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ACCELERATION OF CHARGED PARTICLES IN A PLASMA

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THE search for new physical principles for the acceleration of charged particles and the realization of these principles was one of the primary trends in the fruitful work carried out by Vladimir Iosifovich Veksler in the field of high-energy physics. He had the good fortune to develop one extremely important idea, the idea of phase focusing,* which lies at the basis of present day accelerator physics. This discovery represented a really new departure in the field of particle acceleration and made it possible for accelerators to become one of the fundamental tools in the development of the physics of elementary particles. The originator of this fundamental idea having done much for its realization, then carried on a continuous search for new methods of acceleration. It appears to the present author that this difficult field of work, which in many instances can lead to unfruitful results, was one of the basic sources of satisfaction to Veksler. In the opinion of the present author the remarkable idea of phase focusing does not represent Veksler's last word in the field of accelerator physics; this idea, with further development by researchers and subsequent investigators will lead to a still more radical change in this field of physics.

Keeping in touch continuously with the development of physics Veksler very quickly realized the possibilities that might be made available by the use of an electron-ion plasma for the acceleration of charged particles. In one of the new principles of acceleration that he proposed, the coherent method, attention was first directed to the possibility of using a plasma for the acceleration of charged particles.[1-4] The idea of coherent acceleration as proposed by Veksler is well known and there is no need to consider it in detail here except to point out one extremely important feature: in this method one makes use of precisely those features of a plasma that are of greatest value for accelerator physics. These features are the possibility of obtaining high electric fields, the possibility of accelerating quasi-neutral plasmoids and the possibility of acceleration of heavy particles indirectly by their entrainment by electrons which are accelerated by rf fields. Furthermore, in coherent methods which embody acceleration by forces that are quadratic in the field there is no need to maintain synchronism between the wave and the accelerated particles and no need for, localization of the acceleration fields in those regions in which the accelerated particles move. In addition, acceleration of longitudinal oscillators is possible. It then becomes clear that coherent methods provide a foundation which allows us to go far beyond the limits of the actual concrete method of acceleration. At the present time, from among the various coherent methods of acceleration, the greatest investigational effort has been given to radiation acceleration of a plasma. Investigations in this field are still in the exploratory stage. In work carried out under the direction of M. S. Rabinovich at the Physics Institute, Academy of Sciences (FIAN) (experimental and theoretical) and by M. L. Levin at the Research Institute, Academy of Sciences (RIAN) (theoretical) the general theory of the radiation method has received further development. The work has been centered on the question of producing a stable shape for the plasmoid and providing the focusing required for extended acceleration; a detailed investigation has also been made of the accelerating forces that act on the plasmoid. Great interest attaches to work carried out by A. V. Gurevich and V. P. Silin[9] who, in addition to investigating the displacement of the accelerated bunch as a whole, have also investigated the internal motion of the charges and have shown that acceleration of a plasmoid affects the particles in such a way that those particles are accelerated which lie within a skin depth of the plasma. Under these conditions there occurs a "mixing" of particles since the forward and trailing edges of the plasmoid change places periodically. The first experimental results on radiation acceleration of a plasma in a traveling-wave field have already been obtained in the Soviet Union (FIAN, M. S. Rabinovich and his colleagues) and in Japan; results with inhomogeneous RF fields have been reported in the USA and in the USSR, and work with combined inhomogeneous rf fields and static fields has been reported in France. In particular, during a recent visit of Professor Consoli and his colleagues we have learned of successful theoretical and experimental progress in this work.

Experiments carried out in these first investigations, performed with low-power rf generators, has shown that ion energies of the order of tens of kilo-electron volts can be achieved. The basic problem now appears to be the use of high rf powers for radiation acceleration and the production of rf and static fields which will provide the required focusing and the stability for the plasmoids.

The present trend toward plasma techniques in the new methods of acceleration of charged particles now shares a place with methods based on the use of self-stabilizing electron-ion beams as proposed by G. I. Budker,[5] and the use of longitudinal plasma waves excited by electron beams and waves in a plasma waveguide (FTI AN UkrSSR)-Physico-technical Inst. Ukr. Acad. Sci.). We shall not dwell here on the well-known and extremely interesting method of self-stabilizing beams, but shall present the basic results of investigations on plasma acceleration carried out in the FTI AN UkrSSR. The idea behind these investigations derives from the work of Veksler and the theoretical and experimental research was discussed many times with

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*The principle of phase focusing was discovered by V. I. Veksler in 1944; it was discovered independently by McMillan in 1945.

him. These extremely valuable discussions have, to a large degree, stimulated the development of our own work.

Turning to the method of acceleration of charged particles in a plasma by means of longitudinal waves, excited by electron beams, and waves in a plasma waveguide, excited by external high-frequency sources as proposed by the author[8] and developed by him and his colleagues* in the FTI AN UkrSSR, we first wish to make some general remarks concerning linear accelerators.

As is well-known, conventional linear accelerators are capable of producing accelerated particles of high energy. This is particularly true of linear accelerators in which electrons with energies of the order of 20 GeV can be obtained with pulsed currents of the order of 25 mA. It appears that accelerators with energies up to 40 GeV will be built shortly.† Smaller steps have been taken in the development of linear proton accelerators; however, even at the present time there are proton accelerators-injectors- which are very useful devices. In this connection, further development of linear accelerators and the exploitation of all their potentialities, in particular the production of high-current accelerators of heavy particles and electrons, will require new methods of linear acceleration. In a linear accelerator, in which the charged particle traverses the acceleration system only once, further increases in energy can be achieved in only two ways: one, by increasing the strength of the accelerating field, which presently is of the order of 100–150 kV/cm, and two, by increasing the length of the accelerator which is already of the order of 3 km for the Stanford accelerator. But if the field strength is increased by an order of magnitude (with a corresponding increase in energy), in existing accelerating systems the loss of RF power would be of the order of 200–300 MW/m. The flux of rf energy under these conditions would reach values of the order of 300 MW. Even if we neglect possible breakdown at these field strengths at the surface of metal waveguides ($E \sim 10^6$ V/cm) it becomes clear that increasing the energy by increasing the strength of the accelerating field in the existing linear accelerators is not a feasible approach. On the other hand, increasing the length of an accelerator to tens of kilometers is an equally unattractive approach. The physical reason behind these difficulties lies in the fact that metal waveguides are used in the accelerating system. The absorption of electromagnetic energy in the walls of the waveguides is proportional to $H_\varphi^2 \sigma^{-1/2}$ (H_φ is the magnetic field in the wave and σ is the conductivity). In the microwave region for metals we find that $\sigma \sim 1/\nu$; the collision frequency ν is large so that σ cannot be made large. A significant reduction in the loss of rf energy can be realized by other methods. The most obvious is the use of superconducting waveguides, for which rf energy losses can be reduced by four or five orders of magnitude. This result follows from theory and from numerous experiments. For linear accelerators with field strengths typical of those presently being used, superconducting acceleration systems would be extremely effective. Furthermore, they would make it possible to operate the accelerator as a cw device. However, a serious obstacle to the use of superconducting waveguides and resonators in accelerating systems with high accelerating electric fields is the possibility that the superconductivity can be destroyed by strong rf magnetic fields associated with the waves that accelerate the particles. As is well-known, the use of existing superconducting alloys with high values for the critical static magnetic fields is not yet possible in the rf region. Hence, in accelerators with strong fields the use of superconductors will become feasible only if it is possible to produce superconductors with high critical magnetic fields in the microwave region. It is also possible to set up accelerating fields with configurations such that the intensity of the magnetic field $\tilde{H}$ associated with the mode is small at the surface of the superconductor for a specified value of the acceleration field $\tilde{E}$. There exists the possibility of using metal waveguides in the ordinary and superconducting states by means of focused electromagnetic energy. One version of this solution of the problem is the use of converging cylindrical waves (polarized in such a way as to have a high electric field component in the direction of particle motion) in which the field strength at the axis of the system is much larger than at the surface of the waveguide. The phase velocity of a mode of this kind must obey the condition $v_\varphi \sim c$. Calculations indicate that a mode of this kind can be produced by means of a waveguide filled with gas like a cylindrical diffraction grating, which is excited externally. Another, more radical solution to the problem, is to give up the use of metal waveguides altogether and achieve acceleration by means of electromagnetic fields propagating in media in which it is possible to excite high electric fields with small loss of rf energy and minimum particle energy loss. In the case of acceleration by forces that are linear in the electric field, synchronization requires that the phase velocity of the wave satisfy the relation $V_\varphi \lesssim c$; also, the wave must have an electric field component in the direction of motion of the particle. In the case of acceleration by forces that are quadratic in the field these two requirements no longer appear.

In existing linear accelerators it is not possible to achieve longitudinal and radial stability simultaneously by means of the accelerating fields alone. This feature is especially important for heavy-particle accelerators and high-current electron accelerators. For this reason, great interest attaches to those media in which it is possible to achieve radial and longitudinal stability simultaneously. This requirement is necessary not only in order to achieve high currents but also to concentrate the electromagnetic energy in the radial and longitudinal directions. The achievement of high field strengths for reasonable values of the electromagnetic energy flux and loss is possible only when the energy can be concentrated in the limited regions in

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*The theoretical work has been carried out by V. D. Shapiro, V. I. Kurilko, V. I. Shevchenko, V. B. Krasovitskiĭ, M. F. Gorbatenko, and in the initial stages N. A. Khizhnyakov; the experimental work has been carried out by A. K. Berezin, L. I. Bolotin, E. A. Kornilov and their colleagues and up to 1965 by I. F. Kharchenko.

†In cyclic accelerators one cannot avoid the limitation due to electron radiation (this is a fundamental theoretical limitation) so that linear accelerators become the principal method of acceleration for electrons.

which the accelerated particles actually move.

Media that satisfy these conditions to a significant degree are the electron-ion plasma, and unneutralized electron and ion beams. At the present time great interest also attaches to the use, for acceleration purposes, of nonlinear active media[13] in which the energies stored in the atoms of the medium is very large and can be transformed into a form suitable for acceleration by waves (for example, longitudinal waves). The motion of particles in such media, or in channels in such media, might provide effective particle acceleration.

We shall now consider in detail the use of a plasma for particle acceleration. In contrast with metal waveguides, in which the waves propagate in vacuum and the wave properties are determined by reflections from metal surfaces, in most plasma waveguides the waves propagate in the plasma itself (there are also waveguides that formed from channels in a plasma). The possibility of using such waveguides for particle acceleration is based on the fact that the waveguide properties (for the microwave region) appear at relative low plasma densities. We note that the density of electrons in a metal is many orders of magnitude greater than that required to provide wave properties in the microwave region. As is well known, in order for a medium to have a noticeable effect on wave propagation it is necessary that its characteristic frequency be comparable with the wave frequency. In the present case this means that the Langmuir (or Larmor) frequency of the plasma must be of the order of the frequency of the propagating wave $\omega_p \sim \omega$. For the region in question, this condition can be satisfied at densities of the order of 10^9-10^{13} cm^{-3}. Under these conditions the loss of rf energy due to binary collisions of plasma particles is small. At low collision frequencies $\nu \ll \omega$ the loss of rf energy is proportional to $\sigma \sim (\omega_p^2/\omega^2)\,\nu_{\mathrm{Coul}}$ so that a further reduction can be achieved by heating the plasma to temperatures of the order of 10–100 eV, which leads to an appreciable reduction in the frequency of Coulomb collisions $\nu_{\mathrm{Coul}} \sim n_p/T.^{3/2}$ For densities of 10^9-10^{13} cm^{-3} the Bohr loss due to the passage of the particles through the plasma is proportional to the plasma density and is insignificant, $dE/dx \sim 10^{-5}-10^{-6}$ eV/cm. This means that the plasma is transparent to the accelerated particles. Depending on the acceleration conditions, the particles can divide up into individual bunches. In this case, because of the inversion of the coherent acceleration effect described by V. I. Veksler, the rf losses can increase greatly. However, even for currents of the order of tens of amperes these losses are still insignificant. In order to build high-current electron and proton accelerators, which will be the real problem as far as industrial usage is concerned, it will be necessary to provide strong radial and phase stability simultaneously. We note that this problem does not exist for usual electron accelerators, which are low-current devices. An interesting feature of plasma acceleration is the possibility of simultaneously achieving radial and longitudinal stability. This statement holds for anisotropic gyrotropic media in which $\epsilon_z/\epsilon_r < 0$. This condition follows from the condition $\nabla\cdot D = 0$ in which case the radial component

$$E_r \approx -r\frac{\varepsilon_z}{\varepsilon_r}\frac{\partial E_z}{\partial z};$$

whence the frequency of the radial oscillations

$$\Omega_r^2 \approx \frac{\varepsilon_z}{\varepsilon_r}\frac{\partial E_z}{\partial z}.$$

Since the longitudinal stability is proportional to the electric field gradient, the frequency of the longitudinal oscillations $\Omega_z^2 \sim -\partial E_z/\partial z$ (the minus sign arises because the particle trapped by the wave must "fall" into a region of lower field). Hence, when $\epsilon_z/\epsilon_r < 0$ we find $\Omega_2^2 > 0$ and $\Omega_z^2 > 0$ and, in contrast with the vacuum case, it is possible to have radial and phase stability simultaneously. In this case, as the calculations indicate, the radial stability is much stronger than in vacuum:

$$\Omega_r^2 = -\frac{\varepsilon_z}{\varepsilon_r}(1-\varepsilon_r\beta^2)(1-\beta^2)^{1/2}\frac{\pi e E_0}{m\beta\lambda}.$$

If the wave is sinusoidal the longitudinal stability given by the field E_z is the same as in the vacuum case. However, in the plasma it is very possible to excite nonlinear waves with sharp leading edges; in this case the longitudinal stability increases greatly. For longitudinal focusing by nonlinear waves in a plasma it is possible to achieve higher densities and currents of accelerated particles since the frequency of phase oscillations can be very large:

$$\Omega_{\mathrm{ph}}^2 \sim \omega_p^2\frac{m}{M}\left(\frac{v_{\mathrm{ph}}}{v_T}\right)^{1/2},$$

while the limiting density of accelerated particles is determined by the condition $\omega_\lambda^2 = \Omega_{\mathrm{ph}}^2$ where ω_λ is the plasma frequency of the plasmoids.

In considering acceleration of particles in a plasma it is important to note the following: it is possible to excite waves with strong electric fields low densities 10^9-10^{13} cm^{-3}. We find $E_{\max} \sim \sqrt{4\pi n m v_{\mathrm{ph}}^2}$ and for reasonable densities one can achieve millions of volts per centimeter. An order-of-magnitude estimate can be made very simply on the basis of the fact that for electrostatic fields $E^2/8\pi \sim nmv^2/2$; the maximum velocity is given by $v \approx v_{\mathrm{ph}}$ because at high velocities one encounters opposing beams of electrons in the plasma. This same relation can be obtained by elementary methods if we note that in the one-dimensional case

$$\frac{\partial E}{\partial x} \sim \frac{E}{\beta_{\mathrm{ph}}\lambda} \approx 4\pi e n;$$

since $n_{\max} \approx n_0$ and the frequency of excited longitudinal waves $\sim \omega_0$, from which the relation given above follows immediately. It should be noted that as the density increases the associated wave length is reduced and $E_{\max}$ increases.

We have indicated a number of advantages of acceleration in a plasma. However, there are still a number of serious obstacles in the way of realizing plasma acceleration. First of all, it is necessary to produce a stable, highly ionized plasma. In contrast with metals and dielectrics, the properties of a plasma can be changed by the waves that propagate in it; hence it is necessary to keep the possibility of nonlinear effects in mind. For these reasons in some cases it is necessary to choose the plasma parameters and the wavelengths in such a way as to avoid the appearance of nonlinear effects. This requirement can be stated in the form

$$\frac{eE\lambda}{2\pi mc^2\beta_{\text{ph}}} \ll 1$$

($eE\lambda$ is the energy acquired by a particle in a wavelength).

On the other hand, since we have seen that nonlinear effects can play a helpful role in increasing acceleration efficiency, it would be desirable to learn to control them. Comparing the requirements on the parameters and properties of a plasma used in accelerators and the properties of plasmas used in controlled thermonuclear fusion research one sees that the requirements are much less stringent in the former case. When a plasma is used for acceleration purposes its waveguide properties are determined primarily by the electrons in the plasma. Hence the ion component can, to a considerable degree, be arbitrary; the problem of ion heating, which is so important for the controlled thermonuclear reaction, is not important here. In working with accelerators in the pulsed mode the problem of stability is relieved considerably since a number of stabilities simply do not have time to develop. Typical plasma densities and currents for the aceleration case are relatively small so that the growth rates for instabilities and the diffusion rates are also reduced considerably.

In acceleration systems that use waveguides formed by unneutralized electron and ion beams one generally avoids a large number of instabilities and the problem of maintaining the necessary plasma parameters is alleviated.* Work on controlled thermonuclear fusion, has already resulted in plasmas with parameters that approach those required for acceleration purposes.

An important question which must be answered both theoretically and experimentally is that of determining whether it is possible to excite the waves required for plasma acceleration. Second, and no less important, is the problem of producing a stable plasma. Finally, it is necessary to determine whether high-intensity waves can be excited in a plasma: the ordered motion of the plasma electrons in the fields of these waves can lead to additional instabilities, primarily the two-stream instabilities that have been predicted theoretically by A. I. Akhiezer, Ya. B. Faĭnberg, D. Bohm and E. Gross.[10]

The most effective method of exciting longitudinal waves in a plasma at the present time is that proposed and developed at the FTI AN UkrSSR, in which excitation is achieved by electron beams that are injected into the plasma.[8,15] It has been shown both theoretically[11] and experimentally[14] (by investigations of the nonlinear stage of the beam-plasma interaction) that 30% of the beam energy goes into the excitation of oscillations, 30% into heating of the plasma, and 15% into heating of the beam.† It is interesting to note that in the transition from beams with powers of tens of kilowatts to a beam with the power of 600 kW which has been carried out recently, there has been no change in the relative fraction of the energy going into wave excitation.

The intensity of the oscillations excited in the plasma by means of beams is already approximately 200 kW. At the present time experiments are being initiated with beam powers of the order of 10 MW and preparations are being made for beams with a power of 100 MW (pulse length $2-5 \times 10^{-7}$ sec). If the relative fraction of the energy going into wave excitation remains the same the problem of wave excitation in a plasma can be regarded as solved.

If the waves excited by electron beams in a plasma are to be used efficiently for particle acceleration, it is necessary that these waves be regular, that is to say, they must have fixed frequency and phase velocity. Among other things, as we have indicated above, in the interaction of an initially unmodulated beam with a plasma an appreciable part of the energy can go into heating of the plasma and the beam. The resultant smearing of the electron distribution function leads to a strong broadening of the spectrum of excited waves, both in frequency and phase velocity, and the oscillations become randomized. For this reason it is of importance to find a method of narrowing the spectrum and compressing it in frequency and phase velocity (wave number). This problem has been solved both theoretically and experimentally by preliminary modulation of the beam, that is to say, by requiring that the initial perturbation at the input of the system be of regular nature. Preliminary modulation inhibits the development of two-stream instabilities with frequency and wavelength different from that of the modulation; hence this method damps instabilities over a wide range of frequency and wave number. At the same time, it guarantees the development of instabilities at frequencies and wavelengths specified by the modulation. Moreover, in this method a part of the initial amplitude spectrum is at a level appreciably greater than the fluctuation spectrum.[15]

The theory indicates[16] that in contrast with the interaction of an unmodulated beam with a plasma, in which case the wave numbers of the excited waves (and more importantly, their phase velocities) can assume any values $k \leq \omega_p/v_0$, in a modulated beam the spectrum of wave numbers is highly compressed:

$$\omega_p \left(1 - \frac{a}{l}\right)^{1/2} \leqslant kv_0 \leqslant \omega_p$$

(a is the dimension of the bunch produced in the beam by modulation and l is the distance between bunches). This result holds for the case

$$\mu = \frac{\omega_p a}{\omega_1 l} \gg 1$$

(ω_1 is the plasma frequency of the particles in the bunch and $\omega_1 \ll \omega_p$). When $\mu \ll 1$, corresponding to a highly inhomogeneous distribution of density in the modulated beam (narrow bunches), there is a further compression of the spectrum of unstable waves and the instability appears only within the narrow range

$$\omega_p + \omega_1 (1 - \mu^{1/2}) < kv_0 < \omega_p + \omega_1 (1 + \mu^{1/2}).$$

In this case the maximum growth rate

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*For large currents of accelerated particles the waveguide properties required for acceleration can be produced by untrapped particles through which the trapped particles move, that is to say, the waveguide system itself can be a beam injected into the accelerator.

†The investigation of the nonlinear stage of the interaction of smeared-out beams with a plasma can be carried out by means of quasilinear theory.[12]. The case of monoenergetic beams is considered in [11].

$$\gamma_{\max} \sim \left(\frac{a}{l}\right)^{1/2} \omega_1^{1/2} \omega_p^{1/2}$$

is reduced considerably as compared with the interaction of a continuous beam with a plasma for a beam density equal to the mean value of the density in the modulated beam $\gamma_{\max} \sim (a/l)^{1/3} \omega_1^{2/3} \omega_p^{1/3}$.

In Figs. 1 and 2 we show the frequency spectra for waves excited by the interaction of a nonmodulated beam (Fig. 1) and a modulated beam (Fig. 2), interacting with a plasma. It is evident from these curves that premodulation of the electron beam leads to compression of the frequency spectrum (half-width approximately 3 MHz instead of 70 MHz in the absence of modulation).[13] Since the resolution of the spectrum in wave number (phase velocity) is achieved by the fact that the modulated signal of coherent nature produces the most favorable conditions for excitation of waves with k corresponding to the modulation, it is reasonable that this can be achieved for interaction of an initially unmodulated beam with a layered or spatially periodic plasma.

As we have indicated above, for acceleration purposes the nature of the excited waves is of great importance (coherent or random). The most effective method of experimentally investigating these questions is through the measurement of space and time correlation functions or direct observation of the shape of the excited fields and by Fourier analysis (in this case the phase relations are retained). Knowing the correlation function, by means of the Wiener-Khinchin relation we can determine the spectral densities of the radiation in k and ω, which then appear as the basic characteristics of the turbulent plasma. The results of experimental investigations for the case of both low power and intense electron beams are shown in Figs. 3–6. In Figs. 5 and 6 we show the autocorrelation functions* for an initially unmodulated and a modulated electron beam (25 A, 25 kV, current pulse length 4.5 μsec, plasma density (6–8) × 10^{11} cm^{-3}, longitudinal magnetic field 1000–2000 Oe).[17]

We note that the monotonic decay of the correlation

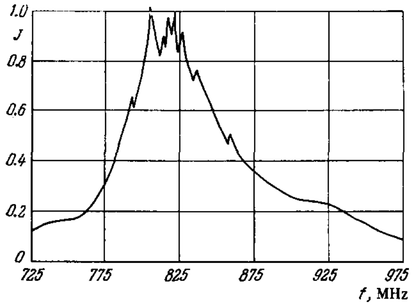

FIG. 1. Frequency spectrum for an unmodulated beam. The beam energy is 25 keV, the beam current is 25 A, the pulse length is 4.5 μsec, the plasma density (6 – 8) × 10^{11} cm^{-3}, the longitudinal magnetic field is 2000 Oe. The beam power is 625 kW.

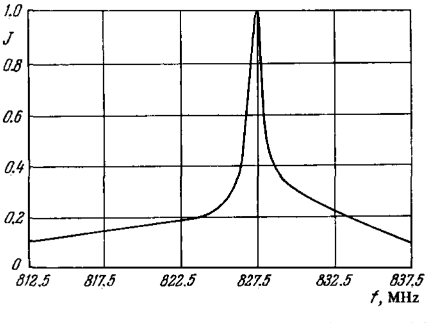

FIG. 2. Frequency spectrum for a modulated beam with the same parameters for the beam and plasma. The power of the initial modulation is 10 kW. The total rf power is not reduced.

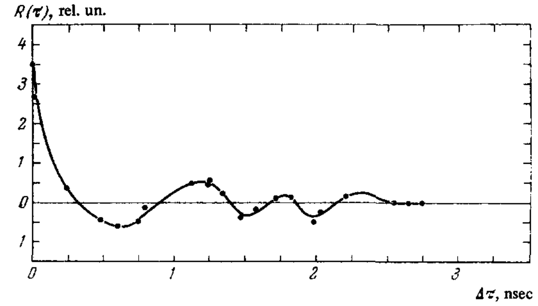

FIG. 3. Time auto-correlation function for oscillations excited by an electron beam in a plasma. The beam energy is 5 keV, the beam current is ~ 100 mA, the magnetic field ~ 1000 Oe, the plasma density ~ 6 × 10^{11} cm^{-3}[19a].

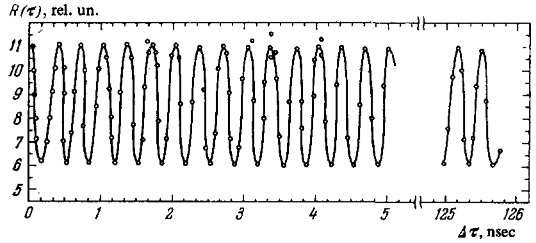

FIG. 4. The time auto-correlation function for oscillations excited by a beam modulated at a frequency of 3000 MHz.[19a]

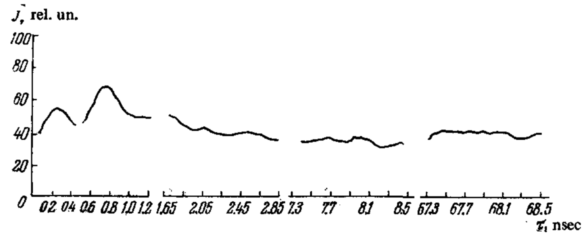

FIG. 5. Auto-correlation functions for unmodulated electron beam.

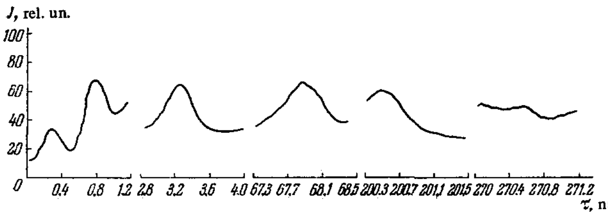

FIG. 6. Auto-correlation functions for modulated electron beam.

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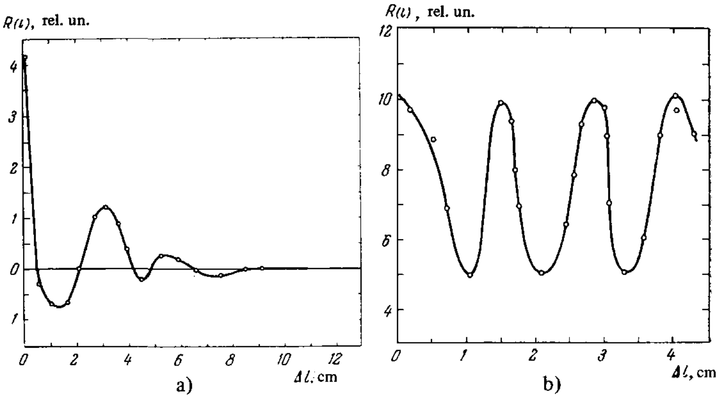
*More exact measurements carried out in the range $\tau = 0 - 0.8$ nsec show that these functions, as expected, reach a peak when $\tau = 0$.



a) b)

FIG. 7. Space auto-correlation functions for oscillations excited by a beam: a) no external modulation of the beam, b) beam modulated at a frequency 3000 MHz ($E_e \sim 5$ kV, $I_e \sim 100$ mA, $H \approx 1000$ Oe, $n_p = 6 \times 10^{11}$ cm^{-3}.

function corresponds to the excitation of irregular random oscillations while the oscillating correlation function corresponds to regular oscillations. The presence of even weak modulation leads to a strong compression of the frequency spectrum and regularization of the oscillations. For acceleration purposes it is very important to have a compressed spectrum of phase velocities for the excited waves (Figs. 7 and 8; these measurements are carried out as yet only for low power levels).

The results of the experimental investigations indicate that premodulation of beams is an effective method of compressing the spectrum of excited oscillations in both ω and k and is a method for conversion of random oscillations of a plasma into regular oscillations, that is, a method suitable for exciting waves needed for acceleration by regular fields. It should be noted that the investigation of collective plasma interactions carried out for the purposes of developing new methods of plasma acceleration has yielded a very effective method for heating electrons and ions in devices used for controlled thermonuclear fusion research and has also lead to a new possibility for the generation and amplification of strong regular and random oscillations.* It has been shown at the FTI AN UKrSSR that the interaction of the beams with a plasma leads to very intense plasma electron bunches whose energy is appreciably greater than that of the initial beam. Further development of this research both here and abroad indicates that by means of beam-plasma interactions in magnetic traps it is possible to heat plasma electrons to temperatures greater than 100 keV. Interesting results in this direction have been achieved in the turbulent heating method proposed by E. K. Zavoĭskiĭ in which an important role is played by the collective interactions of the beam-plasma system. The actual mechanism by which electrons are heated and accelerated in the fields that arise in the collective beam plasma interaction in magnetic traps appears to be a mechanism very much like the stochastic-acceleration mechanism proposed by V. I. Veksler together with Burshtein and Kolomenskiĭ[6] (further development of the idea of stochastic acceleration was carried out in interesting papers by Kolomenskiĭ and Lebedev[7]). At the present time an analysis is being made of the quasi-linear theory of acceleration of particles in a plasma by stochastic longitudinal and electromagnetic waves and the damping of stochastic fields in a plasma associated with the transfer of energy to the plasma field.[20] In acceleration by an electromagnetic field there is an invariant

$$w = v_\perp^2 + v_z^2 - 2v_z\frac{\omega}{k_z} = \text{const}$$

(this relation expresses the conservation of particle energy in the reference system fixed in the wave). Hence, particles which at $t = 0$ have the thermal velocity $v \lesssim \sqrt{T/m}$ uniformly fill (in the course of time) a rather narrow region in velocity space, a spherical layer of radius $\omega/k_z \gg \sqrt{T/m}$ and thickness $\sim\sqrt{T/m}$. As a result of this process, in a time $t \sim \tau W/n_0 m(\omega^2/k^2)$ (τ is the correlation time, and W is the energy of the electromagnetic field, n_0 is the plasma density) a significant fraction of the plasma particles (as compared with n_0) acquire an energy $\gtrsim m\omega^2/k^2$ which significantly exceeds the thermal

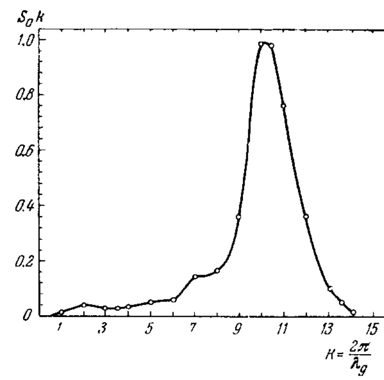


FIG. 8. Spectral density of the energy in the electric field (the curve is obtained for the conditions in Fig. 7a).[17,18]

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*There is a large number of experimental investigations of collective interactions of charged particle beams with a plasma. In addition to those mentioned directly, the reader is directed to those listed in [30]. We will not have the opportunity here to discuss all the interesting results in the references.

energy. We may note that important investigations of the process of stochastic acceleration have also been carried out by V. N. Tsytovich.

Another important problem, whose solution will be needed for effective acceleration of particles in a plasma, is that of providing stability. The importance of this problem for plasma accelerators can be seen from the fact that anomalous diffusion of a plasma, which can arise in the development of instabilities, leads to an increase in cross section. In this case, in order to achieve high field strengths it then becomes necessary to provide a much higher flux of rf power. It should be remembered that the excitation of rf waves required for acceleration does not lead to anomalous diffusion since these waves exhibit only a weak interaction with the ions. Hence, we are considering primarily the question of low-frequency instabilities. In theoretical and experimental work carried out at the FTI AN UkrSSR it is shown that there are methods of suppressing instabilities. Without considering the large number of instabilities and their superficial differences, we can say that they are due to three or four basic mechanisms. Hence it might be hoped that the process of removing them might be universal to some degree. As we have indicated above, modulation of the beam leads to the suppression of high-frequency instabilities. Further experimental investigations show that this process also leads to a suppression of a wide spectrum of low-frequency instabilities, thus reducing appreciably the anomalous diffusion and the area of the cross section of the plasma (Figs. 9 and 10). At the moment we still do have a definitive explanation of these results, but they are evidently associated with the fact that the low-frequency instabilities arise as a result of nonlinear interactions of the high-frequency oscillations; hence, suppression of the high-frequency oscillations causes suppression of the low-frequency oscillations. This result is supported by the observed experimental relation

$$\Delta\omega_{\mathrm{rf}} = \omega_{\max\ \mathrm{lf}}.$$

Microinstabilities due to nonuniformity of the plasma lead to anomalous diffusion.[21-25] It has been shown both theoretically and by preliminary experimental work there is also a possibility of suppressing these by means of high-frequency fields. (We note that the possibility of stabilization of hydrodynamic instabilities by rf fields has been studied by Osovets.) It is important to point out that here we are not considering the removal of effects associated with the development of instabilities, but the suppression of the instabilities themselves. The possibility of suppression of microinstabilities has much in common with the method of suppression of instabilities in a plasma based on the use of self-phasing.[26] In the presence of self-phasing the interaction of particles of the plasma with electromagnetic fields which grow by virtue of the instability leads to the interaction of these fields with a longitudinal oscillator rather than with the free particles, as in the absence of self-phasing. The intensity of the oscillations that are excited in the development of an instability is determined by the energy transferred by the particles to the growing wave; the energy transferred by the moving oscillator, with the exception of resonance cases, will be appreciably smaller than for free particles so that the growth rate is reduced appreciably. Calculations show that this reduction is $\sim(\gamma/\Omega_{\mathrm{ph}})^2$ where Ω_{ph} is the frequency of the phase oscillations. In stabilization of microinstabilities in an inhomogeneous plasma by an external rf field* the role of the frequency of phase oscillations is played by the frequency of the stabilizing rf field. Calculations show[28] that the application of the high-frequency field and the related pressure

$$-\frac{e^2}{2m\Omega^2}\nabla\langle \mathbf{E}_0\mathbf{E}_{1z}\rangle$$

(the angle brackets denote an average over the rf period while E_{1z} is the rf part of the field; because of the inhomogeneity of the external field E_0 there is no higher term $\sim E_0^2$ under the average sign) leads to an

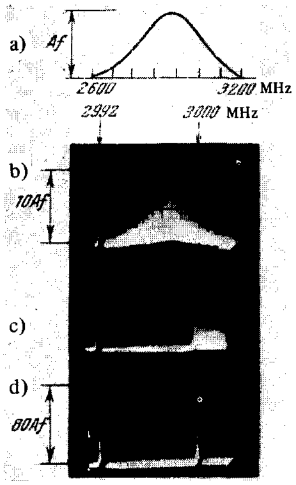

FIG. 9. Spectrum of high-frequency oscillations as a function of the depth of modulation of the electron beam (modulation frequency 3000 MHz). a) $\alpha = 0$; b) $\alpha \approx 0.06 + 0.09$; c) $\alpha = 0.11$; d) $\alpha = 0.15$, f = 2992 is the frequency marker at beginning of sweep of the IV-46 analyzer. Sweep width is 10 MHz.[19a]

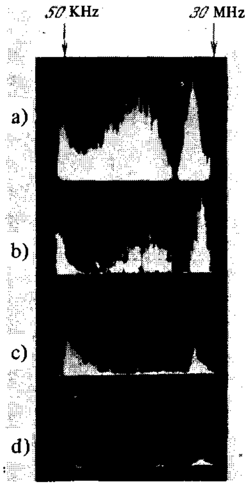

FIG. 10. Spectrum of low-frequency oscillations as a function of depth of modulation of the electron beam. a) $\alpha = 0$; b) $\alpha \approx 0.06 + 0.09$; c) $\alpha \approx 0.11$; d) $\alpha = 0.15$.[19a]

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*The suppression of the two-stream instability by means of uniform RF fields has been considered by Aliev and Silin.[27].

increase in the frequency of oscillation at which the instability develops. As the frequency increases the value of the stabilizing term increases in the expression for the growth rate

$$\gamma \sim \alpha \frac{\partial f_0}{\partial x} + \beta \frac{\partial f_0}{\partial W};$$

the stabilizing term

$$\frac{\partial f_0}{\partial W}\left(\frac{\omega}{k_z}\right) \approx -\frac{m_e \omega}{T k_z} f_0(0).$$

It follows that to suppress drift instabilities it is adequate to use relatively small electric fields for which the parameter $a_e = eE_0/m_e\Omega^2$, which determines the displacement of the electron in the rf field, is of order $(k/k_z)\lambda_{De}$ (λ_{De} is the electron Debye radius, k and k_z are the magnitudes of the wave vector for the drift oscillations and its component in the direction of the rf field).

Future detailed experimental verification must give the answer as to how effective this method of stabilization will be. Going beyond the importance of the question of providing stability, the basic problem in plasma methods of acceleration is the solution of the problem of exciting strong waves in the plasma. We have already seen no relative reduction in intensity of excitation in going from beams with power of tens of kilowatts to beams of the order of 600 kW. The next stage in experiments that are already under way will be the excitation required for acceleration by waves with electron beams of powers of 10 and 100 MW. Simultaneously, experimental work is being carried out to study the possibility of increasing the strength of the excited waves $E^2/8\pi$ to values larger than the energy density in the beam $nmv^2/2$. This possibility can be realized by continuous injection of particles into a plasma.[29] In this kind of injection the excitation of oscillations is maintained by continuous injection of new groups of fast electrons into the plasma. If the group velocity of the waves v_{gr} is somewhat smaller than the beam velocity v_0 the energy lost by the beam to the excitation of waves accumulates in a transition layer at the boundary of the plasma. In this case the energy density of the field becomes very large and can exceed the energy density in the beam:

$$\frac{\sum_k |E_k|^2}{4\pi} \approx \frac{2}{15} \frac{n_0 m v_0^4}{v_T^2},$$

that is to say, v_0^2/v_{gr}^2 times greater than the energy density in the beam.

It is interesting to note that the formation of this layer leads to the production of a shock wave.

The results given here for theoretical and experimental investigations of longitudinal and electromagnetic waves excited by means of electron beams moving in a plasma have indicated the efficiency of this method of excitation. Going from beams with powers of tens of kilowatts to a power of 600 kW we have seen a proportional increase in the power of the excited waves. If the new experimental investigations on beams with powers of 10 MW and future experiments with beams of a 100 MW do not show any new effects the problem of excitation of waves required for acceleration can be regarded as solved. As far as the general state of plasma acceleration is concerned, it appears to us that these methods are developing more slowly than would be desirable. However, the work carried out up to this time has verified the validity of the basic ideas and the difficulty in realizing them is, to a considerable extent, technological and should be overcome in the course of time. The time for achieving these and other new methods of acceleration would be much shorter if we were fortunate enough to have Vladimir Iosifovich Veksler with us.

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