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INTERACTION OF ELECTRON STREAMS WITH ELASTIC LATTICE WAVES

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PROBLEMS connected with interactions between the electron streams and various waves present in some system have recently aroused great interest. One of the causes of such an increased interest in these phenomena is that under certain conditions amplification or generation of various types of waves, both electromagnetic and acoustic, is possible in a system with electron streams. Physically, the reason for the amplification and generation of waves in such systems is the same, namely the Cerenkov radiation of some wave or another by a charge moving with a "superwave" velocity.[1,2]

The first investigations of such phenomena were started relatively long ago (principally as applied to a gas-discharge plasma). In an electron-ion plasma, propagation of many types of waves is possible (especially in the presence of an external magnetic field; see [3]), and the presence of electron streams or beams leads as a rule to a growth, i.e., intensification or generation, of various waves in the system.[4,5] In a gas-discharge plasma, in final analysis, this leads to development of instability.

The study of analogous phenomena in solids, in semiconductors and semimetals, in which the electron or "hole" streams can be produced by means of external fields or with the aid of concentration or temperature gradients, began only relatively recently. By now, a relatively large number of published papers is devoted to the study of various instability phenomena produced by electron streams in solids. These papers can be subdivided, quite arbitrarily, into two groups: papers in which excitations of the plasma types are investigated,[6] and papers devoted to the buildup of various "lattice" waves by the electron screens. A characteristic feature of the papers of the first type is that in excitations of the plasma type the crystal lattices play a secondary role, namely, the carrier streams excite a particular wave that exists in the electron-hole plasma itself; the second group, to the contrary, is characterized by the fact that the lattice plays here an active role, and the electron streams excite or intensify some lattice wave.*

In order to narrow down the circle of problems under consideration, we shall discuss only the second of these two large and independent problems, namely, we shall investigate problems connected with intensification of elastic waves in a lattice.

The intensification of elastic waves was first observed experimentally by Hutson, McFee, and White,[7] who observed intensification of a transmitted ultrasonic wave in piezoelectric semiconducting crystals of cad-mium sulfide when the drift velocity of the electrons exceeded the sound velocity.

The physical process occurring upon amplification or generation of acoustic waves by a drifting stream of electrons in a solid consists in the following: the transmitted elastic wave leads to a redistribution of the electrons in space in such a way that regions of increased electron density are produced, i.e., a space charge is produced. If the space charge is now made to drift (for example, by means of an external field) in the wave direction at a supersonic velocity, then, in analogy with the free electrons, the space charge will emit phonons.

This is the physical nature of the effect of amplification in the region of low frequencies, when the amplification or generation of the elastic waves is due to Cerenkov radiation of the space charge produced by the wave itself. At high frequencies, when the period of the wave is too short to permit production of space charge, the amplification of the acoustic waves is due to the Cerenkov radiation of individual electrons, in analogy with two-stream instability in a gas-discharge plasma.

In the first part of the review we shall consider the qualitative picture of amplification of acoustic waves in a solid in two limiting cases, when the carrier mean free path is small or large compared with the ultrasound wavelength.

We shall show that in both cases, the possibility of amplification of sound waves in the system is determined completely by the Cerenkov condition for the radiation of phonons by the drifting electrons.

The second part is devoted to the derivation of dispersion equations that describe the propagation of sound waves in conducting bodies with different character of the interaction with the conduction electrons. The state of the electron-hole plasma itself is not specified concretely here, and consequently that part of the dispersion equation, which determines the interaction with the electrons, contains in a natural manner the dielectric tensor of the medium.

In essence, the nub of all the processes considered here lies in the calculation of the dielectric tensor of the crystal in the presence of directed streams of charged particles. For this reason, the third part of the paper is devoted entirely to the calculation of dielectric constants of a medium in the presence of drift, in various cases: low and high frequencies, influence of the magnetic field, cyclotron and geometric resonances, quantum oscillations, etc. It should also be noted that the dielectric tensor of the medium determines not only the plasma part of the damping or amplification of the elastic waves, but also many other phenomena, including also those of purely plasma nature, and therefore its calculation for different cases is of independent interest. The expressions derived for the dielectric constants of the medium are then used to investigate effects

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*We must emphasize once more that such a subdivision of the excitations into "plasma" and "lattice" branches is quite arbitrary and is somewhat meaningless. The latter occurs in the vicinity of the so-called coupled acousto-electromagnetic waves.

of amplification (and associated phenomena) under various conditions for different types of interaction between the sound waves and the electron-hole carrier plasma.

1. QUALITATIVE ANALYSIS OF THE EFFECT OF AMPLIFICATION. INTRODUCTORY REMARKS

As already mentioned, it was observed experimentally that acoustic waves become amplified under conditions when the carrier drift velocity exceeds the wave velocity.[7] To illustrate the physical nature of this effect, let us consider the propagation, in a medium, of a plane longitudinal electric wave

$$E_{\sim}(z, t) = E_{\sim}e^{i\omega t - iqz}, \quad E_{\sim} \parallel q. \tag{1.1}$$

The electric field of the wave $E_{\sim}$ performs in a unit time a work $V \operatorname{Re} \sigma_{\sim} E^2$ on the medium; here $\operatorname{Re} \sigma_{\sim}$ is the conductivity of the medium to the alternating field of the wave, and V is the volume of the system. The sign of the work is determined by the sign of the conductivity $\operatorname{Re} \sigma$; if $\operatorname{Re} \sigma_{\sim} > 0$, then the work performed on the medium is positive and the transmitted wave attenuates, giving up its energy to the medium; if $\operatorname{Re} \sigma_{\sim} < 0$, then the medium gives up energy to the wave and the amplitude of the transmitted wave increases. Thus, in order to calculate the character of the interaction of the wave (1.1) with the medium, at least qualitatively, it is necessary to determine the sign of the conductivity of the medium to the alternating field of the wave.

We shall show that a plasma medium in which there is directional motion of charged particles with a velocity exceeding the phase velocity of the wave $v_{ph} = \omega/q$, has negative conductivity. To find the conductivity of the medium, it is necessary to determine the response of the system to the longitudinal wave (1.1), i.e., to find the current induced by the wave in the system. Let us assume for concreteness that the directed motion of the particle in the medium is produced by an external constant electric field E_d. Then we obtain for the ac component of the current in the linear approximation in the wave field

$$j_{\sim} = en_0v_{\sim} + en_{\sim}v_d; \tag{1.2}$$

here n_0 is the equilibrium value of the electron density,* $n_{\sim}(z, t)$ is the small deviation of the concentration from the equilibrium value, produced by the wave, v_d is the electron directional velocity due to the action of the external field E_d, and $v_{\sim}(z, t)$ is the alternating, i.e., vibration, velocity of the electron in the electric field of the wave. As seen from (1.2), the current induced by the wave in the system consists of two parts: the first term in (1.2), $en_0v_{\sim}$ is the current of the oscillatory motion of the electrons, which is usually taken into account in conductivity theory; the second term in (1.2), $en_{\sim}v_d$, is the current due to the ordered motion of the space charge. To determine $n_{\sim}(z, t)$, we use the continuity equation

$$e\frac{\partial n}{\partial t} + \operatorname{div} j = 0. \tag{1.3}$$

Assuming for simplicity that the electron drift v_d is directed along the wave, i.e., $v_d \parallel q$, we get from (1.3)

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*We shall refer most frequently to electrons, but all the statements pertain to holes as well.

$$n_{\sim}(\omega, q) = n_0 \frac{qv_{\sim}}{\omega - qv_d}. \tag{1.4}$$

In the simplest case, the vibrational electron velocity $v_{\sim}$ is equal to $\mu E_{\sim}$, where μ is the electron mobility.

In the general case (especially at high frequencies), the mobility μ is a complex quantity, but at present we are interested only in the qualitative picture of the phenomenon, and furthermore at low frequencies, so that the mobility μ can be regarded as pure real. Substituting (1.4) in expression (1.2) for the current, we obtain[8]

$$j(\omega, q) = \sigma_{\sim}(\omega, q)E_{\sim}, \quad \sigma_{\sim}(\omega, q) = \frac{\sigma_0}{1 - (v_d/v_\phi)}; \tag{1.5}$$

Here $\sigma_0 = en_0\mu$ is the dc conductivity.

The obtained formula (1.5) for the conductivity shows that $\operatorname{Re}\sigma_{\sim}(\omega, q)$ reverses sign when

$$v_d > \frac{\omega}{q}, \tag{1.6}$$

i.e., when the electron drift exceeds the phase velocity of the wave.

The condition under which the conductivity of the plasma medium reverses sign is none other than the condition for Cerenkov radiation,[3] namely, the electron drift velocity must exceed the phase velocity of the developing wave. The role of the Cerenkov radiator is played here not by one particle, as is usually the case, but by an assembly of particles—a local bunch of charged particles of like sign, produced by the wave itself. Whereas in ordinary Cerenkov radiation the electromagnetic wave is emitted by each electron separately (provided the electron velocity exceeds the phase velocity of the wave in the medium), in our case the wave is radiated by the space charge moving as a unit under the influence of the external constant field. This is the mechanism amplifying the low-frequency waves if the space charge can be formed within a time equal to the period of the wave. If the wave is of high frequency and no space charge is produced in the wave, then the amplification is due to the Cerenkov radiation of the individual electrons moving with "superwave" velocity.

Let us make a few remarks concerning the applicability of the expression (1.5) for the conductivity. When $v_d = v_{ph}$, as seen from expression (1.5), infinite resonance takes place. In fact, when $v_d = v_{ph}$, formula (1.5) is not applicable. This follows directly from the condition $n_0 \gg n_{\sim}$, which leads to the requirement $|1 - (v_d/v_{ph})| \gg v_{\sim}/v_{ph}$, and by the same token excludes the singularity (infinite resonance) at $v_d = v_{ph}$. As will be shown below, at resonance, the current is finite as a result of the diffusion of the space charge, electron-phonon collisions, collisions with impurities, etc., and also as a result of other factors, which can be taken into account only in a more rigorous analysis of the problem. The simple formula (1.5) for the conductivity of the medium is valid only at low frequencies, when thermal motion of the particles can be neglected.

The experimentally produced directional velocity of the electrons in solids usually does not exceed 10^6–6 $\times 10^7$ cm/sec, so that only relatively slow waves can be amplified. The question of just which wave can propagate and be amplified in the system depends on the properties of the medium itself and calls for a special analysis in each individual case. It is immediately

clear, however, that the most suitable for amplification are acoustic waves, the velocity of which in solids usually amounts to 10^5–10^6 cm/sec. The interaction between an acoustic wave and a drifting stream of carriers can be realized in various ways, but this interaction is most effective in piezoelectric semiconducting crystals, where the wave passing through the crystal is accompanied by an electric field. It is precisely in piezoelectric semiconducting crystals, cadmium sulfide, where the amplification of ultrasound was first observed in supersonic electron motion.[7]

The expression previously derived for the conductivity of the electron gas is valid only at low frequencies, much lower than the effective electron-collision frequency. One can, however, advance simple considerations indicating that free electrons moving with supersonic velocity can likewise emit phonons. The reasoning in this case differs little from the case of the ordinary (photon) Cerenkov radiation.[1,2]

We shall consider the case of degenerate semiconductors or semimetals at absolute zero temperature, when the Fermi distribution function of the electrons is a pure "step." In the case of equilibrium electron distribution, in spite of the fact that the characteristic velocities of the "excited" electrons are of the order of the Fermi velocity, and consequently are much higher than the velocity of sound, no phonons are generated in the system, since all the states of the electrons with energy lower than the Fermi energy are occupied. The situation is different in the case of a non-equilibrium system. Assume that electron drift with velocity v_d was produced inside the semiconductor by means of some mechanism. The presence of the electron drift causes the Fermi function to shift by the vector v_d in velocity space, and therefore all the states with $|v - v_d| < v_F$ will turn out to be occupied, and the states with $|v - v_d| > v_F$, i.e., outside the Fermi sphere, are free. In such a system, phonons can be generated provided the electron drift velocity v_d satisfies a definite requirement. Let us determine this requirement.

The initial electron velocity v_i (prior to the emission of the phonon) and the final velocity v_f (after the phonon emission) should satisfy the inequalities

$$|v_i - v_d| \leqslant v_F, \qquad |v_f - v_d| \geqslant v_F,$$

from which we get

$$|v_i - v_d| \leqslant |v_f - v_d|. \tag{1.7}$$

The initial and final electron velocities, in addition, are connected by the energy and momentum conservation laws

$$\tfrac{1}{2} mv_i^2 = \tfrac{1}{2} mv_f^2 + \hbar\omega, \qquad mv_i = mv_f + \hbar q, \tag{1.8}$$

where $\hbar$ is Planck's constant. Substituting in (1.7) the values of v_i and v_f obtained from (1.8), we get the condition that must be satisfied by the electron drift velocity:

$$q v_d > \omega, \tag{1.6'}$$

i.e., the process of phonon emission by electrons becomes possible if the Cerenkov-radiation condition is satisfied.

Similar reasoning can be used in the case of finite temperature.[9,10] To this end, let us consider the sim-

plest form of kinetic equations for the number of phonons N_q[11] in the stationary case

$$v_{gr}\frac{dN_q}{dx} = \frac{2\pi}{\hbar} \Lambda^2 (q) \sum_p [f_{p+q}(1 - f_p)(N_q + 1) \\ - N_q f_p (1 - f_{p+q})] \delta (\varepsilon_{p+q} - \varepsilon_p - \hbar\omega); \tag{1.9}$$

Here f_p—electron distribution function, p—quasimomentum, ε_p—electron energy, $\Lambda(q)$—characteristic constant of interaction between the electrons and phonons, and v_{gr}—group velocity of the sound waves in the x direction. For simplicity we assume here that the drift velocity is parallel to x and therefore the growth of the phonons takes place in the same direction.

If the electron distribution is described by a Fermi function with a certain temperature T, then the number of phonons $N_q^{(0)}$ in equilibrium with the electrons is determined by a Planck distribution with the same temperature T. It is precisely under these conditions that the "collision integral" in (1.9) vanishes and the total derivative is $dN_q/dt = 0$.

Let now the electron distribution function be a shifted Fermi function

$$f_p = (e^{\frac{\varepsilon_p - pv_d - \varepsilon^0_F}{\varkappa T}} + 1)^{-1}. \tag{1.10}$$

Then the number of phonons in equilibrium with the shifted function (1.10) should seemingly be determined by the "shifted" Planck distribution:

$$N_q(v_d) = [e^{\frac{\hbar(\omega(q) - qv_d)}{\varkappa T}} - 1]^{-1}. \tag{1.11}$$

Actually, as will be shown later, when $\omega > q \cdot v_d$ the number of phonons is described by the distribution (1.11), but when the Cerenkov condition (1.6) is satisfied the phonons no longer have the stationary distribution (1.11).* This statement can be deduced rigorously directly from the solution of Eq. (1.9):

$$N_q(x) = N_q(x = 0) e^{-\frac{\gamma_q x}{v_{gr}}} + \frac{\sum_p f_{p+q}(1 - f_p) \delta(\varepsilon_{p+q} - \varepsilon_p - \hbar\omega)(1 - e^{-\frac{\gamma_q x}{v_{gr}}})}{\sum_p (f_{p+q} - f_p) \delta(\varepsilon_{p+q} - \varepsilon_q - \hbar\omega)}, \tag{1.12}$$

where the growth (damping) increment γ_q is

$$\gamma_q = \frac{2\pi}{\hbar} \Lambda^2 (q) \sum_p (f_{p+q} - f_p) \delta(\varepsilon_{p+q} - \varepsilon_p - \hbar\omega), \tag{1.13}$$

N_q (x = 0) is the number of phonons specified on the boundary x = 0. It follows from (1.13) that the sign of the coefficient γ_q is determined by the sign of the difference $q \cdot v_d - \omega$, namely, if $(\omega - q \cdot v_d) > 0$, then $\gamma > 0$ and as $x \to \infty$ the phonon distribution goes over to the stationary value (1.11); on the other hand, if $(\omega - q \cdot v_d) < 0$, i.e., the condition for the Cerenkov radiation is satisfied, then $\gamma_q < 0$, and as $x \to \infty$ the number of phonons in the system will increase without

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*If we change over to a reference frame in which the electrons are at rest in the mean, then the distribution function (1.10) goes over into the usual Fermi distribution, in equilibrium with which the phonons have a Planck distribution at the same temperature. Going back to the laboratory frame, we obtain the distribution (1.11) [12]. It is easy to show that this reasoning is valid only when the electron drift velocity is smaller than the velocity of sound. Indeed, for the phonons the sound velocity plays the role of the limiting velocity, and therefore we can obtain from such a reasoning the distribution (1.11) only when $v_d < v_{ph}$.

limit. We see here that we get not only amplification of the "boundary" phonons, but also internal generation of spontaneous phonons, which grow along the drift direction. We have imposed no condition whatever on the electron temperature, so that the latter statement is· valid also for nondegenerate semiconductors and semimetals.

The foregoing qualitative investigation of the amplification of acoustic waves in semiconductors and semimetals covers two limiting cases: on the one hand, the case of low frequencies, when the electron mean free path in the solid is small compared with the wavelength, and on the other hand, the case of an infinitely large mean free path. This analysis, of course, is only illustrative in character, since it describes only the qualitative character of the phenomenon. In order to study these effects more rigorously and in the entire frequency region, it is necessary to construct a dispersion equation that describes the propagation of the elastic waves in solids with allowance for their interaction with the electron-hole plasma of the carriers.

2. PROPAGATION OF ELASTIC WAVES IN SEMI-CONDUCTORS

a) General Remarks

The presence of electrons and holes in a solid greatly influences the processes that occur in the crystal lattice, particularly the character of propagation of acoustic waves. The character of this influence is determined, first, by the interaction of the conduction electrons with the lattice vibrations and, second, by the state of the electron-hole plasma itself. If the electron-hole plasma is in equilibrium, i.e., the real part of the conductivity is positive, then its interaction with the lattice vibrations leads to a damping of elastic waves in solids;* on the other hand, if the plasma is not in equilibrium, the waves may sometimes become amplified rather than damped.

The distribution of elastic waves in crystals is described by the equations of the theory of elasticity[20,21]

$$\rho \frac{\partial^2 u_i}{\partial t^2} - \lambda_{iklm} \frac{\partial u_{lm}}{\partial x_k} = f_i, \qquad (2.1)$$

where ρ is the lattice density, u the displacement vector, λ_{iklm} the elastic-modulus tensor,

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)$$

the strain tensor, and f_i the volume force that determines the interaction between the phonons and the electron-hole plasma.

Thus, the problem of the interaction of elastic waves with conduction electrons reduces to a determination of the explicit form of the force f. The determination of the force f for different interaction mechanisms entails certain difficulties, and we shall therefore consider this question in greater detail in this section.

The interaction between carriers and lattice vibrations can be realized by several methods. In semiconductors and semimetals with an "ionic" core (i.e., those whose lattice consist of charged ions of like sign), the interaction is realized via the electric field accompanying the longitudinal wave in the lattice. The force f has in this case the simplest form

$$f = \rho_{(l)} E, \qquad (2.2)$$

where $\rho_{(l)}$ is the lattice charge density and E is the self-consistent electric field produced in the sound wave.

If the semiconductor crystal has piezoelectric properties, then the propagating elastic wave will be also accompanied by an electric field resulting from the piezoelectric properties of the medium. The expression for the force in this case is[20]

$$f_i = \beta_{l,\ ik}\frac{\partial E_l}{\partial x_k}. \qquad (2.3)$$

Finally, a direct interaction between the electrons and the lattice vibrations via the deformation potential is also possible: the transmitted wave deforms the lattice, leading to a change of the electron energy in the conduction band (or of the hole in the valence band). In a reference frame connected with the moving lattice, the electron energy can be written in the form

$$\varepsilon(p, r) = \varepsilon_0(p) + \Lambda_{ik}u_{ik}; \qquad (2.4)$$

Here $\varepsilon_0(p)$ is the electron energy in the absence of the wave, and Λ_{ik} is the tensor of the electron-phonon interaction constants.* Formula (2.4) signifies that the electrons exert on the lattice a force

$$f_i = \sum_{\alpha=e,\ h} \frac{\partial}{\partial x_k}\Lambda_{ik}^{(\alpha)}\int f^{(\alpha)}(r, p, t)\,d^3p. \qquad (2.5)$$

where $f^\alpha(r, p, t)$ is the carrier distribution function ($\alpha = e$ corresponds to electrons, $\alpha = h$ to holes).†

We note one feature of the interaction of acoustic waves with electrons and holes in the presence of external fields, magnetic or electric.

Since the sound wave induces a current in the conducting medium, this leads, in a magnetic field, to an additional interaction between the electron and the lattice vibrations. This interaction is due to the induction fields which are produced when the conductor deformed by the sound wave crosses the flux lines of the magnetic field. An expression for the force acting on the lattice in this case can be obtained from the general expression for the stress tensor in a magnetic field (see [20], Sec. 34)

$$\sigma_{ik} = \sigma_{ik}^{(0)} + \frac{\mu}{4\pi}\left(H_iH_k - \frac{1}{2}\delta_{ik}H^2\right),$$

where $\sigma_{ik}^{(0)}$ is the stress tensor in the absence of a mag-

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*The first indication that the conduction electrons play an essential role in the absorption of acoustic waves in metals was obtained experimentally by Bommell [13] (see also [14]). The first qualitative explanation of this phenomenon was given by Pippard [15]. The theory of absorption of ultrasonic metals was subsequently developed by many authors (see, for example, [16-19]).

*Strictly speaking, it is necessary to add to expression (2.3) the term
$$\frac{\partial u}{\partial t}\frac{\partial \varepsilon(p)}{\partial p},$$
which corresponds to allowance for the Stewart-Tolman effect [22]. It is easy to show [16], however, that this term is always small compared with those taken into account; their ratio is always of the order of the ratio of the sound velocity to the characteristic electron velocity, and is consequently smaller than 10^{-2}.

†The values of Λ_{ik} in formulas (2.4) and (2.5) may, generally speaking, not coincide (for more details see [23,24]).

netic field, μ is the magnetic permeability of the medium, which for simplicity is assumed here to be a scalar quantity independent of the lattice deformation. The volume force due to the magnetic field is then:

$$f = \frac{\mu}{4\pi}[(\text{rot } H)\, H] = \frac{\mu}{4\pi}\left[\left(\frac{1}{c}\frac{\partial D}{\partial t} + \frac{4\pi}{c} j\right) H\right], \qquad (2.6)^*$$

where D is the electric-induction vector, j is the conduction current induced in the medium by the sound wave, and H is the external magnetic field (in the derivation of (2.6) we used Maxwell's equations to eliminate curl H). Formula (2.6) without the displacement curve can be obtained in very simple manner also directly by averaging the values of the Lorentz forces exerted on the conduction electrons by the microscopic field h. The "induction" force (2.6) leads ultimately to a renormalization of the interaction, and also to a certain additional interaction,[24,25] which in most cases is small.

In the presence of an external electric field, an additional interaction between the electrons and the lattice vibrations is produced by the electrostriction effect,[25] or, which is the same, by the dependence of the dielectric constants of the medium on the deformation.[26] We shall show that this case can be reduced formally to the piezoelectric interaction, in which the piezoelectric modulus is determined by the electrostriction in an external electric field.[27] This is best demonstrated by using the expression for the free energy per unit volume[20]

$$\mathscr{F} = \mathscr{F}^{(0)} - \frac{1}{2}\lambda_{iklm}u_{ik}u_{lm} - \beta_{i,kl}E_i u_{kl} - \frac{1}{2}a_{iklm}u_{lm}E_iE_k - \frac{1}{8\pi}\epsilon_{ij}^{(0)}E_iE_j, \quad (2.7)$$

here a_{iklm} is the electrostriction-constant tensor and $\epsilon_{ij}^{(0)}$ is the dielectric tensor of the lattice (the explicit forms of these tensors and the number of independent constants are determined completely by the symmetry of the crystal). Assuming that the electric field E consists of E_d—the external field—and $E_\sim$—the field induced by the sound wave—we obtain from (2.7) the volume force acting on the lattice:

$$f_k = (\beta_{i,kl} + a_{imkl}E_{d_m})\frac{\partial E_{\sim i}}{\partial x_l}, \qquad (2.3')$$

i.e., the additional interaction proportional to the external field reduces to the piezoelectric interaction with effective piezoelectric modulus

$$\tilde{\beta}_{i,kl} = \beta_{i,kl} + a_{imkl}E_{dm}.$$

When sound waves propagate in solids, the wave energy becomes dissipated, first because of thermal conductivity (the presence of deformation leads to the appearance of a temperature difference), and second because of internal friction or viscosity. For single crystals (but not for polycrystals), the absorption due to thermal conductivity is small compared with the absorption due to viscosity (for a transverse wave it is in general equal to zero). At low temperatures, however, the situation is much more complicated.[28,29]

Viscous absorption of sound waves can be taken into account by adding to the volume force acting on the system (in Eq. (2.1)), on the right side, a "dissipative force" or a "friction force" f^{diss}, which, as is well

known,[21] is given by

$$f_i^{diss} = \eta_{iklm}\frac{\partial^2 u_{lm}}{\partial t\, \partial x_k},$$

where η_{iklm} is the viscosity tensor. We shall not write out this force each time, but we shall remember that whenever we deal with complete amplification (or attenuation) of sound waves in a solid, it is necessary to take into account f^{diss} besides the plasma part. The damping increment due to the viscosity, for example for longitudinal waves, is

$$\text{Im}\, q = -\eta\frac{\omega^2}{\rho v_s^3} < 0 \qquad (\eta = \eta_{xxxx}, \ u\| x\| q). \qquad (2.8)$$

In many-valley semiconductors and semimetals, when the energy of the electron as a function of its quasimomentum has several minima, the character of the interaction of the ultrasonic waves with the electrons can change appreciably. Under the influence of the elastic deformation of the lattice, the electrons may execute transitions from one minimum to the other, and this leads to additional absorption of the sound waves, to a decrease of the space charge in the wave, and this in turn ensures a stronger interaction between the phonons and the electrons even at high density of the latter.[30,31] In particular, as shown by Weinreich et al.[32] with germanium and silicon as examples, transitions between individual minima (the so-called intervalley transitions) ensure the possibility of experimentally observing the acoustoelectric effect and, as recently observed by Pomerantz,[33] the very effect of amplification of ultrasonic waves in germanium. It should nevertheless be noted that a consistent theory of phonon-electron interaction in many-valley semiconductors encounters considerable difficulties (see [31,34,35,36]).

b) **Dispersion Equations of the Propagation of Sound Waves in Piezoelectric Semiconductors**

For piezoelectric semiconductors, the force f is determined by formula (2.3), and the electric field by Maxwell's equations

$$\text{rot rot } E + \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial}{\partial t} j,$$

in which the induction vector is

$$D_i = -4\pi\frac{\partial\mathscr{F}}{\partial E_i} = \epsilon_{ij}^{(0)}E_j - 4\pi\beta_{i,kl}u_{kl}.$$

Assuming that the quantities u and E depend on the coordinates and on the time in accordance with the plane-wave law $\exp(i\omega t - iq\cdot x)$, we obtain from (2.6) a dispersion equation for the propagation of elastic waves in piezoelectric semiconductors in the absence of a magnetic field

$$\text{Det}\left|\rho\omega^2\delta_{ij} - \lambda_{iklj}q_lq_k - \frac{4\pi\omega^2}{c^2q^2}\beta_{l,im}\Gamma_{lp}^{-1}\beta_{p,js}q_sq_m\right| = 0, \qquad (2.10)$$

where

$$\Gamma_{ij} = \delta_{ij} - \frac{q_iq_j}{q^2} - \frac{\omega^2}{c^2q^2}\epsilon_{ij}(\omega, q),$$

$$\epsilon_{ij} = \epsilon_{ij}(\omega, q) = \epsilon_{ij}^{(0)} + \frac{4\pi}{i\omega}\sigma_{ij}(\omega, q).$$

If we neglect the solenoidal components of the field (which is possible if $\left|\frac{\omega^2}{c^2q^2}\epsilon_j(\omega, q)\right| \ll 1$), then

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$^*[(\text{rot } H)H] \equiv (\text{curl } H) \times H.$

$$\Gamma_{ij}^{-1} = \frac{c^2 q^2}{\omega^2} q_i q_j / \varepsilon_{ps}(\omega, q) q_p q_s$$

and from (2.10) we get the dispersion equation first obtained by Shaposhnikov[35] (see also [36-41]):

$$\mathrm{Det} \left| \rho\omega^2 \delta_{ij} - \lambda_{iqqj} q^2 - \frac{4\pi\beta_{q, iq}\beta_{q, jq}}{\varepsilon_{qq}(\omega, q)} q^2 \right| = 0, \qquad (2.11)$$

where

$$a_{q\,lm} = q_i a_{ilm}/q$$

denotes the contraction of the corresponding tensor with the vector q.

The dispersion equations (2.9) and (2.10) were obtained for acoustic waves interacting only with conduction electrons; on the other hand, if the crystal contains also holes, then these can be accounted for by making the formal substitution

$$\varepsilon_{ij}(\omega, q) \to \varepsilon_{ij}^{(0)} + \frac{4\pi}{i\omega} [\sigma_{ij}^{(e)}(\omega, q) + \sigma_{ij}^{(h)}(\omega, q)],$$

where $\sigma_{ij}^{(h)}(\omega, q)$ is the complex conductivity tensor of the holes.

In the presence of an external magnetic field H, in addition to the fact that the dielectric constant of the medium is a function of H, i.e., $\varepsilon_{ij} = \varepsilon_{ij}(H)$, it is also necessary to take into account in the equation of motion (2.1) the magnetic force (2.6). It is easy to see that the induction mechanism of the interaction differs from zero only when "solenoidal" components of the electric field are present.

The additional "magnetic" interaction, as a rule, is always smaller than the purely piezoelectric interaction at all reasonable values of the magnetic field. On the other hand, if the crystal symmetry and the chosen propagation directions and the polarization of the sound wave are such that the acoustic wave is not accompanied by a prolonged electric field, then the piezoelectric interaction decreases sharply (by an approximate factor $(v_s/c)^2$), and then the relative role of the induction mechanism becomes less significant. Although this case is of little interest, in view of the exceedingly small coupling constant (of the order of $(v_s/c)^2\zeta^2$, where ζ^2 is the square of the constant of the electromechanical coupling), a situation is nevertheless possible in which this interaction can play an important role. This occurs in those cases when any of the acoustic branches crosses the electromagnetic branch. The dispersion equation of the electromagnetic waves in the plasma medium, as is well known,[1,4] is of the form

$$\mathrm{Det} \left| q^2 \delta_{ij} - q_i q_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, q) \right| = 0, \qquad (2.12)$$

and in our notation it is given by Det $| \Gamma_{ij} | = 0$. The latter determines precisely those values of the frequency ω and of the wave vector q at which Γ_{ij}^{-1} formally diverges.

The presence of an interaction between the waves (even an arbitrarily weak one) leads under such conditions to a change of the spectrum of the entire system. This process can be represented illustratively in the following manner. Assume that initially there are two branches: acoustic, $\omega = v_s q$, and electromagnetic, $\omega = \omega(q)$, which is determined from the solution of Eq. (2.12). These are shown in Fig. 1a in the absence of interaction. When the interaction between the acoustic and electromagnetic waves is "turned on," the spec-
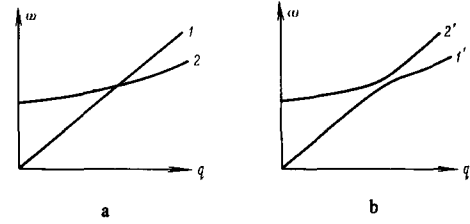

FIG. 1. Realignment of the spectrum of the system when the interaction is "turned on." 1, 2—acoustic and electromagnetic branches respectively; 1', 2'—after realignment of the spectrum.

trum of the system changes and takes the form shown in Fig. 1b. It is clear that such a change of the spectrum of the system is possible only if the acoustic and the electromagnetic waves intersect.* (We recall that it is meaningful to consider acoustic waves only in the region $\omega \lesssim 2\pi v_s/a$, where a is the lattice constant.) The solution of Eq. (2.12) turns out to be most frequently in the form of "fast" waves, which have no intersection points with the acoustic branch. In the presence of a magnetic field, however, anomalously "slow" waves can appear in the spectrum (2.12) (see [3,4,43-47]), and then the situation described above becomes perfectly realistic. We shall not consider such a possibility in detail, and therefore the entire reasoning is valid only far from the roots of Eq. (2.12).

If the medium in which the sound wave propagates is not piezoelectric, then Eq. (2.10) goes over into the well known equations of elasticity theory,[21] which describe the propagation of elastic waves in crystals (for details see [48]). This equation has three roots which are in general different. The latter means that for each direction in the crystal there are three different elastic-wave propagation velocities, and it is possible to separate these waves into purely longitudinal and purely transverse ones only for crystals having a definite symmetry, and only along certain directions. The anisotropy of the elastic-modulus tensor and the presence of a piezoelectric effect in a medium that has both temporal and spatial dispersion make it impossible to separate the waves into purely longitudinal and purely transverse ones: each direction of the wave vector q corresponds to a wave whose displacement vector has components both parallel and perpendicular to the wave propagation direction. If it is assumed that the direction of wave propagation in the crystal is chosen such that there is no "coupling" of oscillations as a result of the anisotropy of the elastic moduli, then the separation of the waves into purely longitudinal and purely transverse is still impossible, owing to the presence of dispersion of the longitudinal electric constant of the medium $\varepsilon_{qq}(\omega, q)$ in which the piezoelectric effect is present. If we introduce an electromechanical constant characterizing the ratio of the density of the electric energy to the density of the elastic energy in the acoustic wave:

$$\zeta_{\parallel} = \left(\frac{4\pi\beta_{q, qq}^2}{\rho v_s^2 \varepsilon_0} \right)^{1/2}, \quad \zeta_{\perp} = \left(\frac{4\pi\beta_{q, q\varkappa}^2}{\rho v_s^2 \varepsilon_0} \right)^{1/2} \quad \left(\varkappa = \frac{u}{u} \right)$$

for the longitudinal and transverse waves respectively,

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*In the presence of damping (amplification) in any one of the oscillation modes, the question of realignment of the spectrum of the system turns out to be more complicated.

then the quantity determining the coupling of the oscillations, owing to the dispersion of $\epsilon_{qq}(\omega, q)$, turns out to be proportional to the fourth power of the electromechanical-coupling constant. Since ζ^2 is of the order of 3×10^{-2} even for such a relatively strong piezoelectric as CdS,[38-40,49] it is clear that the coupling of the oscillations can be neglected in most cases. If it is assumed further that the direction of wave propagation in the crystal is chosen such that there is likewise no coupling of the oscillations as a result of the anisotropy of the elastic moduli then the dispersion equation (2.11) breaks up into dispersion equations for the longitudinal and transverse waves, and these equations are the ones used for the most part in the study of plasma intensification of sound waves in piezosemiconductors.[7,9,20,25, 26,37-41] We present this equation for the case of a longitudinal wave:

$$\omega^2 - q^2 v_{||}^2 \left(1 - \zeta_{||}^2 \frac{\epsilon_0}{\epsilon_{qq}(\omega, q)}\right) = 0. \tag{2.13}$$

As seen from the last expression, in order to find the damping or amplification, or else to determine the change of the phase velocity of the sound waves as a result of the interaction with the conduction electrons, it is necessary to determine the longitudinal component of the dielectric tensor of the medium. As will be shown later, $\epsilon_{qq}(\omega, q)$ describes completely the plasma part of the damping (amplification) of the waves for all types of interaction between the lattice vibrations and the carrier plasma.

c) Dispersion Equation for the Propagation of Ultrasound in Intrinsic Semiconductors and in Semimetals

Let us consider the propagation of elastic waves in intrinsic semiconductors and semimetals with equal electron and hole densities. We assume that there is no piezoelectric effect, so that the interaction of the acoustic waves with the electron-hole plasma can be realized only via the deformation potential. (Direct interaction via the Coulomb field is obviously impossible in crystals with equal electron and hole densities, since the lattice has no charge.)

Let us transform the expression for the Fourier transform of the force (2.5):

$$f_i(\omega, q) = -4\pi i q_k \sum_{\alpha=e,h} \Lambda_{ik}^{(\alpha)} n_{\sim}^{(\alpha)}(\omega, q), \tag{2.14}$$

where $n_{\sim}(\omega, q)$ is the Fourier transform of the nonequilibrium carrier density induced in the crystal by the sound wave. In the derivation of (2.14)) it was assumed that the tensor of the electron-phonon interaction constants does not depend on the quasimomenta of the carriers. To determine the nonequilibrium carrier density $n_{\sim}^{(\alpha)}(\omega, q)$ it is necessary to construct the general expression for the current induced by the acoustic wave in the system. In the case of piezosemiconductors the analogous problem was solved in relatively simple fashion: since the interaction is produced in the main only via the electric field, it follows that $j_i = \sigma_{ij} E_j$. For semiconductors and semimetals, besides the electric field produced as a result of violation of the local quasineutrality of the electron-hole plasma, the carriers are acted upon by an additional force due to the deformation potential and the magnetic field. Assuming that there is

no magnetic field (we shall return to this question later). The total current is written in the form

$$j_i^{(\alpha)} = \sigma_{ij}^{(\alpha)} \left(E_j \mp \frac{1}{e} \Lambda_{lk}^{(\alpha)} \frac{\partial u_{lk}}{\partial x_l}\right), \tag{2.15}$$

where the lower sign (plus) corresponds to holes and the upper (minus) to electrons. Maxwell's equations in this notation are given by

$$\Gamma_{ij} E_j = -\frac{4\pi i \omega}{e c^2} (\Lambda_{qp}^{(e)} \sigma_{iq}^{(e)} - \Lambda_{qp}^{(h)} \sigma_{iq}^{(h)}) u_p, \tag{2.16}$$

where ϵ_{ij} and Γ_{ij} must be taken to mean the total tensor

$$\epsilon_{ij}(\omega, q) = \epsilon_0 \delta_{ij} + \frac{4\pi}{i\omega}(\sigma_{ij}^{(e)} + \sigma_{ij}^{(h)}).$$

Using the continuity equation for the electron and hole components of the currents in (2.15), we can obtain the non-equilibrium value of the electron and hole concentrations, which determine the force (2.14):

$$n_{\sim}^{(e)}(\omega, q) = \frac{q^3}{e^2 \omega} \sigma_{qq}^{(e)} \Lambda_{qm}^{(e)} u_m - \frac{4\pi i q}{e^2 c^2} \sigma_{qj}^{(e)} \Gamma_{js}^{-1} (\sigma_{sq}^{(e)} \Lambda_{qm}^{(e)} - \sigma_{sq}^{(h)} \Lambda_{qm}^{(h)}) u_m. \tag{2.17}$$

With the aid of (2.1), (2.14), and (2.17) we can easily construct the dispersion equation of the ultrasonic waves in intrinsic semiconductors and semimetals. We shall not present its complete form, since it is too complicated, and confine ourselves only to particular but most important cases.

If $|(\omega^2/c^2 q^2)\epsilon_{ij}| \ll 1$, the solenoidal components of the electric field in (2.16) can be neglected, and then the additional plasma term in the dispersion equation will depend only on the longitudinal components of the dielectric constants of the medium:

$$\omega^2 - q^2 v_s^2 = \frac{4\pi i q^4 \epsilon_0}{e^2 \omega \rho} \frac{1}{\epsilon(\omega, q)} \left\{\sigma^{(e)} \Lambda^{(e)2} + \sigma^{(h)} \Lambda^{(h)2} + \frac{4\pi}{i\epsilon_0 \omega} \sigma^{(e)} \sigma^{(h)} (\Lambda^{(e)} + \Lambda^{(h)})^2\right\}. \tag{2.18}$$

The dispersion equation (2.18) is presented for the case of longitudinal waves, and therefore ϵ, σ, and Λ should be taken to mean quantities of the type ϵ_{qq}, σ_{qq}, and Λ_{qq}. From (2.16), in particular, it follows that even if direct interaction with one type of carrier, say holes, does not occur, i.e., $\Lambda^{(h)} = 0$, the influence of the holes on the character of the propagation of the sound waves still remains. Physically this is due to the fact that local electron density is violated in the acoustic waves, and this in turn leads to the appearance of a Coulomb field with which the holes already interact. If we determine the imaginary part of the wave vector q, which determines the absorption of the solid wave in the medium, then we get for $\Lambda^{(h)} = 0$

$$\text{Im} \, q = -\frac{2\pi \omega^2 \Lambda^{(e)2}}{e^2 v_s^5 \rho} \frac{1}{|\epsilon(\omega, q)|^2} \left\{\sigma^{(e)'} \left|\epsilon_0 + \frac{4\pi \sigma^{(h)}}{i\omega}\right|^2 + \frac{16\pi^2 \sigma^{(h)}}{\omega^2} [(\sigma^{(e)'})^2 + (\sigma^{(e)'})^2]\right\}. \tag{2.19}$$

We have used here the universal symbols $\sigma' = \text{Re} \, \sigma$ and $\sigma'' = \text{Im} \, \sigma$. Expression (2.19) means that in equilibrium medium (for which $\sigma^{(e)'}$, $\sigma^{(h)'} > 0$) we have $\text{Im} \, q < 0$ — the sound wave can only attenuate. On the other hand, if the expression in the parentheses of (2.17) is negative, then the wave builds up. The latter occurs only when at least one of the real parts of the conductivity is negative, i.e., one of the components of the medium is not in equilibrium. Thus, if negative electron conductivity is produced via the Cerenkov mechanism, the influence of the holes becomes even stronger, for now they de-

termine even the very possibility of intensification of the waves in the medium.

Assume that $|4\pi\sigma^{(e,h)}(\omega, \mathbf{q})| \gg \omega$; it then follows from the dispersion equation (2.18) that

$$\operatorname{Im} q = -\frac{2\pi\omega^3 |\Lambda^{(e)} + \Lambda^{(h)}|^2}{e^2\rho v_s^5} \operatorname{Re}\left(\frac{\sigma^{(e)}\sigma^{(h)}}{\sigma^{(e)} + \sigma^{(h)}}\right), \quad (2.20a)$$

$$\frac{\Delta v_s}{v_s} = -\frac{2\pi\omega^2 |\Lambda^{(e)} + \Lambda^{(h)}|^2}{e^2\rho v_s^5} \operatorname{Im}\left(\frac{\sigma^{(e)}\sigma^{(h)}}{\sigma^{(h)} + \sigma^{(e)}}\right). \quad (2.20b)$$

These expressions will be determined later, after we obtain the longitudinal conductivity of the medium $\sigma(\omega, \mathbf{q})$.

We stop now to discuss the influence of the magnetic field on the character of propagation of the elastic waves in conducting bodies. It is clear first of all that in a magnetic field the longitudinal dielectric constant of the medium becomes a function of the magnetic field, but as already noted above this is not the only influence of the magnetic field. In a magnetic field, the latter is acted upon, besides the electron pressure (2.5), also by the force (2.6). The deformed lattice now exerts on the electrons and on the holes a force

$$\mathbf{F} = -e\left(\mathbf{E} + \frac{1}{c}[\mathbf{u}\mathbf{H}] + \frac{1}{c}\frac{\partial}{\partial \mathbf{x}}\Lambda_{ik}u_{ik}\right), \quad (2.21)$$

which in turn leads to a change in the expression for the current (2.15). The first and last terms of (2.21) are obvious; as to the term $(1/c)\mathbf{u}\times\mathbf{H}$, it is the consequence of the transformation of the coordinate system[23,24,28] (we recall that the spectrum of the electrons (2.4) is specified in a reference frame that is at rest relative to the lattice,[23,28] whereas the entire analysis carried out in the laboratory system). Depending on which of the last two terms of (2.21) predominates, the interaction between the electrons and the phonons can be realized either via the deformation potential or via the "induction" mechanism.

We can construct further dispersion equations describing the propagation of acoustic waves in the presence of a magnetic field; they are quite complicated and will not be presented here.

3. DIELECTRIC CONSTANT OF AN ELECTRON-HOLE PLASMA IN THE PRESENCE OF DRIFT

d) General Remarks. Conductivity of the Medium at Low Frequencies

We have obtained above dispersion equations for the propagation of acoustic waves in conducting bodies; it is seen from these equations that the character of the interaction of the elastic waves with the carriers is determined completely by the complex dielectric tensor of the medium, i.e.,

$$\varepsilon_{ij}(\omega, \mathbf{q}) = \varepsilon_{ij}^{(0)} + \frac{4\pi}{i\omega}\sigma_{ij}(\omega, \mathbf{q}); \quad (3.1)$$

Here $\varepsilon_{ij}^{(0)}$ is the dielectric constant of the lattice, which henceforth can be regarded as isotropic ($\varepsilon_{ij}^{(0)} = \delta_{ij}\varepsilon_0$), and $\sigma_{ij}(\omega, \mathbf{q})$ is the complex conductivity of the plasma carriers. Since the foregoing analysis was confined to cases in which no acousto-electromagnetic[43] waves were produced, only the longitudinal component $\sigma_{||}(\omega, \mathbf{q}) = (q_i q_j/\sigma_{ij}(\omega, \mathbf{q}))/q^2$ of the tensor $\sigma_{ij}(\omega, \mathbf{q})$ need be known in order to consider the phenomena of amplification

(attenuation) of acoustic waves. We shall therefore be interested below in the calculation of only the longitudinal component $\sigma_{||}(\omega, \mathbf{q})$, under the condition that directional motion of the particles exists in the system.

Calculations of the electronic dielectric constants of the medium as applied to a gas-discharge plasma were undertaken relatively long ago (see [3,4]), and principal attention was paid to various types of instabilities occurring in a gas-discharge plasma containing electron beams.[4,5,51-54] An essentially similar problem has to be solved in a solid-state plasma, the only difference being that the collisions of the electrons and of the holes with the "lattice" play a more important role here than in a gas-discharge plasma. At low temperatures, for example, when the distribution of the electrons or the holes becomes degenerate, new effects appear (geometric resonance, quantum oscillations of conductivity), which do not occur in an ordinary gas-discharge plasma.

It will be shown below that the real part of the conductivity of the electron or hole components of a solid-state plasma becomes negative when the drift velocity exceeds the phase velocity of the wave. This condition is a rather general property of the medium, regardless of how the carrier density is produced; it will be demonstrated by means of a number of examples, that this condition is satisfied also in the presence of different resonances in the magnetic field.

Inasmuch as the longitudinal dielectric constant of the medium always enters in the dispersion equations (2.11), (2.18), and (2.28), independently of the type of interaction between the phonons and the electrons, it is convenient to consider, for the calculation of $\epsilon_{||}(\omega, \mathbf{q})$ the following model problem: assume that the system is acted upon by a plane longitudinal electric wave (1.1) and it is necessary to find the response of the system to this action, i.e., the current induced by the wave in the medium.

We start with the simplest case, when the electron or hole plasma can be described by the equations of hydrodynamics of a charged liquid

$$mn\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v}\right) = e\mathbf{E} + \nabla P - mn\nu\mathbf{v}, \quad (3.2a)$$

$$\frac{\partial n}{\partial t} + \operatorname{div} n\mathbf{v} = 0; \quad (3.2b)$$

here $n(\mathbf{x}, t)$ is the electron density, T is the pressure of the electron gas, $\mathbf{v}(\mathbf{x}, t)$ is the hydrodynamic velocity of the electrons, $\nu = 1/\tau$ is the effective collision frequency, $\mathbf{E} \sim \mathbf{E_d} + \mathbf{E_\sim}(\mathbf{x}, t)$ is the electric field in the medium. Using further the usual procedure of linearizing Eqs. (3.2) with respect to the small deviation from equilibrium values, produced by the wave, it is easy to obtain an expression for the hydrodynamic current $\mathbf{j_\sim}(\omega, \mathbf{q}) = e(n\mathbf{v})_\sim$, from which follows directly an expression for the longitudinal conductivity of the medium[8,55,56]

$$\sigma_{||}(\omega, \mathbf{q}) = \frac{\sigma_0}{1 - (\mathbf{q}\mathbf{v_d}/\omega) - i(q^2 c_T^2/\omega\nu)}, \quad (3.3)$$

where $\mathbf{v_d} = e\mathbf{E_d}\tau/m$ is the electron drift velocity, and $v_T = \sqrt{T/m}$ is the thermal velocity. In the derivation of (3.3) it was assumed that the pressure is $P = nT$, just as for an ideal gas, and by assumption the temper-

ature of the electrons T does not depend on the coordinates or on the time.* In the case when the electron gas is in the degenerate state, i.e., $T \ll \epsilon_F^0$, where ϵ_F^0 is the Fermi energy, the pressure gradient will be:[57]

$$\nabla P = \frac{m}{3} v_F^2 \nabla n_\sim, \qquad (3.4)$$

where v_F is the Fermi velocity, and the final expression for $\sigma_\parallel(\epsilon, q)$ will differ from (3.3) only in the fact that the thermal velocity of the electrons v_T is replaced by $v_F/\sqrt{3}$.

As expected, formula (3.3) differs from the elementary expression (1.5) only in the finite width of the resonance at $v_d = v_{ph}$. Expression (3.3) can be obtained from a kinetic analysis of the problem,[53] by using expansion of the distribution function in Legendre polynomials.[3,55,58,59] From this it is possible to obtain in particular, a criterion for the validity of formula (3.3):

$$\left| \frac{1 - (q v_d)}{\omega} \right| \gg \frac{q^2 v_T^2}{\omega v}. \qquad (3.5)$$

We note that allowance for the dependence of the collision frequency on the electron velocity (if we disregard the special case of heating of the electron gas by the electric field (see [80]), does not lead to a qualitative change of formula (3.3).[55] It can be shown[55] that when account is taken of the dependence of the collision frequency on the velocity, the presence of directional motion of the electrons in the medium leads to a change in the sign of the real part of the conductivity; the drift velocity is then determined in terms of a certain effective collision frequency, which of course depends on the predominant type of scattering. The form of the $\nu(v)$ dependence greatly influences the temperature dependence of the conductivity.

e) Dielectric Constant of the Medium at High Frequencies. Kinetic Analysis

To investigate also the damping and amplification of acoustic waves in semiconductors and semimetals at high frequencies, let us calculate $\sigma_\parallel(\omega, q)$ with the aid of the kinetic equation. Instead of a real semiconductor, we consider a gas of electrons (holes) of density n_0, placed in a homogeneous positive "background" having the same density. The distribution function of such an electron gas is determined from the solution of the kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \frac{\partial f}{\partial \mathbf{v}} + \mathrm{St}\, f = 0, \qquad (3.6)$$

where $\mathbf{F}$ is the force acting on the electron, $\mathbf{F} = -e(\mathbf{E}_d + \mathbf{E}(x,t))$. The change of the distribution function due to the collisions will be taken into account with the aid of the relaxation time

$$\mathrm{St}\, f = -\frac{f - f_{0s}}{\tau}, \qquad (3.7)$$

where f_{0s} is the spherically-symmetrical part of the distribution function, and τ is a constant that does not depend on the energy. In such an approach, the collisions that lead to the appearance of the electron-hole pairs, and also to recombination and capture processes, are not taken into account, so that formally all the conclusions are valid for an infinitely long carrier lifetime.

The presence of an elastic wave in the crystal leads to violation of the local equilibrium value of the electron density, and therefore the distribution function f_0, which is spherically symmetrical in velocity space, should be determined by the local value of the Fermi energy, i.e.,*

$$f_{0s}(\mathbf{r}, \mathbf{v}, t) = f_0(\epsilon_F(\mathbf{r}, t)). \qquad (3.8)$$

It is convenient to expand (3.8) in a series in the small perturbation produced by the wave:

$$f_{0s}(\mathbf{r}, \mathbf{v}, t) = f_0 + \Delta\epsilon_F \frac{\partial f_0}{\partial \epsilon},$$

where f_0 is the Fermi distribution and $\Delta\epsilon_F$ is the change of the Fermi energy as a result of the formation of space charge in the wave:

$$\Delta\epsilon_F = \epsilon_F(\mathbf{r}, t) - \epsilon_F^0 \approx \frac{2}{3n_0} \epsilon_F^0 n_\sim(\mathbf{r}, t). \qquad (3.9)$$

The complete distribution function $f(\mathbf{r}, \mathbf{v}, t)$ will be represented in the form

$$f(\mathbf{r}, \mathbf{v}, t) = f_0 + f_{01} + \varphi(\mathbf{r}, \mathbf{v}, t), \qquad (3.10)$$

where f_{01} is the current increment due to the external electric field, and $\varphi(\mathbf{r}, \mathbf{v}, t)$ is the distribution function component due entirely to the wave, and therefore proportional to $\exp(i\omega t - iqz)$. To calculate the response of the system to the wave (1.1), i.e., to determine the current

$$j_\sim(\mathbf{r}, t) = e \int \mathbf{v}\varphi(\mathbf{r}, \mathbf{v}, t) d^3v, \qquad (3.11)$$

it is necessary precisely to find the explicit form of the functions $\varphi(\mathbf{r}, \mathbf{v}, t)$. This problem is usually solved by successive approximations in terms of the electric field. One first determines f_{01}—the term linear in the "weak" constant field $\mathbf{E}_d$—and then $\varphi(\mathbf{r}, \mathbf{v}, t)$. The field $\mathbf{E}_d$ is "weak" in the sense of the electron drift and it is much smaller than the characteristic electron velocity (thermal or Fermi). †Substituting in (3.11) the function

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*This is valid only at low frequencies, when $\omega \ll \tau_s^{-1}$, where τ_ϵ is the relaxation time of the energy (temperature) of the system. On the other hand, if the frequency ω is of the order of τ_ϵ^{-1}, then it is necessary to consider, besides Eq. (3.2), also the equation for the energy balance, from which the nonequilibrium addition to the temperature is determined. It is easy to see that in this case the real part of the conductivity will no longer reverse sign when condition (1.6) is satisfied, since additional dissipation of the electric energy appears as a result of the temperature oscillations.

*We disregard here the effect of phonon dragging of the electrons, considered by Holstein [61]. Expression (3.8), generally speaking, should be written in the form $f_{0s}(\mathbf{r}, \mathbf{v}, t) = f_0(\mathbf{v} - \mathbf{u}(\mathbf{r}, t))\epsilon_F(\mathbf{r}, t)$, in accordance with the fact that the electron distribution function is symmetrical in the reference frame in which the lattice is at rest [62]. It is easy to see, however, that for semiconductors, allowance for these terms due to the dragging effect in the expansion of f_{0s} makes a small contribution to the interaction compared with other mechanisms, and is therefore disregarded from now on.

† In a constant electric field, generally speaking, the electron gas becomes heated, and therefore the analysis of the kinetic equation (3.6) with a collision integral in the form of the τ-approximation (3.7) calls for certain caution. Nonetheless, it can be shown that even when account is taken of the heating, the main results remain in force provided the electron temperature is taken to mean the effective temperature $T = T_l[1 + (1/3)(v_d^2/v_s^2)]$, where T_l is the lattice temperature [3,58,57]. For the method used here it is important only that the kinetic equation in a constant field have a stationary solution and $f_0 \gg f_{01}$. Since the kinetic equation (3.6) with the τ-term (3.7) does not contain the equation for the stationary function f_0, we are justified in choosing for f_0 an equilibrium function (Boltzman or Fermi) with an effective temperature that depends on the electric field.

$\varphi(\mathbf{r}, \mathbf{v}, t)$ obtained in this manner for the longitudinal (i.e., z-component) of the current, we obtain[9,63]

$$j_{\sim}(\omega, \mathbf{q}) = a(\omega, \mathbf{q}) E_{\sim} - en_{\sim}(\omega, \mathbf{q}) V; \qquad (3.12)$$

here

$$a(\omega, \mathbf{q}) = -e^2\tau \int \frac{\partial f_0}{\partial \varepsilon} \left[\frac{v_z^2}{Q} \left(1 + (1+Q)\frac{i\tau q v_d}{Q^2} \right) + \frac{v_z v_d}{Q^2}(1+Q) \right] d^3v, \qquad (3.13)$$

$$V(\omega, \mathbf{q}) = -\frac{2}{3} \frac{\varepsilon_F^0}{n_0} \int \frac{\partial f_0}{\partial \varepsilon} \left[v_z \left(1 + \frac{i\tau q v_d}{Q} \right) + \frac{v_d}{Q} \right] d^3v, \qquad (3.14)$$

where $Q = 1 + i\omega\tau - i\tau q v_z$, and for simplicity we have confined ourselves to the case of a longitudinal wave $\mathbf{u} \parallel \mathbf{q}$.

The expression for the current (3.12) is similar in structure to formula (1.2) and has a simple physical meaning. The first term in (2.12) is the current induced by the electric field of the wave under the condition that no space charge is produced in the wave $(en_{\sim}(\mathbf{r}, t) = 0)$. The second term is the space-charge current or the diffusion current.

By eliminating from (3.12) the non-equilibrium electron density with the aid of the continuity equation, we obtain the final expression for the current

$$j_{\sim}(\omega, \mathbf{q}) = \sigma_{||}(\omega, \mathbf{q}) E_{\sim}$$

and for the electron conductivity

$$\sigma_{||}(\omega, \mathbf{q}) = a(\omega, \mathbf{q}) \frac{\omega}{\omega - qV}. \qquad (3.15)$$

Let us consider first the expression for $\sigma_{||}(\omega, \mathbf{q})$. Assuming the electron gas to be strongly degenerate, and calculating the integrals (3.13) (using in this case the fact that $\partial f_0/\partial \varepsilon \sim \delta(\varepsilon - \varepsilon_F^0)$, we obtain the exclusive form of the expression for $\sigma_{||}(\omega, \mathbf{q})$).[63] In the general case it is very cumbersome, so that we shall confine ourselves only to two limiting cases: $ql \ll 1$ and $ql \gg 1$, where $l = v_F\tau$ is the electron mean free path. In the former case, when $ql \ll 1$, the result, naturally, assumes the form (3.3), in which v_T^2 is replaced by $v_F^2/3$. In the region of large frequencies, when $ql \gg 1$, we have[63]

$$\sigma_{||}(\omega, \mathbf{q}) = \frac{3\sigma_0}{(ql)^2} \left[\frac{\pi}{2} \frac{(\omega\tau)^2}{ql} \left(1 - \frac{qv_d}{\omega} \right) + i\omega\tau \right] \qquad (3.16)$$

and consequently the real part of (3.16) reverses sign when the Cerenkov condition is satisfied.

In the case when the electron gas is not degenerate, integration of expressions (3.13) is much more complicated. The longitudinal conductivity can be determined in this case by the Enskog method, by expanding the distribution function in Hermite polynomials.[64,65] Just as in the degenerate case, the real part of the conductivity reverses sign when the Cerenkov-radiation condition is satisfied.

f) Dielectric Constant of a Plasma Medium in a Magnetic Field. Geometrical and Cyclotron Resonances

Let us find the dielectric constant of the medium in the presence of a magnetic field. We choose the coordinate system such that the z axis is directed along the magnetic field H, the y axis along the electric field $\mathbf{E_d}$, and the x axis along the direction of wave propagation.

The drift velocity is in this case also directed along x and equals $v_d = cE_d/H$ ($\Omega\tau \gg 1$). The force acting on the electron will now be

$$\mathbf{F} = -e\left(\mathbf{E_d} + \mathbf{E_{\sim}} + \left[\frac{\mathbf{v}}{c}\mathbf{H} \right] \right).$$

The solution of the kinetic equation (3.6) is best sought in the form of an integral along the phase trajectories:[66,67]

$$f(\mathbf{r}, \mathbf{v}, t) = \int_{-\infty}^{t} f_0(\mathbf{r}', \mathbf{v}', t) e^{\frac{t-t'}{\tau}} \frac{dt'}{\tau}, \qquad (3.17)$$

corresponding to an adiabatic turning-on of the interaction in the infinitely remote past.[4] We expand $f(\mathbf{r}, \mathbf{v}, t)$ in powers of the small deviation due to the presence of external electric and magnetic fields[9]

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\varepsilon) + \varphi_1(\mathbf{r}, \mathbf{v}, t) + \varphi_2(\mathbf{r}, \mathbf{v}, t), \qquad (3.18)$$

where*

$$\varphi_1(\mathbf{r}, \mathbf{v}, t) = \frac{\partial f_0}{\partial \varepsilon} \int_{-\infty}^{t} [\Delta\varepsilon - \varepsilon_F' + \varepsilon_F^0] e^{\frac{t-t'}{\tau}} \frac{dt'}{\tau}, \qquad (3.19)$$

$$\varphi_2(\mathbf{r}, \mathbf{v}, t) = \frac{1}{2} \frac{\partial^2 f_0}{\partial \varepsilon^2} \int_{-\infty}^{t} [\Delta\varepsilon - \varepsilon_F' + \varepsilon_F^0]^2 e^{\frac{t-t'}{\tau}} \frac{dt'}{\tau}. \qquad (3.20)$$

The prime in (3.19) and (3.20) denotes that the corresponding quantity should be taken at the instant of time t', $\Delta\varepsilon = \varepsilon(\mathbf{v}') - \varepsilon(\mathbf{v})$ is the change of the electron energy due both to the constant electric field and to the electric field of the wave. Using the equations of motion of charged particles in crossed electric and magnetic fields (see, for example [68], Sec. 22) and going over to Fourier-transformed quantities, we obtain an expression for the current induced in such a system by a longitudinal electric wave. This expression is of the form (3.12), but the coefficients $a(\omega, \mathbf{q})$ and $V(\omega, \mathbf{q})$ will be equal to[9,67]

$$a(\omega, \mathbf{q})$$
$$= -\frac{3\sigma_0}{iql} \left\{ -\frac{v_d}{v_F} + \frac{1}{qr_l} \sum_{n,m} \frac{J_m\left(qr_l \frac{v_d}{v_F}\right)}{\lambda + i(n-m)\Omega\tau} \left[\frac{\lambda n g_n}{qr_l} - \frac{v_d}{v_F}(1 + i\omega\tau) J'_{2n}(2qr_l) \right] \right\} \qquad (3.21a)$$

$$V(\omega, \mathbf{q}) = \frac{\Omega}{q} \sum_{n,m} \frac{J_m\left(qr_l \frac{v_d}{v_F}\right)}{\lambda + i(n+m)\Omega\tau} \left\{ n g_n \left(qr_l - \frac{v_d}{v_F} J'_{2n}(2qr_l) \right) \right\}, \qquad (3.21b)$$

where $\lambda = 1 + i\omega\tau(1 - qv_d/\omega)$, r_l is the Larmor radius, $r_l = v_F/\Omega$ ($\Omega\tau \gg 1$), $J_m(x)$ is a Bessel function, and $g_n(x)$ is a weakly oscillating function introduced by Cohen et al.:[69]

$$g_n(x) = \int_0^{\pi/2} J_n^2(x \sin\theta) \sin\theta \, d\theta. \qquad (3.22)$$

In the derivation of (3.21) it was assumed that $(v_d/v_F) \ll 1$ and $(\omega/qv_F) \ll 1$, so that the squares of these quantities could be neglected in comparison with unity. (In semimetals and in semiconductors, the ratio of the speed of sound to the Fermi velocity of the electron is

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*The need for taking into account the increment φ_2 which is quadratic in the field is due to the cross term $E_{\sim}E_d$.

of the order of 10^{-2}–10^{-3}.) Formulas (3.21) are valid for the strongly degenerate case, when the derivative of the equilibrium distribution function with respect to the electron energy is close to a δ-function of the argument $(\epsilon - \epsilon_F^0)$.

It is easy to see that from (3.21) it is possible to obtain the longitudinal conductivity at low frequencies $ql \ll 1$, but in a strong magnetic field $\Omega\tau \gg 1$. In the result we again arrive at formula (3.3), except that the collision frequency ν will be replaced by the effective collision frequency $\nu_{eff} = (\Omega^2 + \nu^2)/\nu \approx \Omega^2/\nu$, which depends on the magnetic field, and the drift is $v_d = cE_d/H$.

1) Geometric resonance. If the wavelength of the perturbation (1.1) acting on the system turns out to be of the same order as the Larmor radius of the classical orbit of the electron in the magnetic field, then conditions are produced for geometrical resonance in the wave absorption. The geometrical resonance is possible only in degenerate semiconductors and semimetals, when the revolution radii of all the electrons in the magnetic field are practically equal; on the other hand, if the electron is in a non-degenerate state, then the thermal velocity spread of the electrons causes their radii in the magnetic field to be different, and consequently this resonance vanishes as a result of averaging over the velocities. Thus, in the region of geometric resonance

$$T \ll \epsilon_F^0, \quad qr_l \approx 1. \tag{3.23}$$

Investigating the values of the coefficients $a(\omega, q)$ and $V(\omega, q)$ at qr_l of the order of unity, we obtain for the longitudinal conductivity[67]

$$\sigma_{||}(\omega, q) = \frac{3\sigma_0}{(ql)^2}\left\{\frac{\lambda\left[1 - g_0(qr_l)\right]}{1 - \frac{qv_d}{\omega} - \frac{i}{\omega\tau}\left[1 - g_0(qr_l)\right]}\right\}. \tag{3.24}$$

We note first that the real part of (3.24) reverses sign when the Cerenkov condition $v_d > \omega/q$ is satisfied. As a function of the magnetic field, (3.24) experiences resonant bursts determined by the oscillations of the function $g_0(x = qr_l)$ (Fig. 2). These conductivity oscillations ensure resonant changes in the amplification or absorption of longitudinal waves of a degenerate electron plasma.

2) Cyclotron resonance. When the electron revolution frequency in the magnetic field $\Omega = eH/mc$ coincides with the frequency ω of the wave, cyclotron resonance is produced in the absorption of the waves. For an ordinary gas-discharge plasma, these phenomena have been known for a long time (see, for example, [3], Sec. 12, and the references therein); research on the cyclotron resonance in solid-state plasma began much later.[70]*

If the medium is under the influence of both a constant magnetic field and an electric field, which leads to electron drift, resonant amplification of the waves should be expected in place of resonant absorption. Obviously, the condition for resonant cyclotron absorption $\Omega = \omega$ should be replaced, owing to the presence of the electron drift, by the condition $\Omega = \omega'$, where $\omega'' = \omega\left[1 - (v_d/v_{ph})\right]$ is the Doppler-shifted wave frequency.

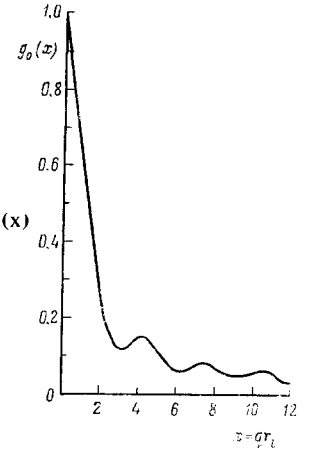*Cyclotron resonance in the absorption of acoustic waves was first predicted by Mikoshiba [71].

FIG. 2. Behavior of the function $g_0(x)$ (according to [69]).

Unlike the geometrical resonance, which is possible only in degenerate semiconductors and semimetals, cyclotron resonance is possible also in nondegenerate ones. This is connected with the simple circumstance that the frequency of revolution of the electrons in the magnetic field (in the nonrelativistic approximation!) does not depend on the electron velocity.

Near cyclotron resonance $qr_l = v_F\omega/v_{ph}\Omega$, and since v_{ph} is of the order of the speed of sound, we get $qr_l \gg 1$. This makes it possible to use the asymptotic value of the expression (3.21a) at large qr_l. As a result we obtain for the conductivity[67]

$$\sigma_{||}(\omega, q) = \frac{3\pi}{2}\sigma_0\frac{(\omega\tau)^2}{(ql)^3}\left(1 - \frac{qv_d}{\omega}\right)\mathrm{cth}\,\frac{\pi\lambda}{\Omega\tau}. \tag{3.25}$$

For the nondegenerate case, as can be readily shown, expression (3.25) remains in force; it is merely necessary to replace the coefficient $3\pi/2$ by $2\sqrt{\pi}$, and to take the mean free path $l = v_F\tau$ to mean the quantity $l = v_T\tau$, where v_T is the thermal velocity of the electrons.

Depending on the magnetic field, the real part of the conductivity (3.25) experiences a number of resonances, the positions of which are determined by the condition

$$n\omega' = n\left(1 - \frac{v_d}{v_\Phi}\right)\omega = \Omega \qquad (n = 1, 2, 3, \ldots). \tag{3.26}$$

Thus, in crossed electric and magnetic fields the resonance condition depends on the electric field, leading to a resonant change of the conductivity as a function of the drift field (Figs. 3a and b).
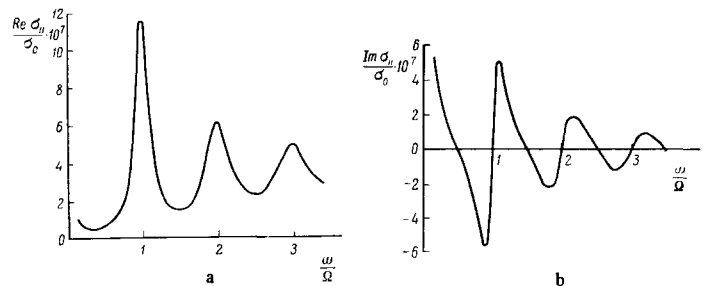

FIG. 3. Dependence of the conductivity on the ratio ω/Ω under conditions of cyclotron resonance. $\tau = 10^9$ sec, $\omega = 10^{10}$ sec^{-1}, $v_d = 0$; the values of $\mathrm{Re}\sigma_{11}(\omega, q)$ at $v_d = 2v_s$ will differ only in sign from the values at $v_d = 0$.

In a strong magnetic field, when the condition $\Omega\tau \gg 1$ is satisfied, the energy spectrum of the electrons ceases to be continuous and Landau levels connected with the quantization of the circular motion of the electrons appear.[72,73] If, in addition, the thermal energy of the degenerate electron gas T is smaller than the energy $\hbar\Omega$ ($T \ll \hbar\Omega$), then it becomes possible to observe a number of effects connected with the quantization of the electron energy. Without taking into account the spin, the spectrum of the free electrons is

$$\varepsilon_{n,\,k_z} = \hbar\Omega\left(n + \frac{1}{2}\right) + \frac{\hbar^2 k_z^2}{2m}, \tag{3.27}$$

where n is a positive integer that determines the Landau quantum level, k_z is the projection of the quasimomentum of the electrons on the direction of the magnetic field. Let us consider the conservation law when a quantum of energy $\hbar\omega$ is absorbed by the electron

$$\left(n + \frac{1}{2}\right)\hbar\Omega + \frac{\hbar^2 k_z^2}{2m} + \hbar\Omega = \left(n' + \frac{1}{2}\right)\hbar\Omega + \frac{\hbar^2}{2m}(k_z + q_z)^2. \tag{3.28}$$

In the case of a sufficiently strong magnetic field ($\Omega \gg \hbar k_z q_z/m$), for small wave vectors $q_z \ll k_z$, it follows from (3.28) that the absorption process is possible only when $n' = n$. Then $\hbar k_z/m = \omega/q_z = v_{ph}/\cos\theta$, where θ is the angle between the direction of propagation of the wave and the magnetic field. The latter condition denotes that the wave is absorbed predominantly only by those electrons that move in phase with the wave. All the electrons that take part in the kinetic processes should have an energy on the order of the Fermi energy, "smeared out" by an amount T, and on the other hand, the energy should satisfy the condition (3.27). These two rules permit the wave vector k_z to assume only values corresponding to the condition $\varepsilon_{n,k_z} \approx \varepsilon_F^0$, accurate to the value of the thermal smearing of the Fermi level. When the magnetic field changes, the position of the allowed intervals of k_z changes, and the effective number of electrons moving in phase with the wave also changes. This leads to corresponding oscillations in the conductivity of the medium. When the system contains electrons moving in a definite direction, the oscillations of absorption[74] give rise to analogous oscillations of wave amplification. As above, the problem reduces to finding the complex conductivity tensor of the medium under the Landau quantization conditions in the presence of directional motion of the electrons (for details see [75-81]).

g) Dielectric Constant of a Medium in the Presence of Impurity Centers

Experimental investigations of the amplification and generation of acoustic waves in piezosemiconductors have shown that the agreement between theory and experiment can be greatly improved by taking into account in the analysis the processes of adhesion and capture of carriers from the conduction bands by various impurity centers.[82,83] We shall therefore consider below the problem of calculating the longitudinal dielectric constant of a medium in the presence of impurity centers. Depending on the number and on the character of impurity centers, many various cases are possible here, and an analysis of all these cases would take up much

space. We therefore consider below only a model problem for a neutral impurity center, the levels of which lie in the forbidden band at a depth ε_t; their concentration is N_t. In the hydrodynamic model, the kinetic equation that determine the processes of ejection and capture of the electrons, will obviously be[84]

$$\frac{\partial n}{\partial t} + \frac{1}{e}\operatorname{div}\mathbf{j}_e = g_c - r_c + G, \tag{3.29a}$$

$$\frac{\partial p}{\partial t} - \frac{1}{e}\operatorname{div}\mathbf{j}_p = g_v - r_v + G, \tag{3.29b}$$

$$\frac{\partial n_t}{\partial t} = r_c - r_v + g_v - g_c, \quad \operatorname{div}\varepsilon_0\mathbf{E} = 4\pi e(p - n - n_t), \tag{3.29c}$$

where n is the electron density in the conduction band, p is the hole density, and n_t is the concentration of the imperfections that capture the electron. The first term on the right sides of (3.29a) and (3.29b) describe the thermal excitation of the captured carriers, with

$$g_v = (N_t - n_t)N_v vS_p e^{-\frac{\varepsilon_g - \varepsilon_t}{T}}, \qquad g_c = n_t N_c vS_n e^{-\frac{\varepsilon_t}{T}}, \tag{3.30}$$

where S_p is the cross section for the capture of a free hole by a center occupied by an electron, S_n is the cross section for the capture of a free electron by an empty impurity center, and N_c and N_v are the effective densities of states in the conduction and valence bands, respectively. The terms

$$r_c = n(N_t - n_t)vS_n, \qquad r_v = pn_t vS_p \tag{3.31}$$

take into account the decrease of the number of free carriers as a result of their capture by the imperfections, G is the density of the carriers generated by the light per unit time,

$$\mathbf{j}_e = -e\mu_n n\mathbf{E} - eD_n\nabla n, \qquad \mathbf{j}_p = e\mu_p p\mathbf{E} + eD_p\nabla p -$$

are respectively the electron and hole currents (μ_n, μ_p —mobilities, D_n, D_p—diffusion constants), and ε_g is the width of the forbidden band.

To calculate $\varepsilon_{\parallel}(\omega, \mathbf{q})$, as above, it is necessary to find the response of the system, i.e., the current produced by the action of the alternating field of the wave (1.1) on the system. Linearizing the system (3.29) with respect to the small perturbation due to the wave (1.1), and eliminating the alternating density of the electrons at the impurity centers, we obtain

$$\omega(1 + \xi)n_- - \frac{q}{e}j_e = 0, \tag{3.32}$$

where the parameter ξ, which characterizes the influence of the impurity centers, is equal to

$$\xi = f_0 vS_n N_t \frac{i\omega + v_p + N_t(1 - f_0)vS_p}{(i\omega + v_n)[N_t(1 - f_0)vS_p + i\omega] + i\omega v_r}; \tag{3.33}$$

here

$$v_n = vS_n(n_0 + N_c e^{-\frac{\varepsilon_t}{T}}), \qquad v_p = vS_p(p_0 + N_v e^{-\frac{\varepsilon_g - \varepsilon_t}{T}}), \qquad f_0 = \frac{N_t - n_t^0}{N_t}$$

and the hole mobility was set equal to zero for simplicity. The physical meaning of the quantities v_n, v_p, and $N_t vS_n f_0$, $n_t^0 vS_p$ can be readily established. It is easy to see that as a result of linearization of the system (3.29), in the general case there appear four characteristic dynamic times of the system: $\tau_1 = v_n^{-1}$ is the time of ejection of the electron from the impurity center, $\tau_2 = N_t f_0 vS_n$ the time of capture of electron by center, $\tau_3 = n_t^0 vS_p$ the time of capture of hole by center, and $\tau_4 = v_p^{-1}$ the time of thermal ejection of hole from center into the valence

band. It is important that these times depend both on the electron density in the conduction band and on the temperature of the system.

The equilibrium values of the concentrations n_0, p_0, n_t^0 are determined by the system (3.29), if the left sides of the equations are set equal to zero. From these equations we can determine the explicit form of the dependence of the concentrations n_0, p_0, and n_t^0 on the quantity G, which is proportional to the illumination of the sample.

From (3.32) and from the definition of the current we can easily obtain a general expression for the conductivity of the medium.

$$\sigma_{\parallel}(\omega, q) = \frac{\sigma_0 (1 - \frac{\cdot}{\cdot})}{\omega(1 + \frac{\cdot}{\cdot}) - q\mu_n E_d - iq^2 r_T^2 \tau} .$$ (3.34)

It follows from (3.33) and (3.34) that the influence of the impurity centers strongly changes the response of the system to the variable field of the wave. It is seen that the condition under which the real part of the conductivity becomes negative and consequently amplification is possible in the system, reduces to the Cerenkov condition, in which, however, the carrier drift velocity $v = \mu_{eff}(\omega, \ldots) E_d$ is determined in terms of a certain effected mobility, that depends in a rather complicated manner on the presence of the impurity centers. As a rule, $\mu_{eff} \ll \mu$, and therefore the threshold value of the electric field, at which amplification becomes possible in the system, is appreciably increased. It is also important that this threshold value of the electric field now turns out to depend on the electron density both in the conduction band and at the impurity centers. Physically this process is connected with the fact that if the electron has time to interact with the impurity center during a time of the order of the period of the wave (for example, it can be captured and be ejected, or recombined), then it loses the directional part of the momentum obtained from the field, and thus the average directional velocity of the electrons decreases.

The presence of impurity centers leads also to a change of the effective conductivity of the medium, due to the redistribution of the bound charges among the impurity centers that capture them; this in turn leads to a decrease of the amplification in the system.

We have considered before a model of a neutral impurity center with one level, characterized by four dynamic times; the model of a charged center with two characteristic times is considered in [85,86] (see also [87,88,89]).

4. AMPLIFICATION OF SOUND WAVES

h) General Remarks. Amplification of Sound Waves in Piezosemiconductors

We have obtained above dispersion equations describing the propagation of sound waves in conducting bodies with different character of the interaction between the lattice vibrations and the electrons or holes. The additional terms that appeared in the dispersion equation as a result of allowance for interaction with the carriers contain the longitudinal electric constant of the plasma medium. Expressions for the complex conductivity of the medium in the presence of drift were then obtained in Ch. 3 for different limiting cases,

namely low and high frequencies near the geometric and cyclotron resonances in a quantizing magnetic field in the presence of impurity centers. It is now clear that to investigate the character of the propagation of the sound waves, their damping and amplification, the change of the phase velocity, etc., it suffices to substitute the obtained expressions for $\sigma_{\parallel}(\omega, q)$ in the dispersion equations. The resultant number of possibilities, meaning formulas, will then be quite large, and an investigation of all of them would require much space. Taking this circumstance into account, and also the fact that such an analysis entails no difficulty and the reader can successfully perform it independently, we confine ourselves below to a consideration of only a few very general cases, which apparently are simplest to obtain in experiments.

The first to observe the effect of amplification of ultrasonic waves as a result of supersonic motion of electrons in CdS piezosemiconducting crystals were Hutson, McFee, and White.[7]* CdS crystals are perhaps the most suitable material for the investigation of the amplification effect, for besides having a large piezoeffect (approximately ten times larger than quartz) they are also photosensitive, so that by varying the illumination of the crystal it is possible to vary the carrier density in a sufficiently wide range (by approximately 10^5–10^6 times for CdS). In the experiments, the amplification in CdS was investigated at frequencies up to 1000 MHz, corresponding to the region $ql \ll 1$. Therefore the expression for the growth increment is obtained from the dispersion equation (2.11) by substituting in the latter the longitudinal conductivity of the medium, defined by formula (3.3):[7,25,26]

$$\text{Im } q = -\frac{1}{2} \zeta_{\perp}^2 \frac{\omega_0^2 \tau}{v_{s\perp} \varepsilon_0} \frac{1 - \frac{v_d}{v_{s\perp}}}{\left(1 - \frac{v_d}{v_{s\perp}}\right)^2 + \frac{\omega_0^4 \tau^2}{\omega^2 \varepsilon_0^2}(1 + q^2 r_D^2)^2} ;$$ (4.1)

here $\zeta_{\perp}$ is the constant of electromechanical coupling for transverse waves, $v_{s\perp}$ is the velocity of the transverse waves, and r_D is the Debye radius. Consequently, when the electron drift velocity exceeds the sound velocity, the electron absorption gives way to amplification; if this amplification exceeds the lattice absorption (see formula (2.9)), then the transmitted elastic wave will grow.

The change of the phase velocity of the sound waves is determined in similar fashion

$$\frac{\Delta v_{s\perp}}{v_{s\perp}} = -\frac{1}{2} \zeta_{\perp}^2 \varepsilon_0 \frac{\omega_0^2 \tau^2 (1 + q^2 r_D^2) \frac{v_T^2}{v_{s\perp}^2} + \left(1 - \frac{v_d}{v_s}\right)^2}{\varepsilon_0^2 \left(1 - \frac{v_d}{v_{s\perp}}\right)^2 + \frac{\omega_0^4 \tau^2}{\omega^2}(1 + q^2 r_D^2)^2} .$$ (4.2)

For CdS crystals, the relative changes in the velocity can reach 1%.[92]

The experimental setup of Hutson et al.[7] for the observation of electronic amplification in CdS is shown in Fig. 4. The sound wave was fed through an acoustic guide of fused quartz to the CdS crystal, to which a constant electric field was applied with the aid of a quartz converter. The resultant signal was registered by a

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*The possibility of amplification of ultrasonic waves and semiconductors was pointed out even earlier (see, for example, [90,91], and also [8]).
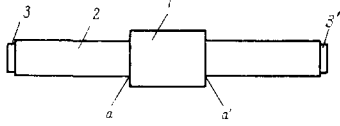
FIG. 4. Diagram of setup used in experiments on the amplification of ultrasound [7,93,98]. 1—Piezosemiconductor crystal, a, a'—electrodes for application of the drift field, 2—sound guide for fused quartz, 3, 3'—quartz converters for the radiation and reception of sound waves.

FIG. 6. Dependence of gain (damping) of transverse ultrasonic waves on the ratio of the drift velocity to the sound velocity [93]. Solid curve—theoretical, dashed—experimental (CdS crystal, length 8 mm, $\omega = 2\pi \times$ 16.5 MHc).

second quartz converter. The orientation of the crystal was chosen such that the transverse elastic wave was accompanied by a longitudinal electric field. The experimentally obtained dependence of the amplitude of the arriving sound pulse on the applied external field is shown in Fig. 5. The reversal of the sign of the electronic absorption occurred in a field of 700 V/cm, amounting to a mobility of 285 cm²/V-sec for a transverse sound wave (the Hall mobility in CdS is of the order of 300 cm²/V-sec[93]), i.e., in splendid agreement with the theory. As a result of the experiment with a crystal 7 mm long, a gain of 18 dB was obtained at a frequency of 15 MHz, and 38 dB at 45 MHz. The possibility of technical application of the amplification of ultrasound in piezosemiconductors has greatly stimulated the experimental investigations in this region. [93-111] The results show that the theory in the main describes the experiment quite well; Fig. 6, for example, shows for comparison two plots, one theoretical based on formula (4.1), and the other experimental.[93]*

The increment of the gain (4.1) as a function of the frequency reaches a maximum at $\omega = \omega_0 v_S/\sqrt{\epsilon_0} \, v_T$, i.e., at a Debye screening radius of the order of the wavelength.[25] The largest values of the gain increment (for all other optimal conditions) is $1/2\zeta_\perp^2 \omega_0/\sqrt{\epsilon_0} \, v_T$ $= 1/2\zeta_\perp^2 \omega/v_{S\perp}$. For CdS crystals, for example, $\zeta_\perp^2$ $= 0.018$, $v_{S\perp} \approx 1.8 \times 10^5$ cm/sec ($\zeta_\parallel^2 = 0.02$, $v_{S\parallel} \approx 4.3$ $\times 10^5$ cm/sec; see [38,39,112,113]), and therefore the maximum possible gain at 45 MHz amounts to about 130

dB/cm. Investigations have shown, however, that a serious obstacle to an extensive use of this effect are the appreciable losses to the conversion of the electromagnetic oscillations into acoustic oscillations, and in addition, the appreciable level of the acoustic noise resulting from the spontaneous generation of phonons.[7,93] Nonetheless, Chubachi, Wada, and Kikuchi,[109] using barrier-layer converters[114-117] (both for the conversion of electric oscillations into acoustic ones and as electrodes for the application of the constant drift field), obtained a net gain in a CdS crystal.*

Most piezosemiconductors, including cadmium sulfide, are characterized by very high carrier collision frequencies (at any rate at room temperature[118,119]) and therefore it is very difficult to obtain in the experiments the conditions $\Omega\tau > 1$ and $\Omega \sim \omega$, under which cyclotron resonance is possible.† Nonetheless, at helium temperatures, as shown by experiments on cyclotron resonance with electromagnetic waves,[118,119] the relaxation time τ in CdS turns out to be of the order of 10^{-11} sec, and therefore the condition $ql \gg 1$ (the free path in CdS under these conditions is of the order of 2×10^{-5} cm) is satisfied at frequencies $\omega \gg 10^{10}$ sec^{-1}, which is perfectly feasible in experiments with sound waves. We therefore present also the electronic part of the gain increment in the region $ql \gg 1$, when the conductivity is determined by formula (3.16):

$$\mathrm{Im}\, q = -\frac{1}{3}\zeta_\parallel^2 \frac{\varepsilon_0 \frac{\omega_0}{v_{s\parallel}}\left(1-\frac{v_d}{v_{s\parallel}}\right)(qr_D)^3}{\left(1-\frac{v_d}{v_{s\parallel}}\right)^2 + \frac{4}{\pi^2}\left(\frac{v_F}{v_{s\parallel}}\right)^2\left(1+\frac{\varepsilon_0}{3}q^2 r_D^2\right)^2}, \quad (4.3)$$

where $r_D = v_F/\omega_0$. Expression (4.3) has been derived for a longitudinal wave, and therefore $v_{s\parallel}$ is the velocity of the longitudinal wave. If the electron gas is not degenerate, then the gain increment is obtained from (4.3) by making the substitution $v_F \rightarrow \sqrt{3v_T}$. A characteristic feature of expression (4.3) is the fact that it does not depend on the relaxation time τ. The last term

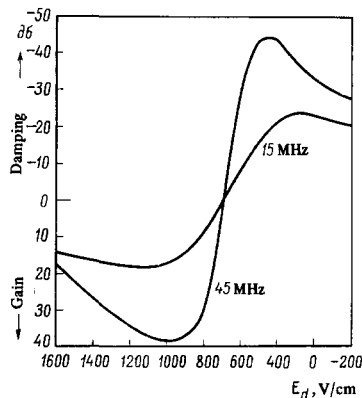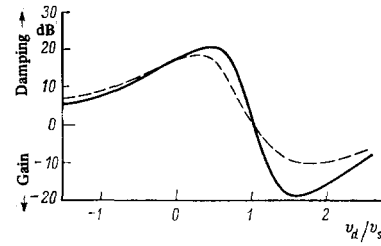
FIG. 5. Experimentally observed [7] dependence of the gain on the drift field (CdS crystal length 7 mm).

*In comparing theory with experiment, it is necessary to bear in mind the fact that in CdS crystals there is an appreciable number of "adhesion and capture" centers for carriers, and these greatly influence the kinetics of the electrons, and consequently influence the gain. If account is taken of the influence of these centers, then the agreement between theory and experiment is greatly improved [82,83,102].

*The temperature dependences of the gain and of the resistance of the sample, measured also in [109], from room temperature up to 380° C, have confirmed the prediction made in [39]—the damping of acoustic waves increases with increasing temperature (see also [92]). The gap energy obtained from these measurements for CdS (2.30 eV) agrees well with the optical data (2.36 eV).

†Cyclotron resonance in absorption of electromagnetic waves in CdS was realized at helium temperatures in [120,121]. The electromagnetic-wave frequency was about 10^{10}–10^{11} Hz. Since generation and reception of acoustic waves of such frequencies were also realized experimentally [122,123], cyclotron resonance in CdS at hypersonic frequencies is thus apparently feasible in principle.

in the denominator of (4.3) is always larger than the first, and therefore the dependence of the increment on the drift velocity represents an almost straight line: the gain increases linearly with increasing velocity of the electron stream.

We now compare the gain increments in piezosemiconductors in the regions $ql \ll 1$ and $ql \gg 1$. Since expressions (4.1) and (4.3) depend differently both on the frequency and on the drift velocity, such a comparison is best carried out at the maximum increments values attainable at a fixed frequency, with all other parameters (carrier density, drift velocity) already at their optimal values. Putting in (4.1) $v_d = v_s [1 + (2\omega_0^2\tau/\omega\epsilon_0)]$, which corresponds to the optimal values, and similarly in (4.3), we obtain for the ratio of the increments in the nondegenerate case

$$\gamma \equiv \frac{\max \operatorname{Im} q_{ql \ll 1}}{\max \operatorname{Im} q_{ql \gg 1}} \approx \frac{6\pi}{\sqrt{\epsilon_0}} \frac{v_T}{v_s} \frac{1}{\omega_0\tau}, \qquad (4.4)$$

where in the case of (4.3) it is assumed that $|1 - (v_d/v_s)| \sim 1$. For crystals of the CdS and CdSe type at room temperatures we have $v_T/v_s \sim 10^2$, $\tau \approx 10^{-13}\text{--}10^{-14}$ sec^{-1}, and $\omega_0 \approx 3 \times 10^5 \sqrt{n_0}$ sec^{-1}, so that for all reasonable values of the carrier densities we have $\gamma \gg 1$, and consequently crystals with $ql \ll 1$ are more effective for amplification. As already mentioned, the difference in gain between the regions $ql \ll 1$ and $ql \gg 1$ is connected with the difference between the amplification mechanisms: in the region $ql \ll 1$ the wave is amplified by the space charge drifting with supersonic velocity, and in the region $ql \gg 1$ it is amplified by individual "almost free" electrons, and in this sense this case corresponds more closely to the mechanism of two-stream instability in a gas-discharge plasma.[4,51-54]

It follows from (4.4) that the gain in the region $ql \ll 1$ may greatly increase with increasing collision frequency $\nu = \tau^{-1}$ compared with the gain in the region $ql \gg 1$. In this connection, notice should be taken of one feature of the amplification of sound waves in a magnetic field,[111,124] When no conditions for any resonances have been satisfied as yet. Whereas in the absence of a magnetic field the gain determined by (4.3) is small, with increasing magnetic field, when $\Omega\tau \gg 1$, the effective carrier collision frequency increases and it is possible to go over gradually into the region $ql_{\mathrm{eff}} \approx ql(\Omega\tau)^{-2} \ll 1$, where the gain increment is large. As shown in Sec. g, the longitudinal conductivity of the medium in the region where $\Omega\tau \gg 1$ and $ql \ll 1$ reduces to formula (3.3), but with the formal substitution $\nu \to \Omega^2/\nu$. This means that the gain increment will be determined by formula (4.1), so that we obtain in lieu of (4.4)

$$\gamma_H \approx \gamma(\Omega\tau)^2 \gg \gamma. \qquad (4.5)$$

At the same time, of course, the threshold value of the electric field in which the amplification takes place also increases (see also [125,126]).

Experimental investigations of acoustic waves in piezosemiconducting InSb crystals* have shown that in a strong magnetic field the gain is well described by

formula (4.1), with the substitutions $v_d \to cE/H$ and $\nu \to \Omega^2$.[111]

So far, no one has succeeded in observing experimentally geometrical and cyclotron resonances or quantum oscillations accompanying amplification in piezosemiconductors. However, the use of piezosemiconducting crystals as InSb, GaAs, Te, and high-efficiency film hypersound converters,[127-130] which operate also at low temperatures, down to helium temperatures,[127] gives grounds for hoping that these experiments will be realized. We do not present here the corresponding formulas for the gain and for the change in phase velocity, noting that they can be readily derived from expressions (2.11), (3.24), and (2.25).

The presence of impurity centers, especially in such piezosemiconductors as CdS and CdSe, greatly influences the character of the amplification of the ultrasonic waves in these crystals. To take them into account it is necessary to substitute in the dispersion equation (2.11) the value of the dielectric tensor obtained from (3.34). Of course, for a more complicated model with two or three types of impurity centers, the dielectric constant of the medium is a more complicated function. It is essential that the impurity centers lead, as a rule, to a decrease of the effective mobility of the carriers, and therefore the critical electric field in which amplification is possible increases sharply, and in some cases, especially at low frequencies, when $\omega < \nu_n, \nu_p$ (see (3.33) and (3.34)) amplification is practically impossible. This is physically connected with the fact that within a time on the order of the period of the wave the carrier has time to be captured by the impurity center or to recombine with the hole.

A comparison of the results of the experiments on amplification in CdS and CdSe with the simplest models of impurity centers, carried out in [99,102-104,109,116,131], has shown that allowance for the processes of adhesion or capture even within the framework of the simplest models with one type of impurity center, leads to better agreement between theory and experiment.*

We note that in comparing the theory of the amplification with the experimental results we have used throughout the model of an isotropic solid, although, on the other hand, such crystals as CdS, CdSe, and ZnO are elastically anisotropic. As shown recently by Y. Kikuchi et al.,[134] allowance for the anisotropy of the elastic and piezoelectric properties leads to the conclusion that the most favorable direction for the amplification of sound waves in CdS is not perpendicular to the C_6 axis, but at an approximate angle $\pi/6$.

i) Acoustoelectric Effect in Piezosemiconductors

In the acoustoelectric effect, when a wave passes

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*The piezoelectric constant for InSb was first measured in [127] and was found to be $|e_{14}| = 0.06 \pm 0.005$ C/m^2. Accordingly, the electromechanical coupling constant for longitudinal waves along the $\langle 111 \rangle$ direction is $\eta_\parallel^2 = 3.6 \times 10^{-4}$

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*In this connection we point out one circumstance. The presence of impurity centers for photosensitive CdS and CdSe centers sometimes leads to the phenomenon of temperature quenching of photoconductivity [84], and in a strong electric field, when the Joule heating is appreciable, it makes the electric field distribution and the sample carrier density inhomogeneous [132]. Under these conditions, the picture of the amplification already changes appreciably, since the initially homogeneous sample, owing to the presence of impurity centers, becomes inhomogeneous. Moreover, it can be shown [133] that under certain conditions this leads to the possibility of amplification of acoustic waves in the continuous regime [108].

through a crystal and drags electrons with it, part of the wave momentum is transferred to the electrons, so that a potential difference is produced at the terminals of an open-circuited semiconductor; if the terminals of the semiconductor are joined by an ordinary conductor, then an acoustoelectric current is produced in the circuit.* We shall consider this effect only in piezoelectric semiconductors, where it can reach appreciable magnitudes.

The expression for the acoustoelectric current in a piezosemiconductor can be obtained by calculating the average force exerted on the electrons by the sound wave. It is not at all essential to specify concretely the very state of the electron-hole plasma of the carriers, and it suffices to assume that the medium (lattice plus carriers) is described by a known dielectric tensor of the medium.[136] Such an approach has the advantage that it makes it possible to describe the phenomenon in many cases for which the form of the tensor $\epsilon_{||}(\omega, q)$ is known (see Ch. 3).

If an acoustic wave propagates through the crystal, then the piezoeffect gives rise to an electric field $E(r, t)$ and the force acting on the electron is

$$f(r, t) = -eE(r, t). \qquad (4.6)$$

Under the influence of the alternating force $f(r, t)$, the electron density in the conduction band will vary from point to point, and the average force acting on the electrons in a unit volume will be

$$\langle F(x, t) \rangle = \langle f(r, t) n(r, t) \rangle; \qquad (4.7)$$

here $n(r, t)$ is the electron density, and the angle brackets $\langle \ldots \rangle$ denote averaging over time and space in scales greatly exceeding the period and the wavelength, respectively. Using the fact that the electric field produced in the sound wave is potential and that $E(r, t) = -\nabla\varphi(r, t)$, where $\varphi(r, t)$ is a scalar potential, and using the relations

$$\varphi(\omega, q) = \frac{4\pi\beta_{i, kl} q_h}{q^2 \varepsilon_{||}(\omega, q)} u_l(\omega, q) \qquad (4.8)$$

and

$$n_{\sim}(\omega, q) = \frac{1}{4\pi e} q^2 \varphi(\omega, q) + 4\pi\beta_{i, kl} q_i q_h u_l(\omega, q), \qquad (4.9)$$

we obtain for the force $F(x, t)$, after simple transformation,[136]

$$\langle F \rangle = \zeta^2 S \left(\frac{q}{2v_s}\right) \text{Im} \left(\frac{\varepsilon_0}{\varepsilon_{||}(\omega, q)}\right), \qquad (4.10)$$

where $S = \rho v_s \omega^2 u_0^2 / 2$ is the acoustic-energy flux density and u_0 is the amplitude of the sound wave (in the general case it is a slow function of the coordinates and of the time). Obviously, the dc current flowing through a unit area of the short-circuited semiconductor will be

$$J^{ac} = \frac{e}{m} \tau \langle F \rangle = -\mu S \zeta^2 \frac{q}{2v_s} \text{Im} \left(\frac{\varepsilon_0}{\varepsilon_{||}(\omega, q)}\right), \qquad (4.11)$$

where μ is the mobility. Comparing (4.11) with the expression for the electron damping or amplification of the sound waves in piezosemiconductors (4.1), we see that Weinreich's relation[32] is satisfied independently

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*The idea of such an effect was apparently first advanced by Parmenter [135].

on the form of the tensor of the dielectric constant of the medium.*

If the semiconducting crystal is open-circuited, then the acoustic emf will obviously be

$$\varepsilon^{ac} = -\frac{1}{en_0} \int_0^l \langle F \rangle \, dx = -\frac{S_0}{en_0 v_s} \frac{\gamma_{el}}{\gamma_{el} + \alpha} (1 - e^{-(\gamma_{el} + \alpha)l}), \qquad (4.12)$$

where $\gamma_{el} = \zeta^2 q \text{ Im} (\varepsilon_0/\varepsilon_{||}(\omega, q))$ and α are respectively the plasma and lattice components of the sound-wave damping, and l is the crystal length. In the derivation of (4.12) we used the fact that $S(x) = S_0 \exp(-(\gamma_{el} + \alpha)x)$, where S_0 is the acoustic-energy flux at the point $x = 0$.

When a directed flux of carriers is present in the semiconductor, at a drift velocity exceeding the phase velocity of the sound wave, γ_{el} becomes negative and consequently the sign of the acoustic emf is reversed. It is important that such a change of the sign of the acoustic emf does not depend on the magnitude of the viscous absorption of the sound waves in the crystals, and from this point of view an experimental investigation of the acoustoelectric effect is preferable to an investigation of the gain, for it makes it possible, independently of the viscous absorption of the sound, to determine the threshold of amplification of the sound waves with sufficient accuracy.

An investigation of the dependence of the acoustic emf on the drift field in CdSe crystals has shown that the theory, in the main, describes the experiment well.[136] The acoustoelectric effect is presently under thorough experimental investigation by many authors,[32,33,136,138] and turns out to be an even effect in ZnS crystals.[141,142]

j) Amplification in Nonpiezoelectric Semiconductors

We now consider the amplification of elastic waves in semiconductors and semimetals in which there is no piezoeffect.† The interaction of the sound waves with the electrons can be realized in this case either via the deformation potential, or if the lattice is charged, via the Coulomb field accompanying the longitudinal wave in such a lattice.

Let us start the analysis with the simplest case, when there is one type of carrier and the interaction with the acoustic wave is via the deformation potential. The gain increment is then determined from expression (2.19), where $\sigma^{(h)}(\omega, q)$ must be set equal to zero. If $ql \ll 1$ and there is no magnetic field, then the electronic

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*Nonetheless, this still does not mean that the Weinreich relation has a universal character (see, for example [137]).

†The possibility of amplifying ultrasound in semiconductors and semimetals via direct interaction between the electrons and the phonons became immediately obvious following the experimental work of Esaki [143,144], who observed bending of the current-voltage characteristic in bismuth in crossed electric and magnetic fields. Although the generation of sound waves was not observed directly in Esaki's experiment [153], nevertheless the fact that the current oscillated at a frequency approximately equal to the natural frequency of the oscillations of the sample and occuring only when the Cerenkov condition $v_d > v_s$ was satisfied, it was possible to deduce that acoustic oscillations were indeed generated in the system [144]. The theoretical papers [9,25,66,74,75,146-148] that followed [143] indeed have demonstrated the possibility of amplification of acoustic waves by a supersonec carrier flux.

conductivity is determined by formula (3.3) and consequently[25]

$$\mathrm{Im}\, q = -\frac{2\pi\omega^2\Lambda^2\varepsilon_0}{\rho v_s^5 e^2}\; \frac{\sigma_0\left(1-\frac{v_d}{v_s}\right)}{\varepsilon_0^2\left(1-\frac{v_d}{v_s}\right)^2+\frac{\omega_0^4\tau^2}{\omega^2}(1+q^2r_D^2)^2}. \quad (4.13)$$

As in the case of a piezosemiconductor, the largest gain is reached when the drift velocity is

$$v_d = v_s\left(1\pm\frac{\omega_0^3\tau}{\omega\varepsilon_0}(1+q^2r_D^2)\right), \quad (4.14)$$

where the upper sign corresponds to the largest gain and the lower sign to the largest damping. At this value of the drift, expression (4.13) becomes

$$\mathrm{Im}\, q = \pm\frac{\Lambda^2\omega^3\varepsilon_0}{4e^2\rho v_s^6}\; \frac{1}{1+q^2r_D^2}. \quad (4.15)$$

In semiconductors and semimetals, the electron-phonon interaction constant is usually of the order of several electron volts, $\rho\approx 5$ g/cm^3, $v_S\approx 2\times10^5$ cm/sec, and $\varepsilon_0\approx 10$. At these values of the parameters, assuming $qr_D\approx 1$, we get from (4.15) that Im q ≈ 1 cm^{-1} at a frequency $\omega/2\pi\approx 5\times10^9$ Hz. If we compare the gain due to the deformation potential and the gain due to the piezoelectric effect, then the ratio of the increments, in the case when the drift satisfies condition (4.14), is

$$\frac{[\mathrm{el.\text{-}phon.}]}{[\mathrm{piezo}]} = \frac{\omega^2\Lambda^2\varepsilon_0}{\zeta_{\parallel}^2\rho v_s^4 e^2}.$$

Thus, the gain due to the electron phonon interaction is effective only at high frequencies. For example, when $\Lambda = 3\times10^{-11}$ erg, $v_S = 5\times10^5$ cm/sec, $\zeta_{\parallel}^2\approx 0.02$, $\rho\approx 5$ g/cm^3, and $\varepsilon_0\approx 10$, the gains in piezosemiconductors and impurity semiconductors or semimetals become comparable at a frequency $\omega\approx 3\times10^{11}$ sec^{-1}. At the same frequency, the largest value of the increment (4.15) is approximately 10^3 cm^{-1}, corresponding to a gain of 8.3×10^3 dB/cm.

At high frequencies, when $ql\gg 1$, the conductivity is determined by expression (3.16), and we get for the gain increment[64]

$$\mathrm{Im}\, q = -\frac{\omega^3\Lambda^2\varepsilon_0}{12e^2\rho v_s^3}\; \frac{(qr_D)^2\left(1-\frac{v_d}{v_s}\right)}{\left(1-\frac{v_d}{v_s}\right)^2+\frac{4}{\pi^2}\left(\frac{v_F}{v_s}\right)^2\left(1+\frac{\varepsilon_0}{3}q^2r_D^2\right)^2}. \quad (4.16)$$

Just as in piezosemiconductors (see (4.3)), expression (4.16) does not depend on the electron relaxation time τ.

Direct amplification of hypersonic waves in semiconductors was effected by Pomerantz[73] in n-Ge single crystals doped with arsenic. If we now attempt to compare the experimental results with the predictions of the theory, we must bear in mind the following very important circumstance. Expressions (4.13) and (4.16) describe amplification in simple semiconductors, where the energy of the electrons as a function of the quasi-momentum has only one simple minimum located at the center of the first Brillouin zone. Therefore the acoustic wave propagating in such a crystal will be accompanied by the appearance of space charge. A manifestation of this effect is the fact that at large carrier densities the increments (4.13) and (4.16) reach a maximum and then begin to decrease sharply in proportion to $1/n_0$.

The situation is entirely different in the so-called multivalley semiconductors and semimetals, where the

energy of the electron as a function of its quasimomentum has several minima located at different points of the Brillouin zone. The direction of propagation of the wave and the polarization vector can be chosen such that the acoustic wave is not accompanied by the appearance of space charge. The total compensation of the space charge will occur when the shift of the energy of the electrons in one group of minima is equal and opposite in sign to the change of the energy of the electrons in the other. Such a case is precisely realized in n-Ge when the transverse sound wave propagates in the ⟨100⟩ direction with the polarization along the ⟨010⟩ direction. The attenuation (meaning also the gain) of such a sound wave increases monotonically with increasing concentration,[32,33] and consequently, in spite of the weak interaction via the deformation potential, the gain can reach an appreciable value. The gain increment for the transverse wave, when total compensation of the space charge takes place, is equal to[33]

$$\mathrm{Im}\, q = -\frac{\omega_0^3\Lambda^2\omega}{36\pi\rho v_T^3 v_T^2}\; \frac{\omega\tau_R}{1+[1-(v_d/v_s)]^2\,\omega^2\tau_R^2}\left(1-\frac{v_d}{v_s}\right); \quad (4.17)$$

here Λ is the deformation potential for the transverse wave, τ_R is the effective relaxation time with allowance for the intervalley scattering ($\tau_R^{-1} = \tau_{iv}^{-1} + q^2D^2$, where τ_{iv} is the time of intervalley scattering), and D is the diffusion constant. The values for τ_R as functions of the carrier density n_0 and the temperature T are known for germanium from the acoustoelectric effect.[32,33]

Using (4.10), we can easily calculate that for germanium ($n_0 = 10^{16}$ cm^3, $\omega = 10^{11}$ sec, $\tau_R = 10^{-11}$ sec, T = 10° K, and $v_d = 3v_S$), the gain amounts to approximately 800 dB/cm. Nonetheless, experiment[32] yielded a gain of 20 dB/cm.* Such a discrepancy between theory and experiment is apparently connected with the fact that such current densities in the sample are accompanied by impact ionization processes, the electron plasma becomes strongly heated, and therefore the true values of the parameters in (4.17) are known. It is also possible that the space charge is not completely cancelled out in the wave, and this greatly decreases the gain increment.

Let us examine now amplification in intrinsic semiconductors and semimetals, where the dispersion equation for the distribution of the acoustic waves is of the form (2.18). For a low-frequency wave, the plasma part of the attenuation (gain), due to either electrons or holes, is determined by (2.20a). If there is no magnetic field, and there is only an electric field leading to supersonic motion of one of the types of carrier, say the electrons, then the amplification of the sound wave can take place only in exceptional cases, when the electronic part of the amplification exceeds the attenuation introduced by the holes. In order for amplification to take place in the

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*The experiment was performed in the following manner: a current pulse of the order of 20 A was applied to a germanium sample oriented in the ⟨100⟩ direction (sometimes in the ⟨110⟩ direction), with length 1 cm and approximate area 0.01 cm^2. The converters used were ferromagnetic films deposited on polished faces of the sample; in a constant magnetic field, owing to the magnetostriction effect, these films absorbed and emitted phonons at a frequency $\omega/2\pi = 9\times 10^9$ Hz. The experiment was performed at liquid-helium temperature. Besides amplification of the pulse traveling through the sample, the experiment revealed also generation of phonons (acoustic noise).

system both as a result of the electronic and as a result of the hole components of the plasma, it is necessary to apply crossed electric and magnetic fields in such a way that the electrons and holes drift in the propagation direction of the sound wave. In this case, the magnetic field should be strong and $\Omega\tau \gg 1$ both for electrons and holes. We shall assume further that the conditions for the quantization of the electron orbits by the magnetic field are not satisfied, i.e., $T \gg \hbar\Omega^{(e)}$, $\hbar\Omega^{(h)}$ (here, as everywhere, the indices e and h denote quantities pertaining to electrons or holes, respectively). If now $ql^{(e)}$ and $ql^{(h)} \ll 1$, then the electron and hole conductivities are determined by expression (3.3), where $v_d = cE_d \times H/H^2$, and the collision frequency ν should be taken to mean the effective collision frequency, which depends on the magnetic field:

$$\nu_{eff}^{(e)} = \Omega_{(e)}^2 \tau_{(e)},$$

$$\nu_{eff}^{(h)} = \Omega_{(h)}^2 \tau_{(h)}.$$

Substituting the electron and hole conductivities in (2.28), we obtain the plasma part of the damping (gain) of an ultrasonic wave propagating in an intrinsic semiconductor or semimetal:

$$\mathrm{Im}\, q = -\frac{\omega^2(\Lambda_{(e)} + \Lambda_{(h)})^2}{2e^2\rho v_s^3} \frac{\sigma_0^{(e)}\sigma_0^{(h)}}{\sigma_0^{(e)} + \sigma_0^{(h)}} \frac{1 - \frac{v_d}{v_s}}{\left(1 - \frac{v_d}{v_s}\right)^2 + \left[\frac{2q^2 P_F^2}{3\omega e^2 H^2} \frac{m_{(e)} m_{(h)} c^2}{(m_{(e)}\tau_{(h)} + m_{(h)}\tau_e)}\right]^2},$$

(4.18)

where P_F is the Fermi momentum, which is the same for electrons as for holes if the concentrations are equal. Expression (4.18) describes the amplification in the region of low frequencies where, besides the condition $ql \ll 1$, it is also necessary to satisfy the requirements $|4\pi\sigma_{\parallel}^{(e)}(\omega, q)| \gg \omega\epsilon_0$ and $|4\pi\sigma_{\parallel}^{(h)}(\omega, q)| \gg \omega\epsilon_0$. Formula (4.18) is valid for semiconductors and semimetals with a Fermi surface that does not differ too much from spherical, and therefore, naturally describes the amplification and attenuation of longitudinal waves only.

The amplification of sound waves in crossed electric and magnetic fields was investigated experimentally by Toxen and Tansal[149] (see also [145,150,151]) on bismuth single crystals. A transverse acoustic wave with frequency $\omega/2\pi = 15$ MHz propagated along the bisector axis, the electric field was applied in the binary direction, and the magnetic field changed in the bisector plane from the trigonal axis to the binary axis. The experimentally observed gain (approximately 14 dB/cm) occurred at a drift velocity $v_d = cE_d/H \gtrsim 1.4 \times 10^5$ cm/sec, which corresponds precisely to the velocity of the transverse waves in bismuth. In addition to the amplification of the transmitted sound wave, generation of "acoustic noise" was also observed, thus confirming the statement made by Esaki[143] that the bending of the current-voltage characteristic is connected with the generation of acoustic phonons in the system.[145,149] Unfortunately, it is difficult to carry out a more complete comparison of theory with experiment, for the following reasons. First, it is easy to show that the transverse wave propagating along the bisector axis in bismuth should not be accompanied by the appearance of space charge, and therefore formula (4.18) is not applicable

directly to this case. The situation here is almost analogous to that with germanium,[33] but is further complicated by the presence of a strong magnetic field and by the presence of two types of carriers. Second, the experiment itself (in the opinion of its authors) requires additional verification and more thorough and deep investigations.

Let us discuss now resonant phenomena that can occur in the amplification of sound waves in semiconductors and semimetals situated in a magnetic field.[67,74, 77,79]

If conditions (3.23) are satisfied, a geometrical resonance in the amplification of the waves is possible. Corresponding expressions for the gain increments can be obtained from (2.19) and (2.29), in which the values of the longitudinal conductivity of the medium (3.34) are substituted under conditions of geometrical resonance.

Cyclotron resonance[67,69] occurs when the conditions (3.26) are satisfied, and in this case the conductivity is expressed by formula (3.25). For example, for semiconductors and semimetals, when the interaction is via the deformation potential and there are two types of carriers with identical parameters, we obtain from (2.20) and (3.26)

$$\mathrm{Im}\, q = -\frac{\pi\omega\omega_0^2\Lambda^2}{12\rho e^2 v_s^2 v_F^3} \frac{(qr_D)^2!}{\left(1 + \frac{\varepsilon_0}{3}q^2 r_D^2\right)^2} \mathrm{Re\, cth}\left(\frac{\pi\lambda}{\Omega\tau}\right).$$ (4.19)

It is easy to see that this expression reverses sign when the Cerenkov-radiation condition is reversed, i.e., when $v_d > v_s$. As already noted above, resonance takes place when $n\omega' = \Omega$, where $\omega' = \omega[1 - (v_d/v_s)]$ is the wave frequency modified by the Doppler effect. This means, in particular, that the gain as a function of the drift field also changes in resonant manner.

The difficulties encountered in the experimental investigation of these resonances in the amplification regime are quite appreciable, and have not yet been overcome. As to such resonances in the attenuation of sound waves, they were investigated many times experimentally in various substances.[152,153] If the inequalities T $\ll \hbar\Omega < \epsilon_F^2$ are satisfied, then conditions are created for the observation of quantum oscillations in the amplification of sound waves.[72-79,154] The gain increment can be obtained in this case from formulas (2.12) and (2.29) by substituting in them the expression for the longitudinal conductivity of the medium under the Landau quantization conditions, i.e., (3.40). We shall not present these cumbersome formulas, and confine ourselves only to general remarks. If the electron drift exceeds the velocity of the sound wave, then in place of the oscillations in the damping there appear analogous oscillations in the amplification of the sound waves. In spite of the fact that the amplitude of the oscillations of the conductivity itself, i.e., Re $\sigma_{\parallel}(\omega, q)$ can reach appreciable magnitude, nonetheless the amplitude of the oscillations in the gain (or damping) is most frequently greatly reduced. This is connected with the fact that the attenuation (or gain of ultrasound in semiconductors and semimetals) is not directly proportional to the real part of the conductivity, and is determined by more complicated expressions (see (2.11), (2.19), (2.20), (2.29)). This in turn is explained by the fact that in semiconductors

and in semimetals with almost spherical Fermi surface (these are the only ones considered here) the sound wave is always accompanied by the appearance of space charge.* On the other hand, if the equal-energy surface is characterized by several minima, then the inter-minima transitions for transverse waves with definite polarization and along definite directions lead to cancellation of the space charge. In this case the growth increment (as was explained with a particular example, see (4.17)) may turn out to be a monotonic function of the carrier density. A rigorous analysis of this problem for the amplification of sound waves in the presence of a magnetic field and near the resonances has not yet been performed. However, if the wave is longitudinal, then regardless of the form of the equal-energy surface, it will be accompanied by a space charge, and consequently the formulas presented above qualitatively describe the amplification and attenuation of the longitudinal waves. This explains, in particular, the oscillations in the variation of the phase velocity of longitudinal sound waves in a quantizing magnetic field,[79,131] due to oscillations of the real part of the conductivity.

Like the classical resonances, quantum oscillations in the amplification of sound waves have not yet been experimentally investigated, but damping oscillations are presently under thorough study (see [155-160]).

If an appreciable intensification of the sound wave occurs in a semiconducting or semimetal crystal, then the increasing deformation field can greatly change the characteristics of the electrons, particularly their spectrum.[12] In a spatially-periodic quasistationary field of acoustic waves, the continuous spectrum of the electron energies breaks up into a number of alternating allowed and forbidden bands (we are speaking here of the energy of electron motion in the direction of wave propagation). At sufficiently large sound-wave amplitude, the produced forbidden bands may turn out to be impenetrable to the electrons, and this greatly influences the character of the behavior of the electron in electric and magnetic (especially quantizing[161]) fields. If the band is sufficiently narrow, such that the electron upon acquiring energy in the electron field, reaches the top of the band without colliding a single time, then when reaching the first forbidden band, the electron experiences Bragg reflection and begins to oscillate in the allowed band with a frequency $\omega \approx eE_d\lambda/\hbar$, where λ is the wavelength of the sound, which determines principally the width of the forbidden band.[12] Under such conditions the system should probably radiate electromagnetic waves intensively.

The main difficulty encountered in an experimental investigation of the amplification effects is the release of large thermal power in the amplification region (for example, in the experiment of Hutson, McFee, and White[7] the released power was approximately 100 W/cm³). Calculation, however, shows that it is possible to reduce greatly the power released in the volume of

the semiconductor in the case of amplification of flexural waves in a thin piezosemiconducting plate[162] (see also [168]) and in the case of amplification of unique surface waves in a layered system consisting of a thin piezoelectric layer and a layer of a high-mobility semiconductor[164] (see also [165]).

The amplification of acoustic waves in layered structure has not yet been realized experimentally, apparently owing to the technological difficulties in constructing the layered system.* At the same time, direct amplification of Rayleigh surface waves in CdS crystals has already been realized[167] (see also [168]).

5. GENERATION OF ACOUSTIC WAVES IN PIEZO-SEMICONDUCTORS

k) Kinetic Equation for Phonons

Besides amplification of an acoustic signal introduced into the semiconducting crystal from the outside, it is also possible to generate spontaneous oscillations (acoustic noise) under conditions when the carrier drift velocity exceeds the phase velocity of the corresponding wave. In the investigation of the qualitative picture of the amplification we have already encountered the generation phenomenon: the second term in formula (1.12) describes a noise flux that grows in space. We shall now consider this phenomenon in greater detail as applied to piezosemiconductors, where the effects of generation of acoustic waves become most strongly manifest.

To describe the effects of generation of acoustic waves in piezosemiconductors, it is obviously necessary to obtain an equation of the type (1.9), describing the evolution of the mean-square fluctuating amplitude of the acoustic noise for a non-equilibrium medium with carrier drift. The method of obtaining such a "kinetic equation" as applied to pure plasma problems is well known[169-170] and corresponds essentially to the derivation of the sign of the conservation of energy in a dispersive medium (see the book of Agranovich and Ginzburg[172]).

Let us consider again the equations of motion of the lattice (2.1) and Maxwell's equations (2.9) with accounts taken also of the spontaneous (random) sources: spontaneous oscillations of elastic stresses $\sigma_{ik}^{(S)}(\mathbf{r}, t)$ and spontaneous oscillations of the electric current $j_i^{(S)}(\mathbf{r}, t)$

$= \frac{1}{4\pi} \frac{\partial}{\partial t} D_i^{(S)}(\mathbf{r}, t)$:

$$\rho \frac{\partial^2 u_i}{\partial t^2} - \lambda_{iklm} \frac{\partial u_{lm}}{\partial r_k} - \mu_{iklm} \frac{\partial^2 u_{lm}}{\partial t \, \partial r_k} - \beta_{l,ik} \frac{\partial E_l}{\partial r_k} = \frac{\partial \sigma_{ik}^{(s)}}{\partial r_k}, \quad (5.1)$$

$$\varepsilon_0 \frac{\partial E_i}{\partial t} - 4\pi \int_{-\infty}^{t} dt' \int d^3r' \sigma_{ij}(\mathbf{r}-\mathbf{r}', t-t') E_j(\mathbf{r}', t') - 4\pi \beta_{i,kl} \frac{\partial u_{kl}}{\partial t} = \frac{\partial D_i^{(s)}}{\partial t}. \quad (5.2)$$

Going over to the wave-packet representation, i.e., separating two characteristic scales of variation of the properties of the system, namely a small-scale variation and a large-scale variation, we can show (for more details see [173]) that the system (5.1) and (5.2) for the spectral density of the phonon radiation energy

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$$\mathscr{E}^\alpha(\omega, q;\, r,\, t) = -\omega \frac{\partial}{\partial \omega} \operatorname{Re} \{L_{ij}(\omega,\, q)\, e_i^\alpha e_j^\alpha\}\, |\, u^\alpha(r,\, t;\, \omega,\, q)\, |^2, \quad (5.3)$$

where

$$L_{ij}(\omega,\, q) = -\rho\omega^2 \delta_{ij} + [\lambda_{iklj} - i\omega\mu_{iklj}]\, q_k q_l - \frac{4\pi\beta_{p,\,ip}\beta_{l,\,jm}q_l q_m q_p q_k}{q^2 \varepsilon_{\|}(\omega,\, q)} \quad (5.4)$$

and $u_i(\omega, q;\, r, t) = \sum_\alpha u^\alpha e_i^\alpha$ (e_i^α—unit vector of polariza-
tion), leads to the following "kinetic equation"

$$\frac{\partial}{\partial t}\, \mathscr{E}^\alpha + v_g \frac{\partial}{\partial r}\, \mathscr{E}^\alpha + 2\gamma^\alpha(\omega,\, q)\, \mathscr{E}^\alpha = Q^\alpha(\omega,\, q); \quad (5.5)$$

here v_g is the group velocity of the waves, and

$$\gamma^\alpha(\omega,\, q) = -\frac{\operatorname{Im} \{L_{ij}(\omega,\, q)\, e_i^\alpha e_j^\alpha\}}{\frac{\partial}{\partial \omega} \operatorname{Re} \{L_{ij}(\omega,\, q)\, e_i^\alpha e_j^\alpha\}} \quad (5.6)$$

is the growth increment of the waves with polarization α. The right side of (5.5) determines the spontaneous generation of phonons by a system at a finite temperature. To find the explicit form of $Q^\alpha(\omega, q)$ it is necessary to determine the correlation functions of the random inductions and random elastic stresses, which enter in the right sides of the system (5.1) and (5.2). As shown in [174] (see also [175]), for nonequilibrium medium with drift

$$Q^\alpha(\omega,\, q) \approx \omega\delta(\omega - qv^\alpha) \left\{ e_i^\alpha e_j^\alpha \xi_{ij} + \frac{2\omega}{\pi}\, \frac{T_e \gamma_{el}^\alpha(\omega,\, q)}{\omega - \mu(qE_d)} \right\} \quad (ql \ll 1),\, (5.7)$$

where

$$\xi_{ij} = \frac{T_l}{\pi\rho}(\mu_\perp q^2 \delta_{ij} + (\mu_\| - \mu_\perp)\, q_i q_j),\quad \gamma_{el}^\alpha(\omega,\, q)$$

$\gamma_{el}^\alpha(\omega, q)$ is the electronic part of the gain increment, T_e and T_l are the temperatures of the electrons and of the lattice respectively, and $\mu_\|$ and $\mu_\perp$ are the components of the viscosity tensor in an elastically-isotropic medium, describing respectively the damping of the longitudinal and transverse waves.

Let us consider now the simplest solutions of (5.5). In the absence of electron drift ($E_d = 0$), the medium is in equilibrium and the dependence on the coordinates and on the time disappears. Then the spectral density of the radiation energy is

$$\mathscr{E}(\omega) = \sum_\alpha \int \mathscr{E}^\alpha(\omega,\, q)\, d^3q = \frac{\omega^2}{2\pi} \left(\frac{1}{v_\|^3} + \frac{2}{v_\perp^3} \right) \left(\frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\frac{\hbar\omega}{T}} - 1} \right), \quad (5.8)$$

i.e., it is determined by the Planck radiation in thermodynamic equilibrium, regardless of the form of the tensor $\varepsilon_{ij}(\omega, q)$. In the stationary case, when the electron drift velocity is directed, say, along x, a solution for the spectral density of the radiation energy, which increases in space, integrated over the wave vectors q, is:

$$\mathscr{E}_\omega = \frac{\omega^2}{4\pi v_\|^3} \int_0^{\pi/2} \frac{d\theta \sin\theta}{2\pi} \left(1 - e^{-\frac{2\gamma^{(\|)}(\theta)}{\cos\theta}\, x} \right)$$

$$\times \left(\frac{\omega}{v_\| \gamma^{(\|)}(\theta)} \right) \left(\frac{\omega\mu_\|}{2\rho v_\|^3}\, T_l + T_e \frac{\gamma_{el}^{(\|)}(\theta)}{\omega - \mu q E_d \cos\theta} \right) + \frac{\omega^2}{4\pi v_\perp^3} \int_0^{\pi/2} \frac{d\theta \sin\theta}{2\pi} \left(1 - e^{-\frac{2\gamma^{(\perp)}(\theta)}{\cos\theta}\, x} \right)$$

$$\times \left(\frac{\omega}{v_\perp \gamma^{(\perp)}(\theta)} \right) \left(\frac{\omega\mu_\perp}{2\rho v_\perp^2}\, T_l + T_e \frac{\gamma_{el}^{(\perp)}(\theta)}{\omega - \mu q E_d \cos\theta} \right)$$

$$+ \int \mathscr{E}(x = 0)\exp\left(-\frac{2\gamma^{(\|)}}{\cos\theta}\, x \right)\, dq, \quad (5.9)$$

where $v_\|$ and $v_\perp$ are the velocities of the longitudinal and transverse acoustic waves respectively, θ is the

angle between the vectors q and E_d, and $\gamma^{(\|)}$ and $\gamma^{(\perp)}$ are the damping decrements per unit length for the longitudinal and transverse waves. In the derivation of (5.9) it was assumed for simplicity that only $\beta_{x,xx} \neq 0$, and the remaining $\beta_{i,kl} = 0$. The integration over the wave vector q was carried out with the aid of the theory of residues, and the poles of $\varepsilon_\|^{-1}(\omega, q)$ were disregarded since they usually correspond to strongly attenuating solutions.* The last term in (5.9) corresponds to the radiation of waves "from the boundary." For a free boundary, on which the surface forces are equal to zero and the displacement vector is specified, this term will equal

$$2\rho\omega^2\, |\, u_x(x = 0)\, |^2\, e^{-\gamma^{(\|)}(\theta = 0)\, x}, \quad (5.10)$$

which corresponds to amplification of the longitudinal wave "from the boundary."

The kinetic equations for the phonons make it also possible to determine the time of establishment of the noise in the piezosemiconductor when the drift is "turned on." If the initial spectral density of the noise at t = 0 has the Planck distribution (5.8), then the solution of (5.5) is given by

$$\mathscr{E}^\alpha(r,\, t;\, \omega,\, q) = \begin{cases} \dfrac{Q^\alpha(\omega,\, q)}{\gamma^\alpha(\omega,\, q)}(1 - e^{\gamma^\alpha(\omega,\, q)\, t}) + \mathscr{E}_0 e^{\gamma^\alpha(\omega,\, q)\, t}, & t < t_0 = \dfrac{x}{v_q \cos\theta}, \\[2ex] e^{-\gamma^\alpha_l t_0}(t - t_0) + \dfrac{Q^\alpha(\omega,\, q)}{\gamma^\alpha(\omega,\, q)}(1 - e^{-\gamma^\alpha(\omega,\, q)\, t_0}), & t > t_0, \end{cases} \quad (5.11)$$

where f is the limiting value of the energy density at x = 0 (see formula (5.10)). We see that the characteristic time of establishment of the stationary solution (5.11) is determined by the time of travel of the phonons from the boundary x = 0 to the point under consideration.

1) Spectral and Angular Distribution of Generated Noise

Expression (5.9) describes the spectral density of the generated noise in a crystal of length x. We see that the first term in (5.9) describes the radiation of longitudinal waves and the second the radiation of transverse waves. Substituting in (5.9) the value of the dielectric tensor (3.3) for the case $ql \ll 1$, we obtain the explicit form of the expression for the spectral density of the noise, which will be investigated below. On the other hand the case $ql \gg 1$, and also effects in a magnetic field, can be considered analogously.

It is seen from (5.9) that the directivity pattern of the phonon emission is a rather complicated picture, which is quite difficult to obtain even for the simplest case when only $\beta_{x,xx} \neq 0$ (of course, without resorting to numerical methods). We shall therefore consider below only the case when $|\gamma(\theta)x/\cos\theta| \gg 1$, and then the decisive factor in the angular dependence of the generation is the behavior of the exponential in formula (5.9). The opposite case, when the exponential can be expanded in a series, is quite simple to analyze, but less interesting, since it corresponds to weak acoustic-noise radiation energy fluxes.

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*In crossed electric and magnetic fields acoustoplasma waves may appear [43], and in this case it is necessary to take into account the poles of $\varepsilon_\|^{-1}(\omega, q)$.

Let us consider first only longitudinal waves, when $\beta_{x,xx} \neq 0$, and the remaining components of the piezotensor are equal to zero. For crystals of the type CdS and CdSe, the symmetry of which is C_{6v}, this corresponds to the case when the electron drift velocity is directed along the C_6 axis. Then from the expression

$$\gamma^{(II)}(\theta) = \frac{\omega}{2v_{\parallel}}\left(\frac{\omega\mu_{\parallel}}{\rho v_{\parallel}^2} + \zeta_{\parallel}^* \cos^6\theta \operatorname{Im}\left(\varepsilon_0\varepsilon_{\parallel}^{-1}\left(\omega, \frac{\omega}{v_{\parallel}}, \cos\theta\right)\right)\right), \quad (5.12)$$

substituting formula (3.3) for $\epsilon_{\parallel}(\omega, \mathbf{q})$ we see that generation is possible only inside the cone $\theta < \theta_0$, where θ_0 is the limiting angle. An investigation shows[173] that the limiting angle first increases with increasing drift velocity of the carriers, and reaches a maximum, and then begins to decrease, approaching zero, and consequently at a certain drift velocity the generation stops. We emphasize that we are referring here to radiation at a certain fixed frequency, and therefore the vanishing of the generation at a considered frequency still does not mean that there is no generation at other frequencies.

Let us consider now the directivity pattern of the emission of transverse waves (as before, only $\beta_{x,xx} \neq 0$), for which the increment is given by

$$\gamma^{(\perp)}(\theta) = \frac{\omega}{2v_{\perp}}\left(\frac{\omega\mu_{\perp}}{\rho v_{\perp}^2} + \zeta_{\parallel}^* \frac{v_{\parallel}^2}{v_{\perp}^2} \sin^2\theta \cos^4\theta \operatorname{Im}\left(\frac{\varepsilon_0}{\varepsilon_{\parallel}\left(\omega, \frac{\omega}{v_{\perp}}, \cos\theta\right)}\right)\right). \quad (5.13)$$

It is seen from (5.13) that the generation begins ($\gamma^{(\perp)}(\theta) \leq 0$) at a carrier drift velocity slightly exceeding the velocity of the transverse waves, and the radiation directivity patterns constitute two lobes. These lobes first become broader with increasing drifts, and then narrow down to a certain limiting angle, not equal to the initial one, after which the generation vanishes. As in the case of longitudinal waves, this behavior of the generation cone for transverse waves is connected with the decrease of the electronic growth increment with increasing drift velocity.

Let us consider now the case when $\beta_{x,xz}$ (or $\beta_{x,xy}$) differs from zero, and all the remaining $\beta_{i,kl} = 0$. For crystals of the type CdS, CdSe, or ZnO this corresponds to a drift velocity direction perpendicular to the C_6 axis. It is immediately evident that the symmetry of the radiation disappears in this case, and an additional azimuthal dependence appears, thus greatly complicating the directivity pattern of the radiation. As to the longitudinal waves, the directivity pattern of the radiation at a fixed azimuthal angle φ corresponds to the case of emission of transverse waves at $\beta_{x,xx} \neq 0$.

Let us stop now to discuss the behavior of the spectral density of the radiation energy as a function of the carrier drift velocity. It is clear that the character of this dependence is determined by two factors: first, by the change of the directivity pattern of the radiation, and second, by the dependence of the increment itself on the drift velocity. Since the directivity patterns of the emission of the longitudinal and transverse waves are different, the character of the $\mathscr{E}_{\omega}(E_d)$ dependence will also be different for the longitudinal and transverse waves. If the crystal is not too long, so that $|\gamma(\theta)x/\cos\theta| < 1$, then the exponential in (5.9) can be expanded in a series, and then, neglecting for simplicity the viscous absorption, we obtain for the transverse waves

$$\mathscr{E}_{\omega} = \frac{T_e\omega^2 x\sigma_0\zeta_{\parallel}^2 v_{\parallel}^2}{\pi\varepsilon_0 v_{\perp}^6}\int_0^{\theta_0}\frac{\sin^3\theta\cos^3\theta\,d\theta}{\left(1 - \frac{v_d}{v_{\perp}}\cos\theta\right)^2 + \frac{\omega_0^4}{\omega^2 v^2}(1+q_{\perp}^2 r_D^2)^2}, \quad (5.14)$$

where θ_0 is the limiting angle determining the generation cone, $\cos\theta_0 = v_{\nu}/v_d$. It is seen from (5.14) that $\mathscr{E}_{\omega}$ as a function of the drift first increases, principally as a result of the broadening of the integration limits, and then decreases when the first term in the denominator of (5.14) exceeds the last term. The spectrum of the generated frequencies then first increases (as $\omega \to 0$) in proportion to ω^4, and near

$$qr_D \ll 1, \quad \omega_0^4/\omega^2 v^2 \ll |(1 - \frac{v_d}{v_{\perp}}\cos\theta)|$$

it increases in proportion to the square of the frequency, becoming practically independent of the frequency when $qr_D \gg 1$. However, at a certain frequency, determined by the viscous absorption, the generation ceases. Here, unlike the theory of amplification of monochromatic signals, the maximum radiation intensity is no longer determined by the condition $qr_D \sim 1$.

Let us consider now the case of crystals with large lengths x, when $|\gamma(\theta)x/\cos\theta| \gg 1$ and the exponential of (5.9) cannot be expanded in a series. In the general case the behavior of $\mathscr{E}_{\omega}(n_0, T_e, \omega, E_d)$ is rather complicated, so that we shall consider only the case $\theta = 0$, which corresponds to an experiment in which the noise is registered by some converter which receives only normally incident waves. Apparently, this was precisely the case in the experiment of [176]. In the absence of viscous absorption we have for transverse waves

$$\mathscr{E}_{\omega}^{(\perp)} \approx \frac{T_e\omega^2}{4\pi v_{\perp}^3}\frac{\exp(-\gamma^{(\perp)}(\theta=0)x)}{\left(\frac{v_d}{v_{\perp}}-1\right)} \quad (v_d > v_{\perp}) \quad (5.15)$$

and analogously for longitudinal waves at $\beta_{x,xx} \neq 0$. It follows from (5.15) that $\mathscr{E}_{\omega}$ as a function of the frequency first increases like $\omega^2 \exp(\alpha\omega^2)$, and then, when $\gamma^{(\perp)}(\omega)$ begins to decrease and the viscous damping begins to increase, a transition from the dependence (5.16) to formula (5.14) takes place.

A direct experimental study of the spectral and angular distributions of the noise was made for CdS and ZnO crystals by Zemon and co-workers[177-180] (see also [181-183]) using methods of Brillouin scattering of light.[194]

When the theory is compared with the experimental results, it is necessary to bear in mind a very important circumstance. As a rule, the results of measurements of the spectral and angular distributions of the generated acoustic noise are made in the acoustoelectric domain, where the inhomogeneity in the distribution of the electric field is large. Since, as shown above, the spectral composition of the radiation is quite sensitive to variations of the electric field, it is necessary to know, when comparing the results of the experiments with the theory, the true value of the distribution of the electric field in the domain. This distribution must be substituted in formulas (5.9), (5.14), and (5.15), and the comparison must be made with allowance for this circumstance. Unfortunately, no such thorough analysis has been performed to date. Nonetheless, it should be noted that both experiment and theory show that the maximum intensity of radiation of acoustic phonons does not occur at the frequency $\omega \approx v_s/r_D$ as in the case of amplification of a wave with a stationary phase.

m) Effects Connected with Generation of Acoustic Noise

1) Current saturation. An investigation of the processes of amplification of acoustic waves in piezosemiconductors and semimetals has shown that an important role is played by various effects connected with the generation of acoustic noise,[94,95,105-107,185] principal among which is the sharp change of the current under conditions of acoustic instability.[185,186]* Figure 7a shows a typical current-voltage characteristic of piezoelectric semiconductors, obtained from experiments on CdS and ZnO.[195,196] The sharp break in the characteristic occurs at an electric field value exceeding the threshold value corresponding to generation of phonons in the sample. A similar break was observed in the semimetal bismuth[143,144] in the presence of a strong $(\Omega\tau > 1)$ magnetic field (Fig. 7b). The threshold value of the field in bismuth also corresponds to the condition of the appearance of acoustic instability.[143] These phenomena were qualitatively explained by Hutson[186] (see also [187-189]), who has shown that the break in the current-voltage characteristic is due to the acoustoelectric current produced by sound waves that generate drift carriers. In piezosemiconductors in which there is no magnetic field, the resultant acoustoelectric current is opposite in direction to the ohmic current, and therefore the current-voltage characteristic bends downward (Fig. 7a). In bismuth,[143,144] where the electrons and holes drift in a third direction in crossed electric and magnetic fields, the acoustoelectric currents due to both the electrons and the holes are directed along the ohmic current, adding up to the latter, so that the current-voltage characteristics bend upward (Fig. 7b).

It is possible to obtain an expression for the current in piezosemiconductors under conditions of acoustic instability by averaging the exact expression

$$\langle j(r, t)\rangle = e\langle n(r, t) v(r, t)\rangle = en_0(x) v_d(x) + e\langle n_\sim(r, t) v_\sim(r, t)\rangle, \quad (5.16)$$

where $n_\sim(r, t)$ and $v_\sim(r, t)$ are rapidly oscillating quantities connected with the acoustic noise. By determining further, from the hydrodynamic equations for the electrons, the connection between the drift field $E_d(x)$ and the hydrodynamic velocity $v_d(x)$, and in addition expressing the average $\langle n_\sim(r, t) v_\sim(r, t)\rangle$ in terms of the intensity of the growing fluctuations, it is readily shown that

$$j(V) = \frac{\sigma_0 V}{L} + \frac{\mu}{L}\int_0^L F(x)\,dx; \quad (5.17)$$

here V is the potential difference across a sample of length L, and

$$F(x, t) = \sum_\alpha \int \frac{d\omega\, d^3q}{(2\pi)^4} \frac{q}{\omega} \gamma_{el}^\alpha(\omega, q)\, \varepsilon^\alpha(\omega, q; r, t) \quad (5.18)$$

is the density of the acoustoelectric force exerted on the electrons by the growing noise. Under generation conditions $\gamma_{el} < 0$, and therefore $F < 0$. This means that under conditions of acoustic instability there will be subtracted from the ohmic current $\sigma_0 V/L$ the acoustoelectric current, which is described by the last term of formula (5.17). Since the threshold of generation of

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*In piezosemiconductors this phenomenon was first observed by Smith [185]; the same effect was first observed by Esaki in a strong magnetic field in bismuth [143].
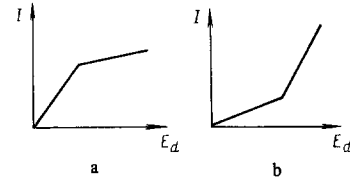

FIG. 7

acoustic phonons in the crystal coincides with the velocity of the transverse waves, the deviations from the ohmic value of the current should always begin in a field corresponding to generation of transverse acoustic waves. This fact indeed takes place in a large number of experiments.[93,106,185,190-193] At small growth increments, when $|\gamma_L| < 1$, it follows from (5.14) that the acoustoelectric current subtracted from the ohmic increases in proportion to the length of the crystal L; this circumstance was observed in CdS by Japanese investigators.[193]

Experiments aimed at observing the saturation of the current are usually carried out in a pulsed regime, by applying to the sample a field for a time of the order of several dozen microseconds. Assuming that the phonon travel time through the crystal is much shorter than the time of establishment of the field, and consequently of the drift velocity, it is easy to determine the time evolution of a current pulse passing through the sample if the initial applied voltage pulse is rectangular. Substituting in (5.17) and (5.18) the nonstationary solution (5.11), we obtain respectively for the instants of "switching on" and "switching off"

$$j(t) = \sigma_0 \frac{V}{L}$$
$$+ \mu \int \frac{d\omega\, d^3q}{(2\pi)^4} \frac{\mathscr{F}(\omega, q)}{\gamma(\omega, q)} \left\{ \frac{tv_{gx}^{(\perp)}}{L} + (1 - e^{\gamma(\omega, q)t})\left(1 - \frac{tv_{gx}^{(\perp)}}{L} - \frac{v_{gx}^{(\perp)}}{L\gamma(\omega, q)}\right)\right\},$$
$$0 \leqslant t \leqslant \frac{L}{v_g^*}, \quad (5.19)$$

$$j(t) = \mu \int \frac{d\omega\, d^3q}{(2\pi)^4} \frac{\mathscr{F}_0(\omega, q)}{\gamma(\omega, q)} e^{-\gamma_0(\omega, q)t}\left\{1 - \frac{tv_{gx}^{(\perp)}}{L}\right.$$
$$\left. + \frac{v_{gx}^{(\perp)}}{L\gamma(\omega, q)}\left(e^{\frac{\gamma(\omega, q)}{v_{gx}^{(\perp)}}(l - tv_{gx}^{(\perp)})} - 1\right)\right\}, \quad t > t_1 > \frac{L}{v_{gx}^{(\perp)}},$$

where

$$\mathscr{F}(\omega, q) = Q^{(\perp)}(\omega, q)\gamma_{эл}^{(\perp)}(\omega, q),$$

$\mathscr{F}_0$ and γ_0 are the values of $\mathscr{F}(\omega, q)$ and $\gamma(\omega, q)$ at zero electron drift velocity, and $t = t_1$ is the instant when the field is turned on. Thus, the current in the circuit differs from zero in the interval $t_1 < t < t_1 + L/v_{gx}^{(\perp)}$, and this is the current due to the "echo" of the acoustic phonons in the crystal. The "echo" effect was first observed experimentally by Maines and Paige[194] in sincle-crystal CdS.

2) Occurrence of inhomogeneity. The acoustic noise growing in the crystal makes it impossible for the distribution of the electric field and for the concentration in the crystal to remain homogeneous, and these two quantities must then depend on the coordinates (and also on the time in the nonstationary case). Nonetheless, in the analysis of all the effects it was tacitly assumed that the system remains homogeneous. This means that the formulas obtained above pertain only to small regions, much smaller than the scales of the inhomogeneities.

On the other hand, since the scale of the inhomogeneity is as a rule much larger than the characteristic length of the amplified or generated acoustic signal, it can be assumed that these formulas pertain also to an inhomogeneous medium whose properties vary slowly. To determine such a quantity as, say, the gain increment it is already necessary to use formulas of the geometrical-optics type:

$$A(L) = A_0 \exp \int_0^L \mathrm{Im}\, q(x)\, dx, \tag{5.20}$$

where A_0 is the amplitude of the sound signal at the boundary $x = 0$. In particular, owing to the redistribution of the electric field in the crystal, the Cerenkov condition may already turn out to be satisfied not in the entire crystal. Indeed, from the equation for the electric field

$$E_d(x) = \frac{V}{L} + \frac{1}{en_0}\frac{1}{L}\int_0^L F(x)\, dx - \frac{1}{en_0} F(x) \tag{5.21}$$

it follows that at the start of the crystal, at small values of x, the value of the electric field (and with it the values of the drift velocity of the electrons) is much smaller than the value averaged over the sample; to the contrary, in the other end of the crystal, at $x \approx 4$, the electric field is stronger. Since an increase of the electric field increases the acoustic-phonon density, causing the field to increase, the system as a whole, as shown by experiment, tends to an inhomogeneous distribution of the electric field and of the phonon density, with a maximum that lies as a rule near the anode part of an n-type crystal.* The appearance of such an inhomogeneity in the crystal changes the character of the amplification of the acoustic waves,[198-200] but it affects most strongly the amplification of the longitudinal acoustic waves, whose velocity is larger than that of the transverse waves, so that it is necessary to have a stronger electric field for their amplification. On the other hand, the threshold of noise production, and consequently of inhomogeneity production, regardless of the crystal orientation, always corresponds to the velocity of the transverse acoustic wave; therefore, the inhomogeneity occurs before amplification of the longitudinal waves becomes possible. Thus, the character of the behavior of the amplification of the longitudinal and transverse waves should be different in the experiments.

3) Current oscillations. Acoustic domains. Experiments with crystals in the regime when acoustic waves are generated have shown that in some cases the saturation of the current does not occur immediately, but only after several oscillations in time. Sometimes the system goes over in general to the regime of continuous oscillations. The first to report such oscillations in CdS crystals was Smith,[185] after which these oscillations were observed in various compounds.[106,191,195,201-207] A rough estimate of the period of the oscillations has

shown that in some experiments[106] it is equal to approximately double the time of passage of the amplified sound wave through the crystal, in others[185,206,207] it is simply equal to the time of passage of the sound waves through the crystal, and in still others[201] it is apparently not connected in a simple manner with the time of passage of the sound waves through the crystal, but is determined principally by the volume properties of the sample, particularly by the distribution of the carrier density and of the field. The proposed explanation of the oscillations of the first type, according to McFee, is that when the drift field is suddenly turned on, piezoelectric sound waves are excited on the end faces of the crystal. The waves propagating in the drift direction will be amplified, and by the instant of arrival to the anode end of the crystal, their amplitude reaches the maximum value. The acoustoelectric field connected with these waves is also maximal at the instant of arrival of a packet of the wave at the crystal anode, and the current through the crystal is minimal. Some of the acoustic waves are then reflected, and the process can continue until the ultrasonic flux is displaced back and forth over the crystal. In order for the oscillations to become continuous, the total gain after passage of the sound wave in the forward direction must be sufficiently large to offset all the losses in the crystal. If this condition is not satisfied, the oscillations are damped, and their amplitude should depend on the intensity of the acoustic flux generated during the initial voltage jump. It is obvious that with decreasing growth time of the crystal voltage, the amplitude of the oscillation should increase. McFee confirmed this experimentally. In addition, he observed noise pulses of sound, due to the non-reflected part of the acoustic flux. As expected, these pulses coincide in time with the current minima. If the piezosemiconductor crystal is now placed in an oscillating circuit (just as piezoelectric quartz is connected for frequency stabilization) and a drift field is applied to it, then electric oscillations should be produced in the circuit. Experiments with CdS reveal such a generation.[208,209]

The explanation of the oscillations of the second type, with a period approximately coinciding with the time of travel of the sound through the sample, is based on the idea of the occurrence of acoustic domains, or regions of strong fields and relatively large acoustic-wave density, moving in the crystal. The energy density fluctuations of the acoustic noise lead to local changes in the acoustoelectric force and in the space charge associated with it. If the total voltage on the crystal is maintained constant, then on regions with low noise intensity the voltage will be smaller, and this will lead to a corresponding decrease of the drift belocity of the electrons. The decrease of the drift velocity of the electrons leads to a decrease of the intensity of generation of phonons in the non-domain regions and to an opposite effect in the domain itself. In final analysis, this leads to a still larger increase of the phonon fluctuations in the region of the domain,[210] and thus the system may turn out to be unstable. Direct probe measurements under generation conditions[211-214] have confirmed this model. Unfortunately, there is still no more complete and rigorous theory of these phenomena (see [215,216]).

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*The situation in crystals that are initially inhomogeneous is much more complicated (see [196]).

6. CONCLUSION

Among the questions that should have also been discussed in the review are various nonlinear effects, which occur both in the amplification of monochromatic acoustic signals and in the generation of spontaneous noise oscillations. This is a very interesting branch of acoustoelectronics, just beginning to be developed (and is applicable primarily to piezosemiconductors, where nonlinear effects are large); we confine ourselves only to references to original papers in which these questions are considered.[217-224] Nonlinearities can be of two types: electronic and elastic, connected with the nonlinear character of the equations of elasticity theory. It is easy to see[217,218] that for piezosemiconductors the electronic nonlinearity plays a decisive role, whereas the anharmonicity becomes appreciable at much larger lattice deformations. Electronic nonlinearity should give rise, upon amplification of a sound wave with frequency ω, to higher harmonics 2ω, 3ω, etc.; these phenomena were already observed in experiment.[219,220]

The amplification of sound waves by streams of charged particles, which we considered above, is one part of a more general problem of the interaction of electron streams with waves in solids. Recently, a relatively large number of papers have been devoted also to these problems, but since their analysis outside the scope of our paper, we shall mention only the original papers where these problems are considered. We note that sound waves can also be generated and amplified with the aid of streams of charged particles passing over a surface or in a gap of a piezoelectric dielectric; the theoretical analysis of these questions is the subject of [224,227]. Experimentally, this phenomenon was first observed by Jamanouchi and Shibayama.[228]

Besides acoustic waves, there can exist in solids also spin waves,[229] waves corresponding to optical lattice vibrations,[57] and also waves corresponding to different types of "plasma" excitations of an electron-hole plasma.[6] The possibility of amplifying spin waves by electron beams has been considered in [230], and the amplification of optical oscillations in solids have been the subject of [231,232]. Spin and optical oscillations are characterized by relatively larger velocity of excitation propagation than sound waves, and it is still not clear how to produce experimentally the conditions necessary for the increase of the velocity of charged-particle streams in solids. As to the buildup of plasma excitations in solids by means of electron streams, there are already many experimental investigations besides the theoretical ones (see the review [6]).

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