac{\operatorname{Im}\langle K_S\,|\,K_L\rangle}{\operatorname{Re}\langle K_S\,|\,K_L\rangle}, \tag{36}$$

where η_{+-} and η_{00} are known parameters[43,13]

$$\left.\begin{aligned} \eta_{+-} &= \frac{(\pi^+\pi^-\,|\,T\,|\,K_L)}{(\pi^+\pi^-\,|\,T\,|\,K_S)} = \frac{\varepsilon + \varepsilon'/\sqrt{2}}{1 + \omega/\sqrt{2}}, \\ \eta_{00} &= \frac{(\pi^0\pi^0\,|\,T\,|\,K_L)}{(\pi^0\pi^0\,|\,T\,|\,K_S)} = \frac{\varepsilon - \sqrt{2}\varepsilon'}{1 - \sqrt{2}\omega}, \end{aligned}\right\} \tag{37}$$

and

$$R = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^+\pi^-)} \cong \frac{1}{2}.$$

To see the sensitivity of the relation (36), we note that if there is CPT (T) invariance

$$\operatorname{Im}\langle K_S\,|\,K_L\rangle = 0 \quad (\operatorname{Re}\langle K_S\,|\,K_L\rangle = 0)\ \text{and}\ \varphi_\varepsilon^{CPT} \cong \frac{\pi}{4}\ \left(\varphi_\varepsilon^T \cong -\frac{\pi}{4}\right).$$

In a more exact formulation[42] the relation (36) contains in the left member the argument of the quantity

$$Z = \sum_F (F \mid T \mid K_S)^* (F \mid T \mid K_L), \tag{38}$$

where the sum is taken over all decays common to K_S and K_L. Evidently the main contribution to Z is that from the decay into two pions, $Z_{2\pi}$. The experimental data on the quantity Z will become more accurate with further study of the violation of CP invariance in various decays of K^0 mesons. At present it is very important to determine the phase of the parameter ϵ; this will make possible a comparison with the relation (36).

A comparison of the contributions to Z from the decays $K^0 \to 3\pi$ and $K^0 \to 2\pi$, $Z_{3\pi}$ and $Z_{2\pi}$, can also be used to test CPT invariance.[44] If this invariance holds $(n\langle K_S|K_L\rangle \neq 0)$

$$\frac{\operatorname{Re} Z_{3\pi}}{\operatorname{Re} Z_{2\pi}} = \frac{\Gamma_L(3\pi) + \Gamma_S(3\pi)}{\Gamma_L(2\pi) + \Gamma_S(2\pi)}. \tag{39}$$

If T invariance holds

$$\frac{\operatorname{Im} Z_{3\pi}}{\operatorname{Re} Z_{2\pi}} = \frac{\Gamma_L(3\pi) + \Gamma_S(3\pi)}{\Gamma_L(2\pi) + \Gamma_S(2\pi)}. \tag{40}$$

VII. CONCLUSION

At present there exists convincing experimental evidence in favor of the correctness of the assumption of CPT invariance of the strong and electromagnetic interactions and the weak interactions which do not change the strangeness. Purely weak interactions are also CPT invariant. Semileptonic or strangeness-changing weak interactions must be studied with greater accuracy. It must also be noted that in all only some aspects of CPT invariance have been subjected to test. We have no single case of accurate measurements which test the relation (4) without summation over the spin directions. It is unquestionably worth while to make tests of such relations for conjugate processes with reversed spin directions, and of predictions of the type of Eq. (16). Future experiments with K mesons will allow us to test CPT invariance with greater accuracy.

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