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PLASMA RADIATION MECHANISMS IN ASTROPHYSICS

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INTRODUCTION

THE main problem of astrophysics is the analysis of the emission spectra of celestial bodies. The development of this problem as an independent discipline, and its further development, were therefore very closely connected with progress in atomic physics, without which the interpretation of the optical spectra, and at times also the radio spectra, is inconceivable. The term "astrophysics based on atomic theory" was at one time very widely used.

Recently, however, especially in connection with the vigorous development of radioastronomy, a new branch of physics, namely plasma research, has been of increasing importance in astrophysics. One can even speak of "astrophysics based on plasma theory." There is every reason for expecting certain problems in plasma physics to find more ready application in astrophysics than in terrestrial laboratories. In fact, cosmic plasma can be regarded as: more "pure" (smaller influence of the admixture of neutral atoms), more homogeneous (the spatial inhomogeneity scales are larger by many orders of magnitude than the Larmor radii and wavelengths), more varied in its properties (e.g., it frequently contains admixtures of epithermal and even relativistic particles), etc., than the "terrestrial" plasma. All this means that many problems in plasma physics, which are under terrestrial conditions only of certain theoretical interest, can find a concrete application in astrophysics. In this connection we wish to draw a certain historical analogy. Many atomic phenomena were first observed under cosmic conditions, and their study was continued in terrestrial laboratories only later, and not in all cases at that (the Fraunhofer spectrum, forbidden transitions of "coronium" and "nebulium").

From the theoretical point of view, likewise, certain problems of astrophysics preceded the "terrestrial" problems (the ionization-equilibrium equation, the radiation-transport equation). The same pertains to a considerable degree to plasma physics. In spite of the important role of plasma in astrophysics, many possibilities afforded by modern plasma theory are hardly ever used by astrophysicists. This is probably connected with the fact that the astrophysicists are "scared" by the complicated relations of the theoretical papers on plasma, and frequently by the non-obvious nature of the plasma phenomena. To enable astrophysicists to become acquainted with certain problems of plasma theory, we shall attempt in the present review to develop the physical principles and present convenient formulas for the description of one of the most important (from the point of view of astrophysics) sections of modern plasma physics. There are many problems in plasma physics that should attract the attention of astrophysicists, but for the interpretation of the spectra of celestial bodies it is probably most important to consider the problem of conversion of plasma waves into electromagnetic ones.

Radioastronomic observations have shown that cosmic sources contain very intense radiation mechanisms, which do not always lend themselves to reduction to the already known and simple mechanisms of bremsstrahlung (electron-ion collisions, cyclotron and synchrotron mechanisms). On the other hand, plasma in general and cosmic plasma in particular are very easily excited and enter a turbulent state. The energy density of turbulent plasma pulsations can be quite large, and its conversion into electromagnetic waves can serve as a powerful source of radiation in a wide frequency interval. These plasma radiation mechanisms are the subject of the present review.

1. FORMULATION OF THE PROBLEM IN THE THEORY OF PLASMA RADIATION MECHANISMS

By virtue of some instability (one of the best known mechanisms is two-stream instability), plasma waves of various types can be excited in a plasma, namely: electron (or Langmuir) waves with frequency close to

$$\omega_{0e}=\sqrt{\frac{4\pi e^2 n_e}{m_e}}, \tag{1.1}$$

where n_e is the electron density; electron plasma waves with frequency close to gyrofrequency

$$\omega_{He}=\frac{eH}{m_e c} \tag{1.2}$$

(where H is the external magnetic field; to this end it is necessary to satisfy the condition $\omega_{He} \gg \omega_{0e}$); ion plasma waves with frequency $\omega_{0i} = \omega_{0e}(m_e/m_i)^{1/2}$; ion acoustic waves ($\omega_s = kc_s$, $c_s^2 = T_e/m_i$) with frequency much smaller than ω_{0i} (ion waves can be excited only in a non-isothermal plasma, when the electron temperature T_e is much higher than the ion temperature T_i —by at least a factor of three); Alfven and magnetosonic waves with frequency smaller than $\omega_{Hi} = \omega_{He} m_e/m_i$; whistlers with frequencies $\omega_{Hi} \gg \omega \ll \omega_{He}$; and longitudinal ion waves with frequency on the order of ω_{Hi}.

Figure 1 summarizes the results of investigations of the spectra of plasma and electromagnetic waves in the case of greatest interest for astrophysical applications, $\omega_{He} \ll \omega_{0e}$ (see the reviews [28–30]).

In a homogeneous plasma, in the linear approximation, these waves, and also electromagnetic radiation, are the so-called normal waves, propagating independently of one another and not interacting with one another. In the nonlinear approximation, however, (or else in an inhomogeneous medium), interaction sets in, accompanied by exchange of energy both between waves of the same type but with different wave numbers, and between waves of different types. This means that if suf-

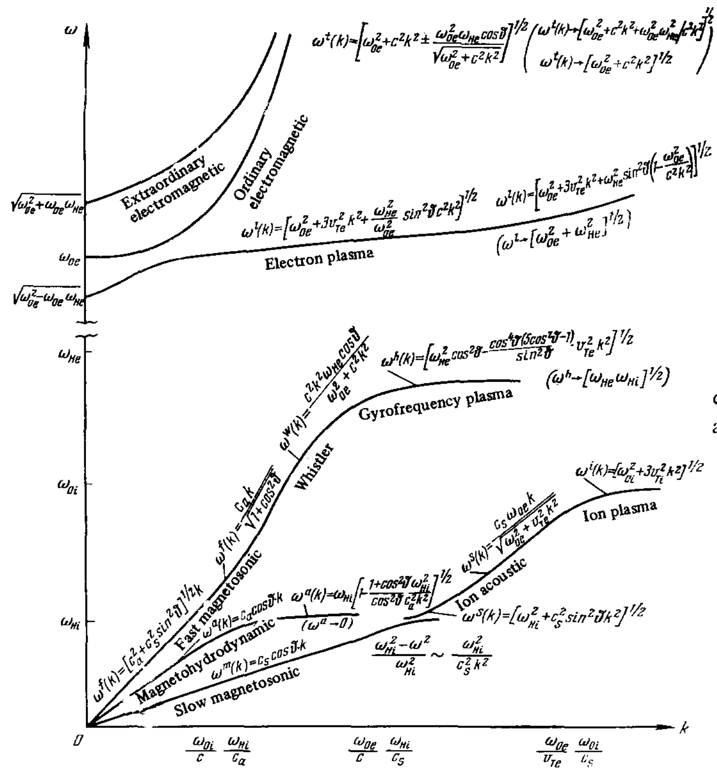

FIG. 1. Dispersion curves of plasma-turbulence waves. Here $\omega_{He} \ll \omega_{0e}$, $c_a \gg c_s$, the parentheses contain spectra at $\theta \to \pi/2$, where θ is the angle between $\mathbf{k}$ and $\mathbf{H}$.

ficiently intense waves of at least one type are excited in the plasma for some reason, the mutual exchange of energy can ultimately lead to radiation of electromagnetic waves, which leave the plasma and which are observed in the form of very powerful radio emission. A special role in the radiation process is played by high-frequency electronic oscillations, but in the presence of fast (say, relativistic) particles the low-frequency ones can also be transformed into radiation.

Thus, a detailed analysis of the plasma radiation mechanisms should include both an investigation of the excitation of various types of waves, and the transformation (conversion) of the plasma turbulence into electromagnetic waves. The first part of this problem—excitation and establishment of the spectrum of plasma turbulence—will not be considered here. On the one hand, this calls for a more specific formulation of the problem, in the sense that a plasma turbulence is excited in different manners in different types of instabilities. On the other hand, many properties of plasma turbulence depend on various concrete properties of the plasma of the object. An example of such a concrete calculation (with allowance for conversion) is contained in [1]. Problems of plasma turbulence are discussed in the reviews [27,7,31,32]. We shall assume here that, as a result of certain causes, turbulence develops in the given plasma at different types of plasma waves, and that the energy density of the plasma turbulence and its distribution over the spectrum of the wave numbers of the plasma pulsations are specified.

A plasma turbulence of type α can be specified either in terms of the Fourier component

$$\mathbf{E}^\alpha_{\mathbf{k},\omega} = \mathbf{E}^\alpha_{\mathbf{k}}\delta(\omega - \omega^\alpha(\mathbf{k})) + \mathbf{E}^{*\alpha}_{-\mathbf{k}}\delta(\omega + \omega^\alpha(-\mathbf{k}))$$

of the electric field of the turbulent pulsations $\mathbf{E}^\alpha_{\mathbf{k}}$, defined by

$$\mathbf{E}^\alpha = \int \mathbf{E}^\alpha_{\mathbf{k}} e^{-i[\omega^\alpha(\mathbf{k})-\mathbf{kr}]}\, d\mathbf{k} = \int \mathbf{E}^\alpha_{\mathbf{k},\omega} e^{-i[\omega^\alpha(\mathbf{k})-\mathbf{kr}]}\, d\mathbf{k}\, d\omega \qquad (1.3)$$

(where $\mathbf{k}$ is the wave vector of the plasma pulsations and $\omega^\alpha(\mathbf{k})$ is the dispersion curve), or else in terms of the density of the number of quanta per unit phase volume $(\langle \mathbf{E}^{*\alpha}_{\mathbf{k}}\mathbf{E}^\alpha_{\mathbf{k}'}\rangle = |\mathbf{E}^\alpha_{\mathbf{k}}|^2\delta(\mathbf{k}-\mathbf{k}))$:

$$N^\alpha_{\mathbf{k}} = \frac{2\pi^2 |E^\alpha_{\mathbf{k}}|^2}{\hbar\omega^\alpha(\mathbf{k})}\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^\alpha)\right]_{\omega=\omega^\alpha(\mathbf{k})}, \qquad (1.4)$$

where the dielectric constant for the waves α is

$$\varepsilon^\alpha(\mathbf{k},\omega) = \frac{c^2}{\omega^2}(\mathbf{k}\mathbf{a}_{\mathbf{k}})(\mathbf{k}\mathbf{a}^*_{\mathbf{k}}) + a^*_{\mathbf{k},i}\varepsilon^\alpha_{ij}(\mathbf{k},\omega)\,a_{\mathbf{k},j}, \qquad (1.5)$$

$\mathbf{a}_{\mathbf{k}}$ is the unit vector of polarization of the wave α, satisfying the normalization condition $(\mathbf{a}^*_{\mathbf{k}}\mathbf{a}_{\mathbf{k}}) = 1$.

Noticeable conversion will take place principally in the case of developed turbulence. Such turbulence, as a rule, will lead to isotropic distribution of the wave vectors (if the influence of the external magnetic field is insignificant). It can therefore be assumed, with sufficient accuracy for astrophysical applications, that the plasma turbulence is isotropic with a spectral function $F_\alpha(\mathbf{k})$ that depends only on the absolute value of the wave number. Incidentally, to simplify the formulas, we shall use in the present section and in the two following ones the quantity $W^\alpha_{\mathbf{k}}$—the turbulence energy density per unit phase volume. We have the following relations:

$$W_\alpha = \int W^\alpha_{\mathbf{k}}\, dk = \int_0^\infty F_\alpha(\mathbf{k})\, dk = \frac{1}{4\pi}\int |\mathbf{E}^\alpha_{\mathbf{k}}|^2\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^\alpha)\right]\frac{dk}{\omega^\alpha(\mathbf{k})} = \int \omega^\alpha(\mathbf{k})\, N^\alpha_{\mathbf{k}}\frac{\hbar\, dk}{(2\pi)^3}. \qquad (1.6)$$

We shall henceforth assume that the function $F_\alpha(\mathbf{k})$ is specified.

Let us turn to the other side of the problem—conversion of the plasma-turbulence waves into electromagnetic radiation. This problem was first formulated and investigated by V. L. Ginzburg and V. V. Zheleznyakov (see [2], where the earlier references are also cited). The problem was subsequently investigated by others.[3-6]

According to [2], two types of conversion of plasma waves into electromagnetic ones are possible: regular conversion in a smoothly-inhomogeneous medium, and scattering by electron-density fluctuations. In fact, both the smooth inhomogeneity of the medium and the fluctuations in the distribution of the parameters alter the conditions for the propagation of the waves, which cease to be strictly independent, and consequently can interact with one another. In the case of propagation in a smoothly-inhomogeneous medium, the conversion coefficient is in general proportional to the ratio of the wavelength λ to the characteristic dimension L of the inhomogeneity.[5] There are also more favorable cases —conversion in those plasma regions, where the geometrical-optics condition is violated, for example where $\epsilon \to 0$. In this case, a transformation coefficient on the order of $(\lambda/L)^{2/3}$ can be obtained.[2] Under cosmic conditions, the ratio λ/L is always very small, and it can be assumed that, with the exception of very special cases, regular conversion does not play a noticeable role under astrophysical conditions.

Conversion in scattering of plasma waves by electron-density fluctuations corresponds to Rayleigh scattering (with conservation of the frequency) or to Raman scattering (with addition of the plasma-wave and density-fluctuation frequencies). The order of magnitude of these effects is determined by the Thomson scattering cross section:

$$\sigma_T n_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 n_e = \frac{\omega_{0e}^4}{6\pi n_e c^4}. \tag{1.7}$$

The conversion coefficients contain, besides σ_T, also factors that depend on the phase velocities of the waves and on the thermal velocities of the electrons, and are different for Rayleigh and Raman scattering.

In scattering by electron-density fluctuations, the very same mechanisms can be described also in the language of the theory of nonlinear effects in a plasma. Rayleigh scattering by thermal fluctuations of the electron density is none other than nonlinear scattering of plasma waves by polarization charges produced by the motion of thermal ions in the plasma. Raman scattering is the process of coalescence of plasma waves with the waves of thermal fluctuations of the electron density.[7,8] By using the methods of nonlinear plasma theory it is much easier to calculate not only the spontaneous conversion, but also the induced conversion (including the absorption of electromagnetic waves as the latter are reconverted into plasma waves). These effects, which are not considered in [2-4,6], change in general the entire picture of the conversion of plasma waves into electromagnetic waves under cosmic conditions. The recently developed methods of nonlinear plasma theory make it possible to consider conversion occurring upon scattering by epithermal particles, wherein high frequencies are radiated, and also upon coalescence and decay of various types of waves existing in a plasma. This makes it possible to expand greatly our knowledge of plasma mechanisms, by increasing the ranges of the radiated frequencies and the radiation powers as a result of the use of induced mechanisms. It must be emphasized that the neglect of induced mechanisms can lead in general to wrong results.

We consider in the present paper the conversion of plasma waves into electromagnetic waves by the methods of the theory of nonlinear processes in a plasma.[7,8] Many of the results are published for the first time. But even in those cases when already known relations are obtained, the methods of nonlinear theory make it possible to visualize more clearly the limitations imposed on the previously derived corresponding formulas. Of particular significance in astrophysics are fast particles, and especially relativistic ones. Therefore special attention is paid to effects of conversion on relativistic electrons and ions.

2. EMISSION COEFFICIENTS OF PLASMA MECHANISMS

The emission coefficients in conversion of plasma waves into electromagnetic waves are best calculated with the aid of the nonlinear-current conductivity tensor.[7,8] The gist of this method is as follows.

Assume that waves of type α_1, α_2, ... propagate in a homogeneous plasma (not necessarily isotropic). The Fourier component of the current excited by these waves, accurate to terms quadratic in the electric field, can be written in the form

$$j_{i,\mathbf{k}\omega} = \sigma_{ij}(\mathbf{k},\omega)\, E^{\alpha}_{j,\mathbf{k}\omega} + j^{(2)}_{i,\mathbf{k}\omega}, \tag{2.1}$$

$$j^{(2)}_{i,\mathbf{k},\omega\alpha_1,\alpha_2} = \sum \int S_{ijl}(\mathbf{k},\omega;\, \mathbf{k}_1,\omega_1;\, \mathbf{k}_2,\omega_2)\, E^{\alpha_1}_{j,\mathbf{k}_1\omega_1} \times E^{\alpha_2}_{l,\mathbf{k}_2\omega_2}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\,\delta(\omega-\omega_1-\omega_2)\, d\mathbf{k}_1\, d\omega_1\, d\mathbf{k}_2\, d\omega_2, \tag{2.2}$$

where σ_{ij} is the tensor of the linear conductivity of the plasma, and S_{ijl} is the tensor of the nonlinear conductivity. To describe the conversion of waves α_1 and α_2 into a wave α, it is sufficient to find the power Q of wave α radiated by the part of the current (2.2) excited by waves α_1 and α_2.

In astrophysics one usually determines the spectral intensity of the radiation per unit frequency interval. The corresponding expression for the emission coefficient (i.e., the energy per unit frequency interval generated in a unit volume and in a unit solid angle) is

$$Q = \int Q_{\mathbf{k}} \frac{dk}{(2\pi)^3}, \quad I_{\omega,\Omega} = \frac{Q_{\mathbf{k}} k^2}{(2\pi)^3 \dfrac{d\omega(\mathbf{k})}{dk}}, \quad I_\omega = \int I_{\omega,\Omega}\, d\Omega, \tag{2.3}$$

Ω is the solid angle of the vector **k**.

In (2.3), **k** is a function of Ω and ω, obtained from $\omega = \omega^{\alpha}(\mathbf{k},\Omega)$. We obtain

$$I_{\omega,\Omega} = 16(2\pi)^4 \int \frac{\omega^{\alpha}(\mathbf{k})\,|[\mathbf{k}\mathbf{S}]|^2\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)}{c^2 k\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha})\right]_{\omega^{\alpha_1}(\mathbf{k}_1)}} \times \frac{\delta(\omega-\omega^{\alpha_1}(\mathbf{k}_1)-\omega^{\alpha_2}(\mathbf{k}_2))\, W^{\alpha_1}_{\mathbf{k}_1} W^{\alpha_2}_{\mathbf{k}_2}}{\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha})\right]_{\omega^{\alpha}(\mathbf{k})}\left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha_2})\right]_{\omega^{\alpha_2}(\mathbf{k})}}\, d\mathbf{k}_1\, d\mathbf{k}_2,$$

$$S_i = S_{ijl} a_j^{\alpha_1} a_l^{\alpha_2}. \tag{2.4}$$

When $\alpha_1 = \alpha_2$ it is necessary to replace the coefficient 16 in (2.4) by 8. We shall use this general formula to determine the emission coefficients of the plasma mechanisms in those cases when the conversion occurs upon coalescence of plasma waves.

Another conversion mechanism is connected with the scattering of plasma waves by polarization charges and the plasma particles themselves.[8] In fact, a charged particle (whether an electron or an ion) moving in a plasma produces a polarization charge moving with the same velocity. The field produced by the polarization charge can correspond to one of two fields of the nonlinear current (2.2). The radiation of the wave α by the current (2.2) then corresponds to the conversion $\alpha_1 \rightarrow \alpha$:

$$I_{\omega,\Omega} = 2(2\pi)^3 \int \frac{\omega^{\alpha}(\mathbf{k}) \, |[\mathbf{k}\Lambda]|^2 \, \delta\,[\omega^{\alpha}(\mathbf{k}) - \omega^{\alpha_1}(\mathbf{k}_1) - (\mathbf{k}-\mathbf{k}_1)\mathbf{v}]}{c^2 k \left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2 \varepsilon^{\alpha_1})\right]_{\omega^{\alpha_1}(\mathbf{k}_1)}} W^{\alpha_1}_{\mathbf{k}_1} f^q_{\mathbf{p}}(\mathbf{p})\, d\mathbf{k}_1 d\mathbf{p}, \quad (2.5)$$

where $f^q_p(p)$ is the distribution function of the momenta of the charges, normalized to a unit phase volume, i.e.,

$$\int f^q_{\mathbf{p}}(\mathbf{p}) \frac{dp}{(2\pi)^3} = n_q,$$

where n_q is the concentration of the corresponding particles,

$$\Lambda_{ij}(\mathbf{k}, \mathbf{k}_1)\,\delta(\omega_- - \mathbf{k}_-\mathbf{v}) = 2S_{ijl}(\mathbf{k}, \omega, \mathbf{k}_-, \omega_-, \mathbf{k}_1, \omega_1)\, E^q_{l, \mathbf{k}_-, \omega_-},$$
$$\mathbf{k}_- = \mathbf{k} - \mathbf{k}_1, \quad \omega_- = \omega - \omega_1. \quad (2.6)$$

$E^q_{l,k,\omega}$ is the field produced by the uniformly moving charge. (For a derivation of (2.6) see [8].)

If the polarization charges are produced by nonrelativistic electrons, then the effect of nonlinear scattering by these charges is of the same order as the usual Compton scattering by the electrons themselves, and both scattering effects partially cancel each other.[7] In this case it is necessary to add to the definition of Λ_{ij} in (2.21) one more term describing the Compton scattering. The effect of this compensation is discussed in detail in [10] (see also [8]). The relative contribution to the radiation of that region in which the compensation takes place is small. Frequently, therefore, it is necessary only to know the regions of $\mathbf{k}_1$ in which there is no compensation, and to make use of (2.5).

Thus, to find the emission coefficient in the plasma mechanisms it remains to determine only the nonlinear conductivity—the tensor S_{ijl}. In the simplest case of an isotropic plasma without a magnetic field, neglecting thermal motion of the electrons and ions (i.e., in the hydrodynamic approximation), we have [8]

$$S_{ijl} = -\frac{e^2\omega_{0e}^2}{8\pi m_e \omega_1^{\alpha_1}\omega_2^{\alpha_2}} \left(\delta_{ij}\frac{k_{2l}}{\omega_2^{\alpha_2}} + \delta_{il}\frac{k_{1j}}{\omega_1^{\alpha_1}} + \delta_{jl}\frac{k_i}{\omega^{\alpha}}\right). \quad (2.7)$$

In the presence of external magnetic fields it is possible to obtain a simple expression for S_{ijl}, provided one of the frequencies is large, $\omega \gg \omega_{He}, \omega_{0e}, kv_{T_e}$:

$$\left.\begin{aligned} S_{1ij} &= \frac{e}{4\pi m_e \omega} \frac{\delta_{1i} + i\frac{\omega_{He}}{\omega}\delta_{2i}}{1 - \frac{\omega_{He}^2}{\omega^2}} [k_{2s}[\varepsilon^e_{sj}(\mathbf{k}_2, \omega_2,) - \delta_{sj}], \\ S_{2ij} &= \frac{e}{4\pi m_e \omega} \frac{\delta_{2i} - i\frac{\omega_{He}}{\omega}\delta_{1i}}{1 - \frac{\omega_{He}^2}{\omega^2}} k_{2s}[\varepsilon^e_{sj}(\mathbf{k}_2, \omega_2) - \delta_{sj}], \\ S_{3ij} &= \frac{e}{4\pi m_e \omega} \delta_{3i} k_{2s}[\varepsilon^e_{sj}(\mathbf{k}_2, \omega_2) - \delta_{sj}]. \end{aligned}\right\} \quad (2.8)$$

The index "e" of the dielectric tensor denotes here that ϵ^e should be taken to mean only that part of the dielectric constant, which is produced by the electrons. In the final expression, it is necessary to take the half-sum

$$\frac{1}{2}[S_{ijl}(\mathbf{k}, \omega; \mathbf{k}_1, \omega_1, \mathbf{k}_2, \omega_2) + S_{ijl}(\mathbf{k}, \omega; \mathbf{k}_2, \omega_2; \mathbf{k}_1, \omega_1)], \quad (2.9)$$

which will be borne in mind throughout.

The relations presented in this section make it possible to calculate the emission coefficients for different plasma mechanisms—conversion of plasma waves of different types into electromagnetic waves upon scattering by charged particles, and also coalescence of different waves of plasma turbulence. Concrete expressions which make it possible to calculate the emission coefficients for the various cases will be given in subsequent sections.

3. INDUCED CONVERSION OF PLASMA WAVES INTO ELECTROMAGNETIC ONES

The formulas obtained above describe the spontaneous mechanism of radiation upon conversion. Yet, as is well known from general radiation theory, each spontaneous process should correspond to an induced process, which can lead to both absorption and induced emission. Obviously, the same should take place also in the conversion of plasma waves into electromagnetic ones.

Induced conversion is accompanied by stimulated emission of a transverse under the influence of an incident transverse wave having the same values of $\mathbf{k}$ and ω as the radiated transverse wave. Induced absorption of electromagnetic waves is the inverse process—transformation of an electromagnetic wave into a plasma wave.

The effects of induced conversion, which are well known in the theory of nonlinear processes in plasma, have been frequently disregarded in astrophysical applications of plasma mechanisms of radiation. Yet allowance for these effects can greatly alter (and in many cases radically) the entire picture of the phenomena occurring in some cosmic source.

One of the principal features of induced conversion is the possibility of buildup of a radiation wave in the case when the induced radiation prevails over the absorption. In this case an appreciable fraction of the energy of the plasma waves can be converted into electromagnetic radiation. It can be assumed that in powerful sources of cosmic radio emission the mechanism is precisely the buildup produced by induced conversion. On the other hand, if absorption predominates over induced emission, the effectiveness of the spontaneous conversion mechanism is limited and the power of this radiation cannot grow beyond a definite limit with simple increase of the density of the plasma waves.

All these considerations will be discussed in detail subsequently, and estimates will be presented for various astrophysical objects. For the present, we present only the formulas that make it possible to calculate the induced conversion processes.

Formula (2.4) determines the spontaneous electromagnetic radiation produced upon coalescence of two plasma waves with $\mathbf{k}_1$ and $\mathbf{k}_2$. This formula is valid only when the number of electromagnetic waves is sufficiently small (the criterion of smallness will be derived later from a comparison of the optical thickness for the conversion processes with the coefficients of spontaneous emission).

In (2.4) the quantity $|E^{\alpha_1}_{k_1}|^2 |E^{\alpha_2}_{k_2}|^2$ is proportional, in accordance with (1.4) and (1.6), to the product $N^{\alpha_1}_{k_1} N^{\alpha_2}_{k_2}$.

When induced conversion is taken into account, this quantity should be replaced by the sum

$$N_{k_1}^{\alpha_1} N_{k_2}^{\alpha_2} - N_k^{\alpha} N_{k_1}^{\alpha_1} - N_{k_2}^{\alpha_2} N_k^{\alpha}. \quad (3.1)$$

A similar replacement follows from the condition of the balance between the numbers of the different waves.[8]

It follows directly from (3.1) that upon coalescence of two plasma waves the induced conversion leads only to absorption of electromagnetic waves (since the last two terms in (3.1) are negative). In this process one electromagnetic wave with wave vector **k** breaks up into two plasma waves $\mathbf{k}_1$ and $\mathbf{k}_2$, such that $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and $\omega = \omega_1 + \omega_2$. The induced conversion of the plasma waves of different types can, generally speaking, lead to a buildup of waves, which arises in the case when besides the process $\alpha_1 + \alpha_2 \to \alpha$ the process $\alpha_1 \to \alpha + \alpha_2$ is also allowed by the conservation laws. For the latter process we have in lieu of (3.1)

$$N_{k_1}^{\alpha_1} N_{k_2}^{\alpha_2} + N_{k_1}^{\alpha_1} N_k^{\alpha} - N_k^{\alpha} N_{k_2}^{\alpha_2}. \quad (3.2)$$

We introduce the coefficient of induced conversion $\mu(\mathbf{k}, \omega)$, which assumes positive values if absorption takes place, and negative values if the radiation field builds up. In order to bring this quantity in correspondence with the absorption coefficient used in astrophysics, which is calculated per unit length, we write

$$\mu(\mathbf{k}, \omega) = -\frac{1}{N_k v_g} \frac{\partial N_k}{\partial t}\bigg|_{\text{ind. conv.}} \quad (3.3)$$

where v_g is the group velocity of the electromagnetic transverse waves, which depends usually only on $|\mathbf{k}| = \omega/c$, N_k is the number of quanta per unit phase volume of the electromagnetic waves, and $\partial N_k/\partial t|_{\text{ind. conv}}$ is the change in the number of these quanta as a result of induced conversion.

It follows from (3.1) and (3.2) that to determine the coefficient of induced conversion it is necessary to leave out one of the factors $|E_{k_1}^{\alpha_1}|^2$ from formula (2.5), which must be replaced by a factor describing the transition from $|E_{k_2}^{\alpha_2}|^2$ to $N_{k_2}^{\alpha_2}$. It is necessary to distinguish between effects resulting from the coalescence of identical turbulent pulsations ($\alpha_1 = \alpha_2$) and different ones, and also the case when one type of turbulent pulsations α_1 is excited in the plasma, and when several types are excited (e.g., two α_1 and α_2). If one type of pulsations is excited, then in accordance with (3.1) the absorption will be determined by $-2N_k N_k^{\alpha_1}$. As a result we obtain for the absorption coefficient averaged over the polarizations

$$\mu(\mathbf{k}, \omega) = \frac{16(2\pi)^4}{v_g(\omega(\mathbf{k}))} \int \frac{|\mathbf{kS}|^2}{\mathbf{k}^2} \frac{\hbar\omega^{\alpha_1}(\mathbf{k}_1)\,\omega^{\alpha_1}(\mathbf{k}-\mathbf{k}_1)}{\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha})\right]_{\omega(\mathbf{k})}} \times \frac{\delta(\omega(\mathbf{k}) - \omega^{\alpha_1}(\mathbf{k}_1) - \omega^{\alpha_1}(\mathbf{k}-\mathbf{k}_1))\, W_{\mathbf{k}_1}^{\alpha_1} d\mathbf{k}_1}{\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha_1})\right]_{\omega^{\alpha_1}(\mathbf{k}_1)} \left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha_1})\right]_{\omega^{\alpha_1}(\mathbf{k}-\mathbf{k}_1)}} \quad (3.4)$$

The foregoing formulas are valid when the processes of the type $\alpha_1 + \alpha_2 \to \alpha$ and $\alpha_1 \to \alpha + \alpha_2$ $(\alpha_1 \neq \alpha_2)$ do not occur in the transverse-wave frequency interval under consideration. We note that these processes must be taken into account even if only one type of turbulent pulsations, say α_1, is excited. The spontaneous process $\alpha_1 + \alpha_2 \to \alpha$ does not occur in this case, as can be seen directly from (3.1). However, the induced process is possible and yields an absorption $-N_k^{\alpha} N_{k_1}^{\alpha_1}$. At the same time, the process (3.2) leads to a buildup of transverse waves $(+N_{k_1}^{\alpha_1} N_k^{\alpha})$. The latter process has a somewhat different probability than the former. This difference, however, reduces only to a change of the signs of the δ functions reflecting the laws of conservation of energy and momentum of the quanta upon coalescence. Indeed, the process $\alpha_1 \to \alpha + \alpha_2$ differs from $\alpha_1 + \alpha_2 \to \alpha$ only in that the wave α_2 is not absorbed but is emitted. It is therefore necessary to reverse the signs of ω_2 and $\mathbf{k}_2$. We obtain in lieu of (3.4)

$$\mu(\mathbf{k}, \omega) = \frac{16(2\pi)^4}{v_g(\omega(\mathbf{k}))} \int \frac{|\mathbf{kS}|^2}{k^2} \frac{\hbar\omega^{\alpha_1}(\mathbf{k}_1)\,[\omega(\mathbf{k}) - \omega^{\alpha_1}(\mathbf{k}_1)]}{\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha})\right]_{\omega^{\alpha_1}(\mathbf{k}_1)} \left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha_1})\right]_{\omega^{\alpha_1}(\mathbf{k}_1)}} \times \frac{\{\delta(\omega(\mathbf{k}) - \omega^{\alpha_1}(\mathbf{k}_1) - \omega^{\alpha_2}(\mathbf{k}-\mathbf{k}_1)) + \delta(\omega(\mathbf{k}) - \omega^{\alpha_1}(\mathbf{k}_1) + \omega^{\alpha_2}(\mathbf{k}-\mathbf{k}_1))\}}{\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha_2})\right]_{\omega=\omega(\mathbf{k})-\omega^{\alpha_1}(\mathbf{k}_1)}} W_{\mathbf{k}_1}^{\alpha_1} d\mathbf{k}_1. \quad (3.5)$$

Formula (3.5) is transformed in such a way as to imply directly the important conclusion that transverse waves are built up, with frequencies lower than the frequencies of the turbulent pulsations, and waves with frequencies higher than those of the turbulent pulsations are attenuated. In each of these two cases, only one of the δ functions "works."

If two types of turbulence are excited, then it is necessary to take into account also the processes $\alpha_2 \to \alpha + \alpha_1$. Concrete formulas will be given in subsequent sections.

The coefficient of induced conversion in scattering by particles is determined in a somewhat different manner. In this case it is necessary to compare the numbers of particles with momenta p and $\mathbf{p} + \hbar(\mathbf{k} - \mathbf{k}_1)$, where $\hbar(\mathbf{k} - \mathbf{k}_1)$ is the change of the momentum of the particles from which the plasma wave is scattered. Assuming that $\hbar(\mathbf{k} - \mathbf{k}_1) \ll p$, the equation for the balance of the number of waves can be obtained by replacing the distribution function f_p^q by the quantity

$$\hbar N_k(\mathbf{k} - \mathbf{k}_1)\frac{\partial f_{\mathbf{p}}^q}{\partial \mathbf{p}}.$$

The conversion sign (emission or absorption) depends here on the sign of the scalar product $(\mathbf{k} - \mathbf{k}_1)(\partial/\partial\mathbf{p})$. As before, using (3.3) and taking (2.5) into account, we obtain for the absorption coefficient averaged over the polarizations:

$$\mu(\mathbf{k}, \omega) = \frac{2(2\pi)^9}{v_g(\omega)} \int \frac{|\mathbf{kA}|^2}{k^2} \frac{\delta(\omega(\mathbf{k}) - \omega^{\alpha_1}(\mathbf{k}_1) - (\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{\left[\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha})\right]_{\omega(\mathbf{k})} \left[\frac{1}{\omega}\frac{\partial}{\partial\omega}(\omega^2\varepsilon^{\alpha_1})\right]_{\omega^{\alpha_1}(\mathbf{k}_1)}} \times \hbar(\mathbf{k}-\mathbf{k}_1)\frac{\partial f_{\mathbf{p}}^q}{\partial\mathbf{p}} W_{\mathbf{k}_1}^{\alpha_1}\frac{d\mathbf{k}_1\, d\mathbf{p}}{(2\pi)^3}. \quad (3.6)$$

In radioastronomical measurements one uses the definition of the brightness temperature more frequently than the spectral intensity of radiation. We shall therefore present here formulas describing the radiation power in terms of the so-called effective temperatures

$$T_{\text{eff}} = \frac{8\pi^3}{k^2} I_{\omega,\Omega}. \quad (3.7)$$

If there is no scattering of the radiation in the medium, then the brightness temperature in the absence of buildup is

$$T_b = T_{\text{eff}}(1 - e^{-|\mu|L}), \quad (3.8)$$

where L is the dimension of the medium in which the conversion takes place. It follows therefore that T_{eff} determines the maximum brightness temperature that can be obtained in spontaneous conversion.

Thus, the mechanism of spontaneous conversion is best characterized not by the emission coefficient, but by the effective temperature and the absorption coefficient: the former parameter determines the maximum possible radiation, and the latter the condition under which this radiation can be obtained.

In the case when induced radiation prevails, we get in lieu of (3.8) $T_b = T_{eff}(e^{|\mu|L} - 1)$, and the limit of the plasma-wave conversions is imposed by the density of the plasma energy, if the conversion occurs by scattering from the thermal particles. The limiting effective temperature is determined in this case both by the total density of the plasma energy W^{α_1}, and by the minimum wave number $k^{\alpha_1}_{min}$ of the turbulent pulsations or by the minimum wave number of the transverse waves. We have, for example, for Langmuir waves

$$T_{eff} \approx \frac{6\pi^2 W^{\alpha_1}}{k^3_{min}} \approx \frac{6\pi^2 W^{\alpha_1}}{(k^{\alpha_1}_{min})^3}\left(\frac{c}{\sqrt{3}\,v_{Te}}\right)^3. \tag{3.9}$$

As is well known, nonlinear interactions between Langmuir waves are accompanied by transfer of the wave energy into the region of higher phase velocities, i.e., smaller wave numbers. This means that the lower limit of the wave numbers of the plasma turbulence is determined only by the geometry of the medium and by the conditions of emergence of the radiation. The effective temperature can be quite high in this case.

If the conversion occurs as a result of scattering by epithermal or relativistic particles, then the energy of the electromagnetic waves is drawn mainly from the particle energy. The effective temperature can be in this case of the same order as the energy of the radiating particles, and hence also large. On the other hand, if the radiating particles are accelerated by the plasma turbulence,[13] then the effective temperature is determined in this case, too, in final analysis, by the energy density of the plasma turbulence.

The buildup and consequently generation of powerful radiation is possible only when the coefficient of induced radiation $\mu(\mathbf{k}, \omega)$ has a larger absolute magnitude than $\mu(\omega)$, the coefficient of the true absorption in the medium. If the latter is determined by the bremsstrahlung mechanism, then

$$\mu_T(\omega) = \frac{\Lambda}{4\pi(2\pi)^{1/2}}\frac{\omega_{0e}^6}{n_e v_{Te}^3 v_g \omega^2} = \frac{\Lambda}{4\pi(2\pi)^{1/2}}\frac{\omega_{0e}^3}{\omega^2 v_g}\frac{1}{n_e D^3}, \tag{3.10}$$

where $v_{Te} = \sqrt{T_e/m_e}$ is the thermal velocity of the electrons, $D = v_{Te}/\omega_{0e}$ is the Debye radius, and Λ is the Coulomb logarithm. Under cosmic conditions $\Lambda = 40-70$, and therefore the numerical factor $\Lambda/4\pi(2\pi)^{1/2}$ can be omitted. Thus, in order for electromagnetic waves to be built up in the course of the conversion, the density of the energy of the plasma waves should be such that $\mu_T(\omega) \ll |\mu(\mathbf{k}, \omega)|$.

For each concrete mechanism of conversion of plasma waves into electromagnetic ones, we shall present the values of the radiation coefficient, the coefficient of induced conversion (absorption and emission), and where necessary the value of the effective temperature. The formulas will be presented in a form most convenient for use in the interpretation of the observed data on cosmic radio sources.

4. CLASSIFICATION OF PLASMA MECHANISMS IN ACCORD WITH THE EMISSION FREQUENCIES

To simplify the exposition, we present here a classification of plasma mechanisms in accordance with the type of nonlinear processes and in accordance with the frequencies radiated by these mechanisms.

It is convenient to bear in mind in this case the graphical representation of the spectra of the plasma waves for the typical astrophysical conditions $\omega_{He} \ll \omega_{0e}$ (see Fig. 1; if $H^2/8\pi \approx n_e T_e$, then $\omega_{He} \approx (v_{Te}/c)\,\omega_{0e} \ll \omega_{He}$). The condition $\omega_{He} \ll \omega_{0e}$ assumed in Fig. 1 may be violated in a number of astrophysical objects (e.g., on the sun or elsewhere). In this case the corresponding plot of Fig. 1 cannot be used. The parentheses in Fig. 1 show the types of spectra that occur in the limiting case when the wave propagates at an angle close to 90° to the direction of the magnetic field.

a) Emission at a Frequency Close to ω_{0e}

In the transformation of Langmuir plasma waves into electromagnetic waves as a result of scattering by thermal ions or electrons, radiation is generated in the frequency interval

$$\omega_{0e} \leqslant \omega \leqslant \omega_{0e}\left(1 + \frac{3}{2}\frac{v_{Te}^3}{v_p^2}\right), \tag{4.1}$$

where v_p is the phase velocity of the plasma waves. Since v_p should be larger than v_{Te} by at least a factor of three, the interval of the frequencies emitted in this case does not exceed $\omega_{0e}/6$.

Electromagnetic waves with frequency close to ω_{0e} can be generated by coalescence of Langmuir waves (frequency ω^l) with ion-plasma or ion-acoustic waves. Since the frequency of the plasma wave of these types is either smaller than or equal to $\omega_{0i} = \omega_{0e}(m_e/m_i)^{1/2}$, it follows that the electromagnetic radiation generated by such a mechanism, which has a frequency $\omega^l + \omega_{0i} \cong \omega_{0e} + \omega_{0i}$, likewise does not go beyond the interval (4.1). Ion-plasma and ion-acoustic waves can be excited only in a non-isothermal plasma, when $T_e \gtrsim 3T_i$ (here T_i is the ion temperature). Also possible are processes in which plasma waves coalesce with whistlers, which can exist also in an isothermal plasma. Since the maximum whistler frequency is of the order ω_{He}, the emitted frequencies will be of the order of $\omega^l + \omega_{He}$, which in a weak magnetic field also corresponds to emission at frequencies close to ω_{0e}. Coalescence with low-frequency hydrodynamic oscillations, particularly Alfven waves, yields frequencies not larger than $\omega^l + \omega_{Hi}$, i.e., also in the interval (4.1). The case of a strong magnetic field is discussed later.

b) Emission at Frequencies Close to $2\omega_{0e}$ (in a Weak Magnetic Field $\omega_{He} \ll \omega_{0e}$)

Coalescence of two plasma waves with frequencies $\sim\omega_{0e}$ yields one electromagnetic wave with frequency $\sim 2\omega_{0e}$. This mechanism has no competition on the part of the conversion mechanisms in scattering by particles. However, there is no induced emission here (only absorption), and therefore, other conditions being equal, the effectiveness of the conversion at frequencies $2\omega_{0e}$ may be lower than at frequencies ω_{0e}. The width of the frequency interval is here likewise of the order of $(1/3-1/5)\,\omega_{0e}$.

c) Emission at Frequencies Close to ω_{He}

If the plasma is in a strong magnetic field, so that $\omega_{He} \gg \omega_{0e}$, then the plasma waves can be excited both at frequencies close to ω_{He} and at frequencies of the order of $\omega^l \approx \omega_{0e} \cos \vartheta_1$. In either case, conversion of these plasma waves into electromagnetic waves is possible as a result of scattering by thermal ions and electrons.

The dispersion curve for plasma waves with frequency close to ω_{He} is given by

$$\omega^h(\mathbf{k}_1) = \omega_{He}\left[1 + \frac{\omega_{0e}^2 \sin^2\vartheta}{2\omega_{He}^2} + 2\frac{\omega_{He}^2}{\omega_{0e}^2}\left(\frac{v_{Te}}{v_p}\right)^2 \operatorname{ctg}^2\vartheta_1 + \ldots\right]. \quad (4.2)$$

where $v_p = \omega_{He}/k_1$ is, as before, the phase velocity, and ϑ_1 is the angle between the vector $\mathbf{k}_1$ and the direction of the external magnetic field. Formula (4.2) is valid if all the terms in the square bracket are noticeably smaller than unity. It follows from (4.2), however, that the width of the frequency interval of the generated radiation can be in this case relatively larger than in the case of radiation at frequencies ω_{0e}, and can be in principle comparable with ω_{He}. A characteristic feature of conversion at frequencies of the order of ω_{He} is the fact that the refractive index of the generated electromagnetic waves is close here to unity, and therefore there are no difficulties with the emergence of the radiation from the medium.

d) Emission at Frequencies of the Order of $2\omega_{He}$ and $\omega_{He} + \omega_{0e}$

Coalescence of plasma waves of the ω_{He} branch with one another and of plasma waves of the ω_{He} branch with the ω_{0e} branch leads to emission of electromagnetic waves at frequencies $2\omega_{He}$ and $\omega_{He} + \omega_{0e}$. This mechanism is analogous to the mechanism of emission at frequencies $2\omega_{0e}$. For the process $\omega_{He} + \omega_{0e}$, buildup is possible. Coalescence of the wave (4.2) with other low-frequency waves is also possible.

e) Emission at Frequencies Exceeding ω_{0e} or ω_{He}

If the conversion occurs by scattering of plasma waves from epithermal nonrelativistic particles, whose velocity greatly exceeds the velocity of the plasma waves, then the generated radiation has a frequency close to

$$\omega = \omega_{0e} + \mathbf{k}_1\mathbf{v} \leqslant \omega_{0e}\left(1 + \frac{v}{v_p}\right), \qquad \omega = \omega_{He} + \mathbf{k}_1\mathbf{v} \leqslant \omega_{He}\left(1 + \frac{v}{v_p}\right), \quad (4.3)$$

where v is the velocity of the particle (by definition $v \ll c$). The interval of the radiated frequencies is determined by the velocity distribution of the epithermal particles, and can in principle greatly exceed ω_{0e} and ω_{He}.

The indicated mechanisms make it possible to obtain radiation at sufficiently high frequencies, but require the presence of plasma turbulence with small phase velocities. Yet it is known that nonlinear interaction of plasma waves is accompanied by an increase of the phase velocities. Incidentally, if the plasma particle having velocity distribution function is anisotropic, then the phase velocities can also decrease.

f) Emission at Very High Frequencies

Conversion of the plasma waves in scattering by relativistic particles is accompanied by a considerable increase of the frequency:

$$\omega = \frac{\omega_{0e} + (\mathbf{k}_1\mathbf{v})}{1 - \frac{v}{c}\cos\vartheta} \leqslant 2\omega_{0e}\left(1 + \frac{c}{v_p}\right)\left(\frac{\mathcal{E}}{m_q c^2}\right)^2,$$

$$\omega \leqslant 2\omega_{He}\left(1 + \frac{c}{v_p}\right)\left(\frac{\mathcal{E}}{m_q c^2}\right)^2. \quad (4.4)$$

In the general case pulsations of very low frequencies, and even hydrodynamic ones, can be converted on relativistic particles, since for their case we always have $v_p \ll c$ and $\omega \approx 2\omega^{\alpha_1}(c/v_p)(\mathcal{E}/m_q c^2)^2$; here $\mathcal{E}$ is the energy of the particles with mass m_q. If $\mathcal{E} \gg m_q c^2$, then the increase of the frequency can be so large, that even scattering of ion-acoustic and hydromagnetic waves leads to generation of an electromagnetic wave with a frequency exceeding ω_{0e}, capable of emerging from the medium.

The conversion mechanisms noted in items e) and f) can lead to induced radiation, if the distribution functions of the epithermal and relativistic particles are anisotropic (with the plasma turbulence isotropic).

Thus, we see that the plasma mechanisms make it possible to obtain electromagnetic radiation of practically any frequency, if the conditions are suitably chosen. The fact that in many cases induced radiation is possible here, allows us to interpret very powerful sources while making relatively modest demands concerning the physical conditions in these objects. We shall present below formulas for the calculation of the plasma mechanisms of radiation not in the order of their frequencies, as in the present section, but in accordance with the sequence of the conversion mechanisms themselves.

5. CONVERSION OF PLASMA WAVES INTO ELECTROMAGNETIC WAVES UPON SCATTERING BY THERMAL IONS AND ELECTRONS IN AN ISOTROPIC PLASMA

The best known and the frequently used conversion mechanism is scattering by thermal particles.[2,6–8,14] As already noted, scattering by thermal ions is equivalent to scattering by thermal fluctuations of the electron concentration.

We shall consider first the conversion of plasma waves into electromagnetic waves by scattering from polarization charges produced by the motion of ions with charge Ze in an isotropic plasma ($\omega_{He} = 0$), and we shall assume the velocity distribution function of these ions to be Maxwellian. Using the general method described in Secs. 2 and 3, we obtain the following expression for the emission coefficient:

$$I_\omega = \frac{k^2}{v_g}\frac{Z^2\omega_{0e}^2 n_z}{2(2\pi)^{5/2}n_e^2}\left(\frac{T_i}{T_i+T_e}\right)^2\int\frac{[\mathbf{k}\mathbf{k}_1]^2}{k^2k_1^2}\frac{W_{\mathbf{k}_1}^l\,dk_1}{v_{Ti}|\mathbf{k}-\mathbf{k}_1|}e^{-\frac{1}{2}\left[\frac{\omega-\omega^l(\mathbf{k}_1)}{v_{Ti}|\mathbf{k}-\mathbf{k}_1|}\right]^2}, \tag{5.1}$$

where n_Z is the ion concentration, and v_{Ti} is the thermal velocity of the ions. The index l denotes that we are considering Langmuir plasma waves, whose frequency is

$$(\omega^l(\mathbf{k}_1))^2 = \omega_{0e}^2 + 3v_{Te}^2k^2. \tag{5.2}$$

For the electromagnetic waves we have

$$\omega^2(\mathbf{k}) = \omega_{0e}^2 + c^2k^2. \tag{5.3}$$

We confine ourselves here to the case of isotropic turbulence; it is then possible to assume that $[\mathbf{k}\times\mathbf{k}_1]^2 = k^2k_1^2/3$. Recognizing also that $k \ll k_1$ (this follows from (5.2) and (5.3)), we get

$$I_\omega = \frac{k^2}{v_g}\frac{Z^2\omega_{0e}^2 n_z}{6(2\pi)^{5/2}n_e^2}\left(\frac{T_i}{T_i+T_e}\right)^2\int\frac{F_l(\mathbf{k}_1)\,dk_1}{k_1v_{Ti}}e^{-\frac{1}{2}\left[\frac{\omega-\omega^l(\mathbf{k}_1)}{v_{Ti}k_1}\right]^2} \tag{5.4}$$

At a specified spectral function of the plasma turbulence, the integral can be easily calculated. In particular, assuming that the scattering is by thermal protons ($Z = 1$, $n_Z = n_e$), and assuming that the maximum of the energy density of the plasma turbulence occurs at a definite value of the phase velocity v_{p0}, we obtain for the total emission coefficient (integrated over the frequencies) in an isotropic plasma

$$I = 4\pi\int I_\omega\,d\omega \approx \frac{e^4v_{Te}}{m_e^2c^3v_{p0}}n_eW^l, \tag{5.5}$$

which coincides with the well-known expression for the conversion coefficient in scattering by thermal fluctuations.

Thus, the known coefficient of conversion by scattering at the frequency ω_{0e} is valid, generally speaking, for the case of a monochromatic turbulence and determines only the total radiation. Formula (5.4) is valid for any plasma-turbulence energy distribution function with respect to the wave numbers, and makes it possible to calculate the spectral distribution of the converted radiation.

The coefficient of induced conversion is similarly calculated in accordance with the general method of Secs. 2 and 3. For an isotropic turbulence we get

$$\mu(\omega) = \frac{(2\pi)^{1/2}}{6}\frac{Z^2\omega_{0e}n_zT_i}{v_gn_e^3(T_e+T_i)^2}\int(\omega-\omega^l(\mathbf{k}_1))\frac{F_l(\mathbf{k}_1)\,dk_1}{k_1v_{Ti}}e^{-\frac{1}{2}\left[\frac{\omega-\omega^l(\mathbf{k}_1)}{v_{Ti}k_1}\right]^2}. \tag{5.6}$$

It follows therefore in accordance with the known results (see [7,8]) that in the scattering of isotropic plasma waves by isotropically distributed particles, the buildup of the radiation ($\mu(\omega) < 0$) occurs only for conversion in which the frequency is decreased ($\omega < \omega^l$). In this case buildup is possible only for such plasma-turbulence spectra, for which the integral (5.6), with allowance for (5.2), is negative for at least some interval of the frequencies ω. This occurs, for example, when the wave numbers of the plasma turbulence are bounded from below.

Let us consider first the region of induced absorption $\omega > \omega^l(\mathbf{k}_1)$. If $F_l(\mathbf{k}_1)$ does not change here very abruptly, then the integral (5.6) can be readily estimated:

$$\mu(\omega) \approx \frac{\pi^{1/2}}{3}\frac{Z^2\omega_{0e}n_zT_i}{n_e^2(T_e+T_i)}W^l. \tag{5.7}$$

In particular, for scattering by protons in an isothermal plasma ($T_l = T_i = T$, $n_e = n_Z$, $z = 1$) we have

$$\mu(\omega) \approx \frac{\pi^{1/2}}{12}\frac{\omega_{0e}W^l}{v_gn_eT} \approx \frac{(\pi/3)^{1/2}}{12}\frac{\omega_{0e}W^l\langle v_p\rangle}{n_em_ev_{Te}^3c}, \tag{5.8}$$

where $\langle v_p\rangle$ is a certain average phase velocity of the plasma waves (here $v_g = c\sqrt{3}\,v_{Te}/v_p$).

Comparison with (3.10) gives a condition under which the inverse conversion of the electromagnetic waves into plasma waves exceeds the true bremsstrahlung absorption:

$$W^l \gtrsim \frac{3\Lambda}{\sqrt{2}\,\pi^2}\frac{m_e\omega_{0e}^3}{v_{Te}}. \tag{5.9}$$

Finally, estimating the integral in (5.4) in exactly the same manner as in (5.6), we obtain for the effective temperature

$$T_{\text{eff}} \approx T_i\frac{\langle v_p\rangle}{v_{Ti}}. \tag{5.10}$$

We emphasize once more that the spontaneous conversion cannot give a brightness temperature larger than (5.10).

We now consider induced radiation—the buildup of the radiation field at frequencies smaller than ω^l. We assume first that the greater part of the energy in the spectral distribution of the plasma turbulence falls to wave numbers close to a certain value $k_{1,0}$. Then, if

$$\frac{3}{2}\frac{v_{Te}^2k_{1,0}^2}{\omega_{0e}} \gg k_{1,0}v_{Ti}, \quad \frac{3v_{Te}^2}{2v_p} \gg v_{Ti}, \tag{5.11}$$

then the width of the frequency interval in which the buildup takes place is of the same order as $v_{Ti}k_{1,0}$. The buildup coefficient is also determined by formulas (5.7) and (5.8), but with the negative sign. For example, for an isothermal hydrogen plasma

$$\mu(\omega) \approx -\frac{(\pi/3)^{1/2}}{12}\frac{\omega_{0e}W^l\langle v_p\rangle}{n_em_ecv_{Te}^3}. \tag{5.12}$$

The same condition (5.9) determines the possibility of a buildup with allowance for true absorption.

When the inequality opposite to (5.11) holds, i.e., when $v_{Ti} \gg 3v_{Te}^2/2v_p$, the width of the frequency interval is smaller, since ω is limited on the low-frequency side by ω_{0e} (or even by a somewhat larger value); here

$$\Delta\omega \approx \frac{3}{2}\frac{v_{Te}^2}{\omega_{0e}^2}k_{1,0}^2 \approx \frac{3}{2}\frac{v_{Te}^2}{v_p^2}\omega_{0e},$$

and the coefficient of induced conversion for an isothermal hydrogen plasma is

$$\mu(\omega) \approx -\frac{(\pi/3)^{1/2}}{8}\frac{\omega_{0e}W^l}{n_em_ecv_{Ti}}\frac{1}{\langle v_p\rangle}. \tag{5.13}$$

We shall continue a discussion of these formulas in Sec. 12.

Conversion in scattering of plasma waves by thermal electrons will not be discussed in detail. As already noted, the nonlinear scattering by the polarization charge and the Compton scattering by the electron itself are partly canceled out here. However, this cancelation is violated, for example, if the change of the frequency in the conversion is small ($|\omega - \omega^l| \ll k_1v_{Ti}$). In this case the nonlinear scattering predominates over the Compton scattering, and it is possible to use for the calculation of all the coefficients the formulas (5.1)–(5.4) and (5.5), in which v_{Ti} should be replaced by v_{Te}. Recognizing that the interval of the converted frequencies is the same here as in the case of scattering by ions, we find that the coefficients of emission and of induced conversion are smaller by the factors v_{Te}/v_{Ti} and $v_{Te}T_e/v_{Ti}T_i$, respectively.

The effective radiation temperature in spontaneous conversion is determined by the same formula (5.10), in which T_i should be replaced by T_e. Therefore conversion in scattering by thermal electrons must be taken into account in a strongly non-isothermal plasma when $T_e \gtrsim T_i (m_i/m_e)^{1/2}$.

According to the formulas presented above, for conversion in scattering by thermal ions the probability of the change of the frequency by an amount $\Delta\omega$ larger than $v_{Ti}k \approx \omega_{0e}v_{Ti}/v_p$, decreases exponentially with increasing $|\Delta\omega|$. In the case of scattering by electrons, the exponential decrease begins at considerably larger change of frequency: $|\Delta\omega| > v_{Te}k \approx \omega_{0e}v_{Te}/v_p$. Therefore in the frequency variation interval

$$\frac{v_{Ti}}{v_p} \ll \frac{|\Delta\omega|}{\omega_{0e}} \ll \frac{v_{Te}}{v_p} \tag{5.14}$$

the scattering by electrons, although greatly weakened by the compensation, prevails over scattering by ions. The spontaneous-radiation and induced-conversion coefficients are in this case smaller by another factor m_i/m_e than in the case of conversion with change of frequency by $\Delta\omega \approx \omega_{0e}v_{Ti}/v_p$.

6. CONVERSION OF PLASMA WAVES INTO ELECTROMAGNETIC WAVES AT THE GYROFREQUENCY ω_{He} IN SCATTERING BY THERMAL IONS

In a plasma with a strong magnetic field ($\omega_{He} \gg \omega_{0e}$), electron plasma waves are excited on two branches of the dispersion curves: one with frequency close to

$$\omega_{He} + \frac{\omega_{0e}^2}{2\omega_{He}} \sin^2 \vartheta_1$$

(a more complete dispersion relation is given by (4.2)) and the other at a frequency close to $\omega_{0e} \cos \vartheta_1$; here ϑ_1 is the angle between the wave vector of the plasma wave $\mathbf{k}_1$ and the external magnetic field $\mathbf{H}$. The conversion of the first branch of the plasma waves into electromagnetic waves can yield radiation at frequencies of order ω_{He}. In [14], where this problem was first considered, it was assumed that the probability of conversion at frequencies ω_{He} is of the same order as at frequencies ω_{0e}. A direct calculation of the conversion coefficients was carried out in this case in [9]. However, before presenting the results of calculations of the conversion at the frequencies ω_{He}, we note the following.

As is well known, when a beam of fast electrons passes through an isotropic plasma without a magnetic field, plasma waves are excited in it at a frequency ω_{0e}, with increment

$$\gamma \approx \omega_{0e} \frac{n_s}{n_e} \left(\frac{v_s}{\Delta v_s}\right)^2,$$

where n_s is the concentration of the particles in the beam, and v_s and Δv_s are their average velocity and the velocity spread. In a plasma with a magnetic field, under the condition $\omega_{He} \gg \omega_{0e}$, the beam excites plasma waves at frequencies ω_{He}, but the increment is already smaller:[9]

$$\gamma = \omega_{0e} \left(\frac{\omega_0}{\omega_{He}}\right)^3 \frac{n_s}{n_e} \left(\frac{v_s}{\Delta v_s}\right)^2 \sin^2 \vartheta_1, \tag{6.1}$$

where ϑ_i is the angle between the direction of the growing plasma wave and the direction of the external magnetic field. It follows from (6.1) that in a strong magnetic field ($\omega_{He} \gg \omega_{0e}$) the buildup increment of the plasma waves at frequencies of order ω_{He} is in general small, and a large plasma-turbulence energy density can be obtained here only in the case of a sufficiently intense beam.

We now turn to the conversion of plasma waves at the frequency ω_{He} (the corresponding parameters of the plasma turbulence will be denoted by the index h).

In the previously derived formulas for scattering by particles it was assumed that the particles move in straight lines. In the case of an external magnetic field it is necessary, in general, to take into account the helical motion of the particles, when their Larmor radius is smaller than the plasma wavelength. As a result, the expressions obtained under the assumption of linear particle motion are valid if the ion velocities satisfy the inequality

$$\frac{m_i c}{eH} v \gg \frac{1}{k} \approx \frac{v_p}{\omega_{He}}, \quad v \gg \frac{m_e}{m_i} v_p, \tag{6.2}$$

where v_p are the phase velocities of the plasma waves. For thermal ions, the condition (6.2) is equivalent to the requirement $v_p \ll \sqrt{m_i/m_e}\, v_{Te}$ (at $T_i = T_e$). If the phase velocities are larger, then it is necessary to take into account the magnetization of the ions. In the case of scattering by electrons, the condition (6.2) assumes the form $v \gg v_p$, and therefore for thermal electrons the magnetization must always be taken into account in this case.

Another feature is connected with the fact that the emission and induced-conversion coefficients depend in this case on the angle ϑ between the wave vector of the electromagnetic wave and the direction of the external magnetic field.

We note finally that for a quasilongitudinal propagation of the electromagnetic waves in a magnetoactive plasma, the refractive index is

$$n^2(\omega) = 1 - \frac{\omega_{0e}^2}{\omega(\omega \pm \omega_{He} \cos\vartheta)}, \tag{6.3}$$

and if $\omega \approx \omega_{He} \gg \omega_{0e}$, then $n^2(\omega) \approx 1$, with the exception of the case of the extraordinary wave traveling strictly along the field. We can therefore assume here $v_g \approx c$.

An analysis of the general formulas of Secs. 2 and 3 shows that in those cases when the magnetization of the ions can be neglected, the formulas for the coefficients of spontaneous and induced conversion turn out to be the same as (5.4) and (5.6), in which it is only necessary to add the factor $(1 + \cos^2\vartheta)/2$ and to replace $\omega^l(\mathbf{k}_1)$ by $\omega^h(\mathbf{k}_1)$, where $\omega^h(\mathbf{k}_1)$ is now defined by (4.2), and not by (5.2) as in the case of conversion at frequencies close to ω_{0e}. Since no lower limit is now imposed by the refractive index on the frequency of the converted radiation, the conversion takes place in the frequency interval

$$|\Delta\omega| = |\omega - \omega^h(\mathbf{k}_1)| \leqslant k_1 v_{Ti} \approx \omega_{He} \frac{v_{Ti}}{\langle v_p\rangle}, \tag{6.4}$$

where, as before, $\langle v_p \rangle$ is the average phase velocity of the plasma waves. When $\Delta\omega < 0$ we have wave buildup, and at $\Delta\omega > 0$ absorption, the increment or decrement having the same absolute magnitude:

$$\mu(\omega) \approx \pm \frac{\pi^{1/2}}{3} \frac{Z^2\omega_{0e}^2 n_z T_i}{cn_e^2 (T_e + T_i)^2 \omega_{He}} W^h. \tag{6.5}$$

In particular, for a hydrogen isothermal plasma

$$\mu(\omega) \approx \pm \frac{\pi^{1/2}}{12} \frac{\omega_{0e}^2 W^h}{cn_e T \omega_{He}}. \tag{6.6}$$

In the case of spontaneous radiation, the effective temperature is defined as before by formula (5.10).

Thus, the coefficients of induced conversion at frequencies ω_{He}, other conditions being equal, are smaller by a factor ω_{He}/ω_{0e} than the corresponding conversion coefficients at the frequencies ω_{0e}. The coefficients of spontaneous conversion at the frequency ω_{He}, however, are larger by a factor ω_{He}/ω_{0e} (owing to the presence of the additional term k^2 in (5.1)).

In real astrophysical condition we have most frequently $\omega_{He} \ll \omega_{0e}$. If the opposite inequality is satisfied, then usually ω_{He} is not much larger than ω_{0e}. Therefore the conversion coefficients at both ω_{He} and ω_{0e}, or more accurately, when ω_{0e} and ω_{He} are comparable, become approximately of the same order at frequencies

$$\omega_\infty = \frac{\omega_{0e}^2 + \omega_{He}^2}{2} + \sqrt{\frac{(\omega_{He}^2 + \omega_{0e}^2)^2}{4} - \omega_{0e}^2 \omega_{He}^2 \cos^2\vartheta_1}. \tag{6.7}$$

Consequently, the considered conversion mechanism at $\omega_{0e} \sim \omega_{He}$ can result in continuous radiation in a wide frequency interval, comparable with ω_{0e} and ω_{He} themselves. It is important to emphasize that there is a much greater field here for induced radiation, since the difference $|\omega - \omega^h(\mathbf{k}_1)|$ is not limited to the small quantity $|\omega_{0e} - \omega^l(\mathbf{k}_1)|$.

As already noted, conversion on electrons or ions, in the case of large plasma-wave phase velocities, depends on the helical motion of the scattering particles. This problem is also considered in [9].

Finally, we note that buildup at frequencies close to ω_{He} is possible only if the coefficient of the true gyroresonance absorption at these frequencies is noticeably smaller than (6.5) or (6.6). In order of magnitude, the coefficient of gyroresonance absorption for ordinary waves, at frequencies close to ω_{He} and $2\omega_{He}$, is approximately the same:[2]

$$\mu_g(\omega_{He}) \approx \mu_{2g}(2\omega_{He}) \approx \frac{\pi}{c} \frac{\omega_{0e}^2}{\omega_{He}} \frac{v_{Te}^2}{c^2} \sin^2\vartheta \tag{6.8}$$

in the frequency interval $\Delta\omega/\omega_{He} \approx v_{Te}/c$. Recognizing that the conversion does not occur exactly at the frequency ω_{He}, but at the frequency

$$\omega_{He} + \frac{\omega_{0e}^2}{2\omega_{He}} \sin^2\vartheta_1 \approx \omega_{He} + \frac{1}{4}\frac{\omega_{0e}^2}{\omega_{He}},$$

we find that gyroresonance absorption can be significant only if the inequality

$$\frac{\omega_{0e}^2}{4\omega_{He}} \leqslant \omega_{He} \frac{v_{Te}}{c} \tag{6.9}$$

is fulfilled. Under cosmic conditions, the magnetic field is rarely so strong.

But even in this region, its magnitude can be made sufficiently small by decreasing the angle ϑ. Comparing (6.8) and (6.6), we obtain the conditions under which gyroresonance absorption can be neglected:

$$W^h \gg n_e T_e \frac{v_{Te}^2}{c^2} \sin^4\vartheta. \tag{6.10}$$

It follows from this, incidentally, that the growth of the radiation field at frequencies close to ω_{He} leads to generation of predominantly directed radiation of the ordinary wave.

7. CONVERSION OF PLASMA WAVES INTO ELECTROMAGNETIC WAVES IN SCATTERING BY EPITHERMAL NONRELATIVISTIC IONS

There are grounds for expecting to find frequently in a plasma an admixture of epithermal particles, the velocity of which exceeds v_{Ti} or even v_{Te}, but is much lower than that of light. The estimated[15] relative concentration of such particles in interstellar space and in nebulas is of the order of 10^{-7}. Recent space-rocket research has revealed the presence of a large number of epithermal particles in interplanetary space.

As already noted (Sec. 4), conversion of epithermal particles leads to generation of radiation with frequencies

$$\omega = \omega_1(\mathbf{k}_1) + \mathbf{k}_1\mathbf{v}_q, \tag{7.1}$$

where $\omega_1(\mathbf{k}_1)$ is the frequency of the plasma waves and $\mathbf{v}_q$ is the velocity of the particle. For example, if $\omega_1 \approx \omega_{0e}$, then the frequency ω turns out to be in the interval from ω_{0e} to $\omega_{0e}[1 + (v_q/v_p)]$. Thus, this mechanisms can yield a continuous radiation spectrum in a sufficiently broad frequency interval, provided the phase velocities of the plasma waves are small.

If the plasma turbulence is isotropic and the distribution of the epithermal-particle velocities is also isotropic, then such a radiation mechanism can only be spontaneous. It is easy to see that in this case the effective radiation temperature is of the order of the average energy of the epithermal particles: $T_{eff} \cong m_q \langle v_q^2 \rangle$, where m_q is the particle mass. Strictly speaking, the effective temperature depends also on the radiation frequency. In fact, let us assume $\omega \gg \omega_{0e}$, then $\omega = k_1 v_q \cos\vartheta$. If, in addition, the particle spectrum decreases with increasing energy, then T_{eff} is determined by the minimum v_q, which for a specified ω corresponds to $\cos\vartheta = 1$, i.e.,

$$v_q = \frac{\omega}{k_1} = \frac{\omega}{\omega_{0e}} v_p.$$

Thus,

$$T_{eff} = m_q v_p^2 \left(\frac{\omega}{\omega_{0e}}\right)^2 \tag{7.2}$$

and T_{eff} increases rapidly with frequency. It should be borne in mind that the latter formula holds only if $\mu \gg \mu_T$ and $v_q \ll c$.

The coefficient of induced conversion (absorption) can be readily determined by a general method in the same manner as in Sec. 5 for conversion on thermal ions. Inasmuch as the velocity distribution function of the epithermal particles is unknown in any case, we confine ourselves only to an estimate of the absorption coefficient in the case of inverse conversion on ions. We present here the result for the case when the number of ions decreases sufficiently rapidly with energy, and the effect of absorption is determined by the ions with the minimum velocity $v_q = v_{min}$, capable of taking part in the conversion processes:

$$\mu \approx \frac{Z^2 \omega_{0e} n_q}{n_e^2 m_q c v_p^2} \left(\frac{\omega_{0e}}{\omega}\right)^5 W^l. \tag{7.3}$$

In the case $\omega_1 \approx \omega_{0e}$ we have here $v_p \approx \omega_{0e}/k_1$. On the other hand, if $\omega_1 \approx \omega_{He}$, i.e., a strong magnetic field is present, then $v_p = \omega_{He}/k_1 \sin \vartheta$ and

$$n_q = \int_{\omega/k_1}^{\infty} f(\mathbf{v}_q)\, d\mathbf{v}_q$$

is the number of ions taking part in the process. The dependence of the absorption coefficient on the frequency is determined, strictly speaking, both by the plasma-turbulence spectrum and by the velocity distribution of the epithermal particles. It is easy to see that when k_1 increases $\omega_{min} = \omega/k_1$ decreases, i.e., n_q increases, and consequently the principal role is assumed by the maximal k_1 excited in the plasma-turbulence spectrum. At $k_1 \approx k_{max} \approx \omega_{0e}/v_{Te}$, the value of $\mu(\omega)$ in a strong magnetic field on the ω_{He} branch is smaller by a factor ω_{He}/ω_{0e} than on the ω_{0e} branch in a weak magnetic field. We note that for the electrons the result for T_{eff} can be obtained from the same formula as for ions, and is therefore smaller by a factor m_i/m_e. However, the increment or the absolute magnitude of the radiation intensity are influenced by compensation effects, and the estimate differs from (7.3). For $\omega_1 \approx \omega_{0e}$ we have

$$\mu \approx \frac{\omega_{0e} n_q}{n_e^2 m_e c^3} \left(\frac{\omega_{0e}}{\omega}\right)^3 W^l \tag{7.4}$$

and for $\omega_1 \approx \omega_{He}$ ($\omega_{He} \gg \omega_{0e}$) we obtain in (7.4) an additional factor $(\omega_{0e}/\omega_{He})^2$.

As already noted, in the case of an isotropic velocity distribution of the epithermal particles, only spontaneous radiation takes place and reaches saturation as a result of self-absorption. However, if the epithermal particles form a beam or are anistropically distributed, we get here, besides the spontaneous radiation, also a buildup of radiation. Assume that all the particles of this beam have the same velocity v_s (both in magnitude and in direction). We then obtain for the coefficient of induced radiation, in the case of scattering by an electron beam

$$\mu(\omega) = -\frac{2\pi n_s \omega_{0e}^2}{n_e m_e c v_g v_s} \frac{\omega_{0e}^2}{\omega^2} \int [(\cos^2 \vartheta_1 - \sin^2 \vartheta_1)^2 + 2\cos^4 \vartheta_1] W_{k_1}^l \frac{dk_1}{k_1 c}, \tag{7.5}$$

where ϑ_1 is the angle between the direction of motion of the beam and the wave vector of the plasma turbulence. For an isotropic plasma turbulence we have

$$\mu(\omega) = -\frac{32\pi n_s \omega_{0e}^2}{15 n_e^2 m_e c v_g v_s} \frac{\omega_{0e}^2}{\omega^2} \int F_l(\mathbf{k}_1) \frac{dk_1}{k_1 c} \approx -\frac{32\pi n_s \omega_{0e} v_p}{15 n_e^2 m_e c^2 v_g v_s} \frac{\omega_{0e}^2}{\omega^2} W^l, \tag{7.6}$$

where v_g, as before, is the group velocity of the electromagnetic waves. Similar formulas can be obtained for ion beams, the increment being of the order of (7.3). We recall that an appreciable increase of the frequency compared with ω_0 occurs only under the condition $\omega_{0e}/k_1 \ll v_s$. It must be borne in mind, however, that owing to the nonlinear interaction the phase velocities of the plasma-turbulence waves increase on the average. But in the presence of anisotropy (at least in the presence of this beam), a decrease of the phase velocities of the plasma waves is also possible, so that one cannot exclude the possibility that a sufficiently intense nonrelativistic beam of electrons or ions can generate and build up in a plasma radiation in a frequency interval of the order of $\omega_{0e} v_s / v_{Te}$.

8. CONVERSION IN COMPTON SCATTERING OF PLASMA WAVES BY RELATIVISTIC ELECTRONS

Even more effective (from the point of view of increasing the frequency) is the conversion occurring upon scattering by relativistic electrons. Since, in accord with (2.12), the condition

$$\omega - \omega_1(\mathbf{k}_1) = (\mathbf{k} - \mathbf{k}_1)\mathbf{v} \tag{8.1}$$

should be satisfied in the case of scattering, it follows that, recognizing that $\omega = k/n(\omega)$, where $n(\omega)$ is the refractive index, and that $v \approx c$,

$$\omega \approx \frac{kv}{1 - \frac{v}{c} n(\omega) \cos\vartheta} \leqslant \frac{\omega_1 c}{v_p} \frac{1}{\frac{1}{2}\left(\frac{m_e c^2}{\mathscr{E}}\right)^2 + \frac{\omega_{0e}^2}{2\omega^2}}. \tag{8.2}$$

If the second term in the denominator of (8.2) can be neglected, then

$$\omega \leqslant 2 \frac{\omega_1 c}{v_p} \left(\frac{\mathscr{E}}{m_e c^2}\right)^2, \tag{8.3}$$

where ω_1 should be taken to mean ω_{0e}, ω_{He}, or ω_{0i}, depending on the type of plasma turbulence. The second term in the denominator of (8.2) is small if

$$\frac{\mathscr{E}}{m_e c^2} > \frac{v_p}{c} \frac{\omega_{0e}}{\omega_1}. \tag{8.4}$$

This condition is always satisfied for relativistic particles if $v_p \ll c$ and $\omega_1 = \omega_{0e}$. On the other hand, if ω_1 is quite small and $v_p \omega_{0e}/c\omega_1 > 1$, then we obtain from (8.3) a condition that limits from below the energy of those particles, conversion on which yields electromagnetic radiation capable of emerging from the medium

$$\frac{\mathscr{E}}{m_e c^2} \gg \left(\frac{v_p}{c} \frac{\omega_{0e}}{\omega_1}\right)^{1/2},$$

a condition which is certainly satisfied if (8.4) holds. If $\omega \ll \omega_{0e} \mathscr{E}/m_e c^2$, then the frequency of the converted radiation is

$$\omega \approx \frac{\omega_{0e}^2}{\omega_1(\mathbf{k}_1)} \frac{v_p}{c}. \tag{8.5}$$

Radiation at this frequency can emerge from the medium only if the frequency of the plasma-turbulence waves is sufficiently low ($\omega_{0e} v_p/\omega_1 c > 1$; for example, $\omega_1 \approx \omega_{0i}$, or even Alfven and other hydromagnetic waves). On the other hand, the condition $\omega \ll \omega_{0e} \mathscr{E}/m_e c^2$, under which (8.5) is valid, assumes the form (8.4). Thus, if $\omega_{0e} v_p/\omega_1 c > 1$, then only electrons of sufficiently high energies radiate, and the radiation frequency is limited from below by the condition (8.5) ((8.5) is the minimum radiated frequency). For relativistic electrons, the coefficient of Compton scattering is much larger than the coefficient of nonlinear scattering by the polarization charge. We therefore neglect here the nonlinear-scattering effect. The second-rank tensor Λ_{ij} for Compton scattering is given in [8]. Contracting it with respect to one index with the wave vector of the plasma turbulence, we obtain the current conductivity vector of the Compton scattering of the plasma waves by relativistic electrons

$$\Lambda = \frac{ie^2}{(2\pi)^3 m_e k_1} \frac{\sqrt{1-v^2/c^2}}{[\omega-(\mathbf{kv})]^2} \{\mathbf{v}[(\mathbf{kk}_1) - c^{-2}\omega(\mathbf{k}_1\mathbf{v})] + \mathbf{k}_1[\omega-(\mathbf{kv})]\}. \quad (8.6)$$

This vector coincides, apart from a numerical factor, with the vector β obtained in [10], also for Compton scattering by relativistic electrons. Substituting (8.6) in (2.5), we obtain the radiation coefficient for conversion in scattering by relativistic electrons. The plasma turbulence will be assumed, as before, isotropic. For the particle-velocity distribution functions we choose several approximating expressions, since its true form is not known exactly under cosmic conditions.

1) Isotropic velocity distribution. Here the distribution function depends on the particle energy ($\mathcal{E} = cp$). The number of relativistic electrons per unit volume with energy from $\mathcal{E}$ to $\mathcal{E} + d\mathcal{E}$ will be denoted by

$$dn(\mathcal{E}) = 4\pi\mathcal{E}^2 f_0(\mathcal{E})\, d\mathcal{E}. \quad (8.7)$$

For $f_0(\mathcal{E})$ one frequently chooses the power-law function

$$f_0(\mathcal{E}) = \frac{K_0}{\mathcal{E}^\alpha}. \quad (8.8)$$

In place of the index α, we shall use below also the index $\gamma = \alpha - 2$. The total number of electrons with energy exceeding a certain value $\mathcal{E}_0$ will be

$$n(>\mathcal{E}_0) = \frac{4\pi K_0}{(\alpha-3)\mathcal{E}_0^{\alpha-3}}. \quad (8.9)$$

2) Beam of relativistic electrons moving in the same direction. The number of particles with momentum vector in the interval from $\mathbf{p}$ to $\mathbf{p} + d\mathbf{p}$ is

$$dn(\mathbf{p}) = f_1(\mathbf{p}_{\parallel})\,\delta(\mathbf{p}_{\perp})\, d\mathbf{p}_{\parallel}\, d\mathbf{p}_{\perp}, \quad (8.10)$$

where $\mathbf{p}_{\parallel}$ is the momentum component along the beam, and $\mathbf{p}_{\perp}$ is perpendicular to the beam. Assuming that $f_1(\mathbf{p}_{\parallel})$ is a power-law function, $f_1(\mathbf{p}_{\parallel}) = K_1\mathcal{E}^{-\alpha}$ (since $p_{\parallel} = \mathcal{E}/c$), we have for the number of particles with energy larger than $\mathcal{E}_1$

$$n(>\mathcal{E}_1) = \frac{K_1}{(\alpha-1)\mathcal{E}_1^{\alpha-1}}. \quad (8.11)$$

We note incidentally that in order to obtain a buildup of the radiation field, it is not at all essential that the energy spectrum of the relativistic particles have a steep cut-off some place on the low-energy side. It can go over smoothly into the energy spectrum of the epithermal particles, and then also of the thermal particles.

3) In some cases it is also possible to obtain not too complicated formulas for an anisotropic particle velocity distribution in the form

$$f_e(\mathcal{E}, \varphi) = f_0(\mathcal{E})\,[1 + \eta(\mathcal{E})\cos\varphi], \quad (8.12)$$

where $f_0(\mathcal{E})$ is the isotropic distribution and $\eta(\mathcal{E})$ also depends only on the particle energy, and φ is the angle between the particle velocity vector and a certain preferred direction, say, the line of sight to the observer. The indicated choice of formulas describing the velocity distribution of the relativistic particles makes it possible to obtain not very complicated formulas, suitable for an analysis of all the cases of conversion that can be encountered at the present time in astrophysical problems.

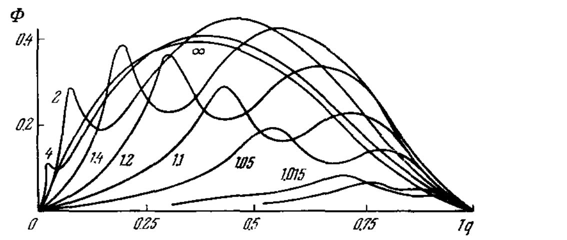

FIG. 2. The function $\Phi(\mathcal{E}, q)$ vs. the parameter q at different values of the energy. The presence of two maxima corresponds to the combination of Compton and nonlinear scattering. The numbers at the curves determine the values of $\gamma = \mathcal{E}/m_ec^2$.

Using (8.6) and the particle distribution functions presented above, we can easily determine from formulas (2.11) and (3.6) the parameters of the plasma mechanisms of radiation due to conversion in scattering by relativistic electrons. We present only the final expressions for $v_p \ll c$.

a) Spontaneous Conversion of Plasma Waves on Anisotropically Distributed Relativistic Electrons

The radiation coefficient is

$$I_\omega = \frac{\omega_{0e}^4}{4m_e^2c^4 n_e^2 c^3\omega} \int \mathcal{E}^2 f_0(\mathcal{E})\, d\mathcal{E} \int\limits_{k_1 > \frac{\omega}{2c}\left(\frac{m_ec^2}{\mathcal{E}}\right)^2}^{\infty} \Phi(\mathcal{E}, q) F_l(\mathbf{k}_1)\, dk_1. \quad (8.13)$$

where the dimensionless function $\Phi(\mathcal{E}, q)$ depends on the energy and on the dimensionless parameter

$$q = \frac{\omega}{k_1}\frac{c-v}{cv} \approx \frac{\omega}{2k_1c}\left(\frac{m_ec^2}{\mathcal{E}}\right)^2. \quad (8.14)$$

Plots of the function $\Phi(\mathcal{E}, q)$ for different values of $\mathcal{E}$ are given in Fig. 2, which is taken from [10], where the conversion of plasma waves on relativistic electrons was calculated for the first time. It should be noted that at low energies, the compensation of the Compton and nonlinear scattering is important, and therefore Fig. 2 presents the functions $\Phi(\mathcal{E}, q)$ with allowance for this effect.

The following analytic expression was obtained for the function $\Phi(\infty, q)$ in the case of ultrarelativistic particles:[10]

$$\Phi(\infty, q) = \frac{8}{3}q\{(1-q)^3 - 3q^2(1 - q + \ln q)\}. \quad (8.15)$$

The function $\Phi(\infty, q)$ has a maximum at $q = 0.30$ ($q < 1$). This means that in scattering by electrons of definite energy $\mathcal{E}$, the largest contribution to the conversion of plasma waves into electromagnetic waves with a given frequency ω is made by plasma waves with wave numbers

$$k_1 \approx 1.5\frac{\omega}{c}\left(\frac{m_ec^2}{\mathcal{E}}\right)^2. \quad (8.16)$$

Unlike the synchrotron mechanism, where the radiation spectrum is determined in practice only by the energy distribution of the electrons, the radiation spectrum depends here both on the electron energy distribution and on the distribution of the phase velocities of the plasma waves. From the point of view of the interpretation of the radio emission of cosmic sources, this circumstance has an important advantage. Apparently, under cosmic conditions the energy distribution function of the nonrelativistic electrons cannot vary rapidly. In addition, this function has, as a rule, a smooth character. Therefore, certain rapidly changing singularities of the spectrum of a number of cosmic sources, the interpretation of which in the framework of the synchrotron mechanism calls for equally rapid and abrupt changes of the electron-energy distribution function, can be interpreted in this case as being due to the change of spectrum of the plasma turbulence even in the case of a stationary distribution function of the relativistic-electron energies.

Further transformations consist of substituting the expressions (8.8), (8.9), and (8.15) in (8.13) and evaluating the integrals. This can be easily done, but in order not to write out here the complicated expressions, we present a formula for the radiation intensity at frequencies ω satisfying the condition that the value of $\mathcal{E}$ from (8.15) lies, for all the plasma-turbulence wave numbers, in the energy interval $\mathcal{E}_0 < \mathcal{E} < \infty$, for which the energy spectrum of the particles (8.8) is defined. We then obtain

$$I_\omega = b(\alpha)\frac{\omega_{0e}^4 n(>\mathcal{E}_0)}{c^3 n_e^2 \omega}\left[\left(\frac{\mathcal{E}_0}{m_e c^2}\right)^2 \frac{2c}{\omega}\right]^{\frac{\alpha-5}{2}} \int F_l(\mathbf{k}_1)\frac{dk_1}{k_1^{\frac{5-\alpha}{2}}}\left(\frac{\mathcal{E}_0}{m_e c^2}\right)^2, \tag{8.17}$$

where the numerical factor is

$$b(\alpha) = \frac{\alpha-3}{6\pi}\left[\frac{1}{\alpha-3} - \frac{3}{\alpha-1} + \frac{2}{\alpha+3} + \frac{6}{(\alpha-1)^2}\right]. \tag{8.18}$$

If the indicated condition is not satisfied (ω is too small), then the radiation spectrum in this frequency region is determined either by the distribution of the particles of lower energy, or decreases sufficiently rapidly in accordance with the dependence of $\Phi(\infty, q)$ on q and with the plasma-turbulence spectrum. Formula (8.17) shows that here, too, the radiation spectrum has a power-law character. In this case, too, as in the case of synchrotron radiation, the exponent in the spectrum (the spectral index) equals $(\alpha - 3)/2 = (\gamma - 1)/2$.

Incidentally, the frequency of the plasma waves is not determined in (8.17) and (8.18). These formulas are valid both for conversion of waves with $k_1 \approx \omega_{0e}/v_p$, and for the conversion of plasma waves with $k_1 \approx \omega_{He}/v_p$. Therefore in a magnetoactive plasma with plasma turbulence, the relativistic electron radiates "twice": synchrotron radiation at the frequency

$$\omega \approx 0.29\omega_{He}\left(\frac{\mathcal{E}}{m_e c^2}\right)^2 \tag{8.19}$$

and plasma-mechanism radiation at the frequency

$$\omega = \omega_{0e}\frac{2c}{\langle v_p\rangle}\left(\frac{\mathcal{E}}{m_e c^2}\right)^2,$$

if $\omega_{0e} \gg \omega_{He}$ or

$$\omega \approx \omega_{He}\frac{2c}{\langle v_p\rangle}\left(\frac{\mathcal{E}}{m_e c^2}\right)^2, \tag{8.20}$$

if $\omega_{He} \gg \omega_{0e}$. The relative intensity of the two mechanisms is of the order of the ratio of the densities of the magnetic and plasma energies, multiplied by $c/\langle v_p\rangle$.

Thus, the conversion of plasma waves on isotropic relativistic electrons is of interest, from the astrophysical point of view, principally because in the case of a low-temperature plasma with plasma turbulence at low phase velocities it can lead to generation of high-frequency radiation. Another important factor is the possibility of radiation in a plasma without a magnetic field. In all other cases this mechanism is less effective than the synchrotron mechanism, since it still requires excitation of intense plasma turbulence.

Induced conversion on isotropically distributed relativistic electrons leads only to the absorption of electromagnetic waves. The corresponding absorption coefficient is calculated by means of the formula

$$\mu(\omega) = \frac{\pi^2\omega_{0e}^4}{2\omega^2 n_e^2 c^2}\int \mathcal{E} f_0(\mathcal{E})\, d\mathcal{E} \int\limits_{k_1 > \frac{\omega}{2c}\left(\frac{m_e c^2}{\mathcal{E}}\right)^2}^{\infty} \Phi_1(\mathcal{E}, q)\, F_l(\mathbf{k}_1)\frac{dk_1}{k_1}. \tag{8.21}$$

For ultrarelativistic particles we have

$$\Phi_1(\mathcal{E}, q) = \frac{8}{3}\left[(1-q)^3 + 3q(1 - q^2 + q\ln q)\right], \tag{8.22}$$

where q is defined as before by (8.14). We shall not stop to analyze these formulas. For a rough estimate of $\mu(\omega)$ we can use the well known relation

$$\mu \approx \frac{I_\omega}{\mathcal{E}k^2}\cdot 2\pi^2 \approx \frac{I_\omega c^2}{\mathcal{E}\omega^2}\cdot 2\pi^2.$$

b) Induced Conversion of Plasma Waves on a Beam of Relativistic Electrons

Whereas the conversion of plasma waves on isotropically distributed relativistic electrons, as shown above, does not offer special advantages compared with the usual synchrotron mechanism, in the case of scattering by an anisotropic beam the situation is radically altered. The point is that a possibility arises of obtaining induced conversion, i.e., buildup of the radiation field.

We recall that for synchronous buildup it is necessary to have a sufficiently high plasma concentration,[16] and in this case the buildup is possible in any case at relatively low frequencies

$$\omega \approx \left(0.24\frac{\omega_{0e}^3}{\omega_{He}}\frac{\mathcal{E}}{m_e c^2}\right)^{1/2}. \tag{8.23}$$

Scattering of plasma waves by a beam of relativistic particles is capable of yielding buildup at very high frequencies

$$\omega \approx 2\omega_{0e}\frac{c}{v_p}\left(\frac{\mathcal{E}}{m_e c^2}\right)^2$$

and calls in this case for a much lower concentration of either relativistic particles or plasma electrons.[17] To be sure, it is necessary to have here plasma turbulence and anisotropy of the relativistic-particle velocities.

The radiation and induced-conversion coefficients are calculated as before in accordance with the general formulas, with allowance for (8.6) and for the anisotropic distribution function. Unfortunately, however, in the general case it is practically impossible to average $[\mathbf{k} \times \Lambda]^2$ over the angles. The case of induced conversion by relativistic beams was considered in [18] using a general expression for the plasma-turbulence waves. We present here simpler and more convenient formulas for the calculation of induced conversion of an isotropic plasma turbulence, first for the case of scattering by a beam of electrons with a distribution function (8.10) and (8.11), followed by the corresponding formulas for the case of the distribution function (8.12). We shall not calculate the coefficient of spontaneous radiation, for if necessary it can be estimated by means of formula (8.17). The main interesting feature of our problem is the possible buildup of radiation at high frequencies.

Substituting (8.10) and (8.6) in the general expression for the induced radiation and expanding $[\mathbf{k}\times\Lambda]^2$ in powers of the angle ϑ between the line of sight (vector $\mathbf{k}$) and the beam direction (vector $\mathbf{v}$), we obtain after averaging with the aid of a δ-function with respect to the angle between $\mathbf{k}_1$ and $\mathbf{v}$

$$\mu(\omega,\vartheta)=-\frac{\pi\omega_{0e}^4}{n_e^2c^2\omega^2}\frac{\mathscr{E}^2(m_ec^2)^2}{[(m_ec^2)^2+(\mathscr{E}\vartheta)^2]^2}\times\int\frac{\partial f_1}{\partial\mathscr{E}}d\mathscr{E}\int\limits_{k_1>\frac{\omega}{2c}\left[\vartheta^2+\left(\frac{m_ec^2}{\mathscr{E}}\right)^2\right]}F_l(\mathbf{k}_1)\frac{dk_1}{k_1}\left\{\left[1-\frac{\omega^2}{4k_1^2c^2}\left(\vartheta^2+\left(\frac{m_ec^2}{\mathscr{E}}\right)^2\right)^2\right]\times\frac{(m_ec^2)^4+(\mathscr{E}\vartheta)^4}{[(m_ec^2)^2+(\mathscr{E}\vartheta)^2]^2}+\frac{\omega^2\vartheta^2}{k_1^2c^2}\left(\frac{m_ec^2}{\mathscr{E}}\right)^2\right\}. \quad (8.24)$$

Let us consider the dependence of the coefficient of induced conversion on the angle ϑ. For electromagnetic waves traveling strictly along the beam ($\vartheta = 0$) we have, after integrating by parts,

$$\mu(\omega,0)=\frac{\pi\omega_{0e}^4}{n_e^2m_e^2c^6\omega^2}\int f_1(\mathscr{E})\,\mathscr{E}\,d\mathscr{E}\int\limits_{k_1>\frac{\omega}{2c}\left(\frac{m_ec^2}{\mathscr{E}}\right)^2}F_l(\mathbf{k}_1)\frac{dk_1}{k_1}\left[1+\frac{\omega^2}{2k_1^2c^2}\left(\frac{m_ec^2}{\mathscr{E}}\right)^4\right] \quad (8.25)$$

These waves always attenuate. Formula (8.25) is approximately valid so long as $\vartheta < m_ec^2/\mathcal{E}$.

For the case $m_ec^2/\mathcal{E} \ll \vartheta \ll 1$, we have from (8.24) (also after integrating by parts)

$$\mu(\omega,\vartheta)=-\frac{2\pi m_e^2c^2\omega_{0e}^4}{n_e^2\omega^2\vartheta^4}\int\frac{f_1(\mathscr{E})\,d\mathscr{E}}{\mathscr{E}^3}\int\limits_{k_1>\frac{\omega\vartheta^2}{2c}}F_l(\mathbf{k}_1)\frac{dk_1}{k_1}\left[1-\frac{\omega^2\vartheta^4}{4k_1^2c^2}\right]. \quad (8.26)$$

In this case we have buildup of the radiation field. With increasing angle ϑ, the increment decreases and becomes minimal at $\vartheta = \pi/2$. For the latter case we have directly from the exact expression

$$\mu\left(\omega,\frac{\pi}{2}\right)=-\frac{\pi m_e^2c^2\omega_{0e}^4}{4n_e^2\omega^2}\int\frac{f_1(\mathscr{E})\,d\mathscr{E}}{\mathscr{E}^3}\int\limits_{k_1>\frac{\omega}{c}}F_l(\mathbf{k}_1)\frac{dk_1}{k_1}\left[1-\frac{\omega^2}{k_1^2c^2}\right]. \quad (8.27)$$

An approximate dependence of $\mu(\omega, \vartheta)$ on the angle is shown in Fig. 3.

In the calculation of the integrals with respect to energy in (8.26)–(8.27) it must be borne in mind that the coefficients of induced conversion are determined from the specified direction, i.e., from the specified value of the angle ϑ. Therefore, in order that radiation in a given direction be built up at a frequency ω, it is important that the scattering take place from all electrons with energy larger than $\mathcal{E}_\vartheta \approx m_ec^2/\vartheta$. The minimum radiation frequencies are limited by the condition $\omega < 2ck_{1\,max}/\vartheta^2$, where $k_{1\,max}$ is the maximum value of the wave numbers of the plasma turbulence.

Using (8.11) and assuming that $\vartheta > m_ec^2/\mathcal{E}_1$, where $\mathcal{E}_1$ is the lower limit of the energy spectrum of the relativistic electrons, we obtain from (8.26)

$$\mu(\omega,\vartheta)=-\frac{\alpha-1}{\alpha+2}\frac{2\pi\omega_{0e}^4n\left(>\frac{m_ec^2}{\vartheta}\right)}{n_em_ec^4\vartheta\omega^2}\times\int\limits_{k_1>\frac{\omega\vartheta^2}{2c}}F_l(\mathbf{k}_1)\frac{dk_1}{k_1}\left[1-\frac{\omega^2\vartheta^4}{4k_1^2c^2}\right]. \quad (8.28)$$

The upper limit of the spectrum of induced conversion, as already noted, is $2ck_{1\,max}\vartheta^2$. On the other hand, it

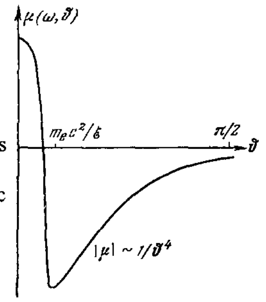

FIG. 3. Approximate plot of the coefficient of induced radiation (absorption) as a function of the angle ϑ in scattering of isotropic plasma waves by a monoenergetic beam of relativistic electrons.

follows from (8.2) that the lower limit of the obtained frequencies under the considered conditions ($1 \gg \vartheta \gg m_ec^2/\mathcal{E}_1$) is $2ck_{2\,max}/\vartheta$. Therefore, denoting by v_{p0} the phase velocity of the plasma waves at a maximum energy of the turbulent pulsation, and assuming that the spectrum of the wave numbers of the plasma turbulence is not too broad, we find that the radiation frequency at which induced buildup conversion takes place is

$$\omega_\vartheta \approx \omega_{0e}\frac{2c}{v_{p0}\vartheta^2}. \quad (8.29)$$

The induced conversion coefficient is in this case

$$\mu(\omega_\vartheta,\vartheta)=-\frac{\alpha-1}{\alpha+2}\frac{\omega_{0e}v_{p0}^3\vartheta^3}{n_e^2m_ec^6}W^ln\left(\mathscr{E}>\frac{m_ec^2}{\vartheta}\right). \quad (8.30)$$

The smaller ϑ, the larger the frequency, but on the other hand the smaller the radiation-field buildup coefficients, both as a result of the factor ϑ and as a result of the decrease of the number of electrons capable of leading to conversion at the given frequency.

Formula (8.30) makes it possible to estimate readily the possibility of induced conversion in all astrophysical applications of interest.

c) An Analytic Formula for the Conversion Coefficients Can Be Obtained Also in the Case of the Distribution Function (8.12)

We have for the induced-radiation coefficient

$$\mu(\omega)=\frac{\pi^2\omega_{0e}^4}{2n_e^2c\omega^2}\int f_0(\mathscr{E})\,\mathscr{E}\,d\mathscr{E}\int\limits_{k_1>\frac{\omega}{2c}\left(\frac{m_ec^2}{\mathscr{E}}\right)^2}\Phi_2(\mathscr{E},q)F_l(\mathbf{k}_1)\frac{dk_1}{k_1c}, \quad (8.31)$$

where the function $\Phi_2(\mathcal{E}, q)$ in the case when $\mathcal{E} \gg m_ec^2$ is

$$\Phi_2(\mathscr{E},q)=\frac{8}{3}\left\{(2+3\eta\cos\varphi)(1-q)^3+6q(1-q^2)-3(1-q)q^2\eta\cos\varphi+3q^2\ln q\,(2+\eta\cos\varphi)\right\}. \quad (8.32)$$

The function η can also depend here on the energy. If we assume η = const and assume expression (8.8) for $f_0(\mathcal{E})$, we obtain

$$\mu(\omega)=b_2(\alpha,\varphi)\frac{\omega_{0e}^4n(\mathscr{E}>\mathscr{E}_0)}{n_e^2\mathscr{E}_0c^2\omega^2}\left[\frac{2c}{\omega}\left(\frac{\mathscr{E}_0}{m_ec^2}\right)^2\right]^{\frac{\alpha-2}{2}}\int F_l(\mathbf{k}_1)\frac{dk_1}{k_1^{\frac{4-\alpha}{2}}}, \quad (8.33)$$

where the function $b_2(\alpha, \varphi)$ is given by

$$b_2(\alpha,\varphi)=\frac{\pi}{2\alpha(\alpha-1)(\alpha+2)(\alpha+1)^2}\times\{2\alpha^4+4\alpha^3+2\alpha+\eta\cos\varphi(3\alpha^4+5\alpha^3+2\alpha+2)\}. \quad (8.34)$$

Formulas (8.33) and (8.34) are valid under the same conditions as formulas (8.17)–(8.19). Buildup of the radiation field is obtained here when $\eta > 2/3$ and $\varphi < 0$. This formula can be used to estimate the buildup of the radiation field in the case of weak anisotropy. In this case, however, the buildup of the radiation field occurs mainly at lower frequencies and in directions opposite to the direction of motion of the weakly anisotropic beam.

Certain considerations regarding the application of the formulas obtained in this section will be given in Sec. 12.

9. CONVERSION OF PLASMA WAVES INTO ELECTROMAGNETIC WAVES IN NONLINEAR SCATTERING BY RELATIVISTIC IONS

Apparently, under cosmic conditions the number of relativistic protons exceeds the number of relativistic electrons. Consequently, (at least in certain cases), the conversion occurring during scattering by the polarization charge of relativistic protons, which is also accompanied by an increase of frequency, may turn out to be appreciable.

So long as the energy of the relativistic ion is smaller than

$$\mathscr{E} < m_i c^2 \frac{m_i}{m_e}, \tag{9.1}$$

the nonlinear scattering predominates. The conductivity vector for this case was obtained in [10]. We present directly the final expression for the coefficient of induced radiation in the case of isotropic turbulence and isotropic velocity distribution of the relativistic ions [25]

$$\mu(\omega, \mathbf{k}) = -\frac{Z^2\omega_{0e}^4\pi^2}{4n_e^2}\int\limits_{\mathscr{E}_{\text{кр}}}^{\infty} \frac{W_{k_1}k_1\,dk_1}{k^4\frac{1}{\omega}\frac{\partial}{\partial\omega}\omega^2\varepsilon\Big|_{\omega=\omega_1(\mathbf{k})}}\varphi\left(\mathscr{E}, \frac{\omega_1}{k}\right)\mathscr{E}^2\frac{d}{d\mathscr{E}}f(\mathscr{E})\,d\mathscr{E},$$

$$\varphi\left(\mathscr{E}, \frac{\omega_1}{k_1}\right) = \ln\frac{1+\frac{\omega_1}{k_1}}{\psi+\frac{\omega_1}{k_1}} + \frac{\psi+\frac{\omega_1}{k_1}}{1+\frac{\omega_1}{k_1}} - 1, \quad \psi(\mathscr{E}, \omega) = \frac{\omega}{2k_1}\left(\frac{m_i^2c^4}{\mathscr{E}^2} + \frac{\omega_{0e}^2}{\omega^2}\right). \tag{9.2}$$

The lower limit of integration with respect to energy is determined by the same relation $\omega = -\mathbf{k}_1 \cdot \mathbf{v}/(1 - (v/c)\cos\vartheta)$, and consequently the minimal energy of the relativistic ion capable of converting a plasma wave into an electromagnetic wave with given frequency ω is

$$\mathscr{E}_{\text{cr}}^2 = \frac{m_i^2c^4}{\frac{2k_1}{\omega} - \frac{\omega_{0e}^2}{\omega^2}}. \tag{9.3}$$

We shall calculate the integral with respect to energy in (9.2) for the power-law spectrum (8.8) and (8.9), it being assumed that $\mathcal{E}_{\min}$ from (9.2) is larger than $\mathcal{E}_0$—the lower limit of the energy spectrum of the relativistic ions at a given ω and at all values of the wave numbers of the plasma waves containing an appreciable fraction of the plasma-turbulence energy; we obtain

$$\mu(\omega) = \frac{(\alpha-1)Z^2\omega_{0e}}{8\alpha(\alpha+1)}\left(\frac{\omega_{0e}}{\omega}\right)^3\left(\frac{2k_1}{\omega}\right)^{\frac{\alpha+1}{2}}\frac{W^{\alpha}n_1(\mathscr{E} > \mathscr{E}_{\text{cr}})}{n_e^2 m_i c^2}. \tag{9.4}$$

We must emphasize the rapid decrease of the coefficient of induced conversion (absorption) with increasing frequency—approximately like $\omega^{-(\alpha+8)/2}$, which leads in the case of $\alpha \approx 4$ (the case most frequently encountered under cosmic conditions) to a frequency dependence $\mu \sim \omega^{-6}$. In our case, therefore, it is difficult to obtain noticeable conversion at high frequencies.

Formula (9.4) determines the absorption that takes place in a plasma with isotropic turbulence and with isotropically distributed relativistic ions. If the relativistic ions form an anisotropic particle beam, then induced buildup is also possible.

To estimate the buildup increment at a given frequency, we can also use formula (9.4). If the beam-particle energy spectrum is unknown, then for the roughest estimate we can assume $\alpha \approx 3$ and take n ($\mathcal{E} > \mathcal{E}_0$) simply to mean the total number of relativistic ions. Leaving out also the numerical factor, we obtain for the increment of buildup at the frequency ω

$$\mu(\omega) \approx -\frac{\omega_{0e}^4}{c\omega^3}\left(\frac{2\omega_{0e}c}{\omega v_{p0}}\right)^2\frac{W^l n_i}{n_i^2 m_i c}. \tag{9.5}$$

The effect decreases rapidly in this case with increasing frequency.

Thus, conversion on relativistic ions, if the number of the latter is sufficient, can yield both spontaneous and induced radiation (i.e., buildup in the case of an anisotropic distribution of the relativistic-ion velocity), but predominantly at low frequencies. Incidentally, if the plasma is low-temperature and dense (large ω_{0e} and small v_{p0}), a noticeable effect on high frequencies is also quite feasible.

10. CONVERSION OF LOW FREQUENCY PLASMA WAVES INTO ELECTROMAGNETIC WAVES IN SCATTERING BY EPITHERMAL AND RELATIVISTIC PARTICLES

It was already noted that the increase of frequency in scattering by epithermal and relativistic particles can cause the conversion of even very low-frequency ion-plasma, ion-acoustic, Alfven, and magnetosonic waves into electromagnetic waves to generate radiation in the frequency region where the plasma is transparent. We consider here this problem very briefly.

The scattering of ion-plasma and ion-acoustic longitudinal waves by relativistic electrons is described by the same relations as the scattering of electron-plasma waves, which was considered in detail in Sec. 8. It is only necessary to take k_1 to mean the wave numbers of the corresponding turbulence. We note that in ion-plasma waves k_1 can be even larger, by a factor $\sqrt{T_e/T_i}$ (compared with the Langmuir waves), and consequently the frequency of the converted radiation increases additionally by the same factor.

The effect of conversion in scattering of ion-acoustic waves is probably small, since the radiation coefficient decreases with the frequency of the ion-acoustic waves ω_s like ω_s^2/ω_{0i}^2 compared with the scattering of ion-plasma waves, and the frequency ω_s in turn decreases as a result of the nonlinear wave interaction.

Incidentally, in the scattering of ion-acoustic waves by nonrelativistic (but epithermal) particles, an appreciable increase of the cross section takes place. Therefore, if the phase velocities of the ion-acoustic waves are so small that $\omega \approx \omega_s v/v_p$ (where v is the velocity of the scattering particles) falls in the region of transparency of the plasma, then noticeable conversion can be expected.

We present here a formula for the calculation of the coefficient of induced radiation (buildup if there is anisotropy in the velocity distribution of the epithermal electrons, or absorption in the case of isotropy of the epithermal electrons) for the case of an isotropic ion-acoustic turbulence with energy density W^s and average wave phase velocity v_s (close to the velocity of sound):

$$\mu(\omega) = \pm \frac{2\pi\omega_{0e}W^s}{n_e^2 m_e v_s^2}\left(\frac{\omega_s}{\omega_{0i}}\right)^2\left(\frac{\omega_{0e}}{\omega}\right)^5 n\left(v > \frac{\omega}{\omega_s}v_s\right), \quad (10.1)$$

where $n\,(v > \omega/\omega_s \cdot v_s)$ is the number of epithermal electrons with velocity v larger than $\omega v_s/\omega_s$.

In scattering by epithermal ions, it is necessary to replace m_e in the denominator of (10.1) by m_i, and $n\,(>v)$ should be taken to mean accordingly the number of epithermal ions with velocities larger than $v_s\omega/\omega_s$.

For conversion in scattering of ion-acoustic waves by relativistic ions, we get in place of (10.1)

$$\mu(\omega) = \pm \frac{4\pi\omega_{0e}W^s}{n_i^2 m_e c^3}\left(\frac{\omega_{0e}}{\omega}\right)^3\left(\frac{2\omega_s c}{\omega v_s}\right)^{1/2} n\left(\mathcal{E} > m_i c^2\left(\frac{\omega v_s}{2\omega_s c}\right)^{1/2}\right), \quad (10.2)$$

where now $n\,(\mathcal{E} > \mathcal{E}_0)$ is the number of relativistic ions with energy larger than the specified value $\mathcal{E}_0$. The conversion in scattering of ion-acoustic waves by relativistic electrons (Compton scattering), and also effects of conversion of Alfven and magnetosonic waves in scattering by electrons and ions, have not yet been considered.

11. CONVERSION IN COALESCENCE AND DECAY OF PLASMA WAVES

In Secs. 5–10 we considered a group of plasma radiation mechanisms connected with the conversion of plasma waves upon scattering by particles. We now consider a second group of plasma radiation mechanisms, connected with nonlinear processes of decay and coalescence of waves.

The theory of these processes, including those that lead to generation of electromagnetic radiation, has been considered in great detail in reviews and monographs. We therefore confine ourselves here only to a brief summary of the formulas useful for astrophysics, all the more since the method of obtaining these formulas has already been described in Secs. 2–4.

First, however, we must make a number of general remarks. In coalescence of two plasma-turbulence waves of the same type, having small phase velocities $v_p \ll c$, electromagnetic radiation is generated only if their wave vectors are opposite in direction. If $v_p \gg c$ for one of the waves, then for the other wave we must have $v_p = c/\sqrt{3}$. The coalescence of two waves of the same type with wave velocities $v_p \gg c$ cannot generate electromagnetic radiation.

Induced processes of nonlinear interaction of two plasma waves of the same type with an electromagnetic wave lead only to the absorption of the electromagnetic waves, but a nonlinear interaction of different waves can lead to a buildup of the radiation field. We now proceed to concrete formulas.

a) Coalescence of Two Langmuir Plasma Waves at Frequencies ω_{0e}, with Generation of Electromagnetic Radiation at Frequency $2\omega_{0e}$

This process was considered many times within the framework of the theory of scattering by thermal fluctuations (see, e.g., [2]) and within the framework of the theory of nonlinear processes.[3,7,8] For the total radiation coefficient in the case of an isotropic plasma turbulence we have

$$I(2\omega_{0e}) = 2\pi\int I_\omega\, d\omega = \frac{2\sqrt{3}\,\omega_{0e}^4}{5\pi n_e m_e c^5}\int F_l^2(\mathbf{k}_1)\frac{dk_1}{k_1^2} \quad (11.1)$$

for $k_1 \gg \omega/c$. An estimate of the radiation coefficient in terms of $\langle v_p\rangle$ (the average phase velocity of the plasma waves) yields

$$I(2\omega_{0e}) \approx 10\,\frac{\omega_{0e}[\langle v_p\rangle]^3}{n_e m_e c^5}[W^l]^2. \quad (11.2)$$

The width of the electromagnetic-radiation spectrum is determined primarily by the width of the plasma-wave spectrum.

b) Spontaneous Emission in the Coalescence of Langmuir and Ion-plasma or Ion-acoustic Waves

This process was also considered many times both within the framework of the theory of nonlinear processes[7,8] and in Raman scattering of ion-plasma waves by thermal fluctuations.[19] An approximate estimate of the total radiation coefficient yields

$$I(\omega_{0e} + \omega_s) \approx \frac{\omega_{0e}W^l W^s}{n_e m_e c^2 v_{Te}^2}[\langle v_p\rangle]^2; \quad (11.3)$$

here v_p is, as before, the phase velocity of the electronic plasma waves.

c) Spontaneous Radiation Upon Coalescence of Electronic Plasma Waves at Frequencies ω_{He} in a Plasma with a Strong Magnetic Field[9]

The total reflection coefficient is

$$I(2\omega_{He}) = \frac{8\pi^3}{3}\frac{\omega_{0e}^2}{n_e m_e c^3}\int F_h^2(\mathbf{k}_1)\,dk_1. \quad (11.4)$$

From this we get the estimate

$$I(2\omega_{He}) \approx 10^2\,\frac{\omega_{0e}^2[W^h]^2}{n_e m_e c^3 \omega_{He}}\langle v_p\rangle. \quad (11.5)$$

As before, we assume here that $v_p \ll c$, but the formula is valid, in order of magnitude, also when $v_p \approx c$.

d) Induced Conversion (Absorption) in the Decay of an Electromagnetic Wave with $\omega \approx 2\omega_{0e}$ Into Two Langmuir Waves

The method for calculating such processes is described in Sec. 3. Since this problem has not yet been considered concretely, we present here more detailed relations. The δ-function entering in (3.4) gives the connection between the wave numbers of the electromagnetic and plasma waves:

$$\delta(\omega - \omega^l(\mathbf{k}_1) - \omega^l(\mathbf{k}-\mathbf{k}_1)) = \delta\left(\omega - 2\omega_{0e} - \frac{3v_{Te}^2}{2\omega_{0e}}(\mathbf{k}^2 + (\mathbf{k}-\mathbf{k}_1)^2)\right)$$
$$= \delta\left(\omega - 2\omega_{0e} - 3v_{Te}^2\frac{k_1^2}{\omega_{0e}}\right). \quad (11.6)$$

Consequently, radiation at the frequency ω is absorbed by the plasma waves with wave numbers $k_1 = (1/\sqrt{3}\,v_{Te})[\omega_{0e}(\omega - 2\omega_{0e})]^{1/2}$. Calculating now the coefficient of induced conversion (absorption), we get

$$\mu(\omega) = \frac{8\pi^2}{9\sqrt{3}}\frac{\omega_{0e}F_l(\mathbf{k}_1)k_1}{n_e m_e c v_{Te}^2}\Bigg|_{k_1 = \frac{[\omega_{0e}(\omega - 2\omega_{0e})]^{1/2}}{v_{Te}\sqrt{3}}}. \quad (11.7)$$

At a specified distribution of the wave numbers of the plasma turbulence, formula (11.7) determines completely the absorption coefficient at all frequencies.

We now find the effective radiation temperature for conversion in coalescence. Inasmuch as above we determined not I_ω but the total radiation coefficient $I = 4\pi\int I_\omega d\omega$, it is necessary, when substituting in (3.7), to use $4\pi\int\mu(\omega)d\omega$ in place of $\mu(\omega)$ (a value $5\omega_{0e}^2/c^2$ can be used for k^2). Ultimately we get

$$T_{\text{eff}}(\omega) = \frac{9\sqrt{3}}{100\pi}\frac{\omega_{0e}^{3/2}v_{Te}}{(\omega-2\omega_{0e})^{1/2}c^2}\frac{\int F_l^2\frac{dk_1}{k_1}}{\int F_l k_1\,dk_1}. \tag{11.8}$$

Going over to plasma-turbulence phase velocities and using the estimate

$$\frac{\omega-2\omega_{0e}}{\omega} \approx \left(\frac{v_{Te}}{v_p}\right)^2, \tag{11.9}$$

we obtain

$$T_{\text{eff}} \approx 0{,}1\left(\frac{v_p}{c}\right)^2\left(\frac{v_p}{v_{Te}}\right)^3 W^l D^3, \tag{11,10}$$

where $D = v_{Te}/\omega_{0e}$ is the Debye radius.

Formulas (11.7) and (11.10) make it possible to calculate Raman scattering of plasma waves with allowance for their absorption. For estimates, $F_l(k_1)$ can be replaced by $W^l/\Delta k_1$, where Δk_1 is the interval of the plasma-turbulence wave numbers. Assuming $\Delta k_1 \approx k_1 \approx \omega_{0e}/v_p$, we get

$$\mu(\omega) \approx \frac{6\omega_{0e}}{n_e m_e c v_{Te}^2} W^l. \tag{11.11}$$

The applicability of these formulas is discussed in Sec. 12.

e) Buildup of Electromagnetic Waves at Frequency ω_{0e} as a Result of a Decay into Plasma and Langmuir Waves and into Ion-acoustic Waves (The Process $l \to t + s$)

This nonlinear process is also considered in detail in [7,8]. An electromagnetic wave with frequency somewhat higher than ω_{0e} decays into a plasma wave with lower frequency and into an ion-acoustic wave. The process can occur only in a non-isothermal plasma. An estimate of the coefficient of induced radiation gives

$$\mu(\omega) = -\frac{\pi}{32}\left(\frac{4m_e}{3m_i}\right)^{1/2}\frac{\omega_{0e}W^l v_p}{n_c m_e v_{Te}^3 v_g}\frac{k_1}{\Delta k_1} \approx 3\cdot 10^{-3}\frac{\omega_{0e}W^l v_p^2}{n_e m_e c v_{Te}^4}. \tag{11.12}$$

Comparison with (5.7) shows that in a strongly non-isothermal plasma this process can compete with buildup in scattering by ions.

12. PLASMA MECHANISMS OF RADIO EMISSION IN RADIO ASTRONOMY

Up to now, relatively few papers were published devoted to the use of plasma radiation mechanisms for the interpretation of radio astronomical observation data. Without claiming completeness, we note here only the main trends of the already developed concepts concerning cosmic plasma radiation mechanisms, and indicate the possibilities of further investigations based on the theory developed in the preceding sections.

It must be emphasized here that an analysis of the "operation" of a plasma radiation mechanism under various conditions must of necessity be based on a discussion of as complete a volume of observational information as possible. This cannot be done within the framework of the present review, so that we shall frequently have to confine ourselves only to the general description.

a) Plasma Radiation Mechanisms on the Sun

The first historically, and so far still the most important applications of plasma mechanisms of radiation were devoted to the interpretation of the sporadic radio emission from the sun. It was observed almost immediately that the intensity of this radiation is in many cases much larger than the thermal limit that follows from the theory of bremsstrahlung. And although in many cases it was possible, at least qualitatively, to make use of the synchrotron mechanism, it became clear that new concepts are necessary concerning the nature of the sporadic radiation of the sun.

From many considerations, both theoretical and based on the interpretation of the observations, it can be regarded as proved that various types of plasma instabilities frequently developed in the plasma of the solar corona (particle beams, shock waves). These lead to the appearance of rather intense plasma turbulence in the solar corona at different frequencies.[1,2,19,20]

The conversion of plasma turbulence in the solar corona into radio waves indeed explains most of the phenomena in the sporadic radio emission of the sun. An investigation of this conversion was initiated by V. L. Ginzburg and V. V. Zheleznyakov.[2] According to them, three types of conversions are significant in the solar corona: regular conversion on smooth inhomogeneities of the solar-corona plasma, conversion in Rayleigh scattering of plasma waves by thermal fluctuations of the electron density (in the language of nonlinear plasma theory—conversion upon scattering by thermal ions), and conversion in Raman scattering, also by thermal fluctuations of the electron density (coalescence of the plasma turbulence wave with the plasma wave of thermal fluctuations). In all the cases under consideration, the probability of conversion (the ratio of the flux of the electromagnetic waves to the flux of the plasma waves) is of the order of 10^{-7}–10^{-5}.

To interpret the observed data on the sporadic radiation of the sun, the energy density of the plasma waves was chosen such as to ensure the observed radio emission flux at the value of the conversion coefficient given above. On the other hand, knowing the energy density of the plasma turbulence, it is also possible to estimate the characteristics of the instability (e.g., the concentration of the particles in the beam). Of course, it is necessary to explain here also other and more subtle characteristics of the sporadic radiation of the sun (polarization, directivity, frequency drift, complicated structure of the frequency spectrum). We are unable to discuss these questions within the framework of the present review, and note only the main difficulties and shortcomings of such investigations.

It must be emphasized first that so far we have considered principally the direct spontaneous conversion of plasma waves into electromagnetic ones. It was assumed that any radio-emission intensity can be interpreted by simply assuming a large energy density of the plasma waves. This is obviously not the case. If we remain within the framework of spontaneous conversion then, as already emphasized in Sec. 3, the brightness temperature of the radio emission cannot exceed a definite limit.

It was shown in Sec. 5 that in scattering by thermal ions (this is the main conversion mechanism under the conditions of the solar corona) the limit of the brightness temperature of the spontaneous emission is

$$T_{\text{eff}} \approx \frac{v_p}{v_{Ti}} T_i \approx 10^{9°}. \qquad (12.1)$$

The plasma-turbulence energy should then exceed ($\mu \gtrsim 10^{-9}$ cm^{-1})

$$W^l \gg 20 \frac{n_e}{\omega_0 v_p} \approx 3 \cdot 10^{-10} \text{ erg/cm}^3 \qquad (12.2)$$

which is eight orders of magnitude larger than the density of the thermal energy in the plasma nT (but not of the thermal energy of the plasma waves). No increase in the density of the plasma energy in excess of the limit (12.2) can increase the brightness temperature above (12.1), provided of course we stay within the framework of the spontaneous mechanism.

Actually, in many cases the brightness temperature of individual phenomena in the sporadic radiation of the sun does not exceed 10^9 deg. But a larger brightness temperature, which reaches sometimes values on the order of 10^{15} deg, is observed sufficiently frequently. It is obvious that in these cases buildup takes place in induced radiation.

But it is clear even without these remarks that induced radiation should prevail over spontaneous radiation as soon as the energy density of the plasma pulsations becomes larger than the limit (12.2), which is much higher than the energy density of the thermal pulsations of the plasma waves. To be sure, with increasing growth of the phase velocities of the plasma waves, more and more of the energy of the plasma waves is pumped over into the region of frequencies that are very close to ω_{0e}, which in turn decreases also the frequency interval in which induced radiation takes place. Nevertheless, at least at the initial stage of development of the turbulence, the induced radiation should be very effective.

Strictly speaking, in investigations of induced radiation it is impossible to regard the energy density of plasma waves as independent of the energy density of the electromagnetic radiation, since constant energy exchange takes place between the plasma waves and the electromagnetic waves, as soon as these energy densities become quantities of the same order.

This means that in such an investigation of induced conversion it is necessary to solve the self-consistent problem of excitation of plasma turbulence and of the transfer of its energy to the electromagnetic radiation. An example of such a formulation and solution of the problem is given in [1], which contains formulations and solutions of the kinetic equations describing the excitation of a plasma turbulence by a two-stream instability, the redistribution of energy along the spectrum of the plasma waves, and mutual conversion of plasma and electromagnetic waves.

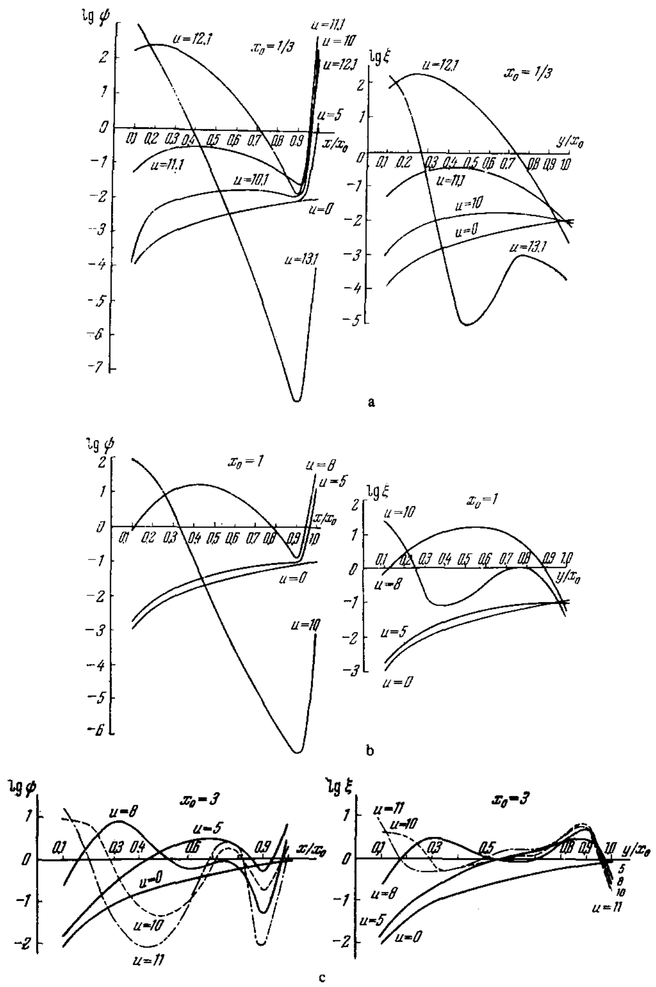

FIG. 4. Spectra of plasma (left) and electromagnetic (right) turbulence energy excited by a beam of particles in a plasma. The abscissas x/x_0 and y/x_0 represent the ratio of the wave numbers k^l and k^t respectively of the longitudinal and transverse waves to ω_{0e}/v_s – the wave number of the waves excited directly by the beam. The ordinates are the dimensionless spectral energy dnesities, ψ is the ratio of the spectral density of the plasma waves to $W_0 v_s/\omega_{0e}$, and ξ is the ratio of the spectral density of the electromagnetic waves to $W_0 v_s/\omega_{0e}$, where

$$W_0 = \frac{9\,(T_e + T_i)^2}{\sqrt{2\pi}\, T_e T_i} \frac{n_s m_e v_{Te}^4}{v_s v_{Ti}} \frac{\gamma}{\omega_{0e}}.$$

Different curves correspond to spectra produced at different times t following the start of the turbulence excitation, u = 2γt, and γ is the increment of the two-stream instability.

$$x_0 = \frac{4}{3} \frac{v_{Te}^2}{v_{Ti} v_s}.$$

This calculation is described in detail in [1].

An electronic-computer solution of these kinetic equations yielded the spectra of the plasma turbulence and of the electromagnetic radiation during various instants of time; these are shown in Fig. 4. The characteristic time of development of the spectra is determined both by the increment of the two-stream instability and by the time of energy redistribution over the spectrum.

It is seen from Fig. 4 that the transfer of energy into the region of very large phase velocities leads, on the one hand, to a decrease in the radiation emerging from the medium, and on the other hand stabilizes the beam, stopping the further generation of the plasma waves. In final analysis, however, the turbulence attenuates also in the region $k \to 0$, owing to the influence of the collisions, making it possible for the plasma instability to become replenished at a phase velocity close to the beam velocity. Such bursts, which are superimposed on each other and have a characteristic period on the order of the time of electron-ion collisions, are actually observed in different types of sporadic radiation from the sun.

The maximum density of both the plasma and electromagnetic energy attained in this process is of the order of

$$W \approx W^l \approx 10^2 \left(\frac{v_s}{\Delta v_s}\right)^2 n_s T_e, \tag{12.3}$$

where n_S is the concentration of the electrons in the beam that produces the plasma turbulence; v_S is the velocity of the beam, and Δv_S is the dispersion of the beam particle velocities. Assuming $T_e \approx 10^6$ deg (10^{-10} erg) and $(N_S/\Delta v_S)^2 \approx 10$, we find that even a flux with a concentration on the order of 10 cm^{-3} produces an electromagnetic-energy density on the order of 10^{-6} erg/cm^3. According to (3.4), this energy density at $k_{min} \approx \omega_{0e}/c$ corresponds to an effective temperature on the order of 10^{16}–10^{17} deg.

Of course, this estimate shows only that, on the one hand, allowance for induced radiation in the analysis of the sporadic radiation is absolutely essential, and on the other hand this effect eliminates also many difficulties in the modern interpretation of phenomena of the sporadic radiation of the sun. In particular, the foregoing example makes it possible to reconcile the high intensity of the solar-radiation bursts with the small number of electrons emitted thereby, as recorded by space rockets.

One more difficulty in modern theoretical notions concerning the sporadic radiation from the sun is as follows. According to the so-called plasma hypothesis, which is the basis for the entire theory (employed above), the sporadic radiation is generated at the plasma frequency ω_{0e} or $2\omega_{0e}$ in the corresponding region of the corona. However, observations of the localization of the sources of the sporadic radio emission of the sun shows that as a rule their frequency exceeds ω_{0e} for that region of the corona in which the sources are located. It can be assumed, of course, that the electron beams pass, for example, through a denser region (corona beam). The increase of the concentration can be attributed also to the passage of shock waves. Nevertheless, the frequency increase is so appreciable, that this effect cannot be explained by simply assuming that the plasma density is high.

This problem can be resolved within the framework of the plasma radiation mechanisms described here by assuming that the conversion of the plasma waves into electromagnetic ones occurs on ions or electrons of the same beam that produces the plasma turbulence. Since the particle velocity distribution is not isotropic, induced conversion with increase of frequency takes place here.

Such an assumption, however, contains one singularity. According to the results of Sec. 7, a noticeable increase of the frequency in scattering by epithermal (but nonrelativistic) particles takes place only when the phase velocities of the plasma waves are noticeably smaller than the velocities of the scattering particles. Yet the beam itself builds up plasma waves with a phase velocity equal to the beam velocity, and scattering by thermal ions can only increase the phase velocities.

This difficulty can be circumvented by taking into account the fact that when the plasma waves are redistributed in a non-isotropic medium (and the beam produces anisotropy of the particle-velocity distribution in the main plasma, too) the phase velocities of the plasma waves can also decrease. Of course, a detailed calculation of the excitation of the plasma waves in a plasma with a beam is necessary, with allowance for the influence of the beam on the redistribution of the plasma waves over their spectrum both in the direction of higher and in the direction of lower wave numbers; it is possible, however, to obtain here conditions under which the phase velocities decrease.

The conversion of plasma waves with small phase velocities in scattering by beam electrons leads to radiation of high frequencies. The buildup coefficient is given by the formula (7.7), which can be written in the form

$$\mu(\omega) \approx -\frac{2\pi n_s \omega_{0e}}{n_e^2 m_e c^3} \left(\frac{\omega_{0e}}{\omega}\right)^3 W^l, \tag{12.4}$$

since $v_S/v_p = \omega/\omega_{0e}$. Assuming $\omega \approx 4\omega_{0e}$ and $\mu(\omega) \approx 10^{-10}$ cm^{-1}, we obtain the condition for the buildup

$$\frac{n_s \omega_{0e} W^l}{n_l^2} \gtrsim 3 \cdot 10^{-5} \text{ erg/sec} \tag{12.5}$$

at $n_e \approx 10^8$ cm^{-3} it is difficult to satisfy this condition, but at $n_e \approx 10^7$ cm^{-3} (the more rarefied regions of the corona) we have $n_S W^l \gtrsim 10$ erg/cm^6. If we assume that the plasma-wave energy density generated by the beam is determined as before by (12.3), we get from this $n_S \approx 10^4$ cm^{-3} and $W^l \approx 10^{-3}$ erg/cm^3 (at a thermal energy density also $n_e T \approx 10^{-3}$ erg/cm^3). Generally speaking, this is possible, although the concentration of the particles in the beam is too high.

We emphasize once more that we have noted here only the feasibility, in principle, of increasing the frequency in conversion by fast particles. It is possible that a more complete analysis will either worsen the conditions and make this effect nonrealistic as applied to the sporadic radiation of the sun, or, to the contrary, will facilitate the conditions under which the effect under consideration is possible.

In concluding this section we make one more remark. The mechanisms explaining the sporadic radiation of the sun were used, with some modifications, also to interpret the sporadic radiation of Jupiter [2] or the very low-frequency radiation generated in the earth's magnetosphere. Since in either case the concentration of the free electrons is low and the magnetic field, to the contrary, may be sufficiently large, it is probable here that $\omega_{He} \gg \omega_{0e}$, and the plasma waves are excited at the gyrofrequencies. Therefore, to analyze the conversion of these waves into electromagnetic waves it is necessary to use the results of Sec. 6.

Obviously, induced conversion should predominate here, too (especially for Jupiter, where the effective temperature should be very low, since the ion temperature is also low). According to (6.6) we have for the buildup coefficient

$$|\mu(\omega)| \approx \frac{\pi^{1/2}}{12}\frac{\omega_{0e}^2 W^h}{cn_e T\omega_{He}} \approx \frac{eW^h}{HT}. \tag{12.6}$$

Assuming $\mu > 10^{-8}$ cm^{-1}, H = 15 Oe, and T = 3×10^{-14} erg (180°), we find the energy density of the plasma waves in the atmosphere of Jupiter, during the bursts of its radiation, should be $W^h \gtrsim 3\times 10^{-12}$ erg/cm^3. If the plasma turbulence is produced by a beam, then, taking (6.1) into account, we obtain in lieu of (11.3)

$$W \approx W^h \approx 10^2\left(\frac{v_s}{\Delta v_s}\right)^2\left(\frac{\omega_{0e}}{\omega_{He}}\right)^2 n_s T. \tag{12.7}$$

This yields $n_s \gtrsim 0.3\,(\omega_{He}/\omega_{0e})^2$ cm^{-3}. It is difficult to say anything definite concerning the concentration of epithermal particles in the atmosphere of Jupiter. Incidentally, the plasma turbulence is most likely excited here by shock waves.[2,21]

b) Plasma Radiation Mechanisms in Quasars

Among the many problems that quasars have posed in modern astrophysics, there is also the problem of interpreting the rapid variability of the radio emission in the cm and even in the mm bands.

The problem consists in the following. In some quasars (and possibly in many of them) the radio emission in the millimeter and centimeter bands changes by a factor of several times within several months. This means that the radiating region cannot have optical dimensions larger than this period, i.e., on the order of 3×10^{17} cm. On the other hand, their radiation intensity in this same band is so large that it corresponds to an electromagnetic energy density up to 1 erg/cm^3 (brightness temperature on the order of 10^{15} and even 10^{17}deg).

Disregarding for the time being the explanation of the variability of the radiation, we note the following. If the radiation mechanism is spontaneous, then the energy of the radiating particles should be of the order of 10^{11}–10^{13} eV and higher. On the other hand, synchrotron radiation of such electrons falls in the millimeter band (let alone the centimeter band) only if the magnetic field intensity is of the order of 3×10^{-8} Oe, which is little likely. In addition, too many relativistic particles are needed to produce an optical thickness comparable with unity.

The combination of an anomalously weak magnetic field with an equally anomalous large number of relativistic electrons of tremendous energy makes the hypothesis of spontaneous synchrotron radiation unacceptable.[14] In [14] it was proposed to interpret the radio emission of variable quasars as being due to conversion of plasma waves into electromagnetic ones at gyrofrequencies. Inasmuch as the frequencies are very high, it is necessary to assume a strong magnetic field (H ≈ 200 Oe). In that paper, only spontaneous conversion was considered; obviously that cannot explain in any manner the tremendous brightness temperature, since the limit of the effective temperature is $T_i v_p/v_{Ti} \approx 10^8$ deg (in quasars, probably, $T_i \approx 10^4$ deg).

Of course, induced conversion at the gyrofrequencies is possible. In fact, assuming in (11.6) $\mu \gtrsim 10^{-17}$ cm^{-1}, T ≈ 10^{-12} erg, and H ≈ 200 Oe, we find that the optical thickness of the medium exceeds unity already at an energy $W^h \gtrsim 2\times 10^{-18}$ erg/cm^3. But the difference between the obtained value of W^h and the observed density of the electromagnetic energy is too high here to obtain consistent values. For such a turbulence to be excited by a particle beam it is necessary, according to (11.7), to have a particle concentration $n_s \approx 10^8\,(\omega_{He}/\omega_{0e})^2$ cm^{-3}, which is likewise excluded.

A much more effective mechanism is that of synchrotron instability[16]—induced radiation at the frequency (8.23). The advantages of this mechanism are determined by the fact that it does not call for the presence of plasma turbulence and anisotropy in the distribution of the relativistic-particle velocities. But it still has considerable shortcomings. First, buildup at mm and cm wavelengths requires either a very large concentration of the main plasma (and consequently a high plasma temperature, for otherwise the bremsstrahlung absorption coefficient is large), or else a very weak magnetic field. A large number of relativistic electrons is required to make the optical thickness in the buildup region comparable with unity, namely

$$n\left(\mathscr{E} > m_e c^2\frac{5\omega_{0e}}{3\omega_{He}}\right) \gtrsim \frac{40H}{eL}\left(\frac{\omega_{0e}}{\omega_{He}}\right)^5 \approx \frac{n_e^{5/2}}{30H^4L}, \tag{12.8}$$

where L is the dimension of the medium.[23] For a typical variable quasar $n_e \approx 10^8$ cm^{-3}, H ≈ 6×10^{-2} Oe, and n (> 450 MeV) ≈ 2.5×10^7 cm^{-3}. These values are sufficiently large. A more favorable combination of values is $n_e \approx 10^9$ cm^{-3}, H ≈ 0.6 Oe, and n (> 150 MeV) ≈ 10^6 cm^{-3}. But even under these conditions the fast-particle concentration is too high.

From our point of view the most effective mechanism for the interpretation of the radio emission of a variable quasar is induced conversion of plasma waves into electromagnetic waves scattered by a beam of relativistic electrons (Sec. 8). To estimate the effect, we shall use formulas (8.29) and (8.30).

We assume the following values of the parameters. Let $n_e \approx 10^8$ cm^{-3}; then $\omega_{0e} \approx 6\times 10^8$ sec^{-1}, and to generate a frequency $\omega \approx 10^{11}$ sec^{-1} it is necessary to have radiation at an angle $\vartheta \approx 0.5\sqrt{c/v_p}$. If $c \approx v_p$, then $\vartheta \approx 6°$. Putting in (8.30) $\alpha = 4$ and $|\mu| \gtrsim 10^{-17}$ cm^{-1}, we get

$$W^l n(\mathscr{E} > 10 m_e c^2) \gtrsim 10^{-2}\ \text{erg/cm}^3 \tag{12.9}$$

The energy density of the plasma oscillations probably does not exceed the density of the thermal energy, i.e., $W^l \lesssim n_e T \approx 10^{-4}$ erg/cm^6. Therefore ~10^2 electrons with very low energy on the order of 10^7 eV in 1 cm^3 suffice to ensure buildup of the radiation field in induced conversion on relativistic electrons. These values can be improved further by assuming that the phase velocity of the plasma waves is much smaller than the velocity of light. Finally, if the plasma temperature in the region of generation of the plasma radiation exceeds 10^4 deg (which is perfectly feasible), then W^l can be increased and accordingly the concentration of the relativistic particles can be decreased.

The main purpose of the foregoing remarks was to point out the importance of plasma radiation mechanisms for the interpretation of phenomena in quasars. Of course, we must have a comprehensive account of all the observational data, and also a solution of the theoretical self-consistent problem of excitation of plasma turbulence by a relativistic beam with allowance for conversion on the particles of the beam itself.

In conclusion, it remains to note once more that in this section we touched upon only certain problems involving the application of plasma radiation mechanisms in astrophysics. We can mention also the small-size source in the Crab nebula, exploding stars, etc. Finally, an important factor in plasma mechanisms is their connection with the plasma scattering of electromagnetic waves.[24,22]

c) Problems of Using Plasma Radiation Mechanisms in Astrophysics

Mention should be made perhaps of a number of problems arising when plasma radiation mechanisms are employed. First, there is no doubt that plasma turbulence, just as synchrotron radiation for relativistic electrons, leads to power-law emission spectra if the spectra of the relativistic electrons are power-law, and the connection between the exponent in the energy spectrum and the spectral index is the same as for synchrotron radiation. It is of interest also to compare the electron energy loss in the plasma radiation mechanisms and in synchrotron radiation. The sum of these losses is of the form

$$\frac{d\mathscr{E}}{dt} = -\frac{16\pi}{3}\frac{e^4}{m_e^2 c^3}\frac{\mathscr{E}^2}{m_e^2 c^4}\left(\frac{H^2}{8\pi} + \frac{1}{6}W^l\right). \qquad (12.10)$$

It can thus be stated that plasma turbulence plays the role of a certain effective magnetic field. The radiation is possible also at H = 0. An important question, however, is why relativistic electrons must have a power-law spectrum. This problem is apparently connected with the mechanisms whereby the relativistic electrons are accelerated. An important role in this problem is played by plasma turbulence.[13] At the same time, such a turbulence can be excited by intense radio emission generated by relativistic electrons. Thus, the circle closes, as it were, and it becomes possible to formulate and solve a self-consistent problem concerning the distribution of relativistic electrons and the electromagnetic radiation excited by them. Apparently a solution of this problem could yield information concerning the parameter $8\pi W/H^2$ in various radio sources given the spectral index of their radio emission.[33] Since the magnetic field can be estimated independently, it becomes possible by the same token to estimate such an important parameter as the energy density of the plasma turbulence.

Second, plasma mechanisms, which can easily explain the rapid variability of the radio emission from a number of sources, when applied to stationary radiating sources raise the question of the nature of such a stationary behavior. Apparently, such a stationary behavior, if it exists at all, can be analogous to the stationary turbulence which calls for a continuous flow of energy over the spectrum. Apparently a situation is possible wherein the principal mechanism of turbulence dissipation is the conversion of the turbulent energy into electromagnetic radiation. Such a possibility exists in a strong magnetic field $\omega_{He} \gg \omega_{0e}$.

Third, finally, for a more complete understanding of the nature of plasma mechanisms, it is important to clarify the question of the distribution of the turbulent energy over the wave numbers of the turbulent pulsations. An example of such a calculation is given in [26]. The mechanism for establishing a stationary turbulence spectrum in a plasma has many features in common with the mechanism of stationary turbulence in liquids. It is connected with the generation of oscillations in one region of wave numbers of the turbulent pulsations, their transfer to another region of wave numbers, where they are annihilated as a result of various dissipation mechanisms.[26]

It can be assumed that the solution of the mentioned problems will yield additional information for the interpretation of the radiation of cosmic objects, and also information concerning the character of plasma turbulence under cosmic conditions.

Note added in proof. In [34] we derived a formula, which is convenient for estimates but is quite rough, and which makes it possible to estimate the buildup of electromagnetic waves (the maser effect) in a turbulent plasma, using only the observed value of the electromagnetic-radiation energy density. The estimates show that, by using only the observational data, it is possible to ascertain the presence of a maser effect in conversion of plasma waves on relativistic electrons in quasars and in other objects having a large radiation power.

In [33] is contained a calculation of the effects of conversion on relativistic electrons in waves with $v_p \gg c$ (in the present review, $v_p \ll c$). In this case the radiation spectrum ceases to depend on the turbulence distribution over the wave numbers. The total electron energy loss is determined by (12.10), where W is the total turbulence energy, including $v_p \ll c$ and $v_p > c$. A distribution of turbulent pulsations with a maximum at $v_p \gg c$ is possible, in accordance with [26], when the turbulence is intense.

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