SOVIET PHYSICS

USPEKHI

A Translation of Uspekhi Fizicheskikh Nauk

SOVIET PHYSICS USPEKHI (Russian Vol. 97, Nos. 1 and 2) JULY-AUGUST 1969

MESOATOMIC PROCESSES AND MODEL OF LARGE MESIC MOLECULES

539.126

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Usp. Fiz Nauk 97, 3–36 (January, 1969)

I. INTRODUCTION

MANY experiments have been performed recently aimed at the study of processes occurring when negatively charged mesons are stopped in matter. Interest in these investigations is caused by two circumstances.

First, mesoatomic and mesomolecular processes in matter determine to a considerable degree the subsequent stages of capture of negative mesons by atomic nuclei. In particular, mesoatomic processes cause nuclear capture of π^- and K^- mesons by protons to occur predominantly from the s-states of the excited $p\pi^-$ and pK^- mesic atoms. For μ^- mesons, the mesoatomic processes in hydrogen[1] (transitions between levels of the hyperfine structure of the mesic atoms and formation of $pp\mu^-$ mesic molecules) increase 3–4 times the probability of nuclear capture of μ^- mesons by protons. It was precisely this circumstance which made it possible to determine the relative sign of the V and A variants of the weak interaction $(\mu\nu)$ (pn) for measurements of the probability of μ^--meson capture in hydrogen.

On the other hand, the construction of more perfect experimental apparatus has made it possible to establish and to investigate in detail such effects as the influence of the chemical bond between the atoms of the matter on the probability of nuclear capture of π^- mesons.[2,3] From the point of view of earlier notions regarding the mechanism of mesoatomic processes, observation of effects of this kind was somewhat unexpected, since the chemical properties of substances are determined by the external electronic shells, the dimensions of which ($\sim 10^{-8}$ cm) greatly exceed the effective radius of the nuclear forces ($\sim 10^{-13}$ cm) or the size of the mesic atoms ($\sim 10^{-11}$ cm) from the levels of which the nuclear capture of the mesons takes place.

Attempts to interpret the indicated facts led to an appreciable refinement of the picture of the capture of negative mesons in matter. In particular, it was established that an appreciable fraction of the mesons is captured in chemical compounds not at the levels of isolated mesic atoms, but at levels belonging to the entire molecule as a whole. Such complexes, in which the dimensions of the meson orbits are comparable with the dimensions of the molecules of the substance (they exceed by hundreds of times the characteristic mesoatomic distances) were called "large mesic molecules."[4] Within the framework of the model of large mesic molecules, it was possible to explain the regularities and the singularities of the process of the capture of π^- mesons in chemical compounds.[2,3] Moreover, predictions based on this model turned out to be in good agreement with the subsequent series of experiments on the capture of π^- mesons in different hydrogen-containing substances.[5,6]

It turned out later that the same model can be used to explain many singularities of the structure of the mesic-x-ray series produced when μ^- mesons are captured in chemical compounds; these series were observed earlier in experiments by V. Zinov, A. Konin, A. Mukhin et al.[7-10] Of great importance for the verification of the model of large mesic molecules and the mechanism of π^--meson capture in hydrogen were experiments on the capture of π^- mesons in gas mixtures.[11] We note also that the observed connection between the chemical properties of matter and the processes of meson absorption in nuclei makes it possible to use these processes for the investigation of the chemical structure of substances.[12]

In this review we report systematically the results of experimental and theoretical investigations of the absorption of negative mesons in matter.

II. SUCCESSIVE STAGES OF THE PROCESS OF ABSORPTION OF MESONS IN MATTER

The first consistent picture of the processes that occur upon slowing down and capture of mesons in matter was developed in the early sixties following the investigations by Fermi and Teller,[13] Wightman et al.,[14] Panofsky, Aamodt, and Hadly,[15] Day, Snow, and Sucher,[16] Eisenberg and Kessler,[17] Leon and Bethe,[18] and others (see also [19] and the review [1]). According to the notions developed in these papers, the process of

deceleration and capture of mesons by the nucleus occurs in several stages, of which the principal ones are:

1) Slowing down of the mesons from velocities $v \sim c$ to velocities $v \sim \alpha c$ (of the order of the velocities of the external atomic electron; $\alpha = 1/137$ is the fine-structure constant).

2) Slowing down of the mesons from velocities $v \sim \alpha c$ to thermal velocities, and capture from the continuous spectrum into the discrete spectrum, i.e., transition to the highly-excited state of the mesic atom or mesic molecule.

3) Processes of de-excitation (transition of the mesons from the high orbits ($n \gg 1$) to relatively low ones), and in particular a) radiative transitions (the transition energy is carried away by the γ quantum); b) Auger transitions in the isolated atom (the energy is carried away by the atomic electron).

4) Reactions between nuclei and mesons at low levels of the mesic atoms.

The meson absorption picture is as simple as that in the case of isolated mesic atoms. In matter, in collisions between mesic atoms and other atoms, there can also occur the reactions:

5) Transfer, formation of mesic molecules, etc. which in many cases greatly change the course of stages 3) and 4). Let us examine each of these stages in greater detail.

1. Slowing Down Stage

The time of deceleration of the mesons from a velocity $v \sim c$ to $v \sim \alpha c$ (of the order of the velocities of the atomic electrons) is inversely proportional to their ionization energy loss $d\mathcal{E}/dx$.[20] For relativistic particles ($\mathcal{E} \gg Mc^2$) we have

$$-\frac{d\mathcal{E}}{dx} = LZ \cdot 4\pi a_0^2 mc^2 \left(\ln \frac{2\mathcal{E}^2 m}{IZM^2c^2} - 1\right), \qquad (1)$$

where m and M are respectively the masses of the electron and the meson, a_0 is the Bohr radius, c is the velocity of light, L is the number of nuclei with charge Z per cm^3, and I is the average ionization potential of the electrons in the atom. For nonrelativistic mesons with kinetic energy $T = \mathcal{E} - Mc^2 \ll Mc^2$ we have

$$-\frac{d\mathcal{E}}{dx} = LZ \cdot 2\pi a_0^2 mc^2 \frac{Mc^2}{T} \ln\left(\frac{4Tm}{IZM}\right). \qquad (2)$$

It is seen from formulas (1) and (2) that in both cases the decelerating ability of the substances in the velocity interval $c > v > \alpha c$ is approximately proportional to Z, i.e., to the total number of electrons in the atom. The time τ of deceleration of the mesons in condensed substances is approximately equal to

$$\tau = 10^{-9} - 10^{-10} \text{ sec} \qquad (3)$$

and in gases it is 10^3 times larger. In hydrogen, the process of deceleration determines the specific mechanism of adiabatic ionization, indicated by Fermi and Teller.[13] The time of deceleration of the mesons in liquid hydrogen from a velocity $v \sim c$ to $v \sim \alpha c$ was calculated by Wightman,[14] and turned out to be of the same order of magnitude as (3).

2. Transition from the Continuous to the Discrete Spectrum

Very little is known concerning the deceleration of the mesons and concerning the mechanism of energy loss in the velocity interval $\alpha c \geq v \geq 0$, and also concerning the method whereby the mesons go from the continuous spectrum into the discrete spectrum. It is usually assumed that mesons with such energy acquire in condensed substances quite rapidly (within a time 10^{-13}–10^{-14} sec) velocities corresponding to the average energy of the thermal motion at the given temperature T:

$$v_T \approx \left(\frac{3kT}{M}\right)^{1/2} \qquad (4)$$

(when $T = 300°$ K, $v_T \approx 0.8 \times 10^6$ cm/sec for the μ meson.). Further meson energy loss and the transition from the region of the continuous spectrum (energy $E > 0$) to the discrete spectrum ($E < 0$) depends significantly on the properties of the target material, for small batches of energy are transferred in this case from the meson to the atom. Only in metals, where the meson can give up energy to the conduction electrons, is the process of meson deceleration sufficiently obvious.[13]

Let us mention other possible mechanisms of energy loss: excitation of vibrations of the molecule[21] or of the entire crystal lattice, adiabatic capture,[22] and transfer of energy in the excitation of collective oscillations to the entire atomic shell,[23] etc. It is still unclear, however, which of these mechanisms prevails and how the capture probability depends on the individual features of the material. Therefore all the estimates of the probability of meson capture from the continuous spectrum into the discrete one are based on the Fermi-Teller model,[13] in which the process of energy loss by the meson is regarded as the motion of the meson in a degenerate gas of electron shells, with transfer of energy to the conduction electrons. The distribution of the density of the electron gas in the atom is described by the Thomas-Fermi model. Under these assumptions, the following formula was obtained for the rate of energy loss:

$$\frac{dE}{dt} = \frac{1}{t_0} \frac{L^{-1}E^{3/2} + 4\pi e^2 Z b^{3/2}[1-(0{,}8/x_0)]}{L^{-1}E^{1/2} + 3{,}96 e b^{5/2} Z^{-1/3} x_0^2}, \qquad (5)$$

where $t_0 = 2.4 \times 10^{-17}/\mu$ sec, $\mu = M/m$, $b = (9\pi^2/128)^{1/3} a_0 = 0.47 \times 10^{-8}$ cm, e is the electron charge, and x_0 is determined from the relation

$$\frac{1}{L} = \frac{4}{3}\pi b^3 \frac{x_0^3}{Z}.$$

Extrapolation of formula (5) to the value E = 0 (i.e., to the boundary of the continuous spectrum) leads to the well known Fermi-Teller "Z-law": in a mixture of substances or in a chemical compound, the probability of a meson "landing" on the level of different isolated mesic atoms is proportional to the value Z of the charge of the nucleus (more accurately, to $Z^{2/3}$).

Even within the framework of the Fermi-Teller model, the extrapolation of formula (5) to E = 0 is not justified.* In addition, in the case of deceleration in di-

*Strictly speaking, such an "impulse" approximation for the description of the process of meson deceleration is valid only so long as their de Broglie wavelength $\lambda = h/Mv$ is much smaller than the dimension of the atom. When $v \geq \alpha c = 2.2 \times 10^8$ cm/sec, this condition is indeed satisfied, for in this case (for the μ meson) $\lambda_\mu \leq 0.5 \times 10^{-10}$ cm $\ll 10^{-8}$ cm. But when $v = v_T = 0.8 \times 10^6$ cm/sec we have $\lambda_\mu \sim 10^{-8}$ cm, which is already comparable with the dimensions of the atoms. As always, an exact "wave" analysis of the problem is necessary in these cases.

electrics and in gases, the meson cannot lose arbitrarily small batches of energy, since in these cases the electrons are separated from the continuous spectrum by a finite energy gap. We note also that in substances with small Z, the Thomas-Fermi approximation itself is not very suitable. In the case of hydrogen it is not applicable at all, and therefore Fermi and Teller proposed for hydrogen a different, adiabatic capture mechanism,[13] according to which the negative meson penetrating through the electron shell leaves the atom with zero energy when the meson comes within a distance r ~ 0.64 a.u. of the proton.[14] If the energy of the incoming meson is in this case smaller than the binding energy of the electron in the hydrogen atom ($I = -\frac{1}{2}$), then the meson can be captured by the mesic-atom levels $N > N_0$ the limiting orbit N_0 being determined from the condition

$$-\frac{\mu^*}{2N_0^2} = -\frac{1}{2}$$

($\mu^* = \mu/[1 + (\mu/M_p)]$ is the reduced mass of the meson-proton system). From this we get for N_0 the estimate

$$N_0 \approx \sqrt{\mu^*}, \qquad (6)$$

which yields for μ^-, π^-, and K^- mesons respectively N_0 = 14, 15, and 26. These estimates served as the starting point for all the subsequent investigations. We shall discuss them in greater detail in Ch. IV.

3. Mesic-Atom De-excitation Processes*

In an isolated mesic atom, the transition of the meson from highly-excited states ($n \gg 1$) to states with lower energy occurs with emission of γ quanta or with transfer of energy to the electron of the atomic shell (Auger transitions). These transitions obey the usual selection rules with respect to the orbital angular momentum l and the projection m for dipole transitions: $\Delta l = \pm 1$ and $\Delta m = \pm 1$ and 0. The selection rules cause the de-excitation of the mesic atom produced in a highly excited state with large l to proceed via cascade transitions. In this case competition exists between the radiative and Auger transitions.

The probability of the radiative transition is

$$w_R = \frac{4}{3}\frac{\langle d\rangle^2}{\hbar^4 c^3}(\Delta E)^3,$$

where $\langle d \rangle$ is the matrix element of the dipole moment between the states (nlm) and (n′l′m′) of the mesic atom and

$$\Delta E = \frac{Z^2 M e^4}{2\hbar^2}\left(\frac{1}{n'^2} - \frac{1}{n^2}\right).$$

We see therefore that the probability w_R is appreciable only for transitions with large ΔE, but such transitions are hindered by the selection rules with respect to l, which actually limit the value of ΔE, since transition from a state with specified (nlm) is possible only to states with $n' \geq l - 1$. As a result, the total time of the cascade is greatly lengthened.

The dimensions of the mesic atoms are much smaller than the dimensions of the electron shells of the atoms, and therefore the ratio of the probabilities of the Auger transitions and of the radiative transitions is equal to the usual conversion coefficient k for the dipole nuclear transition. When $\eta = Ze^2/\hbar v \ll 1$ (v—velocity of the Augur electron) we have

$$k = Z^3\alpha^4\left(\frac{mc^2}{\Delta E}\right)^{7/2}.$$

It follows therefore that the Auger transitions are the main transitions in the case of transitions between highly-excited mesic-atom levels (small ΔE), whereas transitions to lower levels of the mesic atom (large ΔE) are in the main radiative. The ratio of the radiative and Auger transitions depend strongly on Z. Since $w_R \sim Z^4$, and the probability of the Auger transitions depends little on Z, in atoms with large Z the radiative transitions are decisive.

Eisenberg and Kessler,[17] starting from the estimates (6) for the initial orbit N_0 of the meson, calculated the cascade in an isolated mesic atom under different assumptions concerning the initial distribution of the mesons over the states with different l. The relative contribution of radiative and Auger transitions to the total probability of the de-excitation process is considered in [24].

4. Nuclear Capture of Mesons

When the mesons reach one of the lower levels during the de-excitation of the mesic atom, nuclear reactions between the mesons and the nucleons become possible. So far, the principal role was played by electromagnetic properties of the mesons, and the differences in the behavior of these processes in the case of μ^-, π^-, and K^- mesons were determined only by the difference of their masses. Now, however, differences in the character and the strength of the interactions between the nucleons and the μ^-, π^-, and K^- mesons come into play, although general regularities exist here, too.

In particular, if the meson is located at the level (nlm) of the mesic atom, then the rate Γ of any reaction of the meson with the proton of the nucleus Z is proportional to the probability w of finding the meson in the vicinity of this nucleus:

$$w = \int_V |\psi_{nl}(\mathbf{r})|^2 d\tau,$$

where the integration extends over the entire volume V of the nucleus. Since we have when $r \to 0$

$$\psi_{nl}(r) = \frac{1}{(2l+1)!}\sqrt{\frac{(n+l)!}{(n-l-1)!\,2n}}\left(\frac{2Z\mu}{n}\right)^{3/2}\left(\frac{2Z\mu r}{n}\right)^l P_{lm}(\theta)\, e^{im\varphi},$$

and the radius of the nucleus is

$$R = Z^{1/3} r_0, \qquad r_0 = \frac{1.5\cdot 10^{-13}\ \text{cm}}{a_0} \approx 3\cdot 10^{-5},$$

it follows that

$$w = \frac{2^{2l+2}}{(2l+3)(2l+1)!\,(2l+1)!}\frac{(n+l)!}{(n-l-1)!\,n}\frac{Z^{4+\frac{8}{3}l}}{n^{2l+3}}(\mu r_0)^{2l+3}. \qquad (7)$$

Capture of π^- mesons by nuclei in the light π^- mesic atoms occurs from the s-states (l = 0), therefore

$$w = \frac{4}{3}(\mu r_0)^3\frac{Z^4}{n^3}. \qquad (8)$$

In the case of hydrogen, the probability of absorption of

*In this review, the term "de-excitation" is taken to mean the "process of transition of the mesons from the excited states of the mesic atom (or mesic molecule) to lower ones."

the π^- meson by a proton, according to calculations by Fray,[25] is

$$\Gamma = \frac{1{,}6 \cdot 10^{15}}{n^3} \text{sec}^{-1}. \quad (9)$$

For K^- mesons and heavy nuclei, the rate of nuclear capture from other states of the ZK^- mesic atom ($l \neq 0$) may become comparable with the rate of the radiative transitions $nl \to 1s$. In this case the fraction of the mesons captured from the 1s state decreases strongly.[26]*

The μ^- mesons interact with the protons much less intensely, in accordance with the reaction

$$\mu^- + p \to n + \nu. \quad (10)$$

The rate α_Z of this reaction on the K orbit of a $Z\mu^-$ mesic atom for light nuclei ($Z < 10$) is [27]

$$\alpha_Z \approx 200 Z^4 \text{sec}^{-1}. \quad (11)$$

At large values of Z, the influence of the nuclear dimensions comes into play, and the dependence of the constant α_Z on Z weakens in such a way, that $\alpha_Z \approx 10^7$ sec^{-1} for all nuclei with charges $Z \gtrsim 50$. Since the rate α_Z and α_0 of the μ^--meson decay

$$\mu^- \to e^- + \nu + \bar{\nu} \quad (12)$$

are much smaller than the rates of the cascade transitions, both reactions (10) and (12) proceed for the 1s state of the μ-mesic atom, and the number of mesons S(t) decreases like

$$S(t) = S_0 \exp\{-(\alpha_Z + \alpha_0) t\}, \quad (13)$$

where $\alpha_0 = 0.45 \times 10^6$ sec^{-1}.

5. Processes Occurring in Collisions Between Hydrogen Mesic Atoms and Atoms of Matter

Among the mesic atoms of various elements, mesic atoms of hydrogen and of its isotopes occupy a special position, owing to their electroneutrality and small dimensions ($\sim 0.5 \times 10^{-8} n^2/\mu$ cm). Thus, the dimensions of the $p\mu^-$ mesic atom in the ground state (n = 1) are $\sim 2.5 \times 10^{-11}$ cm. Owing to these properties, the hydrogen mesic atoms, like neutrons, penetrate freely inside the electron shells of the atoms and can come quite close to the nuclei of the substance. This in turn leads to many singularities in the processes of de-excitation and nuclear capture, and in particular greatly decreases the time of the cascade transitions.

a) De-excitation processes. In an isolated hydrogen mesic atom, the meson can go from high levels to the ground level only by radiative transitions. For the $p\pi^-$ mesic atom, the time of the fastest transition, $2p \to 1s$, is 6×10^{-12} sec, but the total cascade time from the level (nl) to the level 1s is much larger ($\sim 10^{-9}$ sec), since the transition is hindered by the orbital-momentum selection rules $\Delta l = \pm 1$ for dipole radiation. On the other hand, if the hydrogen mesic atom wanders in the substance and falls inside the electron shells of the atoms, it may give up its energy to the electron of the shell and eject it from the atom. According to calculations by Leon and Bethe[18] this process (external Auger effect) reduces the cascade time in the hydrogen mesic atom to 10^{-11}–10^{-12} sec.*

b) Nuclear capture. The second effect which is specific only for the mesic atoms $p\pi^-$ and pK^-, and which leads to a sharp increase of the probability of the nuclear capture of π^- and K^- mesons, was indicated by Day, Snow, and Sucher.[16] According to the calculations by Fray[25] the nuclear reactions

$$\pi^- + p \to n + \pi^0, \qquad \pi^0 \to 2\gamma, \quad (14)$$

$$\pi^- + p \to n + \gamma \quad (15)$$

proceed with a noticeable probability not only from the ground state (n = 1), but also from excited ns states ($n > 1$, $l = 0$). But it follows from (7) that the nuclear reactions (14) and (15) are strongly suppressed if the meson is in a state with nonzero orbital angular momentum ($l \neq 0$). And inasmuch as the mesic atoms are produced to a greater degree in states with $l \neq 0$, the probability of capture of an isolated mesic atom from high levels is negligibly small. The situation changes if the mesic atom of the hydrogen is in a substance. In this case the hydrogen mesic atoms, penetrating the electron shells of the atoms, can come close to the nuclei of other atoms. The linear Stark effect for the levels of the $p\pi^-$ mesic atom, which are degenerate in the orbital angular momentum, in the Coulomb field of the nuclei leads to a "mixing" of states with different l and to transitions between them. Therefore the hydrogen mesic atoms, which initially were in the state with $l \neq 0$, spend some time during the collision process in a state with $l = 0$, from which the probability of nuclear capture is quite large. Both these processes, namely the external Auger effect and the Day-Snow-Sucher mechanism decrease effectively the lifetime of the π^- meson in hydrogen. The experimentally measured lifetime in hydrogen $\tau_H = (2.3 \pm 0.6) \times 10^{-12}$ sec[28] is smaller by two orders of magnitude than the lifetime in helium $\tau_{He} = (3.6 \pm 0.7) \times 10^{-10}$ sec.[29] The calculations of Leon and Bethe[18] with allowance for both indicated mechanisms[16] yield for the lifetime of the π^- meson in hydrogen a value $\tau_H \approx 3.5 \times 10^{-12}$ sec, which is quite close to the experimentally obtained value.†

c) Transfer processes. In collisions between mesic atoms of hydrogen and nuclei of other atoms, there is one more possible process, namely the transfer of the mesons from the protons to the nuclei Z with formation of $Z\mu^-$ or $Z\pi^-$ mesic atoms. Such a transfer is irreversible and leads to the suppression of the charge-exchange process (14) in the presence of other nuclei Z.‡

*In hydrogen mesic atoms there is a special mechanism of s-capture on high levels ns (see Sec. 5).

*We mention one other possible de-excitation mechanism [15,28], wherein the collisions between the $p\pi^-$ mesic atom and the H_2 molecule can lead to dissociation of the latter, and the dissociation energy ($\sim$ 1eV) is equal to the energy of the cascade transition.

†The theoretical value of τ_H consists of three parts: the time of slowing down from velocities $v \sim \alpha c$ to capture in the discrete spectrum, the de-excitation processes, and the nuclear capture. According to estimates by Day [16], the π^- mesons moving with velocity $v \sim \alpha c$ fall on the $p\pi^-$ mesic atom orbit with $n \approx 15$ already within a time $\tau_1 \approx 1.2 \times 10^{-12}$ sec. According to the calculations of Leon and Bethe [18], nuclear capture of the π^- meson by a proton takes place at a time $\tau_2 = (2.3^{+1.4}_{-0.7}) \times 10^{-12}$ sec later. The total time is $\tau_H = \tau_1 + \tau_2 \approx 3.5 \times 10^{-12}$ sec.

‡The capture of a π^- meson by a nucleus Z leads not to the reaction (14), but to disintegration of the nucleus. The probability of the reaction (14) for all nuclei Z does not exceed 10^{-4}, with the exception of He^3 and D, for which this probability amounts to 0.155 and $< 10^{-3}$ respectively [30].

In a study of the reaction (14) in substances containing hydrogen, Panofsky et al.[15] observed that it can be detected only in pure hydrogen H_2, whereas in lithium hydride LiH and in polyethylene CH_2 it is suppressed to a level $< 10^{-2}$. The authors of [15] attributed the results of their experiments to transfer of the π^- mesons in accordance with the reaction

$$p\pi^- + Z \rightarrow Z\pi^- + p. \qquad (16)$$

Actually the transfer of mesons to the nuclei Z as the result of mesic-atom collisions is significant only in mechanical mixtures and gases, whereas in chemical compounds such as LiH there is a much more effective transfer mechanism (see Ch. IV).

d) Collisions of $p\mu^-$ mesic atoms. The transfer of π^- mesons from $p\pi$-mesic atoms to nuclei Z is effective only on high orbits ($n > 1$), i.e., so long as the transfer rate exceeds the rates of the cascade transitions, and the nuclear capture is small. This in turn is possible only if the concentration of the nuclei Z is sufficiently large. The μ^- mesons interact with the nuclei much less strongly than the π^- mesons, and therefore the lifetime of the μ^- meson in the $p\mu^-$ mesic atom is much longer than the duration of all the cascade transitions, and at small impurity concentrations the μ^- meson has time to reach the K orbit, and the entire subsequent transfer process occurs from the ground state of the $p\mu^-$ mesic atom. Such a process was investigated in detail experimentally[31,32] and theoretically.[19] It was established as a result that the constant of the transfer of the μ^- meson from the K shell of the $p\mu^-$ mesic atom in collisions with the nucleus Z is approximately proportional to Z.

In addition to the transfer process, there occur in collisions of $p\mu^-$ mesic atoms the formation of mesic molecules of hydrogen and its isotopes, nuclear catalysis, depolarization of μ^- mesons, and other processes which we shall not discuss here (see, for example, the review [1]).

III. NEW EXPERIMENTAL RESULTS

We have described the picture of mesic-atom processes as seen in the early sixties, when intense beams of mesons, and new experimental apparatus, with high resolution and efficiency, were developed. This has made it possible to start systematic quantitative investigations of the processes of slowing down and absorption of mesic atoms in matter. It turned out here that the new experimental results cannot be explained within the framework of the previously developed theoretical concepts, which must now be reviewed.

1. Verification of the Z-Law

The first step was a thorough experimental verification of the Fermi-Teller Z-law, since indications of this violation have been known for a long time (see, for example [33]). The results of this verification were analyzed by Baijal et al.[34] In an attempt to fit the capture probability to a power law of the type Z^s, it turned out that the exponent s varies in a wide range for different substances, from −1 to 1.5 (Fig. 1). Bobrov et al. attributed the systematic deviations from the Z-law ob-

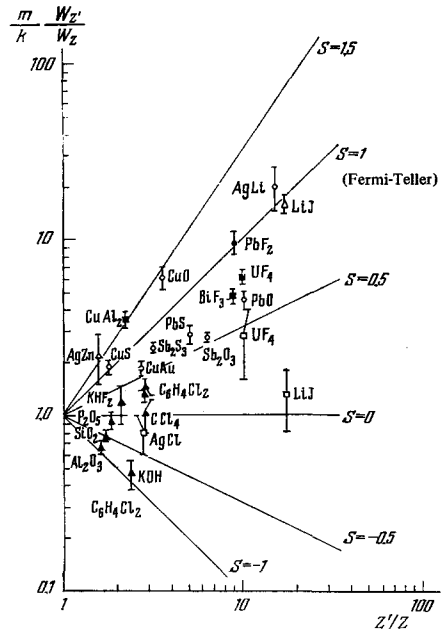

FIG. 1. Results of verification of the Z-law in absorption of μ^- mesons in chemical compounds of the type Z'_kZ_m [33,34]. Curves—the functions $(m/k)\,(Z'/Z)^S$ for different s. The value s = 1 corresponds to the Fermi-Teller Z-law. W_Z—probability of μ^-meson capture by the atom Z.

served by them[35] to the different electron affinities of the chemical elements making up the compound.

In the experiments indicated above, the indicator of the μ^- meson capture by the Z nucleus was the reaction (10). A shortcoming of this procedure is that at large Z the constants α_Z in formula (13) depend little on the value of the charge Z, and therefore in compounds of the type Z'_kZ_m with close values of Z′ and Z it is difficult to separate the exponentials of the decay (13). Experiments on the mesic-x-radiation in chemical compounds[36] are free of this shortcoming, since the characteristic series of the different elements are distributed over the energy scale like Z^2, and can be easily separated from one another. In addition, the stage of the cascade transitions is closer to the start of the chain of the mesic-atom processes.

This measurement procedure was used in a number of investigations by V. Zinov et al.[7-10] for the investigation of the capture of μ^- mesons in chemical compounds and mixtures of substances. The experimental setup used for this purpose is shown in Fig. 2. The μ^- mesons, produced with the synchrocyclotron of the nuclear-problems laboratory of Joint Institute for Nuclear Research, pass through a number of scintillation counters and decelerating filters and are stopped in the investigated target. The stopping of the μ^- mesons in the target was reliably identified by using a special electronic system that recorded the increase of the ionization loss of the μ^- meson (2) with decrease of its residual range. The mesic-x-radiation was detected with a spectrometric scintillation counter with NaI crystal, whose pulses were fed to the input of a multichannel pulse-height analyzer. The investigations have shown that in mixtures of inert gases the Z-law holds satisfactorily, although small systematic deviations are observed

(the probability of the μ^- meson capture by mesic-atom orbits of different elements, calculated per unit charge, decreases weakly with increasing Z). At the same time, the Z-law is not satisfied in the Ar + CO_2 mixture: the capture probability in CO_2 is approximately double the calculated value.[8] We shall return to a discussion of these results in Ch. V, after describing the new model of mesic-atom processes.

All the foregoing experiments on the capture of μ^- mesons make it possible to draw the following general conclusion: the Fermi-Teller Z-law is as a rule violated. This, however, does not mean that the Z-law is violated during the state of the initial landing of the mesons, inasmuch as only the total probability of several successive stages of the meson-absorption process (up to the nuclear capture) was measured in the described experiments, and this probability can differ greatly from the probability of the initial landing.

2. Capture of π^- Mesons in Hydrogen-Containing Substances

The greatest deviations from the Z-law were observed in a series of experiments[2-3,5-6] on the capture of π^- mesons in hydrogen-containing substances of the type (Z_mH_n). In the first investigations of this process [15] it was established that the probability of capture of the stopped π^- mesons by hydrogen nuclei in accordance with the reaction (14) is suppressed to such an extent, that the employed apparatus did not make it possible to register it. The experimental setup used in [2,5,6] for the investigation of the reaction (14) is shown in Fig. 3. The π^--meson beam produced in the synchrocyclotron of the nuclear-problems laboratory of the Joint Institute for Nuclear Research passed through scintillation counters and decelerating filters, after which the π^- mesons were stopped in the investigated target. The γ quanta produced in the reaction (14) as a result of the decay of the π^0 mesons were registered by two Cerenkov total-absorption spectrometers. To suppress the contribution from the charge exchange of π^- mesons in flight, the scintillation counters operated in the stoppage-detector regime.[2] Owing to the high efficiency and low sensitivity to the background of the extraneous radiation, the apparatus made it possible to register the reaction even when its probability was suppressed to a value $\sim 10^{-4}$. This made it possible to register capture of π^- mesons in hydrogen-containing substances of the type Z_mH_n, to measure the probability of this process, and to establish its dependence on the charge Z, and also to establish

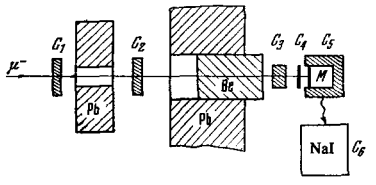

FIG. 2. Experimental setup used for the investigation of μ^--meson capture [7-10]. μ^-—beam of negative muons, Be—decelerating filter, C_1 —C_4—scintillation counters separating the stoppings of μ^-mesons in the target, C_5—scintillation counter for anticoincidence, C_6—spectrometric scintillation counter, M—investigated target, Pb—lead shield.

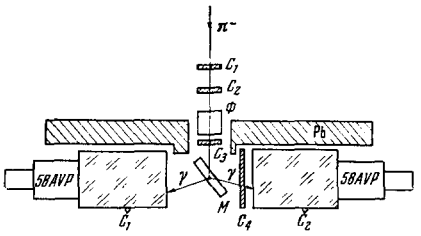

FIG. 3. Diagram of experimental setup for the investigation of the capture of μ^-mesons in hydrogen-containing substances [2]. μ^-—beam of negative pions with momentum 170 MeV/c. C_1—C_4—scintillation counters, $\check{C}_1$, $\check{C}_2$—Cerenkov total-absorption spectrometers, M—investigated target, Pb—lead shield, F—decelerating filter.

many other regularities. In addition, a less intense transfer of π^- mesons from hydrogen to heavy atoms in gas mixtures was observed.[11]

The described experiments can be separated into two groups: experiments on the capture of π^- mesons in condensed chemical compounds and in gas mixtures. We shall discuss below the main results obtained in these experiments.

3. Capture of π^- Mesons in Chemical Compounds

In these experiments, the probability of W capture of π^- mesons by hydrogen nuclei in chemical compounds of the Z_mH_n type was measured.* The following regularities were observed:

1) The probability W depends strongly on the charge Z of the nucleus of the atom that is bound chemically with the hydrogen, and obeys the empirical law

$$W \approx a_L \frac{n}{m} Z^{-3}, \qquad (17)$$

where a_L are quantities that depend on the number L of the period of the atom in the periodic table. Within the periods, the coefficients a_L are approximately constant, with $a_{L+1}/a_L \approx 3$ for periods I–IV.[5]

It is seen from (17) that in chemical compounds of the type Z_mH_n the capture of π^- mesons by hydrogen (14) and (15) is strongly suppressed. Thus, $W \approx 3.5 \times 10^{-2}$ for LiH and $W \approx 3.5 \times 10^{-3}$ for H_2O.

2) The probability W is practically independent of the density or even of the aggregate state of the substances. Thus, in one of the experiments[37] the density of ethylene C_2H_4 changed by a factor of 110, but W remained constant at the same time.

3) W does not decrease even when heavy elements that are not bounded to hydrogen chemically are added to the hydrogen-containing compound as impurities. For example, when NaI and LiCl are dissolved in methyl alcohol CH_3OH (proportion $9CH_3OH$ + NaI and 3.15 CH_3OH + LiCl), the value W remains the same as in pure alcohol.[37]

4. Capture of π^- Mesons in Gas Mixtures

In the study of the transfer of π^- mesons from $p\pi^-$ mesic atoms to nuclei of heavy atoms in gas mixtures of the type H_2 + Z, other singularities of the process of

*In pure hydrogen $W_{H_2} = 1$.

absorption of π^- mesons by hydrogen nuclei are observed:[11]

1) The transfer of π^- mesons to nuclei is less intense than in the case of chemical compounds.

2) It depends little on the charge Z of the impurity.

3) The transfer probability Q is determined only by the relative concentration

$$C=\frac{C_Z}{C_H}, \tag{18}$$

and at fixed C it does not depend on the absolute concentrations C_Z and C_H of the nuclei Z and H.

4) At small concentrations of the impurities Z ($C \ll 1$) the transfer of π^- mesons to Z nuclei is quite small,[38] which is quite unlike the analogous process for μ^- mesons (for example, in liquid hydrogen the transfer of μ^- mesons by impurities is close to 100% already at concentrations $C \sim 0.001$).[31]

5. Preliminary Discussion

Among the foregoing results, special attention should be paid to two facts: the sharp Z dependence of the probability $W_{Z_mH_n}$ of the capture of π^- mesons by hydrogen in chemical compounds of the type Z_mH_n, and the independence of the transfer probability Q_{H_2+Z} in H_2 + Z gas mixtures of the concentrations C_Z and C_H at a fixed C. Let us consider initially the first group of experiments (Sec. 3) and attempt to answer the following fundamental question: at what stage do the violations of the Z-law occur and which of the stages of the mesic-atom processes, (1–5 of Ch. II) determines the foregoing singularities, and which of the existing theoretical models can explain them?

Within the framework of the Fermi-Teller scheme, the Z-dependence on the nuclear reaction (14) is determined by process (2) of Ch. II (the transition of the meson to the discrete spectrum). The Fermi-Teller Z-law results in this case naturally, since Z electrons of the atoms take part in the slowing down of the meson. For large Z in metals and monatomic gases, this assumption is apparently fully justified, but in the case of absorption of mesons in hydrogen it is almost certainly inapplicable, because the Thomas-Fermi model does not hold for hydrogen, and also because the final probability of the nuclear capture in hydrogen depends on the collisions with other atoms. Theoretical[19] and experimental[11,31-32] data indicate that the constants of the transfer of mesons from hydrogen to nuclei of other substances depend little on the charge Z (approximately like Z). Therefore the processes of transfer of π^- mesons from hydrogen to Z nuclei in collisions cannot explain such a sharp ($\sim Z^{-3}$) suppression of the reaction (14) in chemical compounds. In addition, if it is assumed that the cause of the sharp Z-dependences is the mechanism of transfer during collisions (process 4 of Ch. II), then even small admixtures of very heavy elements (for example iodine in the CH_3OH + NaI mixtures) should greatly suppress the reaction (14). This, however, was not observed. Finally, the probability of transfer within the framework of the described model depends on the absolute concentration C_Z of the nuclei and should change with varying density, and all the more with varying aggregate states of the substance. Experiments confirm that such a "density effect" is practically nonexistent.[37]

Thus, a preliminary analysis of the experiments shows that earlier models cannot explain the observed regularities, for which a different explanation must be sought.

IV. MODEL OF LARGE MESIC MOLECULES

Before we describe the new scheme of the processes of meson absorption in matter, let us emphasize the following fundamental experimental fact, which this scheme must explain: the sharp Z-dependence of the probability of π^--meson capture by hydrogen nuclei in hydrogen-containing substances is connected precisely with the fact that the hydrogen atom enters into a chemical compound with the other atoms. Thus, for example, for equal concentrations of hydrogen and nitrogen nuclei, the probability of the reaction (14) in hydrazine N_2H_4 is smaller by a factor of 30 than in the equivalent mechanical mixture of nitrogen with hydrogen $N_2 + 2H_2$.[12]

This fact, as well as the entire aggregate of the results of the first group of experiments (Ch. II, Sec. 2) can be explained by assuming that on going over from the continuous spectrum to the discrete spectrum a fraction of the mesons is captured first not by the mesic-atom levels of the isolated mesic atoms $p\pi^-$ and $Z\pi^-$, but by the mesomolecular orbitals of the entire Z_mH_n system as a whole. Such an assumption is equivalent to asserting the possibility of existence of a stable $Z_m\pi^-H_n$ system, consisting of nuclei, an electron shell, and a meson, "smeared out" over a tremendous area $\sim 500\,a_\mu$ ($a_\mu = \hbar^2/Me^2$ is the mesoatomic unit of length).

At first glance this assumption does not seem likely, since the chemical bond between the atoms is provided by electrons at distances on the order of the atomic unit of length $a_0 = 0.53 \times 10^{-8}$ cm, and "from the point of view of the meson" these distances are tremendous (the mesoatomic unit of length of the π^- meson is $a_\mu = 2 \times 10^{-11}$ cm, and in terms of these units the hydrogen-molecule dimension is R = 371; for all the remaining substances R = 400–600).

However, an analysis of the motion of the mesons on high orbitals shows that $Z_m\pi^-H_n$ systems of this type can actually exist.

1. Fundamental Premises of the Model

According to Wightman's estimate,[14] the π^- meson in adiabatic capture by hydrogen falls on a level with quantum number $N_0 = 15$, and this level is already located inside the K-orbit of the hydrogen-atom electron. It was assumed here that the second proton in the hydrogen molecule H_2 cannot influence this estimate. Since the distances between the nuclei are very large units of a_μ, such an assumption seemed natural. A quantitative analysis, however, shows that this is not the case and the influence of the second nucleus must be taken into account.

To demonstrate this, let us consider a system consisting of a proton, a nucleus Z, and a meson (Fig. 4). The potential energy of the meson in the field of two immobile charge centers is

$$U=-\frac{1}{r_1}-\frac{Z}{r_2}, \tag{19}$$

where r_1 and r_2 are the distances from the meson to the proton and to the nucleus Z respectively (Fig. 5). The

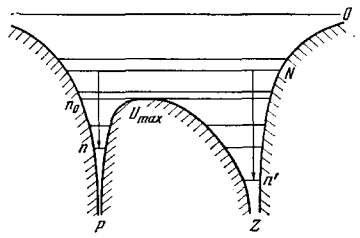

FIG. 4. Level scheme in the $p\pi^-Z$ system. n and n' – levels of isolated mesic atoms $p\pi^-$ and $Z\pi^-$; N–common levels of the system $p\pi^-Z$, n_0–level at which the energy of the meson is $E_n = U_{max}$, i.e., it equals the height of the potential barrier between the nuclei p and Z.

maximum value of the potential (19) is reached at the point

$$r_1 = R/(1+\sqrt{Z}), \quad r_2 = R - r_1$$

and equals

$$U_{max} = -\frac{(1+\sqrt{Z})^2}{R}. \tag{20}$$

The energy of the meson bound in the $p\pi^-$ mesic atom on the n-th orbit in the presence of the nucleus Z at distances $R \gg 1$ is [39]

$$E \approx -\frac{1}{2n^2} - \frac{Z}{R}. \tag{21}$$

In order for the level to belong to an isolated mesic atom $p\pi^-$, it is necessary to satisfy the condition

$$E < U_{max}. \tag{22}$$

From this we obtain an estimate for the maximum value of n_0, at which the meson level can be regarded as belonging to the mesic atom of hydrogen in the presence of another nucleus Z in the molecule:[4]

$$n_0 = \left[\frac{R}{2(1+2\sqrt{Z})}\right]^{1/2} \tag{23}$$

When $1 \leq Z \leq 10$, we have in Z_mH_n compounds R = 400–600 and n_0 = 5–7. For the hydrogen molecule (Z = 1, R = 371) we have n_0 = 8, which is much smaller than the value given by Wightman (N_0 = 15). Thus, the quantum numbers n of the levels of the $p\pi^-$ mesic atoms in the $p\pi^-Z$ system do not exceed the value n_0. As seen from Fig. 4, when $n > n_0$ a phenomenon arises similar to the vanishing of lines in the Stark effect in strong fields, namely, the barrier drops to such a degree that the meson is pulled out by the field of the nucleus Z. Therefore the levels of the $p\pi^-$ mesic atom with quantum numbers $n > n_0$ are common to the entire $p\pi^-Z$ system.*

Taking these considerations into account, as well as the analysis presented in the preceding chapters, we can propose the following sequence of processes occurring when π^- mesons are absorbed by bound hydrogen nuclei:[4]

1) When the π^- mesons go over from the continuous spectrum into the discrete spectrum, some of them are captured by the mesic-atom levels n' of the isolated mesic atoms $Z\pi^-$, and others fall on the mesomolecular orbitals N of the entire molecule as a whole, i.e., a system $Z_m\pi^-H_n$ is produced.

2) This is followed by radiative and Auger transitions from the common levels N to the separated levels n and n' of the mesic atoms $p\pi^-$ and $Z\pi^-$ respectively. We note immediately that in the $Z_m\pi^-H_n$ system, as compared with the isolated mesic atoms, the probability of radiative transitions from high molecular levels N to the mesic-atom levels n and n' should appreciably increase. Indeed, for the N levels the field is no longer centrally symmetrical, and therefore the radiative transitions $N \to n$ and $N \to n'$ do not obey the selection rules with respect to the orbital angular momentum $\Delta l = \pm 1$, which greatly increase the cascade time in isolated mesic atoms.

3) After the meson has gone over to the lower isolated levels of the $p\pi^-$ mesic atom, the mesons are transferred during the collisions from the relatively low orbits (n = 6 – 3) of the mesic atom $p\pi^-$ to the nuclei Z of other atoms. Let us consider each of these stages in more detail.

2. Capture of Muons From the Continuous to the Discrete Spectrum

At the present time there are no calculations that permit an estimate of the fraction of the mesons captured by the common levels N of the mesic molecule, nor was the mechanism of such a capture investigated. In other words, with the exception of metals, we do not know how small batches of energy (~0.02 eV), possessed by the thermal meson prior to capture, are transferred when the mesons go from the continuous spectrum to the discrete spectrum. It is clear, however, that in practice only interaction with the electrons of the atom shell can lead to capture of a meson. For any mechanism of capture of a thermal meson in the discrete spectrum, the probability of releasing an energy much higher than the initial kinetic energy of the free meson is low. Therefore the binding energy of the captured meson is approximately equal to the binding energy of the ejected electron. This in turn signifies that their orbits are geometrically similar, i.e., the orbit of the captured meson is in the vicinity of the former orbit of the electron.

The formulated principle of geometrical similarity of the electron and meson orbits* is equivalent in a certain sense to the assumption that the Fermi-Teller Z-law is valid during the stage of capture of mesons from the continuous spectrum to the discrete one (Ch. II, Sec. 2), since it follows therefore that the total probability of observing the captured meson in any region of the molecule is proportional to the number of electrons in this region. In particular, it follows from it also that on going from the continuous spectrum into the discrete one the meson cannot be captured on a level of the isolated mesic atom $p\pi^-$, since the single electron of the hydrogen atom belongs now to the valence shell of the Z_mH_n molecule. Let us estimate on the basis of this hypothesis the fraction of the mesons captured by the Z_mH_n

*An exact calculation [40] confirms the estimate (23). Allowance for screening of the charge Z by electrons in complex atoms cannot change this estimate greatly, since n_0 depends little on Z.

*Experimental confirmations of this hypothesis are reported in Ch. V.

molecule that falls on the common molecular levels of the $Z_m\pi^-H_n$.

Let the total number of captured mesons be S_0. Then only S_1 mesons fall on the common orbits, with

$$S_1 = a(Z)\,S_0, \qquad a(Z) < 1. \tag{24}$$

Since the number of valence electrons in the Z_mH_n system is 2n, and the total number of electrons is mZ + n, we obtain for the coefficient a(Z)

$$a(Z) = a_L n/(mZ+n)\ . \tag{25}$$

The value of the coefficient a_L is determined by many factors: the character of the bond between the atoms, the ionization energy, etc. For example, for a covalent bond the coefficient a_L should be larger than for an ionic bond, since in the former case the density of the electron cloud between the atoms is higher. One should also expect the magnitude of the coefficient a_L to depend strongly on the structure of the electron shells of the atoms, and since on going from the period L of the periodic table to the period L + 1 the structure of the outer electronic shells changes jumpwise, these changes should lead to a jumpwise change of the coefficients a_L.

Let us estimate the value of a_L, taking into consideration only the character of the bond between the atoms. This means that a_L is determined only by the distribution of the density ρ of the valence electrons, effecting the coupling between the atoms H and Z. By definition

$$\rho = |\psi|^2, \qquad \psi = (\psi_K + \lambda\psi_H)/\sqrt{1+\lambda^2}, \tag{26}$$

where ψ is the wave function of the valence electrons, and ψ_c and ψ_i are the normalized wave functions of the covalent and ionic structures:

$$\psi_H = \psi_H(1)\,\psi_H(2),$$
$$\psi_K = \frac{1}{\Omega}[\psi_H(1)\,\psi_Z(2) + \psi_H(2)\,\psi_Z(1)]. \tag{27}$$

From the normalization condition $\int |\psi_c|^2 d\tau_1 d\tau_2 = 1$ we obtain

$$\Omega^2 = 2(1+s^2), \qquad s = \int \psi_H(1)\,\psi_Z(1)\,d\tau_1 = \int \psi_H(2)\,\psi_Z(2)\,d\tau_2. \tag{28}$$

From (26), neglecting the interference between the covalent and ionic structures, we get

$$\rho = (1-\sigma)\,\rho_K + \sigma\rho_H, \tag{29}$$

where $\sigma = \lambda^2/(1+\lambda^2)$ is the degree of ionicity of the bond,[41] which is equal to the probability of finding both valence electrons near the atom (Z or H) whose electronegativity X is larger. The degree of ionicity of the bond can be calculated from the empirical formula[41]

$$\sigma = 0.16\,|X_Z - X_H| + 0.035\,|X_Z - X_H|^2. \tag{30}$$

The coefficient a_L is equal to that part of the electron "cloud" of the two bond electrons, which is concentrated near the proton $(1/(1+s^2))$ and in the space between the nuclei p and $Z(2s^2/(1+s^2))$. From (26)–(29) we obtain

$$a_L = 2\sigma + (1-\sigma)\left(1 + \frac{s^2}{1+s^2}\right), \tag{31}$$

where the first term in (31) should be omitted if the electronegativity of the hydrogen is smaller than that of the atom Z ($X_H < X_Z$, as for example in the H_2O molecule). For most compounds $s^2/(1+s^2) \approx 0.3$, i.e., approximately 1/7 of the electron density from each bound electron is contained in the overlap region.[41] We note that in the derivation of formula (25) we have in essence assumed that the electrons of the different shells in the atom make equal contributions to the meson slowing-down and capture processes. In the general case this assumption is apparently incorrect, but it can be made for elements of the period II.

Using the electronegativity series,[41] we can use formula (31) to calculate the coefficients a_L for the elements of the period II. It is seen from Table I, where the results of these calculations are presented, that the coefficients a_L vary little inside the period II of the periodic table. It follows also from (31) that in the case of a purely covalent bond $\sigma = 0$ we have $a_L = 1.3$, and that $a_L = 0$ in the case of a purely ionic bond $\sigma = 1$, provided $X_H < X_Z$. For the hydrogen molecule H_2 we have $a_I = 2$.*

3. Cascade Transitions

Let us trace the subsequent fate of the captured mesons. If the meson falls from the very beginning on separated mesic-atom levels n and n′ of the system $Z_m\pi^-H_n$ (see Fig. 4), then the subsequent picture of the processes is quite analogous to the cascade in the isolated mesic atoms $p\pi^-$ and $Z\pi^-$, namely, after losing energy, the mesons reach one of the lower s-states, from which nuclear capture is possible.

According to the geometrical-similarity hypothesis, when the mesons are captured they simply "replace" the electrons in the Z_mH_n molecule. Unlike the remaining atoms, the hydrogen in a chemical compound has no electrons other than the bond electrons, and therefore the meson cannot be captured on separated levels n of the $p\pi^-$ mesic atom. Therefore the hydrogen nucleus can absorb only such a meson, which found its way during the stage of the initial capture on levels N that are common to the entire $Z_m\pi^-H_n$ system, and only subsequently "fell" to the levels n of the isolated mesic atom $p\pi^-$.

Table I

Substance	Z	W exp, 10^{-3}	a_L exp	a_L, Calculated from (31)	Substance	Z	W exp, 10^{-3}	a_L exp	a_L, Calculated from (31)
LiH	3	35±4	1.26±0.15	1.4	N_2H_4	7	5.9±0.7	1.30±0.15	1.1
$B_{10}H_{14}$	5	12.6±1.4	1.44±0.17	1.3	H_2O	8	3.5±0.6	1.12±0.20	0.9
CH	6	5.1±0.6	1.28±0.15	1.2	NaH	11	2.4±0.4	3.6±0.6	—
CH_2	6	13.2±1.5	1.90±0.22	1.2	CaH_2	20	2.5±0.3	11.0±1.4	—

*Actually the uncertainty in the calculations of the coefficient a_L is quite large, but the latest experimental results agree satisfactorily with our estimates (see the table).

Let w_{Nn} and $w_{Nn'}$ be the probabilities of the transitions from the common level N to the separated levels n and n′ of the mesic atoms $p\pi^-$ and $Z\pi^-$ respectively (see Fig. 1). Then the fraction of the mesons S_2, which "falls" from the common mesomolecular level N to the separated levels n of the mesic atom $p\pi^-$ is equal to

$$S_2 = \frac{w_{Nn}}{w_{Nn} + w_{Nn'}} S_1. \tag{32}$$

The transition probabilities W_{Nn} and $w_{Nn'}$ are determined by two processes, namely radiative transitions and Auger transitions. For the Auger transitions, the conversion coefficients are large only at small transition energies ΔE,[24] i.e., for transitions between close mesomolecular levels. Such a process does not lead in most cases to a transition of the mesons from the common levels to the separated ones. The probability of radiative transitions is proportional to $(\Delta E)^3$, so that it is convenient for the meson to go over to the lowest of the possible mesic-atom levels. In the isolated mesic atom, this is prevented by the selection rules with respect to the orbital angular momentum $\Delta l = \pm 1$. But in the $Z_m\pi^-H_n$ system, as already noted, the central symmetry of the field is violated, and therefore the selection rules with respect to l no longer hold.* Thus, the main contribution to the transition probabilities w_{Nn} and $w_{Nn'}$ from the common mesomolecular level of the $Z_m\pi^-H_n$ to the separate levels of the mesic atoms $p\pi^-$ and $Z\pi^-$ should be given by the radiative transitions.

Let us calculate first the probability $w_{Nn'}$. As is known,[42] in mesoatomic units ($\hbar = M = e = 1$)

$$w_{Nn'} = \frac{4}{3}\alpha^3\omega_0\mu A_{Nn'}\, ce\kappa^{-1},$$
$$A_{Nn'} = (E_N - E_{n'})^3 |\, r_{Nn'}\,|^2, \tag{33}$$

where $\omega_0 = 4.1 \times 10^{16}$ sec^{-1}, $\mu = M/m$, E_N and $E_{n'}$ are the energies of the levels N and n′, and $r_{Nn'}$ is the matrix element of the dipole transition between the states N and n′:

$$r_{Nn'} = \int \psi_N r \psi_{n'} d\tau. \tag{34}$$

Let us consider first the transitions to the ground state (n′ = 1). In this case, as shown by estimates,[4] the main contribution to the integral (34) is made by the region of distances $r_2 \sim 1/Z$ (see Fig. 5). In this case

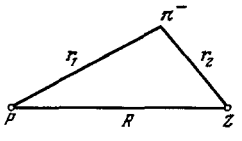

FIG. 5. The pπ–Z system.

expression (34) for the transitions N → n′ assumes the following form:

$$r_{Nn'} \approx \int d\mathbf{r}_2 \psi_N(\mathbf{r}_2)\, r_2 \psi_{n'}(\mathbf{r}_2), \tag{35}$$

where the region of integration can be limited to a sphere of radius $r_0 \approx 1/Z$. The direction $r_{Nn'}$ coincides with the internuclear axis, and the remaining components of the matrix element (34) are ≈ 0.[4]

The wave function $\psi_{n'}(r)$ of the ground state is[42]

$$\psi_{n'}(r) = \frac{Z^{3/2}}{\sqrt{\pi}} e^{-Zr} \tag{36}$$

(we put henceforth $r_2 = r$). The common levels E_N of the $Z_m\pi^-H_n$ system are located in the region of the outer electron shells of the atom, at distances $r \approx r_N$ from the nucleus. In this region we have for all the atoms (regardless of the value of the charge Z of the nucleus) $r_N \approx \mu$, $E_n \approx -1/2\mu$, and the wave function of the level E_N is

$$\psi_N \approx \frac{1}{r\sqrt{\mu p}}, \tag{37}$$

where $p = \sqrt{2(E - U(r))}$ is the quasimomentum of the meson. These relations follow from the Thomas-Fermi model, which have been sufficiently well verified experimentally and are the consequences of the screening of the charge of the nucleus by the internal electrons.[43]

From the condition of the continuity of the wave function at distances $r \sim 1/Z$ from the nucleus, it follows that

$$\psi_N \approx \left(\frac{Z}{\mu}\right)^{1/2}. \tag{38}$$

We note that this formula takes into account in a somewhat unusual manner (without introducing Z_{eff}) the influence of the screening of the nucleus Z on the wave function ψ_N. If there is no screening, then $Z\psi_N \approx (Z/N)^{3/4}$ near the nucleus, and since for a meson moving with energy $E_N \sim -1/2\mu$ at distances $r_N \sim \mu$ from the nucleus Z we have $N^2 \sim Z\mu$, it follows that when $r \sim 1/Z$ its wave function, unlike (38), is

$$\psi_N \approx \left(\frac{Z}{\mu}\right)^{3/4}. \tag{39}$$

Using (38) and (36), we obtain approximately for the matrix element (34)

$$r_{Nn'} \approx \pi r_0^4 \left(\frac{Z}{\mu}\right)^{1/2} \frac{Z^{3/2}}{\sqrt{\pi}} = \left(\frac{\pi}{\mu}\right)^{1/2} Z^{-2}. \tag{40}$$

When $N \gg 1$ we have $E_N - E_{n'} \approx \frac{1}{2}(Z/n')^2$, and the transition probability $w_{Nn'}$ at n′ = 1 is equal to

$$w_{Nn'} \approx \frac{\pi}{6}\alpha^3\omega_0 Z^2 \approx 0.8 \cdot 10^{10} Z^2 \text{ sec}^{-1}. \tag{41}$$

The probability w_{Nn} is obtained from (41) by putting Z = 1 and n′ = n = 1.* It follows from (41) that when n = n′ = 1 we have

$$\frac{W_{Nn'}}{W_{Nn}} = Z^2, \tag{42}$$

and from (32) and (24) we get

$$S_2 = \frac{S_1}{Z^2 + 1} \approx \frac{a(Z)}{Z^2} S_0. \tag{43}$$

By definition, the probability W of the capture of a π^- meson by a hydrogen nucleus is

*In a diatomic molecule, the selection rules with respect to the magnetic quantum number m remain in force.

*In the scheme considered here, the radiative transitions to the levels $n' > 1$ are suppressed, since $(\Delta E)^3 \sim (n')^{-6}$, and after substituting $r_0 \sim n'/Z$ in (40) we obtain $w_{Nn'} \sim 1/n'$. In addition, as shown by estimates [4], transitions with a change of the magnetic quantum number m ($\Delta m \neq 0$) are also strongly suppressed.

$$W_{Z_mH_n} = \frac{S_2}{S_0} = a_L \frac{n}{mZ+n} \frac{1}{Z^2+1}. \quad (44)$$

Using (44), (43), and (25), we obtain ultimately

$$W_{Z_mH_n} \approx a_L \frac{nZ^{-2}}{mZ+n}. \quad (45)$$

For more complicated substances such as $Z'_kZ_mH_n$ (for example, C_2H_5OH and others), the foregoing scheme of the processes leads to the relation

$$W_{Z'_kZ_mH_n} = \frac{a'_L\nu'(Z')^{-2} + a_L\nu Z^{-2}}{kZ'+mZ+n}, \quad (46)$$

where ν' and ν are the numbers of the bonds of the atoms Z' and Z with the hydrogen ($\nu' + \nu = n$). When $\nu' = (0)$ and $k = 0$ we have $\nu = n$, and we return to formula (45).

V. MODEL OF LARGE MESIC MOLECULES AND NEW EXPERIMENTS

The model of mesoatomic processes in chemical compounds, described in the preceding chapter, was based on two statements:

1) The distribution of the mesons over the levels of the mesic molecule $Z_m\pi^-H_n$ and of the mesic atom $Z\pi^-$ on going from the continuous spectrum to the discrete spectrum is determined by the structure (25) of the coefficient a(Z).

2) Further redistribution of the mesons from the common mesomolecular levels N, which lie in the region of the valence electrons of the molecule, to the levels n and n' of the isolated mesic atoms is due to radiative transitions (see Fig. 4).

1. Z-Dependence of the Probability W in Chemical Compounds

These two assumptions suffice to explain many features of the experimental results noted in Ch. III. In particular, they explain the results of the group of experiments[2,3,5,6,37] aimed at studying the reaction $\pi^- + p \rightarrow n + \pi^0$ in chemical compounds of the Z_mH_n type (Sec. 1 of Ch. III), namely, the sharp suppression of the probability W with increasing Z, and the independence of W of the density and of the aggregate state of the substance, and also of the admixture of heavy elements. As seen from Fig. 6, the experimentally obtained values of the reduced probability

$$P = \left(\frac{m}{n} + \frac{1}{Z}\right) W_{Z_mH_n} \quad (47)$$

for binary compounds Z_mH_n of period II of the periodic table agree with the relation

$$P = a_L Z^{-3}, \quad (48)$$

that follows from (45). The coefficients a_L determined from these measurements are equal to*

$$a_I = 2, \quad a_{II} = 1.28 \pm 0.15, \quad a_{III} = 3.6 \pm 0.6, \quad a_{IV} = 11.0 \pm 1.4. \quad (49)$$

We note that these values of the coefficients a_L were obtained from experimental data under the assumption that formula (45) holds for all periods of the periodic table. It was actually verified only for the elements of period II.*

Recently, the model of large mesic molecules was subjected to a more detailed test.[5,6] In particular, the probability W was measured for more complicated substances of the type $Z'_kZ_mH_n$. It turned out that formula (46) describes the experimental data well (see Fig. 6 and Table I). In complex organic compounds, formula (46) likewise holds (Table II). Formula (46) is valid also in the case of mixtures of two hydrogen-containing substances (for example 40% C_2H_5OH + 60% H_2O).

However, this formula cannot explain the difference between the values of a_L for such compounds as CH and CH_2, for which $W_{CH_2}/2W_{CH} = 1.30 \pm 0.13$,[2] and not 7/8 as follows from (45). It is quite probable that this effect is connected with differences between the conditions of the initial capture of the mesons on the mesic-molecule levels.†

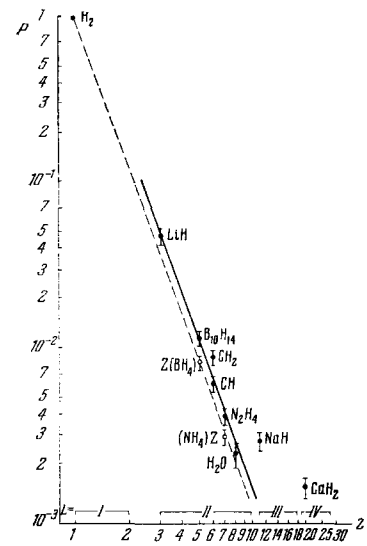

FIG. 6. Dependence of the reduced probability P (formula (47)) on Z for compound of the type Z_mH_n and $Z'_kZ_mH_n$. The solid and dashed straight lines correspond to relations (4.8) with $a_L = 1.28$ and 1. [5].

Table II

Substance	W_{exp}, 10^{-3}	W, calculated from (46), 10^{-3}
CH_3OH	7.5±0.9	7.0±0.8
C_2H_5OH	8.1±0.9	7.7±0.9
C_3H_6O	6.6±0.8	6.7±0.8
C_5H_7OOH	4.2±0.5	4.9±0.6

*The value $a_I = 2$ has been obtained from formula (44) with allowance for the normalization condition $W_{H_2} = 1$, and also for the equalities m = 0, n = 2 and Z = 1.

*One of the possible causes of such a behavior of the coefficients a_L may be that when the mesons are captured in the discrete spectrum in heavy atoms, the electrons of the different shells give unequal contributions to the slowing down of the mesons. Therefore the structure (24) of the coefficient a(Z) may differ from (25). We note that the growth (49) of the coefficients a_L is limited by the condition $a(Z) \leq 1$, which means that all the mesons are captured by the common levels of the system $Z_m\pi^-H_n$.

†The differences in the conditions of capture in CH_2 and CH may be connected with the different character of the hybrid bonds in these compounds. The contribution of the p-orbital to the valence bonds of the molecule CH_2 (sp^2 hybridization) is larger by 1.3 times than in the case of CH (sp hybridization) [41].

Finally, we note one more consequence of the results of [5,6]. The compounds investigated in these studies, of the type $(NH_4)Z'$ and $Z'(BH_4)$ differ only in the atom (or group of atoms) with charge Z'. This atom is bound to the ions $(NH_4)^+$ and $(BH_4)^-$ by an ionic bond and therefore, according to the main premises of the model, takes no part in the meson transfer. Formula (46) assumes for such compounds the form

$$W = a_{II}\frac{4Z^{-2}}{Z'+Z+4}. \quad (50)$$

It follows from this that the quantities 1/W for the indicated compounds should depend linearly on Z':

$$1/W \sim Z' + \text{const}. \quad (51)$$

As seen from Fig. 7, the experimental data confirm this

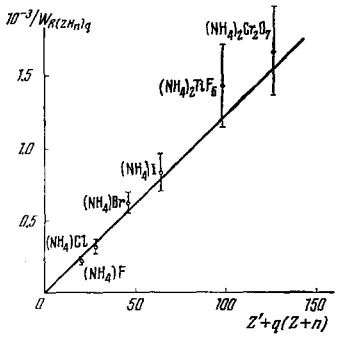

FIG. 7. Results of experimental verification of formulas (51) and (52) for compounds of the $R(ZH_4)_q$ type. Line—relation (51) calculated for $a_L = 1$; Z'—total charge of the nuclei of the group of atoms R.

conclusion. The latter signifies that the Z-law is actually fulfilled during the initial landing, and offers evidence that our approach is valid.

2. Features of Capture of π^- Mesons in Symmetrical Systems

In the preceding section it was shown that formula (46) is well satisfied for a large class of compounds. However, for substances of the type $R(ZH_n)_q$, which contain q identical hydrogen-containing groups connected with a certain group of atoms R, it gives values that are overestimated compared with the experimental ones by a factor q.[6] This regularity is general and does not depend on the type of the identical groups (Table III), i.e., the capture process occurs as if not only the Z atom which is directly coupled with the hydrogen atom, but all the Z atoms of the remaining (q – 1) groups were to take part in the "removal" of the π^- meson from the hydrogen atom.

This effect can be naturally explained within the framework of the quantum-mechanical theory of resonance. Indeed, since all the Z_mH_n groups are identical, the meson captured initially by one of them will subsequently be distributed among all the groups as a result of the quantum-mechanical exchange effects* (in analogy with π electrons in the benzene molecule). However, the group in which the meson was initially captured is singled out among the other Z_mH_n groups, since it has lost one electron following the capture of the meson. This loss does not violate the condition that the groups be identical, since the form of the potential wells is determined mainly by the atoms Z. Now, however, atoms Z from all the q groups take part in the capture, whereas capture by the hydrogen atom is possible only in the initial group, since in the remaining (q – 1) groups this capture is prevented by the electrons screening the hydrogen nuclei.* Thus, in a system with q identical groups the effects of quantum-mechanical resonance should lead to a suppression of the probability of the π^- meson capture by bound hydrogen by a factor of q compared with formula (46), which takes the following form for symmetrical systems of the type $R(Z_mH_n)_q$:

$$W_{R(Z_mH_n)_q} = a_L\frac{nZ^{-2}}{Z'+q(mZ+n)}; \quad (52)$$

here Z' is the total charge of the nuclei of the group of atoms R. As seen from Table III, formula (52) describes

Table III

Substance	W exp, 10^{-4}	q	$\frac{W \text{ calc. from (46)}}{W \text{ exp}}$	W, calculated from (52), 10^{-4}
KOH	6.7±2.1	1	1.1±0,3	7.2±0.8
$Ca(OH)_2$	5.7±1.4	2	1.9±0.5	5.3±0.6
$Cd(OH)_2$	3.3±1.2	2	1.9±0.7	3.1±0.4
$Ba(OH)_2$	2.5±0,7	2	2.2±0.6	2.7±0.3
$Al(OH)_3$	6.3±1.8	3	2,4±0.7	5.0±0.6
$(NH_4)F$ *)	46±7	1	1.1±0,2	42±4
$(NH_4)Cl$ *)	32±6	1	1.1±0.2	29±3
$(NH_4)Br$ *)	16±2	1	0.9±0.1	18±2
$(NH_4)I$ *)	12±2	1	0.9±0.2	12.8±1.3
$(NH_4)_2Cr_2O_7$ *)	6.0±1.1	2	2.2±0.4	6.5±0.7
$(NH_4)_2TiF_6$*)	7.0±1.4	2	2.4±0.5	8.3±0.9
$Ba(CH_3COO)_2$	10,5±1.4	2	1.7±0.3	9.0±1.0
$Cd(CH_3COO)_2$	8.7±1.3	2	2.2±0.3	9.7±1.1
$Pb(CH_3COO)_2$	6±2	2	2.0±0.7	6.1±0.8
$C_2H_4Cl_2$	13±2	2	2.2±0.3	14±1

*In the calculation of W for compounds of the type $R(NH_4)_q$ we assumed $a_L = 1.0 \pm 0.15$, in accord with measurements [5] for ammonium salts. For all the remaining compound we assumed $a_L = 1.28 \pm 0.15$, which is the average over period II. [5].

well the experimental results for a large class of substances: bases, acids, and basic salts, and also for organic halides.

3. Capture of μ^- mesons in chemical compounds.

The model of large mesic molecules was proposed earlier[4] to explain the singularities of the process of absorption of stopped π^- mesons in substances of the Z_mH_n type. Recent experiments[7-10,44] were devoted to the processes of absorption of μ^- mesons in various

*At average barrier heights ~ 0.1 – 0.2 eV and at distances ~ 1 – 2 a.u., the exchange time for π^- mesons is $\tau \sim 10^{-12}$ sec, which is shorter than the time of radiative transitions to the hydrogen ($\sim 10^{-10}$ sec, see (41)).

*For clarity we have used a simplified scheme. Actually, the assumption that the initial Z_mH_n group is singled out is unnecessary, since not only the π^- meson, but all the (q – 1) electrons from the hydrogen atoms of the other groups take part in the exchange interaction between all the groups. These electrons will hinder the capture of the meson by the hydrogen, and as a result the total probability of finding the meson near any of the hydrogen atoms is equal to $\rho_\pi/(\rho_\pi + \rho_e) = 1/q$, since in the subsystem $(Z_mH_n)_q$ there is one meson ($\rho_\pi = 1$) and (q – 1) electrons preventing the capture ($\rho_e = q - 1$). Thus, the probability W will be suppressed in such compounds by a factor q compared with formula (46).

chemical substances. It is of interest to analyze the results of these experiments from the point of view of the proposed model, and to consider the consequences ensuing from this model.

We first stop to discuss the results of an investigation of the structure of the mesic-x-ray K series in metals and chemical compounds,[9,36] which occurs in cascade transitions of μ^- meson following its capture by high levels of the mesic atoms. The experimental conditions make it possible to separate in the K series the lines K_α (transition 2p → 1s), K_β (transition 3p → 1s), and the K_ν series (contribution of all the remaining transitions np → 1s from the higher levels n of the mesic atom to the ground level 1s). In these experiments, an anomalously large contribution of the K_ν series to the total intensity of the K series was observed at $Z > 20$ (see, for example, Fig. 8). This fact contradicts the calculations of Eisenberg and Kessler,[17] and cannot be understood within the framework of the theory of cascade transitions in the isolated atom. In addition, according to the calculations of [17], the intensity of K_ν series should decrease with increasing Z, whereas the experiments[36] show the inverse (Fig. 8).

The results of these experiments can be explained by assuming that some of the μ^- mesons are captured while slowing down by the high levels in the region of the valence shells, and consequently the entire K_ν series is the result of transitions from these levels to the ground level, bypassing the cascade.* The experimental value of the intensity of the K_ν series in metallic chromium is $J_\nu = 0.22$.[9] Since the effective valence of Cr (Z = 24) is ~5.8,[43] we obtain J_ν the estimate (24) $J_\nu \approx 5.8/24 = 0.24$, which agrees well with the experimental value. We note that in monatomic gases the contribution of K_ν series is smaller (for Ar, for example, $J_\nu \approx 0.07$[10]) and is approximately equal to the calculated value,[17] which also agrees with our notions.

It follows from the same reasoning that the structure of the K series of the ionic compounds CsCl, KCl, etc. should be similar to the structure of the K series of the noble gases, for in this case the density of the electron cloud between the atoms is small, and the number of mesons falling on the common levels is much smaller than in metals or in compounds with a covalent bond. Indeed, an indication to this effect for the case of KCl can be found in [9]. Finally, the change of the intensity of the K_ν series over the period of the periodic table, from the point of view of the model, can be attributed to a change in the number of the valence electrons.

The dependence of the structure of the mesoatomic K series on the singularities of the structure of the external electron shells forming the chemical bond between the atoms is clearly demonstrated also by experiments with metals and their oxides.[9,44] It was observed in these experiments that the intensity of the K_ν series (more accurately, the ratio $J_\nu/(J_\alpha + J_\beta + J_\nu)$ in metals is larger by 1.3–1.6 times than in oxides of these metals (Figs. 9a and b). This fact can be attributed to the change of the effective valence, which in pure metals is higher than in oxides, and also to the shift of the valence electron cloud in the oxide molecule towards the electronegative oxygen.

We call attention to one more singularity of the structure of the K series in chemical compounds and in metals: the x-ray spectra of the mesic atoms of these sub-

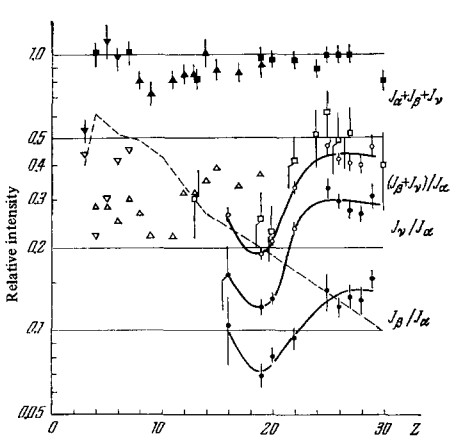

FIG. 8. Structure of mesic-x-ray K series at different nuclear charges Z. Δ, ▽, □, ○–intensity ratio $(J_\beta + J_\nu) / J_\alpha$ [17,33]; dashed–results of calculations of [17]; ●–J_ν / J_α and J_β / J_α [17].

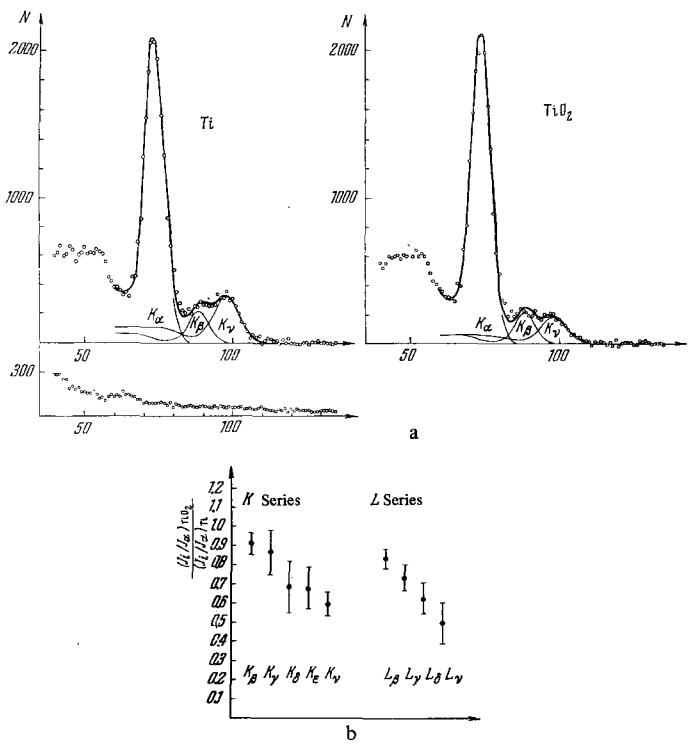

FIG. 9. a) Structure of K series of titanium in pure Ti and in TiO_2 [9]. The abscissas represent the numbers of the channels of the pulse-height analyzer (see Fig. 2). N–number of counts in the channel. The contribution of the K_ν in Ti is larger than in TiO_2. b) Structure of K_ν and L_ν series of Ti and TiO_2, investigated in [44] with the aid of germanium detectors (corresponding for the K series to Fig. 9a, obtained with an NaI detector). We see that not only the total intensity J_ν of the entire K_ν series (Fig. 9a), but also the intensities of the individual lines of the K_ν series (transitions np → 1s) and of the L_ν series (transitions np → 2s) are larger in Ti compared with TiO_2.

*According to the notions of [4], this radiation should be polarized along the axis of a linear molecule.

stances reveal the distinct dip in the region between the lines K_β and K_ν, i.e., there are no transitions np → 1s from the levels n = 4–6 in the spectrum of the K series (Fig. 10). Within the framework of our model this phenomenon is naturally explained, since it is precisely these levels that lie in the transition region between the common levels of the mesic molecule and the lower levels n and n′ of the isolated mesic atoms (see Fig. 4).

The influence of the outer electron shells on the capture of μ^- mesons in chemical compounds was convincingly demonstrated in [7], where this phenomenon was investigated in a wide range of variations of the charges Z of nuclei of the atoms forming the compound. In these experiments they measured the ratio of the probabilities f(Z) for the absorption of the μ^- meson by the Z atom and by the oxygen atom in oxides of the type Z_mO_n:

$$f(Z)=\frac{n}{m}\frac{J(Z)}{J(8)}, \tag{53}$$

where J(8) and J(Z) are the intensities of the K series in oxygen and in an atom with charge Z, respectively. The periodic character of the f(Z), which duplicates the structure of the periodic system of the elements, is clearly seen from Fig. 11, which represents the results of these experiments; the positions of the minima of f(Z) correspond to the alkali metals.

On the whole, however, the deviations from the Z-law observed in Z'_kZ_m systems were not as large as in hydrogen-containing compounds of the Z_mH_n type. From the point of view of the model, the absence of sharp Z-dependences in the Z'_kZ_m systems containing no hydrogen is due to the fact that the violations of the Z-law in this case are connected only with the redistribution of the mesons initially falling on the common mesomolecular levels. Since such mesons constitute a relatively small fraction of the total number of captured mesons, the violations of the Z-law in Z'_kZ_m systems cannot be large. In many cases experiments confirm this conclusion.[33]

The two assumptions on which the model of large mesic molecules is based (concerning the mechanism of landing and subsequent transitions see Ch. IV) lead to the conclusion that the mesons should be predominantly absorbed by nuclei of atoms with large charges Z. The results of experiments on absorption of μ^- mesons by oxides of different substances,[7] however, contradict this conclusion: in these compounds, more mesons are almost always absorbed by the oxygen atom than by an atom of another substance (see Fig. 11), regardless of the charge of its nucleus:

$$f(Z):Z/8<1. \tag{54}$$

A possible cause of such a predominant capture of mesons by oxygen may be the ability of oxygen of forming negative ions, particularly in compounds with alkali metals. The cross section for the capture by mesic-atom levels of oxygen may increase in this case as a result of adiabatic transitions of the meson from the continuous spectrum to the discrete one.[28] The aforementioned singularity of oxides of alkali metals can lead to the appearance of dips on the f(Z) plot at the beginning of each period (see Fig. 11).

VI. MESON TRANSFER PROCESSES

The transfer of mesons from hydrogen mesic atoms to Z nuclei of atoms of other substances did not play a noticeable role in the previously considered processes. In some cases, however, it is decisive.

1. Transfer of μ^- Mesons

The large lifetime of the μ^- meson and the small rates of its capture by hydrogen nuclei cause the overwhelming number of μ^- mesons at small concentrations of the impurity Z to be transferred to the nuclei Z from the K orbit of the hydrogen mesic atom.

$$p\mu^- + Z \rightarrow Z\mu^- + p. \tag{55}$$

The rate of μ^--meson transfer is

$$\omega = \sigma v C_Z, \tag{56}$$

where σ is the transfer cross section, v the relative collision rate, and C_Z the concentration of the Z nuclei. At small v we have $\sigma \sim 1/v$, i.e., σv = const. This makes it possible to introduce a reduced transfer constant λ_Z:

$$\lambda_Z = \sigma v C_H^0; \tag{57}$$

here $C_H^0 = 4.25 \times 10^{22}$ nuclei/cm³ is the density of liquid

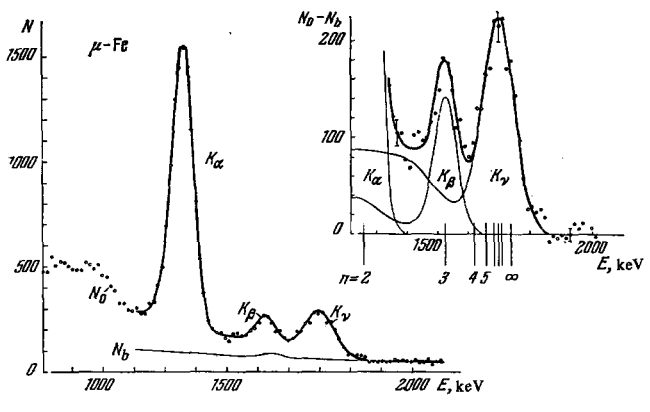

FIG. 10. Structure of K series of iron [36]. E–transition energy, n–number of level. The dip between the lines K_β and K_ν corresponds to n ≈ 4–6.

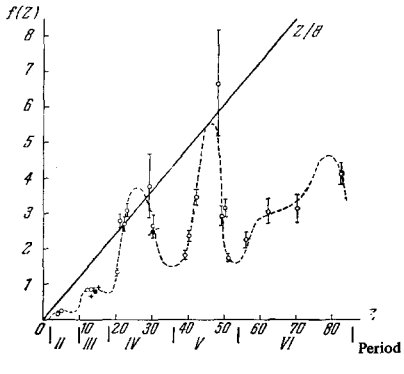

FIG. 11. Relative probability of capture f(Z) = (n/m)J(Z)/J (8) of μ^- mesons by nuclei with charge Z, compared with oxygen in oxides of the type Z_mO_n [7]. The minima of the function f(Z) lie at the starts of the periods of the periodic table. The straight line corresponds to the Fermi-Teller Z-law.

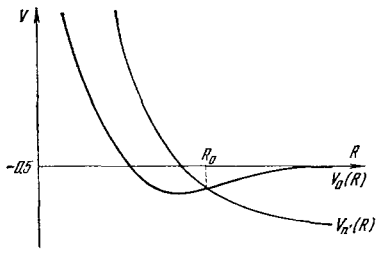

FIG. 12. Scheme of terms of the system $p\mu$-Z [19]. When $R \to \infty$ the term $V_0(R)$ corresponds to the ground state of the system $p\mu^- + Z$, and the terms $V_n'(R)$ correspond to the levels of the system $Z\mu^- + p$.

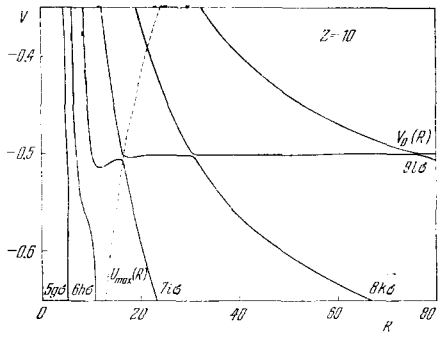

FIG. 13. Picture of terms for the system $p\mu$-Ne [40]. The pseudointersections of the terms $V_0(R)$ and $9l\sigma$, $9l\sigma$, and $8k\sigma$, $8k\sigma$, and $7i\sigma$ occurs at $R_0 = 76, 31, 17$.

hydrogen. Hence

$$\omega = \lambda_Z c_Z, \tag{58}$$

where $c_Z = C_Z/C_H^0$ is the reduced concentration of the nuclei Z. The experimental and theoretical study of the process (55) has been the subject of a number of investigations,[31-32,38,19] according to which λ is approximately proportional to Z and is equal to

$$\lambda_Z = (0.84 \pm 0.04) \cdot 10^{10} Z \text{ sec}^{-1}. \tag{59}$$

Calculation of λ_Z reduces to a solution of the problem of scattering of three particles interacting in accordance with the Coulomb law. The procedure for such calculations[45,19] is best explained in the language of terms (Fig. 12).

Assume that the meson is in the ground state of the mesic atom $p\mu^-$, where its energy in mesic-atom units ($\hbar = M = e = 1$) is $E_0 = -1/2$. At large distances R between the nuclei p and Z, the total energy $V_0(R)$ of the system $p\mu^- + Z$, including the repulsion energy of the nuclei, is equal to

$$V_0(R) = -\frac{1}{2} - \frac{9}{4}\frac{Z^2}{R^4}. \tag{60}$$

Any term V(R) of the $p\mu^-Z$ system is determined by a set of quantum numbers.[39] The system $Z\mu^- + p$ also has its own terms $V_n'(R)$, which in the case when $R \gg 1$ depend on R in the following manner:

$$V_{n'}(R) = -\frac{1}{2}\left(\frac{Z'}{n'}\right)^2 + \frac{Z-1}{R}; \tag{61}$$

here n' is the quantum number of the level of the mesic atom $Z\mu^-$. It follows from (60) and (61) that for certain levels n', at definite values of $R = R_0$, the terms $V_0(R)$ and $V_n'(R)$ intersect. According to the general theory of non-adiabatic transitions,[44] intense transitions are possible at this point from the term $V_0(R)$ to the term $V_n'(R)$, which precisely corresponds to the reaction (55). The μ^- meson then falls on high orbits ($n' \lesssim Z$) of the mesic atom $Z\mu^-$.

Recent calculations[40] have shown, however, that in the systems $Z\mu^- + p$ and $p\mu^- + Z$, the picture of the terms is somewhat different than that assumed in earlier papers.[19] In particular, pseudointersections of the terms were observed at certain points R_0, where term intersection was observed earlier on the basis of the approximate formulas (60) and (61) (Fig. 13). This circumstance apparently does not change very strongly the results of the earlier calculations of λ_Z, but it must be taken into account in further calculations.

In conclusion we note an experiment[10] in which the structure of the mesic-x-ray K series of Ar was investigated in a mixture H_2 + Ar. Usually in the calculation of the transfer constants in such systems it is assumed that the main contribution to the cross section of the transfer of the μ^- meson from $p\mu^-$ mesic atoms to the nuclei Z is made by nonradiative transitions (55) upon intersection (or pseudointersection) of the terms. It follows from [10], however, that approximately half of the mesons is transferred from the levels of the mesic atom $Z\mu^-$, with emission of γ quanta (Fig. 14), i.e., in accordance with the reaction

$$p\mu^- + Z \to Z\mu^- + p + \gamma.$$

So far, there are no correct calculations for the probability of such a reaction. We note that these γ quanta are perfectly analogous to the K_ν series occurring in the transition of μ^- mesons from the common levels of the mesic molecule $p\mu^-Z$, since the effective quantum number of the level N of the system $p\mu^-Z$, from which the transition of the μ^- meson takes place upon collision, is very large and is approximately equal to Z.[39]

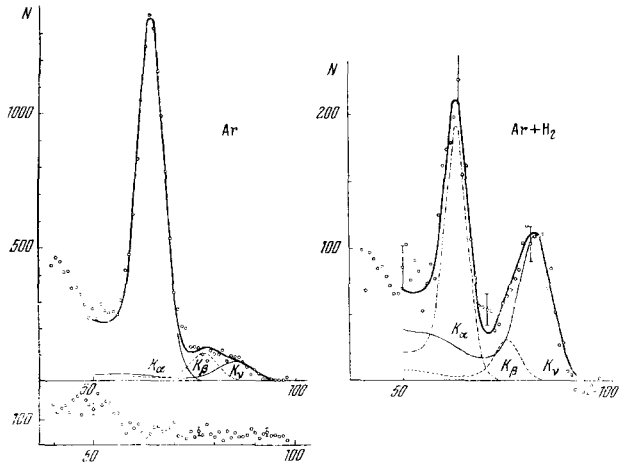

FIG. 14. Structure of μ^--mesic-x-ray K series of Ar in pure Ar and in a mixture of Ar + H_2 [10]. Radiative transitions from high levels of the system $H\mu$ - Ar to the levels of the system μ^-Ar amount to ~ 50%. The symbols are the same as in Fig. 9.

2. Transfer of π^- Mesons

All the preceding arguments are valid only for μ^--mesic atoms. Indeed, the rate of nuclear capture of π^- mesons by a proton from the ground state is quite large (9), and transfer to other systems is possible only for high levels n of the mesic $p\pi^-$ atom, where the transfer rates at relative concentrations $C \sim 1$ are comparable with the probabilities of the cascade transitions and with the rate of nuclear capture. Estimates show that transfer of π^- mesons takes place from levels $n \approx 3-6$ of the $p\pi^-$ mesic atom.*

After these preliminary remarks, let us consider another group of experiments on the measurements of the probability of transfer of π^- mesons Q in gas mixtures $H_2 + Z$ (see Ch. III). In particular, we shall attempt to explain the experimentally obtained dependence of the quantity Q_{H_2+Z} on the concentration of the gases and on the charge Z of the nucleus of the impurity atom.

According to the accepted scheme, during the first stage of the capture the stopped π^- mesons are distributed in the mixture of the gases in accordance with the Z-law.[5-6,8,37,47] During the second stage there are produced mesic atoms $p\pi^-$ with mesons on the orbits $n \leq 7$ (see [23]). This is followed by the transfer processes $p\pi^- + Z \rightarrow Z\pi^- + p$. Let us describe them phenomenologically.

Let the summary velocity γ of the cascade transitions and of the nuclear absorption in the mesic atom $p\pi^-$ be independent of the collisions with other atoms, let the transfer constant of the process be λ. Under these assumptions, the probability of transfer Q is equal to

$$Q=\frac{\lambda c_Z}{\gamma+\lambda c_Z}, \quad c_Z=C_Z/C_Z^0, \quad c_H=C_H/C_H^0, \tag{62}$$

i.e., Q depends on the reduced concentration c_Z of the impurity nuclei. The experimental data[11] contradict this conclusion.

The reason for this discrepancy lies in the fact that the radiative transitions in the isolated $p\pi^-$ mesic atom, as noted in Ch. II, Sec. 5, are not the only process and are not the principal process leading to the rapid drop of the π^- meson to the lower levels of the $p\pi^-$ mesic atom. A much larger contribution to the probability of the drop of the meson downward and of the nuclear capture is made by processes that occur in collisions between the mesic atom $p\pi^-$ and atoms of other substances —the external Auger effect and nuclear capture from higher ns states of the $p\pi^-$ mesic atom in collisions (the mechanism of Day-Snow-Sucher). Both processes obviously depend on the number of collisions of the $p\pi^-$ mesic atom with the Z nuclei and the protons, i.e., on the concentrations c_Z and c_H. In this case γ is proportional to the concentrations c_H and c_Z:

$$\gamma = ac_H + bc_Z, \tag{63}$$

and in lieu of (62) we obtain the expression:[4]

$$Q=\frac{\Lambda C}{1+(\Lambda+\varkappa)C}, \tag{64}$$

where $\Lambda = \lambda/a$, $\kappa = b/a$, $C = c_Z/c_H$. Thus, the value of Q depends only on the relative concentration of the impurity nuclei C, and not on the concentrations c_Z and c_H separately, in agreement with the experimental data (Ch. III, Sec. 4).

A more detailed analysis[4] does not change this conclusion significantly. In particular, there always exists a finite probability of a radiative transition in the mesic atom $p\pi^-$, which does not depend on the collisions with other atoms. Therefore in the general case

$$\gamma = ac_H + bc_Z + \alpha, \tag{65}$$

where α is small and does not depend on c_H and c_Z. The corresponding changes in formula (64) are obvious.

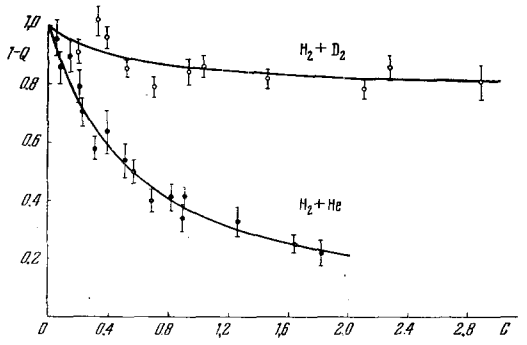

FIG. 15. The dependence of the probability Q of the transfer of π^- mesons from hydrogen to helium and deuterium in H_2 + He and H_2 + D_2 mixtures of the relative concentration C. The curves were calculated from formula (64).

The transfer of π^- mesons in gas mixtures H_2 + Z was investigated[11] for a number of admixtures Z (D_2, He, N_2, Ne, Ar). Transfer of the π^- mesons from the $p\pi^-$ mesic atoms to the nuclei Z was observed (see, for example, Fig. 15). A reduction of the experimental data has shown that

$$\Lambda = (0.7 \pm 0.2)Z \tag{66}$$

(Fig. 16). The reduced transfer constant Λ_Z, which is analogous to the quantity λ_Z for μ^- mesons (59), is equal to*

$$\Lambda_Z \approx 3 \cdot 10^{11} Z \text{ sec}^{-1}. \tag{67}$$

The π^--meson transfer constant Λ_Z is larger by two orders of magnitude than the μ^--meson transfer constant λ_Z determined in experiments at low impurity concentrations C_Z. This means that the transfer of π^- mesons occurs not from the K orbit (as for μ^- mesons), but from levels n = 3-4.†

*If it is assumed that at low velocities of the $p\pi^-$ mesic atoms the cross section of the transfer from the level n is approximately proportional to n^4, then $\lambda_Z \approx 0.8 \times 10^{10} Zn^4$ (see formula (59). Equating this value to the rate of nuclear transfer (9) $\Gamma = 1.6 \times 10^{15}/n^3$, we obtain the lower limit for n. The small level population of the isolated $p\pi^-$ mesic atom with $n > 6$ follows from formula (23).

*The absolute magnitude of the transfer constant $\Lambda_Z = aC_H^0\Lambda$. The coefficient for the recalculation from Λ to Λ_Z was determined on the basis of measurements of the lifetime of the π^-mesons in hydrogen [48], with allowance for the calculations of Leon and Bethe [16]: $aC_H^0 \approx 4 \times 10^{11}$ sec^{-1}.

†This estimate follows from a comparison of formulas (59) and (67) under the assumption that the transfer cross section grows in proportion to n^4.

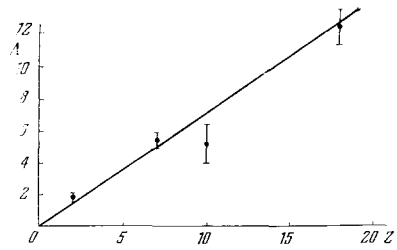

FIG. 16. Z-dependence of the transfer constant Λ of π^- mesons from the mesic atoms $p\pi^-$ to the nuclei Z in gas mixtures of the type $H_2 + Z$ [11] Straight line—relation (66) ($\Lambda = 0.7\,Z$).

κ turns out to be a small quantity ($\kappa < 0.2$–0.5), thus indicating the absence of a noticeable influence of the collisions of the $p\pi^-$ mesic atoms with nuclei Z on the value of γ, i.e., on the rate of meson capture by the proton. In addition, $\kappa \ll \Lambda$, meaning that transfer processes predominate over de-excitation processes in collisions.

In a gas mixture of deuterium with hydrogen, the measured values of the constants Λ and κ are[49]

$$\Lambda = 0.4 \pm 0.1, \quad \varkappa = 1.3 \pm 0.4, \tag{68}$$

and the transfer probability (64) for this mixture exhibits saturation with increasing C already at concentrations $C \sim 1$. In the saturation region, the transfer probability is $Q \approx 0.2$.

The value of Λ (68) in the $H_2 + D_2$ mixture is much smaller than in $H_2 + Z$ mixtures (66). This is due to the fact that in the $H_2 + D_2$ system, unlike the previously considered mixtures, the inverse transfer of π^- mesons from the deuterium to the hydrogen in possible, since the $d\pi^-$ mesic atom (like the $p\pi^-$ mesic atom) is electrically neutral. This process should lead to a decrease of the constant Λ compared with (66). The rates of nuclear capture of the π^- meson in oxygen and in deuterium are approximately equal[48] ($\tau_H = (2.3 \pm 0.6) \times 10^{-12}$ $\sec^{-1}$, $\tau_D = (2.1 \pm 0.5) \times 10^{-12}$ $\sec^{-1}$[48]), but owing to the isotopic difference between the levels of the $p\pi^-$ and $d\pi^-$ mesic atoms, the transfer of π^- mesons from protons to deuterons is more probable than the inverse process. The joint action of these mechanisms leads to the observed value of Λ.

As seen from (68), in the $H_2 + D_2$ mixture $\kappa \approx 1$ and $\kappa > \Lambda$. The former signifies that $a \approx b$ in (63), i.e., the probabilities of the nuclear capture of the π^- meson by the proton in collisions of $p\pi^-$ mesic atoms with hydrogen atoms and deuterium atoms are practically equal. This is natural, since the electromagnetic properties of the atoms H and D are quite close. From the relation $\kappa > \Lambda$ it follows that nuclear capture of the π^- meson by the proton is more probable in collisions between the $p\pi^-$ mesic atom and the deuterium atom than transfer to the deuteron.

VII. CONCLUSION

1. Recent experiments have shown that the processes of slowing down and nuclear absorption of negative mesons in substances depend strongly on the features of the chemical structure of the substance. The observed dependence is quite unexpected, since the dimensions of the outer shells of the atoms greatly exceed the effective radius of the nuclear forces and the dimensions of the mesic atoms. The reason for such a dependence lies in the fact that the landing of the mesons on the excited mesoatomic or mesomolecular levels frequently occurs with transfer of energy to the electrons of the valence shell, and therefore depends strongly on the structure of the latter.

2. A large group of phenomena of this type can be naturally explained within the framework of the model of large mesic molecules. Analysis of the process of the transition of the mesons from the continuous spectrum to the discrete spectrum shows that a fraction of them should be captured initially at high levels belonging not to the individual atoms but to the entire molecule as a whole, and are located in the region of its valence electrons. This gives rise to systems whose dimensions exceed by hundreds of times the characteristic mesic-atom distances. This is followed by the redistribution of these mesons among the mesoatomic levels of the different atoms in the molecule, owing to radiative transitions that depend strongly on the charges Z of the atomic nuclei. This process plays the role of a unique "descent mechanism" that determines the course of the more intense nuclear processes, and can be calculated sufficiently accurately within the framework of the model. The model of large mesic molecules explains quantitatively the regularities observed in the capture of π^- mesons in hydrogen-containing substances.

3. The proposed model explains the law governing the suppression of the probability W of the capture of π^- mesons by hydrogen in chemical compounds of the type Z_mH_n ($W \sim Z^{-3}$), the independence of W of the density and of the aggregate state of substances, and also of impurities of heavy elements.

4. In compounds of the type $R(Z'_kZ_mH_n)_q$ with q identical hydrogen-containing groups, an additional suppression of the probability W by a factor of q takes place as a result of the effects of quantum-mechanical resonance in symmetrical systems.

5. In experiments on the absorption of π^- mesons it has been established that the initial capture obeys the Z-law. An analogous conclusion follows from experiments on the absorption of μ mesons in mixtures of inert gases.

6. Within the framework of the developed concepts, it becomes possible to explain the singularities of the structure of the mesic-x-ray K series, which is produced when μ^- mesons are absorbed in chemical compounds, and particularly the increase of the contribution of the K_ν series to the total intensity of the K series J(Z) with increasing Z, and also the periodic dependence of J(Z) on the charges Z of the elements. It should be noted that the greater probability of landing on oxygen in oxides is not explained by the model of large mesic molecules, and is most probably due to the adiabatic capture mechanism.

7. A study of transfer of π^- mesons from hydrogen to nuclei Z in $H_2 + Z$ gas mixtures has shown that its probability is proportional to Z and depends only on the relative concentration of the Z nuclei. The latter proves that the de-excitation of the $p\pi^-$ mesic atoms and nuclear capture of π^- mesons by protons are determined mainly

by processes of collisions with other atoms (such as the Day-Snow-Sucher mechanism and the external Auger effect). The results of these investigations show that in mechanical mixtures there is a different and much less intense mechanism of the transfer of π^- mesons from $p\pi^-$ mesic atoms to Z nuclei than in chemical compounds, and additionally confirm the model of large mesic molecules.

8. The relation $W \sim a_L(n/m)Z^{-3}$, which follows from the model of large mesic molecules, has been reliably confirmed experimentally for elements of the period II of the periodic table. This makes it possible to predict even now, by using formulas (45), (46), and (52), the probability of absorption of π^- meson by the nucleus of a hydrogen atom bound with an element of period II in a chemical compound.

9. A study of the charge exchange of π^- mesons in hydrogen-containing substances (independently of the details of the theoretical interpretation) makes it possible to distinguish between chemically bound hydrogen and free hydrogen. This sensitive method may prove useful in the explanation of many problems of structural chemistry, kinetics and catalysis of chemical reactions, etc.

The extent to which this unique "meson chemistry" is fruitful will be demonstrated by the future. However, we can already formulate several problems that must be solved in this manner. First of all, it is necessary to investigate more thoroughly the stage of the initial capture of the mesons (experimentally and theoretically). It is quite desirable to verify the observed regularities for elements of other periods, and, in particular, to study the behavior of the coefficients a_L. It is particularly interesting in this respect to study the process of absorption of π^- mesons in isoelectronic molecules (CH_4, NH_3, H_2O, HF), in homological series (C_nH_{2n}-C_nH_{2n}, C_nH_{2n-2}), and in cyclic hydrocarbons.

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Translated by J. G. Adashko