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ON CERTAIN GNOSIOLOGICAL PROBLEMS IN PHYSICS

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IN the March 1968 (Russian) issue of this journal there appeared a translation of E. Wigner's lecture devoted to certain problems of knowledge, and especially to a discussion of the question as to why mathematics is so effective in Physics*. Although this lecture was presented several years ago, it is indicative of the views held by certain theoreticians abroad. Inasmuch as these views have now entered the Soviet scientific community, it is necessary to express certain critical considerations regarding these questions. We do not concern ourselves here with a discussion of the specific studies conducted by Wigner. He has received a Nobel prize for his work on invariance problems and his successful use of group theory in quantum mechanics and for other special work, and is now considered an authority. Nevertheless, an old history, pointed out already by Lenin, has repeated itself here: success in specialized fields of study do not guarantee the substantiation of the philosophical tenets of the scientist. No matter how one regards the philosophical views of Wigner, it is impossible to ignore the fact that his attempt to base himself on his own experience in physics creates the impression among certain of his readers that his views are both new and been substantiated. Wigner's philosophical statements are widely publicized, and are supported in certain circles. This is why we must carefully examine them.

Criticism is an ungrateful enterprise and is of no interest to us. In the present case we are more interested in the theoretical side of the question—the possibility, on the basis of an analysis of the evolution of specific problems in physics, as discussed by Wigner, to present another notion of the laws of nature, the process of establishing scientific theories, grounds for the role of mathematics in knowledge, and other problems of knowledge.

We hope that these problems are of general interest, that they justify the present work of exposing these problems.

I. WIGNER'S VIEWS ON PROBLEMS OF KNOWLEDGE

Wigner discusses in his lecture general questions of the theory of knowledge. One idea is persistently expressed: how surprising and unreasonable are both knowledge and science. The surprise is expressed over many aspects of knowledge.

Wigner asserts, thus, that the "enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it" (2)*. He is surprised also by the peculiarity of mathematical equations that yield unexpected results which "we did not put in" (9). The lecturer is also surprised by theories of physics, since "we do not know why our theories work so well" (14), and also by the fact that "man was capable of relating together thousands of arguments" (14).

He is also surprised by the picture drawn by him, whereby the physicist often gives to his observations a fairly rough mathematical formulation that nevertheless leads in an "unlikely" number of cases to a "surprisingly accurate description of a large class of phenomena." The mathematical formulation is carried out with respect to an idealized problem, and then it turns out unexpectedly that the same mathematical methods can be applied to more complicated problems. In all such results, which are considered to be miraculous and consequently not grounded in logic, Wigner sees "the empirical law of epistemology (i.e. the science of the foundation of knowledge)"; this law, according to the definition by R. G. Sax and Wigner himself, is none other than the "dogmatic creed of theoretical physicists"; Wigner affirms that it "is an integral part of theoretical physics."

It is important to find out whether these opinions on the unreasonableness of knowledge and the appeal to a dogmatic creed are accidental, or even "deliberately pointed" allegorical formulations, or whether they form a connected gnosiological conception.

Let us examine in greater detail Wigner's views.

1. Esthetic Motives for the Development of Mathematics. Laws of Nature as Conditional Statements

In correspondence with the title and purpose of his lecture, Wigner begins his exposition by discussing the question of the power and essence of mathematics. Although he affirms that the "unusual effectiveness" of mathematics in the natural sciences is inconceivable, and verges on the mysterious, he nevertheless attempts to "clarify the role of mathematics in physics." With this purpose in mind, he discusses the question of the substance of mathematics and physics.

According to Wigner, "mathematics is the science of skillful operations with concepts and rules invented just for this purpose" (2).† Abstract ideas are con-

*E. Wigner, 'The Unreasonable Effectiveness of Mathematics in the Natural Sciences', Richard Courant lecture in Mathematical Sciences delivered at New York University, May 11, 1959 in honor of R. Courant's 70-th birthday. [Comm. Pure Appl. Math. 13, 1 (1960)]

*Numbers in parentheses denote pages in the original paper.

†Wigner also characterizes philosophy in the same spirit, quoting the phrase of an unknown author, with whom he agrees: "Philosophy is the abuse of terminology invented specifically for this purpose."

structed as "apt subjects on which the mathematician can demonstrate his ingenuity and sense of formal beauty" (3). The notion of complex numbers, for example, was needed by the mathematician to prove elegant theories in algebraic equations, power series, etc... "The concepts of mathematics are chosen... for their amenability to clever manipulations and to striking, brilliant arguments" (7).

Thus, mathematical theorems and theories are developed out of the inner necessity of mathematicians, and this necessity is of an esthetic character; consequently, the theories lie wholly in the field of subjective thought and are not related to the development of objective logic in nature.

In this case, however, there arises the question of the possibility of mathematically expressing laws of nature, if the latter are determined by a nature which is external to us. It is precisely this situation, namely: a) the subjectivity of mathematical theorems and theories, b) the objectivity of laws of nature, and c) the effectiveness of mathematics in the natural sciences, which Wigner considers surprising and inconceivable. He sees the solution of this problem in the re-examination of the notion of a "law of nature."

Analyzing this notion, Wigner first of all underlines the instances of the relativeness of laws of nature: laws are valid under specific conditions. These conditions (for example, initial conditions on coordinates) cannot be determined with absolute accuracy; laws of probability do not give grounds for accurate predictions, etc... By basing himself on these undisputable facts, Wigner leads the reader to the conclusion that so-called "laws of nature" are always idealizations, whereas the nature of the idealization is determined by the investigator himself. "The principal purpose of the preceding discussion," concludes Wigner, "is to point out that the laws of nature are all conditional statements and they relate only to a very small part of our knowledge of the world" (6) (the itallics are everywhere the author's —S.S.). Modern physics has strengthened this convention and has shown that "even the conditional statements cannot be entirely precise: that the conditional statements are probability laws which enable us only to place intelligent bets on future properties of the inanimate world, based on the knowledge of the present state. They do not allow us to make categorical statements, not even categorical statements conditional on the present state of the world." (6).

Thus, "all laws of nature are conditional statements." Wigner repeats this notion in several very slightly different variations. He does not divulge here his understanding of nature, but, at least, we do not find a direct contradiction of its objectivity or the notion that nature is the creation of our thought processes. Evidently, Wigner's concept is more refined. What, then, is this concept?

We shall attempt to examine it as carefully as possible, but not necessarily in relation to other notions of Wigner. Let us assume that a "philological" rather than a gnosiological meaning is attributed to this formulation, and let us attempt to understand the phrase "all the laws of nature are conditional statements" in the sense in which any formulation of a law of nature tacitly imples on the conditionality of the action of the law under certain circumstances. Such an assertion would be completely rational: the physicist in fact meets constantly with the conditionality of the action of laws of nature. For instance, the law of uniform energy distribution over degrees of freedom is only valid at a sufficiently high temperature at which quantum laws do not hold. The recognition of the conditionality of action of laws of nature is the result of the entire development of science.

Wigner, however, does not favor such a treatment of laws of nature, and labels them conditional statements. If he had in mind the notion of the conditionality of action of the laws, then, first of all, nothing would prohibit him to express this in an obvious manner. Secondly, the conditionality of action of laws does not exclude, but rather assumes the objective nature of laws, as demonstrated by the bounds imposed on their action, and consequently assumes the compulsive character of the laws within these bounds. This would, however, lead Wigner once more to the initial situation which he considers surprising and unreasonable, and whose solution he seeks by changing the understanding of "laws of nature."

This leads us to conclude that Wigner's formulation that "all laws are conditional statements" must be understood in the direct sense. We shall see further that such a conclusion is in complete correspondence with other conclusions by Wigner on the truth criteria of theories, on the multiplicity of theories, etc...

On what, then, is such a treatment of the laws of nature based? Evidently, in the first place on the fact that laws of nature are formulated by man, and since man cannot embrace nature in its unlimited relationships, he considers nature in certain limited aspects ("sections"), applies various degrees of idealization, and, in other words, uses various "methods of observation." The methods of observation depend on the observer, his experiments, intuition, scope and so on, and they determine the character of the formulated "laws of nature."

It is precisely this possibility of varying the method of observation, and consequently the formulation of the law, along with the probabilistic nature of all measurements, which is the foundation of Wigner's statement that laws of nature are conditional statements. Laws are no longer considered as external constraints, but are subjective.

It is this precisely which permits the theoretical physicist to employ mathematics in formulating laws of nature: it is always possible to change the method of observing nature, thereby changing the law of nature, and to choose the appropriate mathematical tool for the new formulation of the law.

2. Physics and Mathematics. The Development of Physics According to Wigner.

Basing himself on the aforementioned notions concerning mathematics and the laws of nature, Wigner presents the development of physics in the following manner: "when the physicist finds a connection between two quantities which resembles a connection well known in mathematics, he will jump at the conclusion that the connection is identical (Wigner's emphasis

—S.S.) with the one discussed in mathematics simply because he does not know of any other similar connection" (8). This identity can even be not justifiable, it may only be a trial, but Wigner warns beforehand the reader that "physicists are irresponsible people" who are willing to gamble.

Then there occurs a miracle: the rough estimate of the physicist leads unexpectedly to the formulation of a precise law. Wigner gives examples of such occurrences in practice. Here is one example in the development of elementary quantum mechanics: "This originated when Max Born noticed that some rules of computation, given by Heisenberg, were formally identical with the rules of computation with matrices, established a long time before by mathematicians. Born, Jordan, and Heisenberg then proposed to replace by matrices the position and momentum variables of the equations of classical mechanics. They applied the rules of matrix mechanics to a few highly idealized problems and the results were quite satisfactory. However, there was, at that time, no rational evidence that their matrix mechanics would prove correct under more realistic conditions." Then occurred the miracle: "The miracle occurred only when matrix mechanics, or a mathematically equivalent theory, was applied to problems for which Heisenberg's calculating rules were meaningless" (9). Physics, Wigner states, "as we know it today would not be possible without a constant recurrence of miracles similar to the one of the helium atom" (10).

3. Subjectivization of Truth Criteria of a Theory. Multiplicity of Theories

Inasmuch as mathematics and physics deal, according to Wigner, with idealized categories of thought, conditional notions and relative assertions, there is no place in such a conception for objective truth criteria of theories. Instead, a subjective criterion is advanced. This is evidenced in numerous cases.

Thus, arguing in favor of his conception, Wigner refers to "Einstein's observation that the only physical theories which we are willing to accept are the beautiful ones" (7). But beauty, as is well known, is an esthetic category.

In the process of losing objective truth criteria, physical theory becomes indeterminate and ambiguous (we are speaking here of physical theories that explain the same phenomena). Wigner underlines this idea of the ambiguity of the theories early in his lecture. His student asks the question: "How do we know that, if we made a theory which focuses its attention on phenomena we disregard and disregards some of the phenomena now commanding our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory? ." Wigner answers:" It has to be admitted that we have no definite evidence that there is no such theory" (7). Further, he states even more explicitly:" We cannot know whether a theory formulated in terms of mathematical concepts is uniquely appropriate. We are in a position similar to that of a man who was provided with a bunch of keys and who, having to open several doors in succession, always hit on the right key on the first or second trial. He became skeptical concerning the uniqueness of the coordination between keys and doors" (2).

Such a conclusion is a direct consequence of the fact that there is no unambiguous truth criterion for a theory; mathematics furnishes for physics a bunch of keys theories, of which almost any key-theory will open the door—one or two tries are sufficient. This is in complete accord with the notion that laws of nature are in essence conditional statements; they do not form external constraints, the key and the lock will fit each other.

4. More General and Less General ("False") Theories

In the same spirit, Wigner discusses the question of theories that are more general and less general. He does not specify their definitions, and only compares them with one another: the more general theory gives a more general perspective. The question of how the more general theory arises is not discussed in the lecture; it is evident from the context, however, that it does not arise on the basis of the generalization of less general theories, inasmuch as Wigner points out that a less general theory can contradict a more general one. In such a case, according to Wigner, it is evidently a false theory.

Since there are no objective truth criteria for theories, it may turn out that that theory we presently consider to be more general is false. Wigner does not deny such a conclusion: "Similarly, it is possible that the theories, which we consider to be "proved" by a number of numerical agreements which appears to be large enough for us, are still false because they are in conflict with a possibly more encompassing theory which is beyond our means of discovery" (12).

Thus, according to Wigner, any theory used with reliability by us at our present stage of knowledge, may turn out to be false. It will never be possible to determine this beforehand.

Wigner knows, of course, that certain physicists—among them famous scientists who contribute to the progress of modern physics—were led by a truth criterion of theories, namely the coincidence of its conclusions with experimental results and the precision of its predictions. He excludes this criterion however. Wigner affirms that even admittedly false theories "which we know to be false give such amazingly accurate results" (12). He gives an example of such false theories "which give, in view of their falseness, alarmingly accurate descriptions of groups of phenomena." Thus, for example, the free-electron theory, "which gives a marvelously accurate picture of many, if not most, properties of metals, semiconductors and insulators" (13).

Thus, an admittedly false theory gives an alarmingly accurate description of certain phenomena! This leads to a gnosiological conclusion: "the free-electron theory raises doubts as to how much we should trust numerical agreement between theory and experiment as evidence for the correctness of the theory" (13).

If there are no objective truth criteria of theories, then the question arises inevitably, namely what then is our justification for our efforts in creating a theory which has a chance of being false? Discussing the possibility of contradictions between future theoretical biology and physical theory, as a result of which "our faith in our theories would be strongly shattered," Wigner states: "the reason that such a situation is conceivable is that, fundamentally, we do not know why our theories work so well. Hence their accuracy may not prove their truth and consistency" (14).

This is Wigner's hopeless conclusion. He knows himself that this conclusion is much too pessimistic. He speaks of the desire to finish the lecture "on a more cheerful note," and concludes: "the miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it" (14).

This concluding "optimism" is inconsistent with the preceding statements made by Wigner. It sounds more like a thanksgiving prayer service to mysterious forces, and is alien to the authentic optimism of the scientist who consistently and ever more profoundly learns about objective nature.

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We have performed a paleontologist's constructive labor, and concluded that these are not arbitrary reservations, nor individual unsuccessful or "deliberately acute" formulations, but an entire conception. All theses are connected with one another: the free creation of mathematical concepts and theories, under the impulse of esthetic necessities; the subjective character of the laws of nature; the negation of truth criteria of physical theories; the possible existence of many non-equivalent theories for the same phenomenon; the notion of "false" theories, no matter how accurately descriptive of the phenomena; the perspective of a recognized theory being turned into a "false" theory.

It is necessary to examine whether the Wignerian treatment of knowledge can be substantiated by the history of scientific progress.

II. PHYSICS AND THE OBJECTIVE SENSE OF THE LAWS OF NATURE

Wigner's concept is not at all the consequence of the development of physics, whether classical or modern. On the contrary, the development of physics and of philosophy confirm another theory of knowledge—the theory arising from the recognition of the objective character of laws of nature, and their reflection in theories as a result of the enormous success of the process of knowledge.

It is from the standpoint of this latter theory of knowledge that one must examine the problems discussed by Wigner*.

These problems can be classed in three groups. The first group is related to the question of the fate and the role of laws of nature in connection with the development of physics. The second group is concerned with the logical sense of theory, its relationship to nature, truth criteria, interdependence of successively deeper theories. Finally, the third group of problems is connected with the discovery of the "secret" of the effectiveness of mathematics. All these groups of problems are closely interrelated.

1. Physical Theories of the 20-th Century and Laws of Nature

Physics as a whole has developed as a complex organism, in which new theories constantly arise, and old theories are generalized. Regularities of a new type (statistical, quantum) are discovered, physical methods ("methods of observation") are developed and modified. How does all this affect the physicist's formulation of the laws of nature? Do the laws vanish without trace as new theories appear—which would be natural, if laws of nature were conditional statements—or are they preserved and transformed as constantly deeper levels of nature are discovered in physics? Is the concept of a "law" at all conserved in connection with the discovery of the role of probability in predictions? Does the fate of a law depend on changes in the methods of observation?

Similar questions were constantly asked by physicists. In the first decades of this century, the notion that new theories of physics refute all or nearly all laws of nature discovered in classical physics was widely held. Illustrations of this are not necessary, they are well known.

If we study, however, the development of the basic theories of physics of the present century, we will see that the theories not only refute the absoluteness of certain notions, but also force essential positions with which the physicist was required to cope as if with external constraints of nature.

When the theory of relativity was formulated, many physicists paid more attention to the unusual notions, namely the conclusion that a substance has no linear dimensions as such, no mass as such, that there is no notion of time flow by itself. According to the theory of relativity, these concepts take on a particular meaning only under particular circumstances; in systems under inertial motion, the quantities related to them depend on the relative velocities of the body and of the system. These conclusions of the theory refuted the traditional concepts of the absoluteness of the named properties. To those physicists who naively related materialism to the recognition that bodies have absolute properties, the refutation of these notions seemed extremely risky; a war which lasted several decades began against the theory of relativity and its author.

But the critics of the theory of relativity did not realize the essential side of the story: <u>the theory was based on objective facts</u>, viz., the independence of the speed of light of the relative motion of the source, the invariance of a series of physical quantities in inertial systems, particularly the covariance of Maxwell's equations of electromagnetism. It is precisely the necessity of taking into account these objective facts occurring in inertial systems which <u>forced</u> the physi-

*We shall also have to present physical facts. We are of course in no doubt that these are well known to Wigner. The point is that when even a prominent physicist raises the problem of the basis of his own gnosiological conception, he uses, in the heat of constructive enthusiasm, the physical facts in his own way. We, however, present them as the basis for another conception, and have in mind a wide circle of readers.

cists to change their concepts of the absoluteness of a number of notions.

Did then the theory of relativity refute by the same token the concept of objectivity of laws of nature discovered by classical physicists? Not at all! Furthermore, the formulation of already well-known physical laws in connection with properties of Galilean space, for which the Lorentz transformations are valid, is precisely the subject of the theory of relativity. All ten integrals of motion, including the energy integral, are valid in this space.

Of course, the laws of classical physics are generalized in this theory. For instance, the classical laws of conservation of energy and conservation of mass are generalized into a single law of conservation of mass-energy as a result of the law of proportionality between mass and energy. This last law, established by theory, became the foundation for the calculation of the energy output in nuclear conversions—a new promising field of modern high-energy physics. Similarly, classical laws of conservation of momentum and kinetic energy are also generalized; in the special theory of relativity they form one law—the law of constancy of four-dimensional momentum vectors.

Thus, the theory of relativity not only did not refute the previously well-known laws of nature, but in fact was based on them as objective and necessary laws, and discovered new laws which cannot be classified by any means as "conditional statements."

More than once it has happened in physics that the laws of nature discovered for a specific set of phenomena, but of a general character, were used towards the discovery of laws specific for new fields of study. This was the case, for example, when Planck was looking for the conditions of equilibrium radiation of an absolutely black body. It is well known that he based himself primarily on the thermodynamic law according to which the entropy of the equilibrium state must be maximal. Indeed, this law was also valid in a new region (although naturally it turned out to be insufficient for the discovery of the laws of radiation, and Planck had to utilize Boltzmann's equation relating the change of entropy with the probability, and the formal computation of "complexes" according to Stirling's formula.) Considering the entropy of radiation per unit volume in a closed region and using the same Boltzmann's equation, Einstein came to the famous conclusion (in 1905) that entropy behaves as if the radiation, at least within Wien's region, consists of individual quanta with energy proportional to the frequency. It is from this concept that arose a whole series of ideas which were the foundation of quantum mechanics. Thus, Planck and Einstein acted in a reasonable manner by basing themselves on thermodynamic laws even in the new field of investigation.

In the field of quantum phenomena, the situation has turned out to be similar. As in the case of the theory of relativity, in the development of quantum mechanics physicists were required to take into account a series of "unusual," "incomprehensible" facts, such as: the discrete spectrum of atomic radiation, the combination principle in the set of radiated frequencies, the "incomprehensible" connection of the energy and momentum with frequency of radiation and wavelength, the discovery of sometime predominantly wave properties and sometime primarily discrete properties of light, etc...

Physicists were compelled to take into account these "whims" of nature. They understood, however, that one should not regard these phenomena of nature as conditional statements, and constructed a theory that generalized all unexplained facts, and related them by one incontrovertible logical system.

At the same time, they had to account in the development of this theory not only for the aforementioned facts, but also for a series of previously discovered laws of nature which were generalized to the new field of atomic phenomena. For example, in the quantum region the laws of conservation of energy and momentum remained valid; in particular, they helped explain the regularities of scattering of light by crystals. Planck's law of radiation energy distribution in a spectrum played a substantial role in the further discovery of quantum laws. In 1909, Einstein showed that the only formula that agrees with this law is that of the fluctuation of energy of a light field consisting of two components, one of which reflects the wave properties and the other reflects discrete properties of radiation. Later (in 1916) Einstein on the basis of this law proved the statistical distribution of elementary momenta in needle-like radiation of atoms.

The transformation of particles of one type into particles of another type is studied in quantum electrodynamics. It is essential that in these processes, which are also unusual for classical physics, objective laws of transformation be sought. They are formulated in the theory of symmetry. In this field, the laws of transformation are considerably more complicated than in transformations studied in classical physics: in their formulation are also included the charge, isotopic spin, strangeness as well as other characteristics. But this does not change the gnosiological sense of the problem: the search for laws of transformation of elementary particles. It can be said that not everything is yet clear, the theory is still in the process of establishment, and occasionally unusual difficulties are encountered. But it is always essential that ways of solving these difficulties are found, and it becomes firmly established that in the field of elementary particles everything occurs according to definite laws of nature.

It follows from this that new theories of physics, no matter how unsettling to our notions, are formulated as a logical generalization of facts forced by nature on the theoreticist; they are based on already known laws of nature, and generalize them; new laws are then discovered, laws which extend our knowledge of nature. All this does not leave room for treatment of laws of nature as conditional statements.

2. "Methods of Observation" and Laws of Nature

During the entire history of physics, the "methods of observation" of the phenomena of nature did in fact change. Let us take for example the gravitational interaction of bodies.

Almost two thousand years ago, Ptolemy had drawn the general picture of motion of the planets and of the sun. This was a picture of fairly complicated planet trajectories. It was not invented, but was rather the result of observations and measurements, and Ptolemy was able to affirm the discovery of a law of nature. It could be said, rather, that this example is a good illustration of the concept that "a law of nature is a conditional statement"; it is particularly clear that Ptolemy's "law of nature" was indeed formed by the method of observation.

This law, however, did not lead to further knowledge, since the "method of observation" in this case did not reveal essential objective relations in the system; it was too inadequate (man is the center of the system). Hence, science repealed the "laws" of Ptolemy, and his work is only remembered now by historians.

More than fifteen hundred years passed before Copernicus, Bruno, Galileo and other scientists and thinkers of the new times overcame the Ptolemaic "method of observation" and found a new one, with the sun as its center. This method of observation differed from the Ptolemaic one in that it corresponded more closely to the actual relationships of the solar system. The three laws of Kepler owe their existence to it. This was an important step in further progress.

But Kepler's laws were connected empirically with the method of observation; there was no proof of their generality, and even the necessity of the interrelationship between all the laws was not clear. The laws were the result of observations, but did not exceed the bounds of kinematic relations.

Newton went farther. His method of observation differed substantially from Kepler's. First of all, he extended Kepler's laws to the earth-moon system, and related the laws of motion of the moon along its trajectory with the Galilean laws of the uniformly accelerated fall of bodies to the earth (the actual motion of the moon was considered to be the result of its constant 'falling' under the action of the earth's attraction from a possible inertial trajectory to the actual one). This required the formulation of the principle of inertia and the laws establishing dynamic relationships. As a consequence, the law of gravitation was formulated, and the possibility of applying it to all bodies with a mass was realized. Kepler's laws, which reflected the kinematic relationships of the motion of planetary bodies, were now considered as the natural, interrelated consequences of a single ('universal') law of gravitation. But, even more important, the establishment of dynamic relationships in Newton's solution led to the formulation of general laws of mechanics which were the basis of the development of macrotechnology.

The important result achieved as a consequence of Newton's method of observation is undisputable, even if we consider only problems of gravitation. But even this method of observation had to be surmounted.

Newton assumed that the forces of gravitation are in essence forces acting at large distances and depend only on the interacting masses and the square of the distance between them. It seemed as if the law of gravitation had achieved its absolute form. It is true that the law in its form did not yet explain, for example, such a phenomenon as the rotation of the elliptical trajectory of Mercury in its plane. But accurate computations, for many years ahead, of actual planetary events, such as eclipses, disguised this "small" inadequacy.

At the same time, another method of observation than Newton's began to be developed with the help of the notion of a continuous field characterized at each point by a gravitational potential φ, related to the gravitation force acting on a unit of mass at that point. At the end of the 18-th and in the first half of the 19-th century this method was worked out in detail by Laplace, Poisson, and others; in particular, the character of the relationship between the gravitational potential and the mass density (Poisson's equation) was clarified.

It seemed at first as if the method of a continuous gravitational field was fully equivalent in all cases to the method of Newton's long-range forces, and that its advantage lay only in that it simplified the computations in the case of many gravitating bodies. Later, however, it turned out that the method of the gravitational field could be greatly generalized, as done by Einstein. Einstein used the notion of the continuous field as the basis for his investigations. Generalizing Poisson's equation and basing himself on the quite general law of nature of the equality of the gravitational and inertial masses, Einstein derived the generalized law of gravitation. The derivation of this law led to many important results. It turned out that Newton's law of gravitation is valid only for weak fields, when the parameter φ/c^2 which characterizes the field is small; for strong fields, however, this law is only an approximation, while the law formulated by Einstein is more precise. Einstein's law explained also the peculiarities of the motion of Mercury. Most important, however, is that Einstein revealed the connection between the law of gravitation and the geometric properties of space-time (its distortion), which depend on the distribution of the masses, and this led to the interpretation of inertial motion as motion along geodesic curves and to the establishment of the interrelationship of gravitational and electromagnetic fields (the bending of a beam of light in a strong gravitational field).

The generalization of Einstein's law of gravitation was an important step in science. It was the result of a new method of observation. Does this transition from one form of the law of gravitation to another confirm the notion that laws of nature are "all conditional statements" that depend on the method of observation of nature?

Not at all! To the contrary, we see that each subsequent method of observation does not contradict the results of the preceding method, but establishes them as a particular case of a more general approach. The method of observation of nature becomes more and more general and leads to an ever deeper knowledge of nature by making it possible to solve a continually larger number of problems in the exploitation of nature.

This means that in the historic development of knowledge, methods of observation of nature become more and more adequate. They are by no means arbitrary, nor subjective, but only reflect the degree of our knowledge of nature. The laws which we thereby discover are in essence objective laws realized in nature itself. The connection between the formulation of the law and the method of observation does not give basis for the enunciation of a new gnosiological tenet: "laws of nature are conditional statements."

Analogous conclusions could be made from an analysis of the development other problems in physics.*)

3. Statistics and Laws of Nature

Wigner considers as one of the arguments in favor of the notion that laws of nature are conditional statements the fact that laws are only of a probabilistic nature; hence, they only permit a mental bet regarding the future properties of non-living nature. In this conception, science is considered as the aggregate of conditional statements, as a total bet with the hope for a miracle.

Undoubtedly, modern science cannot be developed without account of statistical laws. Their role continuously increases with the progress of science in all aspects of knowledge.

However, all of man's activities do not bear the character of a bet. Maneuvers in space and the docking of space vehicles are performed with a great precision, although these processes are carried out along a long chain of various interactions, in each link of which there is a distribution of initial conditions. The process of chain reaction in an atomic bomb occurs in accordance with laws of probability, but the explosion of the atom bomb occurs at an instant of time which differs (as a consequence of statistics!) from the predicted time by no more than 10^{-8} sec, which is quite sufficient to the achievement of the set goal in the macroscopic scale. In these cases, man does not make any bets, but rather performs detailed computations.

Why then is this indisputable fact of the consequences of our knowledge not reflected in the theory of knowledge? Does it not underline in the large scale the existence of a connection between probabilistic laws and laws permitting a forecast? Cannot this connection be observed in a concrete situation? This has been done in physics more than once; Einstein used this method most frequently in the development of the theory of Brownian motion and the theory of radiation of photons by atoms. In particular, Einstein established that the character of absorption and emission of photons by atoms is connected with a specific (Planck) law of distribution of energy density of heat radiation. This relationship can be considered in both the direct and reverse directions. Modern physics has uncovered many analogous connections of statistical laws with uniquely formulated laws. It can be said metaphorically that the existence of this connection shows that nature itself integrates the action of many statistical elements that constitute an entire system. Einstein understood this probably more than any of his contemporaries.

In statistical laws, it is not the question of the "trajectory" of an isolated element of statistics which is essential, but rather the law of distribution of all elements for a given parameter characterizing the motion of the system as a whole. Of course, if we consider the question of the direction of motion of a photon emitted by a single atom under spontaneous transition to a lower level, then the prediction will be probabilistic in nature. If we wish, this would be a "mental bet" on the motion of the photon. But such a bet would not be beneficial. On the other hand, an account of the specific weight of statistical processes of a specific type and their relation to the laws of the system as a whole (in particular, to Planck's law of radiation) led Einstein to the conclusion that, along with absorption and spontaneous emission, there must occur induced radiation (induced by an external field).* As is known, the existence of this radiation was confirmed only after many years, and it has found practical application only in our time (in quantum generators). We note that although the result of the interaction of an atom and the field is determined, according to Einstein, by the statistical correlation of corresponding phases, it nevertheless follows from the general energy balance that the energy of the induced radiation under the given conditions constitutes a definite fraction of the energy.

The notion of statistical laws is as important as the notion of dynamic laws. Many examples can be given in which statistical laws make it possible to solve the problem with sufficient accuracy—not by the method of a bet, but by revealing in the statistical scatter of the elements certain parameters characterizing this scatter. Thus, mathematicians solve the problem of predicting the position of a plane under artillery fire. They use in this the statistical distribution of possible trajectories of the plane (depending, in particular, on the pilot's will) and seek a method of prognosis under which a certain quantity that characterizes the error is minimized. The important problem, especially in radio technology, of the amplification of a useful weak signal against masking noise is also solved on the basis of data on the differences between the statistical characteristics of useful signals and noise. Consequently, statistical laws yield to qualitative and quantitative determinations and do not exclude the very notion of a law.

We cannot stop here in more detail on this important and profound question. It follows from the aforementioned, however, that statistical laws are not at all arbitrary, but that they characterize definite relationships in systems whose behavior under specific conditions can be predicted with definite certainty, and which obey uniquely determined laws. In this manner, the growth in the importance of statistical laws in modern physics does not transform laws of nature into "conditional statements," does not make science a "mental bet," but to the contrary enriches science with the discovery of new natural relationships. Nature excludes arbitrary rules, but permits only clever questions to be asked of it. Hegel pointed out that "a definite development is necessary in order to be able to ask questions (Lenin's emphasis), especially in philosophy, otherwise the answer may be that the question is nonsensical." Lenin thought that this was "well stated"†.

*We shall return to the problem of methods of observation in Ch. III, Sec. 6.

*Ignoring the induced radiation leads not to Planck's formula but rather to Wien's formula for radiation.

†See: V. I. Lenin, Abstract of Hegel's book "Science of Logic," Complete Works, vol. 29, p. 103 (in Russian).

4. Conditionality of Realization of Laws of Nature. Interrelationship Between Laws

By creating new, ever more profound theories that reveal objective relationships of nature, modern physics has contributed much new knowledge to the understanding of the nature of these laws. Thus, physics has revealed that laws of nature do not lose their objective sense even after the establishment of the relativity of notions that enters into the formulation of the laws. Even in the new theories there exist invariant relationships of transformed quantities. Modern physics has also shown the limitations of unambiguously determined relations, the idealization of performing absolutely precise measurements (such as initial values of coordinates) and so on.

The *conditionality of the realization of the laws of nature* is beyond doubt. Physicists have encountered it, for example, during the transition from macrophysics to microphysics: the action of certain laws of classical physics turned out to be valid within limits in which one may neglect the magnitude of the quantum of action.

It is especially important to take into account the conditionality of laws upon transition to a new field of investigation with unusual conditions, such as in the formulation of hypotheses concerning the development of certain cosmic objects in which physical conditions, as has become well known, differ drastically from those encountered so far in science. Is it possible to lean on well known laws of physics in such hypotheses?

In discussing the question of transformation of substances in cosmic objects in which the density changes by billions of times, and the pressure of the gravitational field reaches unheard of values, V. A. Ambartsumyan concludes: "We do not have and cannot have any guarantee that well-known physical laws are also valid under these conditions. It would therefore be completely unsurprising if it turned out that the large difficulties in explaining theoretically a number of nonstationary processes which already exist may in the course of time grow into a direct contradiction of known laws of theoretical physics."*

It is impossible not to consider such a possibility of changes in laws. But even under these conditions which are contrary to our practice, there exist objective laws which physicists and astrophysicists will eventually discover. And, apparently, these scientists will work from already known laws in their search for modifications under new conditions. This is because any future adequate theory, *no matter under which conditions it is formulated* cannot be constructed in contradiction to laws and facts verified at the present stage of learning under known conditions. The latter conclusion stems from the identity of nature and its laws which has found an expression in the principle of correspondence (see below).

Thus, all limitations on the absoluteness of laws of nature cannot suppress the gnosiological conclusion that laws of nature are an objective fact rather than some "regulation" of our subjective perceptions or "conditional statements."

It is necessary to underline one additional phase of the problem. In actuality the physicist never speculates in the spirit of Wigner's pronouncements, as for instance "I have become familiar with Planck's law of radiation. What a clever result, and yet it is only a conditional statement, inasmuch as his formulation is related to measurements, whereas no measurement is absolute."

No, despite the approximate nature of any individual measurement, Planck's law of radiation is accurate, accurate at least to that extent that it permits to establish the quantum nature of light and the statistical law of absorption and emission of photons by atoms. This in turn is confirmed by many other qualitatively different laws, each one of which is also not absolute. This points to the fact that in reality the basis of Planck's law is considerably wider than the direct measurements by Rubens and Curlbaum. And this general interrelationship of laws of nature, verified experimentally, is valid in all cases. The relative imprecision of measurements is thus removed in the establishment of any law. We have already given a few examples of such interrelationships between laws in connection with our discussion of other aspects of the problem.

It is precisely this general connection between the laws of nature, their logical common nature, which explains the fact that amazed Wigner, namely that the mind does not get completely confused in contradictions.

* * *

Thus, a law of nature is not absolute in the sense given by 19-th century physicists. But it is an objective tendency in nature which is realized under specific conditions. In science, the discovery of planets, of chemical elements, elementary particles, or deposits of useful minerals on the basis of predictions based on definite laws are all well known. Does this not confirm the objective sense of law-governed interrelationships in nature? It is precisely the objectivity of laws and the possibility of understanding them which are at the basis of science. There could not be any theory or any science if there were no objective laws of nature. It is the goal of science to discover these laws, rather than to formulate "conditional statements."

It is improper *to confuse the relativity of physical* knowledge with the philosophical problem of the existence of objective laws. This is that same concession to idealism about which V. I. Lenin wrote sixty years ago, by explaining that the origin of this concession is that "physicists don't know dialectics." Listing examples of this tendency towards idealism on the part of physicists, Lenin also touched directly on the question of the relativity and objectiveness of laws of nature: "By refuting the absolute character of the most important and basic laws, (physicists prone to idealism—S.S.) wound up refuting any objective regularity in nature and declaring that laws of nature are purely conditional, "limitations on expectation," "logical necessities," etc..."*

This analysis is still valid today.

*V. A. Ambartsumyan, Contemporary natural science and philosophy (Sovremennoe estestovoznanie i filosofiya), paper at the XIV-th International Congress of Philosophy, Vienna 1968 (Usp. Fiz. Nauk 96, 3 (1968) [Sov. Phys.-Usp. 11, (1969)]

*V. I. Lenin, Materialism and Empirical criticism, Ch. V, Sec 2, Complete Works vol. 18, p. 277 (in Russian).

III. THE OBJECTIVE MEANING OF THEORIES

Objective regularities of nature are discovered through theories. By means of theories one discovers their necessity and limits of application.

In the practical life of a society, the meaning of a theory is so clear that no one asks whether it is necessary, or why it is necessary to formulate them. It seems natural that in enlightened countries lectures are given in theoretical physics, that textbooks are published, journals printed, departments and scientific experimental institutes created; a considerable amount of resources are devoted to the development of these institutions. It is clear to every one that theories are developed in order to understand laws of nature as part of the practical goal of the development of industrial forces.

From this trivial truth important gnosiological conclusions follow, which, however, are not accepted by all theoreticians.

1. Truth Criteria of Theories

If the meaning of theories is that they reflect nature, then there exists an objective truth criterion for theories, which is consequently not of an esthetic nature.

Every important naturalist, not only in the past but also in our time, held that such a criterion is the agreement of the conclusions of the theory with the experimental results, the prediction of new results on the basis of the theory, results which the investigator had not yet experienced (the heuristic power of theories). Einstein was such a scientist.

To cite Einstein as confirming that the only validity criterion of a theory is its beauty means to misunderstand the meaning of Einstein's work as a theoretician, as well as his concepts. Physicists are well acquainted with the fact of how careful Einstein was to lead his theories to the formulation of conclusions that may be verified experimentally. This is true of a number of his theories—Brownian motion, the quantum nature of light, the special theory of relativity, the general theory of gravitation. He turned not infrequently to experimentalists in order to verify the conclusions of some theory just worked out by him.

But the question of truth criteria of theories was placed by him on the theoretical plane. Einstein, in his widely known scientific autobiography (1949) had expounded the principles which he followed in his investigations. Here is what he wrote concerning the truth criteria of theories: "The first criterion is obvious: the theory must not contradict the experimental data. But in so far as this requirement seems obvious, in so far is its use refined. The point is that frequently, if not always, it is possible to preserve the given general theoretical basis if one only adjusts it to reality with the aid of more or less artificial additional assumptions. In any case, the first criterion has to do with the verification of the theoretical basis by available experimental data.

The second criterion has to do not with the relation to experimental data but with the premises of the theory, with what may be called simply, although not completely correctly, the "naturalness" or "logical simplicity" of the premises (the basic concepts and basic relationships among them). This criterion, whose precise formulation bears great difficulties, has always played an important role in the choice among theories and in their evaluation. The problem here is not merely in some enumeration of logically independent premises (if this is at all possible to do unambiguously) but in a kind of weighing and comparison of incommensurable qualities.

Furthermore, of two theories with equally "simple" basic tenets one should choose the one which limits more strongly all possible a priori qualities of the system (i.e., contains the most definite assertions)"*.

This formulation differs from Wigner's in two respects. First of all, Einstein was writing not about a single criterion, but about two, placing in the first place the agreement between theory and experiment; secondly, Einstein proposed as a second criterion the naturalness or logical simplicity of the theoretical premises, which can receive a rational objective interpretation (we cannot develop this theme here) and which is by no means equivalent to the esthetic criterion of beauty.

In the history of natural science, we can cite as an example those scientists who favored the esthetic criterion of theoretical validity. It was not Einstein, but rather Aristotle who was such a scientist, as noted correctly by F. Dyson. Dyson wrote: "Mathematical intuition turns out to be much more frequently conservative than revolutionary, it more frequently binds than loosens. Among the most reactionary in all the history of physics was Aristotle's and Ptolemy's notion of a geocentric system according to which all planetary bodies moved in spheres and circles. Aristotle's astronomy almost completely eclipsed science for 1800 years (from 250 B.C. to 1550 A.D.). This stagnation of science was explained, apparently, by many different reasons, but it is impossible to neglect the fact that the main reason for the popularity of Aristotle's astronomy was the faulty mathematical intuition which said that only spheres and circles are esthetically pleasing' †.

As we can see, Dyson in his estimate of the esthetic criterion contradicts Wigner: he notes the reactionary role of the esthetic criterion as used by Aristotle. Indeed, was it not with the invitation to scientists to read the book of nature itself that the modern revolution in natural science began?

This invitation meant nothing less than an appeal to the objective criterion of truth of a theory. From the time that such a criterion was used as the basis of experimental work, natural science has progressed rapidly.

Of course, the esthetic criterion is also encountered in modern theoretical investigations, especially in such sciences as cosmology. However, the extent to which this criterion is unreliable and unconvincing was shown by Ya. B. Zel'dovich. Noting that in cosmological theories certain authors base themselves on the hy-

*Albert Einstein, Autobiographical Notes, Collected Scientific Papers, vol IV, M., Nauka, 1967, p. 266 (Russ. transl.)

† F. Dyson, Mathematics and physics, Scientific American, 211 (9), 129 (1964).

pothesis that the cosmological constant $\Lambda \equiv 0$, he writes: "either esthetic arguments are advanced, such as: the theory with $\Lambda \equiv 0$ is more beautiful, the formulas are more compact, there exists a particular solution—the empty flat world of Minkowski, or else arguments which remind one of the principle of the economy of thought: why introduce an extra parameter Λ as long as it is not imperative? When authors (cf. the work of E. Salpeter et al., I. S. Shklovskiĭ, N. S. Kardahshev—S.S.) very interested in $\Lambda \neq 0$ appear, the arguments presented above lose their attraction and strength of conviction. It turns out that many authors (cf. the paper by A. L. Zel'manov—S.S.) always considered the scheme with $\Lambda \neq 0$", more beautiful, because of its great generality.*) The author rightly thinks that "the question must be solved on the basis of objective data."

2. On "False" Theories that Describe Phenomena Precisely. The Correspondence Principle

If the value of theories consists in that they reflect nature, if there exists an objective criterion of validity of theories, then Wigner's views, according to which the creation of a more general theory reduces the preceeding less general theory to the position of a "false" theory, are inconsistent with these tenets.

The process of creation of ever more general theories does indeed take place. It is based on the fact that man discovers an ever larger number of relations in nature, while knowledge about nature is deepened. However, at each stage each theory (we are talking here of correct theories) is a generalization of a certain number of discovered relationships in nature, but a small number, and dealing with more external phenomena. Hence, the theory cannot be classified as false merely because there exists a more general theory. If one were to use such a measure, then all of classical physics would be classified as false. But who would agree to do this? Classical physics reflects correctly many regularities of the macroscopic world and is the theoretical basis for macrotechnology, even today when we have the theory of relativity and quantum mechanics. Moreover, classical physics is on the plane of both logic and material technology a necessary step towards the succeeding more general and deeper theory. There are *no reasons to create an artificial* paradox: the theory is false, but for some reason it describes frighteningly precisely (!) certain phenomena! The theory describes them because it is their generalization rather than a random mental construction by a scientist.

Physicists who created modern physics followed more well-grounded paths: while feeling out the contours of a new quantum mechanics, they thought it necessary to establish its genetic relation to classical theory and found a situation in which the more general theory takes on a specific character of the preceding theory; moreover, this relationship between the more general theory and the preceding one turned out to be the guiding line in the search for a new theory and the validity criterion. This relation is formulated in the form of the correspondence principle, and this was accomplished for physicists by Niels Bohr, as previously noted.

It is quite natural that scientists, treating their theories as the reflection of objective regularities in nature, had to arrive at precisely the very notion of the relationship between consecutively developed theories as expressed by the correspondence principle.

Wigner's views, on the other hand, his argument that as new, more general theories appear the older theories become false, contradict the correspondence principle. He did not present and could not present any arguments against this principle, which has been verified in all consecutively developed sciences (such as in the correspondence of relativistic and classical mechanics, non-Euclidean and Euclidean geometries, etc...). We assume, by the way, that Wigner did not have in mind to attack this principle which has received such wide acceptance in physics. We only assert that his gnosiological notions are in contradiction to well known results in physics.

In order that our further critique be clearer, we must briefly describe our notion of the establishment of theories.

3. The Logical Meaning of a Theory; Its Postulates; Its Uniqueness.

A theory arises as the result of the search and formulation of conditions of the logical common nature of relationships discovered in nature and verified experimentally. Let us call these relationships the initial postulates of the future theory. Among initial postulates there may also be found preceding theories which have already been verified. In the final analysis, any theory is a generalization of experiments, observations and experience. Since a theory is such a generalization, it is in some sense more reliable than a single experiment.

Theory of (special) relativity has the following as initial postulates: the independence of the velocity of light of the motion of its source; the principle of the relativity of physical phenomena, such as electromagnetic phenomena, in inertial systems (from which follows in particular the covariance of Maxwell's equations of electromagnetism, whose establishment played an important historical role in the search for postulates of the theory of relativity). These initial postulates are sufficient to conclude the "conditions of their logical common nature"—the transformation of coordinates, time, electrical and magnetic intensities—and to derive all invariants, i.e., to create the theory of relativity. This was the only method of solving the problem which arose in physics at the end of the 19-th century. No other way could lead to an adequate theory. Indeed, from the time of Maxwell's experiments, physicists possessed essentially all the data to generalize the results of mutually contradictory classical experiments in electrodynamics. For twenty years, however, no solution was found. This may be explained precisely by the fact that at that time the correct method of finding a general theory was not yet available. There was an attempt to explain each new result in the light of al-

*Ya. B. Zel'dovich, The Cosmological Constant and the Theory of Elementary Particles UFN 95, 209 (1968) [Sov. Phys.-Usp. 11, 381 (1968)] ready-known notions, by searching for new special reasons to explain the new result. Thus, a negative result was seen in a single experiment—that of Maxwell—in the shortening of the linear dimensions of bodies in the direction of motion (the Lorentz-Fitzgerald contraction).

Only the fundamentally new formulation of the problem by Einstein led to the creation of a new theory, the theory of relativity. This new formulation of the problem consisted in the search for conditions of logical common nature of the initial postulates, chosen by Einstein, whose validity had been established experimentally. It led precisely to a re-examination of a number of notions previously held absolute.

Of course, the correct choice of initial postulates, as well as the subsequent transition to the theory is a complicated affair requiring enormous intuition, experience, and the talent of the scientist, and is most often realized by the collective efforts of an entire generation of scientists. In fact, the search for theories by each individual scientist cannot be placed in a simple logical scheme; often it is determined by tradition, the general conceptions of the scientist and by many psychological factors; the objective logical sense of his research is not always clearly perceived by the scientist; frequently he is influenced by the struggle for the priority of ideas, as a result of which each new discovery leads to the rapid appearance of a new "theory," although perhaps the necessary choice of initial postulates has not yet been found. All this is necessary to understand the complex zig-zags in science. However, the clarification of the logical foundations of the creation of theories is sufficient for gnosiology.

The choice of initial postulates must correspond to definite requirements. Let us name some of them.

Each chosen postulate must be non-repetitive in the sense that it must be a personal specific contribution to the future theory; we shall call this contribution the logical content of the postulate. The entire collection of discovered experimental relationships in the given field of phenomena may be subdivided into a small number of different groups, each one of which comprises relationships that may be different in form but are equivalent in logical content. Naturally, the set of initial postulates includes only one postulate from each group.

Further, the set of initial postulates must be complete in the sense that it must include relationships from all different groups, i.e., the set of postulates must consist of all possible and different in logical content postulative relationships. Only in this case will the theory be a generalization of all phenomena in the new field*.

Of course, other physical relationships may be chosen from each group. But inasmuch as they are taken from the same groups and since their logical content is the same, the new set of initial postulates will be potentially logically equivalent to the previous one. It follows that the formulation of conditions of the logical common nature of this set of postulates will be different from the previous one only in form, but will be equivalent to it.

In light of the summarized notions about the progress of theories, it is possible to evaluate the attempts by Bohm, Vigier and other physicists to construct a new variant of quantum mechanics on the basis of the introduction of "hidden parameters." These attempts are not based on the enrichment of the set of postulates with new postulates with a specific logical content, and which might have arisen from new experiments. Naturally, this effort did not result in a new fruitful theory.

The here-developed notions of theories are intended to explain the place and role of theory in the process of knowledge, and its establishment as a form of objective reality. From the notion of the gnosiological sense of theories, it is not only those conclusions which we employ that follow, but also many others such as the correspondence of theory and concepts, the growth of the latter and the source of their "contradictions," the heuristic power of theories, and so forth, on which we cannot elaborate here*.

4. The Development of Quantum Mechanics

The set of initial postulates which led to the creation of quantum mechanics was assembled over a period of nearly a quarter of a century. We cannot discuss the question in detail here, but wish to mention that the results of many investigations entered into this set of postulates: the discovery of the quantum nature of light and its dual nature; the establishment of stationary states in atoms and transitions between which, both in the simplest and more complicated atoms, result in emission (or absorption) of quanta with energy proportional to the frequency; the combination principle of frequencies of emission of all atoms; the discovery of various types of quantum statistics, including certain "forbidden" states; the establishment of the use of the classical Hamiltonian form in the quantum system, and others.

The choice of this set was a complex and even painful process. Two generations of physicists took part in it. Even an element of drama was present, since its originator, who contributed many new, unusual but experimentally founded basic ideas, was later captivated by another scheme which he thought more simple and realizable for the construction of a universal model, and subsequently turned away from his followers and friends, and for many years protested against the principles of quantum mechanics. It is easy to guess that we are talking here of Einstein.† His direct collaborators Born, Heisenberg and others, remember his many difficult researches, reflections, and doubts in the process of the creation of quantum theory.

*The variations in the estimates of the value of the cosmological constant, described in Ch III, Sec. 1 are merely evidence of the fact that the initial postulates have not yet been chosen in this field (and possibly have not yet all been discovered), and the creation of the theory is by a "trial and error" method.

*In practice, the term "theory" is used in physics in the most different ways, and is often substituted for other more appropriate terms – "hypothesis," "aggregate of notions," etc . . .

†For a more detailed presentation, see: S. G. Suvorov, Einstein's Philosophical Views; Their Relation to His Physical Viewpoints, Usp. Fiz. Nauk 86, 537 (1965) [Sov. Phys.-Usp. 8, 578 (1966)].

This is precisely why we regard the conception given by Wigner about this process as oversimplified.* The point is not even that he "described too briefly" certain pages in the history of physics on the assumption that they are well known; the point is that, in leaning to definite gnosiological conclusions, he presented in too simplified a form the logic of establishing a theory, as a result of which the entire conception based on this logic turned out to be unproven.

5. On the Multiplicity of Theories

Let us examine one more aspect in the process of knowledge discussed by Wigner—the question of the possibility of many different theories explaining one and the same set of phenomena. The distinction between theories pointed out by Wigner is a result of the fact that a different set of initial experimental relationships is chosen for each one. However, there are many unclear points and logical contradictions in Wigner's arguments.

What do the words "... if we turn our attention to phenomena which we now consider deterministic" mean? It is entirely unclear what this new set of "phenomena which we neglected" is. If it satisfies the same requirements of fullness of value of the postulates and completeness of the set as the set "which we now consider deterministic", then the condition of logical common nature, i.e., the formulation of the theory, will be equivalent to the previous one, and consequently both theories will be logically equivalent. Consequently they will, naturally, explain the same number of phenomena, since both sets of initial postulates were not logically different, but were equivalent.

If the first set, however, satisfies the above requirements while the second set is chosen at random (only because we neglect these phenomena) then the new "theory" will be deliberately limited; it will indeed differ from the first theory, but neither will it explain the same group of phenomena. Here is an obvious logical contradiction between the premises and the conclusions.

This is indeed the situation of possibly many theories explaining the same phenomena. If the set of initial postulates is chosen correctly in the sense of the fullness of value of their logical content and the completeness of the set as a whole, then the condition of their logical common nature, i.e., the formulation of the theory, will be logically unique.

Physicists were always, perhaps intuitively, aware of this. Max Born tells how physicists were surprised when in addition to the matrix mechanics of Heisenberg, Born, and Jordan, soon after the publication of their work, there appeared Schrödinger's wave mechanics which successfully solved the same problems, although it apparently differed completely from matrix mechanics. The authors themselves felt this to be a stroke of fate. However, to everyone's satisfaction, it was soon explained that both forms of quantum mechanics were mathematically and logically equivalent; it is a remarkable that the proof of this fact was given by the creator of wave mechanics, Schrödinger, and on top of this before the publication of his classical cycle of papers. So great was the certainty that one and the same postulates given by nature for the use of scientists cannot lead to different theories.

This instructive historical fact is evidence that both schools of physicists first of all, dealt with one and the same objective world of atomic physics, and second, correctly selected from it logically equivalent sets of postulates.

6. Transformation of Theories. Reflections in Various Manifolds

As already noted, the statements above on the uniqueness of theories do not exclude the possible existence of different formulations. All such forms, however, are properly related to one another. In mathematics, there exist systems of transformations of one form of a theory into another. The diversity of transformations points to the different origins of transformed types of theories.

Incidentally, with respect to the two forms of quantum mechanics, it is seen directly that their creation is related to the dual nature of quantum objects. The result was that in the search for a general theory, scientists approached the problem from "both ends": some based themselves on wave notions and sought mathematically formulated conditions under which wave equations yield a discrete spectrum of measured physical quantities characterizing discrete states; others based themselves on the discrete spectrum of frequencies corresponding to atomic transitions and denoted the amplitudes of the probability of these transitions in the form of matrix elements, and later used the analogy with the classical formalism of Hamiltonian equations for the frequency of periodic motion of a system in the form of a partial derivative of its Hamiltonian with respect to action variables. But as it naturally turned out, the essence of theory was not a function of "initial conditions," but of the initial set of postulative relationships as a group, the same group being used in both cases. Consequently, both approaches led to equivalent results, and there arose the possibility of transforming one system of mathematical formalism into the other. In connection with this, Dirac worked out the theory of transformation of the totality of eigenfunctions of Schrödinger's equation into a set of basic vectors of the matrix representation.

Orthogonal transformations in 4-dimensional space-time permit the freedom of choice of any inertial system; at one time the substantiation of such a choice was one of the most important problems in physics. As is well known, these transformations leave the interval invariant.

As noted above, there are a number of quite differ-

*Let us recall Wigner's exposition. Max Born noted that certain of Heisenberg's computational methods coincide with the rules of matrix calculus. After this, three physicists proposed to substitute the values of classical coordinates and momenta by matrices. What if suddenly something happened? Then there came about a miracle. This is the entire process in the creation of a most complicated theory, a theory in which there are so many new and unexpected ideas.

ent transformation systems*. We cannot examine here their character and value; let us only recall that mathematics has worked out different forms of reflection of one manifold into another, creating in this way new forms of equivalent theories. Thus, the dynamics of a system of material points in 3-dimensional Euclidean space may be expressed in the form of the dynamics of one material point in 3N-dimensional configuration space, or in phase space. All similar reflections are isomorphic, but one of them may possess certain advantages, as for example in expressing essential invariant properties of the reflected system, and consequently it may turn out to be more useful in the further development of the theory. Consequently, the search for isomorphic systems of describing Einstein's gravitation theory is fully justified as an example, as is being done by certain investigators†.

However, it is important to underline in the aspect of our analysis that all this difference in forms of reflection does not change the essence of the gnosiological problem: the reflection of one and the same domain of external facts related to a definite set of postulates is always unique in the logical sense. No matter what system of physical variables be used, or no matter how many different reflections be performed, there exists among the forms of the theories a definite unique relationship, showing that the found theory reflects objective laws of nature.

On the gnosiological plane, this relative freedom of choice of the method of reflection plays a rather positive role. It eases and speeds up the search for an adequate theory since it does not uniquely bind the paths of investigation. It can underline more strongly certain peculiarities of the theory and in this way open up a wider perspective for further investigations.

But this freedom of choice is always limited by the fact that the formulated theory must reflect objective law-regulated relationships. We come once more to this conclusion: a theory is not something conditional which depends on conceptions and methods of various schools, or on used physical variables and methods of representation; it is in the logical sense an objective reality and has perfectly precise objective criteria of truth.

IV. ON THE FOUNDATIONS OF THE EFFECTIVENESS OF MATHEMATICS

Wigner speaks of the enormous effectiveness of mathematics in natural sciences as of something unusual, verging on mysticism and with no rational explanation. Meantime, there is no mysticism and nothing unusual in the effectiveness of mathematics. This follows from the analysis of the peculiarities of mathematics and its connection with natural and other sciences.

1. Peculiarity of Mathematics

Mathematics is used to study a variety of logical relations and the hidden consequences. Its peculiarity consists in that it operates with especially abstract and consequently quite movable categories. The study of various types of relationships, conditions of logical compatibility of the most diverse postulates, the deduction of their consequences—all this receives in mathematics an especially flexible and diverse form. This flexibility and diversity of forms is the most important aspect of its power. Because of the abstraction of its subjects, tradition prevails to a lesser extent. Mathematics can be formulated by any combination of postulative relationships, and the consequence of their logical common nature can be studied. This is why new possible relationships are always being sought in it. Thus, a transition occurs to ever more complicated equations whose solutions involve ever more complicated types of functions, such as complex functions; there is a transition from Euclidean geometry to non-Euclidean geometry, from 3-dimensional space to multidimensional space, etc.

Another aspect of the power of mathematics is the fact that all its consequences are derived uniquely from assumed premises. It operates according to laws of logic, in the wide sense of the word. Consequently, if the initial relationships in nature can be expressed in an adequate mathematical form, the mathematics will lead to all logical consequences which the investigator was not able to deduce from direct observations in nature. This often aroused surprise, and quite a few aphorisms were expressed as a consequence (by Kant, Hertz, and many others): "we deduce from equations nothing that was not put into them", "mathematics is more intelligent than mathematicians", and so on. The aphorisms express the emotional relationship to the fact of the effectiveness of mathematics, but of course a logical analysis is lacking.

2. Mathematical Formalism as a Model of System Relationships

The fact of the appearance of new results which apparently were not put into the initial equations is not the perrogative of mathematics alone. The same can be said of the theory of physics, and of theories in general. Theory is not simply the sum of postulative relationships: the requirement of a logical common ground of the chosen group of postulates opens up new relationships which were not present in the postulates in an evident form, but rather in potential form. In other words, the search and formulation of a theory always leads to the discovery of new relationships. Moreover, this is not peculiar of knowledge alone. It is the general law of the material world. In the creation of any system there appears a new property which was not present in the initial free components (it may have existed only in the form of a potential possibility). In physics, these properties of systems were expressed in quantum mechanics especially vividly, proving in this way the failure of the ideas of the mechanistic approach. Especially striking examples of the appear-

*Transformations of one theory into another upon the transition to the limiting value of a characteristic parameter form a special class. Thus, the formalism of the theory of relativity is transformed into the formalism of classical theory as $c \to \infty$, the formalism of non-relativistic quantum mechanics goes over into the formalism of classical theory as $h \to 0$, etc. In this class of transformations we are dealing with theories that reflect objects "at different levels"; at limiting values of characteristic parameters they are simply transformed from one into another.

†See, for example, A. Z. Petrov, On the Modelling of Physical Fields, Report at the 5-th International Conference on Gravitation and the Theory of Relativity (Tbilisi, 1968).

ance of a new property in systems are consciousness, life, human society.

As a matter of fact, the new result may be related not so much to mathematics as to that system of initial postulates whose model is the given mathematical formalism. Mathematics itself does not present us essentially with a new logic which surpasses natural logical relationships present in nature. It receives these relationships in ready form from natural science or from other fields as initial conditions of the problem. This can be verified by the most complicated examples, such as even in the life of society.

Let us take any economic process in which many components participate—manufacture, resources, transportation, market, prices, the labor force, machines, materials, etc. As is well known, only a definite relationship among these elements in the process of manufacture yields an optimal economic effect. Until recently, this problem was solved without the use of modern mathematical methods, but was solved by intuition or previous experience, and in a very approximate form. Mathematics make it possible to construct a model of the entire process and to solve precisely the problem of the optimal relation between the separate elements. Mathematical models make it possible to correctly organize systems of control, supply and manufacture; they provide the freedom from expensive experimentation by substituting for experiments a preliminary computation of the planned process. The use of mathematics is the basis of the rapid progress in this field.

But mathematics is not capable of solving such problems on its own: mathematical models require the determination of the specific weight factors, and the consideration of the economic role of each component. This data cannot be determined from mathematics, it must be found solely on the basis of the knowledge of economic laws.

The same is true of physics. Mathematics cannot lead to the required result in physics if the physicists do not determine correctly the set of postulates, or if the initial principles have not been discovered. In the same way, illnesses cannot be diagnosed mathematically without precise preliminary medical estimates of the role of individual symptoms, and geological prognoses cannot be made mathematically without estimates by specialists of each symptom of a probe. Naturally, such a situation causes in our time the growth of comprehensive investigations in which we find alongside mathematicians the representatives of other sciences.

Thus, the set of postulative relationships determine the logic of relations in the investigated field, and mathematics helps to clarify their consequences*).

*In the text we are speaking of the solution of concrete problems in economics, physics, etc. The above does not mean that mathematics does not cause the evolution of theories on its own by means of investigations into the logical consequences of some combinations of postulative relationships (see Ch. IV, Sec. 1). In such cases, mathematics creates theories of logically possible structures and is well ahead of the discovery of their practical application (the "imaginary geometry" of Lobachevskiĭ, the matrix calculus with non-commutative elements, etc.) But even these facts lead precisely to the same conclusion: Mathematics does not create a new logic above the logical relations existing in nature, it only changes the structure of postulates and clarifies their consequences.

3. Physical Models of Mathematical Problems

In the middle of the 20th century it became clear that not only is mathematics a model of physical processes, but that physics is also capable of creating models of mathematical formalism. Electronic computers in which these models are realized made it possible to not only speed up drastically computation (up to millions of operations per second) but also to yield new results. Thus, computers make it possible to find practically applicable solutions to problems which were considered previously unsolvable. In this manner, physical models of mathematical problems help to develop mathematics. And now mathematicians dream of computers which will perform mathematical operations not be means of semiconductors but on the basis of chemical reactions.

This is again evidence that mathematics is based on the same logic on which other sciences are based, and which is the foundation of the laws of nature.

4. Requirements of Adequateness in Mathematical Formalism

However, logical relations included in the set of postulates and expressing certain relations in nature may be quite varied in nature. On the strength of what was said of logic, a mathematical formalism can be found for each specific set of postulates. This must in fact be a specific rather than arbitrary formalism adequate to the specific set of postulates (or else different in form, but logically equivalent).

This history of physics shows that at each important stage, when new fields were being investigated, the new adequate formalism was either discovered or newly created. Before Newton, mechanical problems were either solved by elementary algebra, as in Hyughen's solution of collision of masses whose velocities had final values and only changed abruptly, or by the aid of geometric representations. By creating the elements of classical mechanics, Newton came to the conclusion that such new notions as velocity and acceleration at a point did not have equivalents in geometric representations; the latter did not give the required freedom for wide generalizations. New requirements pushed Newton to the search for an adequate mathematical formalism, and he found it in the form of differential calculus, in whose development he played an important role.

The development of the theory of motion of continuous media, dealing with quantities whose values varied continuously from point to point and in which were combined the action of various factors, was made possible by an adequate mathematical formalism in the form of the theory of partial differential equations (Euler, Laplace, Poisson etc.). This formalism was later used by Maxwell in his formulation of the theory of electromagnetism, which was quite natural, since analogous requirements were made with respect to the character of the relationships among the physical quantities.

Let us present one more example from which it is seen, on the one hand how physicists, by formulating definite problems, created greater generalizations in mathematics, and on the other, sense how they had to, perhaps by overcoming a certain skepticism, accept a worked out mathematical formalism in so far as it was adequate for the physical problem.

In the special theory of relativity, inertial motion was understood to be straight-line motion; it was assumed that the geometry used in theory was pseudo-Euclidean. Minkowski pointed out that in this pseudo-Euclidean geometry there has appeared a 4-dimensional invariant quantity—an interval in the form of the sum of the squares of differences of the coordinates and time (the latter with negative sign). This made it possible for Minkowski to establish notions of 4-dimensional world, the elements of which ("events") posses a physical reality regardless of the system of computation. These notions, as he pointed out in his famous paper "Space and Time" of September 21, 1908, "were formulated on an experimental-physics basis."

It was natural for Minkowski as a mathematician to place on a purely theoretical basis the question of invariants in cases in which space is regarded no longer as pseudo-Euclidean but as Riemannian space, i.e., in cases when it is distorted in a specific manner, and inertial motion is not along a straight line but rather along a geodesic line. Apparently, in these cases it was impossible to consider only four-component vectors as in the case of pseudo-Euclidean space.

The search for invariants in a curved Riemann space led Minkowski to the investigation of transformations of a multi-component quantity whose components in the new system, are expressed linearly and uniformly by its components in the old-system. Such a quantity is the tensor, which had previously been known to mathematicians. Methods of tensor calculus were used by Minkowski to unify his ideas of "universal geometry" with the Riemann notion of curved spaces.

Einstein, in developing his unified theory of gravitation, went his own way by basing himself in part on the notion of equivalence of the fields of gravitation and acceleration. In the final analysis, he was seeking to solve the same problem as Minkowski, but be based his work on physical postulates and sought a purely physical goal. The basic achievement by Einstein in this problem was that he laid a foundation for the possibility of representing the gravitational field as the deviation of the geometric properties of space-time from pseudo-Euclidean properties. For this it was necessary to find the amount of curvature of space and a generalized measure of the density of gravitating masses. Such uniquely determined measures were indeed the curvature tensor and the energy-momentum tensor.

Minkowski's investigations furthered Einstein's research in this field. Born testifies that in the beginning Einstein was skeptical about Minkowski's work and thought it to be a superfluous mathematical by-product. However, he soon changed his mind when he "penetrated more deeply into the problems of the general theory of relativity in which the mathematical methods of Minkowski turned out to be actually essential*.

Indeed, Einstein later recognized that the "development of the theory of relativity was greatly helped thanks to the mathematical formulation of its foundation given by H. Minkowski" (1915) and in briefly describing his ideas spoke of "the important notions by Minkowski, without which the general theory of relativity... would perhaps have remained in an embryonic state" (1917).*

The classicists of modern physics understood the situation in this way: tensor calculus is precisely the mathematical tool which expresses adequately the notions of the generalized (Einstein) theory of gravitation, that is, the general theory of relativity. Hence it is expressed in the language of tensor calculus.

We wish especially to underline this trait of the interrelationship between physics and mathematics. Physics describes definite relationships in nature and presents a set of postulates, mathematics translates the postulative relationships into its own language and reveals all their hidden consequences. Physics does not adapt its initial notions (its method of examination) to the mathematical formalism, but on the contrary places before mathematics completely definite requirements: to reflect the discovered notions in an adequate mathematical formalism. This was done by Newton, Euler, Maxwell, Minkowski, and Einstein.

The mathematical formalism which is created (or sought out) is precisely the one that corresponds to the entire set of postulates in the given field of physics taken as a whole; consequently, we are dealing not with an arbitrary and single-sided analogy with some particular procedure, as depicted by Wigner in the aforementioned example of the use of matrix calculus in quantum mechanics†. In the case of quantum mechanics, the formalism of matrix calculus turned out to be adequate for the entire set of postulates characteristic of atomic phenomena (see above). The set of postulates of quantum mechanics includes not only the hydrogen atom, but all atoms, including more complicated ones. Consequently, the fact that the formalism of matrix calculus turned out to be also applicable to the helium atom is not at all a surprising miracle, as described by Wigner.

5. Development of Mathematical Notions Through Theories

We must stop on one more problem, the development of mathematical notions.

Wigner points out that abstract mathematical concepts (such as that of complex numbers) were not formulated in order to describe the objects of everyday life. He asserts that these concepts "were so devised that they are apt subjects on which the mathematician can demonstrate his ingenuity and sense of formal beauty" (3). Noting that not all concepts used in theory are included in the initial axioms of the theory, Wigner says that new "concepts outside those contained in the axioms are defined with a view of permitting ingenious logical operations which appear to our aesthetic

*Max Born, "Recollection of Herrmann Minkowski" (Usp. Fiz. Nauk 69, 295 (1959)).

*Albert Einstein, Theory of relativity. Collection of Scientific Papers, vol. I, M., Nauka 1965, p. 421; On the Special and General Theory of Relativity, ibid. p. 559 (in Russian).

†This is the case of the objective process of knowledge taken as a whole; it is another question how the investigator himself understands this.

sense both as operations and also in their results of great generality and simplicity" (3).

The facts indicated here are undoubtedly correct: abstract *concepts are not formulated for the* direct description of objects in the surrounding world; not all concepts used in the theory are already included within its initial axioms.

However, Wigner does not reveal—and perhaps does not know—the mechanism of creation of new mathematical concepts, inasmuch as he speaks of this as a process of "invention of concepts" that precedes the eventual utilization of the concepts*) and assumes that they are determined in order to "permit ingenious logical operations which appear to our aesthetic sense." Obviously, the "appeal to our aesthetic sense" has nothing to do with this. Abstract mathematical concepts are not invented a priori for any reason whatever. The secret, for example, of the creation of the concept of complex numbers is as follows.

The mathematician is never satisfied with the solution of an equation of a particular type, such as $x^2 - a^2 = 0$, since the solution does not exhaust all logical possibilities. Another logical possibility will be an equation of the type $x^2 + a^2 = 0$. If one requires that this equations also have a solution, then by performing an operation similar to the one used in the solution of the first equation we obtain a solution in the form $x = \pm a\sqrt{-1} = \pm ai$. Here i is obtained not as the result of a computational operation, as any real number was historically obtained, but its useful role in equations is analogous to the role of any real number and hence it can also be regarded as a number, even though of a specific type. This was the development of the notion of numbers. The new concept arises as a result of the generalization of the theory. In this process it is not an esthetic criterion which plays a role, but a logical criterion.

Of course, to seek in nature a direct analog to complex numbers would be unreasonable. The cognition role of the complex number a + bi was revealed only after a geometric representation of this number was discovered, in which the real numbers a and b specify the magnitude and direction of a vector, and after it was established that operations on complex numbers obey the same rules as operations on vectors. This led to the development of the theory of functions of a complex variable which turned out to be a mathematical formalism reflecting actual physical relationships in which vector quantities play a role, such as the study of velocity fields in moving media (in particular, the computation of the lifting force of an airplane wing), the calculation of strains in elastic bodies, etc.

Complex functions are at the very core of the formulation of laws of quantum mechanics, as correctly pointed out by Wigner. It is possible to understand the basis for this. It was Einstein, at the time when he gave through his work a powerful stimulus to the development of quantum ideas, who noted the necessity in the future unified theory of "merging wave theory of light with the theory of flow"*). Actually, Einstein even outlined the way of this "merging." As pointed out by Max Born is his Nobel lecture (1954), Einstein *by means of his statistical derivation of Planck's law* of radiation (1916) "graphically demonstrated that the classical concept of intensity of radiation had to be substituted by a statistical concept of transition probability"†. The subsequent development of quantum ideas, including many different experimental results, has indeed demonstrated that it is impossible to avoid *the problem of generalization of discrete-wave properties* in a single theory, and consequently in a single formalism. Such a generalizing formalism had to be based on complex functions. In fact, this formalism must describe the states of a quantum system by means of continuous time dependent wave functions: $\Psi_m = a_m\psi_m(\omega, t)$, $\Psi_n = a_n\psi_n(\omega, t)$. But the transition probability *of the system from one state to another, proportional* to Ψ_m and Ψ_n, does not depend on time. The requirements of independence of the transition probability on time means in essence that the wave functions ψ_m and ψ_n must be complex conjugates, i.e., $\psi_n = \psi_m^*$, since only in that case will $\Psi_m\Psi_n = a_m a_n \psi_m \psi_n^* = \text{const}$. Consequently, it is precisely complex functions which make it possible to express the peculiarities of quantum systems and their dual discrete-wave nature.

Schrödinger's wave equation, in which the first derivative of the wave function with respect to time is proportional to the second derivative with respect to the coordinate, is once again possible only because of the complex character of the wave function.

Inasmuch as an adequate formalism of quantum mechanics must reflect the noted peculiarities of quantum systems, it utilizes complex conjugate functions.

Thus, complex numbers (and complex functions as well) turned out to be not an invention on the basis of esthetic considerations, but rather the result of necessary logical operations; complex numbers are related to reality not directly but through theory which reflects actual relationships in nature; consequently, complex numbers exists not as computation elements but as necessary elements of actual relationships of a definite type. Analogous ideas can be stated about the origin of any other mathematical concept.

This "mechanism" of the development of concepts and the special role of theories must be underlined because the mechanism places on a firm materialistic basis the entire process of knowledge by depriving it of mystery, mysticism, of an aura of miracle, those subjective characteristics which are scattered in great number in Wigner's article.

6. The Heuristic Property of Mathematics

The property of mathematical theory of predicting phenomena not yet observed was noted by many investigators. We already spoke of the conclusions of Kant, Hertz, and others. But whereas these conclusions were

*Wigner even notes an unusual courage in this invention, and once again sees a miracle in that such "recklessness does not lead him into a *morass of contradictions*" (3).

*See the famous paper by Einstein, presented at Salzburg (1909), which Pauli later called "the turning point in the development of theory" (Collection of Scientific Papers, v. III., Paper 19).

†See Max Born, Statistical Interpretation of Quantum Mechanics, in the collection "Physics in My Generation", Pergamon, 1956.

of an emotional character with only surprise being expressed, in our present time this characteristic of mathematics has also received a gnosiological estimate. Apparently it was first seen in the work of Max Born, who introduced the term "mathematical hypothesis"*. Born cited the following examples of such hypotheses: Maxwell's introduction of the expression for the displacement current $1/c\, \partial E/\partial E$ into the equation of electromagnetic fields and the subsequent prediction of electromagnetic waves; further, the introduction by Yukawa of an additional term Φ/a^2 into the wave equation and his subsequent prediction of a new particle, the meson. In both cases there was no experimental evidence for introducing additional terms. Born explains this action as Maxwell and Yukawa's striving to reach the complete representation of physical reality.

In 1944, S. I. Vavilov developed the concept of the value of mathematical hypotheses in knowledge†). Noting the fact that in modern physics the mathematical tool is developed before its physical interpretation. Vavilov explains it by the fact that the physical interpretation is usually based on model concepts; meanwhile, the usual notions and concepts are no longer sufficient to construct a graphic model representation in the new field. Vavilov states that "logic, with its enormous extent and with its transformations into mathematical forms, remains in force and establishes the order of relationship in a new uncomprehensible world, and opens up possibilities of physical predictions". It is a wonder that Vavilov based this idea on the concept by V. I. Lenin: "Categories of thinking are not the gift of man, but the expression of regularities of both nature and man"‡).

A number of scientists note the growth in the role of mathematical hypotheses in recent times. Very recently, physicists were surprised by the fact that the omega-minus particle discovered in 1964 was the result of the search for a particle whose existence and properties (mass, charge, strangeness) were predicted four years before the discovery by Gell-Mann and Neeman, by using mathematical group theory in the systematics of already known elementary particles. The complexity of the physical experiment performed at the Brookhaven Laboratory, and the peculiarity of the behavior of the sought particle demonstrate that without a conscious search and preliminary knowledge of its properties, its rapid discovery as the result of only occasional observations would have been unlikely.

How can then this heuristic role of mathematics be explained? Can it be called on unexplained miracle bordering on mysticism, or perhaps not so extremely, can one consider that in this case the investigator encounters some new field of knowledge in which the appearance of a theory is not regarded as the result of a search for the logical common nature of experimentally confirmed postulates? In other words, can one phrase the question of the history of knowledge anew: first comes the theory, and only later, on its basis, comes the experiment?

We do not think so.

Let us examine more closely in what way the heuristic strength of group theory has been demonstrated in the above case. We shall not discuss the basis of a strict mathematical determination of the notion of groups*); we note merely that as a result of a long historical development, it turned out that group theory is a mathematical formalism that is adequate for a definite widely distributed type of relationships in nature according to which properties of physical systems are transformed; these can be realizable transformations of one system into another, or else, in the general case, such transformations characterize only logically allowed combinations of properties in aggregates of different elements, a factor important for their systematization. Historically, group theory received its biggest impetus in its development from the work of E. S. Fedorov, who used groups of spatial transformations (shifts, rotations, and mirror reflections) to establish in 1895 the possible existence in nature of no more than 230 crystalline forms. All these forms, and only those forms, were found in nature, thus proving the correspondence between group theory and the specific transformations in nature and its effectiveness in the corresponding investigations.

How was it possible then to predict the existence in nature of the omega-minus particle with previously determined properties? All pre-requisites were available for this. On one side, a certain identity of particles with specific properties was already established in physics (Δ quartet, Σ triplet, Ξ doublet; other particles were also known). The initial postulative material was available. On the other hand, the mathematical formalism of group theory was already worked out, and the reality of the reflected relationships and its specific features were proven. The following step consisted in the choice of the SU_3 symmetry type where, upon its assumption, there was missing from among the known particles one particle with specific properties derived from the symmetry type. This choice corresponded to the aforementioned (see p. 371) link in knowledge, whereby "real relationships in nature could be expressed in an adequate mathematical form." The peculiarity in this case was that only one element was needed for complete adequacy. This underlines sharply the hypothetical nature of the formulation of the problem. The subsequent experimental discovery of the hypothetical element confirmed the adequacy of the utilized mathematical formalism. The specific form of this confirmation—the discovery of a particle with the predicted properties—gave it a special effectiveness. But it cannot be forgotten that theory always yields more than is included in the postulates on which it is based, and that it leads to new discoveries (see p. 371). The verification of any theory (or its possible application) by the succeeding practical results is a necessary

*Max Born, Experiment and Theory in Physics, Oxford 1943. The brochure is a somewhat expanded version of the lecture delivered by the author on May 21, 1943.

†S. I. Vavilov, Lenin and Modern Physics, Collected Essays, V. III, M., AN SSSR USSR 1956 (p. 63).

‡V. I. Lenin, Philosophical Notebooks, Complete Works v. 29, p. 83.

*A group presupposes the existence of the following transformations: a) the identity transformation; b) transformations inverse to each non-identity transformation; c) transformations equivalent to two consecutive transformations.

stage in the process of knowledge. It is easy to see that in the given example the successive chain of knowledge is not disrupted: facts and relationships, such as the initial material in the process of knowledge (postulates), followed by theory, followed by theoretical verification. The gnosiological role and place of theory in the entire process of knowledge is also maintained in the given case: theory is always presented as the result of a search for the logical common ground of experimentally verified postulates. This is the source of theory without which there would not be a theory. And having just been verified, the theory appears as the predecessor and herald of any particular experiment.

It is only in this aspect that one can understand the heuristic role of theory as a whole, and of mathematical theory in particular.

7. On the Unity of Objective Logic and Logic of Thought

The above shows sufficiently clearly the logical basis of mathematics and its effectiveness. There can be no question of some unreasonableness of this effectiveness, almost verging on mysticism, and without an explanation.

Consequently, the whole point is that the objective logic of natural processes (among which physical, as well as economic processes, and in general, processes which can be simulated) and the logic of thought (mathematics) have one and the same nature.

This is a uniquely significant fact. In the course of this paper, we more than once alluded to it, trying to make it clear to anyone who had not yet thought of it. But philosophical thinking has grasped this problem a long time ago, which is natural: the unity of objective logic and the logic of thought is the foundation of knowledge in general. Engels expressed this thought particularly strongly. He wrote: "Over all our theoretical thinking there reigns with an absolute force the fact that our subjective thinking and objective world are subject to the same laws, and that therefore they *cannot contradict one another in their results but must* agree with each other. This fact is the unconscious and unconditional postulate in our theoretical thinking"*).

Prominent thinkers investigating the process of knowledge were not able previously to circumvent this problem. It was deeply but individually analyzed at the beginning of the last century by Hegel, who is now undeservedly forgotten in the West in favor of neo-positivism, neo-existentialism, etc... Hegel considered that logic which limited itself only to an outer form was without content. Laws of logic, according to Hegel, are dictated by the content of the material under scrutiny. From this came Hegel's unavoidable transition to dialectic logic. But Hegel, in raising the meaning of thought, was negligent towards nature and thought of reason with a capital letter as a demiurge of all existence. Contemporary dialectic materialism has overturned Hegel's thesis upside down by placing nature as the initiator. But it has retained a healthy seed of Hegel's notion that logical forms are essentially not external but are intimately related to content. In abstracting the "Science of Logic" by Hegel, V. I. Lenin wrote down in his notebook: "Logic is the study not of external forms of thought but of laws of development of all material, natural and spiritual objects, i.e., of the development of the entire concrete content of the world and its knowledge, i.e., the total, sum, conclusion of the history of knowledge in the world"*) (Lenin's emphasis).

But basing itself on the result of the entire world history of knowledge, modern materialism gives a natural scientific basis of the unity of objective logic and of logic of thought. It consists in that a logically thinking being is itself part of nature, a system which comes as the result of the development of nature into its inherent laws.

Within the aspect of this unity it is of interest to look into the historical process of the development of mathematical formalism in physics. Historically, this formalism was created in order to solve problems in mechanics. For centuries, its development was towards ever greater abstraction and generalization—from Newton's differential equations relating mechanical forces with the resulting accelerations, through variational principles (in which time, trajectory, and finally the action were varied) and Lagrange's covariant equation, to Hamilton's variational principle and his equations of motion. In the latter, interrelationships were found between the most general physical quantities by means of transformations of the generalized coordinates q and p and partial derivatives of the Hamiltonian with respect to generalized coordinates. The place of Newton's inertia principle was taken by Hamilton's variational principle and the law of the conservation of energy.

When these generalized forms were discovered, it turned out that they could be applied not only to mechanical forms but also to more complex physical forms of motion and even to quantum processes (in which, for example, the classical Hamiltonian form and the transformed (Poisson brackets are used). In this generalizing process the role of purely mechanical notions (forces acting at a distance, trajectories, etc...) were gradually dropped or erased or transformed. The sense of the variables q and p, which more and more lost connection with purely mechanical notions of coordinates and momentum, was changed and generalized†). Even the concept of mechanics itself was changed, since the term "mechanics" is now used in quantum processes ("quantum mechanics") in which an important role is played by non-mechanical notions—characteristics of state, energy transitions, and so on. Nevertheless, the generalized mathematical formalism makes it possible to also solve mechanical problems, for which purpose it was after all created.

It is now possible, in the light of the estimate of the modern sense of this mathematical formalism, to ex-

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*K. Marx and F. Engels, Works, 2nd edition, vol. 20, M., Gosolitizdat, 1961, p. 581 (in Russian).

*V. I. Lenin, Philosophical Notebooks, Complete Works, vol. 29, p. 84.

†Using the canonical transformation with a generating function $F = \Sigma q_k Q_k$ it can be shown that the generalized variables q and p change place, and consequently lose the meaning given to them by classical mechanics. We note, by the way, that it is difficult to find in the Poisson bracket any traces of purely mechanical representations.

amine in retrospect the historical process of its development not as a formal generalization of laws of mechanical motion or as a search for the solution of the same mechanical problems, but rather as the elucidation of one logic of objective relationships for all physics of logic in nature*).

It is precisely the unity of objective logic and logic of thought which makes it possible for mathematical theory to predict physical phenomena not yet discovered experimentally. As was stated above, the prediction of new facts is characteristic of any integral theory.

The unity of objective logic and logic of thought also explains all puzzles and miracles of knowledge which are no more mysterious and miraculous than nature itself.

V. CONCLUSION

Wigner's gnosiological conception, as exposed in the examined paper and to which we consciously limited ourselves, cannot find verification in the development of science. The foundations of knowledge are certainly not mystical. In practice, the founders of modern physical theories also based themselves on this fact. The materialistic theory of knowledge laid its theoretical foundation.

In conclusion, we must still answer the question as to why we felt it necessary to analyze the conception presented in Wigner's lecture, which recently appeared in the Russian language.

It is said that when someone asked a scientist "You do not believe in God, yet your neighbor does; but why worry about him: after all, life goes on in its own course:?", he answered: "It is of concern to me that my neighbor prays, since God limits logic somewhere, and logic is my instrument of knowledge."

In this story there lies the answer to the question raised above. One perspective of the development of science arises in the aspect that the effectiveness of mathematics is unreasonable, mystical, and a completely different one when its logical foundations are known.

*E. Mach, who worked in particular on the critical history of mechanics and who had a great influence on physicists, did not understand the sense of the generalization described in the text. He saw in all developed forms of mechanical laws only another expression of one and the same fact – two bodies mutually produce in each other accelerations that are inversely proportional to their masses. This idea permeates his whole book (see E. Mach, The Science of Mechanics, Open Court). These are quite primitive notions, characteristic of positivism with its concepts of nature as a purely external description of sensory experiences.

In the first case, it appears as if the success of mathematics is unexpected and accidental, and everywhere this success looks like an unexpected miracle, as we saw in Wigner; in the second case, physics places before mathematics definite problems, widens the field of its investigation, relates it to actual relationships in nature, promotes a deeper solution of mathematical problems, even when they come about as the result of the inherent development of mathematics. And mathematics in turn recompenses these impulses emanating from physics. Instead of miracles, we have nature, its laws and logic. This motion of the logical relationships among sciences leads to their mutual rapid progress.

* * *

Postscript

The present article had already been written when the author laid his hands on E. Wigner's book "Symmetries and Reflections" (1967) in which a number of his methodological papers are printed. An examination of the book reveals that nothing must be changed in the present article. As far as the estimate of Wigner's concepts is concerned the author is satisfied that he correctly caught in the article under scrutiny their basic tendency, as verified by the book. Wigner's creed is very expressly related in the article "Two Forms of Reality": the sole notion of reality which is not only convenient, but absolute, is the content of my consciousness, including my senses; all the rest—objects, scientific concepts, the perception of other people, etc.—is a reality of a second category, synonymous with the usefulness of concepts in our personal thinking and in our communications with others. Thus, "a book is a useful term for the description of certain sensations." Wigner agrees with Margenau that "the world is the sum total of our senses, perceptions and memories." In this way, the meaning of his notion that "all laws of nature are conditional statements" is clearly revealed. Wigner more than once makes fun of materialists, and thus reveals simplified notions of the conception of modern scientific materialism, which is incidentally characteristic of many western critics of materialism. An analysis of this philosophy in connection with the theme of the present article does not seem necessary.

In conclusion I wish to thank R. Ya. Shteĭnman for his valuable advice.

Translated by L. C. Garder