SOVIET PHYSICS USPEKHI VOLUME 12, NUMBER 3 NOVEMBER-DECEMBER 1969

523.038

# COHERENT RADIO-EMISSION MECHANISMS AND MAGNETIC PULSAR MODELS*

V. L. GINZBURG, V. V. ZHELEZNYAKOV, and V. V. ZAĬTSEV

P. N. Lebedev Physics Institute, USSR Academy of Sciences;
Radiophysics Institute of the N. I. Lobachevskiĭ State University, Gor'kiĭ

Usp. Fiz. Nauk 98, 201-236 (June, 1969)

## I. INTRODUCTION

CONTEMPORARY astronomy brings one fundamental discovery after the other. It is sufficient to remind us that in the short period between 1960 and 1967 quasars, intensive radio emission in the OH lines, powerful sources of cosmic x-rays ("x-ray stars"), and the thermal background radiation with a temperature of $T = 2.7°K$ have been discovered. To these astronomical achievements was added in 1968 the discovery of pulsars.

In the 24th of February 1968 issue of "Nature," a note was published[1] about the discovery of pulsars, which for a number of reasons produced a genuine sensation. As early as on the 9th of March a new paper on pulsars appeared and by the end of May 1968 an international symposium was held in the USA devoted to pulsars; some of the results communicated at that symposium were published in [2].

Soon a number of radio-astronomical and optical observatories started to study pulsars. It is therefore not surprising that at the time of writing (December, 1968) there is already much information about pulsars. All the same one cannot yet consider the situation to be clear, neither as far as experimental data are concerned, nor, especially, on the theoretical plane. As far as we are concerned it is therefore too early to do any summing up. At any rate, we had no such aim in mind. We are interested primarily in problems of the mechanism of radio emission by the pulsars and of possible models of pulsars as radio sources. Of course, this is only part of the problem, but this part is just closely connected with radio-astronomy and plasma physics. This side of the problem will be just the main subject of our considerations in what follows. At the same time it seemed appropriate to give here some information about pulsars and also to make some comments which are not directly connected with the problem of their radio emission. In order not to complicate the exposition we refrained from giving any detailed literature survey. References to most papers on pulsars can be found by turning to the issues of "Nature," starting with the issue of 24th February 1968 (the literature published up to the middle of 1968 is given in [2]).

*This paper is mainly based upon a talk given on 26 September 1968 to the General Physics and Astronomy Division of the USSR Academy of Sciences. However, after the publication in October and November 1968 of important new results about pulsars, the paper was appropriately enlarged and changed.

## II. SOME INFORMATION ABOUT PULSARS

At the present time we know about ten thousand sources of cosmic radio emission—different galactic nebulae (among them shells of supernovae), normal galaxies, radio-galaxies, and quasars. The radio flux from most of these sources is constant, or, in a number of cases, changes but by not more than of the order of one per cent per year. In the case of quasars one has observed also larger changes, but all the same these changes in one day, say, are negligibly small. We must mention here that we are talking about the "true" oscillations in the radio flux from the source, and not about the scintillations of the sources caused by the effect of the ionosphere or the interplanetary medium.[3] As regards fast changes in the true radio flux of cosmic sources, up to the discovery of the pulsars they were observed only in the Sun and flare stars.

The most characteristic feature of the pulsars, distinguishing them from other sources, is just the fast and moreover periodic change in their flux. In actual fact, the radio emission $F(t)$ of the pulsars taken at some frequency $\nu$ (with a receiver bandwidth $\Delta\nu \ll \nu$) consists of pulses with a complicated structure and a total length $\Delta\tau$ of the order of 5 to 50 msec, following one another after a time (basic period) $\tau \sim 0.03$ to 2 sec. Undoubtedly, we are dealing here with cosmic radio-sources of a new type.

At the beginning of December 1968 20 pulsars were known which are given in the table. We have no doubt that the table will already be obsolete when our paper appears in print. In fact, at the beginning of February 1969 already 27 pulsars were known. (Note by translator: the number of known pulsars in June 1969 was 37.) However, as we have emphasized, the actual characteristics of the pulsars are given here only for convenience and illustration. The names given to most of the pulsars indicate the place where the pulsar was discovered and their position on the celestial sphere. For instance, the first pulsar discovered is CP 1919, which means the following: Cambridge (C) pulsar (P) with right ascension $19^h19^m$. Only for some of the pulsars detected in Australia do the three letters PSR stand for the word "pulsar." The positions of the pulsars on the galactic sphere ($\alpha$, $\delta$) and their galactic coordinates ($l$, b) do not on the whole indicate, when we take into account the incomplete scanning of the sky, any concentration of sources around some directions. However, the MP pulsars together with some of the PSR pulsars are clearly concentrated near the galactic plane.[4]

Information on pulsars

| Pulsar | Right ascension α | Declination δ | Galactic longitude $l$ | Galactic latitude b | Period τ, sec | $\int N_e\,dl$, pc-cm$^{-3}$ | Average flux at frequency ν (MHz) $\bar{F}_\nu$, flux units* |
|---|---|---|---|---|---|---|---|
| 1. NP 0532 | $05^h31^m30^s.6\pm$ $\pm0^s.43$ | 21°58′.8±1′.0 | 185° | −6° | 0.033 | 58 | 207 (ν = 100) |
| 2. PSR 0833—45 | $08^h33^m38^s.9\pm1^s$ | −45°.4±0.3° | 263°.5 | −2°.8 | 0.089 | 50 | 1.7 (ν = 400) |
| 3. PSR 1929+10 | $19^h29^m52^s\pm1^s$ | 10°52′49″±15″ | 48° | −4° | 0.227 | 8 | 0.2 (ν = 408) |
| 4. MP 1451 | $14^h5^m$ | −68° | 314° | −8° | 0.248 | — | 0.1 (ν = 408) |
| 5. CP 0950 | $09^h50^m28^s,95$ | 08°10′±1′ | 230° | 44° | 0.253 | 3.2 | 0,8 (ν = 81.5) |
| 6. JP 1933+16 | $19^h33^m10^s$ | 16°06′ | 52°.3 | −2° | 0 359 | 143± ±13 | — |
| 7. MP 0736 | $07^h36^m$ | −40° | 254° | −9° | 0.375 | 100 | 0,05 (ν = 408) |
| 8. AP 0823+26 | $08^h23^m52^s\pm20^s$ | 26°48′±5′ | 199° | 32° | 0.53 | 13.7 | 25 (ν = 111) |
| 9. AP 2015+28 | $20^h15^m45^s\pm20^s$ | 28°31′±5′ | 69° | −4°,0 | 0.558 | — | 6.0 (ν = 611) |
| 10. PSR 1749—28 | $17^h49^m48^s.8\pm$ $\pm0^s.3$ | −28°05′57″± ±8″ | 1.6° | −1°.0 | 0.562 | 50.88 | — |
| 11. CP 0328 | $03^h28^m52^s\pm15^s$ | 55°±1° | 145° | 0° | 0.71 | 18± ±1.5 | 0.2 (ν = 81.5) |
| 12. HP 1506 | $15^h07^m40^s\pm30^s$ | 55°30′±1° | 90° | 53° | 0.73 | 15 5± ±1 | 0.3 (ν = 150) |
| 13. MP 0835 | $08^h35^m$ | −40° | 260° | 0° | 0.765 | 120 | 0.1 (ν = 408) |
| 14. MP 1426 | $14^h26^m$ | −66° | 313° | −5° | 0.79 | 60 | 0.1 (ν = 408) |
| 15. MP 1727 | $17^h27^m$ | −50° | 341° | −9° | 0.83 | 140 | 0.1 (ν = 408) |
| 16. CP 1133 | $11^h33^m32^s\pm20^s$ | 17°00′±45′ | 240° | 70° | 1.18 | 6 | 0.3 (ν = 81.5) |
| 17. CP 0834 | $08^h34^m07^s\pm15^s$ | 07°00′±45′ | 220° | 26° | 1 27 | 12±1 | 0.3 (ν = 81.5) |
| 18. CP 0808 | $08^h08^m50^s\pm30^s$ | 75°10′±30′ | 140° | 34° | 1.29 | 10±1 | 1 5 (ν = 81.5) |
| 19. CP 1919 | $19^h19^m37^s\pm0^s.2$ | 21°47′02″±10″ | 56° | 4° | 1.33 | 12.55 | 1.0 (ν = 81.5) |
| 20. PSR 2045—16 | $20^h45^m47^s.6\pm$ $\pm0^s.4$ | −16°27′50″± ±12″ | 30°.5 | −33°.1 | 1 96 | 11.40 | |

*The flux unit is $10^{-26}$ W m$^{-2}$Hz$^{-1}$ = $10^{-23}$ erg cm$^{-2}$ sec$^{-1}$ Hz$^{-1}$. For the pulsar NP 0532 we have given the flux $\bar{F}_\nu$ for the whole of the compact radio source in the Crab nebula; the radio flux of the pulsar itself is appreciably weaker.

The period at which the pulses from the pulsars are repeated is indicated in the table only with three significant figures to make comparison simple; the period is, however, known with enormously larger accuracy. For instance, for the pulsar CP 1919, τ = 1.33730113 sec. With the same accuracy of the order of $10^{-8}$ one has found no changes in the periodicity of the pulsar CP 1919, i.e., its "rate" changes less than by one second per year. For most other pulsars the accuracy with which τ has been measured is slightly less, and also there no change in the period τ has been observed (for the pulsar CP 0328 this statement[5] is made with an accuracy reaching $10^{-9}$ sec*). At the same time different pulses, even ones following immediately upon one another can be completely different both in intensity and shape. The fact that the period of the oscillations of the pulsars is constant must in this connection be understood to mean that the period of the corresponding Fourier-components of the radiation flux is constant.

The shapes of the pulses are different for different pulsars. For instance, the pulse of the pulsar CP 0950 is a single one and a rather short one (Δτ ~ 10 msec); however, at a distance of about 100 msec before this pulse there is a "precursor" which corresponds at a frequency of 408 MHz to a flux of 1.8%, on the average, of the main pulse.[2] In [5] it is noted that on average the pulse width Δτ increases with the pulsar period (i.e., approximately Δτ ∼ τ). For a number of pulsars (CP 1919, CP 0834, CP 1133, and others) it was noted already in a relatively early stage of their study that the pulse often splits up into two or three subpulses. The average shape of the pulse with a fine structure of sub-pulses was determined (see [2,8]). It was assumed that the structure of the pulse reflects the nature of the emitting region and the processes in it. For instance, if the pulsar is a non-rotating dense star emitting from the regions of its magnetic poles[6] there

*We received information at the end of December 1968 that the period of repetition of the pulses τ is slowly increasing (reported to the Texas Symposium on Relativistic Astrophysics). For instance, for the Crab Nebula pulsar NP 0532 the period increases approximately by $10^{-5}$ sec per year, i.e., $\delta\tau/\tau \sim 3 \times 10^{-13}$ and the change per period is $\delta\tau \sim 10^{-14}$ sec; the period doubles over a time of the order of $3 \times 10^3$ yr. For the pulsar CP 0834 $\delta\tau/\tau = (5 \pm 0.8) \times 10^{-15}$ and the period doubles in $8.10^6$ yr; for the pulsars CP 1133, CP 1919, and CP 0950 the periods double in $9 \times 10^6$, $4 \times 10^7$, and $3 \times 10^7$ yr, respectively. For the pulsar PSR 0833-45 which lies in the shell of the supernova Vela X, the period is equal to 0.089 sec and doubles in $2.4 \times 10^4$ yr.[4a]

would naturally occur a splitting into two subpulses (one is dealing with a dipole field, i.e., with two poles), and so on.

The most important fact observed after the discovery of the pulsars[1] was, however, the detection[7] of the regular character of the subpulses and their connection with the presence of a second period $\tau_2$ (we call the time between the pulses the first or basic period $\tau_1 \equiv \tau$). In [7] it was shown, using the pulsars AP 2015+28 and CP 1919 as examples, that at least for those pulsars the radio flux oscillates also with a short period $\tau_2 \ll \tau_1 \equiv \tau$. For AP 2015+28 the period $\tau_2 = 10.6879 \pm 0.0002$ msec, i.e., $\tau_2 \approx 10^{-2}$ sec, while $\tau_1 = 0.558$ sec. For CP 1919 the period $\tau_2 \approx 15.5089 \pm 0.0001$ msec. It is important that the period $\tau_2$ is by far less constant than the period $\tau_1$. To be precise, the phase of the second oscillations changes by $2\pi$ over $4 \times 10^3$ periods $\tau_2$. This means that if we were dealing with a damped harmonic oscillator, we would have a Q-factor

$$Q = \frac{\text{stored energy}}{\text{loss of energy per period}} \approx 4 \times 10^3$$

At the same time, if the main period $\tau_1$ were connected with free oscillations, we would have for it $Q > 10^8$–$10^9$.

The period $\tau_2$ is in practice incommensurable with the period $\tau_1$ and this leads just to the apparent variability of the structure of the pulse. To illustrate at once what is the matter, we give a simple model which would have the properties characteristic for the above-mentioned pulsars. Let us imagine a lighthouse whose radiating "lantern" rotates with a period $\tau_1$ while emitting a light beam with an aperture $\Delta\varphi$. An observer on a ship will then see flashes of the emission from the lighthouse with a period $\tau_1$ and a duration $\Delta\tau \approx \Delta\varphi\,\tau_1/2\pi$. If the brightness of the lighthouse is now modulated as $F = F_0 + a\cos(2\pi t/\tau_2)$, i.e., if the brightness changes with a period $\tau_2$ ("blinking" lighthouse), one will at once observe the characteristics which we want to describe if the periods $\tau_1$ and $\tau_2$ are incommensurable. At the present time it is not yet known whether all pulsars have a second period. Logically there is no reason whatever for this to be the case. Moreover, on the basis of known models one might already assume that there are pulsars of different types. On the other hand, at present we cannot likewise exclude the possibility of a very simple hypothesis, according to which we have observed so far only one kind of pulsar with two periods, having the same characteristics as the model of a simultaneously rotating and blinking lighthouse.*

To avoid misunderstandings we emphasize that when we talk below about the pulse of the radio-emission of the pulsars we have in mind, as before, the whole pulse that repeats with a period $\tau_1 \equiv \tau$. We shall call the different components of the pulse, and in particular the components which have a period $\tau_2$, subpulses.

As we have already emphasized, the amplitudes of the pulses vary strongly. Moreover, at some times the pulses disappear completely (become indistinguishable from the noise in the apparatus) and afterwards appear again. For instance, in the case of CP 1919 at a frequency of 81.5 MHz the pulses were observed[1] on the average during approximately 1 minute and then during about 3 minutes the pulsar "was silent." At higher frequencies the length of the pauses increases in some cases although the ratio of the length of the active phase to the pause length varies, perhaps, little.

The emission of the pulsars is, generally speaking, observed in a wide band of frequencies. For instance, the pulsar CP 1919 has been observed in the range from 40 MHz ($\lambda$ = 7.5 m) to 3000 MHz ($\lambda$ = 10 cm). The spectral radiation flux $F_\nu$ decreases with increasing frequency. If, as is usually done, the spectrum is approximated by a power law $F_\nu = \text{const}\cdot\nu^{-\alpha}$, the spectral index for CP 1919 is $\alpha = 1.5$ (in the frequency range 40 to 400 MHz) and $\alpha = 3.2$ (in the frequency range 400 to 2300 MHz). For the pulsars CP 0834, CP 0950, and CP 1133 we have, respectively, $\alpha$ = 1.2; 1.1; and 1.1; in these cases one did not observe a bend in the spectrum.

The average flux from the pulsars is given in the table and for most cases does not exceed a unit flux, i.e., $10^{-26}\,\text{W m}^{-2}\,\text{Hz}^{-1} = 10^{-23}\,\text{erg cm}^{-2}\,\text{Hz}^{-1}\,\text{sec}^{-1}$. Separate pulses are appreciably stronger: for CP 1919 at a frequency of 81.5 MHz ($\lambda$ = 3.86 m) one has observed[1] pulses with a flux $F_\nu \approx 20 \times 10^{-26}\,\text{W-m}^{-2}\,\text{Hz}^{-1}$ (we are dealing here with the peak value of the spectral flux density averaged in a band of 1 MHz width). Since the radiation flux of pulsars varies strongly, the flux values given in the table are only meant to give a very rough idea of their magnitude.

The pulses occur not simultaneously at different carrier frequencies: the higher the frequency the earlier the pulse occurs. This effect is undoubtedly connected with the retardation of the pulse in the plasma lying on the path from the pulsar to the observer. In practice we are dealing here with the interstellar plasma. Let us explain these statements.

It is well known that when we neglect the influence of the magnetic field and of collisions, which is fully justified in the case of interest to us (see, for instance, [9]), we have $n^2 = 1 - (\omega_L^2/\omega^2)$, where n is the refractive index, $\omega = 2\pi\nu$ is the frequency, $\omega_L = 2\pi\nu_L = (4\pi e^2 N_e/m)^{1/2}$, and $N_e = 1.24 \times 10^4 \nu_L^2$ is the electron density (in the last expression $N_e$ is in cm$^{-3}$ and $\nu_L$ in MHz). In the interstellar medium with $N_e \sim 0.1\,\text{cm}^{-3}$ the frequency $\nu_L \sim 3 \times 10^{-3}\,\text{MHz} = 3 \times 10^3\,\text{Hz}$. Hence we have in the radio-astronomical band with a large margin $\nu^2 \gg \nu_L^2$, and $n = 1 - (\nu_L^2/2\nu^2)$. Moreover, the group velocity of the radio waves $v_{gr} = d\omega/dk = c/(d(\nu n)/d\nu) = cn$ and the time of group retardation is

$$t_{gr} = \int_0^R \frac{dl}{v_{gr}} \approx \frac{R}{c} + \frac{\int_0^R \nu_L^2\,dl}{2c\nu^2}\,.$$

Hence

$$\frac{d\nu}{dt_{gr}} = -\frac{c\nu^3}{\int_0^R \nu_L^2\,dl}\,.$$

Measuring $d\nu/dt_{gr}$ and the frequency $\nu$ we can check

*Unfortunately, we do so far not know of any confirmation of the conclusions of [7] by other observers (however, there is indirect confirmation in [27]). Under the circumstance, the very probable conclusion about the presence of the period $\tau_2$ can not yet be considered to be rigorously proven (see in this connection the note added in proof at the end of this paper).

the relation obtained, $d\nu/dt_{gr} = -\text{const}\cdot\nu^3$. The observations confirm this dependence and this gives us the possibility to determine $\int_0^R \nu_L^2\, dl$ and, hence, the number of particles along the line of sight $\int_0^R N_e\, dl$. The values of $\int_0^R N_e\, dl$ obtained in this way in pc-cm$^{-3}$ units are given in the table.

It is clear from the table that for all known pulsars $\int_0^R N_e\, dl \gtrsim 3$ pc-cm$^{-3}$ $\approx 10^{19}$ cm$^{-2}$. The value of $\int_0^R N_e\, dl$ for the terrestrial ionosphere and for the interplanetary medium is appreciably smaller. The retardation of the signals from the pulsars must thus take place either in interstellar space or in the source itself. The latter possibility is very unlikely, again because of the large values of $\int_0^R N_e\, dl$ (when we restrict ourselves to values of $N_e$ which guarantee that the relation $d\nu/dt_{gr} = -\text{const}\cdot\nu^3$ is satisfied). On the other hand, in the case of the interstellar medium the occurrence of the observed values of $\int_0^R N_e\, dl$ is completely as to be expected, since they lead to reasonable values for the distances to the pulsars.

If we assume that the average value of $N_e$ in the interstellar space is equal to 0.1 cm$^{-3}$ we find for the distance to the pulsar as a first estimate $R = 10\int_0^R N_e\, dl$ pc. Just such an estimated distance has often been used in the literature. It is clear that the true distance remains as yet undetermined; most likely, for most directions the average value of $N_e < 0.1$ and hence $R > R_0$. For instance, according to [10], we have on the average in the arms of the galactic spiral $0.01 \lesssim N_e \lesssim 0.05$ and for H I regions it is reasonable to use the value $N_e \approx 0.03$, whence $R \sim 3R_0 = 30\int_0^R N_e\, dl$.

It is possible, especially for some directions, that the electron density in the interstellar plasma is even lower. For instance, in the spectrum of the pulsar CP 0328 it has been possible to observe[11] the hydrogen absorption line (wavelength 21 cm) and to use the Doppler effect in this line to find the velocity of the absorbing neutral hydrogen clouds. As a result the conclusion was reached that the pulsar CP 0328 was not lying closer than 4.2 kpc = $1.3 \times 10^{22}$ cm. From this it follows that the plasma concentration along the line of sight to the pulsar CP 0328 is not more than $N_e = 0.006$ cm$^{-3}$ since in this case $\int_0^R N_e\, dl = 26.75$ pc-cm$^{-3}$ (see the table and [5,11]).* In [12] the quantity $\int_0^R N_H\, dl$ was determined for our Galaxy in the directions of several pulsars (here $N_H$ is the neutral hydrogen density). Using these values of $\int_0^R N_H\, dl$ the distance R to the pulsars was estimated in [12] under some reasonable assumptions about the degree of ionization of the interstellar medium. This distance turned out to correspond to an average value of the electron density (taking contributions both from the disc and from the halo into account) equal to 0.02 to 0.05. In accordance with what we said earlier it is therefore at the moment appropriate to choose for an estimate of the average distance to the pulsars the distance $R \sim R_0$. However, we shall sometimes, as is also done by other authors for the sake of simplicity put $R \sim R_0$. Of course this value may in each separate case possibly be improved and such a more precise determination has already been done for a number of pulsars.[11,12] Independent of the estimate of pulsar distances connected with an actual choice of the electron density in interstellar space, their localization within the limits of our Galaxy is certain (in any case it can be asserted for most of the pulsars).

Knowing the distance R from the pulsar and the radio flux on the Earth $F_\nu$ we find the spectral radio power $P_\nu = 4\pi R^2 F_\nu$ and the total power (radio brightness) of the pulsar $P = 4\pi R^2 \int F_\nu\, d\nu$, assuming that the emission is isotropic. The average spectral power $\overline{P}_\nu$ and average total power $\overline{P}$ are connected with the averaged (over a period $\tau_1$) emission flux $\overline{F}_\nu$ by similar relations. The observed values of $\overline{F}_\nu$ are given in the table. Using these data one estimates easily that, for instance, for the pulsar CP 1919 at $R = R_0 = 126$ pc the average total power for all frequencies $\nu > 40$ MHz is $\overline{P} \approx 10^{28}$ erg/sec. For "blinking" pulsars giving off an isotropic radiation with $\tau \sim 1$ sec we cannot distinguish between the energy per pulse and the average power: $P \sim 10^{28}$ erg sec$^{-1}$ $\sim 10^{28}$ erg/pulse; for some pulsars $P \sim 10^{29}$ erg/pulse. The length of the pulse $\Delta\tau \sim 50$ msec, and clearly the power in the pulse itself reaches $10^{30}$ erg/sec, although usually $P \sim 10^{29}$ erg/sec. If the distance to the pulsar is $R = 3R_0$, i.e., thrice that taken earlier, we have correspondingly $\overline{P} \lesssim 10^{30}$ erg/sec and $P \lesssim 10^{31}$ erg/sec. This last value is only two or three orders of magnitude less than the solar brightness $L_0 = 3.86 \times 10^{33}$ erg/sec. If we use the distance R = 4.2 kpc for the pulsar CP 0328, its average energy output in the radio band would be, assuming isotropic emission, $2 \times 10^{31}$ erg/sec. During the pulse which has a length of about 7 msec, the instantaneous radio power would be equal to $3 \times 10^{33}$ erg/sec which is comparable with the total solar brightness in the optical band. Even more remarkable is that the radio power of the pulsars exceeds by many orders of magnitude the power of the sporadic solar radio emission. For instance, the total solar radio power during type III bursts is usually not more than $10^{18}$ erg/sec although the maximum power for some type II events may exceptionally reach $10^{23}$ erg/sec.

However, we have in the foregoing assumed that the radiation from the pulsar is isotropic, or to be precise quasi-isotropic like the emission from a dipole. However, in rotating "lighthouse" models we are clearly dealing with an essentially anisotropic emission with an angle of the directional diagram of the emitter of

*The data from [11] about the remoteness of the pulsar CP 0328 have not been confirmed. According to [11a] and other measurements on the 21 cm line the distance to the pulsar CP 0328 is less than 1 kpc.

$\Delta\varphi \sim 2\pi\Delta\tau/\tau_1 \sim 0.1 \approx 6^\circ$ for $\Delta\tau \sim 20$ msec and $\tau \equiv \tau_1 \sim 1$ sec. If the diagram has a cone ("pencil") shape the solid angle corresponding to this diagram is $\Delta\Omega \sim (\Delta\varphi)^2 \sim 10^{-2}$. For a beavertail diagram which is also possible in "lighthouse" models (see below) $\Delta\Omega \sim 2\pi\Delta\varphi \sim 1$. At the same time for a quasi-isotropic emitter $\Delta\Omega \sim 4\pi$. From this it is clear that if the power in a pulse for a quasi-isotropic emitter is equal to $P_p$ and the power averaged over the period $\tau_1$ is $P = P_p\Delta\tau/\tau_1$, in "lighthouse" models both these powers are equal to $P_p\Delta\Omega/4\pi$. One checks easily that in the case of a beavertail diagram $\Delta\Omega/4\pi \sim \Delta\tau/\tau_1$ and that thus the average power of the radio-emission of a "blinking" pulsar with an isotropic diagram and that of a "lighthouse" type pulsar with a beavertail diagram will be approximately the same.* For a "pencil" diagram, however, the average power is smaller by a factor $\tau_1/\Delta\tau$ for the same power in a pulse. For instance, for the pulsar CP 0328 with $\tau_1 \approx 0.71$ sec and $\Delta\tau \approx 7 \times 10^{-3}$ sec it is smaller by a factor $\sim 10^2$. The conclusion that the emission power of the pulsars is by many orders of magnitude larger than the power of the sporadic radio emission of the Sun remains, clearly, unchanged.

One of the most important facts observed[13,2] after the discovery of the pulsars[1] is the presence of a strong polarization of the pulsar radiation. Generally speaking, the polarization is elliptic, but frequently it happens to be linear and sometimes circular. The degree of polarization is in some cases nearly 100% for different subpulses.

We note finally that so far it has not been possible to identify any of the pulsars with a visual object,† i.e., the pulsar is in an optical respect a rather faint star. Such a conclusion is nearly trivial for the photospheres of neutron stars as their surface area is very small and it is in general impossible to observe them in the optical part of the spectrum even at distances of the order of a parsec. As far as white dwarf stars are concerned, they have been observed at distances of the order of 10 to 100 pc provided they are not too cold. The absence of appreciable optical emission from the pulsars indicates thus only that they are not too near and moreover that they are not hot (that means ordinary) white dwarfs or some other kind of known stellar object.

This is the basic information we have about pulsars; we must yet again emphasize that we have only given this for the sake of orientation and convenience, but that we can in no way assign greater significance to it.

*At the same time the energy emitted during a period of pulsation will be different because of the different values of the period: $\tau_1$ for a "blinking" pulsar and $\tau_2$ for a "lighthouse" type pulsar.

†In the middle of January 1969 we received a telegram that the pulsar NP 0532 had been observed in the optical band – the brightness of one of the stars in the Crab nebula changes with a period $\tau_1 = 0.033$ sec (see [63-65]). The pulsar NP 0532 in the Crab nebula differs rather much from all other known pulsars (except perhaps the pulsar PSR 0833-45) and is clearly extremely young. If we are dealing with a neutron star with a very strong magnetic field (see Sec. VII) the emission of a rather large amount of light is a very significant fact.

It is possible for us to assume that the optical emission from a pulsar is incoherent synchrotron radiation of relativistic electrons that form an annular radiation zone in the equatorial part of the magnetosphere of a rotating neutron star (see also the notes added in proof at the end of the paper).

## III. PULSARS AS ASTROPHYSICAL OBJECTS

What is the nature of the pulsars? Already in [1] the assumption was made that we are dealing with the vibrations (radial oscillations) of white dwarfs or neutron stars. After that models of dense rotating stars or close binary systems (i.e., a pair of stars or a star with a satellite)[14] were also proposed. A binary system emits gravitational waves and turns out to be unstable;[15] it is therefore unlikely that binary systems are possible sources of pulsating radio emission.

The period of pulsations of neutron stars does, generally speaking, not exceed $10^{-2}$ to $10^{-3}$ sec.[16,17] After this fact had become clear and prior to publication of [7], the basic period of the pulsars $\tau_1 \equiv \tau$ was connected either with the vibrations of white dwarfs or with the rotational periods of neutron stars (see, for instance, [2,6,18]). The fundamental tone of the radial oscillations of white dwarfs can have a period of $\tau \gtrsim 1$ sec. One can try to connect periods of $\tau \lesssim 1$ sec with overtones. At the same time there arises the natural possibility[6] for a powerful radio emission in the case of a pulsating star and especially of a pulsating magnetic star. However, in the model of a stationary (non-pulsating) rotating star with some "hot spot" localized on its surface as the source of directed radio emission, the mechanism for the radio emission remained unclear. The authors of the present paper concentrated therefore their attention first of all on models of vibrating white dwarfs.[6] We wish to note, however, that we were also interested in hypothetical models of vibrating magnetic neutron stars and models of rotating stars with "pencil" or "beavertail" diagrams (these aspects were reflected in the talk which was the basis of the present paper).

The observation of a second period $\tau_2 \sim 10^{-2}$ sec for two pulsars[7] at once introduced an essentially new element into the whole situation. It is clear that for pulsars with two periods $\tau_1 \lesssim 1$ sec and $\tau_2 \lesssim 10^{-2}$ sec the most probable model is that of a neutron star which pulsates with a period $\tau_2$ and rotates with a period $\tau_1$. Such a synthetic model does not encounter the difficulties inherent in the model of a pulsating white dwarf (smallness of the period $\tau_1$, problem of the structure of the pulses, absence of optical identification). One can also, of course, adduce in favor of this synthetic model the fact that the period $\tau_1$ is so highly constant while the period $\tau_2$ is appreciably less stable (for details see Sec. VII).

We have already mentioned that one should not exclude the existence of different kinds of pulsars corresponding, say, to rotating and vibrating neutron stars, to practically non-rotating neutron stars which vibrate, and, finally, to vibrating and rotating white dwarfs. However, if the emission is not caused by pulsations, but excited in some other way during the rotation of the star,[14,18] we have rotating dense stars as models for the pulsars. However, whichever of these objects we are dealing with, one may suppose that their radio emission is generated in the relatively extended atmosphere of the object which is in a state of a rarefied plasma. We can thus distinguish in the analysis of pulsar models two main problems: an "internal" one connected with a discussion of the star itself, its vibrations, rotation and their coupling, and an "external" one connected with the mechanism of the radio emission and in general with the processes in the atmosphere or, perhaps better, the corona of the star.

Such a division is expedient, since in these problems we will be dealing with completely different conditions. It is sufficient to point out that the average density of the stars we are discussing lies between about $10^7$ to $10^9$ g/cm$^3$ (dense white dwarfs) and $10^{13}$ to $10^{15}$ g/cm$^3$ (neutron stars). On the other hand, radio-emission with a frequency $\omega$ comes from a region where the plasma frequency is $\omega_L = (4\pi e^2 N_e/m)^{1/2} \lesssim \omega$ (for the sake of simplicity we do not consider here the influence of a magnetic field and also do not consider the possibility that the plasma moves with a high velocity). When $\nu = \omega/2\pi = 100$ MHz we get then an electron density $N_e = 1.24 \times 10^4 \nu_L^2 \lesssim 10^8$ cm$^{-3}$, and for the density we have $\rho \lesssim (10^{-23}$ to $10^{-24}) N_e \lesssim 10^{-15}$ to $10^{-16}$ g/cm$^3$.

Of course, the "internal" and the "external" problems are connected: the processes in the corona determine to some extent the state of the main part of the star and reflect the processes which take place there. All the same, the discussion of the radio emission mechanism is not only of interest in itself but also must be an aid in solving the problem of the structure and the different characteristics of the dense parts of the pulsars. It is in this context that our research has been developed as will be shown in what follows (see also [6,6a]).

## IV. COHERENT RADIO-EMISSION MECHANISMS OF PULSARS

Up to very recently galactic and metagalactic radioastronomy was dealing only with incoherent radio emission mechanisms. The main role was there played by the incoherent synchrotron (magnetic-bremsstrahlung) mechanism which is connected with the radiation by relativistic electrons in interstellar magnetic fields; this mechanism explains well such phenomena as the distribution of the radio emission in the Galaxy and the radiation from supernova remnants and radiogalaxies.

In the case of an incoherent mechanism of radio emission the intensity of the radiation by the source is equal to the sum of the intensities of the spontaneous emission from separate elements of the source (excited atoms, radiating electrons, and so on) provided the reabsorption of the radiation in the source is unimportant. This is the case under conditions where the optical thickness of the source $\mu L \ll 1$ ($\mu$ is the reabsorption coefficient and L the size of the source). If $\mu L \gg 1$, the intensity of the radiation from the source is equal to the sum of the intensities of only those separate elements of the source which are situated in a surface layer of thickness $l \approx 1/\mu$, i.e., in a layer with optical thickness equal to unity. The radiation from deeper layers does not get outside the source because of the absorption. From what we have said it is clear that increasing the number of radiating particles per unit volume only leads to an increase in the radiation intensity of the source as long as $\mu L \lesssim 1$. A further increase in the concentration does not lead to an increase in the radiation intensity. The fact is that the reabsorption coefficient $\mu$ is proportional to the concentration of the radiating particles and an increase in concentration therefore leads to a corresponding decrease in the thickness of the effectively radiating layer $l \sim 1/\mu$; as a result the total radiation intensity remains unchanged.

What we have said is particularly clear for the example of thermal radiation produced by particles with an equilibrium energy distribution (this is a special case of an incoherent radiation mechanism). The role of the limiting radiation intensity of the source is here played by the intensity of the radiation from an absolutely black body. The effective radiation temperature $T_{eff}$ is in that case equal to the kinetic temperature T of the radiating particles where $T \sim \mathcal{E}_{av}/k$ ($\mathcal{E}_{av}$ is a characteristic particle energy and k the Boltzmann constant). The estimate $T_{eff} \sim \mathcal{E}_{av}/k$ remains valid also for non-equilibrium distributions of the radiating particles with an energy dispersion $\Delta\mathcal{E} \sim \mathcal{E}_{av}$ as such a distribution is not essentially different from the equilibrium distributions.

As in each actual case for incoherent mechanisms there exists an upper limit for $T_{eff}$ it is clear that such mechanisms are primarily effective for extended sources which are not too dense, i.e., for objects with a comparatively low effective temperature.

Recently, however, several kinds of powerful sources with small angular dimensions have been discovered—quasars, OH emission sources, and, finally, pulsars. The enormous values of $T_{eff}$ in those sources makes an interpretation of them on the basis of incoherent mechanisms improbable—it turned out to be necessary here to include coherent radio-emission mechanisms* which had earlier been discussed only in connection with the Sun, Jupiter and flare stars.[22]

Indeed, for pulsars $T_{eff} \sim 10^{21}$°K and, possibly, even one or two orders of magnitude higher. This value of $T_{eff}$ leads to an estimate of the characteristic energy of the radiating particles of $\mathcal{E}_{av} \sim T_{eff}/k \gtrsim 10^{17}$ eV. Particles with such an energy cannot be present in appreciable numbers in any reasonable pulsar model. The radio-emission mechanism of pulsars can thus not belong to the incoherent class. Such a high value of $T_{eff}$ can be guaranteed only by a coherent radio-emission mechanism in which the above-mentioned upper limit on the value of $T_{eff}$ does not exist.

This last circumstance is connected with the fact that for coherent mechanisms the intensity of the radiation from a source exceeds the sum of the intensities of the spontaneous emission from the separate radiating particles which are present in the source. In the case of distributed sources of radio-emission with dimensions $L \gg \lambda$ ($\lambda$ is the wavelength; only this kind of source is considered in practice in astrophysics) this is achieved owing to the existence in the system of a region with a negative reabsorption coefficient, $\mu < 0$. Values $\mu < 0$, which are realized under relatively sharp deviations from equilibrium in the distributions of the radiating particles in the source, lead to an amplification of the radiation from each element in the source

*With one exception: for quasars the incoherent synchrotron mechanism does not usually encounter serious difficulties for the explanation of the observed intensity. In that case the inclusion of coherent emission mechanisms is possible, but not necessary.[19,20] See also [21] about the limits of applicability of the incoherent synchrotron radio-emission mechanism.

(according to the relation $e^{-\mu l} = e^{|\mu| l}$*) before it leaves its boundaries. In the following we shall apply the term "coherent radiation" in just that sense. There are, of course, coherent sources of another kind (in particular, particle bunches, or "antennae" with dimensions or with one dimension, small compared to the wavelength) but they are hardly of any interest under cosmic conditions.

When discussing the nature of pulsars one assumes usually that the radio-emission comes from an object of dimensions $L \lesssim (1 \text{ to } 5) \times 10^8$ cm. This conclusion is, first of all, based upon the fact that this is just the value of the radius of a white dwarf vibrating with a period $\tau \sim 1$ sec. For neutron stars with a period of vibrations $\tau_2 \sim 10^{-2}$ sec the radius of the dense neutron and plasma part of the star is $r_n \sim 10^7$ cm (see [17]), but the extended plasma shell and also probably the magnetosphere reach a size of $r \gtrsim 10^8$ cm. Secondly, one commonly assumes that the dimension of the radiating region is $L \lesssim c\Delta\tau$, where $\Delta\tau$ is the length of the pulse and c the velocity of light (more exactly, we should have here the group velocity of the waves). If the condition $L \lesssim c\Delta\tau$ is violated the variation in the radio emission will be appreciably smoothed out due to the different retardation of signals from different parts of the extended source. For pulsars $\Delta\tau \sim (1 \text{ to } 5) \times 10^{-2}$ sec and $L \lesssim c\Delta\tau \sim (3 \text{ to } 15) \times 10^8$ cm (if we are dealing with a separate subpulse, its length in the cases we know about is of the order of $\tau_2 \sim 10^{-2}$ sec; it is clear that changing $\Delta\tau$ to $\tau_2$ is inessential in the example given).

It is relevant to note that this estimate cannot be doubted. In the case of a coherent radiation mechanism, when the intensity I increases exponentially, for an appreciable change in I it is necessary to change the parameters of the source in a layer for which $|\mu| l \sim 1$. There will then be no smoothing out, if the signal traverses a distance $l \sim 1/|\mu|$ during a time not more than $\Delta\tau$, i.e., if the following condition is satisfied:

$$l \sim \frac{1}{|\mu|} \lesssim c\Delta\tau. \tag{1}$$

Hence

$$\frac{L}{|\mu| L} \lesssim c\Delta\tau, \quad \text{or} \quad L \lesssim c\Delta\tau |\mu| L. \tag{2}$$

For an effective coherent mechanism $|\mu| L \gg 1$, and hence the inequalities (1) and (2) are appreciably weaker (by a factor $|\mu| L \gg 1$) than the condition $L \lesssim c\Delta\tau$. If, say, $|\mu| L \sim 10^2$, the size of the radio-emission region of the pulsar is $3 \times 10^8 \times 10^2 \sim 3 \times 10^{10}$ cm, i.e., of the order of the size of the Sun. From this it follows that a convincing basis for an estimate of the dimensions of the pulsar itself is in first instance the repetition period of the pulses or subpulses, i.e., the nature of the source. However, one can also say that the dimensions of the region in the source which is sufficiently actively radiating is $l \sim 1/|\mu| \lesssim c\Delta\tau \sim 3 \times 10^8$ cm. This is, of course, also a limitation.

Changing now to a discussion of coherent mechanisms for the generation of the radio emission of pulsars, we note at once that there are two basic variants of such mechanisms.

1) Negative reabsorption is at once realized for electromagnetic waves which freely leave the source. As an example of such a mechanism we may quote the coherent synchrotron mechanism which operates in the system of a "cool" plasma + relativistic electrons in a magnetic field.[20,21,23] For an appropriate choice of the electron energy spectrum (for instance, for a spectrum with a sufficiently narrow maximum) the reabsorption coefficient connected with the synchrotron radiation can in such a case become negative and waves emitted by the separate relativistic electrons in the system are accelerated before they leave the confines of the system.

2) First of all, plasma waves are excited (accelerated) (i.e., the picture of the acceleration of waves in the system refers not to electromagnetic but to plasma waves), and afterwards during the transformation (conversion) process part of the energy of these waves is transformed into the energy of electromagnetic radiation which leaves the source. An example is the acceleration and instability of plasma waves in a "current-plasma" system and the conversion of the waves when their scattering is taken into account (see [19,22,24]).

Variant (1) is attractive because all the electromagnetic energy leaves the source (without conversion processes that diminish the effectiveness of the operation of the generation mechanism). Variant (2) usually leads to an attenuation of the radiation in the conversion process (in the conditions on the Sun[22] the degree of transformation drops to $10^{-6}$). However, for pulsars—for sources with $L \sim 3 \times 10^8$ cm and an effective radiation temperature $T_{eff} \gtrsim 10^{21}$°K—the conversion turns out to be much higher (an energy is transformed into electromagnetic radiation which is comparable with the energy of the excited plasma waves; the same may also be the case for quasars[19]). We are dealing here with the fact that owing to the high density of the plasma waves necessary to produce the powerful electromagnetic radiation in pulsars, the so-called process of induced scattering of plasma waves into electromagnetic radiation is effective here.

The induced scattering effect consists in the fact that the probability for the appearance (in classical language, the intensity) of a scattered wave with wave vector **k** is increased when waves of the same type and with the same wave vector **k** are present. To be precise, for the scattering (transformation) of plasma waves with a wave vector $\mathbf{k}_l$ when they transform into transverse (radio) waves with wave vector $\mathbf{k}_t \equiv \mathbf{k}$, the probability of induced scattering is proportional to the intensity of the radio waves $I_t(\mathbf{k}) \equiv I$. At the same time the probability for the same spontaneous process (the usual scattering) is independent of I (for details see [24]).

Taking into account both the spontaneous and the induced scattering the transport equation for the change in intensity of the radio emission in the source along the beam (length element $dl$) can be written in the form

$$\frac{dI}{dl} = \alpha I_l + (\beta I_l - \mu_c) I; \tag{3}$$

here $I_l$ is the intensity of the plasma waves and $\mu_c$ the absorption coefficient for radio waves taking into account the electron-ion collisions in the plasma. The

*More exactly, according to the relation $\exp(-\int\mu\, dl)$ which is valid when there is no "saturation"; we shall for the sake of simplicity write the optical thickness $\int\mu\, dl$ in the form $\mu l$, understanding by $\mu$ some average value over the characteristic distance $l$ or L. In the case considered $l$ is the distance from the given element to the "edge" of the source.

term $\alpha I_l$ corresponds to the spontaneous scattering (the spontaneous transformation of plasma waves into transverse waves), and the term $\beta I_l I \equiv \mu_S I$ describes the induced scattering at frequencies $\omega \approx \omega_L$.*

Very important are the angular relations which in (3) are reflected in the form of the coefficients $\alpha$ and $\beta$. For the sake of simplicity we shall assume that the plasma waves form a beam, i.e., that their wave vectors are grouped around some value. In fact, this has already been taken into account in (3) as otherwise there should have appeared there integrals over $d^3\mathbf{k}_l$ which correspond to the formation of radio waves with wave vector $\mathbf{k}$ as a result of the scattering of plasma waves with different $\mathbf{k}_l$. Thus, in (3) $I_l$ is some average intensity of plasma waves with a fixed value of $\mathbf{k}_l$ which is the effective value for the packet. In that case $\alpha$ and $\beta$ depend only on the angle $\theta$ between $\mathbf{k}_l$ and $\mathbf{k}$—the wave vector of the radio waves which have been formed (their intensity is $I = I(\theta)$).

We restrict ourselves here to taking into account what is usually the most important part of the scattering, that due to density fluctuations (Rayleigh scattering). For this kind of scattering the frequency of the radiowaves formed is $\omega \approx \omega_l \sim \omega_L = \sqrt{(4\pi e^2 N_e/m)}$. We have then[25]

$$\alpha = \frac{\omega_L^4 \sin^2\theta}{32\pi^2 c^3 v_{ph} n(\omega_l) N_e}, \quad \beta = \frac{\omega_L (m/m_i)^2 (v_{ph}/v_T)^3 \sin^2\theta}{mcv_{ph} v_T^2 n^2(\omega_l) N_e},$$
$$\frac{\alpha}{\beta} = \frac{m(m_i/m)^2 \omega_L^3 n^2 v_T^2}{32\pi^2 c^2 (v_{ph}/v_T)^3} = \frac{m(m_i/m)^2 \omega_L^3 n^5 v_T^2}{96\sqrt{3}\,\pi^2 c^2}; \tag{4}$$

here $v_{ph} = \sqrt{3kT/m}\,(1 - \omega_L^2/\omega_l^2)^{-1/2} = \sqrt{3}\,v_T/n(\omega_l)$ is the phase velocity of the longitudinal waves with frequency $\omega_l$; $n(\omega_l) = \sqrt{(1 - \omega_L^2/\omega_l^2)}$ is the refractive index for transverse waves at the frequency $\omega_l$; $v_T = \sqrt{kT/m}$ is a characteristic thermal velocity for the electrons, and $m_i$ is the mass of the ions.

Apart from a numerical factor $1/2$, the spontaneous scattering can be evaluated by defining it as the scattering by free electrons. The total cross-section for this scattering is well known to be $\sigma_t = \frac{8}{3}\pi(e^2/mc^2)^2 = \omega_L^4/6\pi c^4 N_e^2$. The value of $\alpha$ in (3) which is integrated over the solid angle is thus of the order $\sigma_t N_e c/v_{ph} n(\omega_l)$ (the factor $c/v_{ph} n(\omega_l)$ takes into account the fact that the group velocity of the transverse waves is equal to $cn(\omega_l)$, while the group velocity of the longitudinal waves is $v_{gr} \approx v_{ph} n^2 = \sqrt{3kT/m}\,\sqrt{(1 - \omega_L^2/\omega^2)}$. In accordance with this $\alpha = (3/8\pi)\sigma_t N_e (c/v_e n(\omega_l)) \sin^2\theta$ which is only twice the exact value of $\alpha$ in (4). The occurrence of a dependence on the angle $\theta$ of the form $\sin^2\theta$ is also completely understandable: this is just the shape of the transverse waves emitted by a dipole—an electron vibrating under the influence of a longitudinal wave (the electrical field in this wave is along $\mathbf{k}_l$ and $\theta$ is thus the angle between the direction of the vibrations and the wave vector $\mathbf{k}$ of the emitted wave). As regards the induced scattering, the term $\beta I_l I$ is typical for the description of the interaction of waves when one takes into account the non-linearity of the medium (classical language) and for processes such as induced scattering (quantum language). The analogy with induced scattering allows us also to understand at once that the angular dependence is the same for the coefficients $\alpha$ and $\beta$ (for details see [24]).

The coefficient for the absorption of radio waves due to collisions is equal to (see [9])

$$\mu_c(\omega) = \frac{1 - n^2(\omega)}{cn(\omega)} \nu_{eff}, \quad n(\omega) = \sqrt{1 - \frac{\omega_L^2}{\omega^2}},$$
$$\nu_{eff} = \pi \frac{e^4}{(kT)^2} \sqrt{\frac{8kT}{\pi m}}\, N_e \ln\left(0.37 \frac{\varkappa T}{e^2 N_e^{1/3}}\right) = \frac{5{,}5 N_e}{T^{3/2}} \ln\left(220 \frac{T}{N_e^{1/3}}\right). \tag{5}$$

We have here taken into account the fact that in the cases of interest to us $\omega \gg \nu_{eff}^2$, $n^2 \gg (\omega_L^2/\omega^2)\nu_{eff}/\omega$, where $\nu_{eff}$ is the number of electron-proton collisions (a hydrogen plasma; this restriction is, of course, totally unimportant and is reflected only in the numerical coefficient in the expression for $\nu_{eff}$).

For a uniform plasma and $I_l$ = const the solution of (3) is as follows

$$I = \frac{\alpha I_l}{\mu_c - \beta I_l}\left[1 - e^{(\beta I_l - \mu_c)L}\right], \tag{6}$$

where L is the thickness of the plasma layer (emitting region). The induced scattering is particularly important when the conditions

$$\beta I_l \gg \mu_c, \quad \beta I_l L \gg 1. \tag{7}$$

are satisfied. In that case

$$I(\theta) = \frac{\alpha}{\beta} e^{\beta(\theta) I_l L}. \tag{8}$$

Let us estimate the different parameters, assuming that $N_e \sim 10^8$ electrons/cm$^3$, $T \sim 10^6$°K, $m_i/m \sim 2000$, and that the phase velocity of the longitudinal waves is $v_{ph} = \sqrt{3}\,v_T/n \sim 10^{10}$ cm/sec. We then have $\omega_L = 5.64 \times 10^4\sqrt{N_e} \sim 2\pi \times 10^8$ sec$^{-1}$, $\nu_{eff} \sim 10$ sec$^{-1}$, $v_T \sim 5 \times 10^8$ cm/sec, and $n \sim 0.1$. Moreover, from (4) and (5) we have

$$\mu_c \sim 3 \cdot 10^{-9}\ \text{cm}^{-1} \quad \beta \sim 10^{-10}\sin^2\theta, \quad \alpha \sim 10^{-16}\sin^2\theta, \quad \frac{\alpha}{\beta} \sim 10^{-6}. \tag{9}$$

For a directed plasma wave beam $I_l = W_l v_{gr}$, where $W_l = \overline{E}^2/4\pi$ is the time-averaged plasma wave energy density and $v_{gr} = \sqrt{3kT/m}\,\sqrt{(1 - \omega_L^2/\omega^2)} = v_{ph} n^2$. It is preferable to use for estimates the density $W_l$ rather than $I_l$ since this quantity is not so sensitive to the choice of the parameters. For the values which we have already used $v_{gr} \sim 10^8$ cm/sec, $I_l \sim 10^8\,W_l$ and the first of conditions (7) is satisfied for $\sin^2\theta \sim 1$, provided $W_l \gg 3 \times 10^{-6}$ erg/cm$^3$. The second of conditions (7) is satisfied for regions with dimensions $L \gg 10^3/W_l$. For pulsars both these inequalities are certainly satisfied. On the other hand, for type III solar flares $W_l < 10^{-6}$ erg/cm$^3$ (see [22]) and induced radiation plays no role or, at any rate, is not decisive.

In actual fact, therefore, induced radiation may under pulsar conditions become the main cause for conversion, due to which an (as far as order of magnitude is concerned) total transformation of plasma waves into electromagnetic waves is guaranteed.

The question now arises, of course, what part of the energy the source can transfer in the plasma to the excitation of plasma waves. Let, for instance, a fast electron current in the plasma be such a source. We shall

*In principle there is also possible [24a] another variant of the plasma mechanism in which the plasma eigenfrequency $\omega_L$ (and hence also the frequency of the excited plasma waves) is much lower than the frequency of the radio waves $\omega$. The high values of $\omega$ are then obtained by taking into account the induced scattering of the plasma waves not in the "cold" plasma, but by the relativistic particles in the source.

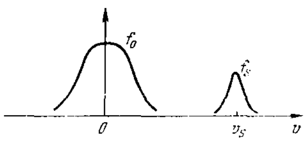


FIG. 1. Initial particle velocity distribution functions in the "main" plasma, $f_0$, and in the current, $f_S$.

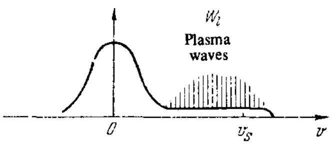


FIG. 2. First stage of the development of the beam instability: a quasi-linear relaxation of the beam distribution function.

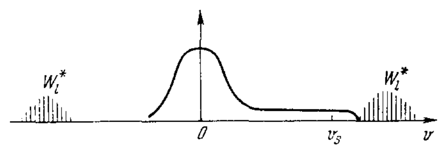


FIG. 3. Second stage of the development of the beam instability: transfer of the energy of the plasma waves into the non-resonance region of the phase velocities due to induced scattering.

show that under the most favorable conditions this part will be of the order of the energy of the current itself.

For the sake of simplicity we consider the case $\omega_L \gg \omega_H = eH/mc = 1.76 \times 10^7 H$, when we can discuss the excitation of plasma waves without taking into account the magnetic field. We assume that the velocity distribution functions for the electron current $f_S$ and for the main plasma $f_0$ at the moment the current is injected have the form schematically indicated in Fig. 1. If the current is injected into the plasma sufficiently fast compared with the characteristic times $t_r$, $t_{ll}$, and $t_{lt}$, which we shall discuss below, a beam instability will develop as follows (the estimates of the times $t_r$, $t_{ll}$, and $t_{lt}$ given below can be found in [24]):

In the first stage of the process, the order of magnitude of the duration of which is determined by the time $t_r$, where

$$t_r^{-1} \approx \frac{\pi\omega_L}{2} \frac{N_S}{N_e} \left(\frac{v_S}{\Delta v_S}\right)^2 \frac{1}{\ln (W_l/W_T)}, \quad (10)$$

we have a quasi-linear relaxation of the beam distribution function (here $N_S$ and $N_e$ are the particle densities in the current and in the plasma, $v_S$ and $\Delta v_S$ are, respectively, the average velocity and the initial dispersion in the velocities in the current, and $W_T$ is the energy density of the thermal plasma fluctuations). After a time t plasma waves are generated in the resonance region of the spectrum with a considerable energy $W_l \approx N_S m v_S^2/3$, increasing the dispersion of the velocities in the beam and flattening the electron distribution function. In this stage there appears a "plateau" in the current distribution function (a region with $\partial f_S/\partial v = 0$) and the generation of plasma waves practically ceases (Fig. 2).

In the second stage the plasma waves are redistributed from the resonance region of the spectrum into the non-resonance region (in the region of phase velocities where there are no beam particles (Fig. 3)). This takes place through induced plasma wave-plasma wave scattering over a period $t_{ll}$ where

$$t_{ll}^{-1} \approx \frac{\omega_L}{4} \frac{W_l^*}{N_e m v_S v_{Ti}}; \quad (11)$$

here $v_{Ti} = v_T\sqrt{m/m_i}$ is the ion thermal velocity and $W_l^*$ is the plasma wave energy density in the non-resonance region of the phase velocities ($W_l^* \approx W_l$).

In the third stage we have the induced conversion of plasma waves into electromagnetic waves. When the conversion is effective, and we explained earlier the conditions for this, practically all the energy is changed into electromagnetic radiation over a period $t_{lt}$, where

$$t_{lt}^{-1} \approx \frac{\omega_L W_l^*}{m N_e v_T^2} \left(\frac{m}{m_i}\right)^2 \left(\frac{v_S}{v_T}\right)^3; \quad (12)$$

here, as before, $v_T = \sqrt{kT/m}$ is the electron thermal velocity.

The process described here of the excitation of plasma waves and their conversion into electromagnetic radiation will be fully completed if the times $t_r$, $t_{ll}$, and $t_{lt}$ are less than the pulse length $\Delta\tau$.

We can neglect the influence of the collisions in the plasma (as was done in the above analysis of the dynamics of the development of the beam instability), provided $\Delta\tau \ll 1/\nu_{eff}$, where $\nu_{eff}$ is the number of collisions given by (5). For pulsars with a sufficiently hot plasma this condition can easily be satisfied: for instance, when $T \sim 10^6$ °K and $N_e \sim 10^8$ electrons/cm$^3$ we have $1/\nu_{eff} \sim 10^{-1}$ sec, while $\Delta\tau \sim (1 \text{ to } 5) \times 10^{-2}$ sec.

The process of the development of an instability and of conversion which we have indicated here takes place if there is no "stabilization" when the time $t_r \ll t_{ll}$. This is usually satisfied for electron currents if the temperature of the main plasma is not too low.[25]

It is, however, not excluded that the currents are ion (proton) currents. In that case $t_r$ will be large compared to $t_{ll}$. The picture of the development of the instability will thus be essentially different: the plateau is not established at once, and the conditions for the development of the beam instability acquire a specifically pulsating character—over a period $t_{ll}$ the waves excited by the current are transferred into the non-resonance region and stabilize the instability. The plasma waves change thus over a period $t_{lt}$ into radio emission. The stabilization ceases and the current again excites a part of the plasma waves; they anew are transferred to the non-resonance region, change into electromagnetic waves, and so on. This kind of process is repeated many times until a plateau appears on the distribution function of the ion current and the generation of plasma waves stops completely. As before, an energy of the order of the kinetic energy of the current $W \sim N_S m_i v_S^2$ changes in such a process into radio emission, but in separate portions. The time for the transfer of one portion of energy is $t \sim t_{ll} + t_{lt} \approx t_{lt}$. For reasonable values of the parameters the value of t can be of the order of $10^{-4}$ sec. It is possible that a fine structure of the radiopulses with a characteristic time $\lesssim 10^{-4}$ sec, observed in [8] and in a number of other papers* is connected with such a pulsating regime

*Another explanation of the pulse fine structure is given in [26]. There it is connected with the spreading out of a short ($\lesssim 10^{-4}$ sec) pulse, generated by the pulsar, through the scattering by inhomogeneties in the corona of the pulsar. The total length of the pulse (tens of msec) is in this point of view determined by the parameters of the scattering region and not by the nature of the generation of the radioemission in the source. (See also [27] about the role of flickering (scintillations) of the radio emission of the pulsar in its own atmosphere.

of the beam instability of ion currents. It will become clear from further estimates that ion currents are preferable over electron currents not only for an explanation of the fine structure of the radiopulse, but also to guarantee a high energy output of the current, especially if the pulsars are situated relatively far away.

From the discussion we have given it follows that coherent mechanisms (in both variants—with or without conversion) can explain the effective radio-emission output from a source with an energy comparable to the initial energy of the radiating particles. This makes it possible to estimate the minimum density of charged particles which is necessary for the generation of the radio emission from the pulsars.

According to the pulsar radio-emission given in Sec. II, the emission power in a pulse is for the case of quasi-isotropic emission for the majority of pulsars $P_p \sim 10^{30}$ erg/sec, and the total energy in a pulse is $\mathscr{E} \sim P_p \Delta\tau \sim 10^{30} \times 3 \times 10^{-2}$ erg $\sim 3 \times 10^{28}$ erg or somewhat larger. If during the period of the pulse an energy comparable to the energy of the electrons, is transferred to the radiation, we have

$$P_p \Delta\tau \sim \mathscr{E} \sim \frac{1}{2} m v_S^2 N_S V, \tag{13}$$

where V is the radiating volume. Assuming that the velocity of the current $v_S$ is of the same order of magnitude as the velocity of light c, that the radiating volume is $V \sim L^3 \sim (5 \times 10^8)^3$ cm$^3$ $\sim 10^{26}$ cm$^3$, we determine the total number of radiating electrons in the source: $N_S V \sim 5 \times 10^{34}$ and also the density of fast electrons in the current: $N_S \sim 5 \times 10^8$ electrons/cm$^3$. From this it is clear that we must in this example have a very dense electron current (with a density $N_S \gtrsim N_e$, the density of the "cold" plasma) in order to explain the observed radio-emission power. If the radiation is not by electrons, but by protons with $v_S \sim c$, the required density for them is three orders of magnitude less: $N_S \sim 10^5$ protons/cm$^3$. These estimates of $N_S$ are, in fact, also valid for the first variant of a coherent mechanism, as then there is no conversion at all. It is also clear that if in the radiating region there is a boost in energy during the time of the pulse, the requirements on $N_S$ become weaker.

Just now we have been discussing only a quasi-isotropic emitter—the pulsar. However, the observed pulsars are in all probability emitting directionally and rotate. As we saw in Sec. II the power in a pulse $P_p$ is then diminished by a factor $4\pi/\Delta\Omega$ for the same value of the observed flux $F_\nu$ ($\Delta\Omega$ is the solid angle subtended by the directional diagram). For a "beavertail" diagram $4\pi/\Delta\Omega \gtrsim 10$, and for a "pencil" diagram $4\pi/\Delta\Omega \gtrsim 10^3$. The energy lost by the pulsar during the period of the pulse $\Delta\tau$ and the minimum particle density $N_S$ necessary for the radiation over a period $\Delta\tau$ will also be lower by the corresponding factor. The values of $4\pi/\Delta\Omega$ given here characterize also the gain which is given by the directionality of the emission by a "lighthouse" type pulsar, as compared with an isotropically emitting, "blinking" pulsar, if we compare the energies lost to radio emission during the period of the pulsations ($\tau_2$ in the first and $\tau_1$ in the second case). This is clear since the loss of energy during a period of the pulsations is for a "blinking" pulsar equal to the energy emitted during the pulse $\Delta\tau$, while in a "lighthouse" type model the latter is close to the energy loss during the period of the vibrations $\tau_2$ (as $\tau_2 \sim \Delta\tau$).

The above comparison of the energy losses during the period of the pulsations for different pulsar models is completely plausible as the pulsation period is just the quantity which determines apparently the characteristic time for the particle acceleration and so on. From the comparison it is clear that models with directional emission require lower values of the particle energy $\mathscr{E} \approx P_p \Delta\tau$ and of the density $N_S$ determined by the relation (13). It is then easier than in the isotropic case to satisfy the condition $N_S \lesssim N_e$ for the density of the radiating electrons, forgetting the possibility of proton (ion) currents. Moreover, we must note that a "beavertail" diagram for a rotating pulsar does not give any advantages in the sense of average energy expenditure on the radiation per unit time (and the particle number necessary for it) as the average radio-emission power remains close in that case to the corresponding quantity for a "blinking" pulsar with a quasi-isotropic diagram. However, a "pencil" diagram gives also here a gain of about two orders of magnitude.

## V. PULSAR MODELS: MAGNETIC PULSATING WHITE DWARFS AND NEUTRON STARS

So far we have discussed the coherent radio-emission mechanisms in a rather general form in which we can apply it to any pulsar model. We now wish to make these models more specific, discussing in first instance models of pulsars which are magnetic white dwarfs or neutron stars. For the sake of simplicity we shall call both these types of stars magnetic dense stars. We shall assume that the stars are non-rotating which is not in agreement with the available observational data (see above). In this connection we note at once that we shall consider "synthetic" models (a pulsating and rotating dense star) in Sec. VII. It is, however, expedient for a number of reasons to discuss first models of non-rotating but pulsating stars. Firstly, the existence of such pulsars is not excluded. Secondly, in the above-mentioned "synthetic" models the radio-emission mechanism may be connected with the pulsations of the stars. The rotation of the star plays then in some sense a subsidiary role and is displayed by the directionality of the radiation diagram. The initial discussion of purely pulsating, but not rotating models is thus appropriate if only on methodical grounds.

If we neglect the influence of the magnetic field the radiation of a pulsating dense star will not be polarized (this conclusion is independent of the actual generation mechanism and is already obvious from symmetry considerations). The observed strong polarization of the radio emission from the pulsars, from linear to circular polarization, may be connected with the presence of fairly strong magnetic fields if the waves are generated in a plasma. The pulsars must thus belong to the class of magnetic dense stars, if they are dense stars. The idea that some dense stars should have a rather large magnetic field is very plausible: this field can and even should be amplified as compared to the field of the star from which the dense star was formed (compared, say, with the field of a star such as the Sun, where H ~ 1 Oe) when the strongly conducting matter of the star is compressed under conditions where the lines of force are "frozen in."

Under "freezing-in" conditions the field changes as $H \sim r^{-2} \sim \rho^{2/3}$, where r is the radius (characteristic size) and $\rho$ the density. From this it follows that when a star is compressed from a size $r \sim r_{\odot} = 7 \times 10^{10}$ cm to a size $r_{wd} \sim 3 \times 10^{8}$ cm or $r_n \sim 10^7$ cm the field increases, respectively, by a factor $5 \times 10^4$ and $5 \times 10^7$. This means clearly that if we start from a field $H \sim 1$ Oe, we arrive, respectively, at fields $H \sim 10^5$ and $10^8$ Oe. If a magnetic type star with $H_0 \sim 10^4$ Oe changes into a neutron star and if it ends up with a radius $r_n \sim 10^6$ cm one may expect even much stronger fields $H_n \lesssim 10^{14}$ Oe (some years ago this possibility has already been mentioned[28,29]). For a relatively light neutron star with a dense core radius $r_n \sim 10^7$ cm and a relatively weak field $H_n \sim 10^6$ Oe at the surface of the dense part of the star, the dipole field would even at a distance $r \sim 3 \times 10^8$ cm have a strength of $H \sim 30$ Oe. Such a field is still very strong from the point of view of emission and wave propagation in the radio band.

In a pulsating magnetic field charged particles must be accelerated. Firstly, this acceleration is realized as a result of "magnetic pumping": when there are no collisions the particle momentum component at right angles to the field $p_\perp$ also changes periodically in a periodic field (the quantity $p_\perp^2/H$ is an adiabatic invariant); when collisions are taken into account this leads to an exponential increase in the average momentum (for details see [30,31]). Secondly, if the field changes not only in time, but also is strongly non-uniform there may appear a very effective cumulative acceleration mechanism.[32] When the surface of the dense magnetic star vibrates sufficiently strongly it is likely that its atmosphere and corona will be strongly turbulent and the magnetic fields will just turn out to be non-uniform and variable. The acceleration of particles in the atmosphere of a magnetic star leads to the formation of radiation belts. The particles from the radiation belts will "be ejected" downwards only in the circumpolar regions. Moreover, if in the polar regions of the star the particles are effectively accelerated they will move outwards in the form of two "jets." These "jets" are a form of "stellar" wind—it is probable that for magnetic stars this wind will always leave predominantly from the polar regions.

The generation of radio emission in a pulsating magnetic dense star may proceed as follows (we indicate three basic models):[6]

Model I. Two fast particle streams leave continuously from the polar regions; the power of the stream is $P_S \sim N_S v_S \epsilon_S S$, where $N_S$ is the density, $v_S$ the particle velocity, $\epsilon_S$ the particle energy, and S the cross section of the stream. Shock waves moving in time with the vibrations of the dense star (or, perhaps, even the vibrations of the atmosphere for the pulsations) are ejected in each period upwards from the "cold" plasma out of the atmosphere of the star (Fig. 4). In the current-plasma system plasma waves are generated which change into the radio waves which are emitted by the pulsar from its polar regions. We have shown above that an energy comparable with the energy of the current can be transferred to the plasma waves while (through induced scattering) also a comparable part of the plasma waves will change into electromagnetic radiation; it thus becomes clear that the radiative power P is of the order of magnitude of the power of the stream $P_S$; this leads to an estimate of the density in the relativistic electron current of $N_S \sim 10^8$ electrons/cm$^3$ when $P \sim 10^{29}$ erg/sec, $v_S \sim c$, $\epsilon_S \sim mc^2 \sim 10^{-6}$ erg, and $S \sim 3 \times 10^{16}$ cm$^2$. The density of a relativistic ion current can be less by three orders of magnitude.

The density in the ejected plasma must in the case of electron currents be not less than $N_e \sim 10^8$ electrons/cm$^3$ (from considerations about the effectiveness of the generation of plasma waves). Moreover, the extension in height $\Delta h$ of the cold plasma must be at least $5 \times 10^6$ cm: estimates show that this is necessary in order that the plasma waves are able to be excited and to be converted into electromagnetic radiation. The total number of electrons and protons ejected in the cold plasma must therefore be of the order of $N_e \Delta h S \gtrsim 10^8 \times 5 \times 10^6 \pi \times (10^8)^2 \sim 10^{31}$ electrons. The number of energetic electrons lost per second from the radiation belts is $N_S v_S S \times 10^8 \times 3 \times 10^{10} \pi (10^8)^2 \sim 10^{35}$ electrons, i.e., it is considerably higher than the emission of a cold plasma.

Model II. Under the action of a shock wave particles in the radiation belts "are ejected" in the circumpolar regions because the adiabatic invariant is no longer an invariant on the front of the collisionless shock wave (Fig. 5). The violation of the adiabatic invariance takes place when the characteristic size of the front of the shock wave is less than the gyroscopic radius of the particle. For instance, when $\omega_L \gg \omega_H$ the front consists of a succession of single waves of width $\delta \sim c/\omega_L$ while the radius of curvature of the electrons is $r_H \sim \epsilon_S/\omega_H mc = (\epsilon_S/mc^2)c/\omega_H$. Since in the case of sufficiently energetic particles $\epsilon_S/mc^2 \gg 1$ and, moreover, $\omega_L \gg \omega_H$, the gyroscopic radius $r_H$ is larger than the width $\delta$ of the separate waves. When such a shock wave passes through the region of the radiation belts we can thus have a breakdown of the adiabatic invariant which is associated with the intensive "ejection" of particles. As a result there occurs in the circumpolar regions of the upper atmosphere in the pulsar model discussed here a strongly non-equilibrium velocity distribution of energetic electrons—or protons (streams of particles to the surface and a shortage of particles with

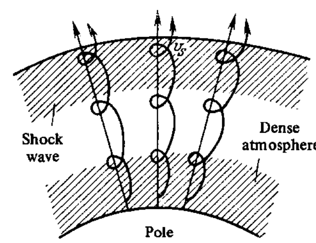

FIG. 4. Magnetic model I.

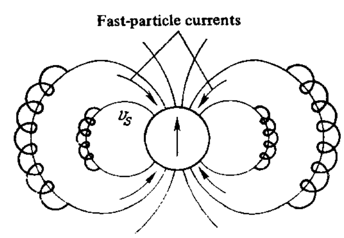

FIG. 5. Magnetic model II.

a small transverse velocity component). Thanks to this plasma waves, which after conversion give radio-emission, are excited due to beam[22] or wedge-type[33,34] instabilities. The coherent synchrotron mechanism[23] which directly gives radio waves may also be active (provided the nature of the energy spectrum and the density of the cold plasma is appropriate); it is very likely that just this mechanism plays a basic role in the generation of the pulsar radio emission.

A variant of model II is one in which the shock wave does not "eject" existing particles from the radiation belts, but accelerates particles which straightaway are "ejected" into the circumpolar regions.

In the case of a beam instability model II is close to model I. It is different in that the generation of energetic particles in model I must be considerably more intensive than in model II as the emerging of particles from the radiation belts in model I takes place continuously during the whole of the period while in model II it only takes place during the pulse. In other words, the rate of generation of particles in the belts can in model II be smaller by 1.5 to 2 orders of magnitude.

We have not considered the wedge-type instability in model II in detail, although results obtained in a study of the very low frequency radiation of the Earth's magnetosphere which appears in roughly the same conditions (but with different parameters of the radiating region and with a different frequency of the radio-emission) may be of great help.

The following parameters of the radiating region would be the best for coherent synchrotron radiation at a frequency $\omega \sim 2\pi\ 10^8\ \mathrm{sec}^{-1}$ (if the amplification at the given frequency $\omega$ takes place in a region of dimensions $L \sim 0.2\,r$, where $r \sim 3 \times 10^8$ cm is the radius of the star or its magnetosphere[6]): density of the "ejected" particles $N_S \sim 3 \times 10^5$ electrons/cm$^3$, their energy $\epsilon_S \sim 10\ mc^2$, the density in the cold plasma $N_e = 2 \times 10^7$ electrons/cm$^3$, $T \gtrsim 2 \times 10^3$ °K, $H \sim 6$ Oe. For these estimates we used equations given in [20]. We need not be disturbed by the fact that the electron density is insufficient to explain the observed radio-emission power during the whole pulse (as particles are flowing downwards from the radiation belts).

Model III. The radiation is generated at the front of the shockwaves due to the excitation of plasma waves and their subsequent transformation into radiowaves (see also [35] in this connection).

If the propagation is at right angles to the magnetic field the shape of the front of a collisionless shock wave is schematically given in Fig. 6. The front consists of successive separate waves with a characteristic size $\delta$ which depends on the density $N_e$ and the strength of the magnetic field H in front of the shock wave. We have then

$$\delta \approx c/\omega_L,$$

if

$$N_e kT \ll H^2/8\pi \ll N_e mc^2;$$
$$\delta \approx c\omega_H/\omega_L^2,$$

if

$$N_e mc^2 \ll H^2/8\pi \ll N_e m_i c^2;$$
$$\delta \approx c/\omega_{Li},$$

if

$$N_e m_i c^2 \lesssim H^2/8\pi$$

We recall that $\omega_L = \sqrt{4\pi e^2 N_e/m}$ is the plasma frequency of the cold plasma, $\omega_{Li} = \omega_L\sqrt{m/m_i}$ and $\omega_H = eH/mc$ is the gyrofrequency for electrons. The plasma waves are generated in the front of the shock wave due to a beam instability which arises because of the drift of the electrons relative to the ions. If $\omega_H \ll \omega_L$ (i.e., $H^2 \ll 8\pi N_e mc^2$), the drift velocity $v_d \approx (H/\sqrt{4\pi m N_e})(M-1)^{3/2}$ and the steady-state plasma wave energy is $W_l \approx N_e m v_d^2/6 \approx H^2(M-1)^3/24\pi$, i.e., of the order of the energy density in the magnetic field (M is the Mach number which in this case is the ratio of the velocity of the shock wave and the velocity of the magnetohydrodynamic waves $v_A = H/\sqrt{4\pi\rho} \approx H/\sqrt{4\pi m_i N_e}$).

This model for the generation has, however, some weak spots. First of all we note that an effective generation of plasma waves occurs only for shock waves which move across the magnetic field. When the angle between the plane of the front of the shock wave and the field H becomes appreciable, the width of the front increases and the conditions for the generation of plasma waves steeply deteriorate. The effective excitation of plasma waves by shock waves can therefore in a pulsar only take place in the equatorial regions. Moreover, even when the shock wave propagates across the magnetic field an effective generation of plasma waves takes place only if $\omega_H \ll \omega_L$ (if the opposite inequality holds the increment of the plasma waves is so small that the plasma waves cannot be appreciably be amplified during the time that a single wave passes through the given layer). However, the variant $\omega_H \ll \omega_L$ is poor because the induced scattering is ineffective at distances of the order of the thickness of the shock wave so that the degree of transformation of the plasma waves into electromagnetic waves steeply decreases. Model III is thus less likely than I or II.

When discussing the different models we used non-relativistic expressions (see, for instance, [13]) and we assumed that the "basic" plasma with density $N_e$, and so on, was non-relativistic. In general, this is all possible only provided the energy density $W_t$ of the radio waves is small compared with the self-energy density of the electrons in the plasma $mc^2 N_e \sim 10^2$ to $10^3$ erg/cm$^3$ for typical values of $N_e \sim 10^8$ to $10^9$ cm$^{-3}$. The radio-emission power $P \sim cW_t S'$ where S′ is the cross section (area) of the radio-emitting region to which we assign as an estimate a value of $S' \sim \pi r^2 \sim 3 \times 10^{17}$ cm$^2$ (a characteristic dimension is $r \sim 3 \times 10^8$ cm). Hence $W_t \sim 10^{-28} P \sim 10^2$ erg/cm$^3$ when $P \sim 10^{30}$ erg/sec. It is probable that for many pulsars $P < 10^{30}$ erg/sec and, possibly, $S'' > 3 \times 10^{17}$ cm$^2$. Under those conditions $W_t \ll mc^2 N_e$. However, for the pulsar CP 0328, for instance, with a "beavertail" diagram we have $P \gtrsim 10^{32}$ erg/sec (see Sec. II) and $W_t \lesssim 10^2$ erg/cm$^3$ only if $S' \gtrsim 3 \times 10^9$ cm$^2$, i.e., $r \gtrsim 3 \times 10^9$ cm. The necessity to approach the problem relativistically is thus by no

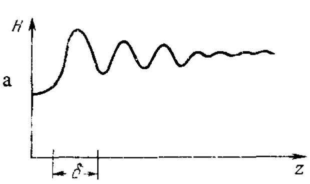

FIG. 6. Structure of the front of a collisionless shock wave propagating across the magnetic field.

means excluded. We shall, however, restrict ourselves to non-relativistic estimates (apart from the estimates for the case of coherent synchrotron radiation, of course).

In the foregoing we were dealing, when discussing the various models, mainly with schemes for the generation and with energy estimates. To compare the models with observations we must also make clear whether we are able to explain a number of actual properties of the pulsar radio emission. The most important problem is here that of the directional diagram of the emission. The authors did originally not pay attention to this point,[6] since they connected the main pulsar period with white dwarf oscillations. The observation of the second period, however, led to the conclusion that the star was rotating and that the polar directional diagram for its emitted radiation was narrow. Turning therefore to the problem of the directional diagrams in the magnetic models discussed above we observed[6a] that the directional diagram can become narrow in these models. We shall consider this problem in Sec. VII.

Here we note that the observed strong changes in the intensity ("fading") of the pulses of the pulsar radio-emission with characteristic times of the order of several seconds and larger are, in model II, possibly connected with a depletion of the radiation belts as a result of the "ejection" of particles from them. It takes some time to store up new particles with sufficiently high energies. In model I the "fading" (disappearance or strong decrease of the intensity of the pulses) time can be assumed to be the time necessary for the cold plasma to flow near the poles replacing the plasma ejected upwards by shock waves.

One must separately discuss the polarization of the radio emission. The role of the stellar rotation is as yet unclear in this connection; it is very well possible that it is unimportant. We shall therefore discuss the polarization problem in the next Sec. VI, neglecting the rotation of the star.

## VI. THE POLARIZATION OF THE PULSAR RADIO EMISSION

One of the most important characteristics of pulsars which needs an explanation is the strong polarization of the radio emission. Usually the polarization is close to being linear, but sometimes also elliptical polarization right up to circular polarization has been observed. When studying radio-emission mechanisms and the propagation of radio waves in the vicinity of the pulsar we must therefore explain not only the appearance of linear polarization (which is particularly difficult), but also the variations in the nature of the polarization. The difficulty here lies in the fact that in the case when a source generates in the plasma a wave of one kind (the extraordinary wave, say), the polarization of the radiation becomes circular, when the wave goes into the interstellar medium following the geometrical optics rules, independent on its character in the source.

The first condition—the generation of a wave of one kind—is probably characteristic for pulsars. We are dealing here with the fact that when the radiation in the source is sharply amplified when not only the condition $|\mu_{1,2}|L \gg 1$ (L is the size of the source, the $|\mu_{1,2}|$ are the amplification coefficients of the ordinary or the extraordinary wave) but also the inequality $|\mu_1 - \mu_2|L \gg 1$ is satisfied, the polarization of the radiation emerging from the source is determined by the character of the polarization of the wave of that type for which the amplification coefficient is largest.[21,36] What we have stated here is clear as under those conditions that particular wave will be predominantly accelerated in the source. (The induced scattering coefficient $\mu_S$ can here also play the role of $\mu$; see Sec. IV*.)

For the emerging of a linearly polarized radiation from the source it is first of all necessary that the ordinary and extraordinary waves in the source are linearly polarized. It is well known that this is realized if the radiowaves propagate quasi-perpendicularly to the magnetic field, and especially in the case when the condition

$$\frac{\omega_H^2}{\omega^2}\,\mathrm{tg}^2\,\alpha\sin^2\alpha \gg \left(1-\frac{\omega_L^2}{\omega^2}\right)^2 \qquad (14)$$

is satisfied; here $\alpha$ is the angle between the magnetic field **H** and the wave vector **k**; details about the quasi-perpendicular and quasi-parallel propagation of radiowaves can be studied in [9,22].

Condition (14) will be violated in a wide range of angles $\alpha$, if $\omega_L^2/\omega^2 \gg 1$ or $\omega_L^2/\omega^2 \ll 1$ and at the same time $\omega_H^2/\omega^2 \ll 1$. However, in the case when at least one of these inequalities is not satisfied, i.e., when $\omega_L^2/\omega^2 \approx 1$ or $\omega_H^2/\omega^2 \gtrsim 1$, the condition for "quasi-perpendicularity" (14) is satisfied in a relatively large range of angles $\alpha$ around $\frac{1}{2}\pi$. In practice this means that inequality (14) holds for the generation of radio-emission in the region $\omega \approx \omega_L$ or $\omega \approx \omega_H$ (when $\omega < \omega_H$ the emerging of the radio-emission from the plasma is inhibited, in particular, because of the absorption in layers where the frequency $\omega$ is equal to the electron gyrofrequency $\omega_H$).

However, the fact that the inequality (14) is satisfied in the generation region is insufficient for the observation of the linear polarization of the pulsar as in the interstellar medium $\omega_H/\omega \ll 1$, $\omega_L/\omega \ll 1$, and inequality (14) turns into its opposite. In the interstellar medium we therefore have a quasi-parallel propagation and both types of waves are circularly polarized. As we have already noted, if the transition from the region of quasi-perpendicular propagation to the region of quasi-parallel propagation takes place while the geometrical optics approximation remains valid the polarization of the waves of one type gradually changes and becomes circular.

The linear polarization after emerging into the interstellar medium is retained only when the geometrical-optics approximation (according to which the polariza-

*To avoid misunderstandings we emphasize that the polarization of incoherent synchrotron radiation (for instance, the radio emission from galaxies, supernova remnants, and so on) is determined by the polarization of the radiation emerging from the different volume elements of the source itself, when there is no appreciable reabsorption ($|\mu_{1,2}|L \ll 1$). Since the synchrotron radiation of each such element is a linearly polarized, the observed radiation must also have a linearly polarized component. The degree of linear polarization is, generally speaking, decreased by the Faraday effect in the source tion of the waves changes in agreement with the change in the parameters of the medium) will be violated before inequality (14) becomes invalid. Such a violation of the geometrical optics approximation is by reference to the polarization characteristics of radiation, when the radiation makes a transition to the rarefied interstellar medium, called the "limiting polarization" effect. It consists in the fact that the polarization of the radiation does not change in the region where geometrical optics is inapplicable and remains the same as before it entered this region.[9,22]

The position of the layer in which geometrical optics is violated is determined by one of the relations (see [22], Sec. 24)

$$\frac{\omega_L^2\omega_H}{\omega^3}\sim\frac{c}{\omega}\frac{1}{L_H},\quad \frac{\omega_L^2}{\omega^2}\sim\frac{c}{\omega}\frac{1}{L_N} \tag{15}$$

depending on which of the characteristic lengths—that of the change in the field $L_H$ or of the change in the density $L_N$—is the smaller. For pulsars it is more likely that $L_H > L_N$ so that our further considerations will be based upon the second equation of (15).

It is clear from (15) that the position of the boundary of the geometrical optics region depends on the actual parameters of the plasma; for instance, when $L_N$ and $L_H$ increase, the boundary is displaced to a layer with smaller values of $\omega_L$, $\omega_H$, and so on. We note that the displacement of the layer (15) can explain the variation in the character of the polarization of the radio emission of the pulsar. If this layer is situated far from the pulsar, in the region of the quasi-parallel propagation where the inequality (14) is changed into the opposite, the polarization of the observed radio emission becomes circular. When the layer (15) is situated in the intermediate region

$$\frac{\omega_H^2}{\omega^2}\operatorname{tg}^2\alpha\sin^2\alpha\sim\left(1-\frac{\omega_L^2}{\omega^2}\right)^2, \tag{16}$$

where waves of both types are elliptically polarized, and the polarization of the radiation received will also be elliptical.

As linear polarization dominates in the radio emission of pulsars, the layer

$$\frac{\omega_L^2}{\omega^2}\sim\frac{c}{\omega}\frac{1}{L_N}=\frac{\lambda}{2\pi L_N} \tag{17}$$

is situated mostly in the region (14) of quasi-perpendicular propagation. We must expect that near the pulsar $\lambda \ll L_N$ and, thus, that at the boundary of the region of applicability of geometrical optics $\omega_L^2/\omega^2 \ll 1$. Taking this inequality into account the criterion (14) becomes

$$\frac{\omega_H^2}{\omega^2}\operatorname{tg}^2\alpha\sin^2\alpha\gg 1. \tag{18}$$

In an appreciable interval of angles $\alpha$ around $\frac{1}{2}\pi$ (for instance, for values of $\alpha \gtrsim 70°$, for which $\tan^2\alpha\sin^2\alpha \gtrsim 10$) this inequality is satisfied, if $\omega_H/\omega \sim \frac{1}{3}$. At a frequency $\omega = 2\pi\nu \sim 2\pi \times 10^8$ sec$^{-1}$ a value of H ~ 10 Oe corresponds to this relation.

The second equation of (15) determines requirements as to the electron density in the transition layer. It is, however, not possible to determine from it independently N and $L_N$, since they occur only as their product $NL_N \sim \omega_L^2 L_N$. It follows from the above-mentioned relation that $NL_N\nu^{-2} \sim 3\times10^{-7}$ cm$^{-2}$ sec$^2$, i.e., $NL_N \sim 3\times10^9$ cm$^{-2}$ at a frequency $\nu \sim 10^8$ Hz. This value of $NL_N$ can be realized in the neighborhood of a pulsar, for instance, with $L_N \sim 10^7$ cm and $N \sim 3\times10^2$ electrons/cm$^3$.

It is very important that the value $NL_N\nu^{-2} \sim 3\times10^{-7}$ cm$^{-2}$ sec$^2$ obtained here leads to an insignificant depolarization of the radio emission due to the Faraday effect in a region of thickness $L_N$ above the layer (16).* Indeed, we need not consider an appreciable depolarization when there is quasi-parallel propagation if the angle over which the plane of polarization is rotated over a distance $L_N$ satisfies the relation

$$\Delta\chi=\frac{4.7\cdot10^4}{\nu^2}L_N NH\cos\alpha<1 \tag{19}$$

(for details see [22], Sec. 23). When H ~ 10 Oe and cos $\alpha$ ~ 1, this inequality will be satisfied, if the quantity $NL_N\nu^{-2} < 2\times10^{-6}$ cm$^{-2}$ sec$^2$. A moment ago we found that at the boundary of the region where geometrical optics is applicable (and certainly in higher layers) $NL_N\nu^{-2} \sim 3\times10^{-7}$ cm$^{-2}$ sec$^2$. This means that the influence of the Faraday effect in the scheme considered here for the generation and propagation of radio-emission in the neighborhood of the pulsar can be neglected.

The linear polarization of the pulsar radio emission (and its variation up to circular polarization) can thus be explained if we assume that the radio emission corresponding to one kind of wave is generated in a region of quasi-perpendicular propagation; moreover, when it goes into the interstellar medium (where the propagation is quasi-parallel) geometrical optics must cease to be valid before the condition of quasi-perpendicular propagation. There is no essential change if the radiation corresponding to one type of wave is generated in a quasi-parallel region but later passes through a layer of quasi-perpendicular propagation before it emerges into the interstellar medium (of course, under conditions such that the emerging into the interstellar medium from the quasi-perpendicular region will take place as it was described a moment ago).

Concluding this section we note that in [37] another explanation of the linear polarization of the pulsar radio emission was proposed which was based upon the effect of the interaction between the ordinary and the extraordinary waves when they pass through a transverse magnetic field. This interaction is connected with the breakdown of geometrical optics in the regions lying on both sides of the layer where $\mathbf{H} \perp \mathbf{k}$ ($\mathbf{k}$ is the wave vector of the radiation; for details see [38] and [22], Sec. 24). The interaction is characterized by a parameter $\delta_0 \backsim LNH^3/\nu^4$ (where L is the characteristic length over which the magnetic field changes its direction in the region considered). If a circularly polarized wave enters in the interaction region, the radiation will be partially linearly polarized when it leaves this region and the degree of linear polarization $\rho_l = 2e^{-\delta_0} \times (1-e^{-2\delta_0})^{1/2}$ is equal to unity when $\delta_0 = \ln\sqrt{2}$ and decreases fast as $\delta_0$ gets away from that value. If we also bear in mind that $\delta_0 \backsim \nu^{-4}$ it becomes clear that

*Notwithstanding the high values of $\omega_H$ and $\omega_L$ in the source and its immediate neighborhood (near the layer corresponding to condition (16)) there is in that region no depolarization because there is no Faraday effect. This is connected with the quasi-perpendicular character of the propagation and the predominant generation in the source of waves of only one kind (ordinary or extraordinary waves).

the frequency-dependence of the interaction effect which leads to the occurrence of linear polarization is very steep. It is unlikely that for different pulsars in the region where the field H is transverse (i.e., where $H \perp k$) the values $\delta_0 = \ln\sqrt{2}$ which alone in the scheme of [37] can lead to the appearance of strongly linear polarization are realized.

Summarizing, we are led to the conclusion that for coherent radio-emission sources (characterized by the presence of wave amplification and of a sufficiently dense plasma) amongst which we have the pulsars, the occurrence of linear polarization and other polarization features is most plausibly explained along the lines of the present section. We are dealing here, to be precise, with the emission of waves in the region of quasi-perpendicular propagation and with the transition to quasi-parallel propagation under conditions where as far as the polarization of the radiation is concerned the geometrical optics approximation breaks down (for the determination of other parameters such as the direction of the ray, geometrical optics remains in general well applicable; for details see [9]).

## VII. SYNTHETIC PULSAR MODELS; PULSATING AND ROTATING MAGNETIC NEUTRON STARS

Until the observation[7] of two periods $\tau_1$ and $\tau_2$ in the radio emission of the pulsars, it was a priori unlikely that one needed to consider simultaneously both rotation and vibrations of a star. But now it is clear that just models of pulsating and rotating stars (which we shall call for simplicity synthetic models) deserve most attention.

The most important element of the synthetic models which lay any claim to a comparison with the available observational data is, however, not that the rotation itself is taken into account, but the narrowness of the directional emission diagram. Indeed, if a pulsating star with a quasi-isotropic (say, a dipole) directional emission diagram rotates, the change in the radio-emission intensity may have two periods but under not even the most peculiar circumstances could we expect the practically total absence of radiation during the greater part of the main period.

The directional diagram must thus be narrow with a characteristic opening angle $\Delta\varphi \sim 2\pi\,\Delta\tau/\tau_1$. Both "pencil" and "beavertail" diagrams are admissible. In both cases, of course, the rotational axis of the star may not coincide with the symmetry axis of the diagram. To be precise, we shall assume that we are dealing with magnetic stars with a rotational axis and a magnetic axis (i.e., with an axis which is equivalent to a magnetic dipole; it is, of course, not necessary that the field is a dipole) which do not coincide. It is then plausible to assume that the axis of symmetry of a "pencil" diagram coincides with the magnetic axis (Fig. 7). For a "beavertail" diagram it is likewise plausible to assume that the magnetic axis is perpendicular to the plane of the diagram, i.e., that the symmetry plane of the diagram coincides with the magnetic equatorial plane (Fig. 8). Asymmetric diagrams are, of course, logically admissible and also diagrams with a symmetry plane which coincides with the plane of a magnetic meridian. In the last case, however, when the symmetry is conserved

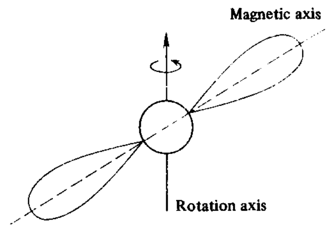


FIG. 7. Synthetic model with a "pencil" directional diagram.

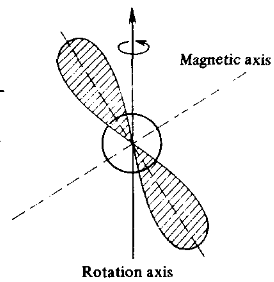


FIG. 8. Synthetic model with a "beavertail" directional diagram. We have cross-hatched the intersection of the diagram with the plane of the figure in which the rotational axis and the magnetic axis lie; the diagram has symmetry around the magnetic axis.

there occurs degeneracy—all meridional planes are equivalent. Therefore, if we select one such plane we must to some extent forego the magnetic symmetry (symmetry with respect to the magnetic axis). It seems to us that there is no basis, at least at this stage, to renounce magnetic symmetry.

We must note that in a synthetic model, owing to the directionality of the radiation, the discontinuous character of the generation, which in the pulsating model guaranteed the "silence" of the pulsar during most of the period $\tau_1$, becomes unnecessary. For instance, in the framework of model II (see Sec. V) the "ejection" of particles from the radiation belts must now take place not sporadically but continuously, guaranteeing thereby the continuous generation of radio emission in the circumpolar regions. The modulation of the radio-emission level with a period $\tau_2$ can then in final reckoning be connected with the periodic variations of the magnetic field of a pulsating star, which change the parameters in the generation region.

In the synthetic model there is also the possibility of a variant of the magnetic model II without the "ejection" of particles from the radiation belts. Instead of in the polar regions the generation of radio emission will then take place in low latitudes, in regions occupied by the radiation belts, provided the conditions for the amplification of synchrotron radiation or for the excitation of plasma waves (taking wedge-type instability or temperature "anisotropy" into account) are realized.

A narrow directional diagram can be formed in a number of cases. For instance, ultrarelativistic particles are more complicated emitters moving in vacuo, and emit basically within the angular limits $\Delta\varphi \sim \sqrt{1-(v^2/c^2)}$ near the direction of their velocity v (of course, within the angular limits $\Delta\varphi$ the diagram may have many lobes, as far as that is concerned, Figs. 7 and 8 are only symbolic). Narrow diagrams are also typical for induced radiation and scattering when there are no saturation effects. In the magnetic models discussed in Sec. V the radio emission appears just either as the result of induced scattering by plasma waves or as the result of induced (coherent) synchrotron radiation.

The amplification coefficient $|\mu|$ of the synchrotron radiation is a maximum for waves propagating at right angles to the magnetic field.[39,40] If the intensity of the radio emission emerging from the generation region is $I = I_0 e^{|\mu| L}$, where $|\mu| L \gg 1$, even for a small deviation of the propagation direction the quantity $|\mu|$ decreases by unity and the intensity by a factor e. From this it is clear that the condition $|\Delta\mu| L \sim 1$ determines the width of the directional diagram in the case of a coherent radiation mechanism. When applied to a magnetic dipole field in which the radio emission is generated in circumpolar or equatorial regions (where the magnetic field is directed at right angles to the plane of the magnetic equator) the diagram will be rather narrow and, moreover, will be a "beavertail" with a symmetry plane coinciding with the plane of the magnetic equator. Models I and II correspond to the same kind of "beavertail" diagrams in the case where the radio emission from the polar regions arises as a result of induced scattering of plasma waves excited by particle currents along the magnetic field. The plasma waves are then mainly also excited along the magnetic field, i.e., along the magnetic axis. We have shown in Sec. IV that the induced transformation is characterized by expression (8) which can be written in the form

$$I = I_0 e^{bL \sin^2 \theta};$$

here $I_0 = \alpha/\beta$, $b \sin^2\theta = \beta I_l$, and $\theta$ is the angle between the magnetic axis (direction of propagation of the plasma waves) and the wave vector of the radio waves. The diagram obtained is clearly maximal for $\theta = \frac{1}{2}\pi$ and will be rather sharp when $bL \gg 1$.

Let us estimate the opening angle $\Delta\varphi$ of this diagram (see [6a]). According to (9) $I_0 = \alpha/\beta \sim 10^{-6}$ and we need only determine the value of bL. For this we note that the pulsar radio-emission flux observed on the Earth $F \sim (r/R)^2 I(\pi/2)$, where r is the radius of the emitting region in the pulsar, R the distance from the pulsar, and $I(\pi/2)$ the intensity of the radio-emission in the pulsar at the maximum of the directional diagram. The spectral density of the flux for pulsars $F_\nu$ is of the order of a flux unit, i.e., $F_\nu \sim 10^{-26}$ W/m$^2$ Hz $= 10^{-23}$ erg/cm$^2$ sec Hz. If the scattering is from density fluctuations (Rayleigh scattering), as is most probable, for a given frequency of the plasma waves the scattered radiation is characterized by a spectral width $\Delta\omega \sim (m/m_i)^{1/2}(v_T/v_{ph})\,\omega_L \sim 10^{-3}\omega_L \sim 10^{-3}\omega$ (we use here the values from Sec. IV).* The width is thus $\Delta\omega \sim 2\pi\Delta\nu \sim 2\pi \times 10^5$ Hz for $\omega \sim 2\pi \times 10^8$ Hz and the flux which is of interest to us $F \sim F_\nu \times \Delta\nu \sim 10^{-18}$ erg/cm$^2$ sec $\sim (r/R)^2 I(\frac{1}{2}\pi) \sim 10^{-26} I(\pi/2)$ for $r \sim 3\times 10^8$ cm and $R \sim 1$ kpc $\approx 3\times 10^{21}$ cm. Hence $I(\pi/2) = I_0 e^{bL} \sim 10^{-6} e^{bL} \sim 10^8$ erg/cm$^2$ sec Hz and $bL = \ln[I(\pi/2)/I_0] \sim 30$. The width of the directional diagram $\Delta\varphi$ can be determined from the relation $I(\frac{1}{2}\pi - \Delta\varphi) = I_0 e^{bL\sin^2(\frac{1}{2}\pi - \Delta\varphi)} = \frac{1}{2} I(\frac{1}{2}\pi)$ and when $\Delta\varphi \ll 1$ we have

$$\Delta\varphi = \sqrt{\frac{\ln 2}{bL}} \sim 0.15 \sim 8^\circ. \tag{20}$$

Since bL depends only logarithmically on the intensities $I(\pi/2)$ and $I_0$, the estimate (20) characterizes relatively well the emission diagram of the pulsars in the models discussed.

According to [40a], the duration of the pulse $\Delta\tau$ decreases for the pulsar CP 1133 with increasing frequency (to be precise, $\Delta\tau = 72$ msec at a frequency of 40 MHz and $\Delta\tau = 40$ msec at a frequency of 430 MHz). If the pulsar CP 1133 has a short period $\tau_2$, for which there are some indications,[7] the two-peaked profile of the form of the pulse averaged over a long time [40a] must obviously reflect the structure of the directional diagram* (in this case the presence of two lobes). Moreover, for models with a rotation $\Delta\varphi$ is proportional to $\Delta\tau$ and the above-mentioned result corresponds to a decrease in $\Delta\varphi$ with increasing frequency. If we use Eq. (20) and assume that for the frequencies considered the intensity of the longitudinal waves $I_l$ and the phase velocity $v_{ph}$ are constant (for Cerenkov excitation of plasma waves $v_{ph} \sim v_S$ where $v_S$ is the velocity of the current), $\Delta\varphi$ depends on $\omega$ as $1/\sqrt{\omega}$ (see also Eq. (4) for $\beta c \omega_L \sim \omega$). This result is rather close to the $\omega$-dependence of $\Delta\tau$ found in [40a]. This does not, of course, as yet show the validity of the model considered by us but at any rate indicates its usefulness.

In the foregoing we neglected when discussing the character and width of the directional diagram the inhomogeneous character of the magnetic field in the generation region and we did not consider the regular refraction of radio waves which must take place in the non-uniform plasma which forms the atmosphere (corona) of the pulsars. When refraction is taken into account the diagram in general narrows and is distorted (becomes non-planar); the inhomogeneity of the magnetic field, on the other hand, leads to a widening of the diagram and can appreciably complicate the character of the directionality of the radiation.†

It is necessary to note also that the narrow directional diagram was obtained above assuming that the intensity of the plasma waves $I_l$ was constant. Moreover, when the intensity of the transverse waves I which occur as a result of the induced scattering of plasma waves is sufficiently large, when $I \sim I_l$, we must take into account the inverse effect of scattering on the plasma wave intensity. For instance, the relation $I = I_0 e^{bL\sin^2\theta}$ ceases to be valid. In fact, we have here a "saturation" effect due to which the intensity of the scattered electromagnetic waves cannot exceed the intensity of the excited plasma waves. If this effect is important the directionality of the scattered electromagnetic radiation can be appreciably less than given by the estimate (20).

We can thus obtain in the magnetic models of dense pulsating stars under well-defined conditions a high directionality of the radio-emission, in particular a "beavertail" diagram with an opening angle $\Delta\varphi \lesssim 10^\circ$ and a plane of the diagram oriented along the magnetic axis. Such a diagram is realized if the radio-emission

*The scattering line width $\Delta\omega$ is the Doppler width for the scattering of waves with frequency $\omega \sim \omega_L$ and phase velocity $v_{ph}$ for ions with a velocity of the order $v_{Ti} = (m/m_i)^{1/2} v_T = \sqrt{kT/m_i}$; for the velocity $v_{ph}$ we must take here the velocity of the longitudinal waves $v_{phl} = \sqrt{3}\, v_T/n$ as in the given case it is less than the velocity of the transverse waves $v_{pht} = c/n$.

*We assume that this is valid also for other rotating pulsars.

†We note in this connection that we obtain in a rotating model a natural explanation of the "precursor" in the pulsar CP 0950 which precedes the main pulse by 100 msec.[61] It may be connected with an additional small lobe of the directional diagram arising due to the complicated character of the magnetic field in the pulsar.

source in the circumpolar region or the region of the magnetic equator consists of plasma waves excited due to a wedge-type instability. The plasma waves occur then mainly across the magnetic field; the most effective scattering will then take place in the direction of the magnetic field and this guarantees the orientation of the directional diagram of the radio-emission along the magnetic axis of the pulsar. Moreover, the estimates given in Sec. V indicate that pulsating neutron stars with a magnetic field $H_n \gtrsim 10^6$ Oe on the surface can emit radiowaves with a power like that observed for pulsars from regions with a radius $r \gtrsim 3 \times 10^8$ cm.

Let us now discuss still a few problems connected with the *identification of pulsars with pulsating and rotating magnetic neutron stars.*

At the present time it is difficult to give any reliable estimate of the density of pulsars as their distances are poorly known, which are necessary to consider the pulsar distribution in the Galaxy and, finally, it is unknown what part of even the relatively close pulsars has already been observed. To get some ideas we assume that 10 pulsars are observed in a disc in the neighborhood of the Sun of thickness 100 pc and radius 500 pc (volume $\sim 10^8$ pc$^3$). If we assume an isotropic or "beavertail" directional diagram for the radio emission of the pulsars their spatial density is then $n_p \sim 10^{-7}$ pc$^{-3}$.* For a "pencil" diagram $n_p \sim (1 \text{ to } 3) \times 10^{-6}$ pc$^{-3}$.

The densities of all stars and of white dwarf stars near the Sun are, respectively, equal to $n_{st} \sim 0.1$ pc$^{-3}$ and $n_{wd} \sim 3 \times 10^{-2}$ pc$^{-3}$ (see [41]). The total number of stars in the Galaxy is $N_{st} \sim 10^{11}$, its effective volume $V_{st} \sim N_{st}/n_{st} \sim 10^{12}$ pc$^3$. The number of neutron stars is unknown but, in principle, it might be comparable with the number of white dwarfs. However, if neutron stars are formed only as a result of supernova outbursts their number in the Galaxy may reach $3 \times 10^8$ (supernovae flare up in the Galaxy on the average once every 30 years, while the age of the Galaxy is of the order of $10^{10}$ yr). This last value corresponds to a density $n_n \sim 3 \times 10^{-4}$ pc$^{-3}$.

All these numbers, although they are rough, indicate that the value of the density of pulsars $n_p \sim 10^{-7}$ pc$^{-3}$ (or even $n_p \sim 3 \times 10^{-6}$ pc$^{-3}$) at least does not seem to be in contradiction with present ideas about the structure of the Galaxy. Using the value $n_p \sim 10^{-7}$ pc$^{-3}$ we conclude that there are $N_p \sim n_p V \sim 10^5$ pulsars (a value $N_p \sim 10^6$ is also still completely acceptable) in the whole of the Galaxy.

Forgetting for the moment the "pencil" diagrams, the average power in the radio emission (radio brightness) of pulsars for the case of a "beavertail" diagram for the emission does not exceed the value $10^{32}$ erg/sec (see the estimate in Sec. II for the pulsar CP 0328). Most probably for most pulsars the power $\overline{P}$ is appreciably less and does not exceed $10^{30}$ erg/sec. The contribution of the pulsars to the radio brightness of the Galaxy can then be estimated by the value $\overline{P}N_p \lesssim 10^{36}$ erg/sec which does not contradict the radio-astronomical data about the radio emission of the Galaxy (its total radio brightness is of the order of $3 \times 10^{38}$ erg/sec; see, for instance [31]). We note also for comparison that the radio-emission power of supernova shells reaches $10^{35}$ erg/sec while the power of the synchrotron radiation from the Crab nebula in all parts of the spectrum is of the order of $10^{38}$ erg/sec.

*The solid angle corresponding to a "beavertail" diagram, $\Delta\Omega \sim 2\pi\Delta\varphi \lesssim 1$ compared to an angle $\sim 4\pi$ for a quasi-isotropic diagram. If the star does not rotate the probability for an observer to hit the directional diagram is thus by a factor $4\pi/\Delta\Omega$ smaller than for a rotating star: for a rotating star with a magnetic axis at right angles to the axis of rotation, a "beavertail" diagram coinciding with the plane of the magnetic equator describes the whole sphere. If the angle between the axes is of the order of unity the diagram describes an appreciable part of the sphere. This leads to the same spatial density of pulsars for an isotropically radiating model as for a rotating model with a "beavertail" diagram.

A neutron star with a main period of pulsations $\tau_2 \sim 10^{-2}$ sec and a rotational period $\tau_1 \sim 1$ sec has the following parameters:[17]

*mass*: $M \sim (0.1 \text{ to } 0.2) M_\odot$ ($M_\odot = 1.99 \times 10^{33}$ g is the Solar mass),

central density: $\rho_c \sim 3 \times 10^{13}$ to $10^{14}$ g/cm$^3$,

radius: $r_n \sim (5 \text{ to } 20) 10^7$ cm,

rotational energy: $W_r \sim 10^{46}$ erg,

energy of the pulsations: $W_p \sim (10^{47} \text{ to } 10^{48})(\delta r/r)^2$ erg, where $\delta r/r$ is the relative change in the radius (at the stellar surface) during its pulsations.*

The mass of such a neutron star is concentrated in its central (mainly neutron) part which is surrounded by a relatively large shell of strongly compressed plasma; we speak no longer about a plasma atmosphere or magnetosphere the dimensions of which turn out to be very large (in the foregoing we assumed that the radio-emission arises at distances $r \gtrsim 3 \times 10^8$ cm from the center of the star).

The damping of the pulsations due to the emission of gravitational waves caused by a coupling between the main radial oscillation and the oscillations with a quadrupole moment is given by [17] a time scale of from $10^5$ to $10^8$ years.

For an average radio-emission power $P \sim 10^{30}$ erg/sec the star loses $\sim 10^{41}$ erg in $3 \times 10^3$ yr. Such an amount of energy corresponds to the energy of free pulsations with a comparatively small amplitude $\delta r/r \sim 10^{-3}$. Hence it is clear that if the energy dissipated in the free oscillations would mainly go into the radio-emission, these oscillations could support the observed level of the radio emission of a pulsar during a relatively long time. However, the low stability of the pulsation period $\tau_2$) of the order of $10^{-4}$; see [7] and Sec. II) indicates that this is not the case: this stability corresponds (for free oscillations) to $Q \sim 10^4$, and free oscillations are damped in a time $Q\tau_2 \sim 10^2$ sec. The situation is thus not reduced to the free oscillations of a neutron star: the presence of the period $\tau_2$ indicates a self-oscillations regime of the pulsar. In contrast to free oscillations the stability of the self-oscillations period is not only dependent on the Q-factor: it is determined also by the so-called "stiffness" of the limit cycle of the self-oscillations, which contains many other parameters besides Q. As an example of another

*Cameron and Cohen [44a] have expressed doubt about the possible existence of such not-very-dense stable neutron stars. In that connection we wish, first of all, to emphasize the necessity to refrain from final conclusions with regard to the kind of neutron stars (or any other dense stars) to be identified with pulsars. Secondly, one sees easily that the main contents of the present paper (a consideration of the radio-emission mechanism and even the models of pulsating and rotating dense magnetic stars) are practically independent of the use of any particular parameters to characterize the dense parts of these stars.

example of self-oscillations in astrophysics we can quote the Cepheids; the stability of the period of change in their brightness is also small. We must in this connection emphasize that the high stability of the repetition period of the pulsar pulses $\tau_1$ (up to $10^8$ and higher) can be used as an argument against the idea that this is a period of self-oscillations: even under laboratory conditions without special precautions it is impossible to obtain such a high constancy of the self-oscillator frequency. The stability of the period $\tau_1$ indicates that one should use its connection with the rotation of the star.

Above we were talking about relatively not-very-dense neutron stars only in connection with the observed period of the pulsations $\tau_2 \sim 10^{-2}$ sec.[7] At the same time it is completely possible and even probable from a priori considerations that there are pulsars with periods $\tau_2 \ll 10^{-2}$ sec, corresponding to denser neutron stars with a mass $M \sim M_\odot$. For such stars $\tau_2 \sim 10^{-3}$ to $2 \times 10^{-4}$ sec and the radius of the neutron core is $r_n \sim 10^6$ cm (see [17]). Perhaps the radio emission of such stars is less powerful than for less dense neutron stars because of the smaller dimensions of the plasma corona.

This is, however, only a suggestion and the question remains on the whole completely open. The same can also be said of a number of other problems, in particular those connected with the evolution and the nature of the stellar magnetic field. Already from the example of the Sun and "normal" (not dense) magnetic stars it is known [42] how complicated and incomplete is the theory of the origin, configuration, and variations of the stellar magnetic field. For dense stars (white dwarfs, neutron stars) the theory of their magnetic properties has so far hardly been developed at all, let alone when their pulsations and rotation are taken into account (there are only a few notes in the literature on this score; see, for instance,[6,18,28,29,37,43]). As far as the general theory of dense stars (in particular, of neutron stars) is concerned many papers and a number of reviews[17,41,44-46] have been devoted to it but the number of unsolved problems in this area is still large. As an example we mention the fact that neutron stars are probably superfluid and possibly also superconducting[47]. Sufficiently cold white dwarfs (the very dense ones have time to cool off[41,48]) may turn out to be superconducting in some surface layer and this leads to a change in the magnetic field of the star.[49] Moreover, nobody has taken into account in any detail such possibilities when studying the cooling-off and pulsations of dense stars.

In the framework of the present paper there is no possibility for a detailed discussion of the general problems connected with the theory and observation of dense stars. We can only ascertain on the basis of what we have said that the identification of pulsars with rotating magnetic neutron stars does not contradict any data or estimates.

It is well possible that the presence of stellar pulsations is not a characteristic property of all pulsars. In the literature pulsar models have also been discussed in which pulsations are not considered at all. The only such model known to us,[18,43] which considers an energy source for the radiation, is a rotating magnetic neutron star. Near the star its magnetosphere is dragged along by the star, but at some distance $r_0 \sim c\tau/2\pi \sim 10^{10}$ cm (in practice this distance may also be equal to $10^9$ cm) where the velocity of the shell $v = 2\pi r_0/\tau$ is equal to the velocity of light, a further dragging-along becomes impossible. At $r \sim r_0$ we may expect the formation of a magneto-turbulent layer and an acceleration of particles. The presence of fast particles in turn may lead to the occurrence of radio emission, in particular, in one of the ways discussed above. As far as we know the radio-emission mechanism for such rotating models has not been considered in more detail.

## VIII. CONCLUSION

The hypothesis of the existence of neutron stars was already made in the thirties soon after the discovery of the neutron. The observation of these stars in the optical spectrum is practically impossible, if we are considering the thermal emission of the photosphere of the star. In this connection particularly great attention was paid to a discussion of the possibility to observe x-ray or neutrino emission from neutron stars. The x-ray techniques open up at the present time many perspectives and possibly some of the observed "x-ray stars" are neutron stars. The appearance of a relatively powerful radio emission from pulsating neutron stars was also proposed.[44,50] Nonetheless, both the discovery of the pulsars[1] and the appearance of a real basis to identify them with neutron stars[7] were unexpected.

So far it is not known whether all pulsars have two periods and it is not at all excluded that there exist different types of pulsars (in particular, not only neutron stars, but also white dwarfs). However, even if we forget about such hypothetical possibilities and consider only pulsars such as CP 1919 and AP 2015+28, studied in [7] (of course, it is not excluded that all pulsars observed belong to this type), it is probable, but certainly not yet proven, that they can be identified with neutron stars. To be precise, we mention as an alternative possibility the quasi-stationary collapsing magnetic star.[28,51-53]

These reservations are felt, on the one hand, due to caution. On the other hand, they are connected with the situation in the Crab nebula. If the pulsar NP 0532 lies indeed within the confines of the Crab nebula it is rather plausible to connect it with the activity of this object. The power of the synchrotron radiation of the Crab nebula $\sim 10^{38}$ erg/sec and probably the power of some kind of energy source situated in the nebula is of the same order. In $10^3$ years this corresponds to an energy output $W \sim 3 \times 10^{48}$ erg. Such an energy might, in principle, turn out to be the kinetic energy of a rotating star but may also have been derived through the diminution of its gravitational or nuclear energy. Moreover, this energy of the order of $3 \times 10^{48}$ erg is too large when we are considering the energy of free pulsations of a neutron star (see Sec. VII and [17,54]) or the magnetic energy $W_M$ contained in the magnetosphere of a neutron star.[28,52,53] For instance, for a field $H \sim 3 \times 10^9$ Oe and a characteristic dimension $r \sim 10^7$ cm, the energy is $W_M \sim (H^2/8\pi) \times 4\pi r^3/3 \sim 10^{39}$ erg. A magnetic neutron star may also have a larger field. However, such neutron stars with a strong field differ already, generally speaking, appreciably from the neutron stars usually considered. It is possible that such a class of neutron stars (field energy comparable with the gravitational energy or, at any rate, substantial when one analyzes the collapse and the whole dynamics of the star) somehow is linked to the abovementioned quasi-stationary magnetic stars.[28,52,53]

There are therefore on various grounds not yet sufficient reasons to assume that pulsars are more or less "normal" neutron stars although such a hypothesis is consistent and rather probable (the pulsar in the Crab nebula is possibly an exception but also that pulsar may well turn out to be a neutron star, though rotating faster than the others).

The discovery of pulsars may also be of interest in connection with the problem of the origin of cosmic rays. Pulsating magnetic stars might accelerate particles to very high energies and their possible role has been noted[55] already in that sense before the discovery of the pulsars. It is, however, yet too early to reach any conclusions about an appreciable role of the pulsars as cosmic ray sources. Indeed, to retain a balance in the framework of galactic models of the origin of cosmic rays there must be formed in the Galaxy on the average cosmic rays with a power of the order of $10^{40}$ to $10^{41}$ erg/sec (see [31]). Therefore, if the number of pulsars in the Galaxy $N_p \sim 10^6$, their contribution as sources of cosmic ray would be important if the power for the generation of cosmic rays were of the order of $10^{34}$ to $10^{35}$ erg/sec per pulsar. Such a power is several orders of magnitude higher than the radio-brightness of the pulsars: $P \lesssim 10^{30}$ to $10^{32}$ erg/sec. The assumption of the effectiveness of pulsars as cosmic ray sources is thus to a large extent a new hypothesis. It is true that the observation of pulsars in supernova shells indicates its usefulness. On the whole this problem remains open and certainly will again be discussed.

In the near future we must clearly expect a fast development of the theory of dense pulsating and rotating stars. The pulsar data enable us to compare to a greater or lesser extent such a theory with observations and this is the main value of the discovery of pulsars for astronomy.

We know, on the other hand, also other problems for the solution of which the use of pulsars is important. We are having in mind here the determination of the number of electrons $\int N_e \, dl$ along the path from the pulsars to the solar system, already discussed in Sec. II, the study of the interstellar magnetic field[56] and of the inhomogeneities which have been observed in the interstellar plasma if one is able to observe the corresponding scintillations.[57-59] We shall here not discuss the use of pulsars, together with other sources such as quasars, for a study of the interplanetary medium (see, for instance, [3,60]) or their observations for several other purposes where one needs point sources or periodically emitting cosmic radio sources.

At present it is already difficult to question that the discovery of the pulsars belongs in significance to a number of outstanding events and does not take second place to any other astronomical achievement in recent years. The study of pulsars and their use will have a large influence on the development of a whole number of trends in contemporary astronomy.

Note added in proof. At the beginning of April 1969 already 31 pulsars were known. Among the pulsars absent from the table we must mention specially the source NP 0527. This pulsar, first of all, has the longest of the known periods, equal to 3.7455 sec, and secondly lies on the celestial sphere near the Crab nebula. This enables us to assume that the pulsar NP 0527 is somehow connected with the nebula (for instance, was formed in the supernova outburst of 1054 and, having a high velocity, is found at the limits of the nebula).

A second (short) period, equal to $\tau_2 = 0.05364 \pm 0.00001$ sec has been observed [62] for the pulsar CP 0808. This confirms the conclusion in [7] about the existence of a second period for a number of pulsars. On the other hand, a neutron star can, apparently, not have a period of the main radial oscillation equal to $5 \times 10^{-2}$ sec (even periods $\tau_2 \sim 10^{-2}$ are to be doubted [44a] in this connection). If this is the case, one must rather consider for an explanation of the period $\tau_2$ oscillations of the atmosphere or of the magnetosphere of the neutron star.

We note that the presence of a second period which could be connected with pulsations has not been shown to exist for the pulsars with the shortest periods NP 0532 and PSR 0835-45. Hence, such oscillations either are absent or their period is so short ($\tau_2 \lesssim 10^{-3}$ sec) that they could so far not have been registered.

The synthetic model discussed in Sec. VII of this paper is the most probable one when a second (short) period exists. However, if the pulsations are not present, we may be dealing with a model of a magnetic neutron star which rotates without pulsations. When there are no pulsations the problem of the acceleration of the particles in the radiation belts is very probably much more complicated. The acceleration can then be caused by motions such as magnetohydrodynamic or shock waves propagating in the magnetosphere of the star. Another possibility is the acceleration in a layer close to the outer boundary of a rotating magnetosphere.[18,66]

As before, the radiation belts may serve as the source for the radio emission, and for the pulsar NP 0532 also of the optical emission, in rotating models. In the radio band we are dealing here with coherent synchrotron radiation and in the optical part of the spectrum with incoherent synchrotron radiation. In such a model the high directionality of the optical radiation of a system of relativistic particles is realized if these electrons are concentrated in the plane of the magnetic equator. Such a model is at the moment analyzed in detail by the authors. For a check of this model the polarization measurements both in the radio and in the optical band are very important (see in this connection the measurements of the polarization of the radio-emission of the pulsar CP 0328 [67]).

The coherent synchrotron mechanism was also discussed in [68] as applied to pulsars. However, in that paper the formula used for the calculation of the power of the coherent radiation of a pulsar is valid only for sources with dimensions $L \ll \lambda$. One source of such size is clearly unable to guarantee the powerful emission from a pulsar. On the other hand, if the magnetosphere of a pulsar contains many sources with dimensions $L \ll \lambda$ where each individual source consists of coherently emitting particles, two cases may occur. In the first all sources are mutually coherent. The formulae in [68] are then inapplicable and the correct procedure to study the coherent synchrotron mechanism must be based on the introduction of the amplification coefficient $\mu$ which is connected with the synchrotron instability (see in this connection [20, 23] and the present paper). In the second case the sources are incoherent one with another, i.e., there is an incoherent emission mechanism in the system, in which particle aggregates concentrated in a volume $L^3$ play the role of separate radiating sources. The emission power from such a system is limited by reabsorption and, not less importantly, the existence of such a system seems to us to be improbable under astrophysical conditions.

[1] A. Hewish, S. J. Bell, J. D. H. Pilkington, P. F. Scott, and R. A. Collins, Nature 217, 709 (1968).
[2] S. P. Maran and A. G. W. Cameron, Phys. Today 21 (8), 41 (1968).
[3] N. A. Lotova, Usp. Fiz. Nauk 95, 293 (1968) [Sov. Phys.-Uspekhi 11, 424 (1968)].
[4] M. I. Large, A. E. Vaughan, and R. Wielebinski, Nature 220, 753 (1968).

[4a] V. Radhakrishnan, D. J. Cooke, M. M. Komesaroff, and D. Morris, Nature 221, 443 (1969).
[5] A. G. Lyne and B. J. Rickett, Nature 219, 1339 (1968).
[6] V. L. Ginzburg, V. V. Zheleznyakov, and V. V. Zaitsev, Nature 220, 355 (1968).
[6a] V. L. Ginzburg and V. V. Zaitsev, Nature 222, 230 (1969).
[7] F. D. Drake and H. D. Craft, Jr., Nature 220, 231 (1968).
[8] H. D. Craft, Jr., J. M. Comella, and F. D. Drake, Nature 218, 1122 (1968).
[9] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma) Moscow, Nauka, 1967 [English translation published by Pergamon Press, Oxford].
[10] S. B. Pikelner, Ann. Rev. Astron. Ap. 6, 165 (1968).
[11] G. De Jager, A. G. Lyne, L. Pointon, and J. E. B. Ponsonby, Nature 220, 128 (1968).
[11a] M. Guélin, J. Guibert, W. Huchtmeier, and L. Weliachew, Nature 221, 249 (1969).
[12] W. L. H. Shuter, V. R. Venugopal, and M. J. Mahoney, Nature 220, 356 (1968).
[13] A. G. Lyne and F. G. Smith, Nature 218, 124 (1968).
[14] J. Ostriker, Nature 217, 1227 (1968).
[15] F. Pacini and E. E. Salpeter, Nature 218, 733 (1968).
[16] K. S. Thorne and J. R. Ipser, Ap. J. Lett. 152, L71 (1968); 153, L215 (1968).
[17] K. S. Thorne, Comments Ap. Space Sci. 1, 12 (1969).
[18] T. Gold, Nature 218, 731 (1968).
[19] V. L. Ginzburg and L. M. Ozernoĭ, Izv. VUZOV (Radiofizika) 9, 221 (1966) [Sov. Radiophys. 9, 153 (1967)]; Ap. J. 144, 599 (1966).
[20] V. V. Zheleznyakov, Astron. Zh. 44, 42 (1967) [Sov. Astron.-AJ 11, 33 (1967)].
[21] V. L. Ginzburg and S. I. Syrovatskii, Ann. Rev. Astron. Ap 7, (1969).
[22] V. V. Zheleznyakov, Radioizluchenie Solntsa i planet (Radio Emission of the Sun and Planets) Moscow, Nauka, 1967 [English translation published by Pergamon Press, Oxford].
[23] V. V. Zheleznyakov, Zh. Eksp. Teor. Fiz. 51, 570 (1966) [Sov. Phys.-JETP 24, 381 (1967)].
[24] V. N. Tsytovich, Nelineĭnye effekty v plasme (Nonlinear effects in a plasma) Moscow, Nauka, 1967; S. A. Kaplan and V. N. Tsytovich, Usp. Fiz. Nauk 97, 77 (1969) [Sov. Phys.-Uspekhi 12, 42 (1969)].
[24a] S. A. Kaplan and V. N. Tsytovich, Astron. Zh. 46, 192 (1969) [Sov. Astron.-AJ 13 (1969)].
[25] V. V. Zheleznyakov and V. V. Zaĭtsev, Astron. Zh. (1969).
[26] L. M. Eroukhimov, Ap. Space Sci. (1969).
[27] R. V. E. Lovelace and H. D. Craft, Jr., Nature 220, 875 (1968).
[28] V. L. Ginzburg, Dokl. Akad. Nauk SSSR 156, 43 (1964) [Sov. Phys.-Doklady 9, 329 (1964)]; V. L. Ginzburg and L. M. Ozernoĭ, Zh. Eksp. Teor. Fiz. 47, 1030 (1964) [Sov. Phys.-JETP 20, 689 (1965)].
[29] L. Woltjer, Ap. J. 140, 1309 (1964).
[30] H. Alfvén and C. G. Fälthammar, Cosmical Electrodynamics, Oxford University Press, 1963, Secs. 2, 7, 4.
[31] V. L. Ginzburg and S. I. Syrovatskiĭ, Proiskhozhdenie kosmicheskikh lucheĭ (Origin of Cosmic Rays) Acad. Sci. USSR Press, 1963 [English translation published by Pergamon Press, Oxford].
[32] S. I. Syrovatskiĭ, Zh. Eksp. Teor. Fiz. 50, 1133 (1966); Astron. Zh. 43, 340 (1966) [Sov. Phys.-JETP 23, 754 (1966); Sov. Astron.-AJ 10, 270 (1966)].
[33] R. F. Post and M. B. Rosenbluth, Phys. Fluids 9, 730 (1966).
[34] V. Yu. Trakhtengerts, Geomagn. i aéronomiya 8, 332 (1968) [Geomagn. Aeronomy 8, 263 (1968)].
[35] Ya. B. Zel'dovich, Astron. Zh. 45, 913 (1968) [Sov. Astron.-AJ 12, 725 (1969)].
[36] V. V. Zhelezhnyakov, Ap. Space Sci. 2, 417 (1968).
[37] M. R. Kundu and S. M. Chitre, Nature 218, 1037 (1968).
[38] V. V. Zheleznyakov and E. Ya. Zlotnik, Astron. Zh. 40, 633 (1963) [Sov. Astron.-AJ 7, 485 (1963)].
[39] V. V. Zheleznyakov and E. V. Suvorov, Zh. Eksp. Teor. Fiz. 54, 627 (1968) [Sov. Phys.-JETP 27, 335 (1968)].
[40] V. L. Bratman and E. V. Suvorov, Zh. Eksp. Teor. Fiz. 55, 1415 (1968) [Sov. Phys.-JETP 28, 740 (1969)].
[40a] H. D. Craft Jr. and J. M. Comella, Nature 220, 676 (1968).
[41] V. Weidemann, Ann. Rev. Astron. Ap. 6, 351 (1968).
[42] P. Ledoux and P. Renson, Ann. Rev. Astron. Ap. 4, 293 (1966).
[43] F. Pacini, Nature 219, 145 (1968).
[44] A. G. W. Cameron, High Energy Astrophysics, Les Houches Lectures, Vol. 3, Gordon and Breach, New York, 1967.
[44a] A. G. W. Cameron and J. M. Cohen, Preprint 1968; Ap. Lett. 3, 3 (1969).
[45] J. A. Wheeler, Ann. Rev. Astron. Ap. 4, 393 (1966).
[46] Ya. B. Zel'dovich and I. D. Novikov, Relyativistskaya astrofizika (Relativistic Astrophysics) Moscow, Nauka, 1968 [English translation to be published by Chicago University Press].
[47] V. L. Ginzburg, Usp. Fiz. Nauk 97, 601 (1969) [Sov. Phys.-Uspekhi 12, 241 (1969)].
[48] J. P. Ostriker and L. Axel, Astron. J. 73 (Suppl.) No. 1357 (1968).
[49] V. L. Ginzburg and D. A. Kirzhnits, Nature 220, 148 (1968).
[50] A. G. W. Cameron, Nature 205, 787 (1965).
[51] F. Hoyle, J. V. Narlikar, and J. A. Wheeler, Nature 203, 914 (1964).
[52] L. M. Ozernoĭ, Astron. Zh. 43, 300 (1966) [Sov. Astron.-AJ 10, 241 (1966)]; Highlights of Astronomy (Ed. L. Perek), D. Reidel Publ. Cy., Dordrecht, 1968, p. 384.
[52a] N. S. Kardashev, Astron. Zh. 41, 807 (1964) [Sov. Astron.-AJ 8, 643 (1964)].
[53] D. Layzer, Nature 220, 247 (1968).
[54] A. Finzi and R. A. Wolf, Ap. J. 153, 835 (1968).
[55] A. G. W. Cameron, Nature 206, 1342 (1965).
[56] F. G. Smith, Nature 218, 435; 220, 891 (1968).
[57] V. L. Ginzburg, Dokl. Akad. Nauk SSSR 109, 61 (1956) [Sov. Phys.-Doklady 1, 403 (1957)].
[58] P. A. G. Scheuer, Nature 218, 970 (1968).
[59] L. M. Erukhimov and V. V. Pisareva, Astron. Circular, No. 489, 11 Nov. 1968.
[60] J. V. Hollweg, Nature 220, 771 (1968).
[61] B. J. Rickett and A. G. Lyne, Nature 218, 934 (1968).

References added in proof

[62] Yu. I. Alekseev, V. V. Vitkevich, and Yu. P. Shitov,

Astron. Circular of the USSR No. 495, 24 Febr. 1969.
[63] W. J. Cocke, M. J. Disney, and D. J. Taylor, Nature **221**, 525 (1969).
[64] R. E. Nather, B. Warner, and M. Macfarlane, Nature **221**, 527 (1969).
[65] R. Lynds, S. P. Maran, and D. E. Trumbo, Ap. J. Lett. **155**, L121 (1969).
[66] T. Gold, Nature **221**, 25 (1969).
[67] R. R. Clark and F. G. Smith, Nature **221**, 724 (1969).
[68] B. J. Eastlund, Nature **220**, 1293 (1968).

Translated by D. ter Haar